

Article

A Neural Approach for Position–Orientation Tracking of the Stewart Platform with Disturbance Suppression

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Abstract

In this paper, the position–orientation tracking of the Stewart platform is investigated. We focus on the target-oriented tracking task, which usually occurs in the application of spotlights and cameras. Specifically, the mobile plate of the Stewart platform is always oriented to an immobile point while tracking a desired path. Based on the velocity-level kinematics and zeroing neural dynamics (ZND), a kinematic tracking model is proposed for the position–orientation tracking task. In addition, a robust ZND (RZND) model is further proposed against two kinds of disturbances. Theoretical analyses are presented to show the convergence properties of the proposed models. Discrete-time algorithms of the models are also developed for the convenient implementation. According to the comparative simulations, both the ZND and RZND algorithms accomplish the position–orientation tracking task without the disturbance, and the disturbance-suppression capability of the RZND algorithm is substantiated.

Keywords: zeroing neural dynamics (ZND); parallel robot; Stewart platform; disturbance suppression; position–orientation tracking

1. Introduction

Robot control, as one of the hottest issues in both automation and industrial applications, continues to attract the attention of numerous researchers [1–4]. Among different kinds of robots, the parallel robot is widely used in industry, medical science, automation, and other fields [5–7], due to its advantages of high stiffness, high loads, and low accumulated error. One of the most popular parallel robots is the Stewart platform, which is a six-degree-of-freedom parallel robot that consists of a fixed base and a mobile plate connected by extensible legs. To achieve accurate and efficient control of the Stewart platform, researchers have conducted extensive studies on its modeling, focusing on developing mathematical models that describe its kinematic and dynamic characteristics. For example, in [8], a prescribed-disturbance-rejection control method was proposed to improve the tracking accuracy and disturbance-rejection performance of a Stewart platform. In [9], a fixed-time sliding-mode composite controller with prescribed performance was developed to achieve accurate task-space tracking of a Stewart parallel mechanism in the presence of modeling uncertainties and external disturbances. In addition, Ref. [10] presented a fast finite-time composite control method with dynamic prescribed perfor-



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mance to achieve smooth, nonovershooting, and accurate task-space tracking of a Stewart parallel mechanism.

As for the control algorithms, the proportional-integral-derivative (PID) algorithm has long been recognized as a powerful and widely used control technique due to its simplicity, effectiveness, and robustness in many systems [11]. However, its tracking performance depends strongly on the selection of controller gains. Appropriate parameter tuning becomes more difficult when the Stewart platform is affected by nonlinear coupling, modeling uncertainties, measurement perturbations, external disturbances, and actuator limitations. Moreover, a set of gains selected for one operating condition may not maintain comparable performance under different trajectories, payloads, or motion ranges. Various tuning and adaptive strategies have been developed to alleviate these limitations [11], but the control performance still depends on the accuracy of the underlying model and the selected tuning mechanism. Sliding-mode control has also been extensively investigated for nonlinear robotic systems because of its robustness against certain matched and bounded disturbances [12,13]. Nevertheless, conventional sliding-mode controllers may generate chattering due to the switching control action. Their performance is also sensitive to the selection of switching gains, which are commonly chosen according to the expected uncertainty or disturbance level. Excessively large gains may cause undesirable oscillations and increase the control effort, whereas insufficient gains may weaken the disturbance-rejection capability. Higher-order sliding-mode and observer-based methods can reduce some of these effects, but their design and implementation still require appropriate assumptions about the disturbance and system dynamics.

Recent studies have illustrated the potential of learning and optimization-oriented methodologies for intelligent and robust control. A model-free Q-learning approach was developed for robust H_∞ output consensus of heterogeneous multi-agent discrete-time systems [14]. Distributionally robust optimization was combined with stochastic self-triggered model predictive control to address uncertain disturbances and state chance constraints while reducing sampling and communication requirements [15]. Moreover, distributed distributionally robust model predictive control was investigated for linear systems with incomplete disturbance-distribution information and coupled chance constraints [16]. These studies provide valuable insights into learning-based adaptation, uncertainty handling, constraint management, and resource-aware control. Incorporating these mechanisms into the RZND framework to address more complex uncertainties and practical constraints is a meaningful direction for future research.

Neural dynamics is a powerful theoretical framework that has found widespread applications in various fields, including control, optimization, pattern recognition, machine learning, neuroscience, and cognitive science [17–21]. One of the most widely studied neural dynamics models is the Hopfield network [22], which is trained to store a set of patterns by adjusting the weights to minimize an energy function, and can then retrieve a stored pattern from a partial or noisy input by following a dynamical trajectory towards a stable state. Generated from the Hopfield network, gradient neural dynamics (GND) is a popular method investigated by researchers. For instance, a GND model was proposed to solve the time-varying nonlinear optimization [23]. In addition, Zhang [24] proposed an improved GND model by introducing additional nonlinearity into a traditional GND model for handling time-varying Lyapunov equations. Yu et al. utilized the GND model to construct a unified position-based visual servo control system [25]. Despite being a concise and feasible neural dynamics method, the GND model may suffer from lower precision due to time-varying problems, which could potentially compromise its performance in real-time computation.

In past decades, zeroing neural dynamics (ZND, also termed Zhang neural dynamics) has been recognized as a classical and effective method for solving various time-varying problems [26–31]. Through previous works, the ZND model has been applied in scientific and engineering problems handling. For example, researchers [27] developed a double integral fuzzy ZND controller to enhance the robustness and adaptability of unmanned aerial vehicle trajectory tracking under unbounded disturbances. In the work [28], a fuzzy adaptive complex-valued ZND model was proposed for a time-varying complex-valued quadratic programming problem. In another work [29], a novel proportional-integral-parameter ZND model was presented to solve the quaternion-valued time-varying linear matrix inequalities problem. In addition, two quaternion-valued ZND models were proposed for dynamic quaternion-valued matrix inversion with the application of mobile manipulator control [30]. In addition, a barrier varying-parameter dynamic learning network was proposed to solve the time-varying quadratic programming with the help of the ZND design formula [31]. Although robust and integral-enhanced ZND models have been investigated for other time-varying problems, their direct application does not address the geometric coupling involved in target-oriented motion. In this task, the desired orientation is generated online from the relative direction between the moving platform and a fixed target. Consequently, platform translation continuously changes the required orientation, and the position and orientation objectives cannot be independently prescribed.

Tracking control is a crucial task in the operation of the Stewart platform. In the field of flight simulation, the Stewart platform is commonly used to provide realistic motion cues to pilots during flight training. Accurate tracking control is essential to simulate various flight conditions and provide an immersive training experience. In addition, the Stewart platform is utilized as the base of radars, requiring accurate orientation to better receive signals. Recent studies have investigated advanced tracking-control methods for Stewart platforms [8–10]. These studies have significantly improved the path-tracking accuracy, constraint handling, and disturbance-rejection capability of Stewart platforms. However, the target-oriented tracking problem considered in this paper involves an additional position–orientation coupling because the desired orientation is generated online according to the relative direction between the moving platform and the target. Consequently, changes in the platform position continuously affect the required orientation. Developing a unified method for this coupled position–orientation tracking task therefore remains meaningful.

Motivated by the above limitations, this paper develops a neural dynamics model for target-oriented position–orientation tracking of the Stewart platform. Unlike GND and adaptive GND (AGND) models that update the platform variables by minimizing instantaneous objectives, the proposed ZND model directly prescribes the evolution of the target-dependent position and orientation residuals. The time variation of the desired path and target-relative direction is explicitly incorporated into the resulting velocity-level model. In particular, the orientation equation is coupled with the platform translation because the desired pointing direction changes as the mobile plate moves. A robust ZND (RZND) model is further developed by introducing integral-augmented residuals rather than merely modifying the convergence gains of the ZND model. This structure is designed to suppress additive disturbances entering the position and orientation error dynamics. Theoretical results are established for unknown constant disturbances and unknown linear time-varying disturbances. Comparative simulations with GND and AGND are conducted to illustrate the performance differences under the considered tracking task and disturbance settings.

The remainder of this paper is organized into four sections. Section 2 introduces the schematic and the kinematics of the Stewart platform. Section 3 introduces the

position–orientation tracking of the Stewart platform investigated in this paper, and proposes the ZND model and the RZND model as well as the interfered models with theoretical analyses. In Section 4, comparative simulations are carried out to evaluate the proposed models with developed discrete-time algorithms. In addition, the GND and AGND algorithms are also developed for comparison. Section 5 concludes this paper.

At the end of this section, the main contributions are listed as below.

1. A target-dependent position–orientation tracking system is constructed for the Stewart platform. The desired orientation is generated online from the relative direction between the moving platform and the fixed target, leading to an explicitly coupled velocity-level ZND model.
2. An integral-augmented RZND model is derived from the coupled residual system. Theoretical results establish nominal convergence, exact rejection of the considered unknown constant additive disturbances, and explicit steady-state error bounds under the considered unknown linear time-varying additive disturbances.
3. The structural distinctions among ZND, RZND, GND, and AGND models are analyzed. The influence of the orientation-error construction on the RZND model is also investigated, showing that an error function suitable for conventional ZND is not necessarily compatible with integral augmentation.

2. Preliminaries

In this paper, a Stewart platform that works in 3-D space with six degrees of freedom is considered, and its schematic is shown in Figure 1. Specifically, the Stewart platform consists of a mobile plate, a fixed base, and six independent retractable legs. In the forward kinematics, the Stewart platform receives the lengths of the six legs as input, and controls the mobile plate of which position and orientation are determined by a translation vector $\mathbf{r}_{ac} \in \mathbb{R}^3$ and a rotation matrix $A \in \mathbb{R}^{3 \times 3}$, respectively. Generally, the translation vector $\mathbf{r}_{ac}$ denotes the actual position of the end-effector, which is installed at the center point of the mobile plate. The rotation matrix A is defined by the Euler angles $\boldsymbol{\theta} = [\theta_x, \theta_y, \theta_z]^T$. For better understanding, coordinate systems are established in Figure 1, and descriptions of symbols used in this paper are presented as below.

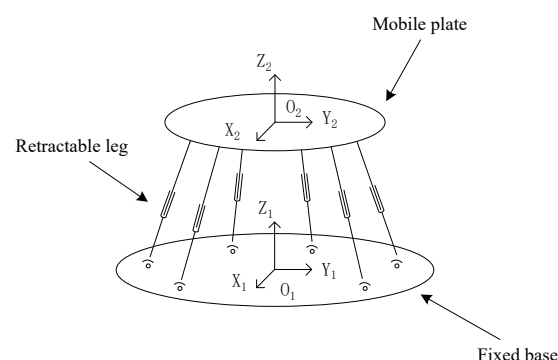


Figure 1. Schematic of Stewart platform.

$\{O_1\}$: Global coordinate system of which origin is at the center point of the fixed base.

$\{O_2\}$: Platform coordinate system of which origin is at the center point of the mobile plate.

$\mathbf{f}_i$: Connection point position in $\{O_1\}$ of the i th leg on the fixed base.

$\mathbf{m}_i$: Connection point position in $\{O_1\}$ of the i th leg on the mobile plate.

$\mathbf{m}'_i$: Connection point position in $\{O_2\}$ of the i th leg on the mobile plate.

$\mathbf{r}_{ac}$: Actual position in $\{O_1\}$ of the end-effector, which is installed at the center point of the mobile plate.

A : Rotation matrix defined by the Euler angles for transformation from platform coordinate to global coordinate.

l_i : Length of the i th leg with $\mathbf{l} = [l_1, l_2, \dots, l_6]$.

For a point $\mathbf{p}' \in \mathbb{R}^3$ in $\{O_2\}$, its corresponding global coordinate $\mathbf{p}$ in $\{O_1\}$ is computed by the following equation:

$$\mathbf{p} = \mathbf{r}_{ac} + A\mathbf{p}', \quad (1)$$

where

$$A = A_z A_y A_x, \quad (2)$$

$$A_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_x & -\sin \theta_x \\ 0 & \sin \theta_x & \cos \theta_x \end{bmatrix},$$

$$A_y = \begin{bmatrix} \cos \theta_y & 0 & \sin \theta_y \\ 0 & 1 & 0 \\ -\sin \theta_y & 0 & \cos \theta_y \end{bmatrix},$$

$$A_z = \begin{bmatrix} \cos \theta_z & -\sin \theta_z & 0 \\ \sin \theta_z & \cos \theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

In light of (1), the transformation from $\mathbf{m}'_i$ to $\mathbf{m}_i$ is presented as

$$\mathbf{m}_i = \mathbf{r}_{ac} + A\mathbf{m}'_i. \quad (3)$$

Subsequently, the i th leg is mathematically expressed as a vector:

$$\mathbf{q}_i = \mathbf{m}_i - \mathbf{f}_i = \mathbf{r}_{ac} + A\mathbf{m}'_i - \mathbf{f}_i, \quad (4)$$

and the length of the i th leg l_i is further obtained as the 2-norm of the leg vector:

$$l_i = \|\mathbf{r}_{ac} + A\mathbf{m}'_i - \mathbf{f}_i\|_2. \quad (5)$$

According to the work of the kinematic modeling of a Stewart platform [9], the kinematic equation of the Stewart platform at velocity level is expressed as

$$\dot{\mathbf{i}} = B \begin{bmatrix} \dot{\mathbf{r}}_{ac} \\ \dot{\boldsymbol{\theta}}_{ac} \end{bmatrix}, \quad (6)$$

where $\dot{\mathbf{i}} \in \mathbb{R}^6$ is the stretch velocity of the legs; $\dot{\mathbf{r}}_{ac}$ is the translational velocity of the end-effector; $\dot{\boldsymbol{\theta}}_{ac}$ is the angular velocity of the mobile plate. In addition, matrix B is computed by

$$B = \begin{bmatrix} \frac{1}{l_1} \mathbf{q}_1^T & \frac{1}{l_1} ((A\mathbf{m}'_1) \times \mathbf{q}_1)^T \\ \frac{1}{l_2} \mathbf{q}_2^T & \frac{1}{l_2} ((A\mathbf{m}'_2) \times \mathbf{q}_2)^T \\ \dots & \dots \\ \frac{1}{l_6} \mathbf{q}_6^T & \frac{1}{l_6} ((A\mathbf{m}'_6) \times \mathbf{q}_6)^T \end{bmatrix}, \quad (7)$$

where “ $\times$ ” denotes the cross product.

Here, B denotes the velocity-level kinematic matrix of the Stewart platform at a general configuration. When the platform moves along a trajectory, its position and orientation become time varying. Accordingly, the quantities contained in B vary with time, and the matrix is denoted by $B(t)$ in the subsequent tracking model.

3. Problem Description, Solution Models and Analyses

In order to lay a basis for further investigation, the problem description and the design procedures of the ZND model and the RZND model are presented in this section.

3.1. Problem Description

Generally, the tracking problem of the Stewart platform is formulated as

$$\phi(\mathbf{l}(t)) = \begin{bmatrix} \mathbf{r}_{ac}(t) \\ \boldsymbol{\theta}_{ac}(t) \end{bmatrix} \rightarrow \begin{bmatrix} \mathbf{r}_{de}(t) \\ \boldsymbol{\theta}_{de}(t) \end{bmatrix}, \quad (8)$$

where $\mathbf{r}_{de}(t) \in \mathbb{R}^3$ and $\boldsymbol{\theta}_{de}(t) \in \mathbb{R}^3$ depict the desired path and the desired posture, respectively. In addition, $\phi(\cdot) : \mathbb{R}^6 \rightarrow \mathbb{R}^6$ is the forward-kinematics mapping function. It is worth pointing out that the forward-kinematics problem of the Stewart platform is hard to solve due to the inherent nonlinearities and complexity. In practice, the actual position as well as the posture $[\mathbf{r}_{ac}^T(t), \boldsymbol{\theta}_{ac}^T(t)]^T$ is often obtained by numerical algorithms such as the Newton–Raphson method, or directly measured by sensors.

In this paper, the position–orientation tracking of the Stewart platform oriented to an immobile point is investigated. In global coordinate system $\{O\}$, for a desired path $\mathbf{r}_{de}(t) \in \mathbb{R}^3$ and an immobile point $\mathbf{d} \in \mathbb{R}^3$ that always stays above the mobile plate, the goal is to track $\mathbf{r}_{de}(t)$ in real time t by the end-effector with the connecting line between the immobile point $\mathbf{d}$ and the end-effector $\mathbf{r}_{ac}(t)$ always vertical to the mobile plate. Thus, the position–orientation tracking problem is formulated as

$$\begin{cases} \mathbf{r}_{ac}(t) \rightarrow \mathbf{r}_{de}(t), \\ \psi(\mathbf{r}_{ac}(t), \boldsymbol{\theta}_{ac}(t), \mathbf{d}) \rightarrow 0. \end{cases} \quad (9a)$$

$$(9b)$$

Therein, $\psi(\cdot, \cdot, \cdot)$ is a scalar function that computes the angle between the connecting line and the normal vector of the mobile plate. Compared with problem (8), the information of the desired posture is hidden in the actual trajectory and the immobile point, which allows the mobile plate to rotate adaptively.

3.2. ZND Model and Analysis

For the Stewart platform, the mobile plate can be in any posture within physical constraints at a certain position. Similarly, the mobile plate can be at any position within physical constraints in a certain posture. Thus, the position–orientation tracking problem can be investigated from two aspects.

For the position part, an error vector is defined to measure the position error:

$$\mathbf{e}_{pa}(t) = \mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t). \quad (10)$$

According to the ZND design formula, i.e., $\dot{\mathbf{e}}_{pa}(t) = -\lambda_1 \mathbf{e}_{pa}(t)$ where $\lambda_1 \in \mathbb{R}^+$, one obtains

$$\dot{\mathbf{r}}_{ac}(t) = \dot{\mathbf{r}}_{de}(t) + \lambda_1(\mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t)). \quad (11)$$

For the orientation part, an auxiliary matrix is introduced as $M = [1, 0; 0, 1; 0, 0] \in \mathbb{R}^{3 \times 2}$, and the direction vector from $\mathbf{r}_{ac}(t)$ to $\mathbf{d}$ is presented as $\mathbf{v}(t) = \mathbf{d} - \mathbf{r}_{ac}(t)$. The two

columns of M represent two linearly independent directions in the plane of the mobile plate. Therefore, the columns of $A(\theta_{ac}(t))M$ span the mobile-plate plane in the global coordinate system. Requiring $(A(\theta_{ac}(t))M)^T \mathbf{v}(t) = \mathbf{0}$ makes the target-direction vector $\mathbf{v}(t)$ perpendicular to the mobile plate and thus parallel to its normal vector. The assumption that the immobile point remains above the mobile plate resolves the directional ambiguity. The orientation error is defined as

$$\mathbf{e}_{or}(t) = (A(\theta_{ac}(t))M)^T \mathbf{v}(t), \quad (12)$$

in which $A(\theta_{ac}(t))$ denotes the time-varying representation of the coordinate transformation matrix described above. On the basis of the ZND design formula, i.e., $\dot{\mathbf{e}}_{or}(t) = -\lambda_2 \mathbf{e}_{or}(t)$ where $\lambda_2 \in \mathbb{R}^+$, one obtains

$$J(\theta_{ac}(t))\dot{\theta}_{ac}(t) = -\lambda_2(A(\theta_{ac}(t))M)^T \mathbf{v}(t) + (A(\theta_{ac}(t))M)^T \dot{\mathbf{r}}_{ac}(t), \quad (13)$$

where

$$J(\theta_{ac}(t)) = \left[\left(\frac{\partial A(\theta_{ac}(t))}{\partial \theta_x(t)} M \right)^T \mathbf{v}(t), \left(\frac{\partial A(\theta_{ac}(t))}{\partial \theta_y(t)} M \right)^T \mathbf{v}(t), \left(\frac{\partial A(\theta_{ac}(t))}{\partial \theta_z(t)} M \right)^T \mathbf{v}(t) \right].$$

Combining (6), (11), and (13) yields the ZND model for the position–orientation tracking:

$$\begin{cases} \dot{\mathbf{r}}_{ac}(t) = \dot{\mathbf{r}}_{de}(t) + \lambda_1(\mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t)), \end{cases} \quad (14a)$$

$$\begin{cases} J(\theta_{ac}(t))\dot{\theta}_{ac}(t) = -\lambda_2(A(\theta_{ac}(t))M)^T \mathbf{v}(t) \\ \quad + (A(\theta_{ac}(t))M)^T \dot{\mathbf{r}}_{ac}(t), \end{cases} \quad (14b)$$

$$\begin{cases} \dot{\mathbf{i}}(t) = B(t) \begin{bmatrix} \dot{\mathbf{r}}_{ac}(t) \\ \dot{\theta}_{ac}(t) \end{bmatrix}. \end{cases} \quad (14c)$$

Since $J(\theta_{ac}(t)) \in \mathbb{R}^{2 \times 3}$, the orientation equation is underdetermined. Within the feasible and nonsingular operating region, it is assumed that

$$\text{rank}(J(\theta_{ac}(t))) = 2.$$

Let

$$\mathbf{b}_{or} = -\lambda_2 \mathbf{e}_{or}(t) + (A(\theta_{ac}(t))M)^T \dot{\mathbf{r}}_{ac}(t). \quad (15)$$

The general solution of the orientation equation is

$$\dot{\theta}_{ac}(t) = J^\dagger(\theta_{ac}(t))\mathbf{b}_{or}(t) + \left[I_3 - J^\dagger(\theta_{ac}(t))J(\theta_{ac}(t)) \right] \mathbf{v}(t), \quad (16)$$

where $J^\dagger(\theta_{ac}(t))$ denotes the Moore–Penrose pseudoinverse and $\mathbf{v}(t) \in \mathbb{R}^3$ is an arbitrary vector. In this study, $\mathbf{v}(t) = \mathbf{0}$ is selected. The validity of the ZND model (14) is guaranteed by the following theorem under the assumption.

Assumption 1. The subsequent theoretical analyses are established under the following operating assumptions. The desired and actual platform motions remain within the reachable workspace of the Stewart platform, and the platform configuration, leg lengths, and leg-length rates satisfy the corresponding physical constraints. The platform operates away from kinematic singularities and Euler-angle singularities. Moreover, the orientation Jacobian satisfies $\text{rank}(J(\theta_{ac}(t))) = 2$, and

its Moore–Penrose pseudoinverse $J^+(\theta_{ac}(t))$ remains bounded. The target-direction vector is also assumed to satisfy $\|\mathbf{v}(t)\|_2 > 0$.

Theorem 1. Consider the position–orientation tracking of the Stewart platform oriented to an immobile point. Let $\mathbf{v}(t)$ denote the connecting line between immobile point $\mathbf{d}$ and end-effector $\mathbf{r}_{ac}(t)$, and $\mathbf{u}(t)$ denote the normal vector of the mobile plate. Suppose that the Stewart platform always works within the physical constraints. With proper hyperparameters λ_1 and $\lambda_2 \in \mathbb{R}^+$, and under the assumption that the kinematic mapping remains feasible and nonsingular, the tracking errors of the ZND model in (14) asymptotically converge to zero. Consequently, the end-effector trajectory $\mathbf{r}_{ac}(t)$ converges to the desired path $\mathbf{r}_{de}(t)$, and the mobile plate orientation $\mathbf{u}(t)$ aligns with the direction $\mathbf{v}(t)$.

Proof. Considering the position error $\mathbf{e}_{pa}(t) = \mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t)$, let $e_i(t)$ denote the i th entry of $\mathbf{e}_{pa}(t)$. According to (14a), one has

$$\dot{e}_i(t) = -\lambda_1 e_i(t) \quad (17)$$

and

$$e_i(t) = e_i(0) \exp(-\lambda_1 t), \quad (18)$$

which leads to

$$\lim_{t \rightarrow \infty} e_i(t) = 0 \quad (19)$$

and

$$\lim_{t \rightarrow \infty} (\mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t)) = \lim_{t \rightarrow \infty} \mathbf{e}_{pa}(t) = \mathbf{0}. \quad (20)$$

Thus, the end-effector trajectory $\mathbf{r}_{ac}(t)$ converges to the desired path $\mathbf{r}_{de}(t)$ as time t goes.

Similarly, the equation (13) can be rewritten as

$$\dot{\mathbf{e}}_{or} = -\lambda_2 \mathbf{e}_{or}, \quad (21)$$

and the following relationship is obtained

$$\lim_{t \rightarrow \infty} (A(\theta_{ac}(t))M)^T \mathbf{v}(t) = \lim_{t \rightarrow \infty} \mathbf{e}_{or}(t) = \mathbf{0}. \quad (22)$$

Since the vector $(A(\theta_{ac}(t))M)^T \mathbf{v}(t)$ is obtained by taking the dot product of $\mathbf{v}(t)$ with the direction vectors of two intersecting lines on the mobile plate, and given that the immobile point $\mathbf{d}$ always lies above the platform, the mobile plate orientation $\mathbf{u}(t)$ converges to the direction of $\mathbf{v}(t)$. $\square$

If the ZND model (14) is perturbed by the positional disturbance, the corresponding polluted ZND model is formulated as

$$\begin{cases} \dot{\mathbf{r}}_{ac}(t) = \dot{\mathbf{r}}_{de}(t) + \lambda_1 (\mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t)) + \sigma_p(t), \\ J(\theta_{ac}(t))\dot{\theta}_{ac}(t) = -\lambda_2 (A(\theta_{ac}(t))M)^T \mathbf{v}(t) \\ \quad + (A(\theta_{ac}(t))M)^T \dot{\mathbf{r}}_{ac}(t) + \sigma_o(t), \end{cases} \quad (23a)$$

$$\quad (23b)$$

$$\begin{cases} \dot{\mathbf{i}}(t) = B(t) \begin{bmatrix} \dot{\mathbf{r}}_{ac}(t) \\ \dot{\theta}_{ac}(t) \end{bmatrix}, \end{cases} \quad (23c)$$

where $\sigma_p(t) \in \mathbb{R}^3$ denotes the additive noise in the position-tracking equation, while $\sigma_o(t) \in \mathbb{R}^2$ denotes the additive noise in the pointing-direction equation. These terms may represent the combined effects of position and orientation measurement errors, kinematic-

parameter uncertainties, numerical differentiation errors, and imperfect execution of the generated velocity commands by the lower-level servo system.

3.3. RZND Model and Analyses

In order to monitor the tracking process and overcome the disturbance, an integrator term is introduced to define a new position error:

$$\epsilon_{pa}(t) = \mathbf{e}_{pa}(t) + \gamma_1 \int_0^t \mathbf{e}_{pa}(\tau) d\tau, \quad (24)$$

where $\mathbf{e}_{pa}(t)$ is defined as (10). By applying the ZND design formula, i.e., $\dot{\epsilon}_{pa}(t) = -\gamma_2 \epsilon_{pa}(t)$, one has

$$\dot{\mathbf{r}}_{ac}(t) = \dot{\mathbf{r}}_{de}(t) + (\gamma_1 + \gamma_2) \mathbf{e}_{pa}(t) + \gamma_1 \gamma_2 \int_0^t \mathbf{e}_{pa}(\tau) d\tau, \quad (25)$$

where γ_1 and $\gamma_2 \in \mathbb{R}^+$. Similarly, a new orientation error is defined as

$$\epsilon_{or}(t) = \mathbf{e}_{or}(t) + \gamma_1 \int_0^t \mathbf{e}_{or}(\tau) d\tau, \quad (26)$$

By applying the ZND design formula, i.e., $\dot{\epsilon}_{or}(t) = -\gamma_2 \epsilon_{or}(t)$, one has

$$\begin{aligned} J(\boldsymbol{\theta}_{ac}(t)) \dot{\boldsymbol{\theta}}_{ac}(t) = & -(\gamma_1 + \gamma_2) \mathbf{e}_{or}(t) - \gamma_1 \gamma_2 \int_0^t \mathbf{e}_{or}(\tau) d\tau \\ & + (A(\boldsymbol{\theta}_{ac}(t))M)^T \dot{\mathbf{r}}_{ac}(t), \end{aligned} \quad (27)$$

where γ_1 and $\gamma_2 \in \mathbb{R}^+$.

Combining (6), (25), and (27) yields the RZND model:

$$\begin{cases} \dot{\mathbf{r}}_{ac}(t) = \dot{\mathbf{r}}_{de}(t) + (\gamma_1 + \gamma_2)(\mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t)) \\ \quad + \gamma_1 \gamma_2 \int_0^t (\mathbf{r}_{de}(\tau) - \mathbf{r}_{ac}(\tau)) d\tau, \end{cases} \quad (28a)$$

$$\begin{cases} J(\boldsymbol{\theta}_{ac}(t)) \dot{\boldsymbol{\theta}}_{ac}(t) = -(\gamma_1 + \gamma_2) \mathbf{e}_{or}(t) - \gamma_1 \gamma_2 \int_0^t \mathbf{e}_{or}(\tau) d\tau \\ \quad + (A(\boldsymbol{\theta}_{ac}(t))M)^T \dot{\mathbf{r}}_{ac}(t), \end{cases} \quad (28b)$$

$$\begin{cases} \dot{\mathbf{i}}(t) = B(t) \begin{bmatrix} \dot{\mathbf{r}}_{ac}(t) \\ \dot{\boldsymbol{\theta}}_{ac}(t) \end{bmatrix}. \end{cases} \quad (28c)$$

The underdetermined orientation equation of the RZND model is handled in the same manner as that of the ZND model, namely, by using the Moore–Penrose pseudoinverse and setting the null-space vector to zero.

Theorem 2. Consider the position–orientation tracking of the Stewart platform oriented to an immobile point. Let $\mathbf{v}(t)$ denote the connecting line between immobile point $\mathbf{d}$ and end-effector $\mathbf{r}_{ac}(t)$, and $\mathbf{u}(t)$ denote the normal vector of the mobile plate. Suppose that the Stewart platform always works within the physical constraints. With proper hyperparameters $\gamma_1, \gamma_2 \in \mathbb{R}^+$, and under the assumption that the kinematic mapping remains feasible and nonsingular, the tracking errors of the RZND model in (28) asymptotically converge to zero. Consequently, the end-effector trajectory $\mathbf{r}_{ac}(t)$ converges to the desired path $\mathbf{r}_{de}(t)$, and the mobile plate orientation $\mathbf{u}(t)$ aligns with the direction $\mathbf{v}(t)$.

Proof. Considering the position error $\epsilon_{pa}(t) = \mathbf{e}_{pa}(t) + \gamma_1 \int_0^t \mathbf{e}_{pa}(\tau) d\tau$ and $\mathbf{e}_{pa}(t) = \mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t)$, let ϵ_i and $e_i(t)$ denote the i th entry of $\epsilon_{pa}(t)$ and $\mathbf{e}_{pa}(t)$, respectively. A Lyapunov function candidate is defined as $\xi(t) = \epsilon_i^2(t)/2$ and its corresponding derivative is

$$\dot{\xi}(t) = -\gamma_2 \epsilon_i^2(t) \quad (29)$$

which falls into one of the following two situations.

1. $\epsilon_i(t) = 0$. In this case, $\epsilon_i(t) = e_i(t) + \gamma_1 \int_0^t e_i(\tau) d\tau = 0$. By differentiating directly, one has

$$\dot{e}_i(t) = -\gamma_1 e_i(t). \quad (30)$$

According to the proof of Theorem 1, the end-effector trajectory $\mathbf{r}_{ac}(t)$ converges to the desired path $\mathbf{r}_{de}(t)$ as time t goes.

2. $\epsilon_i(t) \neq 0$. In this case, $\dot{\xi}(t) < 0$ and $\xi(t)$ will decline gradually down to zero, which implies that $\epsilon_i(t)$ converges to zero and falls into the first situation.

In addition, the convergence of the mobile plate orientation and the whole model (28) can be generalized from the proof of Theorem 1. Thus, the end-effector trajectory $\mathbf{r}_{ac}(t)$ and the mobile plate orientation $\mathbf{u}(t)$ of the RZND model (28) converge to the desired path $\mathbf{r}_{de}(t)$ and the direction of $\mathbf{v}(t)$, respectively. The proof is thus completed. $\square$

Similarly, if the RZND model (28) is perturbed by the disturbance, the corresponding polluted RZND model is formulated as

$$\begin{cases} \dot{\mathbf{r}}_{ac}(t) = \dot{\mathbf{r}}_{de}(t) + (\gamma_1 + \gamma_2)(\mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t)) \\ \quad + \gamma_1 \gamma_2 \int_0^t (\mathbf{r}_{de}(\tau) - \mathbf{r}_{ac}(\tau)) d\tau + \boldsymbol{\sigma}_p(t), & (31a) \\ J(\boldsymbol{\theta}_{ac}(t)) \dot{\boldsymbol{\theta}}_{ac}(t) = -(\gamma_1 + \gamma_2) \mathbf{e}_{or}(t) - \gamma_1 \gamma_2 \int_0^t \mathbf{e}_{or}(\tau) d\tau \\ \quad + (A(\boldsymbol{\theta}_{ac}(t))M)^T \dot{\mathbf{r}}_{ac}(t) + \boldsymbol{\sigma}_o(t), & (31b) \\ \dot{\mathbf{i}}(t) = B(t) \begin{bmatrix} \dot{\mathbf{r}}_{ac}(t) \\ \dot{\boldsymbol{\theta}}_{ac}(t) \end{bmatrix}. & (31c) \end{cases}$$

The performance of the interfered RZND model (31) under two kinds of disturbances is investigated by the following two theorems.

Theorem 3. Consider the position–orientation tracking of the Stewart platform oriented to an immobile point. Under unknown constant disturbances $\boldsymbol{\sigma}_p(t) = \boldsymbol{\zeta}_p \in \mathbb{R}^3$ and $\boldsymbol{\sigma}_o(t) = \boldsymbol{\zeta}_o \in \mathbb{R}^2$, with proper hyperparameters $\gamma_1, \gamma_2 \in \mathbb{R}^+$, and under the assumption that the kinematic mapping remains feasible and nonsingular, the tracking errors of the RZND model in (31) asymptotically converge to zero. Consequently, the end-effector trajectory $\mathbf{r}_{ac}(t)$ converges to the desired path $\mathbf{r}_{de}(t)$, and the mobile plate orientation $\mathbf{u}(t)$ aligns with the direction $\mathbf{v}(t)$.

Proof. By reformulating the position part of the interfered RZND model (31), one has

$$\dot{\mathbf{e}}_{pa}(t) = -(\gamma_1 + \gamma_2) \mathbf{e}_{pa}(t) - \gamma_1 \gamma_2 \int_0^t \mathbf{e}_{pa}(\tau) d\tau - \boldsymbol{\zeta}_p. \quad (32)$$

Let $e_{pa,i}(t)$ and $\zeta_{p,i}$ denote the i th entry of $\mathbf{e}_{pa}(t)$ and $\boldsymbol{\zeta}_p$, respectively. Using Laplace transformation [32] leads to

$$s \mathcal{L}[e_{pa,i}(t)] - e_{pa,i}(0) = -\frac{\zeta_{p,i}}{s} - \frac{\gamma_1 \gamma_2}{s} \mathcal{L}[e_{pa,i}(t)] - (\gamma_1 + \gamma_2) \mathcal{L}[e_{pa,i}(t)] \quad (33)$$

which is further reformulated as

$$\mathcal{L}[e_{\text{pa},i}(t)] = \frac{s(e_{\text{pa},i}(0) - \zeta_{\text{p},i}/s)}{s^2 + s(\gamma_1 + \gamma_2) + \gamma_1\gamma_2}. \quad (34)$$

Evidently, the transfer function is $s/(s^2 + s(\gamma_1 + \gamma_2) + \gamma_1\gamma_2)$, whose poles are

$$s_1 = \frac{-(\gamma_1 + \gamma_2) + |\gamma_1 - \gamma_2|}{2} \quad (35)$$

and

$$s_2 = \frac{-(\gamma_1 + \gamma_2) - |\gamma_1 - \gamma_2|}{2} \quad (36)$$

which both lay on the left half-plane for γ_1 and $\gamma_2 \in \mathbb{R}^+$. According to the final value theorem [32], one has

$$\begin{aligned} \lim_{t \rightarrow \infty} e_{\text{pa},i}(t) &= \lim_{s \rightarrow 0} s \mathcal{L}[e_{\text{pa},i}(t)] \\ &= \lim_{s \rightarrow 0} \frac{s^2 e_{\text{pa},i}(0) - s \zeta_{\text{p},i}}{s^2 + s(\gamma_1 + \gamma_2) + \gamma_1\gamma_2} \\ &= 0. \end{aligned} \quad (37)$$

Therefore,

$$\lim_{t \rightarrow \infty} (\mathbf{r}_{\text{de}}(t) - \mathbf{r}_{\text{ac}}(t)) = \lim_{t \rightarrow \infty} \mathbf{e}_{\text{pa}}(t) = \mathbf{0}. \quad (38)$$

Hence, under the influence of an unknown constant disturbance $\sigma_{\text{p}}(t) = \zeta_{\text{p}} \in \mathbb{R}^3$, as time t goes, the end-effector trajectory $\mathbf{r}_{\text{ac}}(t)$ of the interfered RZND model (31) converges to the desired path $\mathbf{r}_{\text{de}}(t)$.

In addition, the orientation error is defined as

$$\mathbf{e}_{\text{or}}(t) = (A(\boldsymbol{\theta}_{\text{ac}}(t))M)^T \mathbf{v}(t), \quad (39)$$

where

$$\mathbf{v}(t) = \mathbf{d} - \mathbf{r}_{\text{ac}}(t). \quad (40)$$

Differentiating the orientation error gives

$$\dot{\mathbf{e}}_{\text{or}}(t) = J(\boldsymbol{\theta}_{\text{ac}}(t))\dot{\boldsymbol{\theta}}_{\text{ac}}(t) - (A(\boldsymbol{\theta}_{\text{ac}}(t))M)^T \dot{\mathbf{r}}_{\text{ac}}(t). \quad (41)$$

According to the orientation component of the interfered RZND model, one has

$$J(\boldsymbol{\theta}_{\text{ac}}(t))\dot{\boldsymbol{\theta}}_{\text{ac}}(t) = -(\gamma_1 + \gamma_2)\mathbf{e}_{\text{or}}(t) - \gamma_1\gamma_2 \int_0^t \mathbf{e}_{\text{or}}(\tau) d\tau + (A(\boldsymbol{\theta}_{\text{ac}}(t))M)^T \dot{\mathbf{r}}_{\text{ac}}(t) + \sigma_{\text{o}}(t). \quad (42)$$

Substituting the above equation into the derivative of the orientation error yields

$$\dot{\mathbf{e}}_{\text{or}}(t) = -(\gamma_1 + \gamma_2)\mathbf{e}_{\text{or}}(t) - \gamma_1\gamma_2 \int_0^t \mathbf{e}_{\text{or}}(\tau) d\tau + \sigma_{\text{o}}(t). \quad (43)$$

Let $e_{\text{or},i}(t)$ and $\sigma_{\text{o},i}(t)$ denote the i th components of $\mathbf{e}_{\text{or}}(t)$ and $\sigma_{\text{o}}(t)$, respectively, where $i \in \{1, 2\}$. Each scalar orientation-error component satisfies

$$\dot{e}_{\text{or},i}(t) = -(\gamma_1 + \gamma_2)e_{\text{or},i}(t) - \gamma_1\gamma_2 \int_0^t e_{\text{or},i}(\tau) d\tau + \sigma_{\text{o},i}(t). \quad (44)$$

Taking the Laplace transform on both sides gives

$$s\mathcal{L}[e_{\text{or},i}(t)] - e_{\text{or},i}(0) = -(\gamma_1 + \gamma_2)\mathcal{L}[e_{\text{or},i}(t)] - \frac{\gamma_1\gamma_2}{s}\mathcal{L}[e_{\text{or},i}(t)] + \mathcal{L}[\sigma_{\text{o},i}(t)]. \quad (45)$$

Consequently,

$$\mathcal{L}[e_{\text{or},i}(t)] = \frac{se_{\text{or},i}(0) + s\mathcal{L}[\sigma_{\text{o},i}(t)]}{s^2 + (\gamma_1 + \gamma_2)s + \gamma_1\gamma_2}. \quad (46)$$

The denominator satisfies

$$s^2 + (\gamma_1 + \gamma_2)s + \gamma_1\gamma_2 = (s + \gamma_1)(s + \gamma_2), \quad (47)$$

which is stable for $\gamma_1 > 0$ and $\gamma_2 > 0$.

For the unknown constant orientation noise

$$\sigma_{\text{o}}(t) = \zeta_{\text{o}}, \quad (48)$$

the i th component satisfies

$$\sigma_{\text{o},i}(t) = \zeta_{\text{o},i}, \quad \mathcal{L}[\sigma_{\text{o},i}(t)] = \frac{\zeta_{\text{o},i}}{s}. \quad (49)$$

Therefore,

$$\mathcal{L}[e_{\text{or},i}(t)] = \frac{se_{\text{or},i}(0) + \zeta_{\text{o},i}}{(s + \gamma_1)(s + \gamma_2)}. \quad (50)$$

According to the final value theorem,

$$\begin{aligned} \lim_{t \rightarrow \infty} e_{\text{or},i}(t) &= \lim_{s \rightarrow 0} s\mathcal{L}[e_{\text{or},i}(t)] \\ &= \lim_{s \rightarrow 0} \frac{s^2 e_{\text{or},i}(0) + s\zeta_{\text{o},i}}{(s + \gamma_1)(s + \gamma_2)} \\ &= 0. \end{aligned} \quad (51)$$

Since the above result holds for $i = 1, 2$, it follows that

$$\lim_{t \rightarrow \infty} \mathbf{e}_{\text{or}}(t) = \mathbf{0}. \quad (52)$$

The mobile plate orientation $\mathbf{u}(t)$ of the interfered RZND model (31) converges to the direction of $\mathbf{v}(t)$. The proof is thus completed. $\square$

Theorem 4. Consider the position–orientation tracking of the Stewart platform oriented to an immobile point. Under unknown linear time-varying disturbances $\sigma_{\text{p}}(t) = \zeta_{\text{p}}t \in \mathbb{R}^3$ and $\sigma_{\text{o}}(t) = \zeta_{\text{o}}t \in \mathbb{R}^2$, with proper hyperparameters $\gamma_1, \gamma_2 \in \mathbb{R}^+$, as time t goes, the end-effector trajectory $\mathbf{r}_{\text{ac}}(t)$ of the interfered RZND model (31) converges to the desired path $\mathbf{r}_{\text{de}}(t)$ with an error $\lim_{t \rightarrow \infty} \|\mathbf{e}_{\text{pa}}(t)\|_2 = \|\zeta_{\text{p}}\|_2 / (\gamma_1\gamma_2)$. The mobile plate orientation $\mathbf{u}(t)$ of the interfered RZND model (31) converges to the direction of $\mathbf{v}(t)$ with an error $\lim_{t \rightarrow \infty} \|\mathbf{e}_{\text{or}}(t)\|_2 = \|\zeta_{\text{o}}\|_2 / (\gamma_1\gamma_2)$.

Proof. Similar to the proof of Theorem 3, one has the equation after Laplace transformation:

$$s\mathcal{L}[e_{\text{pa},i}(t)] - e_{\text{pa},i}(0) = -\frac{\zeta_{\text{p},i}}{s^2} - \frac{\gamma_1\gamma_2}{s}\mathcal{L}[e_{\text{pa},i}(t)] - (\gamma_1 + \gamma_2)\mathcal{L}[e_{\text{pa},i}(t)] \quad (53)$$

which is further reformulated as

$$\mathcal{L}[e_{pa,i}(t)] = \frac{s(e_{pa,i}(0) - \zeta_{p,i}/s^2)}{s^2 + s(\gamma_1 + \gamma_2) + \gamma_1\gamma_2}. \quad (54)$$

According to the final value theorem [32], one has

$$\begin{aligned} \lim_{t \rightarrow \infty} e_{pa,i}(t) &= \lim_{s \rightarrow 0} s \mathcal{L}[e_{pa,i}(t)] \\ &= \lim_{s \rightarrow 0} \frac{s^2 e_i(0) - \zeta_{p,i}}{s^2 + s(\gamma_1 + \gamma_2) + \gamma_1\gamma_2} \\ &= -\frac{\zeta_{p,i}}{\gamma_1\gamma_2}. \end{aligned} \quad (55)$$

Therefore,

$$\lim_{t \rightarrow \infty} \|\mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t)\|_2 = \lim_{t \rightarrow \infty} \|\mathbf{e}_{pa}(t)\|_2 = \frac{\|\zeta_p\|_2}{\gamma_1\gamma_2}. \quad (56)$$

Hence, under the influence of an unknown linear time-varying disturbance $\sigma_p(t) = \zeta_p t \in \mathbb{R}^3$, as time t goes, the end-effector trajectory $\mathbf{r}_{ac}(t)$ of the interfered RZND model (31) converges to the desired path $\mathbf{r}_{de}(t)$ with an error $\lim_{t \rightarrow \infty} \|\mathbf{e}_{pa}(t)\|_2 = \|\zeta_p\|_2/(\gamma_1\gamma_2)$. Similar to the proof of Theorem 3, one has

$$\mathcal{L}[e_{or,i}(t)] = \frac{s e_{or,i}(0) + s \mathcal{L}[\sigma_{o,i}(t)]}{s^2 + (\gamma_1 + \gamma_2)s + \gamma_1\gamma_2}. \quad (57)$$

For the unknown linear time-varying orientation noise

$$\sigma_o(t) = \zeta_o t, \quad (58)$$

the i th component satisfies

$$\sigma_{o,i}(t) = \zeta_{o,i} t, \quad \mathcal{L}[\sigma_{o,i}(t)] = \frac{\zeta_{o,i}}{s^2}. \quad (59)$$

It follows that

$$\mathcal{L}[e_{or,i}(t)] = \frac{s e_{or,i}(0) + \zeta_{o,i}/s}{(s + \gamma_1)(s + \gamma_2)}. \quad (60)$$

Using the final value theorem gives

$$\begin{aligned} \lim_{t \rightarrow \infty} e_{or,i}(t) &= \lim_{s \rightarrow 0} s \mathcal{L}[e_{or,i}(t)] \\ &= \lim_{s \rightarrow 0} \frac{s^2 e_{or,i}(0) + \zeta_{o,i}}{(s + \gamma_1)(s + \gamma_2)} \\ &= \frac{\zeta_{o,i}}{\gamma_1\gamma_2}. \end{aligned} \quad (61)$$

Since the above result also holds for $i = 1, 2$, one obtains

$$\lim_{t \rightarrow \infty} \|\mathbf{e}_{or}(t)\|_2 = \frac{\|\zeta_o\|_2}{\gamma_1\gamma_2}. \quad (62)$$

The mobile plate orientation $\mathbf{u}(t)$ of the interfered RZND model (31) converges to the direction of $\mathbf{v}(t)$ with an error $\lim_{t \rightarrow \infty} \|\mathbf{e}_{or}(t)\|_2 = \|\zeta_o\|_2/(\gamma_1\gamma_2)$. The proof is thus completed. $\square$

Theorems 3 and 4 reveal different disturbance-rejection properties of the RZND model for the two considered types of additive disturbances. Under unknown constant additive

disturbances, both the position and orientation errors asymptotically converge to zero, corresponding to exact asymptotic disturbance rejection. In contrast, under unknown linear time-varying additive disturbances with nonzero slopes, the position- and orientation-error norms converge to bounded but generally nonzero steady-state values. Therefore, the RZND model suppresses the considered linear time-varying disturbances, and the corresponding steady-state error bounds are inversely proportional to $\gamma_1\gamma_2$.

4. Experimental Verifications

In this section, the discrete-time algorithms of the ZND model (14) and the RZND model (28) are first developed for the convenience of implementation. The discrete-time algorithms of GND model (78) and the AGND model (79) are also developed. Then four groups of computer simulations are conducted to verify the efficacy of the ZND model (14), the RZND model (28), the GND model (78), and the AGND model (79), which are implemented by the corresponding discrete-time algorithms.

In the simulations, the task is to track a triple-ring path of which the starting position and the ending position coincide, with the mobile plate always oriented to the immobile point $[0, 0, 6]^T$. The initial position and Euler angles are $[0, 0, 2]^T$ and $[0, 0, 0]^T$, respectively. The task duration is set as $T = 8$ s and the sampling gap is set as $g = 0.01$ s. Indicators $\|\mathbf{e}_{pa}\|_2$ and $\|\mathbf{e}_{or}\|_2$ are selected to evaluate the performance.

4.1. Discrete-Time Algorithms

For convenience, the Euler method is adopted to discretize the ZND model (14), the RZND model (28), and the GND model (78). Let g denote the sampling gap, and T denote the task duration. The discrete-time algorithms of the ZND model (14) and RZND model (28) are presented in Algorithms 1 and 2, respectively. The integrator term of the RZND model is approximated by the left Riemann sum and one has

$$\begin{aligned}\dot{\mathbf{r}}_{ac}(t) &= \dot{\mathbf{r}}_{de}(t) + (\gamma_1 + \gamma_2)(\mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t)) \\ &\quad + \gamma_1\gamma_2 \sum_{j=0}^{t/g-1} (\mathbf{r}_{de}(jg) - \mathbf{r}_{ac}(jg))g,\end{aligned}\quad (63)$$

where $t/g \in \mathbb{N}$ and the summation term is set to $\mathbf{0}$ if $t = 0$. Notably, other algorithms are omitted as they can be obtained readily by replacing steps 5, 6, and 7 by corresponding formulas.

Algorithm 1: Discrete-time ZND Algorithm for Stewart Platform

1. **Initialize:** Physical modeling parameters of Stewart platform such as $\mathbf{f}_i$ and $\mathbf{m}'_i, i = 1, 2, \dots, 6$
 2. **Input:** Immobile point $\mathbf{d}$, desired path $\mathbf{r}_{de}(t)$, sampling gap g , and task duration T
 3. **For** $k = 0 : T/g$
 4. **Read:** Actual position $\mathbf{r}_{ac,k}$ and posture $\theta_{ac,k}$
 5. **Compute:** $\dot{\mathbf{r}}_{ac,k} = \dot{\mathbf{r}}_{ac}(kg)$ via (14a)
 6. **Compute:** $\dot{\theta}_{ac,k} = \dot{\theta}_{ac}(kg)$ via (14b)
 7. **Compute:** $\dot{\mathbf{l}}_k = \dot{\mathbf{l}}(kg)$ via (14c)
 8. **Compute:** $\mathbf{l}_{k+1} = \mathbf{l}_k + g\dot{\mathbf{l}}_k$
 9. **End**
-

Algorithm 2: Discrete-time RZND Algorithm for Stewart Platform

1. **Initialize:** Physical modeling parameters of Stewart platform such as $\mathbf{f}_i$ and $\mathbf{m}'_i$, $i = 1, 2, \dots, 6$
2. **Input:** Immobile point $\mathbf{d}$, desired path $\mathbf{r}_{\text{de}}(t)$, sampling gap g , and task duration T
3. **For** $k = 0 : T/g$
4. **Read:** Actual position $\mathbf{r}_{\text{ac},k}$ and posture $\theta_{\text{ac},k}$
5. **Compute:** $\dot{\mathbf{r}}_{\text{ac},k} = \dot{\mathbf{r}}_{\text{ac}}(kg)$ via (28a)
6. **Compute:** $\dot{\theta}_{\text{ac},k} = \dot{\theta}_{\text{ac}}(kg)$ via (28b)
7. **Compute:** $\dot{\mathbf{l}}_k = \dot{\mathbf{l}}(kg)$ via (28c)
8. **Compute:** $\mathbf{l}_{k+1} = \mathbf{l}_k + g\dot{\mathbf{l}}_k$
9. **End**

For the ZND model (14), the position-error dynamics and orientation-error dynamics can be uniformly expressed as

$$\dot{\mathbf{e}}(t) = -\lambda \mathbf{e}(t), \quad (64)$$

where $\mathbf{e}(t) \in \mathbb{R}^n$, with $n = 3$ for the position component and $n = 2$ for the orientation component. The parameter λ represents λ_1 or λ_2 , respectively. Applying the forward Euler method with the sampling gap g gives

$$\mathbf{e}_{k+1} = \mathbf{e}_k + g\dot{\mathbf{e}}_k = (1 - g\lambda)\mathbf{e}_k. \quad (65)$$

Therefore, the discrete-time ZND error system is asymptotically stable if

$$|1 - g\lambda| < 1, \quad (66)$$

which is equivalent to

$$0 < g\lambda < 2. \quad (67)$$

Accordingly, the feasible conditions for the position and orientation components are

$$0 < g\lambda_1 < 2, \quad 0 < g\lambda_2 < 2. \quad (68)$$

For the RZND model (28), define the integral state as

$$\mathbf{z}(t) = \int_0^t \mathbf{e}(\tau) d\tau. \quad (69)$$

The corresponding vector error dynamics can be written as

$$\begin{cases} \dot{\mathbf{e}}(t) = -(\gamma_1 + \gamma_2)\mathbf{e}(t) - \gamma_1\gamma_2\mathbf{z}(t), \\ \dot{\mathbf{z}}(t) = \mathbf{e}(t). \end{cases} \quad (70)$$

Using the forward Euler method gives

$$\begin{bmatrix} \mathbf{e}_{k+1} \\ \mathbf{z}_{k+1} \end{bmatrix} = \Phi \begin{bmatrix} \mathbf{e}_k \\ \mathbf{z}_k \end{bmatrix}, \quad (71)$$

where

$$\Phi = \begin{bmatrix} [1 - g(\gamma_1 + \gamma_2)]I_n & -g\gamma_1\gamma_2 I_n \\ gI_n & I_n \end{bmatrix}. \quad (72)$$

The characteristic equation associated with each eigenvalue pair of Φ is

$$\mu^2 - [2 - g(\gamma_1 + \gamma_2)]\mu + (1 - g\gamma_1)(1 - g\gamma_2) = 0, \quad (73)$$

which can be factorized as

$$[\mu - (1 - g\gamma_1)][\mu - (1 - g\gamma_2)] = 0. \quad (74)$$

Therefore, the eigenvalues of Φ are

$$\mu_1 = 1 - g\gamma_1, \quad \mu_2 = 1 - g\gamma_2, \quad (75)$$

with each eigenvalue repeated n times. The Euler-discretized RZND error system is asymptotically stable if

$$|1 - g\gamma_1| < 1, \quad |1 - g\gamma_2| < 1, \quad (76)$$

or equivalently,

$$0 < g\gamma_1 < 2, \quad 0 < g\gamma_2 < 2. \quad (77)$$

The above results show that the sampling gap and gain parameters should be selected jointly. For a fixed sampling gap, excessively large gains may move the discrete-time eigenvalues outside the unit circle and consequently cause oscillation or instability. Conversely, excessively small gains generally lead to slow convergence. A smaller sampling gap enlarges the feasible gain range and reduces the Euler discretization error, but also increases the required computational frequency.

The sampling gap used throughout the simulations is $g = 0.01$ s. All gain–sampling-gap combinations adopted for the ZND and RZND algorithms satisfy the stability conditions in Equations (68) and (77).

4.2. Simulations Without Disturbance

In this subsection, the position–orientation tracking task is accomplished without disturbance. For the ZND algorithm and the GND algorithm, hyperparameters are set as $\lambda_1 = \lambda_2 = 100$. For the RZND algorithm, hyperparameters are set as $\gamma_1 = \gamma_2 = 50$.

The results synthesized by the ZND and the RZND algorithms are shown in Figures 2 and 3, respectively. As illustrated in the two figures, the performance of the ZND algorithm and the RZND algorithm without disturbance is very close. Specifically, the position errors synthesized by the ZND algorithm and the RZND algorithm are both at the magnitude of 10^{-4} m, while the position error synthesized by the RZND algorithm is marginally smaller than that of the ZND algorithm. As shown in Figures 2d and 3d, under zero disturbance the trajectories generated by both the ZND and RZND algorithms coincide exactly with the desired path, thereby underscoring their efficacy for the position–orientation tracking of the Stewart platform.

4.3. Simulations Under Constant Disturbance

In this subsection, the position–orientation tracking task is accomplished under the constant disturbance. The hyperparameters of the interfered ZND and RZND algorithms are set as $\lambda_1 = \lambda_2 = 100$ and $\gamma_1 = \gamma_2 = 50$, respectively. For the position component, the constant disturbance is set to $\sigma_p = [0.1, 0.1, 0.1]^T$, whereas for the orientation component, it is set to $\sigma_o = [10, 10]^T$.

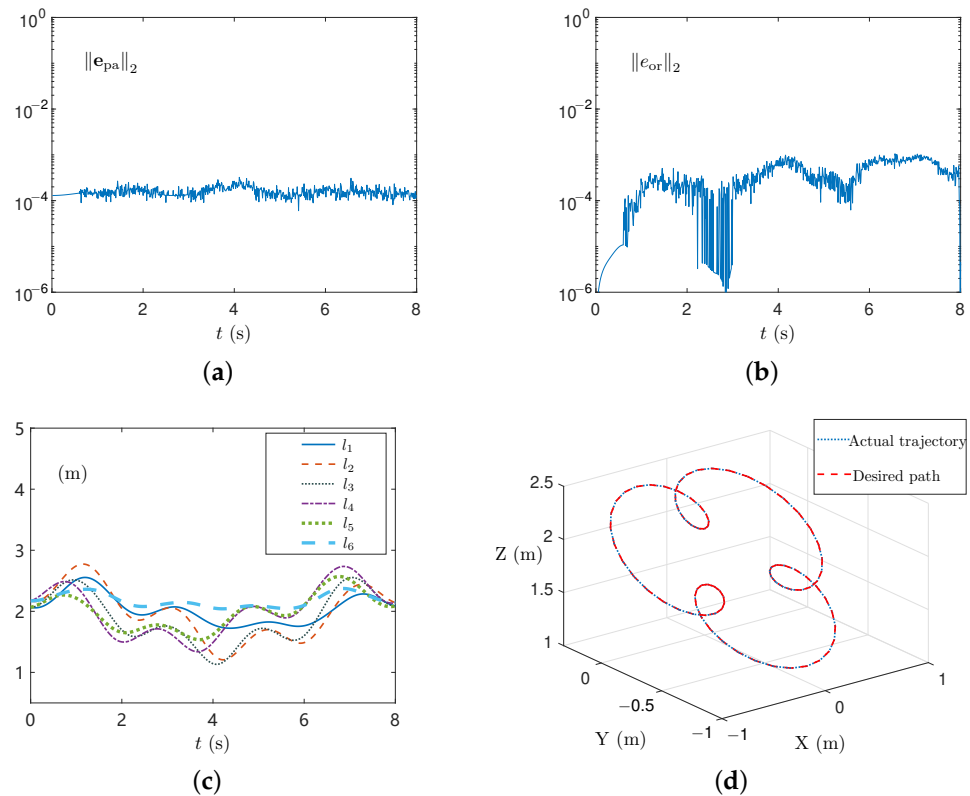


Figure 2. Results synthesized by ZND algorithm without disturbance. (a) Trajectory of position error. (b) Trajectory of orientation error. (c) Trajectories of extensible legs. (d) Actual trajectory and desired path.

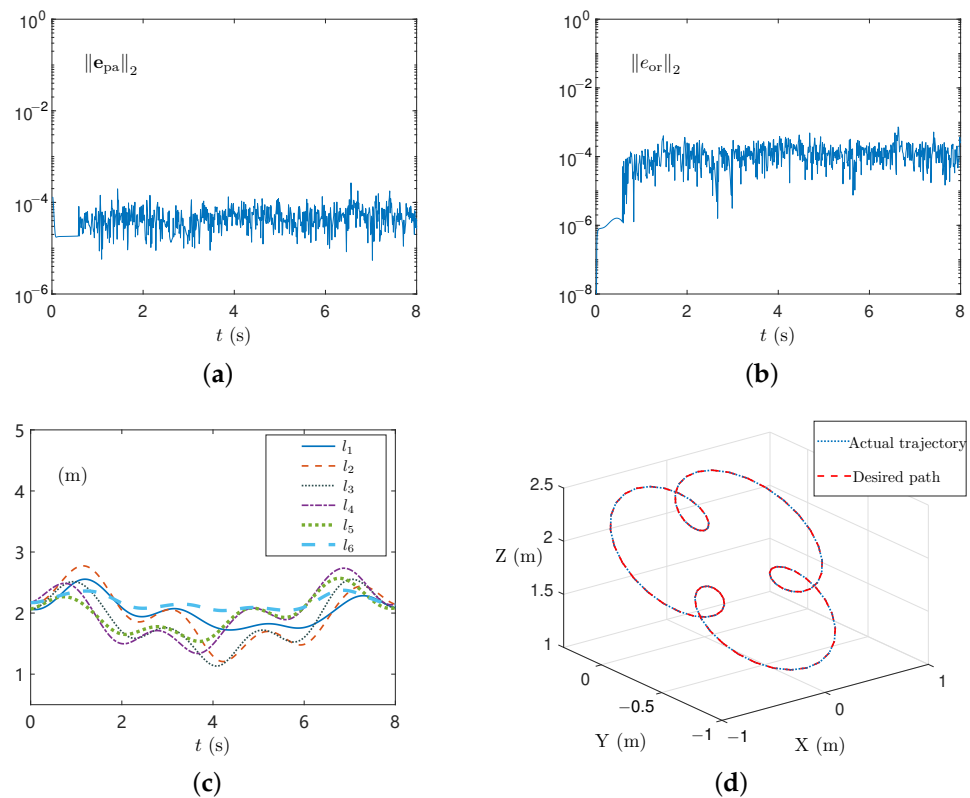


Figure 3. Results synthesized by RZND algorithm without disturbance. (a) Trajectory of position error. (b) Trajectory of orientation error. (c) Trajectories of extensible legs. (d) Actual trajectory and desired path.

The results synthesized by the interfered ZND the RZND algorithms are illustrated in Figures 4 and 5, respectively. As shown in Figure 4, the position and orientation errors synthesized by the interfered ZND algorithm are remarkably influenced by the disturbance, and the corresponding actual trajectory is presented in Figure 4d, which deviates from the desired path. In contrast, the results depicted in Figure 5 illustrate the RZND algorithm's strong robustness against constant disturbance. Compared with the zero disturbance case shown in Figure 3, the RZND algorithm maintains both position and orientation errors on the magnitude of 10^{-4} even under constant disturbance. Moreover, Figure 5d confirms that the tracking trajectory produced by the RZND algorithm still coincides perfectly with the desired path under such disturbance.

To highlight the impact of orientation error on the studied task, Figures 6 and 7 respectively illustrate the motion of the Stewart platform during position–orientation tracking under the ZND and RZND algorithms. In Figure 6, the platform normal vector $\mathbf{u}(t)$, depicted by the black arrow, visibly deviates from the dashed line connecting the platform center to the immobile point, indicating that the orientation of the platform has drifted from the desired value. In contrast, Figure 7 shows that under the RZN algorithm the normal vector $\mathbf{u}(t)$ remains perfectly aligned with the dashed line throughout the entire tracking process, fully satisfying the task requirements.

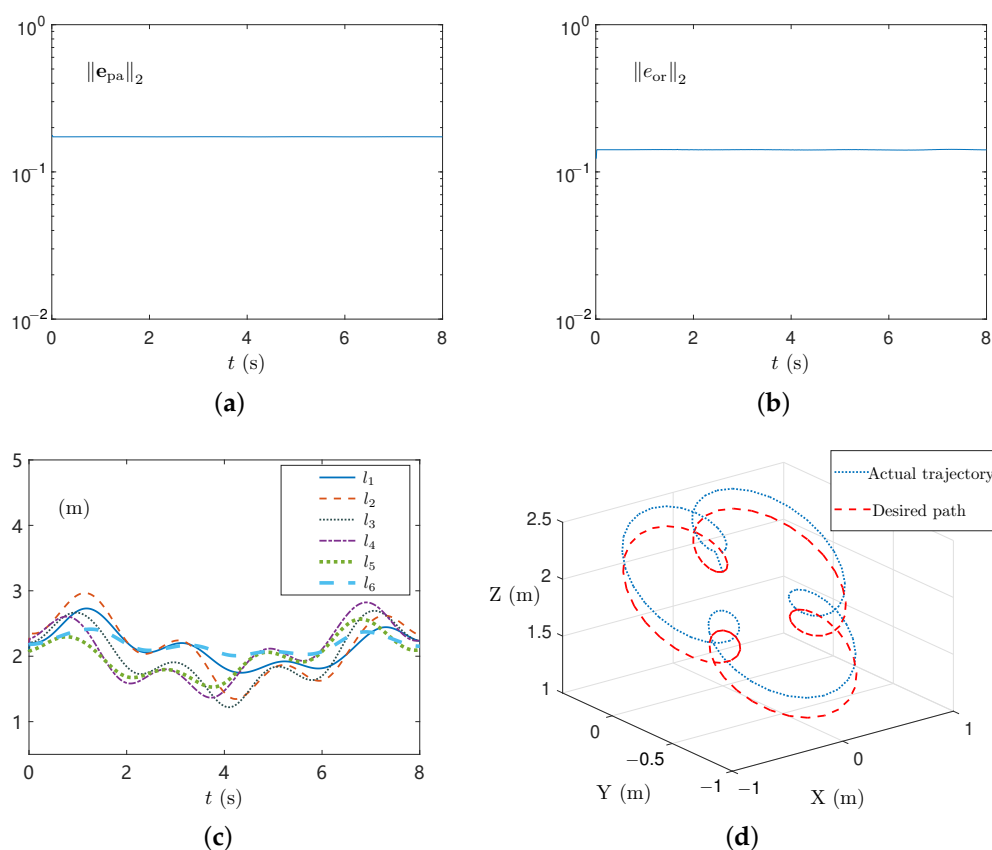


Figure 4. Results synthesized by ZND algorithm with constant disturbance. (a) Trajectory of position error. (b) Trajectory of orientation error. (c) Trajectories of extensible legs. (d) Actual trajectory and desired path.

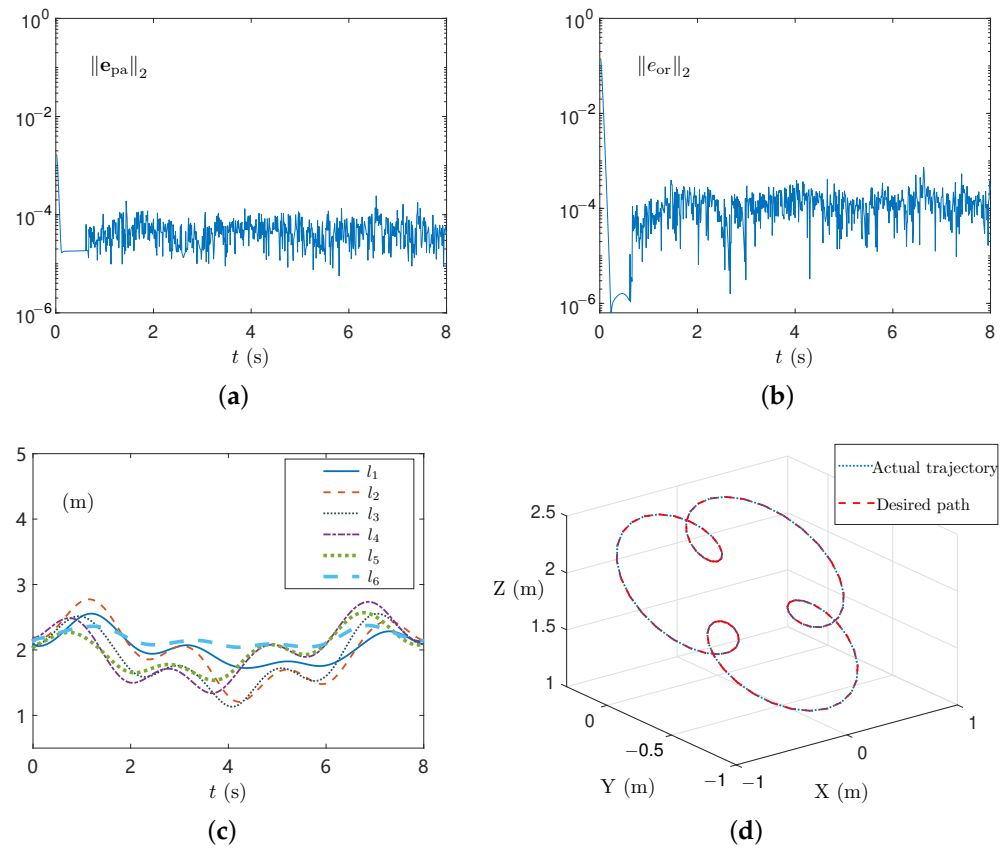


Figure 5. Results synthesized by RZND algorithm with constant disturbance. (a) Trajectory of position error. (b) Trajectory of orientation error. (c) Trajectories of extensible legs. (d) Actual trajectory and desired path.

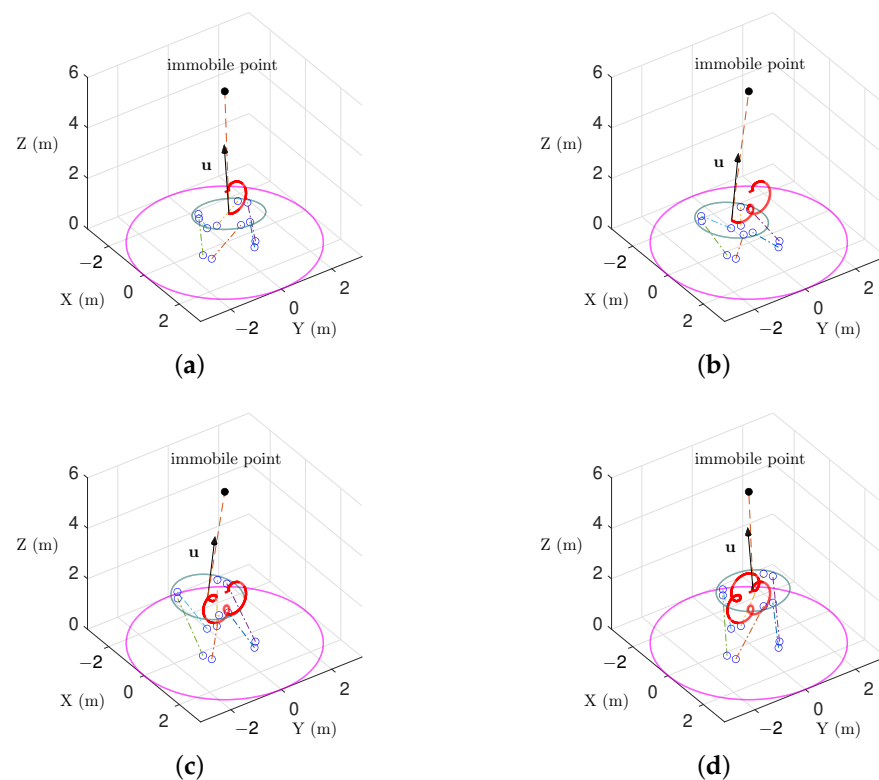


Figure 6. Tracking procedure of Stewart platform synthesized by ZND algorithm with constant disturbance. (a) $t = 2$ s. (b) $t = 4$ s. (c) $t = 6$ s. (d) $t = 8$ s.

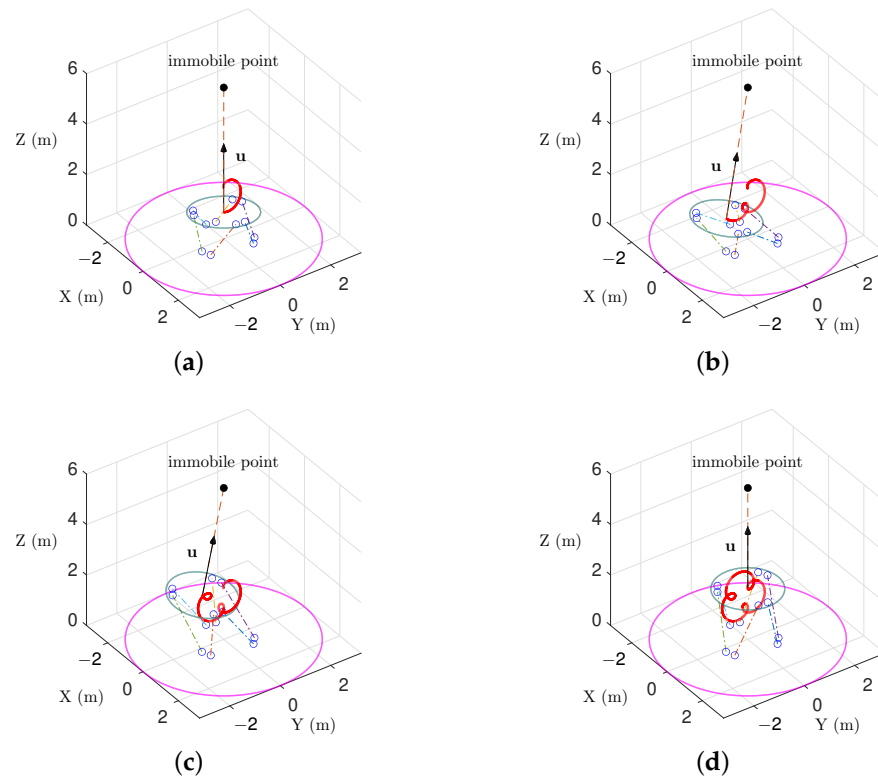


Figure 7. Tracking procedure of Stewart platform synthesized by RZND algorithm with constant disturbance. (a) $t = 2$ s. (b) $t = 4$ s. (c) $t = 6$ s. (d) $t = 8$ s.

Remark 1. In the experiments, Figures 6 and 7 are provided to give a clear, visual verification of how excessive orientation error compromises the tracking task. Under zero disturbance, the platform-motion sequences produced by the ZND and RZND algorithms are virtually identical and therefore are omitted. Likewise, under time-varying linear disturbance, the difference between the corresponding motion sequences mirrors that shown in Figures 6 and 7, so these figures are also omitted for brevity.

4.4. Simulations Under Linear Time-Varying Disturbance

In this subsection, the position–orientation tracking task is accomplished under the linear time-varying interfered. The hyperparameters of the interfered ZND and RZND algorithms are set as $\lambda_1 = \lambda_2 = 100$ and $\gamma_1 = \gamma_2 = 50$, respectively. For the position component, the linear time-varying disturbance is set to $\sigma_p = [0.01t, 0.01t, 0.01t]^T$, whereas for the orientation component, it is set to $\sigma_o = [t, t]^T$.

The results synthesized by the interfered ZND and RZND algorithms are illustrated in Figures 8 and 9, respectively. As illustrated in Figure 8, when the ZND algorithm is subjected to a time-varying linear disturbance that grows with time, both its position and orientation errors increase progressively. The trajectory in Figure 8d visibly diverges from the desired path as the task advances. In contrast, Figure 9 reveals that the RZND algorithm continues to exert sufficient suppression of the time-varying linear disturbance, stabilizing the position and orientation errors at a low level without further escalation over time. Consequently, Figure 9d confirms that the RZND algorithm still tracks the desired path accurately under the influence of such disturbance.

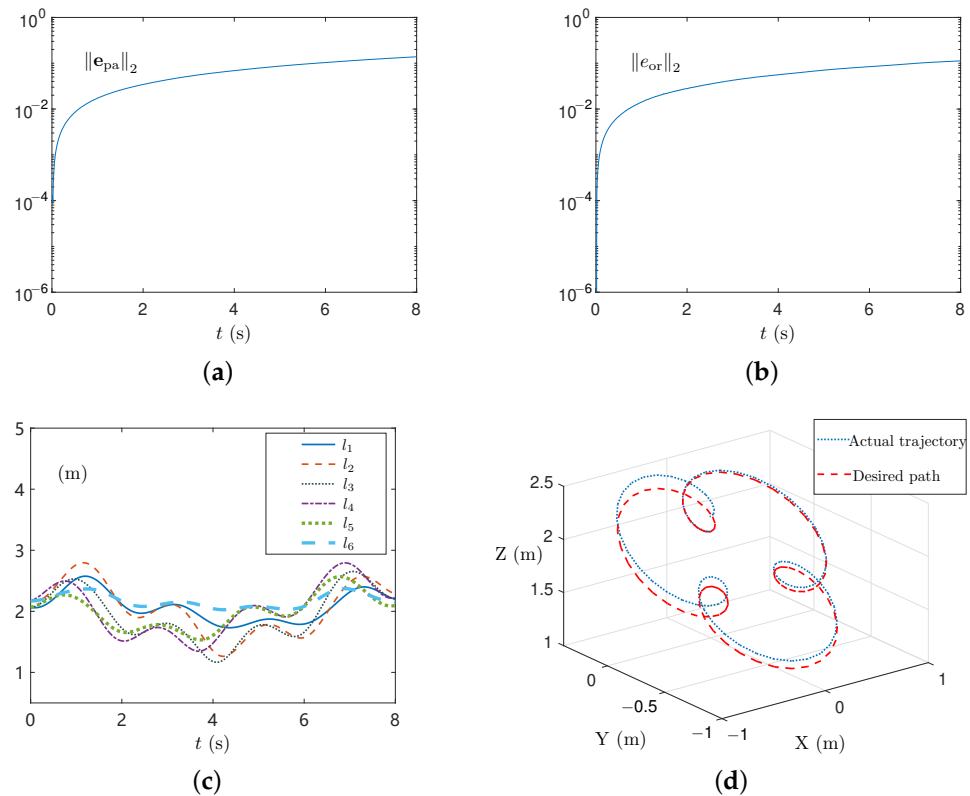


Figure 8. Results synthesized by ZND algorithm with time-varying linear disturbance. (a) Trajectory of position error. (b) Trajectory of orientation error. (c) Trajectories of extensible legs. (d) Actual trajectory and desired path.

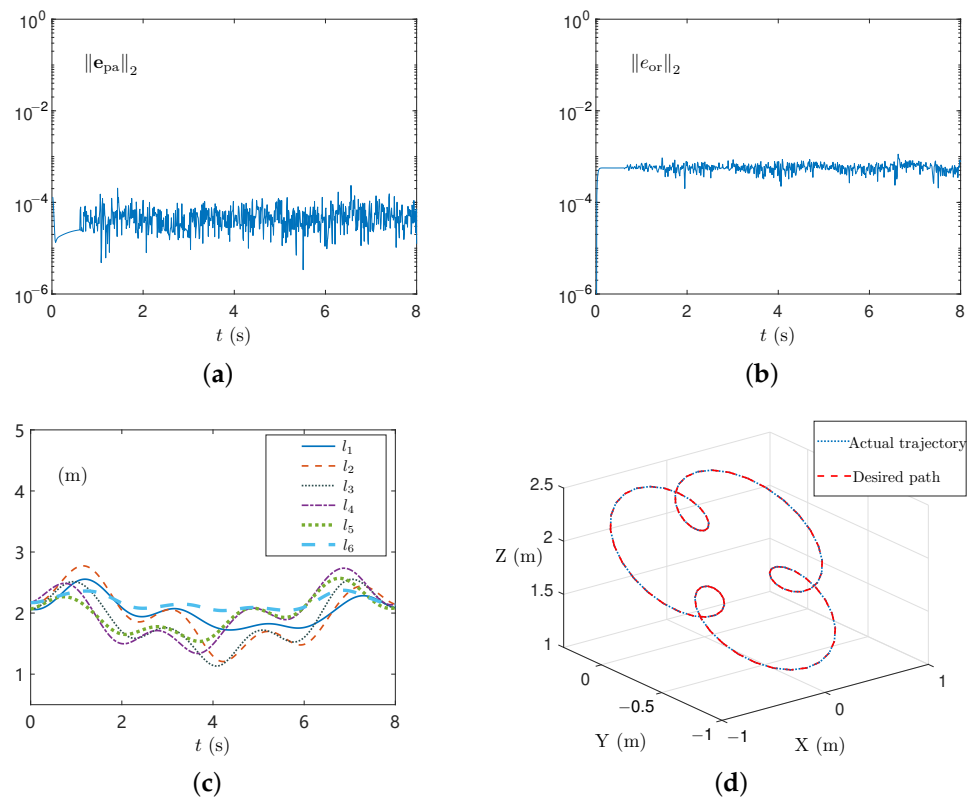


Figure 9. Results synthesized by RZND algorithm with time-varying linear disturbance. (a) Trajectory of position error. (b) Trajectory of orientation error. (c) Trajectories of extensible legs. (d) Actual trajectory and desired path.

Moreover, to investigate the capability of the RZND algorithm to suppress the linear time-varying disturbance, three groups of hyperparameters are chosen that satisfy $\gamma_1 + \gamma_2 = \lambda_1 = 100$. Group 1: $\gamma_1 = 10$, $\gamma_2 = 90$, and $\gamma_1\gamma_2 = 900$. Group 2: $\gamma_1 = 20$, $\gamma_2 = 80$, and $\gamma_1\gamma_2 = 1600$. Group 3: $\gamma_1 = 50$, $\gamma_2 = 50$, and $\gamma_1\gamma_2 = 2500$. The results of this comparative experiment are presented in Figure 10. It is evident that, while fixing the sum of the hyperparameters γ_1 and γ_2 , the capability of the RZND algorithm to restrain the linear time-varying disturbance depends on the production of the hyperparameters γ_1 and γ_2 , which accords precisely with the conclusion drawn in Theorem 4.

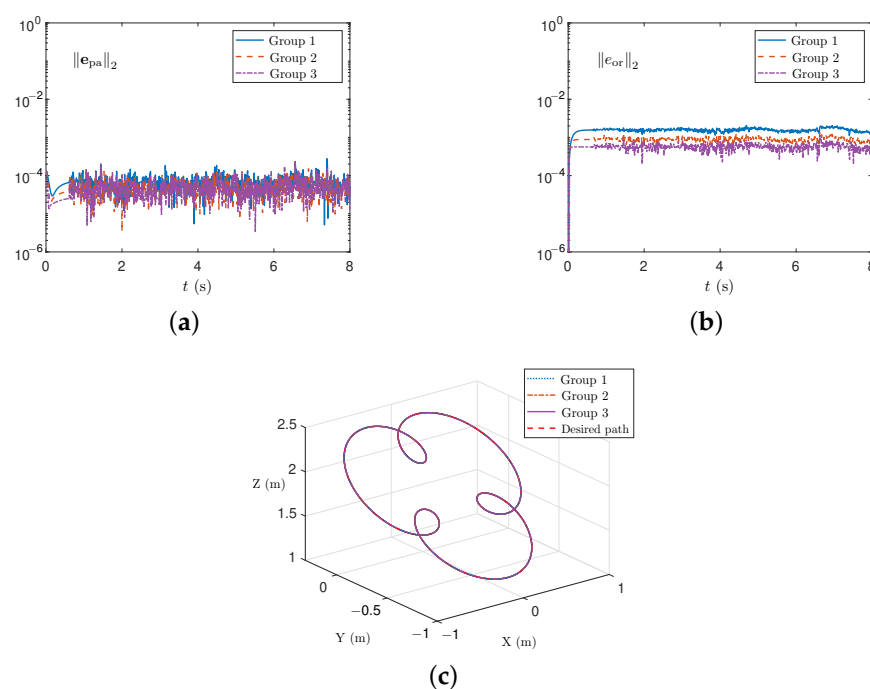


Figure 10. Results synthesized by RZND algorithm with time-varying linear disturbance and different groups of hyperparameters. (a) Trajectories of position error. (b) Trajectories of orientation error. (c) Actual trajectories and desired path.

4.5. Comparisons with Other Models

In this subsection, according to the GND method [23] and the adaptive GND (AGND) method [24], the GND and AGND models for position–orientation tracking of the Stewart platform are respectively presented as

$$\begin{cases} \dot{\mathbf{r}}_{ac}(t) = p_1(\mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t)), \end{cases} \quad (78a)$$

$$\begin{cases} \dot{\boldsymbol{\theta}}_{ac}(t) = -p_2 \frac{\partial \frac{1}{2} \|\mathbf{e}_{or}(t)\|_2^2}{\partial \boldsymbol{\theta}_{ac}(t)}, \end{cases} \quad (78b)$$

$$\begin{cases} \dot{\mathbf{i}}(t) = B(t) \begin{bmatrix} \dot{\mathbf{r}}_{ac}(t) \\ \dot{\boldsymbol{\theta}}_{ac}(t) \end{bmatrix}, \end{cases} \quad (78c)$$

and

$$\begin{cases} \dot{\mathbf{r}}_{ac}(t) = q_1(t)(\mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t)), \end{cases} \quad (79a)$$

$$\begin{cases} \dot{\boldsymbol{\theta}}_{ac}(t) = -q_2(t) \frac{\partial \frac{1}{2} \|\mathbf{e}_{or}(t)\|_2^2}{\partial \boldsymbol{\theta}_{ac}(t)}, \end{cases} \quad (79b)$$

$$\begin{cases} \dot{\mathbf{i}}(t) = B(t) \begin{bmatrix} \dot{\mathbf{r}}_{ac}(t) \\ \dot{\boldsymbol{\theta}}_{ac}(t) \end{bmatrix}, \end{cases} \quad (79c)$$

in which

$$q_1(t) = \frac{1.001|\mathbf{e}_{pa}^T(t)\dot{\mathbf{r}}_{de}(t)|}{\|\mathbf{e}_{pa}(t)\|_2}, \quad (80)$$

$$q_2(t) = \frac{1.1|\mathbf{e}_{or}^T(t)(A(\boldsymbol{\theta}_{ac}(t))M)^T\dot{\mathbf{r}}_{ac}(t)|}{\|J(\boldsymbol{\theta}_{ac}(t))\mathbf{e}_{or}(t)\|_2^2}. \quad (81)$$

Under the three noise-disturbance scenarios, the corresponding results synthesized by the ZND, RZND, GND, and AGND algorithms are presented in Figure 11, with the hyperparameters set to $\lambda_1 = \lambda_2 = 100$, $\gamma_1 = \gamma_2 = 50$, $p_1 = 100$, and $p_2 = 5$. The results illustrate that, in the absence of disturbance, both the ZND and RZND algorithms outperform the GND and AGND algorithms. When disturbances are present, the RZND algorithm consistently delivers the best performance among all four algorithms.

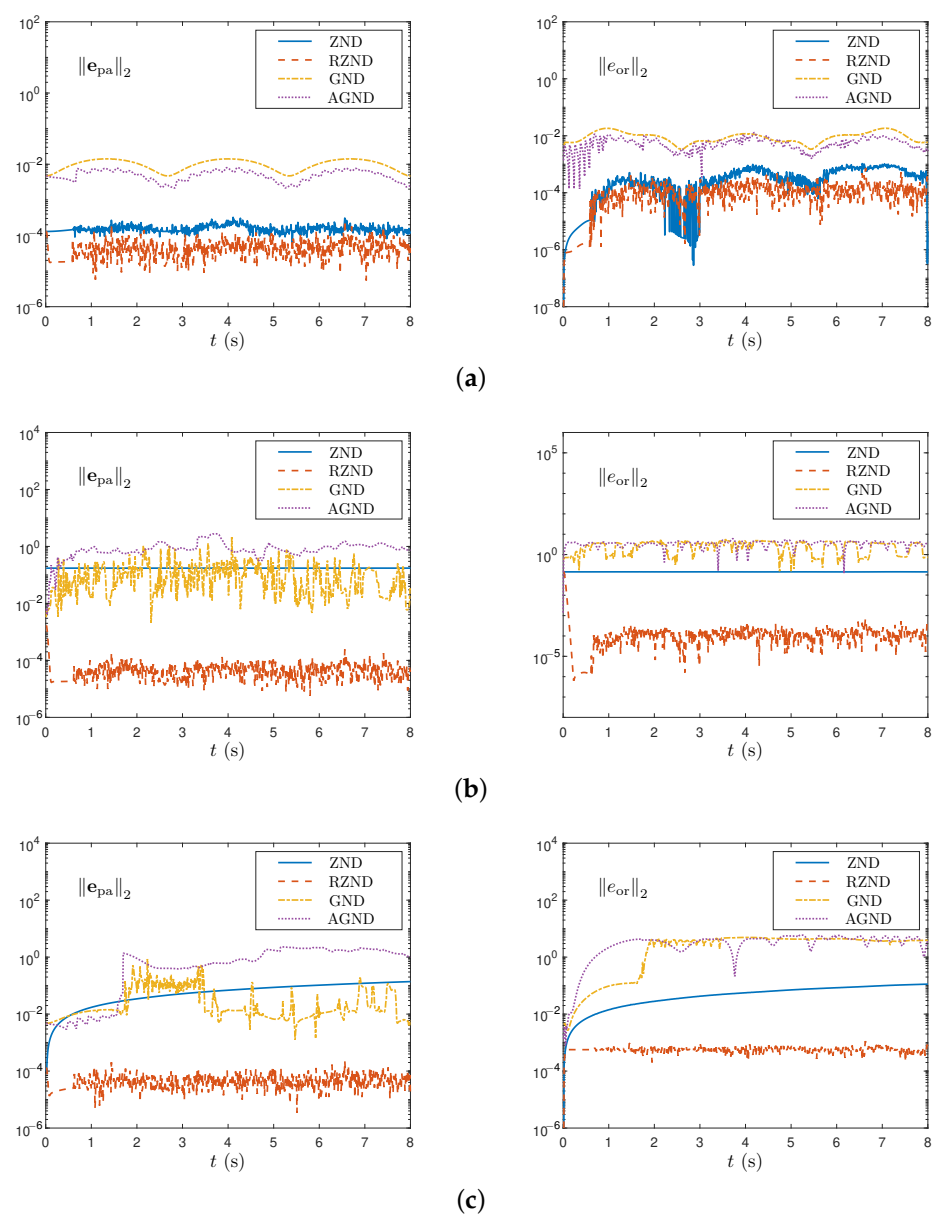


Figure 11. Results synthesized by different algorithms with different disturbance. (a) Trajectories of position errors and orientation errors without disturbance. (b) Trajectories of position errors and orientation errors with constant disturbance. (c) Trajectories of position errors and orientation errors with time-varying linear disturbance.

4.6. Influence of Error Function on RZND Model

In this subsection, we investigate the influence of the error function on the RZND model. For the orientation component of the Stewart platform, another novel scalar error function is introduced based on the vector-angle formula:

$$\tilde{e}_{or}(t) = \mathbf{u}^T(t)\mathbf{v}(t) - \|\mathbf{u}(t)\|_2\|\mathbf{v}(t)\|_2, \quad (82)$$

where the normal vector $\mathbf{u}(t)$ of the mobile plate can be taken directly as the vector $\mathbf{u}' = [0, 0, 1]^T$ in the coordinate system $\{O_2\}$, and its representation in coordinate system $\{O\}$ is given by $\mathbf{u}(t) = A(\boldsymbol{\theta}_{ac}(t))\mathbf{u}'$. According to the ZND design formula, i.e., $\dot{\tilde{e}}_{or}(t) = -\lambda_2\tilde{e}_{or}(t)$, one obtains

$$\begin{aligned} \mathbf{h}^T(t)\dot{\boldsymbol{\theta}}_{ac}(t) &= \mathbf{u}^T(t)\dot{\mathbf{r}}_{ac}(t) - \frac{\mathbf{v}^T(t)}{\|\mathbf{v}(t)\|_2}\dot{\mathbf{r}}_{ac}(t) - \lambda_2(\mathbf{u}^T(t)\mathbf{v}(t) \\ &\quad - \|\mathbf{u}(t)\|_2\|\mathbf{v}(t)\|_2), \end{aligned} \quad (83)$$

in which

$$\mathbf{h}(t) = \begin{bmatrix} \mathbf{u}^T \frac{\partial A^T(\boldsymbol{\theta}_{ac}(t))}{\partial \theta_x(t)} \mathbf{v}(t) \\ \mathbf{u}^T \frac{\partial A^T(\boldsymbol{\theta}_{ac}(t))}{\partial \theta_y(t)} \mathbf{v}(t) \\ \mathbf{u}^T \frac{\partial A^T(\boldsymbol{\theta}_{ac}(t))}{\partial \theta_z(t)} \mathbf{v}(t) \end{bmatrix}, \quad (84)$$

specifically, $\dot{\boldsymbol{\theta}}_{ac}(t)$ is set to $\mathbf{0}$ while $\mathbf{h}(t) = \mathbf{0}$.

The position component remains unchanged, and combining (6), (11), and (83) yields the ensuing ZND-2 model:

$$\begin{cases} \dot{\mathbf{r}}_{ac}(t) = \dot{\mathbf{r}}_{de}(t) + \lambda_1(\mathbf{r}_{de}(t) - \mathbf{r}_{ac}(t)), & (85a) \\ \mathbf{h}^T(t)\dot{\boldsymbol{\theta}}_{ac}(t) = \mathbf{u}^T(t)\dot{\mathbf{r}}_{ac}(t) - \frac{\mathbf{v}^T(t)}{\|\mathbf{v}(t)\|_2}\dot{\mathbf{r}}_{ac}(t) \\ \quad - \lambda_2(\mathbf{u}^T(t)\mathbf{v}(t) - \|\mathbf{u}(t)\|_2\|\mathbf{v}(t)\|_2), & (85b) \end{cases}$$

$$\dot{\mathbf{i}}(t) = B(t) \begin{bmatrix} \dot{\mathbf{r}}_{ac}(t) \\ \dot{\boldsymbol{\theta}}_{ac}(t) \end{bmatrix}. \quad (85c)$$

For the RZND model, the error function of the orientation component is defined as

$$\tilde{e}_{or}(t) = \tilde{e}_{or}(t) + \gamma_1 \int_0^t \tilde{e}_{or}(\tau) d\tau. \quad (86)$$

By applying the ZND design formula, i.e., $\dot{\tilde{e}}_{or}(t) = -\gamma_2\tilde{e}_{or}(t)$, one obtains

$$\begin{aligned} \mathbf{h}^T(t)\dot{\boldsymbol{\theta}}_{ac}(t) &= \mathbf{u}^T(t)\dot{\mathbf{r}}_{ac}(t) - \frac{\mathbf{v}^T(t)}{\|\mathbf{v}(t)\|_2}\dot{\mathbf{r}}_{ac}(t) - (\gamma_1 + \gamma_2)\tilde{e}_{or}(t) \\ &\quad - \gamma_1\gamma_2 \int_0^t \tilde{e}_{or}(\tau) d\tau. \end{aligned} \quad (87)$$

The position component remains unchanged, and combining (6), (25), and (87) yields the ensuing RZND-2 model:

$$\begin{cases} \dot{\mathbf{r}}_{\text{ac}}(t) = \dot{\mathbf{r}}_{\text{de}}(t) + (\gamma_1 + \gamma_2)\mathbf{e}_{\text{pa}}(t) + \gamma_1\gamma_2 \int_0^t \mathbf{e}_{\text{pa}}(\tau) d\tau, & (88a) \\ \mathbf{h}^T(t)\dot{\boldsymbol{\theta}}_{\text{ac}}(t) = \mathbf{u}^T(t)\dot{\mathbf{r}}_{\text{ac}}(t) - \frac{\mathbf{v}^T(t)}{\|\mathbf{v}(t)\|_2}\dot{\mathbf{r}}_{\text{ac}}(t) \\ \quad - (\gamma_1 + \gamma_2)\tilde{e}_{\text{or}}(t) - \gamma_1\gamma_2 \int_0^t \tilde{e}_{\text{or}}(\tau) d\tau, & (88b) \\ \dot{\mathbf{i}}(t) = B(t) \begin{bmatrix} \dot{\mathbf{r}}_{\text{ac}}(t) \\ \dot{\boldsymbol{\theta}}_{\text{ac}}(t) \end{bmatrix}. & (88c) \end{cases}$$

Subsequently, we discretize each model via the Euler method to obtain the corresponding algorithms, then evaluate the performance of the ZND-2 and RZND-2 algorithms under zero disturbance. For the ZND-2 algorithm, the hyperparameters are set to $\lambda_1 = \lambda_2 = 100$. For the RZND-2 algorithm, the hyperparameters are set to $\gamma_1 = 2$ and $\gamma_2 = 5$. The experimental results of ZND-2 are shown in Figure 12. Under the new error function of the orientation component (82), ZND-2 algorithm, like the original ZND algorithm, successfully accomplishes the position–orientation tracking of the Stewart platform in the absence of disturbance. The results for RZND-2 are given in Figure 13. It is observed from Figure 13a,b that both the position and orientation errors grow continuously as the task proceeds. Figure 13c shows that the extensible-leg trajectories exhibit pronounced oscillations during the middle and later stages of the motion from the middle to the latter part of the motion, and Figure 13d shows that the actual trajectory of the platform center diverges sharply from the desired path, underscoring the failure of the RZND-2 algorithm in this task. Furthermore, Figures 14 and 15 depict the tracking processes of the ZND-2 and RZND-2 algorithms, respectively. Figure 14 shows that, throughout the motion, the Stewart platform under ZND-2 consistently keeps its orientation directed toward the immobile point. Meanwhile, the results of Figure 12 confirms the effectiveness of the ZND-2 algorithm under zero disturbance, and the results illustrate the suitability of the error function (82). In contrast, Figure 15 reveals that the platform orientation under RZND-2 deviates dramatically from the immobile point during tracking. Coupled with the evidence in Figure 15, this failure of RZND-2 in a zero-disturbance environment is evident. It is worth noting that selecting larger hyper-parameters γ_1 and γ_2 only exacerbates the degradation, often driving the algorithm to instability or collapse. The reasons for the failure of the RZND-2 algorithm based on error functions (82) and (86) will be analyzed below.

For the error function (82), the angle formula between two vectors is presented as

$$\cos \alpha = \frac{\mathbf{u}^T(t)\mathbf{v}(t)}{\|\mathbf{u}(t)\|_2\|\mathbf{v}(t)\|_2}. \quad (89)$$

It is evident that, provided the immobile point stays above the platform, choosing the platform normal vector in the appropriate direction confines $\cos \alpha$ to the interval $(0, 1]$. Hence, the error function (82) is negative semi-definite. For the ZND model, this negative semi-definiteness does not impair the validity of the model derived from the ZND design formula. In the RZND case, however, the negative semi-definiteness of (82) causes the error function (86) built upon it to decrease monotonically in time. Consequently, the design equation $\dot{\tilde{e}}_{\text{or}}(t) = -\gamma_2\tilde{e}_{\text{or}}(t)$ becomes ineffective; the error accumulates relentlessly and drives the system to instability and eventual collapse. From the above analysis we conclude that, within the RZND framework, the underlying error function (such as (82) in this subsection) must not be (semi-) negative or (semi-) positive definite; otherwise, the RZND error function will monotonically increase in the positive direction or decrease in the negative direction, leading to system failure.

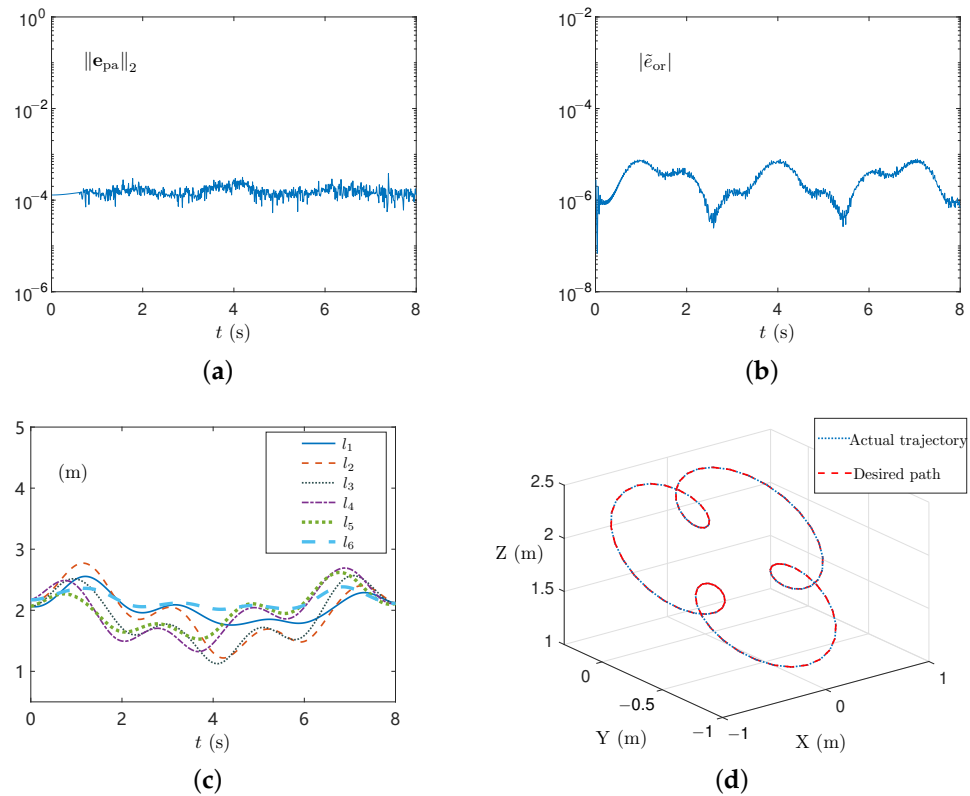


Figure 12. Results synthesized by ZND-2 algorithm without disturbance. (a) Trajectory of position error. (b) Trajectory of orientation error. (c) Trajectories of extensible legs. (d) Actual trajectory and desired path.

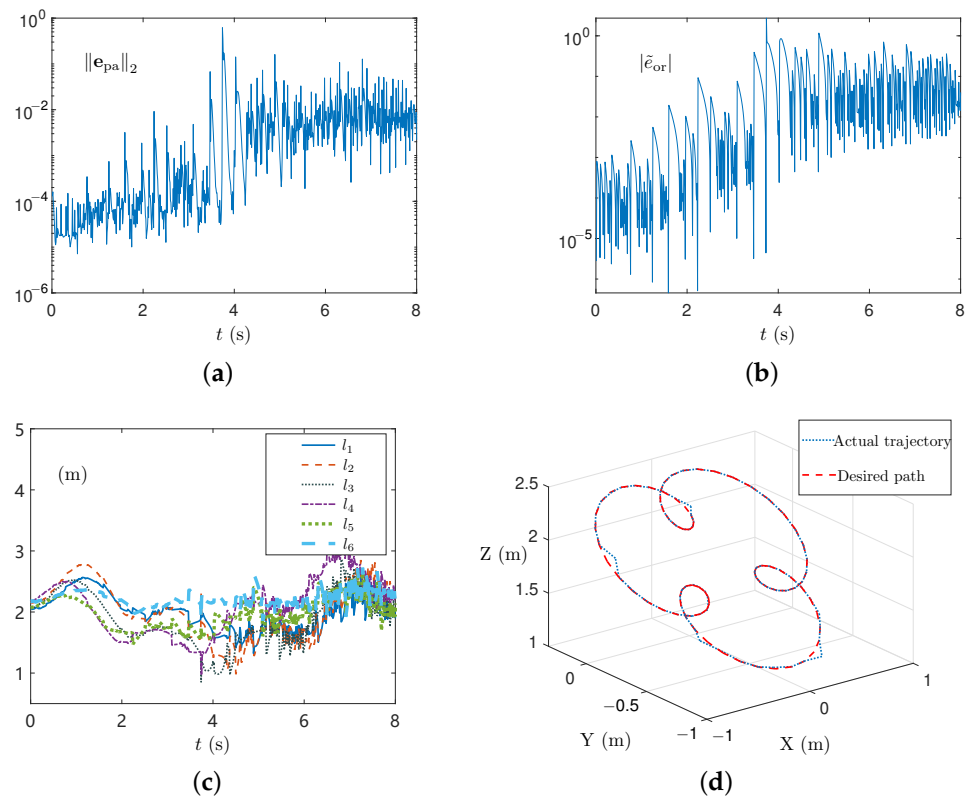


Figure 13. Results synthesized by RZND-2 algorithm without disturbance. (a) Trajectory of position error. (b) Trajectory of orientation error. (c) Trajectories of extensible legs. (d) Actual trajectory and desired path.

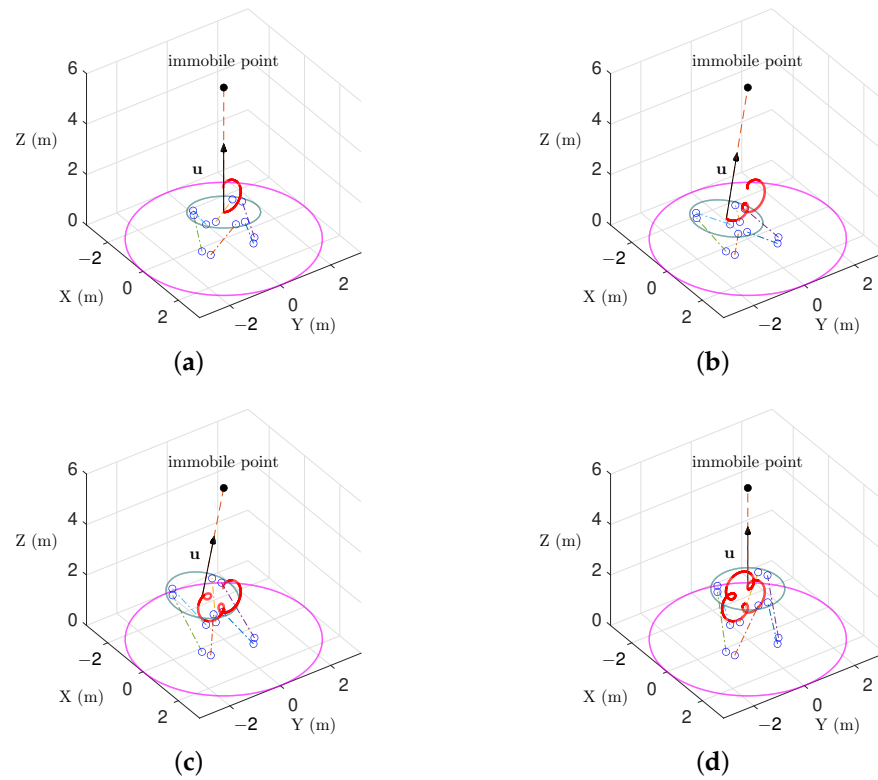


Figure 14. Tracking procedure of Stewart platform synthesized by ZND-2 algorithm without disturbance. (a) $t = 2$ s. (b) $t = 4$ s. (c) $t = 6$ s. (d) $t = 8$ s.

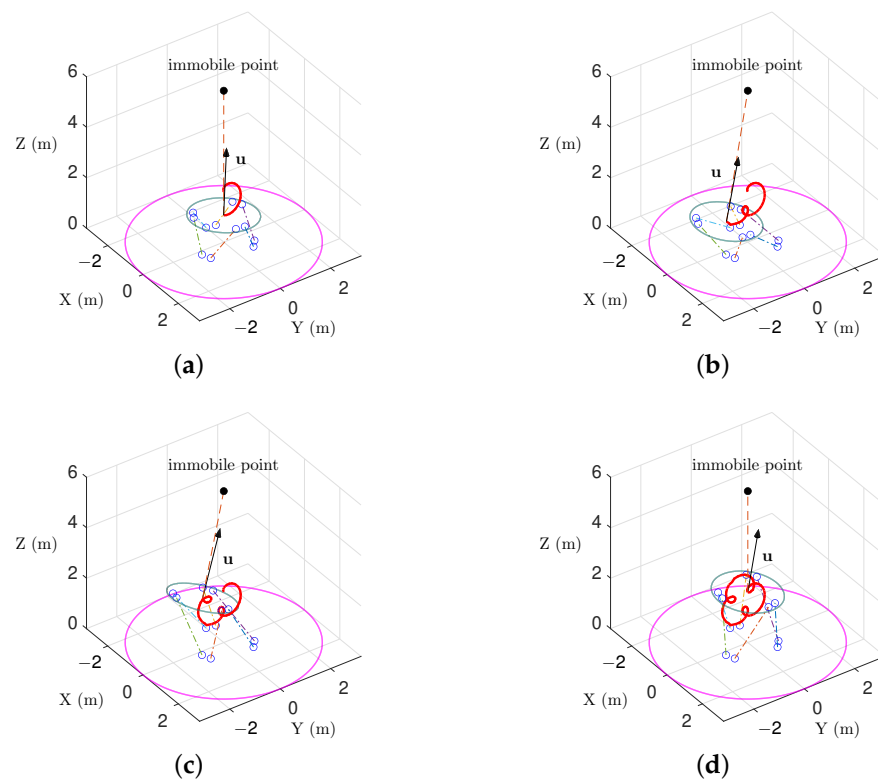


Figure 15. Tracking procedure of Stewart platform synthesized by RZND-2 algorithm without disturbance. (a) $t = 2$ s. (b) $t = 4$ s. (c) $t = 6$ s. (d) $t = 8$ s.

5. Conclusions

In this paper, the position–orientation tracking of the Stewart platform oriented to an immobile point has been investigated. Two new models, i.e., the ZND model and the RZND model, have been proposed to accomplish the position–orientation tracking task under two kinds of disturbances, and their convergence properties have also been studied using theoretical analyses. In addition, the GND and AGND models have been developed for comparison. Corresponding discrete-time algorithms of the four models have been developed for convenient implementation. According to groups of comparative simulations, the ZND and RZND algorithms perform better than the GND and AGN algorithms without disturbance, especially in the part of controlling the orientation of the mobile plate. Furthermore, it has been substantiated that the ZND algorithm is competent to accomplish the position–orientation tracking task without disturbance but fails under constant and linear time-varying disturbances. In addition, the RZND algorithm has been shown effective on disturbance-suppression. Finally, we have examined how the choice of error function affects the RZND scheme. Both experiments and theoretical discussion have confirmed that the underlying error function employed by RZND must not be (semi-) negative or (semi-) positive definite. Otherwise, the error function of the RZND will monotonically increase in the positive direction or decrease in the negative direction over time, inevitably driving the system to collapse. As future work, the proposed RZND model will be extended to more general disturbance conditions, including nonlinear, stochastic, and state-dependent disturbances. Experimental validation on a physical Stewart platform will also be conducted. In addition, learning- and optimization-based techniques may be incorporated to further improve the adaptability and practical applicability of the proposed model.

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