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Evaluating Lean Facility Layout Designs Using a BWM-Based Fuzzy ELECTRE I Method

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Abstract: Selecting a suitable design for a lean facility layout has become a critical issue for a manufacturing company aiming to remove waste or non-value-added activities and implement the optimal facility arrangement. Many quantitative and qualitative criteria with different weights need to be considered in evaluating lean facility layout designs. To address the issue, a Best-Worst method (BWM) based on fuzzy ELECTRE I is introduced to determine the optimal lean facility layout design, in which the BWM is utilized for generating the criteria weights, and an extension of fuzzy ELECTRE I is introduced to identify the most suitable alternative. The signed distance method is employed to defuzzify the fuzzy numbers and obtain discordance matrix values. Based on the subtraction of discordance values from concordance values, a modified fuzzy ELECTRE I is introduced to evaluate alternative lean facility layout designs that can avoid missing information. A numerical example of the evaluation of lean facility layout designs for a manufacturing company is provided to show the potential of the suggested models. Comparative studies are investigated to illustrate the superiority of the suggested method.

Keywords: lean facility layout design; BWM; fuzzy ELECTRE I; signed distance

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1. Introduction

A facility's layout plays a tremendously important role in production system designs. Maniya and Bhatt [1] defined the common facility layout as "an integration of the physical arrangement of machines, materials, departments, workstations, storage areas, and common areas within an existing or proposed facility for processing a product in the most efficient manner". Tompkins et al. [2] noted that facility planning may allow 10–30% of operating cost for changes in the system. Hence, an evaluation of the facility layout design can be conducted from a strategic viewpoint [3]. To maintain and enhance competitiveness while achieving sustainable production, companies need to apply efficiency improvement methods to help reduce costs and increase productivity [4]. Lean facility layout designs can be understood as the physical equipment arrangement in a company aiming to help a facility function productively. When considering alternative layouts, several possible solutions need to be evaluated in parallel to choose the optimal lean facility layout design [4]. The means of selecting a suitable design that enhances competitiveness and supports sustainable production is a critical issue. Many qualitative and quantitative criteria must be defined in order to evaluate designs. The criteria need to be weighted, and the optimal solution should be identified; thus, the selection of a layout design is a multiple-criteria decision-making problem (MCDM). The hybrid application of MCDM with a lean philosophy is still limited [5]. The major target of lean philosophy is to improve production procedures in order to remove waste and non-value-added activities and, instead, facilitate value-adding activities [4], while MCDM is one of the most widely used techniques for evaluating alternatives. The key objective of this study is to propose a

Best-Worst method (BWM)-based fuzzy ELECTRE method to identify optimal lean facility layout designs. In the suggested method, the BWM [6] is obtained to generate the relative importance for the criteria. BWM is not only a straightforward method, but it is also a powerful MCDM technique based on the systematic pairwise comparison of multiple and possibly conflicting criteria [6]. The advantages of BWM over existing MCDM methods are that it uses pairwise comparisons in a manner that requires less comparison data and it also reduces inconsistencies that generally characterize such pairwise comparisons [6]. For these reasons, BWM is used in this paper.

The ELECTRE method was introduced by Roy [7], and various ELECTRE methods have been investigated (ELECTRE I, II, III, IV, TRI and IS). It is in an outranking method, and the critical benefit of such methods is they take ordinal scales while retaining the original verbal meaning; they do not change original scales into abstract scales with an arbitrarily imposed range [8]. With its ability to handle quantitative and qualitative information, ELECTRE I is employed to analyze alternatives in this study. Furthermore, ELECTRE I can be used to create a complex methodology when combined with other multiple-criteria decision-analysis (MCDA) methods [9]. However, a shortcoming of the ELECTRE method is its lack of precision in rating performance and producing weights for criteria [8]. To resolve this issue, fuzzy set theory is an ideal solution that can handle ambiguity and uncertainty in dealing with MCDA problems. Fuzzy set theory is an effective technique for evaluating qualitative criteria, using linguistic values with equivalent fuzzy numbers. In the suggested method, fuzzy ELECTRE I is utilized to select the best lean facility layout design with the human subjective cognition of an ambiguous nature for linguistic evaluation; i.e., fuzzy numbers instead of crisp values are used to evaluate alternatives under the qualitative criteria.

Numerous relevant studies of facility layout problems were reviewed, but few consider the combination of MCDM with a lean philosophy [5]. Shahin and Poormostafa [10] combined quality function deployments with a fuzzy analytic hierarchy process and MCDM to improve and optimize facility layout design. Aiello et al. [11] combined ELECTRE with a non-dominated ranking multi-objective genetic algorithm to solve problems of unequal area facility layouts. Fogliatto et al. [5] used AHP to choose the optimal lean-oriented layout design for a health care facility. AHP, TOPSIS, and fuzzy TOPSIS were suggested by Vadivel and Sequeira [12] in a case study of the Indian postal service to solve a layout design problem.

To the best of our knowledge, the combination of BWM and fuzzy ELECTRE to evaluate lean facility layout designs has not been investigated. Therefore, this study proposes a BWM-based fuzzy ELECTRE I method to select the most appropriate lean facility layout design. This method includes an extension to ELECTRE, which is shown to be effective by conducting a numerical comparison. A numerical application is conducted to display the potentials of the model, and comparisons are provided to show its effectiveness.

The rest of this study is arranged as follows. Section 2 introduces a literature review on lean facility layout design, BWM, and fuzzy ELECTRE. Section 3 introduces the basic concepts of fuzzy set theory. Section 4 presents the proposed model, along with a numerical comparison to show the merits of the extension to ELECTRE. Section 5 uses a numerical example to display the feasibility of the method and includes comparisons to show the advantage of the proposed method. Finally, Section 6 sets out concluding remarks and the future research direction.

2. Literature Review

2.1. Facility Layout Selection

Facility layout can be determined as the arrangement of facilities in a factory by considering attributes or objectives under specific constraints to find the most efficient placement [13]. It is estimated that facility planning and material handling can account for between 20% and 50% of operating costs, and an efficient layout design can reduce costs considerably [2,13]. Additionally, it has been found that applying incorrect layouts and location designs can lose more than 35% in system efficiency [13]. Therefore, layout design selection should be viewed from a strategic perspective. The facility layout design problem has become one of the most active topics, and it has drawn the attention of many researchers [14]. Selecting the best facility layout design involves interconnections between a variety of departments in the company; thus, decision makers have to address many challenges when evaluating alternative layouts [1]. In the facility layout selection process, numerous attributes need to be considered, making this an MCDM issue [15]. There are two groups of criteria that impact facility layout design: Internal criteria depict characterizations inside the boundaries of the organization, and external criteria include factors such as water distribution, fuel distribution systems, and the pattern of traffic flows, etc. [16]. Thus, a facility layout design's evaluation is considered to be a typical MCDM problem, which is based on evaluating alternatives by determining both quantitative and qualitative factors.

Various MCDM methods have been obtained to deal with this issue. For example, Shokri et el. [14] proposed the integration of AHP with the VIKOR method to select the best layout design. Al-Hawari et al. [3] provided a case study of a wood factory and selected the optimal facility layout using an analytic network process (ANP) method based on multiple independent and dependent criteria. Durmusoglu [17] suggested a method for assessing the sustainability of facility layouts using TOPSIS. Abdollahi et al. [18] integrated grey relation analysis and nonlinear programming to choose the optimal facility layout design. Sharma and Sharma [19] used a case study of a manufacturing facility and analyzed alternative layouts based on the AHP methodology. Various MCDM techniques have been investigated, as can be seen in the survey by Besbes et al. [20].

2.2. Lean Facility Layout

The original lean concept emerged in the 1960s from the success of the Toyota Production System in identifying and removing waste or non-value-added activities from the system [21]. The lean concept has been studied extensively. Holweg [22] explained the concept as an operational practice that determines the cost of any goal and identifies as waste to be eliminated any expense that does not create value for the end customer. The primary goal of the lean concept is to enhance the production process by focusing on lead-time and cost minimization to eliminate waste and activities that do not add value, generating only value-adding activities from the perspective of the customer throughout the entire production system [4]. The bases of the lean method and its main tools are described in Zhou [21], Jørgensen and Emmitt [23], and Tortorella et al. [24]. During the process of establishing a lean facility layout, a large number of alternatives need to be assessed and reduced, and the one that is most in line with lean philosophy selected [5]. To address this problem, a BWM-based fuzzy ELECTRE I method is suggested to select the best lean layout design. Lean layout designs have become an interest of various researchers. Alex et al. [25] integrated lean techniques into facility layout design to accommodate new machines and optimize material movement with minimal investment using the systematic layout procedure (SLP). Jia et al. [26] developed a facility layout design system for a production line by conducting an in-depth study of a mathematical model designed to indicate optimization. Kovács [4] built a methodology and combined lean methods and a facility layout design method to produce an efficient layout. Nicholas [27] provided an integrated lean method approach for hospital facilities' redesign. Lista at al. [28] combined guidelines for systematic layout planning and lean manufacturing to provide a new facility layout in a case study for an Indian textile company.

2.3. Best-Worst Method

The AHP technique is one of the powerful methods in prioritizing criteria [29,30]. However, the weakness of AHP is inconsistency in the pairwise comparison matrix, which can produce about incorrect or misleading results [31]. Furthermore, the high workload of pairwise comparisons can be a problem in cases where there is a large number of attributes or alternatives [31]. To address these problems and overcome the drawbacks of the AHP method, Rezaei [6] introduced BWM, which provides pairwise comparison matrices in a particularly structured way that requires few data and helps decision makers by producing more reliable pairwise comparisons. Moreover, BWM requires integer values in two vectors, avoiding the fundamental distance issue involved when using fractions in pairwise comparisons [32]. Given these advantages, BWM was used to produce the weights for criteria used in this study. BWM has attracted the attention of many scholars in various areas. Kheybari et al. [33] employed it to choose the most appropriate province for bioethanol production in Iran. Van de Kaa et al. [34] proposed BWM as a possible method for producing the weights of values in the value-sensitive design of smart meters. Yazdi et al. [35] proposed the integration of BWM and a democratic-autocratic decision-making style to improve the efficiency of conventional failure mode and effect analyses. A novel integrated fuzzy BWM and fuzzy-combined compromise solution (CoCoSo) with a Bonferroni test was introduced by Ecer and Pamucar [36] for selecting suppliers on the basis of sustainability. Sofuoğlu [37] integrated the fuzzy application of BWM and BWM with fuzzy TOPSIS in the selection of non-traditional machining methods. Dwivedi et al. [38] introduced a combination of a balanced scorecard and BWM to evaluate the performance of an insurance company. Ali and Rashid [39] presented a group BWM for the selection of robots. A survey of BWM and its applications can be found in Mi et al. [31].

2.4. Fuzzy ELECTRE

In the ELECTRE method, outranking relationships between alternatives are determined based on concordance (satisfaction) and discordance (dissatisfaction) values [40]. Among ELECTRE methods, ELECTRE I is used to resolve choice problems by establishing a small group of preferred alternatives and then selecting the optimal solution from them [41]. The most popular ELECTRE I method is chosen for comprehensive and global explorations [42]. Moreover, ELECTRE I selects the optimal solution based on a preference for the majority of attributes that do not cause an unacceptable level of dissatisfaction with other attributes [43]. ELECTRE I is one of the earliest multicriteria evaluation methods developed with other advanced methods that is used to select a desirable alternative that meets both the synchronization option over multiple evaluation criteria and the discrepancy option under any preferred criteria [44]. The principles of ELECTRE I are based on the construction of a contradictory and very heterogeneous set of criteria, and quantitative and qualitative consequences that are not only associated with numerical ordinal scales but also are attached with imprecise, uncertain, and ill-determined knowledge of data [45]. The facility lean layout design selection holds heterogeneous and multi-criteria structure, i.e., qualitative and quantitative criteria, uncertain, and indeterminate knowledge; thus, applying the ELECTRE I method to conduct the evaluation is appropriate.

ELECTRE I has been studied by many researchers since its introduction in 1968 and has been applied in many areas. ELECTRE I is suitable for a small set of good alternative selections in such a manner that a best solution may be selected; on the other hand, ELECTRE TRI is designed for assignment problems and ELECTRE II, III, and IV for ranking problems [9]. Moreover, the ELECTRE I method is less involved than other ELECTRE methods [9], which makes it easier to combine with the Best-Worst method to integrate into a comprehensive method. Therefore, the ELECTRE I method is used to evaluate the lean facility layout designs in this study. However, the traditional ELECTRE I method uses crisp numbers for criteria and alternatives for evaluation, which are inadequate as they require knowledge from experts [46]. Furthermore, it is sometimes not possible to provide precise measurements when rating performances in most real-life conditions. Therefore, the linguistic terms represented by fuzzy numbers are ideally suited for rating the performance of alternatives in the place of the crisp values used in ELECTRE methods [46]. Fuzzy set theory is powerful technique for handling vague and ambiguous information. It is very challenging to use conventional quantification to represent complicated situations; thus, linguistic variables (words or sentences) need to be used to solve this issue [47]. Fuzzy ELECTRE has been used in various areas. Akram et al. [48] combined the ELECTRE

I method and the Pythagorean fuzzy ELECTRE I method in a group decision-making environment to select a solid-waste disposal method. Nghiem and Chu [49] suggested applying an AHP-based MCDM method to evaluate conceptual sustainable designs, and their method can also resolve the limitation of losing information in ELECTRE. Vimal et al. [50] proposed an ELECTRE method to select the most suitable layout for an existing healthcare facility. Akram et al. [51] suggested an integrated ELECTRE-I method for risk evaluations under hesitant Pythagorean fuzzy information.

Currently, the membership function for the division of two fuzzy numbers cannot be developed precisely. Therefore, defuzzifying the fuzzy numbers may be the best method for producing discordance values with the fuzzy ELECTRE I method. The Hamming distance has been widely used to produce concordance and discordance values with fuzzy ELETCRE I. However, the drawback of the Hamming distance is a complicated calculation process that considers the summation of the distance between two fuzzy numbers in many small intervals. Therefore, Kumar et al. [52] introduced the Chen distance [53], which considers the distance between three points in a triangular fuzzy number. This method provides simplicity in calculations but it cannot consider the entire membership function as a fuzzy number. To solve these limitations, the signed distance [54] is employed in this study to produce concordance and discordance values. By using the α-cut to produce a closed interval, the signed distance between two fuzzy numbers is computed by the distance of two closed intervals. This method considers the entire membership function of two fuzzy numbers and provides a simple calculation process.

The concept of the modified total matrix that considers the Hadamard product of concordance and modified discordance values was introduced by Ke and Chen [55]. In this method, the modified total matrix is obtained by multiplying concordance and modified discordance values. Despite the merits, their method can cause missing information when values in the concordance matrix or the modified discordance matrix are zero. Moreover, the threshold value can be affected because the modified total matrix cannot fully capture the information. Therefore, the concept of the subtraction of discordance values from concordance values is introduced in this paper to produce the total decision matrix that can overcome the limitations of the Ke and Chen method.

To the best of our knowledge, the combination of BWM and fuzzy ELECTRE I for selecting lean facility layout designs has not yet been investigated. Therefore, this study introduces a BWM-based fuzzy ELECTRE I method to select the optimal lean facility layout design. The contribution of this study is as follows. First, a BWM-based fuzzy ELECTRE I method is proposed to select the most suitable lean facility layout design. Second, the signed distance is applied to defuzzify fuzzy numbers, which considers the entire membership function of a fuzzy number and renders calculations simpler. Third, an approach based on subtracting discordance values from concordance values in ELECTRE is proposed, and a comparative study displays the advantage of the suggested model. Finally, the numerical example of evaluating the lean facility layout design in the manufacturing sector is provided to show the feasibility of the proposed hybrid method, and some comparative studies are included to indicate the effectiveness of the proposed method.

3. Fuzzy Set Theory (FST)

3.1. Fuzzy Sets and Fuzzy Numbers

A fuzzy set $\widetilde{p}$ can be defined as $\widetilde{p} = \left\{ \left(x, \mu_{\widetilde{p}}(x) \right) | x \in P \right\}$, $\widetilde{p}$ can be determined by a membership function $\mu_{\widetilde{p}}$ that associates each element x in P with the interval [0, 1], and P is the universe of discourse [56].

A triangular fuzzy number denoted as $\widetilde{p}\left(p^{l}, p^{\gamma}, p^{u}\right)$ has been used frequently because of its convenience [57]. Membership function $\mu_{\widetilde{p}}(x)$ is presented in Equation (1).

$$\mu_{\widetilde{p}}(x) = \begin{cases} \frac{x-p^{l}}{p^{\gamma}-p^{l}}, & p^{l} \leq x \leq p^{\gamma}, \\ \frac{x-p^{u}}{p^{\gamma}-p^{u}}, & p^{\gamma} \leq x \leq p^{u}, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

3.2. Ranking Fuzzy Number by Signed Distance

Many ranking/defuzzification techniques have been studied for a long period of time. The literature review of the ranking methods can be investigated in Chen [58], Wang et al. [59], Wang and Kerre [60,61], and Chu and Le [62]. In this study, the signed distance [54] method is used due to its simplicity, and it can be applied to both negative and positive fuzzy numbers. Assume the fuzzy numbers $\widetilde{Q}_1 = \left(q_1^{l}, q_1^{\gamma}, q_1^{u}\right)$ and $\widetilde{Q}_2 = \left(q_2^{l}, q_2^{\gamma}, q_2^{u}\right)$; then, the α-cut $(0 \leq \alpha \leq 1)$ of $\widetilde{Q}_1$ and $\widetilde{Q}_2$ can be defined as follows.

$$Q_1^{\alpha} = \left[Q_{1L}^{\alpha}, Q_{1R}^{\alpha}\right] = \left[(q_1^{\gamma} - q_1^{l})\alpha + q_1^{l}, -(q_1^{u} - q_1^{\gamma})\alpha + q_1^{u}\right]$$

$$\widetilde{Q}_2^{\alpha} = \left[Q_{2L}^{\alpha}, Q_{2R}^{\alpha}\right] = \left[(q_2^{\gamma} - q_2^{l})\alpha + q_2^{l}, -(q_2^{u} - q_2^{\gamma})\alpha + q_2^{u}\right]$$

The signed distance [54] is obtained to compute the distance of the two fuzzy numbers as follows:

$$\begin{aligned} d\left(\widetilde{Q}_1, \widetilde{Q}_2\right) &= \frac{1}{2}\int_0^1 \left[Q_{1L}^{\alpha} + Q_{1R}^{\alpha} - Q_{2L}^{\alpha} - Q_{2R}^{\alpha}\right] d\alpha \\ &= \frac{1}{4} \times \left(2 \times (q_1^{\gamma} - q_2^{\gamma}) + q_1^{l} - q_2^{l} + q_1^{u} - q_2^{u}\right) \end{aligned} \tag{2}$$

The ranking of these two fuzzy numbers is $\widetilde{Q}_1 > \widetilde{Q}_2$ if and only if $d\left(\widetilde{Q}_1, \widetilde{Q}_2\right) > 0$; $\widetilde{Q}_1 < \widetilde{Q}_2$ if and only if $d\left(\widetilde{Q}_1, \widetilde{Q}_2\right) < 0$; $\widetilde{Q}_1 = \widetilde{Q}_2$ if and only if $d\left(\widetilde{Q}_1, \widetilde{Q}_2\right) = 0$.

3.3. Linguistic Values

Linguistic variables are the common terms or words used in daily life that do not retain numerical values. These linguistic variables are helpful in solving complicated situations [47] in which crisp numbers are ineffective when presenting real-life contexts in MCDM [63]. However, these terms are not true or false type but are something in between and contain uncertainty or imprecision. To solve vague and imprecise data, linguistic variables described by triangular fuzzy numbers have been applied in this study. For instance, the ratings of alternatives versus qualitative criteria are provided by linguistic values presented by triangular fuzzy numbers, as displayed in Table 1.

Table 1. Linguistic variables presented by triangular fuzzy numbers.

Linguistic Term	Fuzzy Number		
Very low (VL)	0	0.1	0.3
Low (L)	0.1	0.3	0.5
Medium (M)	0.3	0.5	0.7
High (H)	0.5	0.7	0.9
Very high (VH)	0.7	0.9	1

4. Model Establishment

Suppose that a committee including k experts (i.e., $D_t,\ t = 1 \sim k$) has been selected to evaluate m options (i.e., $O_i, i = 1 \sim m$) based on n criteria ($F_j, j = 1 \sim n$) consisting of quantitative and qualitative criteria. The quantitative criteria can be grouped into benefit

(θ_B) and cost (θ_C) criteria. The benefit criteria are improved when larger, and cost criteria are improved when smaller.

Step 1. Obtain criteria weights by the Best-Worst Method.

Supposed that the decision makers choose the best criterion that depicts the most prominent criterion and the worst criterion, which is the least prominent criterion relative to the decision among the set of criteria from their perspective. Pairwise comparison vectors for the best criterion relative to other criteria and other criteria relative to the worst criterion are conducted in the structure by using Equations (3) and (4), respectively:

$$E_B = (e_{B1}, \ldots, e_{Bj}, \ldots, e_{Bn}) \tag{3}$$

$$E_W = (e_{1W}, \ldots, e_{jW}, \ldots, e_{nW}) \tag{4}$$

where e_{Bj} indicates the preference of the best criterion B over criterion j, and e_{jW} indicates the preference of criterion j over worst criterion W.

The BWM model is shown in Equation (5) [64].

$$\begin{array}{l} \min \xi^L \\ \text{s.t.} \\ \left|w_B - e_{Bj} w_j\right| \leq \xi^L, \text{ for all } j \\ \left|w_j - e_{jW} w_W\right| \leq \xi^L, \text{ for all } j \\ \sum\limits_j w_j = 1,\ w_j \geq 0, \text{ for all } j \end{array} \tag{5}$$

The criteria weights $w_1, \ldots, w_j, \ldots w_n$, and ξ^{L*} can be obtained by solving Equation (5). The value of ξ^{L*} can be defined as the consistency ration of the matrix with the characteristic being improved when smaller.

Step 2. Develop decision matrix.

Assume $x_{ij} = \left(x_{ij}^l, x_{ij}^\gamma, x_{ij}^u\right), x_{ij} \in R^+, i = 1 \sim m, j = 1 \sim n$ defined as the rating of option O_i given by the experts versus criterion F_j. Each expert is required to evaluate options versus each qualitative criterion, and these results can be aggregated by the Equation (7). Additionally, the ratings of alternatives versus the qualitative criterion are provided by experts utilizing linguistic terms represented by fuzzy numbers as displayed in Table 1. The qualitative attributes are considered as benefit attribute.

$$\widetilde{x}_{ij} = \left(\frac{1}{k}\right) \times (\widetilde{x}_{ij1} \oplus \ldots \oplus \widetilde{x}_{ijt} \oplus \ldots \oplus \widetilde{x}_{ijk}),\ j \text{ is the qualitative criterion} \tag{6}$$

Step 3. Normalization of values under quantitative criteria.

Because of the impact of different units, the normalized performance rating is needed to convert various criteria measures into comparable measures. Assume that $\widetilde{x}_{ij}$ is the rating of option O_i versus a quantitative criterion. The following formulas are applied to normalize values under quantitative criteria.

$$\widetilde{r}_{ij} = \left(\widetilde{r}_{ij}^l, \widetilde{r}_{ij}^\gamma, \widetilde{r}_{ij}^u\right) = \left(\frac{\widetilde{x}_{ij}^l}{a_j^+}, \frac{\widetilde{x}_{ij}^\gamma}{a_j^+}, \frac{\widetilde{x}_{ij}^u}{a_j^+}\right), a_j^+ = \max_i\left(\widetilde{x}_{ij}^u\right), j \in \theta_B \tag{7}$$

$$\widetilde{r}_{ij} = \left(\widetilde{r}_{ij}^l, \widetilde{r}_{ij}^\gamma, \widetilde{r}_{ij}^u\right) = \left(\frac{a_j^-}{\widetilde{x}_{ij}^u}, \frac{a_j^-}{\widetilde{x}_{ij}^\gamma}, \frac{a_j^-}{\widetilde{x}_{ij}^l}\right), a_j^- = \min\left(\widetilde{x}_{ij}^l\right), j \in \theta_C \tag{8}$$

Step 4. Weighted normalization matrix.

The weighted normalized decision matrix can be computed as follows.

$$\widetilde{v}_{ij} = \left(\widetilde{v}_{ij}^l, \widetilde{v}_{ij}^\gamma, \widetilde{v}_{ij}^u\right) = \left(\widetilde{r}_{ij}^l \times w_j, \widetilde{r}_{ij}^\gamma \times w_j, \widetilde{r}_{ij}^u \times w_j\right) \tag{9}$$

Step 5. Calculate sign distance.

$$d\left(\widetilde{v}_{ij}, \widetilde{v}_{hj}\right) = \frac{1}{4}\left(2 \times \left(\widetilde{v}_{ij}^{\gamma} - \widetilde{v}_{hj}^{\gamma}\right) + \widetilde{v}_{ij}^{l} - \widetilde{v}_{hj}^{l} + \widetilde{v}_{ij}^{u} - \widetilde{v}_{hj}^{u}\right),\ i, h = 1 \sim m, j = 1 \sim n \quad (10)$$

Step 6: Identify concordance and discordance sets.

By the signed distance [54], $\widetilde{v}_{ij} \geq \widetilde{v}_{hj}$ if and only if $d\left(\widetilde{v}_{ij}, \widetilde{v}_{hj}\right) \geq 0$; $\widetilde{v}_{ij} \leq \widetilde{v}_{hj}$ if and only if $d\left(\widetilde{v}_{ij}, \widetilde{v}_{hj}\right) \leq 0$. The concordance and discordance sets can be determined as follows:

$$C_{ih} = \left\{j, \widetilde{v}_{ij} > \widetilde{v}_{hj}\right\} \quad (11)$$

$$D_{ih} = \left\{j, \widetilde{v}_{ij} \leq \widetilde{v}_{hj}\right\} \quad (12)$$

Step 7. Produce concordance and discordance matrices.

The concordance matrix is produced by aggregating the criteria weights in the concordance set. The formula for concordance matrix *Con* can be obtained by Equation (13).

$$Con = [c_{ih}]_{m \times m},\ c_{ih} = \frac{\sum_{j \in C_{ih}} w_j}{\sum_{j=1}^{n} w_j} \quad (13)$$

Herein, the signed distance [54] is used to produce discordance matrix as Equation (14).

$$D = [d_{ih}]_{m \times m},\ d_{ih} = \frac{\max_{j \in D_{ih}}\left\{\left|d\left(\widetilde{v}_{ij}, \widetilde{v}_{hj}\right)\right|\right\}}{\max_{j \in J}\left\{\left|d\left(\widetilde{v}_{ij}, \widetilde{v}_{hj}\right)\right|\right\}},\ J = \{1, 2, \ldots.., n\} \quad (14)$$

Step 8. Develop the total dominance matrix.

The concept of modified discordance matrix was established by Hwang and Masud [65], as shown in Equation (15). In 2012, Ke and Chen [55] proposed the modified total matrix in the ELECTRE method, and they are presented in Equations (15) and (16).

$$D' = \left[d'_{ih}\right]_{m \times m}, d'_{ih} = 1 - d_{ih} \quad (15)$$

$$S = [s_{ih}]_{m \times m}, s_{ih} = c_{ih} \otimes d'_{ih} \quad (16)$$

Despite the merits, their method causes a problem of missing information when using the multiplication of concordance and modified discordance matrices as in Equation (16). If either values in the above matrices are zero, the corresponding result in the modified total matrix is zero. This means that the information of either concordance or modified discordance cannot be fully captured. Furthermore, following Ke and Chen [55], the value of threshold is calculated based on the average of the two smallest values of the modified total matrix; therefore, this threshold also can be affected by missing information. Hence, the shortcoming of Ke and Chen [55] is that it cannot completely describe the information in the modified total matrix, which impacts the threshold value and the modified superiority matrix. To resolve this limitation, this study proposes total dominance matrix using the subtraction of discordance values from concordance values approach. A comparative study is introduced in Section 4.1 to illustrate the merits of the suggested model.

In the original ELECTRE I method, the alternative that has a high value of concordance and low value of discordance will be selected. To avoid missing information, we propose the approach of subtracting discordance values from concordance values to obtain the total dominance matrix, as shown in Equation (17).

$$G = [g_{ih}]_{m \times m},\ g_{ih} = c_{ih} - d_{ih} \quad (17)$$

Step 9. Produce the Boolean matrix.

The Boolean matrix is obtained by Equation (18):

$$U = [u_{ih}]_{m\times m}, \begin{cases} u_{ih} = 1, & if \quad g_{ih} \geq \overline{g} \\ u_{ih} = 0, & if \quad g_{ih} < \overline{g} \end{cases} \tag{18}$$

where

$$\overline{g} = \sum_{i=1}^{m} \sum_{h=1}^{(m-1)} \frac{g_{ih}}{m(m-1)} \tag{19}$$

Step 10. Determine the final result.

If the value of $u_{ih} = 1$ from matrix U, option i is greater than option h and can be denoted as $O_i \rightarrow O_h$.

4.1. Numerical Comparison

Suppose that a company needs to evaluate four lean facility layout alternatives based on four criteria including three benefit criteria (F_1, F_2, F_3) and one cost criteria (F_4). The normalized values of alternatives versus criteria, r_{ij}, $i, j = 1 \sim 4$, and the criteria weights, w_j, $j = 1 \sim 4$, are produced as presented in Table 2. The weighted normalized values, $v_{ij} = r_{ij} \times w_j$, $i, j = 1 \sim 4$, are shown in Table 3. The concordance matrix can be obtained by $c_{ih} = \frac{\sum_{j\in C_{ih}} w_j}{\sum_{j=1}^{n} w_j}$, $C_{ih} = \left\{ j, v_{ij} \geq v_{hj} \right\}$, as shown in Table 4. The discordance matrix can be obtained by $d_{ih} = \frac{\max_{j\in D_{ih}} \left\{ \left| v_{ij} - v_{hj} \right| \right\}}{\max_{j\in J} \left\{ \left| v_{ij} - v_{hj} \right| \right\}}$, $J = \{1, 2, \ldots.., n\}$, $D_{ih} = \left\{ j, v_{ij} < v_{hj} \right\}$, as displayed in Table 5. The modified discordance matrix is produced by using Equation (15), as presented in Table 6. The modified total matrix is determined by Equation (16), as displayed in Table 7. The superiority matrix is calculated by using Equations (18) and (19), as shown in Table 8. Finally, the ranking order is $O_1 > O_2 = O_3 > O_4$, as presented in Table 9. However, by the proposed method, i.e., Equations (17)–(19), the ranking order is $O_1 > O_2 > O_3 > O_4$.

Table 2. The weights and ratings of alternative versus criteria.

	F_1	F_2	F_3	F_4
O_1	1.000	1.000	0.111	1.000
O_2	0.778	0.778	0.333	0.667
O_3	0.556	0.333	0.778	0.400
O_4	0.333	0.333	1.000	0.286
Weight	0.40	0.210	0.220	0.170

Table 3. Weighted normalized values.

	F_1	F_2	F_3	F_4
O_1	0.400	0.210	0.024	0.170
O_2	0.311	0.163	0.073	0.113
O_3	0.222	0.070	0.171	0.068
O_4	0.133	0.070	0.220	0.049

Table 4. The concordance matrix.

	O_1	O_2	O_3	O_4
O_1	-	0.780	0.780	0.780
O_2	0.220	-	0.780	0.780
O_3	0.220	0.220	-	0.570
O_4	0.220	0.220	0.220	-

Table 5. The discordance matrix.

	O_1	O_2	O_3	O_4
O_1	-	0.550	0.825	0.733
O_2	1.000	-	1.000	0.825
O_3	1.000	0.955	-	0.550
O_4	1.000	1.000	1.000	-

Table 6. Modified discordance matrix.

	O_1	O_2	O_3	O_4
O_1	-	0.450	0.175	0.267
O_2	0.000	-	0.000	0.175
O_3	0.000	0.045	-	0.450
O_4	0.000	0.000	0.000	-

Table 7. The modified total matrix.

O_1	O_2	O_3	O_4	O_1
O_1	-	0.351	0.137	0.208
O_2	0	-	0.000	0.137
O_3	0.000	0.010	-	0.257
O_4	0.000	0	0	-

Table 8. The superiority matrix.

	O_1	O_2	O_3	O_4
O_1	-	1	1	1
O_2	0	-	0	1
O_3	0	0	-	1
O_4	0	0	0	-

Table 9. The comparison.

	Ke and Chen	Proposed Method
O_1	1	1
O_2	2	2
O_3	2	3
O_4	4	4

Based on the weighted normalized values in Table 3, the ranking result clearly is $A_2 > A_3$ with the reason that v_{21} $(0.311) > v_{31}$ (0.222), v_{22} $(0.163) > v_{32}$ (0.070), and v_{24} $(0.113) = v_{34}$ (0.068) although v_{23} (0.073) is smaller than v_{33} (0.171). However, the ranking result obtained by Ke and Chen [55] is $O_2 = O_3$ in Table 9; this result is inconsistent with the data displayed in Table 3. However, the proposed approach provides the ranking result $O_1 > O_2 > O_3 > O_4$, which is in line with the data in Table 3. Therefore, the proposed approach can resolve the weakness of Ke and Chen [55] method.

5. Numerical Example

Suppose that a manufacturing firm needs to choose the optimal lean facility layout for a new factory. Therefore, the firm decides to establish a committee including three experts D_t, $t = 1 \sim 3$ to rank four layout options O_i, $i = 1 \sim 4$. To determine the optimal lean facility layout design, assume that the experts have decided to apply fifteen criteria, including the quantitative and qualitative criteria provided by Kovács (2020) [4] in Table 10. Moreover, these quantitative factors can be categorized into cost and benefit factors, as shown in Table 10. Further assume that qualitative criteria are treated as benefit criteria. The four layout alternatives are ranked by the proposed approach as follows.

Table 10. List of criteria.

	Criteria	Symbol	Qualitative	Quantitative
1	Longest cycle time in the whole production process (C, mins)	F_1		x
2	Productivity (B, units/shift)	F_2		x
3	Number of workstations (C, pieces)	F_3		x
4	Number of operators (C, person)	F_4		x
5	WIP inventory (C, %)	F_5		x
6	Space used for assembly (C, m^2)	F_6		x
7	Material workflow (C, UL.m)	F_7		x
8	Travel distance of materials (C, m)	F_8		x
9	Material handling cost (C, %)	F_9		x
10	Labour cost of operators (C, %)	F_{10}		x
11	Reliability of continuous component supply of the production lines (B)	F_{11}	x	
12	Quality of the processes and final products (B)	F_{12}	x	
13	Transparency of the processes (B)	F_{13}	x	
14	Standardisation of the processes (B)	F_{14}	x	
15	Workplace ergonomics and worker's satisfaction (B)	F_{15}	x	

Step 1. The experts select the best criterion and the worst criterion relative to the decision among the set of criteria from their perspective. By Equations (3) and (4) and a 9-point scale (numbers from 1 to 9), pairwise comparisons between the best and the worst criterion to the other criteria are performed, as shown in Tables 11 and 12.

Table 11. The comparison between the best to other criteria.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
F_2 (Best)	2	1	4	4	5	6	3	6	3	4	7	5	8	6	9

Table 12. The comparison between the other criteria to the worst.

Criteria	F_{15} (Worst)
F_1	8
F_2	9
F_3	5
F_4	6
F_5	4
F_6	3
F_7	6
F_8	3
F_9	6
F_{10}	5
F_{11}	3
F_{12}	5
F_{13}	2
F_{14}	2
F_{15}	1

Criteria weights w_j and $\xi^{L*} = 0.045$ can be obtained by solving Equation (5). The value of ξ^{L*} is quite small, indicating that the pairwise comparison matrices are consistent, as displayed in Table 13.

Table 13. The weights of criteria.

Criteria	Weight
F_1	0.126
F_2	0.207
F_3	0.063
F_4	0.063
F_5	0.05
F_6	0.042
F_7	0.084
F_8	0.042
F_9	0.084
F_{10}	0.063
F_{11}	0.036
F_{12}	0.05
F_{13}	0.031
F_{14}	0.042
F_{15}	0.018

Step 2. The performance ratings of four options with respect to 10 quantitative criteria are produced by using Equation (6), as presented in Table 14. The linguistic values with equivalent triangular fuzzy numbers in Table 1 and Equation (6) are obtained to provide the ratings of the four options with respect to 5 qualitative criteria, as also displayed in Table 14.

Table 14. The ratings of the alternatives versus qualitative and quantitative criteria.

		F_1			F_2			F_3			F_4			F_5	
O_1	7	7	7	83	83	83	12	12	12	11	11	11	64	64	64
O_2	8	8	8	75	75	75	14	14	14	10	10	10	70	70	70
O_3	7	7	7	80	80	80	12	12	12	9	9	9	65	65	65
O_4	8	8	8	62	62	62	15	15	15	13	13	13	97	97	97
		F_6			F_7			F_8			F_9			F_{10}	
O_1	190	190	190	348	348	348	55	55	55	90	90	90	85	85	85
O_2	210	210	210	350	350	350	63	63	63	94	94	94	86	86	86
O_3	200	200	200	340	340	340	57	57	57	92	92	92	85	85	85
O_4	190	190	190	365	365	365	66	66	66	98	98	98	89	89	89
		F_{11}			F_{12}			F_{13}			F_{14}			F_{15}	
O_1	0.7	0.9	1	0.7	0.9	1	0	0.1	0.3	0.7	0.9	1	0.5	0.7	0.9
O_2	0.3	0.5	0.7	0.5	0.7	0.9	0.1	0.3	0.5	0.3	0.5	0.7	0.3	0.5	0.7
O_3	0.5	0.7	0.9	0.7	0.9	1	0.5	0.7	0.9	0.7	0.9	1	0.3	0.5	0.7
O_4	0.1	0.3	0.5	0.1	0.3	0.5	0.7	0.9	1	0.1	0.3	0.5	0	0.1	0.3

Step 3–4. The quantitative criteria are normalized by using the Equations (7) and (8), as displayed in Table 15. The weighted normalized fuzzy decision matrix of options with respect to criteria is computed by the multiplication of performance ratings and criteria weights in Equation (9), as presented in Table 16.

Table 15. Normalization of values under quantitative criteria.

	F_1			F_2			F_3			F_4		
O_1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.818	0.818	0.818
O_2	0.875	0.875	0.875	0.904	0.904	0.904	0.857	0.857	0.857	0.900	0.900	0.900
O_3	1.000	1.000	1.000	0.964	0.964	0.964	1.000	1.000	1.000	1.000	1.000	1.000
O_4	0.875	0.875	0.875	0.747	0.747	0.747	0.800	0.800	0.800	0.692	0.692	0.692
	F_5			F_6			F_7					
O_1	1.000	1.000	1.000	1.000	1.000	1.000	0.977	0.977	0.977			
O_2	0.914	0.914	0.914	0.905	0.905	0.905	0.971	0.971	0.971			
O_3	0.985	0.985	0.985	0.950	0.950	0.950	1.000	1.000	1.000			
O_4	0.660	0.660	0.660	1.000	1.000	1.000	0.932	0.932	0.932			
	F_8			F_9			F_{10}					
O_1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000			
O_2	0.873	0.873	0.873	0.957	0.957	0.957	0.988	0.988	0.988			
O_3	0.965	0.965	0.965	0.978	0.978	0.978	1.000	1.000	1.000			
O_4	0.833	0.833	0.833	0.918	0.918	0.918	0.955	0.955	0.955			

Table 16. Weighted normalization matrix.

	F_1			F_2			F_3			F_4		
O_1	0.126	0.126	0.126	0.207	0.207	0.207	0.063	0.063	0.063	0.051	0.051	0.051
O_2	0.110	0.110	0.110	0.187	0.187	0.187	0.054	0.054	0.054	0.057	0.057	0.057
O_3	0.126	0.126	0.126	0.199	0.199	0.199	0.063	0.063	0.063	0.063	0.063	0.063
O_4	0.110	0.110	0.110	0.154	0.154	0.154	0.050	0.050	0.050	0.044	0.044	0.044
	F_5			F_6			F_7			F_8		
O_1	0.050	0.050	0.050	0.042	0.042	0.042	0.082	0.082	0.082	0.042	0.042	0.042
O_2	0.046	0.046	0.046	0.038	0.038	0.038	0.081	0.081	0.081	0.037	0.037	0.037
O_3	0.050	0.050	0.050	0.040	0.040	0.040	0.084	0.084	0.084	0.040	0.040	0.040
O_4	0.033	0.033	0.033	0.042	0.042	0.042	0.078	0.078	0.078	0.035	0.035	0.035
	F_9			F_{10}			F_{11}			F_{12}		
O_1	0.084	0.084	0.084	0.063	0.063	0.063	0.025	0.032	0.036	0.035	0.045	0.050
O_2	0.080	0.080	0.080	0.062	0.062	0.062	0.018	0.025	0.032	0.025	0.035	0.045
O_3	0.082	0.082	0.082	0.063	0.063	0.063	0.011	0.018	0.025	0.005	0.015	0.025
O_4	0.077	0.077	0.077	0.060	0.060	0.060	0.004	0.011	0.018	0.005	0.015	0.025
	F_{13}			F_{14}			F_{15}					
O_1	0.000	0.003	0.009	0.029	0.038	0.042	0.009	0.013	0.016			
O_2	0.003	0.009	0.016	0.029	0.038	0.042	0.005	0.009	0.013			
O_3	0.016	0.022	0.028	0.013	0.021	0.029	0.005	0.009	0.013			
O_4	0.022	0.028	0.031	0.004	0.013	0.021	0.000	0.002	0.005			

Step 5–6. The sign distance values are calculated by Equation (10), as displayed in Table 17. Then, these values are used to identify the concordance and discordance set by Equations (11) and (12), as displayed in Table 18.

Table 17. The sign distance values.

F_1	O_1	O_2	O_3	O_4	F_2	O_1	O_2	O_3	O_4
O_1	0.000	0.016	0.000	0.016	O_1	0.000	0.020	0.007	0.052
O_2	−0.016	0.000	−0.016	0.000	O_2	−0.020	0.000	−0.012	0.032
O_3	0.000	0.016	0.000	0.016	O_3	−0.007	0.012	0.000	0.045
O_4	−0.016	0.000	−0.016	0.000	O_4	−0.052	−0.032	−0.045	0.000
F_3	O_1	O_2	O_3	O_4	F_4	O_1	O_2	O_3	O_4
O_1	0.000	0.009	0.000	0.013	O_1	0.000	−0.005	−0.011	0.008
O_2	−0.009	0.000	−0.009	0.004	O_2	0.005	0.000	−0.006	0.013
O_3	0.000	0.009	0.000	0.013	O_3	0.011	0.006	0.000	0.019
O_4	−0.013	−0.004	−0.013	0.000	O_4	−0.008	−0.013	−0.019	0.000
F_5	O_1	O_2	O_3	O_4	F_6	O_1	O_2	O_3	O_4
O_1	0.000	0.004	0.001	0.017	O_1	0.000	0.004	0.002	0.000
O_2	−0.004	0.000	−0.004	0.013	O_2	−0.004	0.000	−0.002	−0.004
O_3	−0.001	0.004	0.000	0.016	O_3	−0.002	0.002	0.000	−0.002
O_4	−0.017	−0.013	−0.016	0.000	O_4	0.000	0.004	0.002	0.000
F_7	O_1	O_2	O_3	O_4	F_8	O_1	O_2	O_3	O_4
O_1	0.000	0.000	−0.002	0.004	O_1	0.000	0.005	0.001	0.007
O_2	0.000	0.000	−0.002	0.003	O_2	−0.005	0.000	−0.004	0.002
O_3	0.002	0.002	0.000	0.006	O_3	−0.001	0.004	0.000	0.006
O_4	−0.004	−0.003	−0.006	0.000	O_4	−0.007	−0.002	−0.006	0.000
F_9	O_1	O_2	O_3	O_4	F_{10}	O_1	O_2	O_3	O_4
O_1	0.000	0.004	0.002	0.007	O_1	0.000	0.001	0.000	0.003
O_2	−0.004	0.000	−0.002	0.003	O_2	−0.001	0.000	−0.001	0.002
O_3	−0.002	0.002	0.000	0.005	O_3	0.000	0.001	0.000	0.003
O_4	−0.007	−0.003	−0.005	0.000	O_4	−0.003	−0.002	−0.003	0.000
F_{11}	O_1	O_2	O_3	O_4	F_{12}	O_1	O_2	O_3	O_4
O_1	0.000	0.006	0.013	0.021	O_1	0.000	0.009	0.029	0.029
O_2	−0.006	0.000	0.007	0.014	O_2	−0.009	0.000	0.020	0.020
O_3	−0.013	−0.007	0.000	0.007	O_3	−0.029	−0.020	0.000	0.000
O_4	−0.021	−0.014	−0.007	0.000	O_4	−0.029	−0.020	0.000	0.000
F_{13}	O_1	O_2	O_3	O_4	F_{14}	O_1	O_2	O_3	O_4
O_1	0.000	−0.005	−0.018	−0.024	O_1	0.000	0.000	0.016	0.024
O_2	0.005	0.000	−0.013	−0.018	O_2	0.000	0.000	0.016	0.024
O_3	0.018	0.013	0.000	−0.005	O_3	−0.016	−0.016	0.000	0.008
O_4	0.024	0.018	0.005	0.000	O_4	−0.024	−0.024	−0.008	0.000
F_{15}	O_1	O_2	O_3	O_4					
O_1	0.000	0.004	0.004	0.010					
O_2	−0.004	0.000	0.000	0.007					
O_3	−0.004	0.000	0.000	0.007					
O_4	−0.010	−0.007	−0.007	0.000					

Table 18. Concordance and discordance sets.

	Concordance	Discordance
12	1,2,3,5,6,7,8,9,10,11,12,15	4,13,14
13	2,5,6,8,9,11,12,14,15	1,3,4,7,10,13
14	1,2,3,4,5,7,8,9,10,11,12,14,15	6,13
21	4,13	1,2,3,5,6,7,8,9,10,11,12,14,15
23	11,12,14	1,2,3,4,5,6,7,8,9,10,13,15
24	2,3,4,5,7,8,9,10,11,12,14,15	1,6,13
31	4,7,13	1,2,3,5,6,8,9,10,11,12,14,15
32	1,2,3,4,5,6,7,8,9,10,13	11,12,14,15
34	1,2,3,4,5,7,8,9,10,11,14,15	6,12,13
41	13	1,2,3,4,5,6,7,8,9,10,11,12,14,15
42	6,13	1,2,3,4,5,7,8,9,10,11,12,14,15
43	6,13	1,2,3,4,5,7,8,9,10,11,12,14,15

Step 7–8. By Equations (13) and (14), concordance and discordance values are produced as shown in Tables 19 and 20, respectively. Next, the total dominance matrix can be obtained by Equation (17), as shown in Table 21.

Table 19. Concordance matrix.

	O_1	O_2	O_3	O_4
O_1	-	0.864	0.570	0.927
O_2	0.094	-	0.128	0.801
O_3	0.178	0.854	-	0.876
O_4	0.031	0.073	0.073	-

Table 20. Discordance matrix.

	O_1	O_2	O_3	O_4
O_1	-	0.276	0.625	0.451
O_2	1.000	-	0.781	0.559
O_3	1.000	1.000	-	0.123
O_4	1.000	1.000	1.000	-

Table 21. The total dominance matrix.

	O_1	O_2	O_3	O_4
O_1	-	0.588	−0.055	0.476
O_2	−0.906	-	−0.653	0.242
O_3	−0.822	−0.146	-	0.754
O_4	−0.969	−0.927	−0.927	-

Step 9–10. By the Equations (18) and (19), the Boolean matrix is produced as presented in Table 22. According to the Boolean matrix, the final result is $O_1 > O_3 > O_2 > O_4$, as shown in Table 23. Thus, layout design O_1 should be selected.

Table 22. The Boolean matrix.

	O_1	O_2	O_3	O_4
O_1	-	1	1	1
O_2	0	-	0	1
O_3	0	1	-	1
O_4	0	0	0	-

Table 23. The final result.

	Proposed Method	Ke and Chen (2012) [55]	Nijkamp and Van Delft (1977) [66]	Chhipi-Shrestha et al. (2017) [40]	WPM Kabassi (2021) [67]
O_1	1	1	1	1	1
O_2	3	2	3	3	3
O_3	2	2	2	2	2
O_4	4	4	4	4	4

Numerical Comparison

A comparison of the proposed approach with the method of Ke and Chen [55] is conducted using the above numerical example. Following Ke and Chen [55], the modified discordance value is obtained by Equation (15), as shown in Table 24. The total dominance value is produced by multiplying the concordance and modified discordance values using Equation (16), as presented in Table 25. By Equations (18) and (19), a Boolean matrix is obtained, as presented in Table 26, and this leads to the ranking result $O_1 > O_2 = O_3 > O_4$, as shown in Table 23. According to Table 23, the proposed method produces the ranking result $O_1 > O_3 > O_2 > O_4$. Obviously, the suggested model provides more distinguishable result than the method of Ke and Chen [55].

Table 24. The modified discordance matrix.

	O_1	O_2	O_3	O_4
O_1	-	0.724	0.375	0.549
O_2	0.000	-	0.219	0.441
O_3	0.000	0.000	-	0.877
O_4	0.000	0.000	0.000	-

Table 25. The total dominance matrix (Ke and Chen, 2012).

	O_1	O_2	O_3	O_4
O_1	-	0.625	0.214	0.509
O_2	0.000	-	0.028	0.353
O_3	0.000	0.000	-	0.769
O_4	0.000	0.000	0.000	-

Table 26. The Boolean matrix (Ke and Chen, 2012).

	O_1	O_2	O_3	O_4
O_1	-	1	1	1
O_2	0	-	0	1
O_3	0	0	-	1
O_4	0	0	0	-

According to Table 16, the weighted normalized values of alternative O_3 versus criteria F_{11}, F_{12}, F_{14} are smaller than the values of alternative O_2, and the weighted normalized values versus criterion F_{15} of two alternatives are equal; however, the weighted normalized values of alternative O_3 are higher than the values of alternative O_2 under criteria $F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}$ and F_{13}. Therefore, the final result should be $O_3 > O_2$. The ranking result of Ke and Chen [55] is not consistent with the data in Table 16 because of missing information when using the multiplication of concordance and modified discordance values. Thus, the proposed method can overcome the drawback of Ke and Chen's method and provide more distinguishable results. Moreover, a comparison with the methods of Nijkamp and van Delft [66] and Chhipi-Shrestha et al. [40] is shown in Table 23. The ranking result obtained from these two methods is also $O_1 > O_3 > O_2 > O_4$, which is consistent with the proposed method. Following Nijkamp and Van Delft [66] and

Chhipi-Shrestha et al. [40], the alternative with a higher net concordance value and lower net discordance value is given a higher rank. If an alternative has different ranking orders according to its net concordance and net discordance values, the average of ranking orders based on net concordance and net discordance values is used to determine the ranking order of the alternatives. The use of an average operator may not be distinguishable in ranking alternatives and can lead to some alternatives having the same ranking order. In addition, the comparison with the weighted product model (WPM) [67] is also shown in Table 23. The ranking result obtained from the WPM [67] is consistent with the proposed method; however, this method has the limitations of sensitivity to units' ranges and the exaggeration of specific scores [68].

6. Conclusions

The resources available for production are limited, whereas the global population and consumption are increasing. To improve sustainability and maximize the use of resources, manufacturing companies need to concentrate on cost-effective production, final products that are material efficient, and energy efficiency. As a result, lean facility layout design, with the elimination of waste and non-value-added activities to achieve the optimal arrangement of facilities, is of growing interest. In the design process, several alternatives need to be compared and the best alternative should be selected. MCDM methods are effective tools for evaluating lean facility layout designs as they can handle many criteria, both quantitative and qualitative, and consider the importance of weights in the evaluation process.

This study proposes a hybrid method using BWM and fuzzy ELECTRE I for the evaluation of lean facility layout designs, where BWM is used to generate the criteria weights and fuzzy ELECTRE I is used to rank alternatives. The signed distance method is employed to defuzzify the fuzzy numbers and to obtain the discordance matrix. In addition, an extension to fuzzy ELECTRE I based on the subtraction of discordance values from concordance values is used to rank alternatives. The proposed extension can overcome the missing information issue. A numerical comparison shows the advantage of the proposed extension. Furthermore, a numerical example evaluating lean facility layout designs for a manufacturing firm is provided to display the merits of the proposed model, and a comparison shows that the proposed method provides correct and more distinguishable results than other methods. For future investigations, the suggested method can be used to resolve other MCDM issues with a case study; however, different weight derivation methods and different defuzzification methods, etc., could generate different results.

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