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Editorial

"Computational Mathematics and Mathematical Physics"—Editorial I (2021–2023)

Hovik A. Matevossian [1,*] and Francesco dell'Isola [2]

[1] Federal Research Center "Computer Science & Control", Russian Academy of Sciences, Moscow 119333, Russia
[2] DICEAA & International Research Center on Mathematics and Mechanics of Complex Systems (M&MoCS), Università dell'Aquila, Via Giovanni Gronchi 18, 67100 L'Aquila, Italy; francesco.dellisola.me@gmail.com
* Correspondence: hmatevossian@graduate.org

Based on the papers published in the Special Issue of the scientific journal *Axioms*, here we present the Editorial Article "Computational Mathematics and Mathematical Physics", the main topics of which include both fundamental and applied research in computational mathematics and differential equations of mathematical physics.

"Mathematical Physics" is one of the topical sections of *Axioms*.

The benefit of our Special Issue "Computational Mathematics and Mathematical Physics" within the framework of *Axioms* is that we can cover applied, experimental, as well as fundamental sections of modern science in mathematics, physics, and their applications in medicine and space.

In this regard, a brief overview of the content of the Special Issue "Computational Mathematics and Mathematical Physics" carried out in the period 2021–2023, is provided, including a detailed description of the published papers (Contributions 1–12). These articles largely cover the scope and topics that we set out in the description of the Special Issue.

1. Contribution 1

In Contribution 1, using canonical and affine quantization, the behavior of full- and semi-harmonic oscillators is investigated, towards the development of field theories. The results obtained in the paper lead to a valid affine quantization of the half-harmonic oscillator field theory, which points towards further valid quantization of more realistic field theory models.

2. Contribution 2

This paper discusses the dynamics of the discrete-time amensalism model with a focus on the first species, where the authors first obtain the existence and stability of fixed points and conditions for the permanent coexistence of two species. Further, the authors demonstrate the occurrence of a flip bifurcation using the center manifold theorem and the theory of bifurcations. A hybrid control strategy is used to control the flip bifurcation and stabilize unstable periodic orbits embedded in the complex attractor. Finally, numerical simulations confirm the feasibility of the theoretical analysis and reveal some new and exciting dynamical phenomena.

3. Contribution 3

As is known, for some areas of theoretical physics, such as wave mechanics and the theory of oscillations, the solution of a problem is reduced to a problem of eigenvalues. In addition, the unambiguous definition of a mechanical system, i.e., the Hamilton function, through the spectrum of eigenvalues of the linear differential equation associated with it is important.

In cases where a string is vibrating and the boundary conditions are natural, it was shown in [1] that the spectrum of eigenvalues uniquely determines a differential equation that, in Schrödinger's theory, is called the "amplitude equation". The authors in [2] deal

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Citation: Matevossian, H.A.; dell'Isola, F. "Computational Mathematics and Mathematical Physics"—Editorial I (2021–2023). *Axioms* **2023**, *12*, 824. https://doi.org/10.3390/axioms12090824

Received: 21 August 2023
Accepted: 23 August 2023
Published: 28 August 2023

with the problem of determining the Hill equation (or the one-dimensional Schrödinger equation) from its spectrum, as well as deriving the Hill equation from specific properties of its discriminant.

Contribution 3 is devoted to the study of the asymptotic behavior of solutions of the Cauchy problem for a second-order hyperbolic equation with periodic coefficients as $t \to \infty$ in the case when the left end of the spectrum of the Hill operator is nonpositive. To obtain this asymptotic expansion for $t \to \infty$, methods of the spectral theory of differential operators are used, as well as the properties of the spectrum of the Hill operator with periodic coefficients.

In [3], the same authors considered similar questions for the Cauchy problem when the Hill operator is positive. We also note [4,5], in which the asymptotic behavior, as $t \to \infty$, of solutions of the initial-boundary value problem for a second-order hyperbolic equation with periodic coefficients on the semi-axis was obtained.

4. Contribution 4

This paper deals with the problem of goodness-of-fit tests for the weighted generalized quasi-Lindley distribution using ranked-set sampling and simple random sampling techniques.

The critical values and power of each test are obtained based on a simulation study using ranked-set sampling and simple random sampling methods, considering various sample sizes and alternatives. Moreover, these tests are based on the empirical distribution function and sample entropy. Based on the same number of measured units for the various alternatives taken into consideration in this study, it is discovered that the ranked-set sampling tests are more effective than those of their rivals in simple random sampling.

5. Contribution 5

In this paper, the usual particle in a box is turned into a field theory, and its behavior is examined using canonical and affine quantization.

In Contribution 1, the author considered the half-harmonic oscillator field theory, using both canonical and affine quantization procedures. The results of that study led to a valid quantization using affine quantization.

In Contribution 5, the author uses a new version of "quantum field theory", in which the result leads to a valid affine quantization of the particle-in-a-box field theory, which points toward further valid quantization of more realistic field theory models.

6. Contribution 6

The purpose of this study was to obtain regularity results and existence topics regarding an Eyring–Powell fluid. The geometry under study is given by a semi-infinite conduct with a rectangular cross section of dimensions $L \times H$. Starting from the initial velocity profiles (u_1^0, u_2^0) in the xy-planes, the fluid flows along the z-axis subject to a constant magnetic field and Dirichlet boundary conditions.

The global existence is demonstrated in different cases. First, the initial conditions are considered to be squared-integrable; this is the Lebesgue space $(u_1^0, u_2^0) \in L^2(\Omega)$, $\Omega = [0, L] \times [0, H] \times (0, \infty)$. Afterward, the results are extended for $(u_1^0, u_2^0) \in L^p(\Omega)$, $p > 2$.

Finally, the existence criteria are obtained when $(u_1^0, u_2^0) \in H^1(\Omega)$, and a physical interpretation of the obtained bounds is provided, showing the rheological effects of shear thinning and thickening in Eyring–Powell fluids.

7. Contribution 7

A commonly accepted variant of the theory of grand unification is constructed in a space that has a total of eleven dimensions. Four dimensions refer to real physical space–time, three of them are spatial, and one is temporal. The remaining seven correspond to some abstract space, within the framework of which one can introduce such quantum characteristics of elementary particles as isospin, hypercharge, color, flavor, etc. The number of dimensions of that additional abstract space may increase as new conserved quantum

characteristics of elementary particles are discovered. The scheme for the development of the theory is simple; however, one cannot overcome the feeling that it is ad hoc and strained. Several methods of introducing additional dimensions were explored: super-membranes, extending M-theory, F-theory, strings, $12D$ super Yang–Mills, and super-gravity theories, amongst others. These are presented in the review in [6]. From the literature, one can see that the standard theory itself, as well as most alternative approaches, introduces many additional dimensions to form a mathematical structure that is able to support the necessary quantum characteristics. Even if the increase in dimensions is moderate as in [7], the proposed modifications seem arbitrary from the mathematical point of view and look more like a fitting to explain certain features.

In the authors' paper [8], a six-dimensional manifold of symmetric signature $(3,3)$ and a special metric were proposed to construct a unified theory of gravity and electromagnetism.

Here, the authors consider its typical tangential layer, which is the pseudo-Euclidean space $\mathbb{E}_{3,3}$, as a real physical space–time in the microcosm.

The six-dimensional pseudo-Euclidean space $\mathbb{E}_{3,3}$ with signature $(3,3)$ is proposed as a model of real physical space at the sub-particle scale. The conserved quantum characteristics of elementary particles, such as spin, isospin, electric and baryon charges, and hypercharge, are expressed through the symmetries of this space. The symmetries are reflected in the various representations of the metric in $\mathbb{E}_{3,3}$ with the aid of spinors and hyperbolic complex numbers. The properties of the metric allow the number of quarks to be predicted to be equal to 18.

The violation of strong conservation laws in weak interactions is addressed through compactifying the three-dimensional temporal subspace at the sub-particle scale into single-dimensional time at bigger scales, which reduces the symmetry from the spherical to axial type.

8. Contribution 8

Frobenius' theory is widely recognized as a classic theory of solutions of Fuchsian differential equations about their regular singular points. The class of Fuchsian differential equations includes Bessel and Legendre equations.

For Fuchsian equations of second order, the solutions about any regular singular point are largely characterized by the two roots of a so-called indicial polynomial of second degree. In physics, exceptional cases, in which the roots of the indicial polynomial are real and differ by an integer, are of particular interest.

A Frobenius series is an analytical power series multiplied by a power r of the independent variable. A Fuchsian equation always has a first fundamental solution in the form of a Frobenius series, with the power r equal to the smallest root of the indicial polynomial.

In this paper, the authors developed the Frobenius method to solve linear ordinary differential equations with respect to regular singular points. In this case, the key to the Frobenius approach is the study of the derivative with respect to one parameter; this parameter is introduced through powers of generalized power series.

The method is applicable to practically important exceptional cases where the roots of the original equation are equal or differ by a nonzero integer.

The article includes a detailed demonstration of the presented enhanced Frobenius method on Bessel's equation. The method renders Frobenius-type representations that span the general solutions of all Bessel equations, including those of integer and half-integer order. For these latter cases, tandem recurrence relations for the coefficients are constructed.

9. Contribution 9

Black holes radiate their mass and energy in the form of gravitational waves and Hawking radiation, which lead to a decrease in their mass and energy. During the formation of gravitational waves and Hawking radiation, the mass and energy of black holes reduce continuously with the passage of time t. For this reason, the metric tensor of the black hole should depend on time t.

In this paper, a time-dependent term is introduced in the horizon of black hole without losing its symmetry structure by using the approximate Noether symmetry equation. The time-dependent term affects the effective potential, effective force, and all the dynamic features of the black hole. They are discussed for neutral and charged particles. Profiles of the escape velocity of colliding particles are also taken into consideration. A Lyapunov exponent is used to check the stability of the orbits of the black hole. Hawking temperature, Bekenstein entropy, Komar energy, and specific energy at the horizon of the black hole are discussed in this scenario.

10. Contribution 10

As is known, blood, depending on the size of the artery, has Newtonian and non-Newtonian fluid properties in nature, which is very attractive for various research purposes in the field of biomedical engineering and drug delivery.

As shown in some studies, blood behaves like a Newtonian fluid in large arteries, while non-Newtonian behavior is more prominent in narrow blood vessels. Because of this specific behavior of blood, it can be called a Casson fluid. These are also known to behave like fluids when a suitable shear stress is applied; otherwise, they look like a solid body. For better blood circulation, it is necessary to maintain a standard blood viscosity and temperature.

Various studies have found that hybrid nanoparticles are dispersed in blood in order to modify its thermophysical properties.

The movement of biological fluids in the human body is a particularly important field of interest for overcoming growing biomedical challenges. Blood shows different behavior in capillaries, veins, and arteries during circulation.

In the authors' paper, a new mathematical relation was developed for the flow of a nanolayer of biological fluids with the influence of TiO_2 and Ag hybrid nanoparticles. They also explain the engineering phenomena of biological fluids and the role of hybrid nanoparticles in the blood vessel system.

Using similarity transformation, the governing equations were converted into nonlinear ordinary differential equations, and the mathematical problem was solved by employing the numerical shooting method. Plots of momentum, temperature, skin friction coefficient, as well as the Nusselt number for different nondimensionless parameters for lower/upper porous walls of the channel are displayed. It was shown that the walls of the channel showed different results on magnetized physical parameters.

11. Contribution 11

This article is devoted to studying the question of the existence (in time) of weak solutions and the derivation of qualitative properties of such solutions for the nonlinear visco-elastic wave equation with variable exponents and minor damping terms. By using the energy method combined with the Faedo–Galerkin method, the local and global existence of solutions are established. Then, the stability estimate of the solution is obtained by introducing a suitable Lyapunov function.

12. Contribution 12

As is known, along with the Michelson–Morley experiment, investigations on the Sagnac effect (SE) are one of the most important fundamental aspects of relativity theory. The SE has been experimentally studied many times, both for electromagnetic waves and for massive particles. With the advent of laser and optical sensor technologies, the SE has been harnessed in so-called laser or fiber-optic gyroscopes. The theoretical consideration of the SE constitutes a broad variety of interpretations and explanations. The most widespread explanation (which is included in textbooks and can be called standard) is based on a special relativity statement of the constant light velocity, which immediately brings up the correct formula. However, this simplicity overlooks the fundamental issues of the physics involved. Moreover, such consideration is deceptive since the principles of special relativity were originally established only for the inertial frames of reference, whereas the

Sagnac effect is a substantially noninertial one and is not revealed in inertial systems at all. A more elaborate treatment can be applied with the help of a general frame of reference for a rotating observer. Much effort has been paid to incorporating the SE into the frame of general relativity, including the generalization to Finsler or Born coordinates. In the opinion of the authors, ultimately, such approaches somewhat obscure the simple physical meaning of the SE as a purely relativistic effect of special relativity.

The paper shows that the SE can be directly explained within the framework of special relativity theory by the fact that there is a group of nonlinear proper motions of the Minkowski metric, i.e., a group of nonlinear transformations of the pseudo-Euclidean coordinates, which leaves the Minkowski metric invariant. This is a group of plain circular rotations with constant angular speed. Using this group, we can derive the coordinate transformation laws, which connect the inertial system and uniformly circularly rotating one. These laws appear quite similar to Lorenz transformations between inertial systems. Hence, one can use a Lorenz-like transformation as if it were a uniform straight motion with a constant velocity $v = rw$. It should be noted that we treat only narrow cases of motions, i.e., general coordinate transformations to accelerating frames, which do not leave the space–time invariant. It might be reasonable to test whether relativistically invariant theories (i.e., theories whose equations are covariant with respect to the Lorentz transformation) are also invariant with respect to the whole group $SO(1,3)$, which incorporates the group of nonlinear transformations of the metric.

Along with the SE, we consider uniform circular motion in the so-called twin paradox. As well as the explanation of the SE, the commonly accepted treatment of the paradox is simple and based on the constancy of light velocity. Analogously, it overlooks the fundamental physical issues of the space–time metric. The essence of the twin paradox is as follows: Let there be two clocks, one of which moves along a closed path, returning to the original point where the other clock is at rest. If at the initial moment of time both clocks read the same, then at the moment of its return, the moving clock will be found to lag behind the one at rest. This conclusion is based on a nonrigorous heuristic reasoning, invoked by the properties of the Lorentz transformations where a clock at rest always shows a longer time interval than a moving clock. The group of motions of the Minkowski metric constructed above enables one to carry out a more consistent analysis of this effect and cast some doubt on its plausibility. From this analysis, it turns out that the clocks of both observers at the moment of their repeated meeting read the same time. Again, this is true only for narrow cases of uniform circular motions.

We hope that this Special Issue will inspire ambitious young talents to make important and new discoveries in the field of axioms in computational mathematics and mathematical physics.

List of Contributions

1. Klauder, J.R. A Valid Quantization of a Half-Harmonic Oscillator Field Theory. *Axioms* **2022**, *11*, 360. https://doi.org/10.3390/axioms11080360.
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Author Contributions: Both authors contributed equally to the review. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Acknowledgments: We want to express our gratitude to all the authors, reviewers, assistants, associate editors, and editors for their great work on this Special Issue.

Conflicts of Interest: The authors declare no conflict of interest.

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