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Correction

Corrections: Kim, T.; et al. Some Identities for Euler and Bernoulli Polynomials and Their Zeros. *Axioms* 2018, *7*, 56

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1. Corrigendum

The authors, Kim and Ryoo in [1], studied Euler polynomials and Bernoulli polynomials with an extended variable to a complex variable, replacing real variable x by complex variable $x + iy$, and achieved several useful identities and properties.

The authors would like to note that these results can also be derived from a different approach by considering Euler polynomials and Bernoulli polynomials with a pair of two variables, as shown in [2], instead of a complex variable.

For example, Masjed-Jamei, Beyki and Koepf in [2] introduced the new type Euler polynomials given by

$$\begin{aligned} \frac{2e^{pt}}{e^t+1}\cos(qt) &= \sum_{n=0}^{\infty} E_n^{(c)}(p,q)\frac{t^n}{n!}, \\ \frac{2e^{pt}}{e^t+1}\sin(qt) &= \sum_{n=0}^{\infty} E_n^{(s)}(p,q)\frac{t^n}{n!}, \end{aligned} \tag{1}$$

which are considered without a complex variable.

On the other hand, the authors in [1] considered the Euler polynomials and Bernoulli polynomials with a complex variable instead of x variable as follows:

$$\frac{2}{e^t+1}e^{(x+iy)t} = \sum_{n=0}^{\infty} E_n(x+iy)\frac{t^n}{n!},$$

and

$$\frac{t}{e^t-1}e^{(x+iy)t} = \sum_{n=0}^{\infty} B_n(x+iy)\frac{t^n}{n!},$$

which imply the equivalence definitions to Equation (1) as

$$\begin{aligned} \frac{2}{e^t+1}e^{xt}\cos(yt) &= \sum_{n=0}^{\infty} \frac{E_n(x+iy)+E_n(x-iy)}{2}\frac{t^n}{n!} = \sum_{n=0}^{\infty} E_n^{(c)}(x,y)\frac{t^n}{n!}, \\ \frac{2}{e^t+1}e^{xt}\sin(yt) &= \sum_{n=0}^{\infty} \frac{E_n(x+iy)-E_n(x-iy)}{2i}\frac{t^n}{n!} = \sum_{n=0}^{\infty} E_n^{(s)}(x,y)\frac{t^n}{n!}. \end{aligned} \tag{2}$$

and

$$\frac{t}{e^t-1}e^{xt}\cos(yt)=\sum_{n=0}^{\infty}\frac{B_n(x+iy)+B_n(x-iy)}{2}\frac{t^n}{n!}=\sum_{n=0}^{\infty}B_n^{(c)}(x,y)\frac{t^n}{n!},$$
$$\frac{t}{e^t-1}e^{xt}\sin(yt)=\sum_{n=0}^{\infty}\frac{B_n(x+iy)-B_n(x-iy)}{2i}\frac{t^n}{n!}=\sum_{n=0}^{\infty}B_n^{(s)}(x,y)\frac{t^n}{n!}. \quad (3)$$

Here, the authors considered the Euler polynomials and Bernoulli polynomials of a complex variable, by treating the real and imaginary parts separately, which are able to introduce the cosine Euler polynomials, the sine Euler polynomials, the cosine Bernoulli polynomials, and the sine Bernoulli polynomials such as Equations (2) and (3).

After the paper "Some Identities for Euler and Bernoulli Polynomials and Their Zeros in Axioms 2018, 7, 56." by T. Kim and C.S. Ryoo was published, we realized that some results of the paper "A New Type of Euler Polynomials and Numbers in Mediterr. J. Math. (2018) 15: 138." by M. Masjed-Jamei, M.R. Beyki, and W. Koepf were published ahead with some identical results, which are consistent with the ones in the paper [1].

The authors in [1], after the publication, were aware of that Hacéne Belbachir, the reviewer of the paper [2], left the question related to the extension of a variable in Mathematical Reviews (MR3808565) of the American Mathematical Society: "Is it possible to obtain their results by considering the classical Euler polynomials of complex variable $x+iy$, and treating the real part and the imaginary part separately?" The approach in Equation (2) can be an affirmative answer to the question.

Thus, we want to inform our readers that some results of Reference [2] have been published before the paper [1]. In addition, their related works are presented in [3], in which some similar results are shown as their consistent works in [2].

The authors conclusively note that some of the results in both [1,2] are derived from these two different approaches mentioned above.

In addition, the identical results in both [1,2] are listed as follows.

1. Theorem 1 in [1] and Results (13) and (14) in [2] are identical: for $n \geq 0$,

$$E_n^{(C)}(x,y)=\sum_{l=0}^{n}\binom{n}{l}E_lC_{n-l}(x,y) \quad \text{and} \quad E_n^{(S)}(x,y)=\sum_{l=0}^{n}\binom{n}{l}E_lS_{n-l}(x,y).$$

2. Theorem 3 in [1] and Proposition 2.1 in [2] state the same outcome: for $n \geq 0$,

$$E_n^{(C)}(1-x,y)=(-1)^nE_n^{(C)}(x,y) \quad \text{and} \quad E_n^{(S)}(1-x,y)=(-1)^{n+1}E_n^{(S)}(x,y).$$

3. Theorem 4 in [1] and Proposition 2.2 in [2] present identical results: for $n \geq 0$,

$$E_n^{(C)}(x+1,y)+E_n^{(C)}(x,y)=2C_n(x,y) \quad \text{and} \quad E_n^{(S)}(x+1,y)+E_n^{(S)}(x,y)=2S_n(x,y).$$

4. Corollary 1 in [1] and Corollary 2.2 in [2] show matching expressions: for $n \geq 0$,

$$E_{2n}^{(C)}(1,y)+E_{2n}^{(C)}(0,y)=2(-1)^ny^{2n} \quad \text{and} \quad E_{2n+1}^{(S)}(1,y)+E_{2n+1}^{(S)}(0,y)=2(-1)^ny^{2n+1}.$$

5. Theorem 5 in [1] and Proposition 2.3 in [2] have matching results: for $n \geq 0$ $r \in \mathbb{N}$,

$$E_n^{(C)}(x+r,y)=\sum_{k=0}^{n}\binom{n}{k}E_k^{(C)}(x,y)r^{n-k} \quad \text{and} \quad E_n^{(S)}(x+r,y)=\sum_{k=0}^{n}\binom{n}{k}E_k^{(S)}(x,y)r^{n-k}.$$

2. Corrections

In addition, while reviewing our paper, we found some typing errors: Equation (11) should be revised by

$$\frac{E_n(x+iy)-E_n(x-iy)}{2i},$$

and Equation (31) should be also replaced by

$$\frac{B_n(x+iy)-B_n(x-iy)}{2i}.$$

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References

1. Kim, T.; Ryoo, C.S. Some Identities for Euler and Bernoulli Polynomials and Their Zeros. *Axioms* **2018**, *7*, 56. [CrossRef]
2. Masjed-Jamei, M.; Beyki, M.R.; Koepf, W. A New Type of Euler Polynomials and Numbers. *Mediterr. J. Math.* **2018**, *15*, 138. [CrossRef]
3. Masjed-Jamei, M.; Beyki, M.R.; Koepf, W. An extension of the Euler-Maclaurin quadrature formula using a parametric type of Bernoulli polynomials. *Bull. Sci. Math.* **2019**, to appear. [CrossRef]