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A Certain Class of Equi-Statistical Convergence in the Sense of the Deferred Power-Series Method

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Abstract: In this paper, we expose the ideas of point-wise statistical convergence, equi-statistical convergence and uniform statistical convergence in the sense of the deferred power-series method. We then propose a relation connecting them, which is followed by several illustrative examples. Moreover, as an application viewpoint, we establish an approximation theorem based upon our proposed method for equi-statistical convergence of sequences of positive linear operators. Finally, we estimate the equi-statistical rates of convergence for the effectiveness of the results presented in our study.

Keywords: statistical convergence; deferred power-series method; Cesàro summability; Korovkin-type theorem; bernstein polynomials; rate of convergence

MSC: 40A05; 40G15; 41A36

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1. Introduction, Preliminaries and Motivation

The principal edition monograph of Zygmund [1], printed in the year 1935, served as the foundation for the statistical convergence concept. Subsequently, in the year 1951, Fast [2] investigated and studied such concepts in a new direction over sequence space and presented a note on that basis. Later on, Schoenberg [3] independently developed the same concepts on sequence space with some specific fundamental limit concepts. In recent trends of sequence space, the rudimentary idea of statistical convergence has been expanded to a wider class and has becoming a very active research area in the study of various spheres of mathematical analysis, such as theory of approximation, Banach spaces, measure theory, locally convex spaces, summability theory and Fourier analysis, etc.

The concept of statistical convergence is used more frequently today than that of point-wise convergence. The credit of such development goes to two eminent mathematicians, Fast [2] and Steinhaus [3], and this concept makes the convergence analysis much wider. Nowadays, this potential idea has been applied in many disciplines of applied and pure mathematics and analytical statistics as well. In particular, it is very much useful in the study of machine learning, soft computing, number theory, measure theory and probability theory, etc. For some latest works, the interested learners may to refer [4,5].

Suppose $\mathcal{E} \subseteq \mathbb{N}$, and setting

$$\mathcal{E}_m := |\{m : m \leqq n \quad \text{and} \quad m \in \mathcal{E}\}|, \tag{1}$$

the asymptotic (natural) density $\delta(\mathcal{E})$ of $\mathcal{E}$ is defined by

$$\delta(\mathcal{E}) = \lim_{m\to\infty} \frac{1}{m}|\{m : m \leqq n \quad \text{and} \quad m \in \mathcal{E}\}| = a \quad (a \text{ exists and is finite}), \tag{2}$$

where $|\mathcal{E}_m|$ symbolizes the cardinal number (cardinality) of the set $\mathcal{E}_m$.

Definition 1. *A given sequence (x_m) is statistically convergent to ℓ if, for all $\epsilon > 0$,*

$$\mathcal{E}_\epsilon = |\{m \in \mathbb{N} \quad \textit{and} \quad |x_m - \ell| \geqq \epsilon\}|$$

ensures the natural (asymptotic) density zero (see [2,3]). Hence, for every $\epsilon > 0$,

$$\delta(\mathcal{E}_\epsilon) = \frac{|\mathcal{E}_\epsilon|}{m} = 0 \quad (m \to \infty),$$

and let us write it as

$$stat \lim x_m = \ell \quad (m \to \infty).$$

We now recall the deferred Cesàro technique for sequences of real numbers as follows. Let (a_m) and $(b_m) \in \mathbb{Z}^{0+}$ such that

$$a_m < b_m \quad \text{and} \quad \lim_{m\to\infty} b_m = \infty.$$

The deferred Cesàro $D(a_m, b_m)$-mean subjected to the regularity condition (see Agnew [6]) is given by

$$D(a_m, b_m) = \frac{x_{a_m+1} + x_{a_m+2} + x_{a_m+3} + \cdots + x_{b_m}}{b_m - a_m} = \sum_{k=a_m+1}^{b_m} x_k.$$

Definition 2. *Let (a_m) and $(b_m) \in \mathbb{Z}^{0+}$. A sequence (x_m) is deferred statistically (or $\mathrm{stat_D}$) convergent to ℓ if, for every $\epsilon > 0$,*

$$\mathcal{E}_\epsilon = |\{a_m < m \leqq b_m \quad \text{and} \quad |x_m - \ell| \geqq \epsilon\}|$$

ensures the natural (asymptotic) density zero (see [7]). Hence, for every $\epsilon > 0$,

$$\delta(\mathcal{E}_\epsilon) = \lim \frac{|\mathcal{E}_\epsilon|}{m} = 0 \quad (m \to \infty).$$

We write

$$\mathrm{stat_D} \lim_{m\to\infty} x_m = \ell.$$

We now introduce the elementary notion of convergence under the deferred power-series technique.

Let $(q_m) \in \mathbb{R}_+$ with $q_0 > 0$, and suppose that the corresponding deferred power-series, given by

$$q(s) = \sum_{m=a_n+1}^{b_n} q_m s^m,$$

has the radius of convergence R^- such that $0 < R^- \leqq \infty$.

Definition 3. *A given sequence* (x_m) *is convergent under the deferred power-series technique if*

$$\lim_{0<s\to R^-} \frac{1}{q(s)} \sum_{m=a_n+1}^{b_n} q_m s^m x_m = a,$$

where a is real and finite.

We recall that the deferred power-series technique is said to be regular (see [8]) if and only if

$$\lim_{0<s\to R^-} \frac{q_m s^m}{q(s)} = 0 \qquad (\forall\ m \in \mathbb{N}).$$

We next present the statistical convergence of real sequences for the deferred power-series method.

Let $\mathcal{M} \subset \mathbb{N}$. Additionally, let

$$\mathcal{M}_\epsilon = \{a_n < m \leqq b_n \quad \text{and} \quad m \in \mathcal{M}\}. \tag{3}$$

If the following limit:

$$\delta_{\mathrm{DP}}(\mathcal{M}_\epsilon) = \lim_{0<s\to R^-} \frac{1}{q(s)} \sum_{m\in\mathcal{M}_\epsilon} q_m s^m$$

exists, then

$$\delta_{\mathrm{DP}}(\mathcal{M}_\epsilon)$$

is called the DP-density of $\mathcal{M}$.

Definition 4. *A sequence* (x_m) *is statistically convergent to* ℓ *under the deferred power-series technique if, for each* $\epsilon > 0$,

$$\lim_{0<s\to R^-} \frac{1}{q(s)} \sum_{m\in\mathcal{M}_\epsilon} q_m s^m = 0,$$

where

$$\mathcal{M}_\epsilon = \{a_n < m \leqq b_n \quad and \quad |x_m - \ell| \geqq \epsilon\},$$

that is,

$$\delta_{\mathrm{DP}}(\mathcal{M}_\epsilon) = 0 \qquad (\forall\ \epsilon > 0).$$

We write

$$\mathrm{stat}_{\mathrm{DP}} \lim x_m = \ell.$$

The following example illustrates that the statistical convergence and the statistically deferred power-series (or $\mathrm{stat}_{\mathrm{DP}}$) convergence are not comparable.

Example 1. *Let*

$$q_m = \begin{cases} 1 & (m = n^2;\ n \in \mathbb{N}) \\ 0 & (\text{otherwise}) \end{cases}$$

and

$$x_m = \begin{cases} 0, & (m = n^2;\ n \in \mathbb{N}) \\ m, & (\text{otherwise}). \end{cases}$$

It is apparently true that, (x_m) does not converse statistically to 0, but in view of Definition 4, we have

$$\lim_{0<s\to R^-} \frac{1}{q(s)} \sum_{m\in\{a_n<m\leqq b_n:|x_m|\geqq\epsilon\}} q_m s^m = 0,$$

where $a_n = 2n$ and $b_n = 4n$. Consequently, (x_m) converges statistically to 0 in the sense of the deferred power-series technique.

Again, let

$$x_m = \begin{cases} \frac{1}{m} & (m = n^2;\ n \in \mathbb{N}) \\ 0 & (\text{otherwise}), \end{cases}$$

where $a_n = 2n$ and $b_n = 4n$.

It is actually true that (x_m) converges statistically to 0, but we have

$$\lim_{0<s\to R^-} \frac{1}{q(s)} \sum_{m\in\{a_n<m\leqq b_n:|x_m|\geqq\epsilon\}} q_m s^m \neq 0.$$

Thus, clearly, the sequence (x_m) is not statistically convergent under the deferred power-series (stat_{DP}) technique.

In the second half of the nineteenth century, many works about statistical convergence were discussed by a few researchers, such as in the year 1980, Šalát [9] investigated the theory of statistically convergent real number sequences and studied the boundedness properties of such sequences. After that, Fridy [2] discussed the concrete definition of Cauchy criterions of statistical convergence and accordingly established some rudimentary results based on summability means. Subsequently, in the year 1988, Maddox [10] considered the locally convex space for the extensive study of statistical convergence and accordingly established certain relevant results. Gradually, in view of more advanced studies in this direction, Fridy and Orhan [11] presented the lacunary statistical summability means for a sequence of real numbers and obtained some prominent results.

The notion of the fundamental limit concept on statistical Cesàro summability and its applications was first introduced by the eminent mathematician Móricz [12]. Again, Mohiuddine et al. [13] obtained a nice outcome on the statistical Cesàro summability mean with an illustrative example and further proved some associated Korovkin-type theorems. Afterwards, Karakaya and Chishti [14] popularized the elementary idea of statistical convergence via the weighted summability mean, and later in the year 2018, Mursaleen et al. [15] clearly modified this concept and established some fundamental limit theorems. Recently, Baliarsingh et al. [16] introduced and deliberated the notion of an advance version of uncertain sequences via statistical deferred A-convergence and proved some inclusion theorems. Again, in that year, Saini et al. [17] also studied the results on equi-statistical convergence via the deferred Cesàro and deferred Euler summability product means with associated Korovkin-type theorems. Additionally, Saini et al. [18] again studied deferred Riesz statistical convergence of a complex uncertain sequence with its applications; also in that year, Sharma et al. [19] demonstrated the implementations of statistical deferred Cesàro convergence of fuzzy number valued sequences of order (ξ, ω). In the year 2018, Srivastava et al. [20] studied and investigated the idea of sequences that converge equi-statistically based on the deferred Nörlund mean. Subsequently, Parida et al. [21] proposed some results for sequences that converge equi-statistically via the deferred Cesàro mean and accordingly demonstrated the Korovkin-type theorems. More recently, Demirci et al. [22,23] investigated the perception of sequences that converge equi-statistically under the power-series technique and proved some approximation results.

In view of the above-mentioned literature and study, we recall the deferred point-wise statistical convergence, the deferred equi-statistical convergence and the deferred uniform statistical convergence of sequences of functions (see [20,21]).

Let $I \subseteqq \mathbb{R}$, and let $f \in C(I)$, so $f_m \in C(I)$ as well, where $C(I)$ is the class of real-valued continuous functions over I. Additionally, let $\| f \|_{C(I)}$ be the supremum norm.

(a) If, for each $\epsilon > 0$ and for every $x \in I$,

$$\lim \frac{\mathcal{K}_m(x,\epsilon)}{m} = 0 \quad (m \to \infty),$$

where

$$\mathcal{K}_m(x,\epsilon) := |\{m \in (a_n b_n] \quad \text{and} \quad |f_m(x) - f(x)| \geqq \epsilon\}|,$$

then (f_m) is deferred statistically point-wise convergent to f on I. We write

$$f_m \to f \;\; (\text{stat} - \text{pointwise}).$$

(b) If, for each $\epsilon > 0$,

$$\lim_{m\to\infty} \frac{\mathcal{K}_m(x,\epsilon)}{m} = 0 \text{ uniformly with regards to } x \in I,$$

then (f_m) is equi-statistically convergent to f on I. We write

$$f_m \Longrightarrow f \; (\text{equi} - \text{stat}).$$

(c) If, for each $\epsilon > 0$,

$$\lim_{m\to\infty} \frac{\mathcal{D}_m(\epsilon)}{m} = 0,$$

where

$$D_m(\epsilon) = |\{m \in (a_n b_n] \quad \text{and} \quad \| f_m - f \|_{C(I)} \geqq \epsilon\}|,$$

then (f_m) is statistically uniformly convergent to f on I. We write

$$f_m \rightrightarrows f \; (\text{stat} - \text{unifomly}).$$

Now, in order to have some advanced study in line with the uniform convergence of the power-series method, we wish to introduce the deferred power-series technique for sequences of real numbers in the following sense.

It is well known that nearly all of the transformation techniques used in the summability theory have many undesirable characteristics. In particular, the power-series technique of any given positive order having usual bounds and oscillations usually does not always preserve continuous convergence or convergence in the uniform sense. However, the proposed modified power-series transformation technique (that is, the deferred power-series technique) has very useful properties with regard to uniform convergence of the sequence of functions. In particular, the proposed technique is well-behaved in the sense of uniform convergence, which, in fact, are shown diagrammatically in Section 3.

Motivated essentially by the above-mentioned discussions and results, we present our investigations as follows. In Section 1, we discuss the notion of the deferred power-series technique based on statistical convergence. Subsequently, based on the proposed method, we define the point-wise statistical convergence, equi-statistical convergence and uniform statistical convergence of sequences of functions and establish an inclusion relation connecting them. In Section 2, we prove the Korovkin-type theorem, which is based upon our proposed method of equi-statistical convergence of sequences of functions. In Section 3, we present the the geometrical view of equi-statistical convergence of sequences of functions under a suitable example via positive linear operators. In Section 4, we discuss the rates of equi-statistical convergence under the proposed deferred power-series technique for the positive linear operator sequences with respect to the modulus of continuity. In Section 5, based on our main result, we present some concluding remarks and observations and indicate some prospective future scopes in different sequence spaces.

We now suggest the following definitions for the proposed study.

Definition 5. *For all* $\epsilon > 0$ *and* $x \in I$, *if*

$$\delta_{\mathrm{DP}}(\mathcal{K}_m(x,\epsilon)) = \lim_{0<s\to R^-} \frac{1}{q(s)} \sum_{m\in\mathcal{K}_m(x,\epsilon)} q_m s^m = 0,$$

then (f_m) *is point-wise statistically convergent to* f *on* I *under the deferred power-series technique. We write*

$$f_m \to f \quad (\mathrm{stat}-\mathrm{point}_{\mathrm{DP}}).$$

Definition 6. *For all* $\epsilon > 0$, *if*

$$\delta_{\mathrm{DP}}(\mathcal{K}_m(x,\epsilon)) = \lim_{0<s\to R^-} \frac{1}{q(s)} \sum_{m\in\mathcal{K}_m(x,\epsilon)} q_m s^m = 0 \text{ (uniformly in } x),$$

then (f_m) *is equi-statistically convergent to* f *on* I *under deferred power-series technique. We write*

$$f_m \to f \quad (\mathrm{equi}-\mathrm{stat}_{\mathrm{DP}}).$$

Definition 7. *For all* $\epsilon > 0$, *if*

$$\delta_{\mathrm{DP}}(\mathcal{D}_m(\epsilon)) = \lim_{0<s\to R^-} \frac{1}{q(s)} \sum_{m\in D_m(\epsilon)} q_m s^m = 0,$$

then (f_m) *is uniformly statistically convergent to* f *on* I *under the deferred power-series technique. We write*

$$f_m \rightrightarrows f \quad (\mathrm{stat}-\mathrm{uni}_{\mathrm{DP}}).$$

In view of Definitions 5–7, we now propose an inclusion relation, which is supported by several illustrative examples as follows.

Lemma 1. *The implications as mentioned below are true:*

$$\begin{aligned} f_m \rightrightarrows f \quad (\mathrm{stat}-\mathrm{uni}_{\mathrm{DP}}) &\Longrightarrow f_m \to f \quad (\mathrm{equi}-\mathrm{stat}_{\mathrm{DP}}) \\ &\Longrightarrow f_m \to f \quad (\mathrm{stat}-\mathrm{point}_{\mathrm{DP}}). \end{aligned} \tag{4}$$

The implications in (4) *are strict, i.e., the opposite implications in* (4) *are not generally true.*

We provide below the numerical examples to support that the implications are strict as claimed under Lemma 1.

Example 2. *Let* $a_n = 2n$ *and* $b_n = 4n$*, and let*

$$f_m(x) = \begin{cases} -2^m(x - \frac{1}{2^{m-1}}) & (m = n^2, n \in \mathbb{N};\ x \in \mathcal{A}) \\ 2^m(x - \frac{1}{2^m}) & (m = n^2, n \in \mathbb{N};\ x \in \mathcal{B}) \\ 0 & (m = n^2, n \in \mathbb{N}; x \notin A \cup B) \\ m & (\text{otherwise}), \end{cases} \tag{5}$$

where

$$\mathcal{A} = \left[2^{-(m-1)} - 2^{-m}, 2^{-(m-1)}\right] \quad \text{and} \quad \mathcal{B} = \left[2^{-m}, 2^{-(m-1)} - 2^{-m}\right].$$

Suppose also that

$$q_m = \begin{cases} 1 & (m = n^2, n \in \mathbb{N}) \\ 0 & (\text{otherwise}). \end{cases}$$

Clearly, from Definition 4, we have

$$\delta_{\mathrm{DP}}(\{m : m \in (a_n, b_n] \quad \text{and} \quad |f_m - f| \geqq \epsilon\}) = 0.$$

Therefore, for any $x \in I$,

$$\lim_{0<s\to R^-} \frac{1}{q(s)} \sum_{\{a_n < m \leqq b_n \text{ and } |f_m - f| \geqq \epsilon\}} q_m s^m \leqq \lim_{0<s\to R^-} \frac{1}{q(s)} q_{m_0} s^{m_0} = 0.$$

We thus obtain

$$f_m \to f \quad (\text{equi} - \text{stat}_{\mathrm{DP}}) \quad \text{on } I.$$

However, since

$$\| f_m - f \|_{C(I)} \neq 0,$$

(f_m) *neither converges statistically nor uniformly statistically to* 0 *under the deferred power-series technique.*

Example 3. *Let* $I = [0, 1]$ *and let*

$$f_m(x) = \begin{cases} 0 & (m = n^2;\ n \in \mathbb{N}) \\ x^m & (\text{otherwise}), \end{cases}$$

and

$$\lim_{m\to\infty} f_m(x) = f(x) \quad (x \in I),$$

where

$$f(x) = \begin{cases} 0 & x \in [0, 1) \\ 1 & (x = 1). \end{cases}$$

Let

$$q_m = \begin{cases} 0 & (m = n^2;\ n \in \mathbb{N}) \\ 1 & (\text{otherwise}). \end{cases}$$

Then,

$$f_m \to f \quad (\text{stat} - \text{point}_{\text{DP}}).$$

Moreover, if we take $\epsilon = \frac{1}{2}$*, then, for every* $x \in \left(\sqrt[m]{\frac{1}{2}}, 1 \right)$*,*

$$|f_m(x)| = |x^m| > \left| \left(\sqrt[m]{\frac{1}{2}} \right)^m \right| = \frac{1}{2}.$$

Hence, we see that the following statement:

$$f_m \to f \quad (\text{equi} - \text{stat}_{\text{DP}})$$

does not ultimately hold true.

2. A Korovkin-Type Approximation Theorem

In the year 1960, Korovkin [24] proved the traditional Korovkin-type theorem by demonstrating that a sequence $(\mathfrak{L}_m)$ of (positive) linear operators uniformly converges to the function that is to be approximated (see [25]). Following this finding, a number of mathematicians have set out to expand Korovkin's results in a variety of ways and to a variety of contexts, including function spaces, Banach spaces and so on. These advances led to the creation of a theory that are now known as Korovkin-type theorems. This hypothesis has beautiful applications in advance analysis, Fourier series and summability theory. However, the Korovkin-type hypothesis is still in its early stages of research, particularly in the areas where it deals with limit operators other than the identity operator.

Recently, Korovkin-type results have been investigated and studied under various notions of statistical convergence techniques (see [4,26–32]). Furthermore, Balcerzak et al. [33] proposed a stronger result via equi-statistical convergence over the uniform statistical convergence. On the other hand, based upon equi-statistical convergence, different results with various settings have been established by many researchers (see, for example, [20,21,34–38]). In view of some advanced study in this direction, here, we consider the proposed deferred power-series method in establishing a Korovkin-type theorem, which is based upon the prospective concept of equi-statistically convergence of sequences of functions.

Let $\mathfrak{L}$ be a linear operator mapping on $C(I)$, and then, $\mathfrak{L}$ is said to be a positive linear operator if

$$f(x) \geqq 0 \quad \Longrightarrow \quad \mathfrak{L}(f; x) \geqq 0.$$

Recalling certain approximation theorems, in this section, we wish to prove a new Korovkin-type theorem by using our proposed deferred power-series means under equi-statistically convergence of positive linear operator sequences. For establishing the desired theorem, we have considered the test functions (algebraic) $1, x$ and x^2, that is,

$$f_i(x) = x^i \qquad (i = 0, 1, 2).$$

Before introducing the main result, we recall here the traditional Korovkin-type theorem (see [24]), followed by some statistical Korovkin-type theorems in line of the power-series approaches (see [22,23]) as follows:

Theorem 1 (see [24]). *Let* $(\mathfrak{L}_m)$ *be the sequences of (positive) linear operators on* $C(I)$*, and then, for each* $f \in C(I)$*,*

$$\lim_{m\to\infty} \parallel \mathfrak{L}_m(f) - f \parallel_{C(I)} = 0$$

if and only if

$$\lim_{m\to\infty} \parallel \mathfrak{L}_m(f_i;x) - f_i \parallel_{C(I)} = 0 \qquad (i = 0,1,2).$$

Theorem 2 (see [23]). *Let* $(\mathfrak{L}_m)$ *be the sequences of positive linear operators on* $C(I)$*, and then, for each* $f \in C(I)$*,*

$$\text{stat}_{\text{P}} \lim_{m\to\infty} \parallel \mathfrak{L}_m(f;x) - f \parallel_{C(I)} = 0$$

if and only if

$$\text{stat}_{\text{P}} \lim_{m\to\infty} \parallel \mathfrak{L}_m(f_i;x) - f_i \parallel_{C(I)} = 0 \qquad (i = 0,1,2).$$

Theorem 3 (see [22]). *Let* $(\mathfrak{L}_m)$ *be a sequence of (positive) linear operators on* $C(I)$*, and then, for all* $f \in C(I)$*,*

$$\mathfrak{L}_m(f;x) \longrightarrow f \quad (\text{equi} - \text{stat}_{\text{P}}) \text{ on } I \tag{6}$$

if and only if

$$\mathfrak{L}_m(f_i) \longrightarrow f_i \quad (\text{equi} - \text{stat}_{\text{P}})\ (i = 0,1,2). \tag{7}$$

As a primary finding in this investigation, we now establish the following new Korovkin-type theorem.

Theorem 4. *Let* (a_n) and $(b_n) \in \mathbb{Z}^{0+}$*, and let* $(\mathfrak{L}_m)$ *be linear operators on* $C(I)$*. Then, for all* $f \in C(I)$*,*

$$\mathfrak{L}_m(f;x) \longrightarrow f \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I \tag{8}$$

if and only if

$$\mathfrak{L}_m(f_i) \longrightarrow f_i \quad (\text{equi} - \text{stat}_{\text{DP}})\ (i = 0,1,2). \tag{9}$$

Proof. Since

$$f_i(x) = x^i \in C(I) \quad (i = 0,1,2)$$

is continuous, the implication (8) $\Longrightarrow$ (9) is evidently trivial.

Now, to prove the implication (9) $\Longrightarrow$ (8), we suppose that $f \in C(I)(\forall\, x \in I)$, and I being closed, there exists E (a constant) with

$$-E \leqq f(x) \leqq E \quad (x \in I).$$

Consequently,

$$-2E \leqq (f(t) - f(x)) \leqq 2E \quad (x,t \in I).$$

Thus, for every $\epsilon > 0$, $\exists\, \delta > 0$ such that

$$|x - t| < \delta \implies |f(t) - f(x)| < \epsilon \quad (\forall\, x,t \in I). \tag{10}$$

Let us now choose ϑ such that

$$\vartheta = \vartheta(t, x) = t^2 + x^2 - 2tx.$$

We then immediately obtain

$$-\frac{2E}{\delta^2}\vartheta(t, x) \leqq f(t) - f(x) \leqq \frac{2E}{\delta^2}\vartheta(t, x) \quad (x, t \in I), \tag{11}$$

for

$$|t - x| \geqq \delta.$$

Now, from (10) and (11),

$$-\epsilon - \frac{2E}{\delta^2}\vartheta(t, x) \leqq f(t) - f(x) \leqq \epsilon + \frac{2E}{\delta^2}\vartheta(t, x) \quad (x, t \in I). \tag{12}$$

Applying the sequence $\big(\mathfrak{L}_m(1, x)\big)$ of positive operators, which are both linear and monotone, to the inequality (12), we subsequently have

$$\mathfrak{L}_m(1, x)\left(-\epsilon - \frac{2E}{\delta^2}\vartheta(t, x)\right) \leqq \mathfrak{L}_m(1, x)[f(t) - f(x)] \leqq \mathfrak{L}_m(1, x)\left(\epsilon + \frac{2E}{\delta^2}\vartheta(t, x)\right).$$

Here, one can note that x is fixed, and so also, $f(x)$ is a constant number. Thus, clearly, we obtain

$$-\epsilon\, \mathfrak{L}_m(1, x) - \frac{2E}{\delta^2}\mathfrak{L}_m(\vartheta, x) \leqq \mathfrak{L}_m(f, x) - f(x)\mathfrak{L}_m(1, x) \leqq \epsilon\, \mathfrak{L}_m(1, x) + \frac{2E}{\delta^2}\mathfrak{L}_m(\vartheta, x),$$

which, in association with the following obvious identity:

$$[\mathfrak{L}_m(f, x) - f(x)\mathfrak{L}_m(1, x)] + f(x)[\mathfrak{L}_m(1, x) - 1] = \mathfrak{L}_m(f, x) - f(x),$$

yields

$$\mathfrak{L}_m(f, x) - f(x) < \epsilon\, \mathfrak{L}_m(1, x) + \frac{2E}{\delta^2}\mathfrak{L}_m(\vartheta, x) + f(x)[\mathfrak{L}_m(1, x) - 1]. \tag{13}$$

We now see that

$$\begin{aligned}
\mathfrak{L}_m(\vartheta, x) &= \mathfrak{L}_m\big((t - x)^2, x\big) \\
&= \mathfrak{L}_m(t^2 - 2xt + x^2), x) \\
&= \mathfrak{L}_m(t^2, x) - 2x\mathfrak{L}_m(t, x) + x^2\mathfrak{L}_m(1, x) \\
&= [\mathfrak{L}_m(t^2, x) - x^2] - 2x[\mathfrak{L}_m(t, x) - x] + x^2[\mathfrak{L}_m(1, x) - 1].
\end{aligned}$$

Additionally, by using (13), we have

$$\begin{aligned}
\mathfrak{L}_m(f, x) - f(x) &< \epsilon\, \mathfrak{L}_m(1, x) + \frac{2E}{\delta^2}\{[\mathfrak{L}_m(t^2, x) - x^2] \\
&\qquad - 2x[\mathfrak{L}_m(t, x) - x] + x^2[\mathfrak{L}_m(1, x) - 1]\} + f(x)[\mathfrak{L}_m(1, x) - 1] \\
&= \epsilon[\mathfrak{L}_m(1, x) - 1] + \epsilon + \frac{2E}{\delta^2}\{[\mathfrak{L}_m(t^2, x) - x^2] \\
&\qquad - 2x[\mathfrak{L}_m(t, x) - x] + x^2[\mathfrak{L}_m(1, x) - 1]\} + f(x)[\mathfrak{L}_m(1, x) - 1].
\end{aligned}$$

As we know that $\epsilon > 0$ is a very small arbitrary constant, we can write

$$\mathfrak{L}_m(f,x)-f(x)=\left(\epsilon+\frac{2E}{\delta^2}+E\right)|\mathfrak{L}_m(1,x)-1|+\frac{2E}{\delta^2}|\mathfrak{L}_m(t^2,x)-x^2|-\frac{4E}{\delta^2}|\mathfrak{L}_m(t,x)-x|$$
$$\leqq \mathcal{G}\Big\{|\mathfrak{L}_m(1,x)-1|-|\mathfrak{L}_m(t,x)-x|+|\mathfrak{L}_m(t^2,x)-x^2|\Big\}, \tag{14}$$

where

$$\mathcal{G}=\max\left\{\epsilon+\frac{2E}{\delta^2}+E,\frac{4E}{\delta^2},\frac{2E}{\delta^2}\right\}.$$

Next, for $\lambda>0$, we choose $\epsilon>0$ with $0<\epsilon<\lambda$. Consequently,

$$\mathcal{H}_m(x,\epsilon)=\{m\in(a_n,b_n] \quad \text{and} \quad |\mathfrak{L}_m(f,x)-f(x)|\geqq\lambda\}$$

and

$$\mathcal{H}_{i,m}(x,\epsilon)=\left\{a_n<m\leqq b_n \quad \text{and} \quad |\mathfrak{L}_m(f_i,x)-f_i(x)|\geqq\frac{\lambda-\epsilon}{3\mathcal{G}}\right\},$$

we thereafter easily find from (14) that

$$\mathcal{H}_m(x,\epsilon)\leqq\sum_{i=0}^{2}\mathcal{H}_{i,m}(x,\epsilon).$$

We thus obtain

$$\lim_{0<s\to R^-}\frac{1}{q(s)}\sum_{m\in\mathcal{H}_m(x,\epsilon)}q_m s^m\leqq\sum_{i=0}^{2}\lim_{0<s\to R^-}\frac{1}{q(s)}\sum_{m\in\mathcal{H}_{i,m}(x,\frac{\lambda-\epsilon}{3\mathcal{G}})}q_m s^m. \tag{15}$$

Finally, the right-hand side (RHS) of (15) tends to zero under the aforementioned assumption regarding the implication in (9) and by using Definition 6. Therefore, as a result, we obtain

$$\lim_{0<s\to R^-}\frac{1}{q(s)}\sum_{m\in\mathcal{H}_m(x,\epsilon)}q_m s^m=0.$$

Therefore, this implication (8) certainly holds true. Theorem 4 is thus proved. □

3. Geometrical View of Theorem 4

In view of our Theorem 4, we present below an example under certain specific positive linear polynomials, called the Bernstein polynomials. Moreover, for better understanding of the readers, we present their geometrical interpretation by using MATLAB software.

Example 4. *Let $I=[0,1]$, and let the Bernstein polynomials $\mathfrak{B}_m(f;x)$ be such that*

$$\mathfrak{B}_m(f;x)=\sum_{j=0}^{m}f\left(\frac{j}{m}\right)\binom{m}{j}x^j(1-x)^{m-j} \qquad (x\in[0,1])$$

on $C(I)$.

We now denote $\mathfrak{L}_m(f,x)$ as the linear operator sequence under the composition of the Bernstein polynomials and sequences of functions as follows:

$$\mathfrak{L}_m(f,x)=\big(1+f_m(x)\big)\mathfrak{B}_m(f;x) \qquad \big(x\in I;\ f\in C(I)\big), \tag{16}$$

where the sequence (f_m) is given by (5) with

$$q_m = \begin{cases} 1 & (m = n^2;\ n \in \mathbb{N}) \\ \\ 0 & (\text{otherwise}). \end{cases}$$

We then calculate the positive linear operators $\mathfrak{L}_m(f_i, x)$ *for each value of* $i = 0, 1, 2$, *that is,*

$$\mathfrak{L}_m(f_0; x) = \big(1 + f_m(x)\big) f_0(x)$$

$$\mathfrak{L}_m(f_1; x) = \big(1 + f_m(x)\big) f_1(x)$$

and

$$\mathfrak{L}_m(f_2; x) = \big(1 + f_m(x)\big) \left[f_2(x) + \frac{x(1-x)}{m} \right].$$

Since

$$f_m \to f = 0 \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I,$$

for the sequence (f_m) *as defined in Example 2, we have*

$$\mathfrak{L}_m(f_i) \to f_i \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I,$$

for every value of $i = 0, 1, 2$. *Thus, by our Theorem 4, one can see that*

$$\mathfrak{L}_m(f; x) \to f \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I$$

for every $f \in C(I)$.

In view of Figures 1–4, one can easily understand the nature of equi-statistical convergence of the proposed sequence of positive linear operators $\mathfrak{L}_m(f_i; x)\ \ (i = 0, 1, 2)$ for $m = 1, 4, 9, 16$. In Figures 1 and 2 corresponding to $m = 1$ and $m = 4$, we notice the overlapping of the curves under the sequence of positive linear operators (16). However, for $m = 9$ and $m = 16$, we observe the smoothness of the curves in the the corresponding Figures 3 and 4. Thus, the convergence of our proposed sequence of positive linear operators under (16) is well-behaved with the increase in the values of m.

Next, it is interesting to note that the sequence (f_m) as specified in (5) does not statistically uniformly converge to $f = 0$ over I under the deferred power-series method. Thus, clearly, the result of Demirci et al. [22] and the result of Ünver and Orhan [23] do not certainly operate for the operators $\big(\mathfrak{L}_m(f; x)\big)$ in (16). Moreover, since (f_m) does not converge uniformly to $f = 0$ (in the classical sense) on I, the aforementioned traditional Korovkin theorem [24] also does not operate here. Thus, our recommended operators considered in (16) genuinely satisfy Theorem 4.

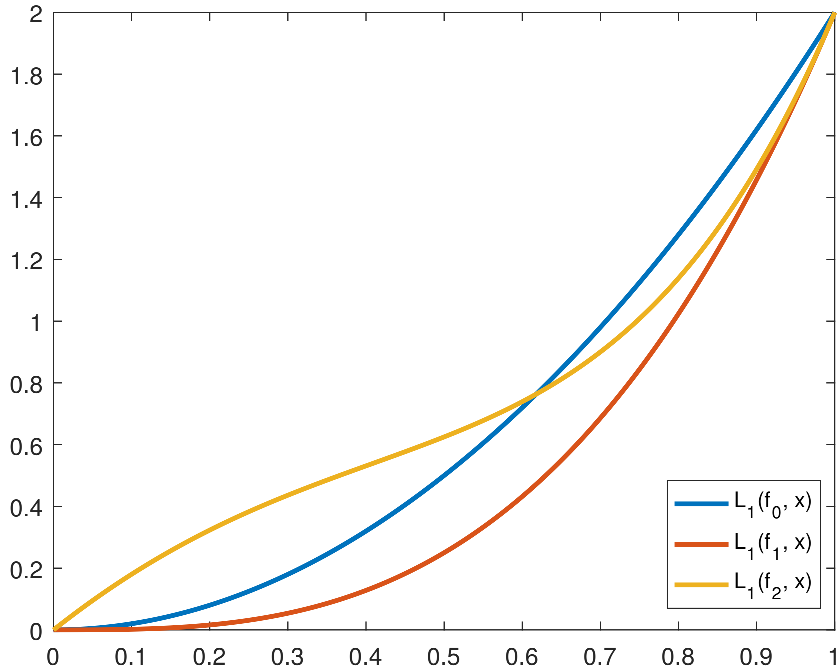

Figure 1. Equi-stat convergence of $\mathfrak{L}_1(f_i; x) \ \ (i = 0, 1, 2)$.

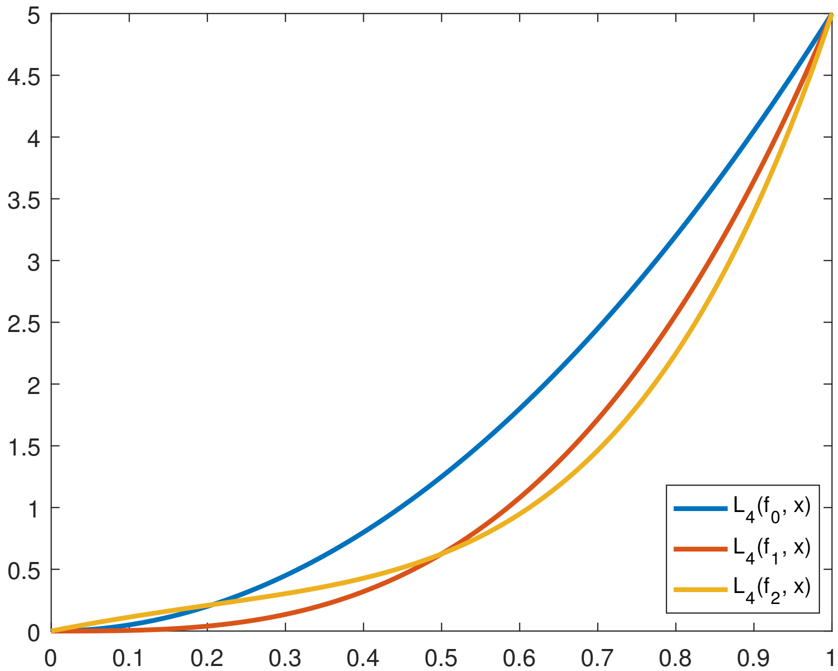

Figure 2. Equi-stat convergence of $\mathfrak{L}_4(f_i; x) \ \ (i = 0, 1, 2)$.

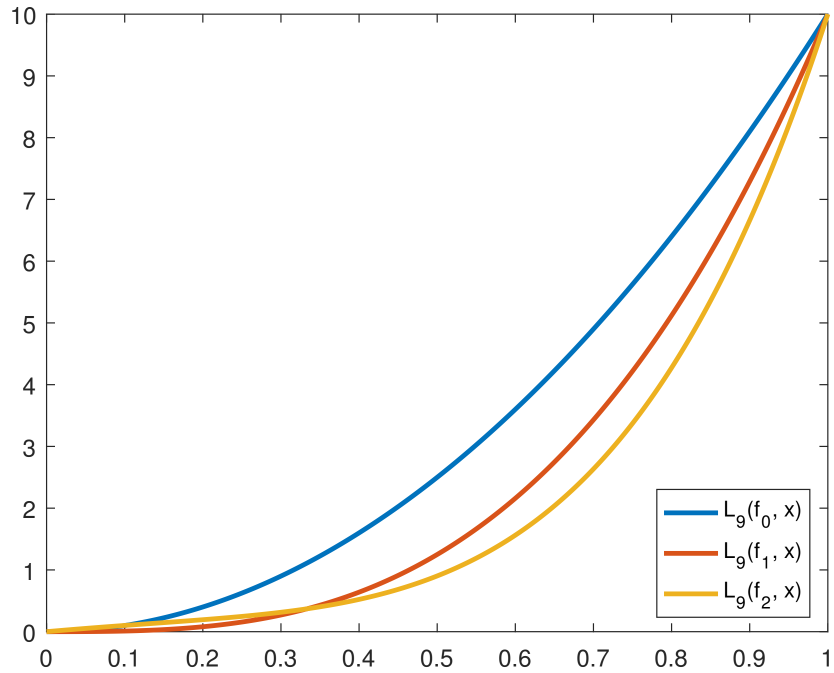

Figure 3. Equi-stat convergence of $\mathfrak{L}_9(f_i; x) \ \ (i = 0, 1, 2)$.

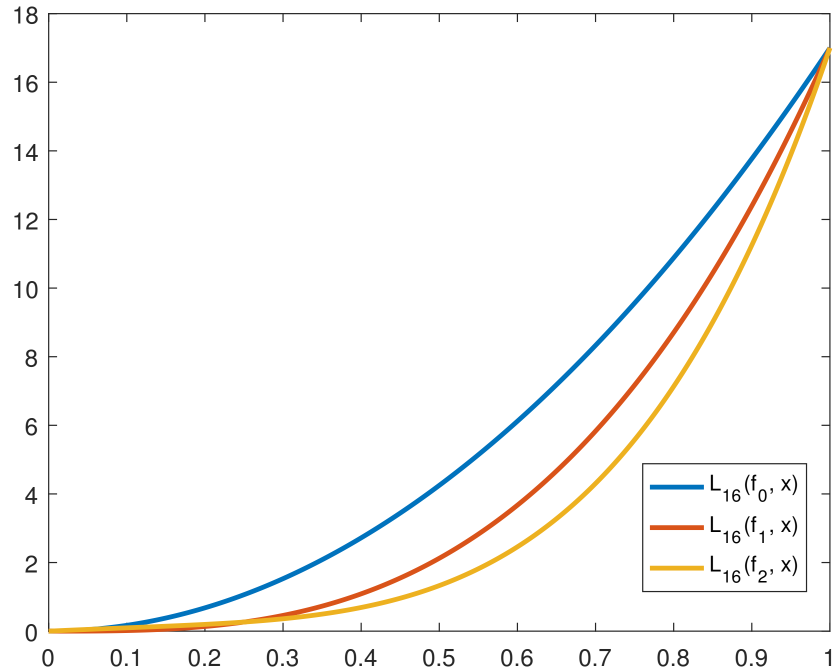

Figure 4. Equi-stat convergence of $\mathfrak{L}_{16}(f_i; x) \ \ (i = 0, 1, 2)$.

4. Rate of DP-Equi-Statistical Convergence

We wish to investigate here the rates of equi-statistical convergence under the proposed deferred power-series technique for the positive linear operator sequences with respect to the modulus of continuity.

Definition 8. *Let (t_m) be a non-increasing sequence (positive). If, for every $\epsilon > 0$,*

$$\lim_{0<s\to R^-} \frac{1}{t_m q(s)} \sum_{m\in\mathcal{K}_m(x,\epsilon)} q_m s^m = 0 \text{ (uniformly) in } x \in I,$$

then (f_m) is equi-statistically convergent under the deferred power-series technique to f with the rate of convergence $o(t_m)$. Symbolically, we write

$$f_m - f = o(t_m) \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I.$$

Before presenting the theorem for equi-statistical convergence rates, we establish Lemma 2 below.

Lemma 2. *Let $(f_m)$and $(g_m) \in C(I)$ with*

$$f_m(x) - f(x) = o(s_m) \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I$$

and

$$g_m(x) - g(x) = o(c_m) \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I.$$

Then, each of the following assertions is satisfied:

(i) $[f_m(x) + g_m(x)] - [f(x) + g(x)] = o(d_m) \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I$

(ii) $[f_m(x) - f(x)][g_m(x) - g(x)] = o(t_m c_m) \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I$

(iii) $\lambda[f_m(x) - f(x)] = o(t_m) \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I$ for any scalar λ

(iv) $\sqrt{|f_m(x) - f(x)|} = o(t_m) \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I,$

where

$$d_m = \max\{t_m, c_m\}. \tag{17}$$

Proof. For the assertion (i) to prove, let $x \in I$ and $\epsilon > 0$, we define the following sets:

$$\mathfrak{A}_m(x,\epsilon) = |\{m \in (a_n, b_n] \quad \text{and} \quad |(f_m + g_m)(x) - (f+g)(x)| \geqq \epsilon\}|,$$

$$\mathfrak{A}_{0,n}(x,\epsilon) = \left|\left\{m \in (a_n, b_n] \quad \text{and} \quad |f_m(x) - f(x)| \geqq \frac{\epsilon}{2}\right\}\right|$$

and

$$\mathfrak{A}_{1,n}(x,\epsilon) = \left|\left\{m \in (a_n, b_n] \quad \text{and} \quad |g_m(x) - g(x)| \geqq \frac{\epsilon}{2}\right\}\right|.$$

Clearly, this yields

$$\mathfrak{A}_n(x,\epsilon) \leqq \mathfrak{A}_{0,n}(x,\epsilon) + \mathfrak{A}_{1,n}(x,\epsilon).$$

Additionally, since

$$d_m = \max\{t_m, c_m\},$$

under the condition (8) of our Theorem 4, we obtain

$$\lim_{0<s\to R^-} \frac{1}{q(s)} \sum_{m\in\mathfrak{A}_m(x,\epsilon)} q_m s^m \leqq \lim_{0<s\to R^-} \frac{1}{q(s)} \sum_{m\in\mathfrak{A}_{0,m}(x,\epsilon)} q_m s^m + \lim_{0<s\to R^-} \frac{1}{q(s)} \sum_{m\in\mathfrak{A}_{1,m}(x,\epsilon)} q_m s^m.$$

Again, by considering the condition (9) of Theorem 4, we have

$$\lim_{0<s\to R^-} \frac{1}{q(s)} \sum_{m\in\mathfrak{A}_m(x,\epsilon)} q_m s^m = 0.$$

The proof of condition (i) is thus completed. Moreover, the remaining conditions (ii) to (iv) are similar to condition (i), so we skip the details involved. This completes the proof of Lemma 2. □

Next, recalling the modulus of continuity $\omega(f,\mu)$ of a function $f \in C(I)$ as

$$\omega(f,\mu) = \sup_{x,t\in I}\{|f(t) - f(x)| : |t-x| \leqq \mu\},$$

we propose the following Theorem.

Theorem 5. *Let* $(\mathfrak{L}_m(f;x)) : C(I) \to C(I)$ *be positive linear operators. Suppose also that each of the following conditions is satisfied*:

(i) $\mathfrak{L}_m(1,x) - 1 = o(t_m) \quad (\text{equi} - \text{stat}_{\text{DP}})$ on I

(ii) $\omega(f,\mu_m) = o(c_m) \quad (\text{equi} - \text{stat}_{\text{DP}})$ on I,

where

$$\mu_m(x) = \sqrt{\mathfrak{L}_m(\vartheta^2;x)} \quad \text{and} \quad \vartheta(t,x) = t - x.$$

Then, for $f \in C(I)$*, the following statement holds true:*

$$\mathfrak{L}_m(f,x) - f = o(d_m) \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I, \tag{18}$$

where d_m *is already mentioned in* (17).

Proof. Suppose $f \in C(I)$ and $x \in I$. We have,

$$|\mathfrak{L}_m(f;x) - f(x)| \leqq \mathcal{M}|\mathfrak{L}_m(1;x) - 1| + \left(\mathfrak{L}_m(1;x) + \sqrt{\mathfrak{L}_m(1;x)}\right)\omega(f,\mu_m),$$

where

$$\mathcal{M} = \|f\|_{C[I]}.$$

This certainly yields

$$\begin{aligned}|\mathfrak{L}_m(f;x) - f(x)| &\leqq \mathcal{M}|\mathfrak{L}_m(1;x) - 1| + 2\omega(f,\mu_m)\\ &\quad + \omega(f,\mu_m)|\mathfrak{L}_m(1;x) - 1| + \omega(f,\mu_m)\sqrt{|\mathfrak{L}_m(1;x) - 1|}.\end{aligned} \tag{19}$$

In view of the requirements (i) and (ii) of Theorem 5 along with Lemma 2, the final inequality (19) leads us to the assertion (18) of our Theorem 5. Hence, Theorem 5 is proved. □

5. Concluding Remarks and Observations

In this section, we present a number of additional remarks and observations pertaining to the numerous findings that we have proved here.

Remark 1. Let $(f_m)_{m\in\mathbb{N}}$ be the sequence of functions given in Example 2. Then, since

$$f_m \to f \quad (\text{equi} - \text{stat}_{\text{DP}}) \quad \text{on } [0,1],$$

we immediately obtain

$$\mathfrak{L}_m(f_i;x) \to f_i \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } [0,1] \quad (i = 0,1,2). \tag{20}$$

Therefore, by applying Theorem 4, we write

$$\mathfrak{L}_m(f;x) \to f_i \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } [0,1] \tag{21}$$

for all $f \in C(I)$. Moreover, since (f_m) is not uniformly statistically convergent to $f = 0$ over $[0,1]$ under the deferred power-series technique, and since it is also not simply uniformly convergent, then the classical Korovkin-type theorem does not impartially operate under our recommended operator in (16). Hence, the above notions shows that our Theorem 4 is a non-trivial generalization of some well-established published results (see [21,24,34]).

Remark 2. *If we substitute $(a_m) = 0$ and $(b_m) = m$ into our main Theorem 4, then the earlier-published results by Demirci et al. [22] and by Ünver and Orhan [23] are deduced. In this sense, we say that Theorem 4 is a non-trivial generalization of the earlier-published results (see [22,23]).*

Remark 3. *In place of the conditions (i) and (ii) in our Theorem 4, we consider the following condition:*

$$\mathfrak{L}_m(f_i,x) - f_i = o(t_{m_i}) \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ over } I. \tag{22}$$

Then, since

$$\mathfrak{L}_m(\vartheta^2;x) = \mathfrak{L}_m(t^2;x) - 2x\mathfrak{L}_m(t;x) + x^2\mathfrak{L}_m(1;x),$$

we can write

$$\mathfrak{L}_m(\vartheta^2;x) \leqq \mathcal{T}\sum_{i=0}^{2}|\mathfrak{L}_m(f_i;x) - f_i(x)|, \tag{23}$$

where

$$\mathcal{T} = 1 + 2\|f_1\|_{C(I)} + \|f_2\|_{C(I)}.$$

It now follows from (22), (23) and Lemma 2 that

$$\mu_n = \sqrt{\mathfrak{L}_m(\vartheta^2)} = o(v_m) \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I, \tag{24}$$

where

$$o(v_m) = \max\{t_{m_0}, t_{m_1}, t_{m_2}\}.$$

Thus, clearly, we have

$$\omega(f,\mu) = o(v_n) \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I.$$

Using (24) in Theorem 4, we immediately see $f \in C(I)$ that

$$\mathfrak{L}_m(f;x) - f(x) = o(v_n) \quad (\text{equi} - \text{stat}_{\text{DP}}) \text{ on } I. \tag{25}$$

In order to obtain the rates of the equi-statistical convergence under the deferred power-series method of $\mathfrak{L}_m(f;x)$ in Theorem 5, we must substitute the condition (22) in Theorem 4 for the conditions (i) and (ii).

Remark 4. *Through this study, we have preluded the conception of statistical convergence in the sense of the deferred power-series technique and presented some new definitions and thereafter established certain new theorems. Next, considering the modulus of continuity, we have estimated the rates of equi-statistical convergence under our proposed deferred power-series method for the positive linear operator sequences.*

Many researchers have considered different summability means on the sequence spaces to prove several approximation results. A list of some articles has been mentioned in the references. Further, combining the existing ideas and direction of the sequence spaces associated with our proposed mean, many new Korovkin-type approximation theorems can be proved under different settings of algebraic and trigonometric functions.

Influenced by a recently published article by Demirci et al. [22], we extract the cognizance of the interested learners concerning the possibilities of establishing some Korovkin-type approximation theorems over the sequence space as well as the probability space. Additionally, in view of the latest result of Paikray et al. [34] and Saini and Raj [18], the consciousness of the curious readers is drawn out for future research pertaining to fuzzy approximation theorems.

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