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C_T-Integral on Interval-Valued Sugeno Probability Measure and Its Application in Multi-Criteria Decision-Making Problems

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Abstract: It is well known that the complexity of the decision-making environment frequently coexists with the diversity of linguistic information in the decision-making process. In order to solve this kind of uncertain multi-criteria decision-making problem, reasonable measures and integrals should be established. In this paper, the discrete expression of the C_T-integral on the interval-valued Sugeno probability measure is proposed. The C_T-integral is the Choquet integral when the t-norm is $T(x, y) = xy$ in the C_T-integral and is a pre-aggregation function. Then, the C_T-integral on interval-valued Sugeno probability measure is applied to solve end-of-life (EOL) strategy in order to determine multi-criteria decision-making problems. Compared with the general Choquet integral, the method proposed in this paper significantly improves the calculation process, that is, the calculation is simpler and the amount of calculation is smaller. A case study was performed in order to validate the effectiveness of this conclusion.

Keywords: fuzzy measure; Choquet integral; C_T-integral; end-of-life strategy; multi-criteria decision-making

Citation: Yang, H.; Shang, L.; Gong, Z. C_T-Integral on Interval-Valued Sugeno Probability Measure and Its Application in Multi-Criteria Decision-Making Problems. *Axioms* **2022**, *11*, 317. https://doi.org/10.3390/axioms11070317

Academic Editors: Graçaliz Pereira Dimuro, Benjamin Bedregal and Junsheng Qiao

Received: 16 April 2022
Accepted: 24 June 2022
Published: 29 June 2022

1. Introduction

In 1954, the French mathematician Choquet introduced capacity theory [1], which is a set function satisfying monotonicity and continuity similar to the Sugeno measure, along with the Choquet integral, which is based on capacity. The Choquet integral is a non-additive measure as well as a nonlinear integral. Afterwards, generalizations of the Choquet integral started to appear; with the work of Sugeno and Murofushi in 1987, a generalized form of the Choquet integral appeared in the literature [2], followed by a corresponding work by Mesiar and Grabisch in 1995. Several other important methods are provided in the literature [3,4], and several boundary conditions have been discussed as well. In 2013, Barrenechea et al. applied the Choquet integral as an aggregation function in a fuzzy rule-based classification system [5]. Then, in 2016, new Choquet integral extensions of the aggregation function appeared in the literature, especially in the research of Lucca et al. [6]. These extensions of the Choquet integral were named, for example, the CC-integral [7], C_F-integral [8], and C_T-integral [9]. For the C_T-integral, the methodology used in these generalizations is simply to replace the product in the Choquet integral, which can be interpreted as the product's t-norm, with other fusion functions with appropriate properties in order to obtain a resulting aggregation-type function.

The Choquet integral is applied in many fields, such as risk evaluation [10], fuzzy systems and control [11], decision-making, and more (see [12–15]). For the multi-attribute decision-making problem ([12,16]), the multi-criteria decision-making problem [15], and the group decision-making problem [13], it can be used as aggregation operators to aggregate. Similarly, the C_T-integral can be applied to multi-criteria decision-making problems; see [17]. Other generalizations of the Choquet integral have seen use in problems involving multi-criteria decision-making (see [18,19]). In this paper, we focus on multi-criteria decision-making problems. Especially in the multi-criteria decision-making problem of end-of-life (EOL) strategy, the Choquet integral is used as an aggregation operator to aggregate data. However, with the emergence of the generalization of the Choquet integral, generalization solves this type of problem more efficiently than the Choquet integral.

With the development of society and the progress of science and technology, more and more people have begun to pay attention to the EOL strategy of products and how to deal with product after use. Many definitions and classifications of EOL strategy have been developed over the decades. EOL strategy has been continuously developed and improved following its initial proposal by Marco et al. in 1994 [20]. The detailed development process of EOL strategies can be followed in the literature [21]. For a product, EOL strategy analysis is more beneficial to the development of both the environment and the economy. Moreover, certain EOL strategies produce a large amount of energy waste, environmental pollution, and cost over the whole product life cycle. Therefore, the theory and method of EOL research has been a concern of many scholars. EOL decision-making depends on many factors which arise from a wide range of stakeholder interests and components, and the view of the results varies by industry and geographical location. As far as we know, the EOL strategy of refrigerator components is an important research area in evaluating EOL decision factors from a comprehensive perspective. Therefore, it is meaningful to use our proposed C_T-integral on interval-valued Sugeno probability measure to solve related uncertain multi-criteria decision-making problems in this paper.

When we solving this kind of multi-criteria decision-making problems, the decision criteria usually interact with each other, and the evaluation values are usually fuzzy linguistic evaluations by experts, often defined as "good", "a little good" etc. These linguistic calculations are usually converted into triangular fuzzy numbers for calculation; however, triangular fuzzy numbers are not easy to calculate. Generally, they are converted into interval values or exact values for operation. Therefore, it is necessary to extend the general C_T-integral to the interval value. In 2016, a new aggregation-like function generalizing the Choquet integral was proposed in [22]. In 2020, Chen et al. proposed the Choquet integral on the interval-valued Sugeno probability measure [23]. The discrete expression of the C_T-integral on the interval-valued Sugeno probability measure is proposed in this paper and applied to solve multi-criteria decision-making problems in the context of determining EOL strategy.

It is well known that in dealing with uncertain multi-criteria decision-making problems, the decision criteria usually interact with each other. The Choquet integral on fuzzy measures based on σ-λ rules can be used to solve such problems effectively. The discrete expression of the C_T-integral on the interval-valued Sugeno probability measure is proposed in this paper. The C_T-integral is the Choquet integral when the t-norm is $T(x,y) = xy$ in the C_T-integral and is a pre-aggregation function. Then, we apply the C_T-integral on interval-valued Sugeno probability measure to determine end-of-life (EOL) strategy as a multi-criteria decision-making problems. Compared with the general Choquet integral, the method proposed in this paper significantly improves the calculation process, that is, the calculation is simpler and the amount of calculation required is smaller. A case study is performed in order to validate the effectiveness of the conclusions.

The remainder of the article is organized as follows. In Section 2, several basic concepts are introduced. Section 3 provides the C_T-integral on the interval-valued Sugeno probability measure. In Section 4, a multi-criteria decision-making problem involving the EOL strategy for a refrigerator component is illustrated as a case study. Our conclusions are presented in Section 5.

2. Preliminaries

In this section, basic concepts are introduced. In the following, $n > 0$, Ω is a non-empty set, and $\mathcal{A}$ is a σ-algebra.

Definition 1 ([24])**.** *The fuzzy measure μ is a set function*

$$\mu : \mathcal{A} \to [0, \infty]$$

with the following properties:

(i) $\mu(\emptyset) = 0$;

(ii) $A \subset B$ *implies* $\mu(A) \leq \mu(B)$;

Fuzzy measure μ *is said to be lower semi-continuous when it satisfies*

(iii) $A_1 \subset A_2 \subset ...$, *implying that* $\mu(\cup_{n=1}^{\infty} A_n) = \lim_{n\to 0} \mu(A_n)$;

Fuzzy measure μ *is said to be upper semi-continuous when it satisfies*

(iv) $A_1 \supset A_2 \supset ...$, *and* $\mu(A_1) < \infty$, *implying that* $\mu(\cap_{n=1}^{\infty} A_n) = \lim_{n\to 0} \mu(A_n)$;

Fuzzy measure μ *is said to be continuous if it satisfies both lower semi-continuity and upper semi-continuity.*

Definition 2 ([25]). *Consider function* $T : [0,1]^2 \to [0,1]$. *Then,* T *is the triangular norm (t-norm for short) if, for all* $x, y, z \in [0,1]$, *the following four axioms are satisfied:*

$(T1)$ *Commutativity:* $T(x,y) = T(y,x)$;

$(T2)$ *Associativity:* $T(x, T(y,z)) = T(T(x,y),z)$;

$(T3)$ *Monotonicity:* $T(x,y) \leq T(x,z)$ *whenever* $y \leq z$;

$(T4)$ *Boundary condition:* $T(x,1) = x$.

The basic definitions of the interval-valued Sugeno probability measure (see [23]) are provided below; R^+ denotes $[0,\infty)$.

Definition 3 ([23]). *Suppose* Ω *is a nonempty set and* $\mathcal{A}$ *is* σ*-algebra on the* Ω. *The set function* μ *is a fuzzy measure based on* σ-λ *rules if*

$$\mu\left(\cup_{i=1}^{\infty} A_{(i)}\right) = \begin{cases} \frac{1}{\lambda}\left\{\Pi_{i=1}^{\infty}\left[1+\lambda\mu(A_{(i)})\right]-1\right\} & \lambda \neq 0, \\ \sum_{i=1}^{\infty} \mu\left(A_{(i)}\right) & \lambda = 0. \end{cases} \tag{1}$$

where $\lambda \in \left(-\frac{1}{sup\mu}, \infty\right) \cup 0$, $A_{(i)} \subset \mathcal{A}$, $A_{(i)} \cap A_{(j)} = \emptyset$ *for all* $i, j = 1, 2...$, *and* $i \neq j$.

Definition 4 ([23]). *Suppose* Ω *is a nonempty set and* $\mathcal{A}$ *a* σ*-algebra on the* Ω, *a set function* $\mu : \mathcal{A} \to R^+$, $\underline{\mu} : \mathcal{A} \to R^+$, $\overline{\mu} : \mathcal{A} \to R^+$, $\underline{\mu}$ *and* $\overline{\mu}$ *satisfying the following conditions:*

(1) $\underline{\mu}(\emptyset) = 0, \overline{\mu}(\emptyset) = 0$;

(2) *if* $A, B \subset \Omega$, *and* $A \subset B$, *then* $\underline{\mu}(A) \leq \underline{\mu}(B), \overline{\mu}(A) \leq \overline{\mu}(B)$;

(3) *for every* $A \subset \Omega$, $\underline{\mu}(A) \leq \overline{\mu}(A)$;

then $\mu = [\underline{\mu}, \overline{\mu}]$ *is an interval-valued fuzzy measure.*

Definition 5 ([23]). *If* $\underline{\mu}$ *and* $\overline{\mu}$ *satisfy the* $\sigma - \lambda$ *rules in (Definition 3), and* $\underline{\mu}(\Omega) = 1, \overline{\mu}(\Omega) = 1$, *then* $\mu = [\underline{\mu}, \overline{\mu}]$ *is called an interval-valued Sugeno probability measure based on* $\sigma - \lambda$ *rules, or simply an interval-valued Sugeno probability measures, and denoted* $g_\lambda = [\underline{g}_\lambda, \overline{g}_\lambda]$.

$L(0,1)$ *denotes the set of all closed subintervals of the unit interval.*

Definition 6 ([23]). *Suppose* Ω *is a finite set and* 2^Ω *is the power set of* Ω, *the set function* μ: $2^\Omega \to [\underline{\mu}, \overline{\mu}] \subset L(0,1)$ *is a regular interval fuzzy measure defined on* 2^Ω *if the following conditions hold:*

(1) $\underline{\mu}(\emptyset) = 0, \overline{\mu}(\emptyset) = 0, \underline{\mu}(\Omega) = 1, \overline{\mu}(\Omega) = 1$;

(2) *if* $D \in 2^\Omega$, $H \in 2^\Omega$, $D \subset H$, *then* $\underline{\mu}(D) \leq \underline{\mu}(H), \overline{\mu}(D) \leq \overline{\mu}(H)$.

Definition 7 ([23]). *Suppose* Ω *is a finite set and* 2^Ω *is the power set of* Ω; *set function* μ : $2^\Omega \to [\underline{\mu}, \overline{\mu}] \subset L(0,1)$, *is a regular* $\lambda-$*interval fuzzy measure defined on* 2^Ω *if the following conditions hold:*

(1) $\underline{\mu}(\emptyset) = 0, \overline{\mu}(\emptyset) = 0, \underline{\mu}(\Omega) = 1, \overline{\mu}(\Omega) = 1;$

(2) *if* $A \subset \Omega, B \subset \Omega, A \cap B = \emptyset$, *then*

$$\underline{\mu}(A \cup B) = \underline{\mu}(A) + \underline{\mu}(B) + \lambda \underline{\mu}(A) \underline{\mu}(B), \tag{2}$$

and

$$\overline{\mu}(A \cup B) = \overline{\mu}(A) + \overline{\mu}(B) + \lambda \overline{\mu}(A) \overline{\mu}(B), \lambda \in (-1, \infty). \tag{3}$$

Theorem 1 ([23]). *If* $g_\lambda = [\underline{g}_\lambda, \overline{g}_\lambda]$ *is an interval-valued Sugeno probability measure, then* g_λ *is a regular* $\lambda-$*interval fuzzy measure defined on* $\mathcal{A}$.

Proof. Refer to the proof of Theorem 3.1.2 in [23]. □

Suppose $X = \{x_1, x_2, ..., x_n\}$ is a finite set. Then, $g_{\lambda_i} = g_\lambda(x_i)(i = 1, 2, ..., n)$ can measure the density.

Theorem 2 ([24,26]). *The parameters* $\lambda = (\lambda_1, \lambda_2)$ *of the regular interval Sugeno probability measure are determined by the following equations:*

$$\prod_{i=1}^{n} \left(1 + \lambda_1 \underline{g}_{\lambda_i}\right) = 1 + \lambda_1, \tag{4}$$

$$\prod_{i=1}^{n} \left(1 + \lambda_2 \overline{g}_{\lambda_i}\right) = 1 + \lambda_2. \tag{5}$$

Proof. Because Theorem 1 and $\lambda = (\lambda_1, \lambda_2)$ take into account that $X = \{x_1, x_2, ..., x_n\}$ is a finite set, $g_{\lambda_i} = g_\lambda(x_i)(i = 1, 2, ..., n)$ is said to be a measure of density. From Equation (1) $\lambda \neq 0$, have $g_{\lambda_i}(\cup_{i=1}^{n}(x_i)) = 1$, that is,

$$\frac{1}{\lambda}\left\{\prod_{i=1}^{n}\left[1 + \lambda_1 g_{\lambda_i}(x_{(i)})\right] - 1\right\} = 1,$$

$$\prod_{i=1}^{n}\left[1 + \lambda_1 \underline{g}_{\lambda_i}(x_{(i)})\right] - 1 = \lambda_1,$$

$$\prod_{i=1}^{n}\left[1 + \lambda_1 \underline{g}_{\lambda_i}(x_{(i)})\right] = 1 + \lambda_1,$$

i.e.,

$$\prod_{i=1}^{n}\left(1 + \lambda_1 \underline{g}_{\lambda_i}\right) = 1 + \lambda_1.$$

□

Then, Equation (5) can be obtained similarly.

3. The C_T-Integral on Interval-Valued Sugeno Probability Measure

Choquet integrals are a natural generalization of the Lebesgue integral; the definition of Lebesgue integrals considers additive measures, whereas the definition of Choquet integrals considers fuzzy measures. In the following, the definition of the discrete Choquet integral is introduced and the discrete expression of the C_T-integral on the interval-valued Sugeno probability measure is proposed. Consider $N = \{1, ..., n\}$ as a finite set.

Definition 8 ([1]). *Suppose $m : 2^N \to [0,1]$ is a fuzzy measure; then, the discrete Choquet integral as regards m is the function $C_m : [0,1]^n \to [0,1]$, defined for all $\vec{x} = (x_1, ..., x_n) \in [0,1]^n$ by*

$$C_m(\vec{x}) = \sum_{i=1}^{n} \left(x_{(i)} - x_{(i-1)} \right) \cdot m\left(A_{(i)} \right), \tag{6}$$

where $\left(x_{(1)}, ..., x_{(n)} \right)$ is an increasing permutation on the input $\vec{x}$, that is, $x_{(1)} \leq ... \leq x_{(n)}$, with the convention that $x_{(0)} = 0$ and $A_{(i)} = \{(i), ..., (n)\}$ is the subset of indices corresponding to the $n - i + 1$ largest components of $\vec{x}$.

Reference [9] mentions recent advances in the generalization of the standard form of the Choquet integral. The method used for these generalizations is simple, and allows for replacing the product operator in Equation (6) with another fusion function of a suitable nature. The C_T-integral is obtained by replacing the product operator of Equation (6) with a t-norm, and is a pre-aggregation function.

Definition 9 ([9]). *Suppose $m : 2^N \to [0,1]$ is a fuzzy measure and $T : [0,1]^2 \to [0,1]$ is a t-norm. Based on the Choquet integral, the C_T-integral is defined as the function $C_m^T : [0,1]^n \to [0,1]$ for all of $\vec{x} = (x_1, ..., x_n) \in [0,1]^n$ by*

$$C_m^T(\vec{x}) = \sum_{i=1}^{n} T\left(\left(x_{(i)} - x_{(i-1)} \right), m(A_{(i)}) \right), \tag{7}$$

where:

(1) $\left(x_{(1)}, ..., x_{(n)} \right)$ is an increasing permutation on the input $\vec{x}$, that is, $x_{(1)} \leq ... \leq x_{(n)}$;

(2) $x_{(0)} = 0$;

(3) $A_{(i)} = \{(i), ..., (n)\}$ is the subset of indices corresponding to the $n - i + 1$ largest component of $\vec{x}$.

The C_T-integral has the averaging and idempotent properties after generalization, as shown below.

Proposition 1 ([6,8]). *Let $T : [0,1]^2 \to [0,1]$ be a t-norm such that $T(x,y) \leq x$ for every $x, y \in [0,1]$. Then,*

$$C_m^T(x_1, ..., x_n) \leq \max(x_1, ..., x_n), \tag{8}$$

for every $(x_1, ..., x_n) \in [0,1]^n$.

Proposition 2 ([6,8]). *Let $T : [0,1]^2 \to [0,1]$ be a t-norm such that $T(x,1) = x$ for every $x, y \in [0,1]$. Then,*

$$C_m^T(x_1, ..., x_n) \geq \min(x_1, ..., x_n), \tag{9}$$

for every $(x_1, ..., x_n) \in [0,1]^n$.

Proposition 3 ([6,8]). *Let $T : [0,1]^2 \to [0,1]$ be a t-norm such that $T(x,1) = x$ and $T(0,y) = 0$ for every $x, y \in [0,1]$. Then, C_m^T is an idempotent function, that is,*

$$C_m^T(x, ..., x) = x. \tag{10}$$

Next, the measurability and integrability of interval-valued functions is introduced.

$R^+ = [0, \infty)$, $I(R^+) = \left\{ r : [\underline{r}, \overline{r}] \subset R^+ \right\}$ is the subset of the interval number, while $P_0(R^+)$ denotes the classes of all non-empty subsets on R^+.

Definition 10 ([27])**.** *Suppose* $(\Omega, \mathcal{A}, \mu)$ *is a non-additive measure space and* $F : \Omega \to I(R^+)$ *is a non-negative measurable interval-valued function on* Ω*,* $A \in \mathcal{A}$*; then, we have*

$$(c)\int_A F\,d\mu = \left\{(c)\int_A f\,d\mu \colon f \in S_F\right\}, \tag{11}$$

where $S_F = \{g|g : \Omega \to R^+$ is a measurable selection on $F\}$*; if* $(c)\int_A Fd\mu \subset I(R^+)$*, then* F *is C-integrable.*

Definition 11 ([27])**.** *F is C-integrally bounded if there exists a C-integral function* $g : \Omega \to P_0(R^+)$ *such that for any measurable selection* $f \in S_F$*,* $A \in \mathcal{A}$ *has*

$$(c)\int_A f\,d\mu \le (c)\int_A g\,d\mu. \tag{12}$$

Theorem 3 ([23])**.** *Let* $(\Omega, \mathcal{A}, \mu)$ *be a non-additive measure space,* μ *be a fuzzy measure,* $A \in \mathcal{A}$*, F be non-negative measurable, and C—be integrally bounded; then, F is C—integrable on A and*

$$(c)\int_A F\,d\mu = \left[(c)\int_A \underline{F}\,d\mu, (c)\int_A \overline{F}\,d\mu\right]. \tag{13}$$

Proof. See [23] in Theorem 3.2.1. □

We know from the previous that the interval-valued function f is C-integrable on A if $(c)\int_A \underline{f}\,d\mu$ and $(c)\int_A \overline{f}\,d\mu$ exists and is bounded.

Suppose $X = \{x_1, x_2, ...x_n\}$ is a discrete set; we can then obtain the following Theorem.

Theorem 4. *Suppose* f *is an interval-valued function on* $X = \{x_1, x_2, ...x_n\}$ *and T is a t-norm. Then, the* C_T*-integral of* f *as regards the interval-valued Sugeno probability measure* g_λ *on X is provided by*

$$(c)\int_X f\,dg_\lambda = \sum_{i=1}^{n} T\big(g_\lambda(X'_i) - g_\lambda(X'_{i+1}), f(X'_i)\big), \tag{14}$$

where $x'_1, x'_2, ..., x'_n$ *is a permutation of* $x_1, x_2, ..., x_n$ *such that* $f(x'_0) \le f(x'_1) \le ... \le f(x'_n)$*,* $f(x'_0) = [0,0]$*,* $X'_i = \{x'_i, x'_{i+1}, ..., x'_n\}$*,* $i = 1, 2, ..., n$*, and* $X'_{n+1} = \emptyset$*.*

Proof. Due to f being an interval-valued function on X, per Theorem 3, we have

$$(c)\int_X f\,dg_\lambda = \left[(c)\int_X \underline{f}\,dg_\lambda, (c)\int_X \overline{f}\,dg_\lambda\right].$$

Note that $\underline{f}$ and $\overline{f}$ are real-valued functions on X, respectively, on account of the continuity and monotonicity of the C_T-integral. Meanwhile, considering the nonnegativity and the monotonicity of the fuzzy measure, we can obtain

$$\begin{aligned}
(c)\int_X f\,dg_\lambda &= \left[\sum_{i=1}^{n} T\Big(g_\lambda(X'_i) - g_\lambda(X'_{i+1}), \underline{f}(X'_i)\Big), \sum_{i=1}^{n} T\Big(g_\lambda(X'_i) - g_\lambda(X'_{i+1}), \overline{f}(X'_i)\Big)\right] \\
&= \sum_{i=1}^{n}\Big[T\Big(g_\lambda(X'_i) - g_\lambda(X'_{i+1}), \underline{f}(X'_i)\Big), T\Big(g_\lambda(X'_i) - g_\lambda(X'_{i+1}), \overline{f}(X'_i)\Big)\Big] \\
&= \sum_{i=1}^{n} T\Big(g_\lambda(X'_i) - g_\lambda(X'_{i+1}), \Big[\underline{f}(X'_i), \overline{f}(X'_i)\Big]\Big) \\
&= \sum_{i=1}^{n} T\big(g_\lambda(X'_i) - g_\lambda(X'_{i+1}), f(X'_i)\big).
\end{aligned}$$

where $x'_1, x'_2, ..., x'_n$ is a permutation of $x_1, x_2, ..., x_n$ such that $f(x'_0) \le f(x'_1) \le ... \le f(x'_n)$, $f(x'_0) = [0,0]$, $X'_i = \{x'_i, x'_{i+1}, ..., x'_n\}$, $i = 1, 2, ..., n$, and $X'_{n+1} = \emptyset$. □

Then, the discrete representation of the C_T-integral on interval-valued Sugeno probability measure is as follows:

$$(c)\int_X f\,dg_\lambda = \left[\sum_{i=1}^{n} T\Big(g_\lambda(X_i') - g_\lambda(X_{i+1}'), \underline{f}(X_i')\Big), \sum_{i=1}^{n} T(g_\lambda(X_i') - g_\lambda(X_{i+1}')), \overline{f}(X_i')\right]. \quad (15)$$

where T can be T_M, T_P, $T_Ł$, T_{DP}, T_{NM}, and T_{HP}. When T is T_M, $T_M(x,y) = \min\{x,y\}$, we have an expression of the C_{T_M}-integral on interval-valued Sugeno probability measure:

$$\begin{aligned}(c)\int_X f dg_\lambda &= \sum_{i=1}^{n} \min(g_\lambda(X_i') - g_\lambda(X_{i+1}'), f(X_i')) \\ &= \sum_{i=1}^{n} \min\Big(g_\lambda(X_i') - g_\lambda(X_{i+1}'), \big[\underline{f}(X_i'), \overline{f}(X_i')\big]\Big) \\ &= \sum_{i=1}^{n} \Big[\min\Big(g_\lambda(X_i') - g_\lambda(X_{i+1}'), \underline{f}(X_i')\Big), \min\Big(g_\lambda(X_i') - g_\lambda(X_{i+1}'), \overline{f}(X_i')\Big)\Big] \\ &= \left[\sum_{i=1}^{n} \min\Big(g_\lambda(X_i') - g_\lambda(X_{i+1}'), \underline{f}(X_i')\Big), \sum_{i=1}^{n} \min\Big(g_\lambda(X_i') - g_\lambda(X_{i+1}'), \overline{f}(X_i')\Big)\right].\end{aligned}$$

4. Application in Multi-Criteria Decision-Making Problems

4.1. Case Study

In this case, the multi-criteria decision-making problem of EOL strategy determination for a refrigerator component was studied. We used the extended form of the Choquet integral, which is the C_T-integral. This multi-criteria decision-making problem considers four primary criteria and fourteen sub-criteria, as shown in Figure 1. Each component of the refrigerator interacts with other components, and includes only the main part and a few small connectors. The case data sets are taken from [21,28]. The decision alternatives include reuse, restructuring, primary recycling, secondary recycling, refuse incineration, and landfill.

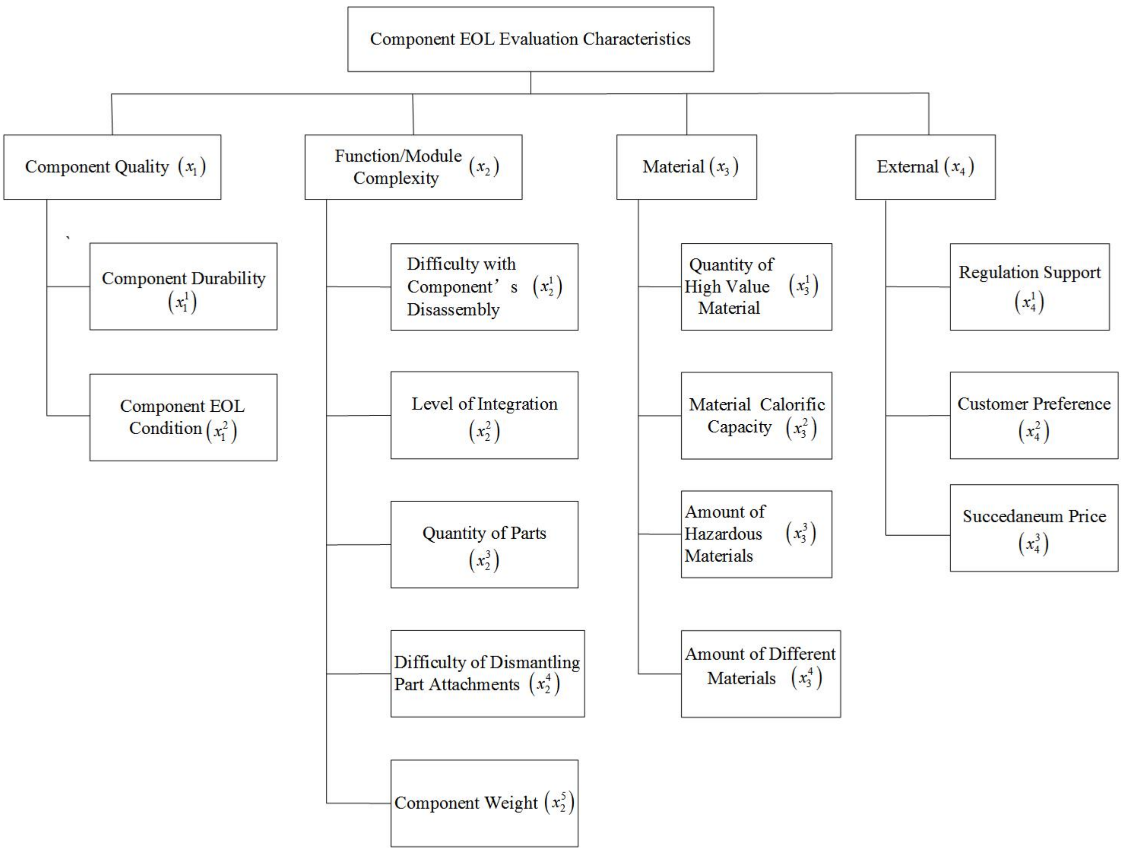

Figure 1. Structure of the decision attributes.

In [12], the language assessment and every attribute are represented by triangular fuzzy numbers. We can evaluate this in natural language and turn it into a triangular fuzzy number. Such an evaluation process requires the participation of participating decision-makers. The language assessment and corresponding triangular fuzzy numbers are provided in Table 1. As an example of the cabinet frame, Table 2 shows the language evaluation of the main criteria and subcriteria, while Table 3 shows the corresponding triangular fuzzy numbers.

Table 1. Linguistic terms and corresponding fuzzy numbers (from [12,23]).

Evaluation/Weighting Terms	Label	Triangular Fuzzy Numbers
Extra poor/Extra unimportant	EP/EU	(0,0,0.1)
Very poor/Very unimportant	VP/VU	(0,0.1,0.2)
poor/unimportant	P/U	(0.1,0.2,0.3)
A little poor/A little unimportant	AP/AU	(0.2,0.3,0.4)
Slightly poor/Slightly unimportant	SP/SU	(0.3,0.4,0.5)
Fair/Middle	F/M	(0.4,0.5,0.6)
Slightly good/Slightly important	SG/SI	(0.5,0.6,0.7)
A little good/A little important	AH/AI	(0.6,0.7,0.8)
good/important	G/I	(0.7,0.8,0.9)
Very good/Very important	VG/VI	(0.8,0.9,1)
Extra good/Extra important	EG/EI	(0.9,1,1)

Table 2. Criteria importance and EOL options for linguistic evaluation with respect to cabinet frame.

Criteria	Weights	Linguistic Evaluation of EOL Options f_{it}^{j}					
		A_1	A_2	A_3	A_4	A_5	A_6
x_1	VG						
x_1^1	VG	G	G	F	VG	VG	VP
x_2	G						
x_2^1	G	G	G	F	F	VP	VP
x_2^2	F	VG	G	G	G	G	P
x_2^3	G	F	VG	VG	F	VG	G
x_2^4	VG	F	G	G	G	F	F
x_2^5	F	G	VG	F	G	P	VP
x_3	VG						
x_3^1	VG	G	P	VG	F	VG	VG
x_3^2	F	P	VP	VG	G	P	G
x_3^3	G	VG	F	G	G	VP	G
x_3^4	P	F	F	G	F	F	G
x_4	F						
x_4^1	F	G	G	F	P	F	F
x_4^2	F	VG	G	VG	P	VG	F
x_4^3	F	VG	VG	VG	VP	VG	VG

Table 3. Triangular fuzzy number evaluation with respect to cabinet frame.

Criteria	Weights	A_1	A_2	A_3	A_4	A_5	A_6
x_1	$(0.8, 0.9, 1)$						
x_1^1	$(0.8, 0.9, 1)$	$(0.7, 0.8, 0.9)$	$(0.7, 0.8, 0.9)$	$(0.4, 0.5, 0.6)$	$(0.8, 0.9, 1)$	$(0.8, 0.9, 1)$	$(0, 0.1, 0.2)$
x_1^2	$(0.4, 0.5, 0.6)$	$(0.7, 0.8, 0.9)$	$(0.7, 0.8, 0.9)$	$(0.8, 0.9, 1)$	$(0.7, 0.8, 0.9)$	$(0.4, 0.5, 0.6)$	$(0, 0.1, 0.2)$
x_2	$(0.7, 0.8, 0.9)$						
x_2^1	$(0.7, 0.8, 0.9)$	$(0.7, 0.8, 0.9)$	$(0.7, 0.8, 0.9)$	$(0.4, 0.5, 0.6)$	$(0.4, 0.5, 0.6)$	$(0, 0.1, 0.2)$	$(0, 0.1, 0.2)$
x_2^2	$(0.4, 0.5, 0.6)$	$(0.8, 0.9, 1)$	$(0.7, 0.8, 0.9)$	$(0.7, 0.8, 0.9)$	$(0.7, 0.8, 0.9)$	$(0.7, 0.8, 0.9)$	$(0.1, 0.2, 0.3)$
x_2^3	$(0.7, 0.8, 0.9)$	$(0.4, 0.5, 0.6)$	$(0.8, 0.9, 1)$	$(0.8, 0.9, 1)$	$(0.4, 0.5, 0.6)$	$(0.8, 0.9, 1)$	$(0.7, 0.8, 0.9)$
x_2^4	$(0.8, 0.9, 1)$	$(0.4, 0.5, 0.6)$	$(0.7, 0.8, 0.9)$	$(0.7, 0.8, 0.9)$	$(0.7, 0.8, 0.9)$	$(0.4, 0.5, 0.6)$	$(0.4, 0.5, 0.6)$
x_2^5	$(0.4, 0.5, 0.6)$	$(0.7, 0.8, 0.9)$	$(0.8, 0.9, 1)$	$(0.4, 0.5, 0.6)$	$(0.7, 0.8, 0.9)$	$(0.1, 0.2, 0.3)$	$(0, 0.1, 0.2)$
x_3	$(0.8, 0.9, 1)$						
x_3^1	$(0.8, 0.9, 1)$	$(0.7, 0.8, 0.9)$	$(0.1, 0.2, 0.3)$	$(0.8, 0.9, 1)$	$(0.4, 0.5, 0.6)$	$(0.8, 0.9, 1)$	$(0.8, 0.9, 1)$
x_3^2	$(0.4, 0.5, 0.6)$	$(0.1, 0.2, 0.3)$	$(0, 0.1, 0.2)$	$(0.8, 0.9, 1)$	$(0.7, 0.8, 0.9)$	$(0.1, 0.2, 0.3)$	$(0.7, 0.8, 0.9)$
x_3^3	$(0.7, 0.8, 0.9)$	$(0.8, 0.9, 1)$	$(0.4, 0.5, 0.6)$	$(0.7, 0.8, 0.9)$	$(0.7, 0.8, 0.9)$	$(0, 0.1, 0.2)$	$(0.7, 0.8, 0.9)$
x_3^4	$(0.1, 0.2, 0.3)$	$(0.4, 0.5, 0.6)$	$(0.4, 0.5, 0.6)$	$(0.7, 0.8, 0.9)$	$(0.4, 0.5, 0.6)$	$(0.4, 0.5, 0.6)$	$(0.7, 0.8, 0.9)$
x_4	$(0.4, 0.5, 0.6)$						
x_4^1	$(0.4, 0.5, 0.6)$	$(0.7, 0.8, 0.9)$	$(0.7, 0.8, 0.9)$	$(0.4, 0.5, 0.6)$	$(0.1, 0.2, 0.3)$	$(0.4, 0.5, 0.6)$	$(0.4, 0.5, 0.6)$
x_4^2	$(0.4, 0.5, 0.6)$	$(0.8, 0.9, 1)$	$(0.7, 0.8, 0.9)$	$(0.8, 0.9, 1)$	$(0.1, 0.2, 0.3)$	$(0.8, 0.9, 1)$	$(0.4, 0.5, 0.6)$
x_4^3	$(0.4, 0.5, 0.6)$	$(0.8, 0.9, 1)$	$(0.8, 0.9, 1)$	$(0.8, 0.9, 1)$	$(0, 0.1, 0.2)$	$(0.8, 0.9, 1)$	$(0.8, 0.9, 1)$

4.2. Case Studies and Solutions

The calculation process for the refrigerator component EOL strategy determination multi-criteria decision-making problem is as follows, in the six steps shown in Figure 2. Figure 1 describes the parameter set and variables. In this example, four primary criteria are considered, denoted as $x_i(i = 1, 2, 3, 4)$, and fourteen sub-criteria, denoted as x_i^j. If $i = 1$, then $j = 1, 2$, if $i = 2$, then $j = 1, 2, 3, 4, 5$, if $i = 3$, then $j = 1, 2, 3, 4$, and if $i = 4$ then $j = 1, 2, 3$. The six EOL choices (Reuse, Remanufacture, Primary Recycling, Secondary Recycling, Incineration, and Landfill) are A_1, A_2, A_3, A_4, A_5, and A_6, respectively.

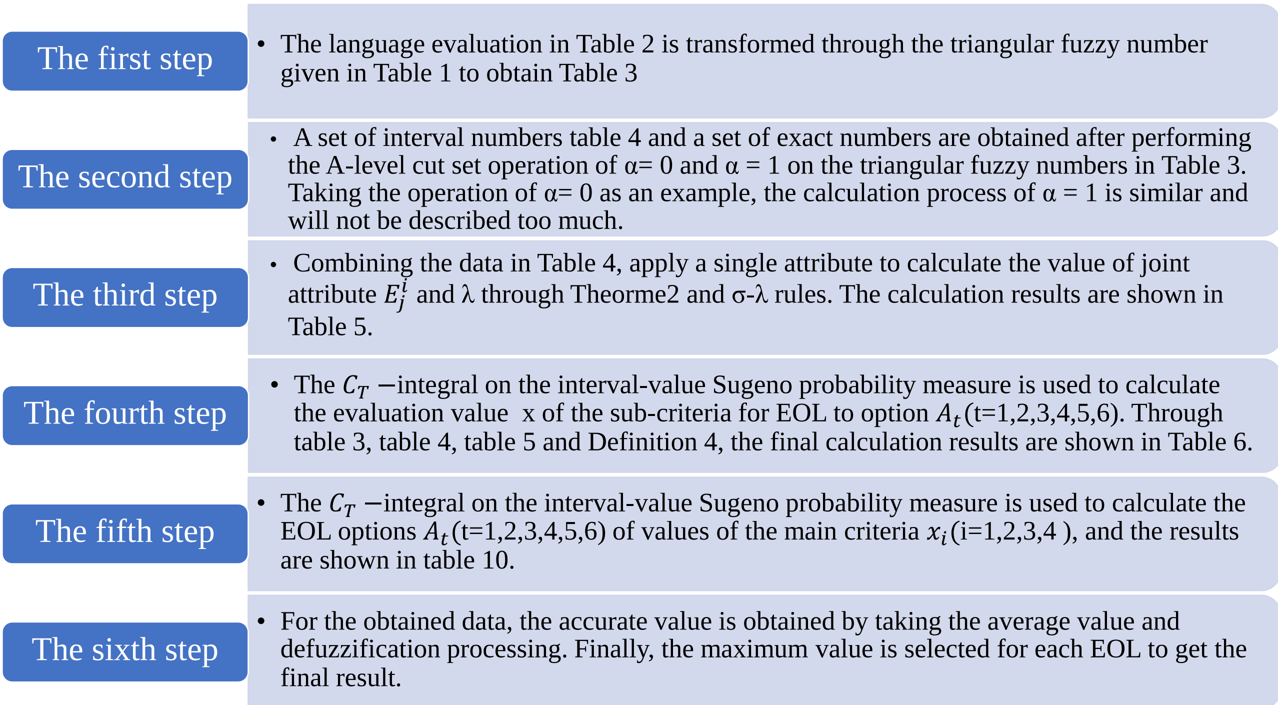

Figure 2. Calculation process.

Taking $\alpha = 0$ as an example, $\alpha = 1$ analogies can be obtained.

The evaluated language information is transformed into triangular fuzzy numbers; Table 2 shows the criteria weights and the linguistic assessment of each EOL strategy. According to Table 1 these can be obtained from the experiment, and Table 3 indicates the corresponding linguistic assessment of the triangular fuzzy numbers.

Second Step: an α-level cut set operation performed on the triangular fuzzy numbers in Tables 3 and 4 are the operation results of $\alpha = 0$; $\alpha = 1$ can be obtained similarly.

Third Step: Combining the data in Table 4, we apply a single attribute to calculate the value of the joint attributes E_i^j and λ through Theorem 2 and $\sigma - \lambda$ rules. The calculation results are shown in Table 5; $E_i^j = \{x_i^j, x_i^{j+1}, ..., x_i^n\}$. If $i = 1$, then $n = 2$, if $i = 2$, then $n = 5$, if $i = 3$, then $n = 4$, and if $i = 4$, then $n = 3$, $1 \le j \le n$. Furthermore, $\underline{g}_\lambda(j) = \underline{g}_\lambda(E_i^j)$, $\overline{g}_\lambda(j) = \overline{g}_\lambda(E_i^j)$.

Table 4. The value of α-level cut set for $\alpha = 0$ of Table 3.

Criteria	Weights	A_1	A_2	A_3	A_4	A_5	A_6
x_1	$[0.8, 1]$						
x_1^1	$[0.8, 1]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0.8, 1]$	$[0.8, 1]$	$[0, 0.2]$
x_1^2	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.8, 1]$	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0, 0.2]$
x_2	$[0.7, 0.9]$						
x_2^1	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0.4, 0.6]$	$[0, 0.2]$	$[0, 0.2]$
x_2^2	$[0.4, 0.6]$	$[0.8, 1]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.1, 0.3]$
x_2^3	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0.8, 1]$	$[0.8, 1]$	$[0.4, 0.6]$	$[0.8, 1]$	$[0.7, 0.9]$
x_2^4	$[0.8, 1]$	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0.4, 0.6]$
x_2^5	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.8, 1]$	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.1, 0.3]$	$[0, 0.2]$
x_3	$[0.8, 1]$						
x_3^1	$[0.8, 1]$	$[0.7, 0.9]$	$[0.1, 0.3]$	$[0.8, 1]$	$[0.4, 0.6]$	$[0.8, 1]$	$[0.8, 1]$
x_3^2	$[0.4, 0.6]$	$[0.1, 0.3]$	$[0, 0.2]$	$[0.8, 1]$	$[0.7, 0.9]$	$[0.1, 0.3]$	$[0.7, 0.9]$
x_3^3	$[0.7, 0.9]$	$[0.8, 1]$	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0, 0.2]$	$[0.7, 0.9]$
x_3^4	$[0.1, 0.3]$	$[0.4, 0.6]$	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0.4, 0.6]$	$[0.7, 0.9]$
x_4	$[0.4, 0.6]$						
x_4^1	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0.1, 0.3]$	$[0.4, 0.6]$	$[0.4, 0.6]$
x_4^2	$[0.4, 0.6]$	$[0.8, 1]$	$[0.7, 0.9]$	$[0.8, 1]$	$[0.1, 0.3]$	$[0.8, 1]$	$[0.4, 0.6]$
x_4^3	$[0.4, 0.6]$	$[0.8, 1]$	$[0.8, 1]$	$[0.8, 1]$	$[0, 0.2]$	$[0.8, 1]$	$[0.8, 1]$

Fourth Step: calculate the evaluation value of the sub-criteria x_j^i regarding the EOL options $A_t(t = 1, 2, 3, 4, 5, 6)$. According to the third step, we know that $(f_{i,t}^j)_\alpha$ stands for the function $f_{i,t}^j$ α-level cut set, while $(f_{i,t}^j)_0$ stands for the 0-level cut set of function $f_{i,t}^j$.

For EOL options A_1 about criteria x_2, $\alpha = 0$.

(1) From Table 4, we can obtain $(f_{2,1}^1)_0 = [0.7, 0.9]$, $(f_{2,1}^2)_0 = [0.8, 1]$, $(f_{2,1}^3)_0 = [0.4, 0.6]$, $(f_{2,1}^4)_0 = [0.4, 0.6]$, $(f_{2,1}^5)_0 = [0.7, 0.9]$.

(2) From Table 4, we can obtain $(g_\lambda)_2^1 = [0.7, 0.9]$, $(g_\lambda)_2^2 = [0.4, 0.6]$, $(g_\lambda)_2^3 = [0.7, 0.9]$, $(g_\lambda)_2^4 = [0.8, 1]$, $(g_\lambda)_2^5 = [0.4, 0.6]$.

(3) From Definition 4, $\underline{f}_{i,t}^j$ and $\overline{f}_{i,t}^j$ stand for the left and right endpoints of the α-level cut set of the function $f_{i,t}^j$ respectively. In order of magnitude $\underline{f}_{i,t}^j$, we have $\underline{f}_{2,1}^3 \le \underline{f}_{2,1}^4 \le \underline{f}_{2,1}^1 \le \underline{f}_{2,1}^5 \le \underline{f}_{2,1}^2$. Then, we have $x_2^{1'}, x_2^{2'}, x_2^{3'}, x_2^{4'}, x_2^{5'}$, which is a permutation of x_2^1, x_2^2, x_2^3, x_2^4, x_2^5; then, $x_2^{1'} = x_2^3$, $x_2^{2'} = x_2^4$, $x_2^{3'} = x_2^1$, $x_2^{4'} = x_2^5$, $x_2^{5'} = x_2^2$.

The value of parameter λ_1 is calculated using Theorem 2, that is $\lambda_1 = -0.993$. Furthermore, we can calculate the weight of the joint attribute according to σ-λ rules and obtain their values as $\underline{g}_\lambda(j) = \underline{g}_\lambda(E_i^j) = \underline{g}_\lambda\{x_i^{j'}, x_i^{(j+1)'}, ..., x_i^{n'}\}$,

$$\underline{g}_\lambda(5) = 0.4, \underline{g}_\lambda(4) = 0.641, \underline{g}_\lambda(3) = 0.896, \underline{g}_\lambda(2) = 0.984, \underline{g}_\lambda(1) = 1.$$

Table 5 lists all measured values and the values of the required parameters λ.

Table 5. The values of fuzzy measure sub-criteria for $\alpha = 0$.

A_1 $\underline{g}_{\lambda}(E^j)\overline{g}_{\lambda}(E^j)$	A_2 $\underline{g}_{\lambda}(E^j)\overline{g}_{\lambda}(E^j)$	A_3 $\underline{g}_{\lambda}(E^j)\overline{g}_{\lambda}(E^j)$
$\lambda_1 = -0.625\lambda_2 = -1$	$\lambda_1 = -0.625\lambda_2 = -1$	$\lambda_1 = -0.625\lambda_2 = -1$
$\underline{g}_{\lambda}(2) = 0.4\overline{g}_{\lambda}(2) = 0.6$	$\underline{g}_{\lambda}(2) = 0.4\overline{g}_{\lambda}(2) = 0.6$	$\underline{g}_{\lambda}(2) = 0.4\overline{g}_{\lambda}(2) = 0.6$
$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$
$\lambda_1 = -0.993\lambda_2 = -1$	$\lambda_1 = -0.993\lambda_2 = -1$	$\lambda_1 = -0.993\lambda_2 = -1$
$\underline{g}_{\lambda}(5) = 0.4\overline{g}_{\lambda}(5) = 0.6$	$\underline{g}_{\lambda}(5) = 0.4\overline{g}_{\lambda}(5) = 0.6$	$\underline{g}_{\lambda}(5) = 0.7\overline{g}_{\lambda}(5) = 0.9$
$\underline{g}_{\lambda}(4) = 0.641\overline{g}_{\lambda}(4) = 0.84$	$\underline{g}_{\lambda}(4) = 0.822\overline{g}_{\lambda}(4) = 0.96$	$\underline{g}_{\lambda}(4) = 0.944\overline{g}_{\lambda}(4) = 1$
$\underline{g}_{\lambda}(3) = 0.896\overline{g}_{\lambda}(3) = 0.984$	$\underline{g}_{\lambda}(3) = 0.967\overline{g}_{\lambda}(3) = 1$	$\underline{g}_{\lambda}(3) = 0.969\overline{g}_{\lambda}(3) = 1$
$\underline{g}_{\lambda}(2) = 0.984\overline{g}_{\lambda}(2) = 1$	$\underline{g}_{\lambda}(2) = 0.984\overline{g}_{\lambda}(2) = 1$	$\underline{g}_{\lambda}(2) = 0.984\overline{g}_{\lambda}(2) = 1$
$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$
$\lambda_1 = -0.975\lambda_2 = -1$	$\lambda_1 = -0.975\lambda_2 = -1$	$\lambda_1 = -0.975\lambda_2 = -1$
$\underline{g}_{\lambda}(4) = 0.7\overline{g}_{\lambda}(4) = 0.9$	$\underline{g}_{\lambda}(4) = 0.1\overline{g}_{\lambda}(4) = 0.3$	$\underline{g}_{\lambda}(4) = 0.4\overline{g}_{\lambda}(4) = 0.6$
$\underline{g}_{\lambda}(3) = 0.964\overline{g}_{\lambda}(3) = 1$	$\underline{g}_{\lambda}(3) = 0.733\overline{g}_{\lambda}(3) = 0.93$	$\underline{g}_{\lambda}(3) = 0.888\overline{g}_{\lambda}(3) = 1$
$\underline{g}_{\lambda}(2) = 0.972\overline{g}_{\lambda}(2) = 1$	$\underline{g}_{\lambda}(2) = 0.972\overline{g}_{\lambda}(2) = 1$	$\underline{g}_{\lambda}(2) = 0.901\overline{g}_{\lambda}(2) = 1$
$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$
$\lambda_1 = -0.443\lambda_2 = -0.904$	$\lambda_1 = -0.443\lambda_2 = -0.904$	$\lambda_1 = -0.443\lambda_2 = -0.904$
$\underline{g}_{\lambda}(3) = 0.4\overline{g}_{\lambda}(3) = 0.6$	$\underline{g}_{\lambda}(3) = 0.4\overline{g}_{\lambda}(3) = 0.6$	$\underline{g}_{\lambda}(3) = 0.4\overline{g}_{\lambda}(3) = 0.6$
$\underline{g}_{\lambda}(2) = 0.729\overline{g}_{\lambda}(2) = 0.875$	$\underline{g}_{\lambda}(2) = 0.729\overline{g}_{\lambda}(2) = 0.875$	$\underline{g}_{\lambda}(2) = 0.729\overline{g}_{\lambda}(2) = 0.875$
$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$
A_4 $\underline{g}_{\lambda}(E^j)\overline{g}_{\lambda}(E^j)$	**A_5** $\underline{g}_{\lambda}(E^j)\overline{g}_{\lambda}(E^j)$	**A_6** $\underline{g}_{\lambda}(E^j)\overline{g}_{\lambda}(E^j)$
$\lambda_1 = -0.625\lambda_2 = -1$	$\lambda_1 = -0.625\lambda_2 = -1$	$\lambda_1 = -0.625\lambda_2 = -1$
$\underline{g}_{\lambda}(2) = 0.8\overline{g}_{\lambda}(2) = 0.1$	$\underline{g}_{\lambda}(2) = 0.8\overline{g}_{\lambda}(2) = 0.1$	$\underline{g}_{\lambda}(2) = 0.4\overline{g}_{\lambda}(2) = 0.6$
$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$
$\lambda_1 = -0.993\lambda_2 = -1$	$\lambda_1 = -0.993\lambda_2 = -1$	$\lambda_1 = -0.993\lambda_2 = -1$
$\underline{g}_{\lambda}(5) = 0.7\overline{g}_{\lambda}(5) = 0.9$	$\underline{g}_{\lambda}(5) = 0.7\overline{g}_{\lambda}(5) = 0.9$	$\underline{g}_{\lambda}(5) = 0.7\overline{g}_{\lambda}(5) = 0.9$
$\underline{g}_{\lambda}(4) = 0.822\overline{g}_{\lambda}(4) = 0.96$	$\underline{g}_{\lambda}(4) = 0.944\overline{g}_{\lambda}(4) = 1$	$\underline{g}_{\lambda}(4) = 0.944\overline{g}_{\lambda}(4) = 1$
$\underline{g}_{\lambda}(3) = 0.969\overline{g}_{\lambda}(3) = 1$	$\underline{g}_{\lambda}(3) = 0.969\overline{g}_{\lambda}(3) = 1$	$\underline{g}_{\lambda}(3) = 0.969\overline{g}_{\lambda}(3) = 1$
$\underline{g}_{\lambda}(2) = 0.984\overline{g}_{\lambda}(2) = 1$	$\underline{g}_{\lambda}(2) = 0.984\overline{g}_{\lambda}(2) = 1$	$\underline{g}_{\lambda}(2) = 0.984\overline{g}_{\lambda}(2) = 1$
$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$
$\lambda_1 = -0.975\lambda_2 = -1$	$\lambda_1 = -0.975\lambda_2 = -1$	$\lambda_1 = -0.975\lambda_2 = -1$
$\underline{g}_{\lambda}(4) = 0.7\overline{g}_{\lambda}(4) = 0.9$	$\underline{g}_{\lambda}(4) = 0.8\overline{g}_{\lambda}(4) = 1$	$\underline{g}_{\lambda}(4) = 0.8\overline{g}_{\lambda}(4) = 1$
$\underline{g}_{\lambda}(3) = 0.827\overline{g}_{\lambda}(3) = 1$	$\underline{g}_{\lambda}(3) = 0.823\overline{g}_{\lambda}(3) = 1$	$\underline{g}_{\lambda}(3) = 0.823\overline{g}_{\lambda}(3) = 1$
$\underline{g}_{\lambda}(2) = 0.846\overline{g}_{\lambda}(2) = 0.927$	$\underline{g}_{\lambda}(2) = 0.908\overline{g}_{\lambda}(2) = 1$	$\underline{g}_{\lambda}(2) = 0.927\overline{g}_{\lambda}(2) = 1$
$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$
$\lambda_1 = -0.443\lambda_2 = -0.904$	$\lambda_1 = -0.443\lambda_2 = -0.904$	$\lambda_1 = -0.443\lambda_2 = -0.904$
$\underline{g}_{\lambda}(3) = 0.4\overline{g}_{\lambda}(3) = 0.6$	$\underline{g}_{\lambda}(3) = 0.4\overline{g}_{\lambda}(3) = 0.6$	$\underline{g}_{\lambda}(3) = 0.4\overline{g}_{\lambda}(3) = 0.6$
$\underline{g}_{\lambda}(2) = 0.729\overline{g}_{\lambda}(2) = 0.875$	$\underline{g}_{\lambda}(2) = 0.729\overline{g}_{\lambda}(2) = 0.875$	$\underline{g}_{\lambda}(2) = 0.729\overline{g}_{\lambda}(2) = 0.875$
$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$	$\underline{g}_{\lambda}(1) = 1\overline{g}_{\lambda}(1) = 1$

(4) The value of the main criteria ($x_i, i = 1,2,3,4$)) regarding the EOL options $A_t(t = 1,2,3,4,5)$ are calculated using the C_T-integral as follows; taking "primary criteria x_2 of EOL options A_1", for example, $\alpha = 0$:

$$
\begin{aligned}
(c)\int \underline{f}_{2,1}^{j} d\underline{g}_{\lambda} &= \min\{\underline{f}_{2,1}^{3}, \underline{g}_{\lambda}(1)\} + \min\{\underline{f}_{2,1}^{4} - \underline{f}_{2,1}^{3}, \underline{g}_{\lambda}(2)\} + \min\{\underline{f}_{2,1}^{1} - \underline{f}_{2,1}^{4}, \underline{g}_{\lambda}(3)\} \\
&\quad + \min\{\underline{f}_{2,1}^{5} - \underline{f}_{2,1}^{1}, \underline{g}_{\lambda}(4)\} + \min\{\underline{f}_{2,1}^{2} - \underline{f}_{2,1}^{5}, \underline{g}_{\lambda}(5)\} \\
&= \min\{0.4 - 0, 1\} + \min\{0.4 - 0.4, 0.972\} + \min\{0.7 - 0.4, 0.896\} \\
&\quad + \min\{0.7 - 0.7, 0.641\} + \min\{0.8 - 0.7, 0.4\} \\
&= 0.4 + 0 + 0.3 + 0 + 0.1 \\
&= 0.8.
\end{aligned}
$$

In the same manner, $(c)\int \overline{f}^{j}_{2,1}\,d\overline{g}_{\lambda} = 1$. Therefore,

$$(c)\int f^{j}_{2,1}\,dg_{\lambda} = \left[(c)\int \underline{f}^{j}_{2,1}\,d\underline{g}_{\lambda}, (c)\int \overline{f}^{j}_{2,1}\,d\overline{g}_{\lambda}\right] = [0.8, 1].$$

Similarly, we can calculate the evaluation values for the remaining sub-criteria, as shown in Table 6.

Table 6. The evaluation value of primary criteria for $\alpha = 0$ with respect to the cabinet frame.

Criteria	Weights	A_1	A_2	A_3	A_4	A_5	A_6
x_1	[0.8, 1]	[0.7, 0.9]	[0.7, 0.9]	[0.8, 1]	[0.8, 1]	[0.7, 0.8]	[0, 0.2]
x_1^1	[0.8, 1]	[0.7, 0.9]	[0.7, 0.9]	[0.4, 0.6]	[0.8, 1]	[0.8, 1]	[0, 0.2]
x_1^2	[0.4, 0.6]	[0.7, 0.9]	[0.7, 0.9]	[0.8, 1]	[0.7, 0.9]	[0.4, 0.6]	[0, 0.2]
x_2	[0.7, 0.9]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.7, 0.9]	[0.8, 1]	[0.8, 1]
x_2^1	[0.7, 0.9]	[0.7, 0.9]	[0.7, 0.9]	[0.4, 0.6]	[0.4, 0.6]	[0, 0.2]	[0, 0.2]
x_2^2	[0.4, 0.6]	[0.8, 1]	[0.7, 0.9]	[0.7, 0.9]	[0.7, 0.9]	[0.7, 0.9]	[0.1, 0.3]
x_2^3	[0.7, 0.9]	[0.4, 0.6]	[0.8, 1]	[0.8, 1]	[0.4, 0.6]	[0.8, 1]	[0.7, 0.9]
x_2^4	[0.8, 1]	[0.4, 0.6]	[0.7, 0.9]	[0.7, 0.9]	[0.7, 0.9]	[0.4, 0.6]	[0.4, 0.6]
x_2^5	[0.4, 0.6]	[0.7, 0.9]	[0.8, 1]	[0.4, 0.6]	[0.7, 0.9]	[0.1, 0.3]	[0, 0.2]
x_3	[0.8, 1]	[0.8, 1]	[0.4, 0.6]	[0.8, 1]	[0.7, 0.9]	[0.8, 1]	[0.8, 1]
x_3^1	[0.8, 1]	[0.7, 0.9]	[0.1, 0.3]	[0.8, 1]	[0.4, 0.6]	[0.8, 1]	[0.8, 1]
x_3^2	[0.4, 0.6]	[0.1, 0.3]	[0, 0.2]	[0.8, 1]	[0.7, 0.9]	[0.1, 0.3]	[0.7, 0.9]
x_3^3	[0.7, 0.9]	[0.8, 1]	[0.4, 0.6]	[0.7, 0.9]	[0.7, 0.9]	[0, 0.2]	[0.7, 0.9]
x_3^4	[0.1, 0.3]	[0.4, 0.6]	[0.4, 0.6]	[0.7, 0.9]	[0.4, 0.6]	[0.4, 0.6]	[0.7, 0.9]
x_4	[0.4, 0.6]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0.1, 0.3]	[0.8, 1]	[0.8, 1]
x_4^1	[0.4, 0.6]	[0.7, 0.9]	[0.7, 0.9]	[0.4, 0.6]	[0.1, 0.3]	[0.4, 0.6]	[0.4, 0.6]
x_4^2	[0.4, 0.6]	[0.8, 1]	[0.7, 0.9]	[0.8, 1]	[0.1, 0.3]	[0.8, 1]	[0.4, 0.6]
x_4^3	[0.4, 0.6]	[0.8, 1]	[0.8, 1]	[0.8, 1]	[0, 0.2]	[0.8, 1]	[0.8, 1]

Fifth Step: The C_T-integral on the interval-value Sugeno probability measure is used to calculate the EOL options (A_1, A_2, A_3, A_4, A_5, A_6) of the values of the main criteria, $x_i (i = 1, 2, 3, 4)$.

For EOL option A_1, $\alpha = 0$; then,

(1) From Table 6, we can obtain $(f_{1,1})_0 = [0.7, 0.9]$, $(f_{2,1})_0 = [0.8, 1]$, $(f_{3,1})_0 = [0.8, 1]$, $(f_{4,1})_0 = [0.8, 1]$.

(2) From Table 6, we can obtain $g_{\lambda}(x_1) = [0.8, 1]$, $g_{\lambda}(x_2) = [0.7, 0.9]$, $g_{\lambda}(x_3) = [0.8, 1]$, $g_{\lambda}(x_4) = [0.4, 0.6]$.

(3) In order of magnitude $\underline{f}_{i,1}, \underline{f}_{i,1} (i = 1, 2, 3, 4)$, there are $\underline{f}_{1,1} \leq \underline{f}_{2,1} \leq \underline{f}_{3,1} \leq \underline{f}_{4,1}$. Then, we have x'_1, x'_2, x'_3, x'_4, which is a permutation of x^1_2, x^2_2, x^3_2, x^4_2, where $x'_1 = x_1$, $x'_2 = x_2$, $x'_3 = x_3$, $x'_4 = x_4$.
Then, the values of the parameters λ and the weights of the joint attributes on the main criterion were calculated in the same way. These are listed in Table 7.

(4) For the EOL options, $A_t (t = 1, 2, 3, 4, 5, 6)$ are calculated using the C_T-integral as follows:

$$\begin{aligned}
(c)\int \underline{f}_{i,1}\,d\underline{g}_{\lambda} &= \min\{\underline{f}_{2,1}, \underline{g}_{\lambda}(1)\} + \min\{\underline{f}_{2,1} - \underline{f}_{2,1}, \underline{g}_{\lambda}(2)\} + \min\{\underline{f}_{2,1} - \underline{f}_{2,1}, \underline{g}_{\lambda}(3)\} \\
&+ \min\{\underline{f}_{2,1} - \underline{f}_{2,1}, \underline{g}_{\lambda}(4)\} + \min\{\underline{f}_{2,1} - \underline{f}_{2,1}, \underline{g}_{\lambda}(5)\} \\
&= \min\{0.4 - 0, 1\} + \min\{0.4 - 0.4, 0.972\} + \min\{0.7 - 0.4, 0.896\} \\
&+ \min\{0.7 - 0.7, 0.641\} + \min\{0.8 - 0.7, 0.4\} \\
&= 0.4 + 0 + 0.3 + 0 + 0.1 \\
&= 0.8.
\end{aligned}$$

In the same way, $(c)\int \overline{f}_{i,1}\,d\overline{g}_{\lambda} = 1$. Therefore,

$$(c)\int f_{2,1}\,dg_{\lambda} = \left[(c)\int \underline{f}_{2,1}\,d\underline{g}_{\lambda}, (c)\int \overline{f}_{2,1}\,d\overline{g}_{\lambda}\right] = [0.8, 1].$$

Similarly, it is possible to calculate the evaluation values for the remaining primary criteria, as shown in Table 8.

Sixth Step: In the process of fuzzy number processing, as the triangular fuzzy numbers cannot be applied directly we must first defuzzify the fuzzy numbers before applying them. There are many methods of defuzzification, and in this study, we choose the mean value method of defuzzification. The mean value was calculated for each of the triangular fuzzy numbers in Table 8 in order to convert the fuzzy number into an exact number.

A similar optimal EOL option for other the components can be obtained as shown in Table 9.

For data comparison, in Tables 1–5 and Tables 10–12 we use the data in [23], while Tables 6, 8 and 9 show the results obtained in this paper. Although the final EOL strategy standards reached by the two are the same, the final data obtained are different. Compared with [23], the data with the method in this paper are more concise.

Table 7. The value of fuzzy measures on the primary criteria for $\alpha = 0$ with respect to the cabinet frame.

A_1 $\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$	A_2 $\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$	A_3 $\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$
$\lambda_1 = -0.993\lambda_2 = -1$	$\lambda_1 = -0.993\lambda_2 = -1$	$\lambda_1 = -0.993\lambda_2 = -1$
$\underline{g}_\lambda(4) = 0.641\overline{g}_\lambda(4) = 0.84$	$\underline{g}_\lambda(4) = 0.822\overline{g}_\lambda(4) = 0.96$	$\underline{g}_\lambda(4) = 0.944\overline{g}_\lambda(4) = 1$
$\underline{g}_\lambda(3) = 0.896\overline{g}_\lambda(3) = 0.984$	$\underline{g}_\lambda(3) = 0.967\overline{g}_\lambda(3) = 1$	$\underline{g}_\lambda(3) = 0.969\overline{g}_\lambda(3) = 1$
$\underline{g}_\lambda(2) = 0.984\overline{g}_\lambda(2) = 1$	$\underline{g}_\lambda(2) = 0.984\overline{g}_\lambda(2) = 1$	$\underline{g}_\lambda(2) = 0.984\overline{g}_\lambda(2) = 1$
$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$
A_4 $\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$	A_5 $\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$	A_6 $\underline{g}_\lambda(E^j)\overline{g}_\lambda(E^j)$
$\lambda_1 = -0.993\lambda_2 = -1$	$\lambda_1 = -0.993\lambda_2 = -1$	$\lambda_1 = -0.993\lambda_2 = -1$
$\underline{g}_\lambda(4) = 0.822\overline{g}_\lambda(4) = 0.96$	$\underline{g}_\lambda(4) = 0.944\overline{g}_\lambda(4) = 1$	$\underline{g}_\lambda(4) = 0.944\overline{g}_\lambda(4) = 1$
$\underline{g}_\lambda(3) = 0.969\overline{g}_\lambda(3) = 1$	$\underline{g}_\lambda(3) = 0.969\overline{g}_\lambda(3) = 1$	$\underline{g}_\lambda(3) = 0.969\overline{g}_\lambda(3) = 1$
$\underline{g}_\lambda(2) = 0.984\overline{g}_\lambda(2) = 1$	$\underline{g}_\lambda(2) = 0.984\overline{g}_\lambda(2) = 1$	$\underline{g}_\lambda(2) = 0.984\overline{g}_\lambda(2) = 1$
$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$	$\underline{g}_\lambda(1) = 1\overline{g}_\lambda(1) = 1$

Table 8. The comprehensive evaluation value of EOL options with respect to the cabinet frame.

Main Criteria	$(c)\int f dg_\lambda$ A_1 A_2 A_3	Crisp Number A_1 A_2 A_3
Overall EOL	$(0.8, 0.9, 1)$ $(0.8, 0.9, 1)$ $(0.8, 0.99, 1)$	0.9 0.9 0.933 *
Main criteria	$(c)\int f dg_\lambda$ A_4 A_5 A_6	Crisp number A_4 A_5 A_6
Overall EOL	$(0.8, 0.9, 1)$ $(0.8, 0.9, 1)$ $(0.8, 0.9, 1)$	0.9 0.9 0.9

* Represents the largest value in each group.

In the above example, we know that Table 10 shows the result of the sub-criteria for the general Choquet aggregation and Table 6 the result of the sub-criteria for the aggregation of C_{T_M}-integral. Tables 11 and 12 contain the final results of the aggregation of the general Choquet integral, while Tables 8 and 9 are the final results of the aggregation of C_{T_M}-integral. By comparing them, we know that the same result as the general Choquet integral can be obtained when the t-norm in the C_T integral is taken as the minimum t-norm (T_M). In this case, the C_T integral is more advantageous than the Choquet integral in terms of calculation efficiency.

Table 9. Refrigerator component relevant closeness RC and appropriate EOL strategy.

Component	A_1	A_2	A_3	A_4	A_5	A_6	Appropriate EOL Strategy
Cabinet frame	0.9	0.9	0.933 *	0.9	0.9	0.9	Primary recycle
Cabinet	0.9	0.9	0.9	0.933 *	0.8	0.9	Secondary recycle
Duct in room	0.9	0.9	0.9	0.9	0.933 *	0.8	Incinerate
Fan unit 1	0.9	0.9	0.933 *	0.8	0.9	0.8	Primary recycle
Fan unit 2	0.933 *	0.8	0.8	0.7	0.9	0.8	Reuse
Evaporator	0.933 *	0.9	0.9	0.8	0.9	0.9	Reuse
Rear board	0.9	0.8	0.933 *	0.8	0.9	0.9	Primary recycle
Compressor	0.9	0.9	0.933 *	0.9	0.9	0.8	Primary recycle
Condenser	0.9	0.9	0.933 *	0.8	0.9	0.8	Primary recycle
Base	0.9	0.933 *	0.9	0.8	0.8	0.9	Remanufacturing
Door 1	0.9	0.9	0.9	0.8	0.933 *	0.9	Incinerate
Door 2	0.933 *	0.8	0.8	0.7	0.9	0.8	Reuse
Gasket 1	0.9	0.9	0.933 *	0.7	0.9	0.8	Primary recycle
Gasket 2	0.9	0.8	0.9	0.933 *	0.9	0.9	Secondary recycle
Door liner 1	0.9	0.9	0.9	0.933 *	0.9	0.8	Secondary recycle
Door liner 2	0.9	0.9	0.8	0.933 *	0.8	0.8	Secondary recycle
Control unit	0.933 *	0.9	0.9	0.9	0.9	0.8	Reuse
Heater	0.9	0.9	0.933 *	0.8	0.9	0.8	Primary recycle
Dryer	0.9	0.9	0.933 *	0.8	0.9	0.9	Primary recycle
Shelf set	0.9	0.8	0.933 *	0.8	0.9	0.9	Primary recycle

* Represents the largest value in each group.

Table 10. The evaluation value of primary criteria for $\alpha = 0$ with respect to the cabinet frame.

Criteria	Weights	A_1	A_2	A_3	A_4	A_5	A_6
x_1	$[0.8, 1]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.56, 0.84]$	$[0.78, 1]$	$[0.75, 1]$	$[0, 0.2]$
x_1^1	$[0.8, 1]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0.8, 1]$	$[0.8, 1]$	$[0, 0.2]$
x_1^2	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.8, 1]$	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0, 0.2]$
x_2	$[0.7, 0.9]$	$[0.709, 0.955]$	$[0.782, 0.996]$	$[0.761, 0.99]$	$[0.691, 0.888]$	$[0.742, 0.99]$	$[0.59, 0.87]$
x_2^1	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0.4, 0.6]$	$[0, 0.2]$	$[0, 0.2]$
x_2^2	$[0.4, 0.6]$	$[0.8, 1]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.1, 0.3]$
x_2^3	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0.8, 1]$	$[0.8, 1]$	$[0.4, 0.6]$	$[0.8, 1]$	$[0.7, 0.9]$
x_2^4	$[0.8, 1]$	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0.4, 0.6]$
x_2^5	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.8, 1]$	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.1, 0.3]$	$[0, 0.2]$
x_3	$[0.8, 1]$	$[0.751, 0.99]$	$[0.317, 0.579]$	$[0.788, 1]$	$[0.648, 0.888]$	$[0.657, 1]$	$[0.78, 1]$
x_3^1	$[0.8, 1]$	$[0.7, 0.9]$	$[0.1, 0.3]$	$[0.8, 1]$	$[0.4, 0.6]$	$[0.8, 1]$	$[0.8, 1]$
x_3^2	$[0.4, 0.6]$	$[0.1, 0.3]$	$[0, 0.2]$	$[0.8, 1]$	$[0.7, 0.9]$	$[0.1, 0.3]$	$[0.7, 0.9]$
x_3^3	$[0.7, 0.9]$	$[0.8, 1]$	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0, 0.2]$	$[0.7, 0.9]$
x_3^4	$[0.1, 0.3]$	$[0.4, 0.6]$	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0.4, 0.6]$	$[0.7, 0.9]$
x_4	$[0.4, 0.6]$	$[0.773, 0.988]$	$[0.74, 0.96]$	$[0.691, 0.95]$	$[0.073, 0.287]$	$[0.692, 0.95]$	$[0.56, 0.84]$
x_4^1	$[0.4, 0.6]$	$[0.7, 0.9]$	$[0.7, 0.9]$	$[0.4, 0.6]$	$[0.1, 0.3]$	$[0.4, 0.6]$	$[0.4, 0.6]$
x_4^2	$[0.4, 0.6]$	$[0.8, 1]$	$[0.7, 0.9]$	$[0.8, 1]$	$[0.1, 0.3]$	$[0.8, 1]$	$[0.4, 0.6]$
x_4^3	$[0.4, 0.6]$	$[0.8, 1]$	$[0.8, 1]$	$[0.8, 1]$	$[0, 0.2]$	$[0.8, 1]$	$[0.8, 1]$

Table 11. The comprehensive evaluation value of EOL options with respect to the cabinet frame.

Main Criteria	$(c)\int f dg_\lambda$ A_1 A_2 A_3	Crisp number A_1 A_2 A_3
Overall EOL	$(0.755, 0.876, 0.99)$ $(0.75, 0.874, 0.989)$ $(0.755, 0.892, 1)$	0.874 0.871 0.882 *
Main criteria	$(c)\int f dg_\lambda$ A_4 A_5 A_6	Crisp number A_4 A_5 A_6
Overall EOL	$(0.757, 0.879, 1)$ $(0.733, 0.858, 1)$ $(0.723, 0.877, 1)$	0.879 0.864 0.867

* Represents the largest value in each group.

Table 12. Refrigerator component relevant closeness RC and appropriate EOL strategy.

Component	A_1	A_2	A_3	A_4	A_5	A_6	Appropriate EOL Strategy
Cabinet frame	0.874	0.871	0.882 *	0.879	0.864	0.867	Primary recycle
Cabinet	0.8859	0.871	0.8644	0.8947 *	0.8457	0.88	Secondary recycle
Duct in room	0.8834	0.8785	0.8725	0.8667	0.8878 *	0.8363	Incinerate
Fan unit 1	0.8787	0.8685	0.8981 *	0.7563	0.8889	0.7908	Primary recycle
Fan unit 2	0.8847 *	0.8051	0.8465	0.7191	0.8633	0.8074	Reuse
Evaporator	0.8974 *	0.8919	0.891	0.8495	0.8677	0.8821	Reuse
Rear board	0.8759	0.8382	0.8974 *	0.8473	0.8954	0.879	Primary recycle
Compressor	0.8873	0.8739	0.8927 *	0.8697	0.8887	0.841	Primary recycle
Condenser	0.8538	0.8683	0.8944 *	0.7563	0.8869	0.834	Primary recycle
Base	0.8741	0.8856 *	0.8729	0.8493	0.8326	0.881	Remanufacturing
Door 1	0.8573	0.8737	0.8737	0.8475	0.8886 *	0.8752	Incinerate
Door 2	0.8954 *	0.8051	0.8196	0.7272	0.8849	0.8093	Reuse
Gasket 1	0.8533	0.8685	0.8939 *	0.6727	0.8868	0.776	Primary recycle
Gasket 2	0.8532	0.8381	0.8964	0.8974 *	0.8914	0.888	Secondary recycle
Door liner 1	0.8851	0.8788	0.8726	0.8987 *	0.8667	0.8364	Secondary recycle
Door liner 2	0.8677	0.8774	0.754	0.8886 *	0.8422	0.7979	Secondary recycle
Control unit	0.8981 *	0.8788	0.8779	0.8667	0.893	0.8364	Reuse
Heater	0.8759	0.8685	0.8944 *	0.7563	0.8889	0.8341	Primary recycle
Dryer	0.8684	0.8736	0.8936 *	0.8382	0.8745	0.867	Primary recycle
Shelf set	0.8566	0.8382	0.8953 *	0.8471	0.8941	0.888	Primary recycle

* Represents the largest value in each group.

5. Conclusions

Through the derivation process and examples provided in the paper, we find that the C_T-integral on the interval-valued Sugeno probability measure is more advantageous than the general Choquet integral on the interval-valued Sugeno probability measure. Moreover, the C_T-integral on interval-valued Sugeno probability measure improves the computational procedure, making the computation simpler and less intensive compared to the general Choquet integral on interval-valued Sugeno probability measure. Furthermore, the C_T-integral is the Choquet integral when the t-norm is $T(x, y) = xy$ in the C_T-integral. In addition, this paper only provides the discrete expression of the C_T-integral on the interval-valued Sugeno probability measure, in particular its application in multi-criteria decision-making problems; its specific properties are not studied, nor are its properties as a pre-aggregation function. These properties of the C_T-integral should be considered in future work in order to obtain a better understand of the C_T-integral and its potential applications.

Although this paper studies C_T-integral, it only studies its applications and characteristics in the context of the interval-valued Sugeno measure. Important research on the C_T-integral has yet to be carried out. For example, the C_T-integral is more widely applicable than the Choquet integral. The calculation intensity of the C_{T_M}-integral o then C_T-integral is less than that od the Choquet integral, thus, whether $T_Ł$, T_{DP}, T_{NM}, and T_{HP} are the same as the C_T-integral is worth studying in the future.

Author Contributions: H.Y., L.S. and Z.G. completed the main study together. L.S. wrote the manuscript, H.Y. checked the proofs process and verified the calculation. All the authors read and approved the last version of the manuscript.

Funding: This work was supported by the Natural Science Foundation of China (No.12161082, No.12061067), the Natural Science Foundation of Gansu Province (No.21JR7RA134), and the University Innovation and Entrepreneurship Fund of Gansu Province (2020A-009).

Data Availability Statement: Data is contained within the article.

Acknowledgments: The authors are very grateful to the anonymous referees.

Conflicts of Interest: The authors declare no conflict of interest.

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