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The Semiclassical Limit of the Gailitis Formula Applied to Electron Impact Broadening of Spectral Lines of Ionized Atoms

Sylvie Sahal-Bréchot

Observatoire de Paris, PSL Research University, Sorbonne Université, Observatoire de Paris-PSL, LERMA, 92190 Meudon, France; sylvie.sahal-brechot@obspm.fr; Tel.: +33-145077442; Fax: +33-145077100

Abstract: The present paper revisits the determination of the semi-classical limit of the Feshbach resonances which play a role in electron impact broadening (the so-called "Stark" broadening) of isolated spectral lines of ionized atoms. The Gailitis approximation will be used. A few examples of results will be provided, showing the importance of the role of the Feshbach resonances.

Keywords: feshbach resonances; electron collisions with ionized atoms; stark broadening; impact approximation; isolated lines; semiclassical-perturbation picture

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1. Introduction

Following the developments of the accuracy of observations in astrophysics and laboratory physics, many needs for atomic data have appeared since the second half of the 20th century, and the needs are always growing. Furthermore, the constant development of powerful computers also stimulates the calculations of atomic data on a large scale. In addition, the access to these atomic data via online databases has become essential since the beginning of the 21st century. In parallel, just after the publication of the fundamental impact broadening theory [1] , the theory and calculations of collisional line broadening showed a great expansion in the 1960s and 1970s, and many developments continued in the decades that followed. In particular, the method developed by S. Sahal-Bréchot [2–6] was inspired by the developments of the theory of electron-atom and electron-ion collisions which has also rapidly progressed since the 1960s. However, compared to theoretical results, experimental results remained few. Therefore, it remained difficult to test the validity of the method and these new calculations by comparison with experimental results.

This method was designed to meet the needs of users. Hence, an approximate method was developed for obtaining numerous data, but accurate enough for most of the needs. Thus, the impact semi-classical perturbation theory for electrons and ions colliding with neutral and ionized atoms was created and developed for isolated lines (neighboring levels do not overlap) and began to be exploited in the 1970s [7], and then exploited on a large scale in the decades that followed. The Stark-B database [8] is intended to implement these published results, and the references to the corresponding publications as well. This work is still continuing today [9].

This approximate method denoted by SCP was shown to be enough precise for the needs: 20 to 30% for the widths, sometimes worse for the shifts: the SCP results agree with the other theoretical methods [10–12], especially most of the quantum close-coupling ones, and also with experimental results, [12] for instance. The uncertainty of the semi-classical-perturbation results is due to the use of the perturbation theory [2,3,6]. However, the use of a symmetrization procedure of the transition probabilities and inelastic cross-sections improve the results [2,3,6]. For the widths, the accuracy is about 20 to 35%, sometimes less at low temperatures, because elastic collisions and, therefore, close collisions are more important, and the perturbation theory becomes not valid any longer. For the shifts, the results can be less accurate than for the widths when they are small, due to negative

interference effects between the upper and lower levels of the studied line. More details can be found in [6] and are not repeated there.

In parallel, accurate quantum close coupling determinations and calculations of inelastic electron–atom and electron–ion cross-sections began to be created and developed in the 1960s and in the decades that followed. Concerning electron–ion collisions, it was established in the 1960s that the inelastic cross-sections are not zero at threshold. In addition, below the threshold of an excited level of an ion of charge Z, the elastic cross-section of the lower level has an infinite number of resonances. These resonances are due to the fact that the colliding electron can be trapped on an excited state of the ion of charge $Z-1$. This gives a a temporary ion of charge $Z-1$, and the trapped electron can be ejected by autoionization. These states form Rydberg series which converge towards the excited state of the ion of charge Z. In order to obtain physical results for the elastic cross-section below this threshold, an average over the resonances must be performed. The discontinuities of the inelastic cross-sections are compensated by the discontinuities of the elastic cross-section, and the final result is continuous. This average leads to an increase of the elastic cross-section under the energy of the threshold of the excited level. The determination of this increase was the basic result of Gailitis, and the so-called Gailitis formula followed [13]. This also concerns the inelastic fine structure cross-sections of a given term. The same result was obtained by means of the quantum defect theory [14], and thus Gailitis resonances could be included automatically in the quantum close-coupling calculations of cross-sections and line broadening since the 1970s, Several publications followed, first based the Schr$\acute{}$odinger equation and relativistic effects treated by perturbations (R-Matrix theory), and more recently based on the Dirac equation. This was not the case for the weak coupling methods of calculations of ion-electron cross-sections (distorted wave or semi-classical in particular): the increase of the cross-sections due to the Gailitis resonances needed to be calculated and added independently. Therefore, an approximate Gailitis formula was successfully developed in the first quarter of the 1970s. using the distorted wave and also the semi-classical approximation. It was applied to the interpretation of the visible and UV spectrum of the solar corona [15,16]. This allowed to interpret the intensity of the green line of Fe XIV and of several EUV lines of the solar corona, in order to determine in particular the local electron density. In fact, the energy levels of the spectral lines of the solar corona are far from the thermodynamic equilibrium (the temperature is very high and the electron density very low). Consequently, for obtaining the intensities of these optically thin lines and for comparing them to observations, the system of statistical equilibrium equations combining excitation and de-excitation by electron collisions and de-excitation by spontaneous emission, the populations of the levels could be obtained. Then, the intensities of these optically thin lines could be calculated. Thanks to the inclusion of the Gailitis resonances in the fine structure cross-sections between the levels of the same given terms, a correct spectroscopic diagnostic was obtained for the first time.

Therefore, this efficient method was adapted to semi-classical Stark broadening calculations in the second half of the 1970s [5], and the results were included in the corresponding numerical code, but without publication of the method.

The purpose of the present paper is to remedy this lack and to revisit the determination of the semi-classical limit of the Gailitis formula [13] applied to electron impact broadening of isolated spectral lines of ionized atoms. This is the object of the following sections. Then, a few examples will be provided, showing the importance of the inclusion of the Feshbach resonances in the line width.

2. Brief Recall: Stark Broadening Widths of Isolated Lines of Ionized Atoms in the Impact Approximation

Following the impact and the isolated line approximations, the line profile is lorentzian [1]. The formula giving the line widths and shifts in the impact approximation can be found in many papers, for example in [1,6,17] for the original formula. Therefore we consider a dipolar radiative transition between an initial level i, $(\gamma_i\ J_i)$, and a final level f, $(\gamma_f\ J_f)$. The γ are the other quantum numbers of the levels: $\gamma = \alpha L$, where αL is the term, L is the total

orbital angular momentum. α contains the configuration and the total spin of the electrons, which is conserved if we consider allowed transitions in LS coupling. The total width at half maximum intensity W reads:

$$\begin{aligned} W = N_P \int_0^\infty \mathrm{d}v\, v\, f(v) \Bigg[& \sum_{\gamma J \neq \gamma_i J_i} \sigma(\gamma_i J_i \to \gamma J, v) + \sum_{\gamma' J' \neq \gamma_f J_f} \sigma(\gamma_f J_f \to \gamma' J', v) \\ & + \sigma(\gamma_i J_i \leftrightarrow \gamma_i J_i, v) + \sigma(\gamma_f J_f \leftrightarrow \gamma_f J_f, v) \\ & - 2 \int_0^\infty \mathrm{d}b\, 2\pi\, b \\ & \times \sum_{M_i\, M_i'\, M_f\, M_f'\, m} (-1)^{2J_f + M_f + M_f'} \begin{pmatrix} J_i & 1 & J_f \\ -M_i & m & M_f \end{pmatrix} \begin{pmatrix} J_i & 1 & J_f \\ -M_i' & m & M_f' \end{pmatrix} \\ & \times \mathrm{Re} \Big\{ \langle \gamma_i J_i M_i | T(v,b) | \gamma_i J_i M_i' \rangle \langle \gamma_f J_f M_f | T^*(v,b) | \gamma_f J_f M_f' \rangle \Big\} \Bigg] . \end{aligned} \tag{1}$$

N_p is the density of the perturbers (electrons in the present paper), v is their velocity, which is integrated over a Maxwell distribution $f(v)$.

$T = 1 - S$ is the transition matrix, S is the scattering matrix, which is symmetric and unitary [1–3,6].

The σ denote the cross-sections. The first line of Equation (1) is the sum of the inelastic ones between the initial level i (resp.final level f) and the perturbing levels γJ (or $\gamma' J'$).

The second line refers to the elastic cross-sections of the initial i and final level f.

The third line is the so-called interference term which is not concerned by the Gailitis resonances.

b is the impact parameter in the semiclassical representation. In the quantum representation, a summation over l, orbital quantum number of the incident electron, replaces the integration over the impact parameter:

$$m\, v\, b = \hbar \sqrt{l(l+1)} \tag{2}$$

and

$$\int 2\pi b\, db = \sum_l \frac{2l+1}{k^2} \pi \tag{3}$$

The Feshbach resonances enter the elastic cross-sections and the inelastic fine structure cross-sections between the levels $\sigma(\gamma_i J_i \to \gamma_i J_i', v)$ and $\sigma(\gamma_f J_f \to \gamma_f J_f', v)$.

In fact, if the fine structure splitting is small compared to the ion–electron interaction potential, and if LS coupling is valid, the line width of each fine structure component is equal of the width of the multiplet [18]. Indeed, if the fine structure splitting ΔE_{FS} is small when compared to the interaction potential, the atomic spin can be decoupled during the collision process. This arises from the Heisenberg uncertainty principle. In other words, if the spin has no time to precess, the fine structure interaction can be ignored during the collision. This was checked in [12]: the relative difference between the widths of the fine structure components of C IV (3s-3p) and P XIII (3s-3p) increases from 0.5% for C IV to 12% for P XIII (in units of wavelengths), but is smaller in units of angular frequency (see below in Section 5.1). This is due to the difference between the wavelengths of the two components, which is more important than the differences between the cross-sections averaged over the Maxwell distribution .

In addition, we will assume LS coupling. Departures from LS coupling become important for very highly ionized atoms, and for heavy elements. For these elements, the atomic structure structure data is generally not sufficiently complete or not very accurate for performing accurate semi-classical calculations of Stark widths. More simple but

approximate methods can sometimes be used. For instance, a number of examples are implemented in the Stark-B database [8].

Therefore, we will neglect departures from LS coupling in the following, and fine structure will not be taken into account. We will consider the Feshbach resonances modifying the elastic cross-sections $\sigma(\alpha_i L_i \leftrightarrow \alpha_i L_i, v)$ and $\sigma(\alpha_f L_f \leftrightarrow \alpha_f L_f, v)$.

3. The Gailitis Formula for the Feshbach Resonances which Increase the Elastic Cross-Section $\sigma(k\alpha L \leftrightarrow k\alpha L)$

Then, inspired by the obtained results for the case of the lines of the solar corona, we will obtain a Gailitis approximate formula for the average of the Feshbach resonances which increase the elastic cross-section $\sigma(k\alpha L \leftrightarrow k\alpha L)$. Atomic units are used throughout the following. We use a quantum formalism.

S is the collisional scattering matrix. $T = 1 - S$ is the transition matrix.

The energy of the incident electron is $k^2/2$.

Concerning angular momenta, upper cases refer to the ion, and lower cases to the colliding electron. Thus, the quantum numbers S, L, M refer to the atom, and s, l, m refer to the incident electron.

αL is an atomic level. L denotes the angular momentum quantum number of the ion, and α denotes the other quantum numbers. $E_{\alpha L}$ is its energy.

We use the quantum coupled representation in this subsection.

The total energy E^T and the total angular momentum L^T of the system ion+electron are conserved during the collision.

$L^T = l + L$

$E^T = \frac{k^2}{2} + E_{\alpha L}$,

and, due to the spherical symmetry of the problem, the matrix elements of S and T do not depend of its projections M^T.

Using [15,16], the contribution of the mean effect of Feshbach resonances to the elastic cross-section $\sigma(k\alpha L \leftrightarrow k\alpha L)$ is given by their Gailitis approximate formula in the so-called "coupled representation". It reads:

$$\Delta\sigma(k\,\alpha L \leftrightarrow k\,\alpha L) = \frac{\pi}{k^2}\frac{1}{2L+1}\sum_{lL^T}\left(2L^T+1\right)(\Delta T)^2, \tag{4}$$

with

$$(\Delta T)^2 = \sum_{\alpha_1 l_1 L_1} \frac{\left|T\left(k\,\alpha\, l\, L\, L^T; k_1\,\alpha_1\, l_1\, L_1\, L^T\right)\right|^2 \left|T\left(k_1\,\alpha_1\, l_1\, L_1\, L^T;\, k\,\alpha\, l\, L\, L^T\right)\right|^2}{\sum\limits_{\alpha_2 l_2 L_2} \left|T\left(k_1\,\alpha_1\, l_1\, L_1\, L^T;\, k_2\,\alpha_2\, l_2\, L_2\, L^T\right)\right|^2}, \tag{5}$$

where index 1 refers for closed shells and index 2 for open shells, and we denote the matrix elements $\left\langle k\alpha l L L^T \middle| T \middle| k_1\alpha_1 l_1 L_1 L^T \right\rangle$ as $T\left(k\,\alpha\, l\, L\, L^T; k_1\,\alpha_1\, l_1\, L_1\, L^T\right)$, and so on in the following.

The elements of the T-matrix are calculated for energies just above the new threshold and extrapolated under this new threshold. This is the fundamental result which signifies that the averaged capture cross-section which enhances the elastic cross-section is equal to the extrapolation of the excitation cross-section under the threshold. This is valid because the distances between the resonances are large compared to their widths, and thus the series do not overlap and interferences can be neglected, [13,14,19].

4. The Semiclassical Limit of the Galitis Formula for the Elastic Cross-Section Entering the Stark Width

Within the semiclassical approximation, the colliding electron is treated classically, and moves on a trajectory characterized by an impact parameter $b = l/k$. Then, the orbital moment l of the electron and its projections m are conserved. Its kinetic energy $k^2/2$ is also conserved. The atomic orbital moment L and its projections M being not conserved, this approximation is valid at high l values and at high energies. Then, the S and T matrices are diagonal in l and their elements do not depend on m.

Therefore, for obtaining the semiclassical limit of the Gailitis quantum formula, we must use the decoupled representation.

The decoupling formula is given by

$$\left| l\,L\,L^T M^T \right\rangle = \sum_{m\,M} C^{lLL^T}_{mMM^T} |l\,m\rangle |L\,M\rangle, \tag{6}$$

where $C^{lLL^T}_{mMM^T} = \left\langle l\,L\,m\,M \middle| L^T M^T \right\rangle$ is a Clebsch–Gordan coefficient in Wigner's notation.

The calculations which only use standard angular algebra are detailed in [20,21] and shown in [16] for the case of the O V coronal lines. In the present paper, we only give the result for the case of the elastic cross-section. See also [15] for the case of the green line.

For the elastic cross-section $\sigma(k\alpha L \rightarrow k\alpha L)$ the semi-classical limit of the Gailitis formula reads:

$$\Delta\sigma(k, \alpha L \rightarrow k, \alpha L) = \sum_{\alpha_1 L_1} \sigma(k_1, \alpha L \rightarrow \alpha_1 L_1) \frac{A(\alpha_1 L_1 \rightarrow \alpha L)}{\sum\limits_{\alpha_2 L_2 < \alpha_1 L_1} A(\alpha_1 L_1 \rightarrow \alpha_2 L_2)} \tag{7}$$

where $A(\alpha_1 L_1 \rightarrow \alpha L)$, and $A(\alpha_1 L_1 \rightarrow \alpha_2 L_2)$ denote the spontaneous transition emission probabilities. $\frac{k_1^2}{2}$ is the incident electron energy just above the $\alpha_1 L_1$ threshold. The levels $\alpha_2 L_2$ are connected to $\alpha_1 L_1$ by permitted transitions.

The result of Equation (7) has to enter Equation (1). For sake of simplicity, we also neglect fine structure in Equation (1).

For that, the contribution of the Feshbach resonances has to be added to the contribution of the elastic collisions to the line width (second line of Equation (1)).

First, we consider the elastic cross-section of the initial level i ($\alpha_i L_i$). The energy of the colliding incident electron is $E = mv^2/2$.

For obtaining the contribution of the resonances to the cross-section $\sigma(\alpha_i L_i \leftrightarrow \alpha_i L_i, E)$, we have to perform a summation over the $i'(\alpha'_i L'_i)$ levels which are above the $i(\alpha_i L_i)$ level, since only excitation can create Feshbach resonances.

The Feshbach resonances play a role for energies E below the threshold of the transition $(\alpha_i L_i \rightarrow \alpha'_i L'_i)$ and are equal to zero above. Under the threshold, and using Equation (7), their contribution is equal to:

$$\sum_{\alpha'_i L'_i} \sigma\left(\alpha_i, L_i \rightarrow \alpha'_i L'_i,\ E\left(\alpha_i, L_i \rightarrow \alpha'_i L'_i\right)\right) \frac{A(\alpha'_i L'_i \rightarrow \alpha_i, L_i)}{\sum\limits_{\alpha L < \alpha'_i L'_i} A(\alpha'_i L'_i \rightarrow \alpha L)} \tag{8}$$

where $E\left(\alpha_i, L_i \rightarrow \alpha'_i L'_i\right)$ is the difference of energy between the two levels, just above the threshold of the level $alpha'_i L'_i$.

The same reasoning can be applied to the final level f: the Feshbach play a role for energies E below the threshold of the transition (in excitation) $(\alpha_f L_f \rightarrow \alpha'_f L'_f)$ and are equal to zero above. Under the threshold, and using Equation (8), their contribution is equal to:

$$\sum_{\alpha'_f L'_f} \sigma\left(\alpha_f, L_f \rightarrow \alpha'_f L'_f,\ E\left(\alpha_f, L_f \rightarrow \alpha'_f L'_f\right)\right) \frac{A(\alpha'_f L'_f \rightarrow \alpha_f, L_f)}{\sum\limits_{\alpha L < \alpha'_f L'_f} A(\alpha'_f L'_f \rightarrow \alpha L)} \tag{9}$$

where $E\left(\alpha_f, L_f \rightarrow \alpha'_f L'_f\right)$ is the difference of energy between the two levels, just above the threshold of the level $alpha'_f L'_f$.

- N.B. In addition, if results of the Gailitis resonances between J-levels would be needed, Equations (5), (7) and (8), must be modified. For that, an additional step, again using standard angular algebra calculations, has to be conducted. We only give the result there: the α and the L and L' which enter Equation (8) must be simply replaced by the γ, J and J'.

5. Examples of Results

The relative importance of the contribution of the Feshbach resonances to the total Stark impact width is exemplified on the example of Ne VIII in [6].

Other examples are given in Tables 1–4. The data are taken from calculations [12,22–24] performed with the SCP numerical code. See also the STARK-B database [8].

Table 1. Results of the SCP code for CII $3d-4f, \lambda = 4267$ Å, electron collisions. Angular frequency units, density $N_p = 10^{18}$ cm^{-3}, temperatures T in Kelvin.

T	0.50×10^4	0.1×10^5	0.50×10^5
Full width at half maximum	0.225×10^{13}	0.178×10^{13}	0.122×10^{13}
Inelastic collision contribution from the upper level	0.124×10^{13}	0.106×10^{13}	0.841×10^{12}
Inelastic collision contribution from the lower level	0.176×10^{12}	0.137×10^{12}	0.114×10^{12}
Feshbach resonances contribution from the upper level	0.835×10^9	0.481×10^9	0.718×10^8
Feshbach resonances contribution from the lower level	0.108×10^{11}	0.733×10^{10}	0.161×10^9
Elastic collisions contribution (without resonances)	0.829×10^{12}	0.581×10^{12}	0.261×10^{12}

Table 2. Results of the SCP code for CIV $2s-2p, \lambda = 1549$ Å, electron collisions. Angular frequency units, density $N_p = 10^{18}$ cm^{-3}, temperatures T in Kelvin.

T	0.10×10^6	0.2×10^6	0.40×10^6
Full width at half maximum	0.396×10^{12}	0.291×10^{12}	0.221×10^{12}
Inelastic collision contribution from the upper level	0.465×10^{11}	0.382×10^{11}	0.346×10^{11}
Inelastic collision contribution from the lower level	0.547×10^{11}	0.663×10^{11}	0.668×10^{11}
Feshbach resonances contribution from the upper level	0.299×10^{11}	0.184×10^{11}	0.944×10^{10}
Feshbach resonances contribution from the lower level	0.823×10^{11}	0367×10^{11}	0.149×10^{11}
Elastic collisions contribution (without resonances)	0.183×10^{12}	0.131×10^{12}	0.951×10^{11}

Table 3. Results of the SCP code for CIV $3s-3p, \lambda = 5805$ Å, electron collisions. Angular frequency units, density $N_p = 10^{17}$ cm^{-3}, temperatures T in Kelvin.

T	0.50×10^4	0.1×10^5	0.30×10^5
Full width at half maximum	0.106×10^{13}	0.70×10^{12}	0.417×10^{12}
Inelastic collision contribution from the upper level	0.133×10^{12}	0.15×10^{12}	0.132×10^{12}
Inelastic collision contribution from the lower level	0.20×10^{10}	0.17×10^{11}	0.615×10^{11}
Feshbach resonances contribution from the upper level	0.187×10^{12}	0.922×10^{11}	0.27×10^{11}
Feshbach resonances contribution from the lower level	0.347×10^{12}	0.226×10^{12}	0.805×10^{11}
Elastic collisions contribution (without resonances)	0.395×10^{12}	0.214×10^{12}	0.116×10^{12}

Table 4. Results of the SCP code for Ar XV $2s^2\ ^1S_0 - 2s3p\ ^1P_1^0, \lambda = 24.7$ Å, electron collisions. Angular frequency units, density $N_p = 10^{20}$ cm^{-3}, temperatures T in Kelvin.

T	0.50×10^6	0.1×10^7	0.2×10^7
Full width at half maximum	0.14×10^{14}	0.101×10^{14}	0.727×10^{13}
Inelastic collision contribution from the upper level	0.663×10^{12}	0.573×10^{12}	0.488×10^{12}
Inelastic collision contribution from the lower level	0.199E + 12	0.287×10^{12}	0.304×10^{12}
Feshbach resonances contribution from the upper level	0.173×10^{12}	0.779×10^{11}	0.332×10^{11}
Feshbach resonances contribution from the lower level	0.582×10^{12}	0.284×10^{12}	0.126×10^{12}
Elastic collisions contribution (without resonances)	0.132×10^{14}	0.920×10^{13}	0.648×10^{13}

As expected, the relative importance of the Feshbach resonances decreases when the temperature increases. These results also show that the relative contribution of the resonances is variable. In fact, it depends on the position of the perturbing levels and of their importance in the calculations. Therefore, it is difficult to predict their importance in advance. In addition, the experimental results are scarce. There are some results for C IV 2s-2p [24,25], but the difference between the experimental, quantum and semi-classical results are not conclusive. There are also results for C IV 3s-3p [12], and the difference between the SCP and experimental results are 10%–20%.

5.1. Case between J-Levels: Example of the Two Fine Structure Components of P XIII 3s-3p

Tables 5 and 6 which concern the two fine structure components of P XIII 3s-3p show that the impact of taking into account the fine structure is negligible:

Table 5. Results of the SCP code for P XIII $3s\,1/2 - 3p\,1/2, \lambda = 1741.3$ Å, electron collisions. Angular frequency units, density $N_p = 10^{18}$ cm^{-3}, temperatures T in Kelvin.

T	0.10×10^6	$0.5\,10^6$	$0.2\,10^7$
Full width at half maximum	0.511×10^{12}	0.243×10^{12}	0.140×10^{12}
Inelastic collision contribution from the upper level	0.716×10^{11}	0.462×10^{11}	0.318×10^{11}
Inelastic collision contribution from the lower level	0.436×10^{11}	0.462×10^{11}	0.337×10^{11}
Feshbach resonances contribution from the upper level	0.200×10^{11}	0.368×10^{10}	0.768×10^{9}
Feshbach resonances contribution from the lower level	0.624×10^{11}	0.840×10^{10}	0.127×10^{10}
Elastic collisions contribution (without resonances)	0.313×10^{12}	0.139×10^{12}	0.725×10^{11}

Table 6. Results of the SCP code for P XIII $3s\,1/2 - 3p\,3/2, \lambda = 1645.8$ Å, electron collisions. Angular frequency units, density $N_p = 10^{18}$ cm^{-3}, temperatures T in Kelvin.

T	0.10×10^6	0.5×10^6	0.2×10^7
Full width at half maximum	0.511×10^{12}	0.244×10^{12}	0.141×10^{12}
Inelastic collision contribution from the upper level	0.727×10^{11}	0.468×10^{11}	0.321×10^{11}
Inelastic collision contribution from the lower level	0.436×10^{11}	0.462×10^{11}	0.337×10^{11}
Feshbach resonances contribution from the upper level	0.191×10^{11}	0.364×10^{10}	0.774×10^{9}
Feshbach resonances contribution from the lower level	0.624×10^{11}	0.840×10^{10}	0.127×10^{10}
Elastic collisions contribution (without resonances)	0.314×10^{12}	0.139×10^{12}	0.727×10^{11}

6. Conclusions

This approximate expression of the Feshbach resonances is consistent with the accuracy of the semiclassical perturbation theory of Stark broadening of spectral lines of ionized atoms. As shown on the above provided examples, their order of magnitude is especially important at low temperatures, but not always. Their semi-classical expression has been implemented in the SCP computer code for many years. Numerous results of calculations are published and a number are implemented in the Stark-B database. These efforts will be pursued for obtaining new results of calculations needed in modeling and spectroscopic diagnostics.

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