

Editorial

# Editorial Conclusion for the Special Issue “New Theory and Applications of Nonlinear Analysis, Fractional Calculus and Optimization”

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Nonlinear analysis has widespread and significant applications in many areas at the core of many branches of pure and applied mathematics and modern science, including nonlinear ordinary and partial differential equations, critical point theory, functional analysis, fixed point theory, nonlinear optimization, fractional calculus, variational analysis, convex analysis, dynamical system theory, mathematical economics, data mining, signal processing, control theory, and many more. For more details, we refer readers to [1–7] and the references cited therein. The rapid development of fractional calculus and its applications during the past thirty years or more has led to a number of scholarly essays on the importance of its promotion and application in physical chemistry, probability and statistics, electromagnetic theory, financial economics, biological engineering, electronic networks, and so forth. Almost all areas of modern science and engineering have been influenced by the theory of fractional calculus. Due to the complexity of the various problems that arise in nonlinear analysis, fractional calculus and optimization, it is not always easy to find exact solutions, which often leads researchers to resort to approximate solutions. Over the past eighty years, optimization problems have been intensively studied, and many scholars have developed various feasible methods with which to analyze the convergence of algorithms and find approximate solutions. For more details about these subjects, we refer readers to research monographs [8–16].

This Special Issue paid greater attention to the new originality and real-world applications of nonlinear analysis, fractional calculus, optimization and their applications. Following a comprehensive review process as per the journal’s policy and guidelines, 10 research papers have been included in this Special Issue and are listed as follows:

- (i). Gao, L.; Yu, G.; Han, W. Optimality Conditions of the Approximate Efficiency for Nonsmooth Robust Multiobjective Fractional Semi-Infinite Optimization Problems. *Axioms* **2023**, *12*, 635. <https://doi.org/10.3390/axioms12070635>

**Summary:** *In this paper, the authors study new optimality conditions and establish saddle point theorems for robust approximate quasi-weak efficient solutions for a nonsmooth uncertain multiobjective fractional semi-infinite optimization problem (NUMFP).*

- (ii). Huang, H.; Došenović, T.; Rakić, D.; Radenović, S. Fixed Point Results in Generalized Menger Probabilistic Metric Spaces with Applications to Decomposable Measures. *Axioms* **2023**, *12*, 660. <https://doi.org/10.3390/axioms12070660>

**Summary:** *This manuscript offers new fixed-point theorems in generalized Menger probabilistic metric spaces. Moreover, the authors provide some nontrivial examples and interesting applications to illustrate the superiority of the results obtained.*

- (iii). Sun, Z.-Y.; Guo, B.-N.; Qi, F. Determinantal Expressions, Identities, Concavity, Maclaurin Power Series Expansions for van der Pol Numbers, Bernoulli Numbers, and Cotangent. *Axioms* **2023**, *12*, 665. <https://doi.org/10.3390/axioms12070665>



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**Summary:** This article mainly has the following innovations and contributions:

- Four determinantal expressions for van der Pol numbers;
- Two new identities for the Bernoulli numbers and van der Pol numbers;
- The increasing properties and concavity of a function involving the cotangent function;
- Two alternative Maclaurin power series expansions of a function involving the cotangent function;
- The coefficients of the Maclaurin power series expansions are expressed in terms of specific Hessenberg determinants whose elements contain the Bernoulli numbers and binomial coefficients.

(iv). Lv, W.; Tian, L. Pricing of Credit Risk Derivatives with Stochastic Interest Rate. *Axioms* **2023**, *12*, 782. <https://doi.org/10.3390/axioms12080782>

**Summary:** In this article, the authors generalize the conventional reduced-form credit risk model for a credit default swap market to deal with a credit derivative pricing problem using the martingale approach.

(v). Chegloufa, N.; Chaouchi, B.; Kostić, M.; Du, W.-S. On the Study of Pseudo  $S$ -Asymptotically Periodic Mild Solutions for a Class of Neutral Fractional Delayed Evolution Equations. *Axioms* **2023**, *12*, 800. <https://doi.org/10.3390/axioms12080800>

**Summary:** The authors investigate the existence and uniqueness of pseudo  $S$ -asymptotically periodic mild solutions for a class of neutral fractional evolution equations with finite delay by applying the fractional powers of closed linear operators, the semigroup theory and some classical fixed-point theorems.

(vi). Jiang, B.; Huang, H.; Radenović, S. Common Fixed Point of  $(\psi, \beta, L)$ -Generalized Contractive Mapping in Partially Ordered  $b$ -Metric Spaces. *Axioms* **2023**, *12*, 1008. <https://doi.org/10.3390/axioms12111008>

**Summary:** This manuscript provides new coincidences and common fixed points in four mappings satisfying  $(\psi, \beta, L)$ -generalized contractive conditions in partially ordered  $b$ -metric spaces, which generalize some recent results in the existing literature.

(vii). Fedorov, V.E.; Plekhanova, M.V.; Melekhina, D.V. On Local Unique Solvability for a Class of Nonlinear Identification Problems. *Axioms* **2023**, *12*, 1013. <https://doi.org/10.3390/axioms12111013>

**Summary:** In this paper, the authors study nonlinear identification problems for evolution differential equations, solved with respect to the highest-order Dzhrbashyan–Nersesyan fractional derivative. Applying the contraction mappings theorem, the authors establish the unique local solvability of the nonlinear identification problems.

(viii). Xie, T.; Li, M. Finite-Time Stability of Impulsive Fractional Differential Equations with Pure Delays. *Axioms* **2023**, *12*, 1129. <https://doi.org/10.3390/axioms12121129>

**Summary:** In this article, the authors introduce a novel concept of the impulsive delayed Mittag–Leffler-type vector function, which improved and generalized the Mittag–Leffler matrix function. The position of the pulse point in this paper is arbitrary, which renders the research more universal. Using the relationship between the Riemann–Liouville fractional derivative and the Caputo fractional derivative, the authors establish new finite-time stability results of impulsive fractional differential delay equations.

(ix). Raza, N.; Fadel, M.; Du, W.-S. New Summation and Integral Representations for 2-Variable  $(p, q)$ -Hermite Polynomials. *Axioms* **2024**, *13*, 196. <https://doi.org/10.3390/axioms13030196>

**Summary:** This manuscript offers various new features of two-variable  $(p, q)$ -Hermite polynomials, such as diffusion equation, differential equations, integral and summation representations.

- (x). Özger, F.; Temizer Ersoy, M.; Ödemiş Özger, Z. Existence of Solutions: Investigating Fredholm Integral Equations via a Fixed-Point Theorem. *Axioms* **2024**, *13*, 261. <https://doi.org/10.3390/axioms13040261>

**Summary:** *The main purpose of this article is to investigate the existence conditions for the solutions of the nonlinear quadratic Fredholm integral equations of the form*

$$\chi(l) = \varrho(l) + \chi(l) \int_p^q k(l, z)(V\chi)(z)dz$$

*in the space  $C_\omega[p, q]$ , where  $\omega$  is a modulus of continuity,  $\chi$  is the unknown function to be determined,  $V$  is a given operator, and  $\varrho, k$  are two given functions.*

In this Editorial, we express our heartfelt appreciation to all authors and reviewers who contributed to this Special Issue. It was with the enthusiasm and spirit of the authors and reviewers that we could make this Special Issue an extraordinary success. The 27 authors of these 10 papers are listed below:

Belkacem Chaouchi (see v);	Naceur Chegloufa (see v);
Tatjana Došenović (see ii);	Wei-Shih Du (see v, ix);
Merve Temizer Ersoy (see x);	Mohammed Fadel (see ix);
Vladimir E. Fedorov (see vii);	Liu Gao (see i);
Bai-Ni Guo (see iii);	Wenyan Han (see i);
Huaping Huang (see ii, vi);	Binghua Jiang, (see vi);
Marko Kostić (see v);	Mengmeng Li (see viii);
Wujun Lv (see iv);	Daria V. Melekhina (see vii);
Faruk Özger (see x);	Zeynep Ödemiş Özger (see x);
Marina V. Plekhanova (see vii);	Feng Qi (see iii);
Stojan Radenović (see ii, vi);	Dušan Rakić (see ii);
Nusrat Raza (see ix);	Zhen-Ying Sun (see iii);
Linlin Tian (see iv);	Guolin Yu (see i);
Tingting Xie (see vii).	

The published contributions to this Special Issue can be divided according to the following scheme considering their main purposes:

- Nonlinear analysis (see ii, iii, v, vii, viii, ix, x);
- Fractional calculus (see v, vi, vii, viii);
- Optimization and analytic number theory (see i, iii, iv).

We hope that researchers and practitioners find these papers interesting and inspirational for future research work in these exciting areas. We also believe that the contributions to this Special Issue provide new insights on several important issues while, at the same time, providing new research problems or avenues that undoubtedly exceed our original aim. Finally, we would like to express our sincere thanks to the Editorial team and the reviewers of *Axioms*, particularly the Editor-in-Chief, Prof. Dr. Humberto Bustince, and the assistant editors, for their great support throughout the editing process of our Special Issue.

**Author Contributions:** Conceptualization, W.-S.D.; methodology, W.-S.D.; software, W.-S.D.; validation, W.-S.D.; formal analysis, W.-S.D.; investigation, W.-S.D.; writing-original draft preparation, W.-S.D.; writing-review and editing, W.-S.D.; visualization, W.-S.D.; supervision, W.-S.D.; project administration, W.-S.D. All authors have read and agreed to the published version of the manuscript.

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