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Statistical Fuzzy Reliability Assessment of a Blended System

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Abstract: Fuzzy sets have been proven to constitute an asset in the evolution of reliability theory in recent decades. Their contribution in addressing the possibility of errors, insufficiency of data, randomness, or fuzziness, either in the system or in the accumulation of any data for the respective system, which is overlooked in the traditional reliability assessment, seems to be quite crucial. The present work deals with the statistical fuzzy reliability evaluation of a blended system that comprises two subsystems. One system contains two components aligned in a parallel configuration, and the other is a 3-out-of-5 system. The reliability of this model is assessed using two approaches to intuitionistic fuzzy sets (IFS), namely, traditional IFS and interval-valued intuitionistic fuzzy sets (IVIFS). Three cases are considered in each approach, which are compared individually as well as with each other. It was established that the IVIFS yield better results than the IFS. The obtained results are displayed in both tabular and graphical forms for better assessment.

Keywords: interval-valued intuitionistic fuzzy sets (IVIFS); intuitionistic fuzzy reliability (IFR); intuitionistic fuzzy sets (IFS); *k*-out-of-*n* system; statistical fuzzy reliability

MSC: 90B25; 62A86; 94D05

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1. Introduction

Technology has advanced tremendously in recent decades, leading to the development of numerous industrial systems. However, these technological advancements are accompanied by complexity in systems. As a result, it is critical that these systems be reliable and durable, which has led to the concept of reliability. Reliability theory is a field that helps in assessing the probability of the operative state of a system for a given time under predetermined conditions. However, reliability computation is a tedious task due to the possibility of errors in obtaining the data or general human errors leading to impreciseness or fuzziness of systems in the real world. This obstacle can be overcome using the fuzzy set theory. The notion of fuzzy sets was first conceptualized by Zadeh [1] to deal with the fuzziness or randomness in countless situations. Extensive research in the field of fuzzy sets has led to various extensions, such as intuitionistic fuzzy sets (IFS), Pythagorean fuzzy sets, hesitant and dual hesitant fuzzy sets, and many more. The different types of sets help in dealing with various kinds of impreciseness or fuzziness in different situations. Traditional fuzzy sets, for instance, merely take the measure of acceptance or membership into account, whereas IFS include both the measures of membership and non-membership into consideration along with a degree of hesitancy. Various new techniques have emerged in IFS in the past few decades for formulating new methodologies or helping in ranking, such as the score (see, e.g., [2]) and accuracy functions (see, e.g., [3]), knowledge distance (see, e.g., [4]), and various multi-criteria decision-making techniques see, e.g., [5]). These techniques have been helpful in reliability estimation. Therefore, since its inception, this theory has proven to be an asset in reliability engineering, as any system in the real world possesses errors and imprecision that cannot be tackled by conventional reliability theory.

Based on the existing literature, it is evident that the reliability assessment of real-world problems cannot be conducted accurately if the fuzziness or possibility of errors, human or otherwise, are not taken into account. Thus, this particular work deals with the statistical evaluation of the fuzzy reliability of a blended system with two different intuitionistic fuzzy approaches. The considered system consists of two subsystems, and their reliability functions are obtained with the assistance of the universal generating function (UGF) procedure. Furthermore, two novel methodologies of IFS are proposed based on the incorporation of intuitionistic fuzzy numbers (IFNs) and interval-valued intuitionistic fuzzy numbers (IVIFNs) into the UGF method to assess the fuzzy reliability. IFS and IVIFS are employed in the reliability computation, as they are capable of considering a higher level of fuzziness than the traditional fuzzy sets, taking into account degrees of acceptance and non-acceptance, with IVIFS using intervals to represent the measures instead of crisp values. The study will also help in determining which set gives better results. Thus, the intuitionistic fuzzy reliability (IFR) and interval-valued intuitionistic fuzzy reliability (IVIFR) are estimated, and the obtained results are compared to determine the better methodology using numerical examples.

2. Literature Review

Numerous researchers have made significant contributions to the field of fuzzy reliability, which is an amalgamation of reliability theory and fuzzy set theory. This section presents some of the work done in the past few years in three subsections based on the three techniques used: fuzzy sets, IFS, and the UGF method.

2.1. Reliability Evaluation Using Fuzzy Set Theory

Utkin and Gurov [6] assessed fuzzy reliability using systems of functional equations by considering variables in the possibility context and validating the results using examples. Bing et al. [7] presented a technique involving the fuzzy linear regression method to analyse the reliability of a system where fuzzy variables ate involved. Dong et al. [8] proposed a simulation-based approach to measure fuzzy reliability to reduce the complexity involved in its calculations. The proposed methodology is useful for situations where the evaluation of fuzzy reliability is complex. Kumar et al. [9] presented a novel technique using interval-valued trapezoidal fuzzy sets to estimate the imprecise reliability, and further applied the methodology to a model of a marine power plant.

Abdelgawad and Fayek [10] utilized fuzzy arithmetic operations to conduct a quantitative fault tree analysis (FTA) for reliability determination, where experts could use linguistic terms instead of exact values. The methodology was demonstrated by the authors with a case study of horizontal directional drilling (HDD). Chandna and Ram [11] applied fuzzy logic, linguistic techniques, and multi-criteria decision-making techniques for the estimation and improvement of fuzzy reliability. Chaube and Singh [12] developed a technique for the evaluation of fuzzy reliability based on the membership function. Wang et al. [13] presented an approach based on sequential optimization and fuzzy reliability for multidisciplinary systems. Furthermore, the authors also proposed a novel collocation method for the appraisal of the fuzzy reliability of systems. Yang et al. [14] evaluated the reliability of multi-state systems (MSS) based on multi-valued decision diagrams under epistemic uncertainty with the help of fuzzy set theory and interval theory. Furthermore, the technique is validated using a case study of a high-speed train bogie system.

2.2. Reliability Evaluation Using IFS

The concept of fuzzy sets has been further extended to intuitionistic fuzzy sets (IFS) by Atanassov [15], which has also been instrumental in evaluating the reliability of numerous systems due to its capability of including a non-membership value along with the membership value for systems having a higher level of hesitancy. Mahapatra and Roy [16] used triangular intuitionistic fuzzy numbers (TIFN) to evaluate the reliability of a system and implemented the technique on a model of a dark room. Kumar et al. [17] introduced trapezoidal intuitionistic fuzzy numbers and their operations to formulate a method for the estimation of the intuitionistic fuzzy reliability of series and parallel systems and applied the methodology to the basement flooding model. Garg et al. [18] utilised the intuitionistic fuzzy concept to optimise the fuzzy reliability by implementing the particle swarm optimisation and genetic algorithm.

Based on evidence theory and intuitionistic fuzzy sets, Song et al. [19] presented a novel method for evaluating sensor dynamic reliability. To demonstrate its practicality and validity, the authors applied the proposed approach to evidence combination and data fusion. Kumar et al. [20] applied the Weibull distribution along with TIFNs to assess the reliability of consecutive k-out-of-n systems, where the reliability of the transition state of the system was computed with the help of the Markov chain method. Akbari and Hesamian [21] developed a novel procedure to estimate the time-dependent reliability of a k-out-of-n system using IFS. Kumar et al. [22] applied the UGF technique along with IFS to compute the system reliability, where the failure rate follows the exponential distribution. Previous findings have shown that another technique that has contributed tremendously to the field of reliability theory is UGF method developed by Ushakov [23].

2.3. Reliability Evaluation Using the UGF Method

Many researchers have incorporated the UGF process with fuzzy theory to compute the reliability of any system more efficiently. Levitin and Lisnianski [24] proposed a novel method for the reliability estimation of an MSS using the UGF method. Furthermore, the authors also conducted a sensitivity analysis. Ding and Lisnianski [25] created a novel technique by implementing the fuzzy concept in the UGF method for the assessment of the reliability of MSSs where the performance rates and their respective probabilities are indicated by fuzzy numbers. An et al. [26] estimated reliability by developing a discrete stress-strength interference model with the assistance of the UGF technique, where the stress and strength random variables are considered discrete instead of continuous. Li and Zio [27] developed a novel method using the UGF method to calculate the reliability of a system of distributed generation. In addition, the authors presented a multiplication operator for merging the UGFs for mechanical deterioration and renewable generation source states into the UGF of renewable generator power output.

Mi et al. [28] extended the concept of UGF using the belief function theory to assess the reliability of MSSs and further extended to the concept of common-cause failures. Meena and Vasanthi [29] proposed a method to evaluate the reliability of a mobile ad hoc network (MANET) by introducing two types of UGFs, node UGF and path UGF, for reliability computation. Jaiswal et al. [30] assessed the fuzzy reliability of a non-reparable weighted k-out-of-n system comprising aleatory and epistemic uncertainties by applying exponential distribution and the UGF process. Kumar and Ram [31] calculated the interval-valued reliability of a sliding window system using the UGF method. Liu et al. [32] produced a novel procedure that incorporates the Bayesian network and the UGF method to analyse the reliability of MSSs. The proposed method simplifies the computational complexities. Li et al. [33] designed a new operator for the simulation of a cold-standby system and analysed its reliability by implementing the GO-FLOW method and the UGF technique.

The remaining article is organised as follows: Section 3 comprises some essential definitions that will be required in the study, followed by a description of the model in Section 4. Section 5 includes the evaluation of the reliability function of the model considered. Section 6 describes the methodology adopted for the estimation of the reliability of the assumed model using the two types of intuitionistic fuzzy methods. In Section 7, the IFR and IVIFR are evaluated based on some examples. Finally, the acquired results are discussed in Section 8, and some concluding remarks are provided in Section 9.

3. Essential Definitions

3.1. Fuzzy Sets

The field of mathematics, in general, gained a valuable tool for coping with the imprecision or fuzziness present in numerous situations with the introduction of fuzzy sets by Zadeh [1]. In a discourse universe, $\widetilde{U}^{s'}$, a set $\overset{\frown}{P}^{\zeta}$ is considered fuzzy if it can be depicted as $\overset{\frown}{P}^{\zeta} = \left\{(p, \mu_{\overset{\frown}{P}^{\zeta}}(p)) : p \in \widetilde{U}^{s'}\right\}$, which implies that a fuzzy set is a compilation of ordered pairs, $(p, \mu_{\overset{\frown}{P}^{\zeta}}(p))$ known as the singleton, where, $\mu_{\overset{\frown}{P}^{\zeta}}(p)$ represents the degree of membership of the element p to the fuzzy set $\overset{\frown}{P}^{\zeta}$. This value ranges only from 0 to 1.

3.2. Intuitionistic Fuzzy Sets

As mentioned earlier, the concept of fuzziness has been further expanded to include intuitionistic fuzzy sets (IFS) by Atanassov [15] to address further hesitation or vagueness occurring in several situations. IFS also includes the measure of non-acceptance or non-membership, along with the measure of acceptance or membership. Considering a set $\overset{\frown}{M}^{\zeta}$ in the universe, $\widetilde{U}^{s'}$ is an IFS if it can be represented as $\overset{\frown}{M}^{\zeta} = \left\{(\widetilde{p}_i, \mu_{\overset{\frown}{M}^{\zeta}}(\widetilde{p}_i), v_{\overset{\frown}{M}^{\zeta}}(\widetilde{p}_i)) : \widetilde{p}_i \in \widetilde{U}^{s'}\right\}$. Here, the quantities of acceptance and non-acceptance of an intuitionistic fuzzy number (IFN) $\widetilde{p}_i = (\mu_{\widetilde{p}_i}, v_{\widetilde{p}_i})$ in $\overset{\frown}{M}^{\zeta}$ are denoted by $\mu_{\overset{\frown}{M}^{\zeta}}(\widetilde{p}_i)$ and $v_{\overset{\frown}{M}^{\zeta}}(\widetilde{p}_i)$, respectively, such that $\mu_{\overset{\frown}{M}^{\zeta}}, v_{\overset{\frown}{M}^{\zeta}} : \widetilde{U}^{s'} \to [0,1]\, \forall\, \widetilde{p}_i$ and $0 \leq \mu_{\overset{\frown}{M}^{\zeta}}(\widetilde{p}_i) + v_{\overset{\frown}{M}^{\zeta}}(\widetilde{p}_i) \leq 1$ for every element. Moreover, $\tau_{\overset{\frown}{M}^{\zeta}}(\widetilde{p}_i) = 1 - \mu_{\overset{\frown}{M}^{\zeta}}(\widetilde{p}_i) - v_{\overset{\frown}{M}^{\zeta}}(\widetilde{p}_i)$ represents the hesitancy measure of $\widetilde{p}_i$.

The IFS can be easily converted to a traditional fuzzy set if the hesitancy measure for each element is 0. Furthermore, it can be transformed to a crisp set if either $\mu_{\overset{\frown}{M}^{\zeta}}$ or $v_{\overset{\frown}{M}^{\zeta}}$ is 0. The former case signifies that $\widetilde{p}_i$ belongs to the IFS, $\overset{\frown}{M}^{\zeta}$, whereas the latter case confirms that the element $\widetilde{p}_i$ is not contained in the IFS, $\overset{\frown}{M}^{\zeta}$.

3.2.1. Operations on IFS

If $\wp = (\mu_{\wp}, v_{\wp})$, $\wp_\alpha = (\mu_{\wp_\alpha}, v_{\wp_\alpha})$, and $\wp_\beta = (\mu_{\wp_\beta}, v_{\wp_\beta})$ are three different IFNs, then some important operations based on IFS are described below [34–36]:

- $\overline{\wp} = (v_{\wp}, \mu_{\wp})$.
- $\wp_\alpha \cup \wp_\beta = (\max\{\mu_\alpha, \mu_\beta\}, \min\{v_\alpha, v_\beta\})$.
- $\wp_\alpha \cap \wp_\beta = (\min\{\mu_\alpha, \mu_\beta\}, \max\{v_\alpha, v_\beta\})$.
- $\wp_\alpha \oplus \wp_\beta = (\mu_\alpha + \mu_\beta - \mu_\alpha\mu_\beta, v_\alpha v_\beta)$.
- $\wp_\alpha \otimes \wp_\beta = (\mu_\alpha\mu_\beta, v_\alpha + v_\beta - v_\alpha v_\beta)$.
- $\kappa\wp = (1 - (1 - \mu_{\wp})^{\kappa}, v_{\wp}^{\kappa})$, $\kappa > 0$.
- $\wp^{\kappa} = (\mu_{\wp}^{\kappa}, 1 - (1 - v_{\wp})^{\kappa})$, $\kappa > 0$.
- The subtraction operator for two IFS was defined by Lei and Xu [2]. The subtraction operator for two IFNs is defined as follows:

$$\wp_\alpha \ominus \wp_\beta = \begin{cases} \left(\frac{\mu_\alpha - \mu_\beta}{1-\mu_\beta}, \frac{v_\alpha}{v_\beta}\right), & \mu_\alpha \geq \mu_\beta \text{ and } v_\alpha \leq v_\beta \\ & v_\beta > 0 \text{ and } v_\alpha\tau_\beta \leq \tau_\alpha v_\beta \\ (0,1) & \text{otherwise} \end{cases}.$$

3.2.2. Score Function and Accuracy Function

The score and accuracy functions are primarily used to compare two IFNs. Traneva and Tranev [36] also presented a new ranking method for intuitionistic fuzzy pairs in intuitionistic fuzzy logic. In this study, the comparison in IFNs is done using the score and accuracy functions. Chen and Tan [37] presented the score function for the ranking of IFNs, whereas Hong and Choi [38] gave the accuracy function for situations where the score function fails. The score function of an IFN $\wp = (\mu_\wp, v_\wp)$ is defined as $\sigma(\wp) = \mu_\wp - v_\wp$, while the accuracy function is defined as $\delta(\wp) = \mu_\wp + v_\wp$. Therefore, if $\sigma(\wp_\alpha)$ and $\sigma(\wp_\beta)$ are score functions, whereas $\delta(\wp_\alpha)$ and $\delta(\wp_\beta)$ are accuracy functions of $\wp_\alpha = (\mu_{\wp_\alpha}, v_{\wp_\alpha})$ and $\wp_\beta = (\mu_{\wp_\beta}, v_{\wp_\beta})$, respectively, then:

- $\wp_\alpha < \wp_\beta$ if $\sigma(\wp_\alpha) < \sigma(\wp_\beta)$.
- If $\sigma(\wp_\alpha) = \sigma(\wp_\beta)$ then,
  1. If $\delta(\wp_\alpha) = \delta(\wp_\beta)$, then $\wp_\alpha = \wp_\beta$.
  2. If $\delta(\wp_\alpha) < \delta(\wp_\beta)$, then $\wp_\alpha < \wp_\beta$.

3.3. Interval-Valued Intuitionistic Fuzzy Sets

As mentioned earlier, the IFS were expanded to interval-valued intuitionistic fuzzy sets (IVIFS) by Atanassov [39], where the measures of membership and non-membership are described as intervals rather than exact values. As a result, the IVIFS can deal with a higher level of randomness or fuzziness. In mathematical terms, an IVIFS $\widetilde{N}^i$ in the universal set $\widetilde{U}^{s'}$ can be expressed as $\widetilde{N}^i = \left\{ < p_i, \widetilde{\hbar}_{\widetilde{N}^i}(p_i), \widetilde{\lambda}_{\widetilde{N}^i}(p_i) > \middle| \, p_i \in \widetilde{U}^{s'} \right\}$, where the values of membership and non-membership are represented by the intervals $\widetilde{\hbar}_{\widetilde{N}^i}(p_i) \subset [0,1]$ and $\widetilde{\lambda}_{\widetilde{N}^i}(p_i) \subset [0,1]$, such that $\sup\widetilde{\hbar}_{\widetilde{N}^i}(p_i) + \sup\widetilde{\lambda}_{\widetilde{N}^i}(p_i) \leq 1 \, \forall \, p_i$.

3.3.1. Interval-Valued Intuitionistic Fuzzy Numbers

For $\widetilde{N}^i$, an IVIFS in $\widetilde{U}^{s'}$ is defined as $\widetilde{N}^i = \left\{ < p_i, \widetilde{\hbar}_{\widetilde{N}^i}(p_i), \widetilde{\lambda}_{\widetilde{N}^i}(p_i) > \middle| \, p_i \in \widetilde{U}^{s'} \right\}$, and an interval-valued intuitionistic fuzzy number (IVIFN) can be denoted as $([\alpha, \beta], [\varpi, \tau])$, where $\widetilde{\hbar}_{\widetilde{N}^i}(p_i) = [\alpha, \beta]$ and $\widetilde{\lambda}_{\widetilde{N}^i}(p_i) = [\varpi, \tau]$ are such that $\beta + \tau \leq 1$ [40].

3.3.2. Operations on IVIFN

If $\ell_1 = ([\alpha_1, \beta_1], [\varpi_1, \tau_1])$ and $\ell_2 = ([\alpha_2, \beta_2], [\varpi_2, \tau_2])$ are two IVIFNs, then some important operations are defined below [40]:

- $\ell_1 \cap \ell_2 = ([\min(\alpha_1, \alpha_2), \min(\beta_1, \beta_2)], [\max(\varpi_1, \varpi_2), \max(\tau_1, \tau_2)])$.
- $\ell_1 \cup \ell_2 = ([\max(\alpha_1, \alpha_2), \max(\beta_1, \beta_2)], [\min(\varpi_1, \varpi_2), \min(\tau_1, \tau_2)])$.
- $\ell_1 \otimes \ell_2 = ([\alpha_1\alpha_2, \beta_1\beta_2], [\varpi_1 + \varpi_2 - \varpi_1\varpi_2, \tau_1 + \tau_2 - \tau_1\tau_2])$.
- $\ell_1 \oplus \ell_2 = ([\alpha_1 + \alpha_2 - \alpha_1\alpha_2, \beta_1 + \beta_2 - \beta_1\beta_2], [\varpi_1\varpi_2, \tau_1\tau_2])$.
- $\kappa\ell_1 = ([1 - (1 - \alpha_1)^\kappa, 1 - (1 - \beta_1)^\kappa], [\varpi_1^\kappa, \tau_1^\kappa])$, where $\kappa > 0$.
- $\ell_1^\kappa = ([\alpha_1^\kappa, \beta_1^\kappa], [1 - (1 - \varpi_1)^\kappa, 1 - (1 - \tau)^\kappa])$, where $\kappa > 0$.
- Zhao et al. [41] gave another important operation of subtraction in IVIFN defined below:

$$\ell_1 \ominus \ell_2 = \begin{cases} \left(\left[\frac{\alpha_1 - \alpha_2}{1 - \alpha_2}, \frac{\beta_1 - \beta_2}{1 - \beta_2} \right], \left[\frac{\varpi_1}{\varpi_2}, \frac{\tau_1}{\tau_2} \right] \right), & if \; \alpha_1 \geq \alpha_2, \, \beta_1 \geq \beta_2, \, \varpi_1 \leq \varpi_2, \tau_1 \leq \tau_2 \\ & \text{and } \varpi_2, q_2 > 0 \, , \, \varpi_1(1 - \alpha_2) \leq \varpi_2(1 - \alpha_1) \\ & \text{and } \tau_1(1 - \beta_2) \leq \tau_2(1 - \beta_1) \\ ([0,0], [1,1]), & \text{otherwise} \end{cases}.$$

3.3.3. Score and Accuracy Function of IVIFNs

For an IVIFN, $\ell = ([\alpha, \beta], [\varpi, \tau])$, its score function, $\widetilde{\sigma}$ is defined as $\widetilde{\sigma}(\ell) = \frac{1}{2}(\alpha - \varpi + \beta - \tau)$, whereas the accuracy function, $\widetilde{\delta}$ is given by $\widetilde{\delta}(\ell) = \frac{1}{2}(\alpha + \varpi + \beta + \tau)$ [40]. The main objective of the two functions is to assist in ranking IVIFNs. Thus, if $\ell_1 = ([\alpha_1, \beta_1], [\varpi_1, \tau_1])$ and $\ell_2 = ([\alpha_2, \beta_2], [\varpi_2, \tau_2])$ are two IVIFNs with their respective score functions, $\widetilde{\sigma}(\ell_1)$ and $\widetilde{\sigma}(\ell_2)$, and respective accuracy functions, $\widetilde{\delta}(\ell_1)$ and $\widetilde{\delta}(\ell_2)$, then,

- $\ell_1 < \ell_2$ if $\widetilde{\sigma}(\ell_1) < \widetilde{\sigma}(\ell_2)$.
- If $\widetilde{\sigma}(\ell_1) = \widetilde{\sigma}(\ell_2)$, then,
  1. $\ell_1 = \ell_2$ if $\widetilde{\delta}(\ell_1) = \widetilde{\delta}(\ell_2)$.

2. $\ell_1 < \ell_2$ if $\widetilde{\delta}(\ell_1) < \widetilde{\delta}(\ell_2)$.

3.4. Universal Generating Function

The UGF method conceptualized by Ushakov [21] has been an asset in numerous fields, including reliability theory, due to its simplicity and efficiency. The u-function for any discrete independent random variable C is expressed as $u(z) = \sum_{\lambda=1}^{\Lambda} \widetilde{p}_\lambda z^{c_\lambda}$, where C contains Λ possible values and $\widetilde{p}_\lambda$ is the probability that $C = c_\lambda$.

In reliability theory, the UGF helps in the evaluation of the reliability of series and parallel systems in a simpler yet more effective manner. For two elements whose respective UGFs are denoted by $u_\kappa(z)$ and $u_\tau(z)$, their UGFs for series and parallel configurations are defined in (1) and (2), respectively:

$$u_\kappa(z) \underset{ser}{\otimes} u_\tau(z) = \sum_{\lambda=1}^{\Lambda_\kappa} p_{\kappa\lambda} z^{c_{\kappa\lambda}} \underset{ser}{\otimes} \sum_{\gamma=1}^{\Upsilon_\tau} p_{\tau\gamma} z^{c_{\tau\gamma}} = \sum_{\lambda=1}^{\Lambda_\kappa} \sum_{\gamma=1}^{\Upsilon_\tau} p_{\kappa\lambda} p_{\tau\gamma} z^{\min\{c_{\kappa\lambda}, c_{\tau\gamma}\}}, \tag{1}$$

$$u_\kappa(z) \underset{par}{\otimes} u_\tau(z) = \sum_{\lambda=1}^{\Lambda_\kappa} p_{\kappa\lambda} z^{c_{\kappa\lambda}} \underset{par}{\otimes} \sum_{\gamma=1}^{\Upsilon_\tau} p_{\tau\gamma} z^{c_{\tau\gamma}} = \sum_{\lambda=1}^{\Lambda_\kappa} \sum_{\gamma=1}^{\Upsilon_\tau} p_{\kappa\lambda} p_{\tau\gamma} z^{\max\{c_{\kappa\lambda}, c_{\tau\gamma}\}}. \tag{2}$$

3.4.1. Algorithm for Evaluation of Reliability of a k-out-of-n System

The concept of UGF has been extensively expanded in the reliability theory by Levitin [42] by formulating algorithms for different types of systems. The algorithm for a k-out-of-n system has been described below:

Step 1. Define the u-functions of every component of the system.

Step 2. Initially assume the reliability to be R = 0 and $U_1(z) = u_1(z)$.

Step 3. For r = 1, 2, . . . , n.

- Attain $U_r(z) = U_{r-1}(z) \underset{+}{\otimes} u_r(z)$.
- If $U_r(z)$ has an expression containing z^k, then it should be removed from $U_r(z)$ and added to R.

The reliability of the system (R) is attained at the termination of the above-mentioned algorithm.

4. Model Description

In the present section, a blended model comprised of two subsystems, A and B, is considered. Subsystem A consists of two components, labeled 1 and 2, which are aligned parallel with each other. Subsystem B consists of five components, labeled I, II, III, IV, and V, connected in a 3-out-of-5 configuration. This implies that subsystem B will operate effectively if at least three of its five components are operational. Furthermore, subsystems A and B are serially aligned with each other, implying that both subsystems A and B are required to be in an operative condition for the entire system to function. Romeu [43] used a similar system to evaluate reliability using crisp values. Here, the work has been extended to a fuzzy system because real-world systems possess impreciseness, inadequacy of data, randomness, or the possibility of errors. Therefore, the fuzzy reliability is estimated using intuitionistic fuzzy approaches. The block diagram of the described system is illustrated in Figure 1.

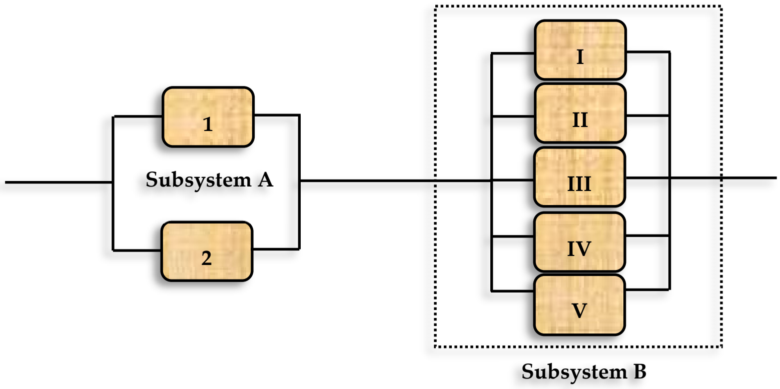

Figure 1. Block diagram of the model.

The main objective of the study is to determine the reliability of the underlying system. For this reason, we need first to determine its reliability function, which is accomplished in the following section.

5. Computation of Reliability Function

The UGF technique has been employed for the evaluation of the reliability function of the system described in Section 3. Firstly, the u-functions of each individual component are obtained. If the reliability of a component r is assumed to be $\widetilde{p}_r$ and the probability of being in failure state is $\widetilde{q}_r = 1 - \widetilde{p}_r$, then the u-functions of each component are obtained as follows:

$$u_1(z) = \widetilde{p}_1 z^1 + \widetilde{q}_1 z^0,\ u_2(z) = \widetilde{p}_2 z^1 + \widetilde{q}_2 z^0,\ u_I(z) = \widetilde{p}_I z^1 + \widetilde{q}_I z^0, u_{II}(z) = \widetilde{p}_{II} z^1 + \widetilde{q}_{II} z^0,$$
$$u_{III}(z) = \widetilde{p}_{III} z^1 + \widetilde{q}_{III} z^0,\ u_{IV}(z) = \widetilde{p}_{IV} z^1 + \widetilde{q}_{IV} z^0, \text{ and } u_V(z) = \widetilde{p}_V z^1 + \widetilde{q}_V z^0.$$

The UGF for subsystem A obtained using Equation (2) is as follows:

$$u_A(z) = \widetilde{p}_1\widetilde{p}_2 z^1 + \widetilde{p}_1\widetilde{q}_2 z^1 + \widetilde{p}_2\widetilde{q}_1 z^1 + \widetilde{q}_1\widetilde{q}_2 z^0. \tag{3}$$

Using the algorithm depicted in Section 3.4.1, the final UGF of subsystem B is acquired as follows:

$$\begin{aligned} u_B(z) = [&\widetilde{p}_I\widetilde{p}_{II}\widetilde{p}_{III} + \widetilde{p}_I\widetilde{p}_{II}\widetilde{p}_{IV}\widetilde{q}_{III} + \widetilde{p}_I\widetilde{p}_{III}\widetilde{p}_{IV}\widetilde{q}_{II} + \widetilde{p}_{II}\widetilde{p}_{III}\widetilde{p}_{IV}\widetilde{q}_I + \\ &\widetilde{p}_I\widetilde{p}_{II}\widetilde{p}_V\widetilde{q}_{III}\widetilde{q}_{IV} + \widetilde{p}_I\widetilde{p}_{III}\widetilde{p}_E\widetilde{q}_{II}\widetilde{q}_{IV} + \widetilde{p}_{II}\widetilde{p}_{III}\widetilde{p}_V\widetilde{q}_I\widetilde{q}_{IV} + \\ &\widetilde{p}_I\widetilde{p}_{IV}\widetilde{p}_V\widetilde{q}_{II}\widetilde{q}_{III} + \widetilde{p}_{II}\widetilde{p}_{IV}\widetilde{p}_V\widetilde{q}_I\widetilde{q}_{III} + \widetilde{p}_{III}\widetilde{p}_{II}\widetilde{p}_V\widetilde{q}_I\widetilde{q}_{II}]z^3 \end{aligned} \tag{4}$$

The UGF of the complete system can be obtained by the aid of (1) as follows:

$$U(z) = u_A(z) \underset{ser}{\otimes} u_B(z). \tag{5}$$

The system reliability is denoted as $R = u'(1)$. Furthermore, assuming that $\widetilde{p}_1 = \widetilde{p}_2 = \rho$ and $\widetilde{p}_I = \widetilde{p}_{II} = \widetilde{p}_{III} = \widetilde{p}_{IV} = \widetilde{p}_V = \Omega$, the final reliability function of the considered system is given as follows:

$$R = \left(2\rho - \rho^2\right)\left(6\Omega^5 - 15\Omega^4 + 10\Omega^3\right). \tag{6}$$

6. Proposed Methodologies

In this section, two types of methodologies involving the IFS are presented. The reliability of the system will be evaluated by implementing two different methods. Firstly, we shall determine the reliability with the aid of IFNs and their operations. Secondly, we shall proceed to determine the desired reliability using the IVIFNs and their respective operations. The adopted methodology has also been explained using a flowchart shown in Figure 2.

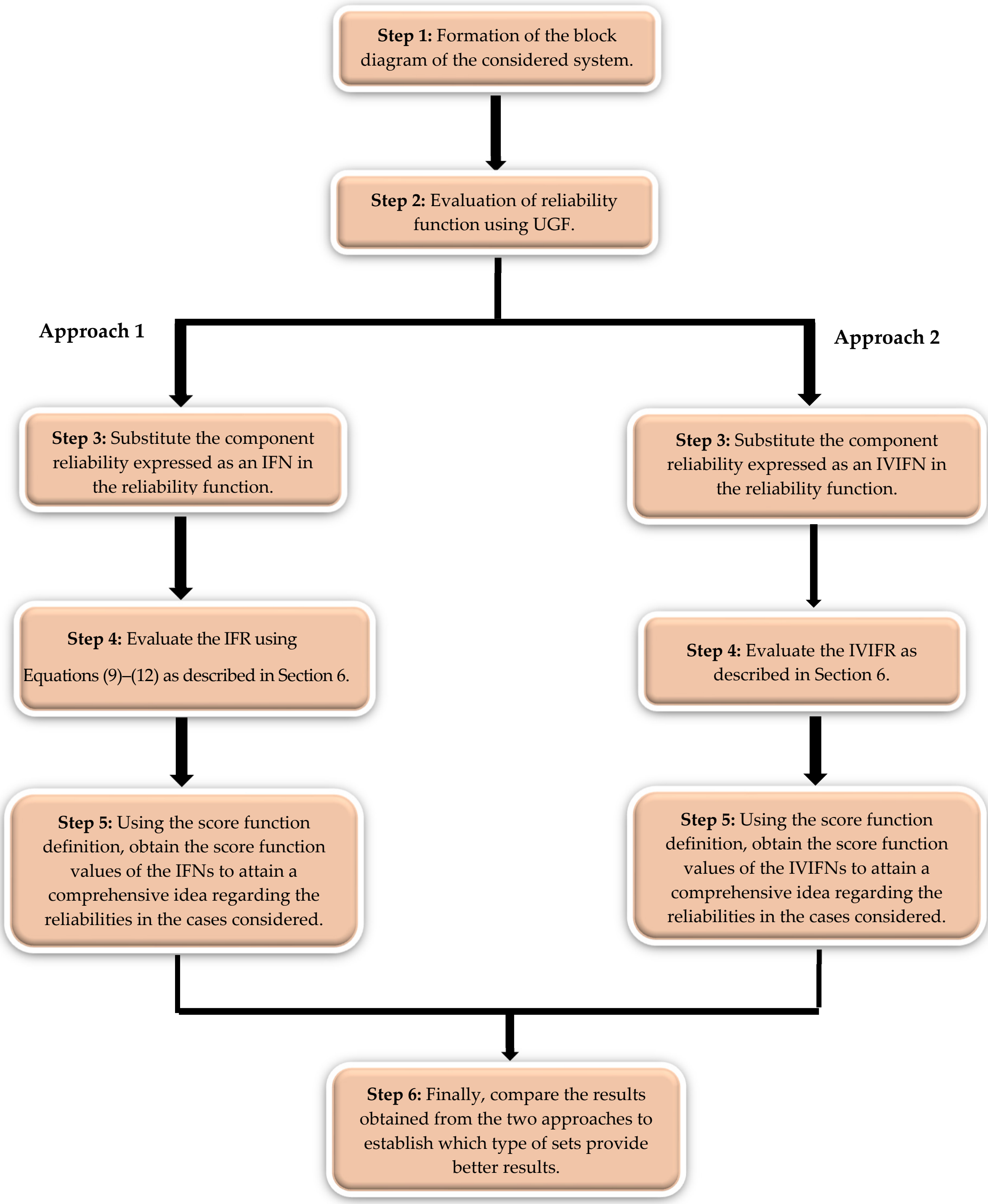

Figure 2. Flowchart of the adopted methodology.

Steps 1 and 2 of the flowchart have already been completed in Sections 4 and 5, respectively. Steps 3 and 4 are explained in this section. Numerical examples have been considered in Section 7 for the complete illustration of the two methodologies, along with steps 5 and 6.

IFN-Based Approach for Fuzzy Reliability Evaluation

The IFNs-based approach for the reliability computation is illustrated as follows. It is evident from Equation (6) that the reliability function has a polynomial form. For convenience, it is assumed that the reliability of each component is equal. The general form of the reliability function can be expressed as $R = \sum_{j=1}^{n} K_j \widetilde{\wp}_i^j$, where $\widetilde{\wp}_i$ is the probability of the operational state of the component. Moreover, assuming that $\widetilde{\wp}_i$ is an IFN defined as the ordered pair, $\widetilde{\wp}_i = (\mu_{\widetilde{\wp}_i}, v_{\widetilde{\wp}_i})$, then the reliability function can be expressed as:

$$R = \sum_{j=1}^{n} K_j (\mu_{\widetilde{\wp}_i}, v_{\widetilde{\wp}_i})^j.$$

With the aid of the operations depicted in Section 3.2.1, the above formula takes the following form:

$$R = \sum_{j=1}^{n} K_j (\mu_{\widetilde{\wp}_i}^j, 1 - (1 - v_{\widetilde{\wp}_i})^j). \tag{7}$$

By applying the operations presented in Section 3.2.1, the following is readily obtained:

$$R = \sum_{j=1}^{n} \left(1 - \left(1 - \mu_{\widetilde{\wp}_i}^j\right)^{K_j}, \left(1 - (1 - v_{\widetilde{\wp}_i})^j\right)^{K_j}\right). \tag{8}$$

For illustration purposes, let us next assume that R contains only two terms, i.e., $n = 2$. Then, $R = K_1(\mu_{\widetilde{\wp}_i}, v_{\widetilde{\wp}_i})^1 + K_2(\mu_{\widetilde{\wp}_i}, v_{\widetilde{\wp}_i})^2$. We consider two distinct cases:

Case (i): If both K_1 and K_2 are positive, then (8) is simplified as:

$$R = \left(1 - \left(1 - \mu_{\widetilde{\wp}_i}^1\right)^{K_1}, \left(1 - (1 - v_{\widetilde{\wp}_i})^1\right)^{K_1}\right) + \left(1 - \left(1 - \mu_{\widetilde{\wp}_i}^2\right)^{K_2}, \left(1 - (1 - v_{\widetilde{\wp}_i})^2\right)^{K_2}\right). \tag{9}$$

We next apply the addition operator mentioned in Section 3.2.1, and the following expression is easily delivered:

$$\begin{aligned} R = \Bigg(&1 - \left(1 - \mu_{\widetilde{\wp}_i}^1\right)^{K_1} + 1 - \left(1 - \mu_{\widetilde{\wp}_i}^2\right)^{K_2} - \left(1 - \left(1 - \mu_{\widetilde{\wp}_i}^1\right)^{K_1}\right)\left(1 - \left(1 - \mu_{\widetilde{\wp}_i}^2\right)^{K_2}\right), \\ &\left(1 - (1 - v_{\widetilde{\wp}_i})^1\right)^{K_1}\left(1 - (1 - v_{\widetilde{\wp}_i})^2\right)^{K_2}\Bigg) \end{aligned} \tag{10}$$

Case (ii): If K_1 is positive while K_2 is negative, then (8) reduces to:

$$R = \left(1 - \left(1 - \mu_{\widetilde{\wp}_i}^1\right)^{K_1}, \left(1 - (1 - v_{\widetilde{\wp}_i})^1\right)^{K_1}\right) - \left(1 - \left(1 - \mu_{\widetilde{\wp}_i}^2\right)^{K_2}, \left(1 - (1 - v_{\widetilde{\wp}_i})^2\right)^{K_2}\right). \tag{11}$$

If the following conditions are satisfied:

$$1 - \left(1 - \mu_{\widetilde{\wp}_i}^1\right)^{K_1} \geq 1 - \left(1 - \mu_{\widetilde{\wp}_i}^2\right)^{K_2} \text{ and } \left(1 - (1 - v_{\widetilde{\wp}_i})^1\right)^{K_1} \leq \left(1 - (1 - v_{\widetilde{\wp}_i})^2\right)^{K_2},$$

such that

$$\left(1 - (1 - v_{\widetilde{\wp}_i})^2\right)^{K_2} > 0,$$

and

$$\left(1 - (1 - v_{\widetilde{\wp}_i})^1\right)^{K_1} \tau_B \leq \left(1 - (1 - v_{\widetilde{\wp}_i})^2\right)^{K_2} \tau_A,$$

where τ_A and τ_B are measures of hesitation of first and second terms of (11), respectively, then by implementing the subtraction operator of IFNs from Section 3.2.1, we readily deduce that the following holds true:

$$R = \left(\frac{\left(1 - \mu_{\widetilde{\wp}_i}^2\right)^{K_2} - \left(1 - \mu_{\widetilde{\wp}_i}^1\right)^{K_1}}{\left(1 - \mu_{\widetilde{\wp}_i}^2\right)^{K_2}}, \frac{\left(1 - (1 - v_{\widetilde{\wp}_i})^1\right)^{K_1}}{\left(1 - (1 - v_{\widetilde{\wp}_i})^2\right)^{K_2}} \right). \tag{12}$$

Thus, the IFR can be obtained using the methodology described above. Furthermore, the methodology for IVIFS can also be derived in a similar manner for the evaluation of IVIFR.

7. Evaluation of IFR and IVIFR

Throughout the following section, examples of IFNs and IVIFNs are presented, wherein the IFR and IVIFR for the system considered in Section 3 are evaluated by the aid of the reliability function given in (6).

7.1. IFR Computation

Here, three examples of IFNs are considered, as shown in Table 1, to evaluate the IFR. It is evident from (6) that the components of subsystems A and B have different reliabilities, namely, ρ and Ω, respectively. Thus, Table 1 shows three pairs of IFNs representing ρ and Ω along with their respective score functions, as they help in comparing the IFNs and hence their respective reliabilities.

Table 1. Component-wise reliabilities in terms of IFNs with respective score functions.

Case	ρ	Score Function	Ω	Score Function
I	(0.70, 0.30)	0.40	(0.60, 0.30)	0.30
II	(0.82, 0.10)	0.72	(0.75, 0.25)	0.50
III	(0.90, 0.05)	0.85	(0.83, 0.10)	0.73

The reliability function obtained in (6) is $R = \left(2\rho - \rho^2\right)\left(6\Omega^5 - 15\Omega^4 + 10\Omega^3\right)$. For Case I, the reliability function is given as follows:

$$\mathrm{R} = (2(0.70, 0.30) - (0.70, 0.30)^2) \times (6(0.60, 0.30)^5 - 15(0.60, 0.30)^4 + 10(0.60, 0.30)^3). \quad (13)$$

We next apply the Equations (9)–(12), and the IFR is readily obtained as (0.46697958101, 0.42795255934).

Following a parallel argumentation, the IFR for Cases II and III can be also delivered. The IFR for all the cases and their respective score functions (rounded to five decimal places) are illustrated in Table 2.

Table 2. IFRs of the system with respective score functions.

Case	Intuitionistic Fuzzy Reliability (IFR)	Score Function
I	(0.46698, 0.42795)	0.03903
II	(0.55251, 0.41912)	0.13338
III	(0.79573, 0.17693)	0.61880

7.2. IVIFR Computation

Throughout the following section, three examples of IVIFNs are considered, wherein the determination of IVIFR is accomplished. As evident from the previous section, two separate values, ρ and Ω, for components of subsystems A and B, respectively, have been considered. Additionally, it must be noted that the underlying examples have the same score function as in the above subsection. Thus, the three pairs of reliabilities of components in terms of IVIFNs with their respective score functions are displayed in Table 3.

Table 3. Component-wise reliabilities in terms of IVIFNs with respective score functions.

Case	ρ	Score Function	Ω	Score Function
I	([0.60, 0.70], [0.20, 0.30])	0.40	([0.50, 0.60], [0.20, 0.30])	0.30
II	([0.77, 0.82], [0.05, 0.10])	0.72	([0.55, 0.60], [0.05, 0.10])	0.50
III	([0.88, 0.90], [0.01, 0.07])	0.85	([0.79, 0.86], [0.08, 0.11])	0.73

For Case I, the reliability function obtained in (6) as $R = \left(2\rho - \rho^2\right)\left(6\Omega^5 - 15\Omega^4 + 10\Omega^3\right)$ becomes:

$$R = (2([0.60, 0.70], [0.20, 0.30]) - ([0.60, 0.70], [0.20, 0.30])^2) \times (6([0.50, 0.60], [0.20, 0.30])^5 - 15([0.50, 0.60], [0.20, 0.30])^4 + 10([0.50, 0.60], [0.20, 0.30])^3), \quad (14)$$

Using the operations given in Section 3.3.2, R can be reduced to:

$$R = ([0.84, 0.91], [0.04, 0.09]) - ([0.36, 0.49], [0.36, 0.51]) \times (6([0.03125, 0.07776], [0.67232, 0.83193]) - 15([0.0625, 0.1296], [0.5904, 0.7599]) + 10([0.125, 0.216], [0.488, 0.657])), \quad (15)$$

By implementing the methodology described in Section 5, the IVIFR of the proposed model for Case I is attained as ([0.3206188837485, 0.4669795807], [0.281447191387, 0.4279525595]).

The IVIFR for Cases II and III can also be attained similarly. The IVIFR for all three cases with their respective score functions (rounded to five decimal places) is shown in Table 4.

Table 4. IVIFR of the system with respective score functions.

Case	Interval-Valued Intuitionistic Fuzzy Reliability (IVIFR)	Score Function
I	([0.32062, 0.46698], [0.28145, 0.42795])	0.03910
II	([0.43368, 0.51097], [0.06861, 0.13840])	0.36882
III	([0.74669, 0.82461], [0.07631, 0.13277])	0.68111

8. Results and Discussion

In the present work, the reliability of the model considered in Section 4 has been evaluated using the two different intuitionistic fuzzy approaches displayed in Figure 2. The IFR and IVIFR have been obtained and shown in Tables 2 and 4, respectively, of the present manuscript. The values of score functions have been calculated for the underlying examples in both cases in order to enable feasible comparisons between them. It is also noticeable that the score function values in both cases are equal, which will be helpful in assessing the competence of both IFS and IVIFS. Table 5 illustrates the reliability in terms of the score function for both of the intuitionistic fuzzy approaches. The same result is also depicted graphically in Figure 3.

Table 5. System reliability (in terms of score function values).

Case	IFR	IVIFR
I	0.03903	0.03910
II	0.13338	0.36882
III	0.61880	0.68111

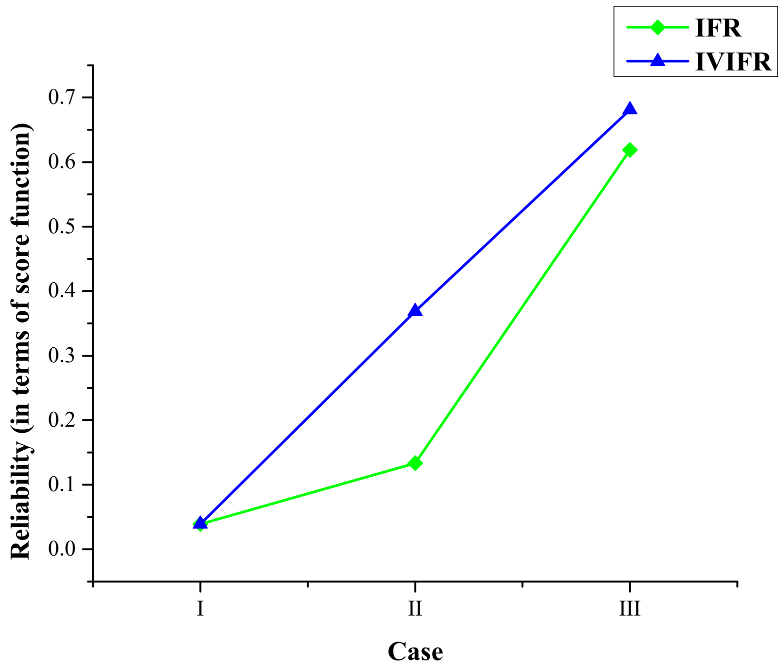

Figure 3. IFR and IVIFR (in terms of score function values).

In all the cases considered, it is evident that the assumed component reliability of Case I is the lowest, while the highest corresponds to Case III in both IFS and IVIFS. In other words, the component reliabilities follow the order Case I < Case II < Case III. In addition, it seems that the resulting system reliabilities follow the same pattern, i.e., Case I < Case II < Case III for both IFR and IVIFR of the system. This signifies that as the component reliability increases, the system reliability also increases, which is evident in both Table 5 and Figure 3. From Figure 3, it can also be seen that the IFR increases significantly from Case II to Case III, whereas the IVIFR increases linearly from Case I to Case III.

When both the IFS and IVIFS approaches are compared, it can be seen that the IVIFS give comparatively better results than the IFS. It is clearly evident from Figure 3 that for all three cases, the IVIFR is higher than the IFR. Although for Case I, they are almost equal, but Cases II and III, the IVIFR is significantly higher than IFR. Therefore, the obtained results establish that the IVIFNs provide better results in comparison to IFNs, and the system reliability increases as the individual component reliability increases in both cases.

9. Conclusions

In the present manuscript, the concept of intuitionistic fuzzy theory was implemented for the statistical computation of the reliability of a blended system constructed in Section 4, as depicted by the block diagram in Figure 1. Two different approaches were applied, namely the conventional IFS and the IVIFS-based approaches. The motive for applying both approaches was to understand the behaviour of reliability when two different types of IFS are used. The benefit of IVIFS against its competitor appears to be its ability to take comparatively higher fuzziness or randomness into consideration. This seems to be true because the acceptance and non-acceptance measures are represented using intervals instead of exact values. From the numerical investigation carried out, it can be concluded that the system reliability increases as the reliability of individual components increases in both cases. Furthermore, the reliability in the case of IVIFS seems to outperform IFS.

This study will be helpful in cases involving higher hesitancy in systems and will help in obtaining an accurate assessment of the system. Despite this, the study is not free from limitations. One of the major limitations is computational complexity for highly complex systems, which needs to be reduced in the future. Additionally, as future potential, both methods considered here can be applied to numerous models or systems for reliability evaluation. Different types of triangular or trapezoidal intuitionistic fuzzy numbers can also be used. This work may also be extended to multi-state systems with the assistance of different types of fuzzy sets.

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