

Article

Operadic and Diagrammatic Semantics of the Greimas Semiotic Square

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Abstract

We develop a categorical and operadic semantics for the diagrammatic proof system underlying the Greimas semiotic square. Building on a prior proof-theoretic formulation, we extract a typed signature of diagrammatic inference rules and construct the corresponding free coloured operad $\mathcal{O}_\Sigma$, whose elements correspond to proof trees. This establishes a precise correspondence between diagrammatic derivations and operadic terms, making explicit the compositional structure implicit in the original system. We then interpret these terms as wiring diagrams in a symmetric monoidal setting, yielding a graphical semantics in which intermediate semantic configurations are represented as flows through a network of operations. Within this framework, the construction of the semiotic square is realised as a single composite operation Ω , obtained by operadic substitution of generators corresponding to negation, implication, and meta-term formation. Finally, we consider semantic interpretations of this structure as algebras of $\mathcal{O}_\Sigma$, yielding a category $\mathbf{Alg}(\mathcal{O}_\Sigma)$, whose morphisms capture structure-preserving translations between interpretations. This provides a formal account of the extensibility of the square across domains such as seme-level analysis, modality, and narratology, and recasts it as a compositional semantic schema rather than a static relational diagram.

Keywords: operads; wiring diagrams; symmetric monoidal categories; diagrammatic reasoning; spider diagrams; Greimas semiotic square; structural semiotics; compositional semantics

MSC: 18M60; 18D10



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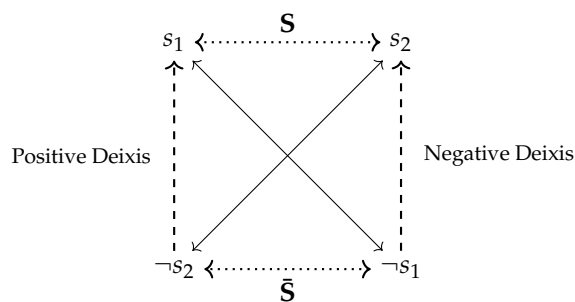
1. Introduction

Diagrammatic reasoning systems provide formal frameworks in which complex relational structures are represented and manipulated through rule-governed transformations. Giacomone et al. [1] and Bakker and Hoffmann [2] have shown how such systems support not only set-theoretic reasoning, but the formalisation of non-classical semantic domains in which meaning arises from structured relations rather than truth-functional evaluation alone. The Greimas semiotic square is a canonical example of such a domain: it encodes a network of oppositional and implicational relations between semantic units, yet the generative procedures by which these relations give rise to higher-order constructions—namely its meta-terms—are typically described only informally.

The semiotic square emerged in the Paris School of structural semiotics, led by A. J. Greimas in the 1960s [3]. The square itself is used as an analytical or heuristic tool for investigating signification within an artefact, text or narrative. Greimas and Rastier [4], p.

88 describe a square as a family of relations between a seme pair (designated s_1 and s_2) and its negation ($\neg s_1, \neg s_2$). A seme (denoted /seme/) is defined as the smallest irreducible unit of meaning that, when in combination, forms a sememe, or the smallest unit of meaning that associates with a morpheme or lexeme, and is thus analogous to a phoneme in phonology.

The aforementioned relations between semes are described as: contrariety ($\langle \cdots \rangle$), contradiction ($\langle \longrightarrow \rangle$) and implication ($\langle - \rightarrow \rangle$), and are commonly visualised through the following diagram, the semiotic square:



(1)

The seme pair of $\{s_1, s_2\}$ is known as the complex axis, and $\{\neg s_1, \neg s_2\}$ the neutral axis. But the four central terms of $\{s_1, s_2, \neg s_1, \neg s_2\}$ also allow for other meta-terms as sememes to be constructed. The procedures for these constructions are found through the “combining of the four simple terms” [5], p. 81. This yields

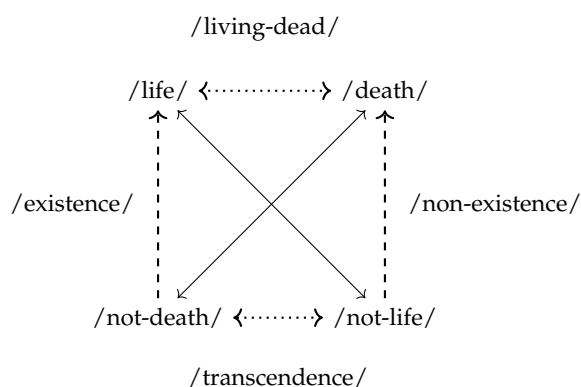
$$\begin{aligned} S &= (s_1 + s_2) \quad \text{complex term,} \\ \bar{S} &= (\neg s_1 + \neg s_2) \quad \text{neutral term,} \end{aligned}$$

as well as the

$$\begin{aligned} \text{Positive Deixis} &= (s_1 + \neg s_2), \\ \text{Negative Deixis} &= (\neg s_1 + s_2). \end{aligned}$$

The Greimas square has become the most utilised tool in the structural semiotics method for identifying and tracking structures of homologation (i.e., seme-wise oppositional pairs as: (s_1, s_2) vs. $(\neg s_1, \neg s_2)$) within narratives, with Greimas and Courtés arguing that it represents “a rigorous formulation of reasoning by analogy” [6], p. 144.

Example 1. In his analysis of the short story “Two friends” (*Deux amis*) by Guy de Maupassant (1850–1893), Greimas [7], p. 5 identifies the seme pair /life/ vs. /death/ as the basis for a partial square that we complete as follows:



(2)

Here, the meta-terms are constructed using the ‘+’ operator so that the hyponym (meta-term) at the **S** position, /living dead/ associates with figurative actors such as the Zombie or Vampire, while the meta-term at the **Ŝ** position, /transcendence/, associates with figurative actors such as Spirits or Ghosts. At the Positive Deixis position is the meta-term /existence/ which carries the axiological value of ‘living’ (i.e., /not-death/ + /life/) and what Greimas and Courtés [6] identify as the actualising modality of ‘being-able-to-do’. Duly, at the Negative Deixis position is the meta-term /non-existence/ (i.e., /not-life/ + /death/) containing the axiological value of ‘dying’ which associates with the virtualising modality ‘not-being-able-to-do.’ Each of the deixes is also associated with the thymic (classematic) categories of “euphoria” (Positive Deixis) and “dysphoria” (Negative Deixis), in turn valorising their diagrammatically implicated positive (s_1) or negative (s_2) terms. The neutral term **Ŝ** is consequently classified as “aphoric” [6], p. 346.

1.1. Diagrammatic Proof System of the Greimas Square

In spite of its widespread popularity in the field of semiotics, we addressed the square’s under-specification in previous work [8] by developing a proof-theoretic reconstruction of the square using unitary spider diagrams. Spider diagrams [9–11] provide a diagrammatic formalism extending Euler/Venn diagrams with existential markers (‘spiders’) and region constraints, allowing the representation of relational and logical structure. Using that formalism, we built on the van Wolde [12], p. 13 “operational account” of the semiotic square as a structure that is traditionally derived from the seme transformations

$$s_1 \mapsto \neg s_1 \mapsto s_2, \quad \text{and} \quad s_2 \mapsto \neg s_2 \mapsto s_1,$$

with the meta-terms as sememes constructed through seme groupings under the ‘+’ operator.

We formalised the square within a semantic universe X whose elements are defined relationally: a subset M as a collection of meta-terms and S_i as subset collections of semes for which $S_1 \cap S_2 = \emptyset$. We also formalized both the core relations of contrariety ($d \#_{\text{diag}} d'$), diagrammatic negation, and diagrammatic implication as well as the meta-term construction procedure through describing semic components of the square as unitary spider diagrams that are transformed according to a system of inference rules:

$$\mathcal{R} = \left\{ \begin{array}{l} \text{ADDFEET, SPLITSPIDER, COMBINE, COPYSPIDER,} \\ \text{ERASESPIDER, IDEMPOTENCY, CONJUNCTIONELIMINATION} \end{array} \right\}.$$

These rules govern elementary transformations of spider diagrams, including decomposition, combination, structural duplication, and conjunction elimination. Collectively, they determine the basic diagrammatic operations from which all derivations in D_{GS} are composed. Full formal definitions are given in Howse et al. [10], Urbas et al. [11].

In Tables 1 and 2, we provide summaries of the derivations of our diagrammatic proof system D_{GS} . In Table 1 we give the unitary spider diagrams that describe the semes of the square as domains and codomains of inference operations that follow the operational account of van Wolde [12]. In Table 2, we show the inferences that associate with the generation of meta-terms of the square under the Greimas operator ‘+’. We denote conjunctive inputs by $d_{\alpha \wedge \beta}$ as shorthand for the ordered pair (d_α, d_β) , indicating the binary configuration from which a meta-term is derived. This notation originates in the proof-theoretic presentation of D_{GS} and should not be interpreted as a single diagrammatic type. In the operadic formulation, meta-term formation is represented explicitly by binary operations. In both tables, accompanying Proof trees (see Appendixes A.1–A.8) serve as witnesses to the compositional structure of semantic change that underlies the Greimas square. A central result of that work was that the construction of the four Greimasian meta-terms can be captured uniformly: i.e., each arises from a conjunctive seme-pair via a fixed

derivation schema, thereby providing a precise, rule-based account of the combinatorial operation underlying meta-term formation. In Figure 1, we provide our diagrammatic reconstruction of the Greimas square using the language of spider diagrams to indicate the existential state of each term in the square at each position. Arrows follow the operational derivation after van Wolde [12] and Greimas [13], though are labelled according to the corresponding witnessed proof tree T_i that we provide in Appendixes A.5–A.8.

Table 1. Primitive diagrammatic entailments for diagrammatic negation and diagrammatic implication in the semiotic square. Each row specifies a typed derivation $d \vdash_{\mathcal{R}} d'$, obtained as a composite of admissible rule instances in $\mathcal{R}$, with the corresponding proof tree given in Appendixes A.1–A.4.

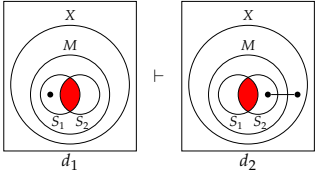
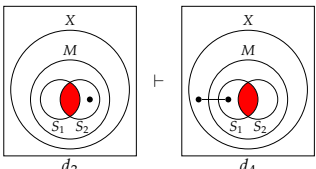
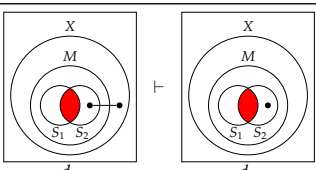
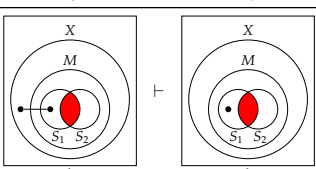
Diagrammatic Entailment $d \vdash_{\mathcal{R}} d'$	Typed Entailment	Description
	$s_1, \neg s_1$	Diagrammatic negation: $s_1 \mapsto \neg s_1$. E.g., /life/ $\mapsto$ /not-life/.
	$s_2, \neg s_2$	Diagrammatic negation: $s_2 \mapsto \neg s_2$. E.g., /death/ $\mapsto$ /not-death/.
	$\neg s_1, s_2$	Diagrammatic implication: $\neg s_1 \mapsto s_2$. E.g., /not-life/ $\mapsto$ /death/.
	$\neg s_2, s_1$	Diagrammatic implication: $\neg s_2 \mapsto s_1$. E.g., /not-death/ $\mapsto$ /life/.

Table 2. Composite diagrammatic derivations for the construction of Greimasian meta-terms under the operator '+'. Each row specifies a typed composite derivation $d_{\alpha\wedge\beta} \vdash_{\mathcal{R}} d_{\alpha\beta}^+$, where $d_{\alpha\wedge\beta} \equiv (d_{\alpha}, d_{\beta})$, and is obtained from admissible rule instances in $\mathcal{R}$. Associated proof trees are given in Appendixes A.5–A.8.

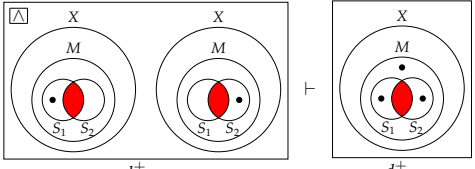
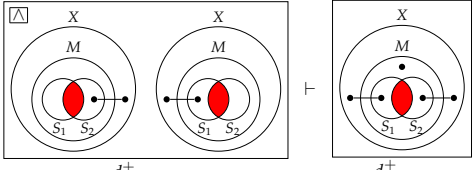
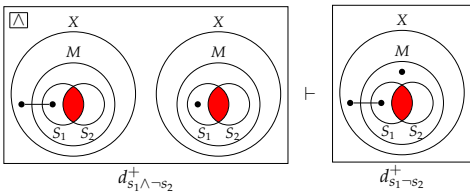
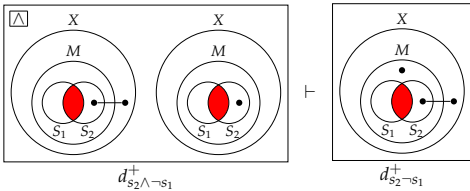
Composite Derivation $d_{\alpha\wedge\beta} \vdash_{\mathcal{R}} d_{\alpha\beta}^+$	Typed Entailment	Description
	s_1, s_2, S	Complex Term, S : $(s_1 + s_2)$. E.g., /life/ + /death/ = /living-dead/.
	$\neg s_1, \neg s_2, \bar{S}$	Neutral Term, $\bar{S}$: $(\neg s_1 + \neg s_2)$. E.g., /not-death/ + /not-life/ = /transcendence/.

Table 2. Cont.

Composite Derivation $d_{\alpha\wedge\beta} \vdash_{\mathcal{R}} d_{\alpha\beta}^+$	Typed Entailment	Description
 $d_{s_1 \wedge \neg s_2}^+ \vdash_{\mathcal{R}} d_{s_1 \neg s_2}^+$	$s_1, \neg s_2$, Positive Deixis	Positive Deixis: ($s_1 + \neg s_2$). E.g., /life/ + /not-death/ = /existence/.
 $d_{s_2 \wedge \neg s_1}^+ \vdash_{\mathcal{R}} d_{s_2 \neg s_1}^+$	$\neg s_1, s_2$, Negative Deixis	Negative Deixis: ($s_2 + \neg s_1$). E.g., /not-life/ + /death/ = /non-existence/

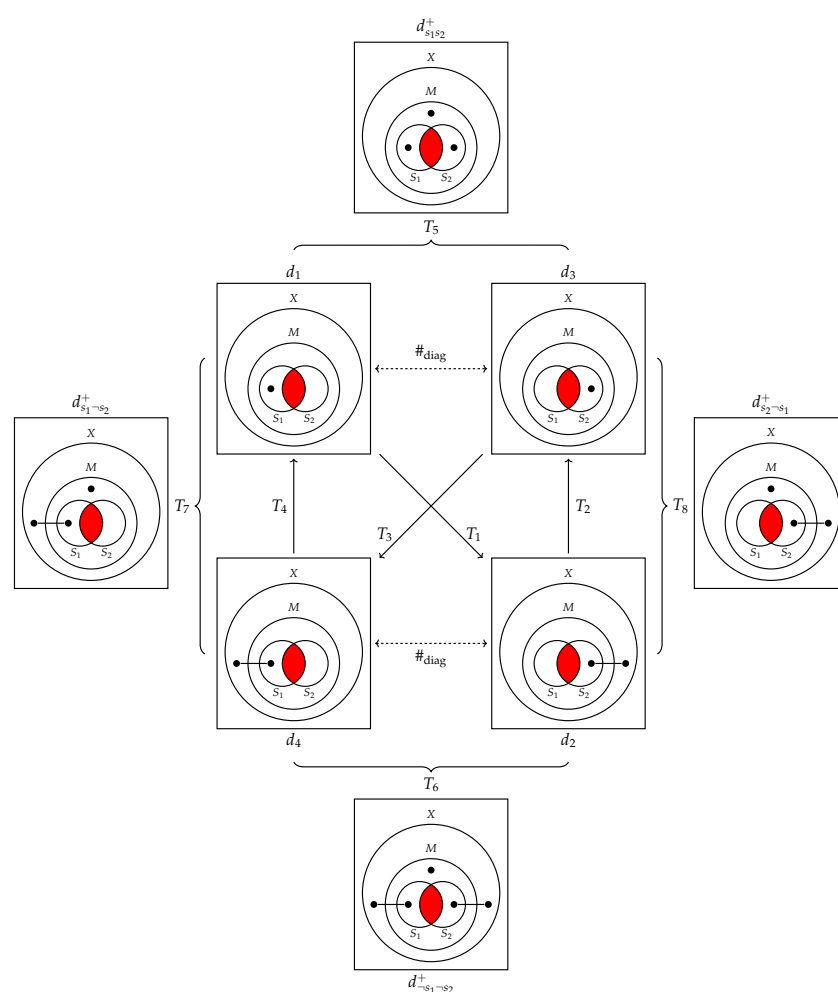


Figure 1. Diagrammatic reconstruction of the Greimas semiotic square in the language of unitary spider diagrams. Each vertex is represented by a unitary diagram encoding the corresponding semic configuration, while the arrows as $T_i : d \vdash_{\mathcal{R}} d'$ denote derivations in the inference system $\mathcal{R}$, indexed by the proof trees T_i . We use ‘ $\vdash$ ’ to indicate binary-input derivations as $d_{a\wedge b} \vdash_{\mathcal{R}} d_{ab}^+$. The relation $d_1 \#_{\text{diag}} d_3$ marks diagrammatic contrariety between s_1, s_2 and their negations $\neg s_1, \neg s_2$ as $d_4 \#_{\text{diag}} d_2$. The figure provides a global summary of the proof-theoretic and semantic structure developed for the Greimas square.

While our proof-theoretic account makes explicit the generative procedures implicit in the traditional formulation of the semiotic square, it leaves open a structural question

concerning the nature of their compositional organisation. The derivations constructed in D_{GS} are built from a fixed collection of inference rules in $\mathcal{R}$, compose into proof trees, and instantiate a uniform pattern underlying the construction of all meta-terms. However, in the previous treatment, this structure is expressed only syntactically—as trees of rule applications—rather than as an abstract algebraic object.

In this paper, we make this compositional structure explicit by showing that our diagrammatic proof system D_{GS} admits a natural operadic semantics, in which proof trees correspond to elements of a free coloured operad $\mathcal{O}_\Sigma$, and the construction of the Greimas square is realised as a composite operation. Under this interpretation, derivations are treated as compositional objects rather than mere witnesses of derivability, and may be represented equivalently as wiring diagrams in a symmetric monoidal setting, where diagrammatic types serve as objects and derivations as morphisms. This reformulation does not extend the underlying diagrammatic system, but re-expresses its compositional content at a higher level of abstraction. In doing so, it makes explicit the algebraic structure implicit in the proof-theoretic account, and provides a unified framework in which the operations of diagrammatic negation, diagrammatic implication, and meta-term formation are understood as components of a single compositional process governing the construction of the square.

1.2. Summary

The paper proceeds as follows. First, we develop a categorical account of the diagrammatic proof system D_{GS} introduced in [8] by extracting its underlying compositional structure and expressing it in operadic terms. We begin by identifying a typed signature Σ of diagrammatic inference rules and constructing the free coloured operad $\mathcal{O}_\Sigma$, whose elements correspond to proof trees in the original system. This establishes a precise correspondence between diagrammatic derivations and operadic terms, thereby making explicit the hierarchical and compositional organisation implicit in the proof-theoretic formulation.

We then reinterpret these operadic terms as wiring diagrams in a symmetric monoidal setting, providing a graphical semantics in which the flow of intermediate semantic configurations is represented explicitly. Within this framework, the construction of the Greimas semiotic square is realised as a single composite operation

$$\Omega : (d_{s_1}, d_{s_2}) \rightarrow d_{\square}, \quad (3)$$

obtained by operadic substitution of generators corresponding to negation, implication, and meta-term formation. This representation shows that the square arises from a uniform compositional process rather than as a static configuration of relations.

Finally, we consider semantic interpretations of this structure as algebras of the operad $\mathcal{O}_\Sigma$ in a symmetric monoidal category $\mathfrak{S}$, and organise such interpretations into a category $\mathbf{Alg}(\mathcal{O}_\Sigma)$. This perspective provides a formal account of the extensibility of the square across different domains—such as seme-level, modal, and actantial analyses—by treating them as distinct but related realisations of a common compositional schema.

1.3. How to Read This Paper

Readers primarily interested in semiotics may focus on Definitions 1 and 2 and Figures 1–6, treating the operadic and categorical constructions as a formal account of compositional structure underlying the Greimas square. Readers with a background in category theory may interpret the paper conversely as a case study in extracting an operadic semantics from a diagrammatic proof system. Section 2 develops the operadic syntax, Section 3 provides its wiring-diagram realisation, and Section 4 introduces semantic interpretations as algebras. Readers unfamiliar with operads may treat $\mathcal{O}_\Sigma$ informally as

a system for recording how smaller derivations are assembled into larger ones; no prior knowledge of operadic homotopy theory or higher category theory is assumed.

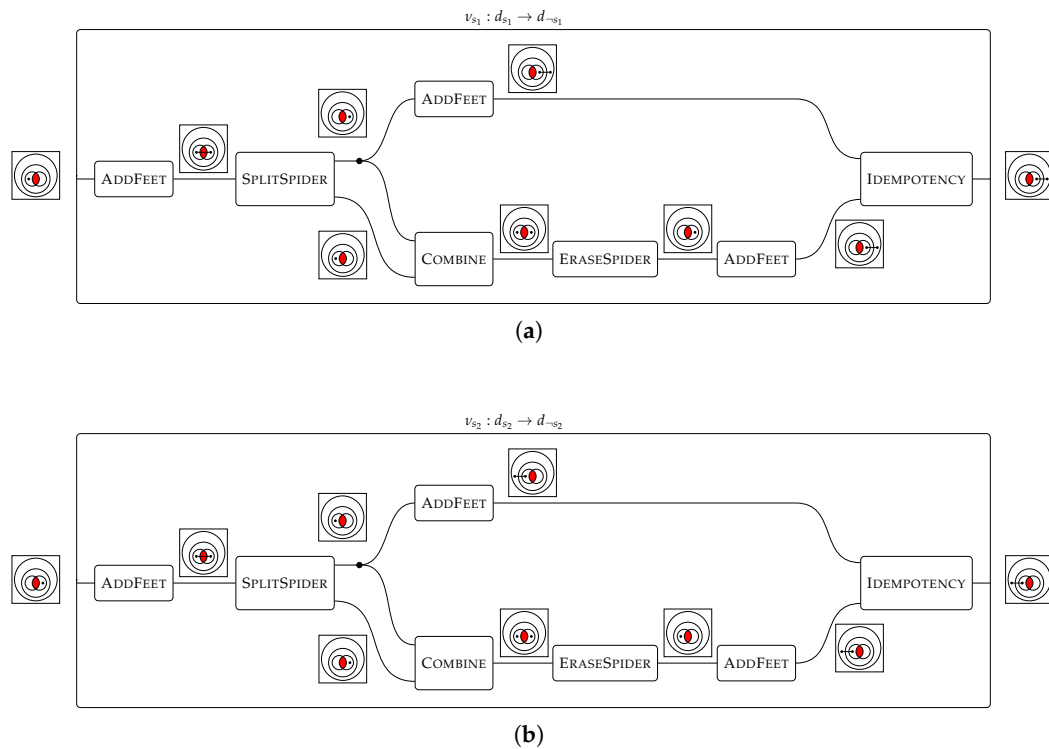


Figure 2. The wiring diagrams for diagrammatic negation. (a) The operation $v_{s_1} : d_{s_1} \rightarrow d_{\neg s_1}$, showing the transformation of the first seme via a fixed sequence of generators. (b) The operation $v_{s_2} : d_{s_2} \rightarrow d_{\neg s_2}$, exhibiting the same compositional structure with different typing. In both cases, the diagrams make explicit the internal sequence of inference rules realising negation.

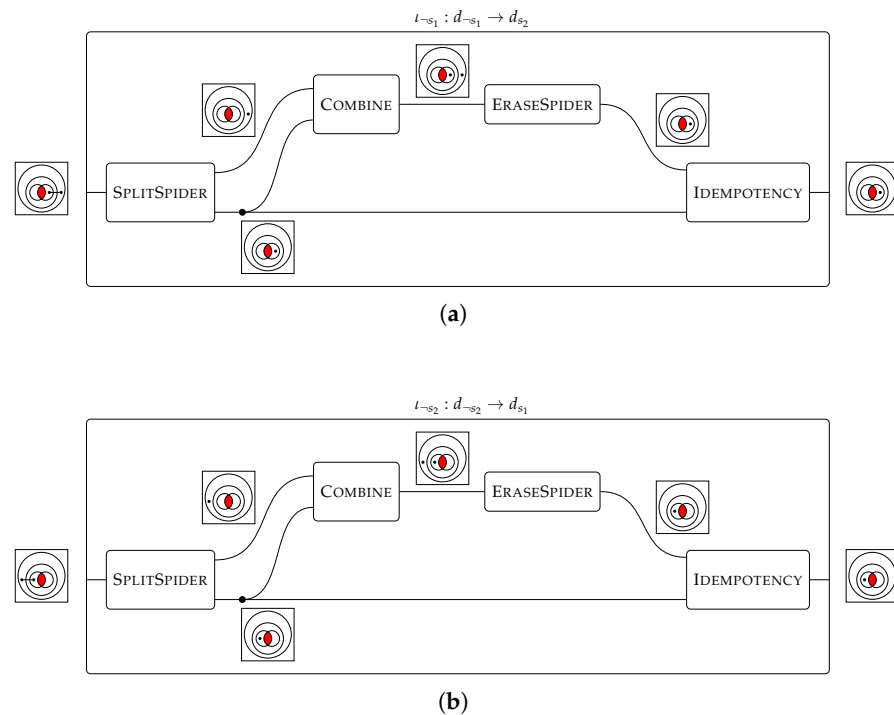


Figure 3. The wiring diagrams for diagrammatic implication. (a) The operation $l_{\neg s_1} : d_{\neg s_1} \rightarrow d_{s_2}$, representing the transformation from negation to implication. (b) The operation $l_{\neg s_2} : d_{\neg s_2} \rightarrow d_{s_1}$. These diagrams encode the collapse of disjunctive structure into a determinate semantic configuration.

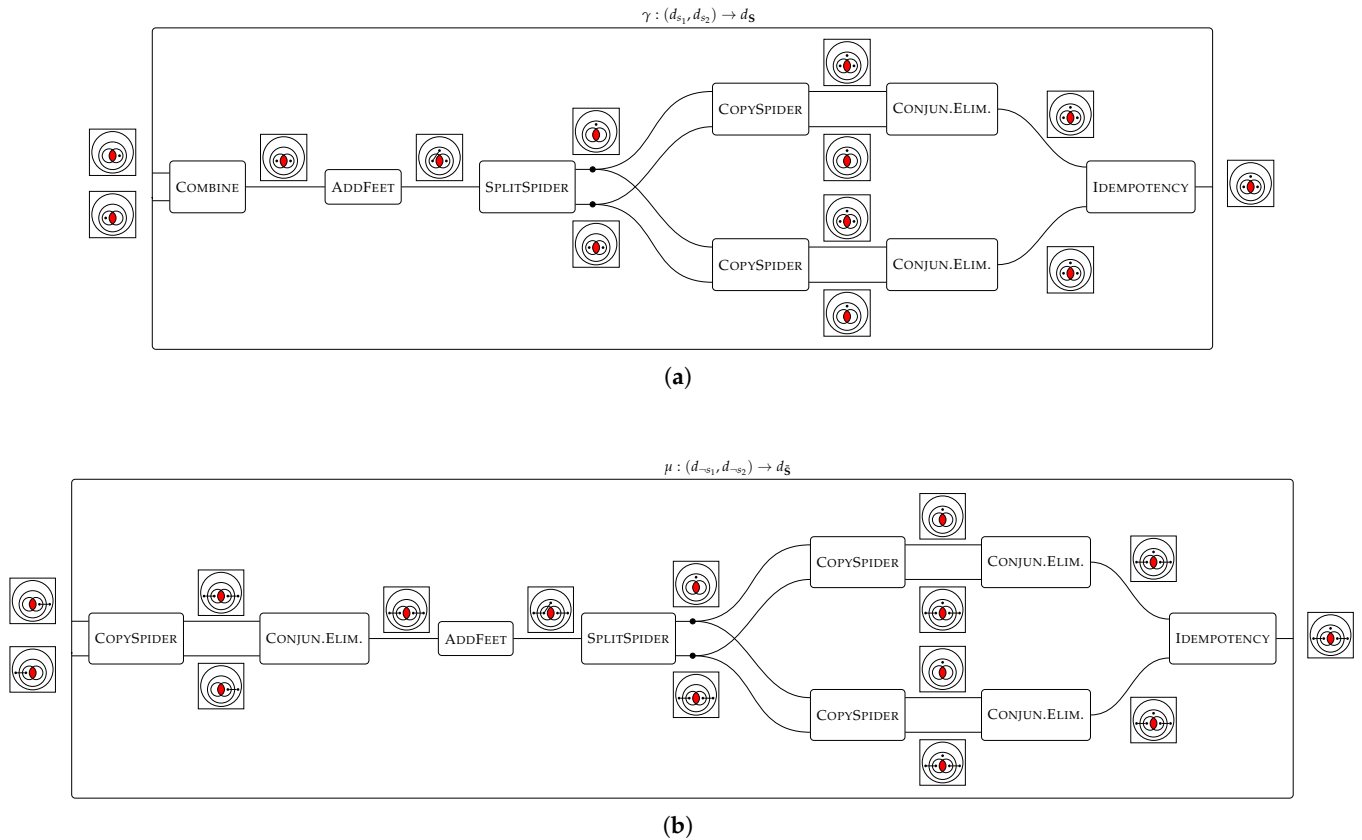


Figure 4. Wiring diagrams for the construction of the complex and neutral meta-terms. (a) The operation $\gamma : (d_{s_1}, d_{s_2}) \rightarrow d_S$ forming the complex term **S**. (b) The operation $\mu : (d_{-s_1}, d_{-s_2}) \rightarrow d_S$ forming the neutral term. Both diagrams exhibit a shared compositional pattern involving duplication and recombination of inputs.

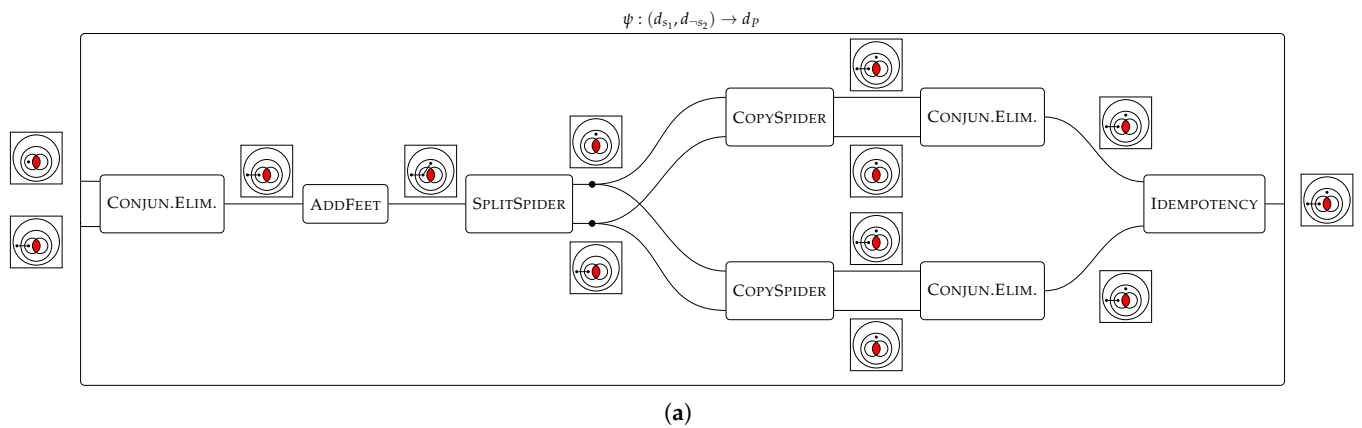


Figure 5. Cont.

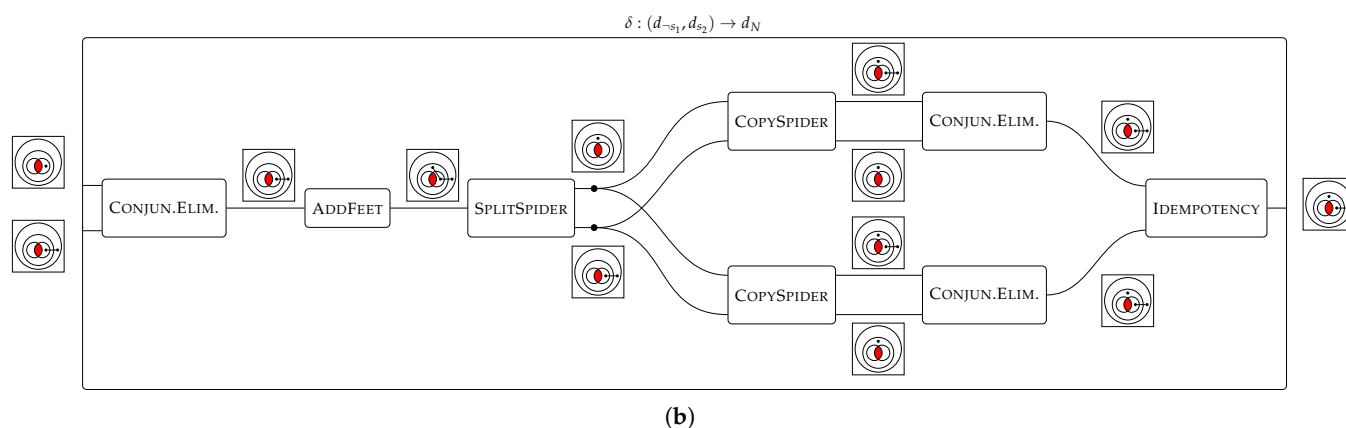


Figure 5. Wiring diagrams for the construction of the deixis terms. (a). The operation $\psi : (d_{s_1}, d_{-s_2}) \rightarrow d_P$ forming the Positive Deixis. (b) The operation $\delta : (d_{-s_1}, d_{s_2}) \rightarrow d_N$ forming the Negative Deixis. These diagrams extend the meta-term construction schema to mixed polarity inputs.

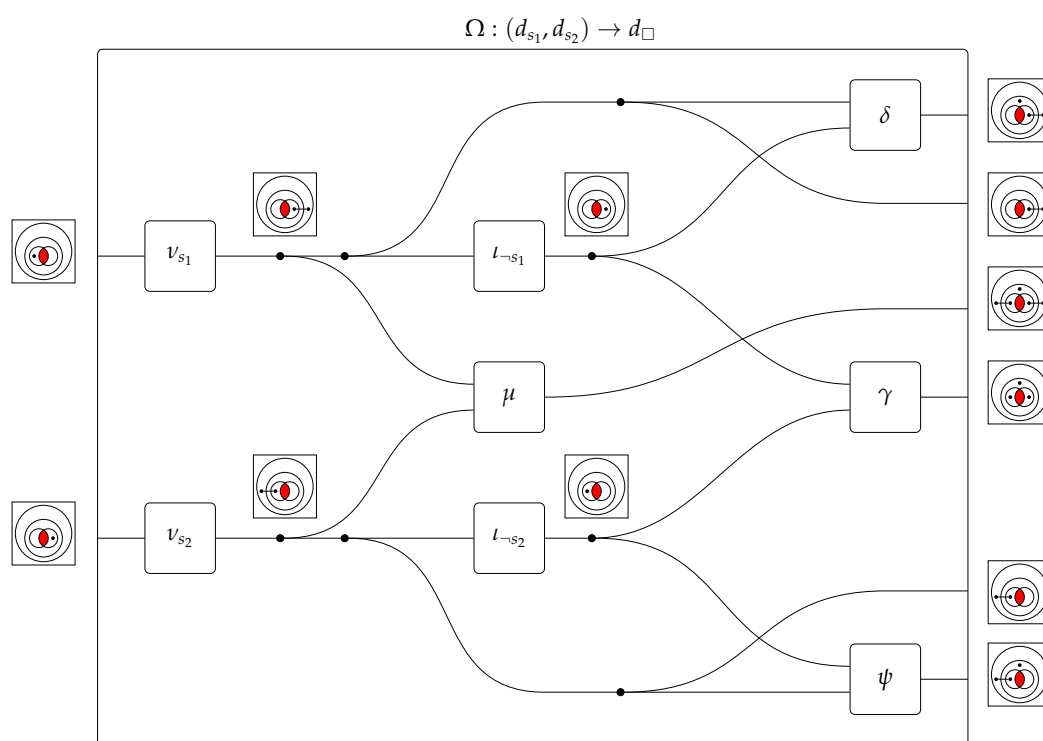


Figure 6. The wiring diagram $\Omega : (d_{s_1}, d_{s_2}) \rightarrow d_\square$ representing the full construction of the Greimas semiotic square. The diagram composes the generators for diagrammatic negation, diagrammatic implication, and meta-term formation into a single operadic term. Each box corresponds to a generator in the signature Σ , while the wiring encodes the flow of intermediate diagrammatic types. The resulting composite realises the square as a hierarchical composition of the operations depicted in Figures 2–5.

2. Diagrammatic Derivations as an Operad

Given that the broader aim of this article is to connect the diagrammatic logic of the semiotic square as a spider diagram calculus to categorical structures, we begin by extracting from our diagrammatic proof system D_{GS} a typed collection of generating operations.

Definition 1 (Diagrammatic types). Let $d = \langle L, Z, Z^*, SI \rangle$ be a unitary spider diagram containing contour labels in the set L , non-shaded zones in Z , shaded zones in Z^* , and spider identifiers $SI = SI(d) \subset \mathbb{N}^+ \times \mathbb{R}(d)$ where $\mathbb{N}^+ = \{1, 2, 3, \dots\}$ indexes spiders, and $\mathbb{R}(d)$ denotes the set of

regions determined by the power set of zones: $\mathcal{R}(d) \subseteq \mathcal{P}(Z)$. Let $\mathcal{D}$ denote the class of unitary spider diagrams occurring as domains and codomains of derivations in $\mathcal{D}_{GS}$. We treat elements of $\mathcal{D}$ as types (or colours), and write

$$\text{col}(\mathcal{D}) := \mathcal{D}. \quad (4)$$

Remark 1. One may quotient $\mathcal{D}$ by diagrammatic equivalence (e.g., up to syntactic normalisation induced by the inference rules). For the purposes of defining a free coloured operad, we work at the level of raw diagrams.

Given that the semes $(s_1, s_2, \neg s_1, \neg s_2)$ and meta-terms (the sememes: $\mathbf{S}, \bar{\mathbf{S}}$, Positive Deixis, Negative Deixis) of the square are represented by unitary spider diagrams encoding relations between semantic components, their interpretation as colours in the operad nevertheless preserves their relational semantics. As Greimas and Courtés suggest, semes are “never substantial, and the seme cannot be defined as the end-term of the relation that one sets up and/or grasps with at least one other term of the same relational network” [6], p. 278. This condition is preserved in $\text{col}(\mathcal{D})$, not merely because diagrams are syntactically distinguishable by their contour labels, but because their types are defined through relational configurations. The contour structure and spider placements encode relations between semantic components, and thus the typing induced by $\mathcal{D}$ reflects differences in relational structure rather than intrinsic properties. In this sense, treating diagrams as colours does not reify semes as isolated entities, but instead preserves their role as positions within a relational network. From this perspective, the inference rules in $\mathcal{R}$ may be understood as operations on relational configurations, transforming one structured arrangement of semes into another.

Definition 2 (Signature of inference rules). Let $\mathcal{R} = \{\text{ADDFEET}, \text{SPLITSPIDER}, \text{COMBINE}, \text{ERASESPIDER}, \text{COPYSPIDER}, \text{CONJUNCTIONELIMINATION}, \text{IDEMPOTENCY}\}$ be the set of inference rules of the diagrammatic proof system $\mathcal{D}_{GS}$. We define a typed signature Σ as follows: For each admissible instance of a rule $r \in \mathcal{R}$, we introduce a generator

$$r : (d_1, \dots, d_n) \rightarrow d', \quad (5)$$

such that

- $d_1, \dots, d_n, d' \in \mathcal{D}$,
- $n \geq 0$ is the arity determined by the rule r ,
- and the typing $(d_1, \dots, d_n) \rightarrow d'$ is determined by the corresponding diagrammatic entailment (cf. Tables 1 and 2).

In particular:

- unary rules such as ADDFEET , ERASESPIDER give operations in the form: $d \rightarrow d'$,
- binary rules such as COMBINE give operations in the form $(d_1, d_2) \rightarrow d'$,
- rules such as SPLITSPIDER may be treated as operations of higher arity depending on their decomposition.

Remark 2. Given the system $\mathcal{D}_{GS}$ consists of the diagrammatic types of $\mathcal{D}$, the inference rules $\mathcal{R}$, and the proof trees generated by their composition, the signature Σ is defined at the level of typed rule instances, not merely rule names. That is, each admissible application of an inference rule determines a generator in Σ . This ensures that all proof trees are well-typed with respect to the colours in $\mathcal{D}$.

We now pass from the signature Σ to the operadic structure generated by it. At the level of semiotic interpretation, this corresponds to moving from individual transformations

between semes to the structured composition of such transformations. For Greimas and Courtés, this framing describes the formulation of syntagmatic units which are “definable by the relations that the constituent elements [semes] maintain among themselves and with the unit that subsumes them” [6], p. 328. As Rastier [14], p. 38 further suggests, this implies a hierarchy that demarcates the microsemantic layer, where semes are defined through localised relations, from the mesosemantic layer, where actants emerge from case valencies established through such semic networks.

In the diagrammatic proof system D_{GS} , derivations are not merely linear sequences but exhibit a branching, hierarchical organisation, as evidenced by the proof trees in Appendixes A.1–A.8. These trees encode how simpler semantic transformations combine to yield more complex configurations, such as the meta-terms of the Greimas square. The appropriate algebraic structure for capturing this mode of composition is that of a coloured operad, in which operations may take multiple typed inputs and are composed by substitution. In this sense, the operad makes explicit the generative organisation implicit in the semiotic system: the construction of sememes from semes is formalised as the composition of typed operations within a structured network of relations.

Definition 3 (Free coloured operad). *Let $\mathcal{O}_\Sigma$ denote the free coloured operad on the signature Σ such that for each tuple of colours $(d_1, \dots, d_n; d')$, the set*

$$\mathcal{O}_\Sigma(d_1, \dots, d_n; d') \quad (6)$$

consists of formal rooted trees built from generators in Σ , whose

- (i) *leaves are labelled by $d_1, \dots, d_n$,*
- (ii) *internal nodes are labelled by generators in Σ ,*
- (iii) *root is labelled by d ,*
- (iv) *typing is respected at each node, such that*
- (v) *composition in $\mathcal{O}_\Sigma$ is given by substitution of trees as*

$$T \in \mathcal{O}_\Sigma(d_1, \dots, d_n; d) \quad \text{and} \quad T_i \in \mathcal{O}_\Sigma(e_{i1}, \dots, e_{ik_i}; d_i), \quad (7)$$

for which their composite is given through

$$T \circ (T_1, \dots, T_n), \quad (8)$$

and is obtained by grafting each T_i into the i -th input of T .

- (vi) *identities are given by trivial trees consisting of a single edge labelled by a colour d .*

No symmetry action is imposed on inputs; composition is determined solely by typed substitution of trees.

The elements of $\mathcal{O}_\Sigma$ are precisely typed trees of inference rules, and therefore provide a canonical syntax for compositional diagrammatic derivations.

Proposition 1 (Proof trees as operadic terms). *Each proof tree T (cf. Appendixes A.1–A.8) in the diagrammatic proof system D_{GS} determines an element*

$$\llbracket T \rrbracket \in \mathcal{O}_\Sigma(d_1, \dots, d_n; d) \quad (9)$$

such that

- *$d_1, \dots, d_n$ are input diagrams at the leaves of T ,*
- *d is the diagram at the root of T ,*

- the structure of T corresponds to the operadic tree defining $\llbracket T \rrbracket$.

This assignment is canonical and uniquely determined by the labelled tree structure of T .

Sketch of Proof of Proposition 1. We argue by induction on the height of a proof tree T in D_{GS} . If T has height 0, then it consists of a single edge labelled by a diagram $d \in \mathcal{D}$, and hence determines the identity operation

$$\text{id}_d \in \mathcal{O}_\Sigma(d; d).$$

For the inductive step, suppose the root of T is labelled by an admissible instance of an inference rule

$$r : (d_1, \dots, d_n) \rightarrow d$$

as a generator in the signature Σ . Let $T_1, \dots, T_n$ be the immediate subtrees attached to the inputs of this root node. By the induction hypothesis, each T_i determines an operadic term

$$\llbracket T_i \rrbracket \in \mathcal{O}_\Sigma(e_{i,1}, \dots, e_{i,k_i}; d_i).$$

Since r is a generator of $\mathcal{O}_\Sigma(d_1, \dots, d_n; d)$, operadic substitution yields

$$\llbracket T \rrbracket = r \circ (\llbracket T_1 \rrbracket, \dots, \llbracket T_n \rrbracket) \in \mathcal{O}_\Sigma(e_{1,1}, \dots, e_{1,k_1}, \dots, e_{n,1}, \dots, e_{n,k_n}; d).$$

The typing conditions imposed by Σ ensure that each substitution is well-defined. Thus, every proof tree determines a unique well-typed element of the free coloured operad. The construction is canonical because it is determined entirely by the labelled tree structure of T , together with the typed rule instances labelling its nodes. In particular, the proof trees for diagrammatic negation in Appendixes A.1 and A.2 determine unary operations, while the meta-term derivations in Appendixes A.5–A.8 determine composite operations whose branching structure realises the uniform schema for meta-term construction. $\square$

Remark 3. The fixed derivation schema underlying Greimasian meta-term formation determines a composite operation in the operad $\mathcal{O}_\Sigma$. In particular, the ‘+’ operation is not treated as primitive, but is realised compositionally through the structured combination of elementary diagrammatic transformations.

While the operad captures the full multi-input structure of diagrammatic reasoning, it also induces a category corresponding to sequential transformations.

Definition 4 (Category of diagrammatic derivations). Let $\mathcal{C}_\Sigma$ denote the category defined as follows:

1. Objects, $\text{Ob} \in \mathcal{C}_\Sigma$ are unitary spider diagrams $d \in \mathcal{D}$,
2. Morphisms are given through

$$\text{Hom}_{\mathcal{C}_\Sigma}(d, d') := \mathcal{O}_\Sigma(d; d'), \quad (10)$$

i.e., the unary fragment of the operad.

3. Composition is induced by operadic composition: that is, given

$$f \in \mathcal{O}_\Sigma(d; d'), \quad g \in \mathcal{O}_\Sigma(d'; d''), \quad (11)$$

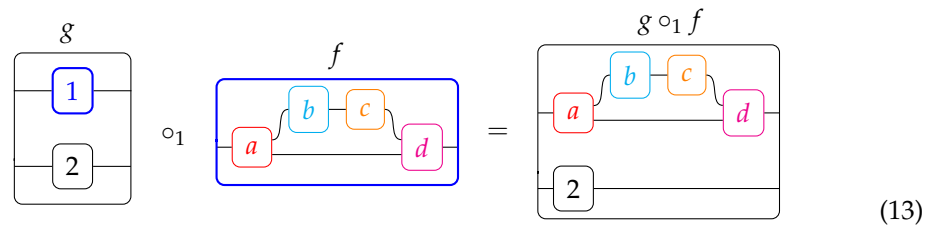
then

$$g \circ f := g \circ_1 f, \quad (12)$$

where $\circ_1$ denotes substitution into the unique input of g .

4. Identities are given by the identity operations in $\mathcal{O}_\Sigma$, i.e., trivial unary trees.

Remark 4. Operadic composition may be visualised diagrammatically as the substitution of one wiring diagram into the input of another:



$$(13)$$

That is, the composition $g \circ_1 f$ is obtained by inserting the wiring diagram f into the box labelled “1” in g . This corresponds categorically to substitution in the operad $\mathcal{O}_\Sigma$, and anticipates the wiring diagram semantics developed in Section 3.

The category $\mathcal{C}_\Sigma$ captures sequential composition of derivations, whereas the full operad $\mathcal{O}_\Sigma$ captures parallel and hierarchical composition via tree substitution. In this sense, the operad provides the primary compositional structure, and the category arises as its unary fragment. In turn, this allows us to interpret Figure 1 as a finite subdiagram in the category $\mathcal{C}_\Sigma$, whose objects are the unitary spider diagrams associated with the semes and meta-terms of the Greimas square, and whose arrows are particular morphisms generated by derivations in D_{GS} . In this sense, the semiotic square is realised not merely as a relational schema, but as a finite compositional fragment of the category of diagrammatic derivations.

3. Wiring Diagram Semantics

In Section 2, we established that the diagrammatic proof system D_{GS} induces a free coloured operad $\mathcal{O}_\Sigma$, whose elements correspond to typed proof trees and whose composition captures the combinatorial structure of diagrammatic derivations. In particular, this operadic structure makes explicit how the Greimas square is generated from a seme pair (s_1, s_2) through a uniform schema of rule composition. However, this description remains syntactic: derivations are represented as trees, and their compositional behaviour is expressed through substitution. In order to make the process of semantic construction more transparent in light of what van Wolde [12] identifies as the flow of intermediate terms through successive operations (i.e., $s_i \mapsto \neg s_i \mapsto s_j$), we now reinterpret these operadic terms as wiring diagrams in a symmetric monoidal setting.

Our motivation stems from a desire to expose the construction of terms within the square via a compositional semantics in which the transformations governed by the inference rules in $\mathcal{R}$, that are duly expressed through the proof tree structures of T_i , serve to expose the compositional organisation underlying the construction of terms in the square. Here, as Greimas and Courtés suggest, the syntagmatic axis is characterised through a network of relations that is defined by linearity and process. It is the “formulation of syntagmatic units (or syntagms), definable by the relations that the constituent elements maintain among themselves and with the unit that subsumes them” [6], p. 328. This naturally leads to the emergence of hierarchies of relations “arranged into successive levels of derivation.” In the following section, we show by construction how wiring diagrams over the signature Σ correspond to elements of the free coloured operad $\mathcal{O}_\Sigma$, for which syntagmatic hierarchies can be described through the composition of diagrams via operadic substitution.

Definition 5. A wiring diagram $\mathcal{W}$ over a signature Σ consists of:

- a finite collection of boxes $X \in \text{Ob}(\mathcal{W})$, each labelled by a generator

$$r : (d_1, \dots, d_n) \rightarrow d'$$

in Σ .

- a set of wires, each labelled by a diagrammatic type $d \in \mathcal{D}$,
- for each box X , finite sets of input and output ports

$$\text{in}(X), \quad \text{out}(X),$$

whose cardinalities match the arity of the corresponding generator,

together with connections assigning:

- each input port of a box to exactly one wire,
 - each wire is the output of at most one port and may be connected to one or more input ports,
- such that:
- the typing of wires agrees with the input/output types of the boxes,
 - and the resulting directed graph is acyclic.

A wiring diagram may have designated external input and output wires, forming its interface. The composition of wiring diagrams is given by connecting outputs to inputs, corresponding to operadic substitution in $\mathcal{O}_\Sigma$.

Remark 5. The correspondence between the semiotic and categorical vocabulary used throughout the paper may be summarised as follows:

Seme	$\leftrightarrow$ Diagrammatic type $d \in \mathcal{D}$	
Transformation	$\leftrightarrow$ Generator in Σ	
Proof tree	$\leftrightarrow$ Operadic term	(14)
Construction of a meta-term	$\leftrightarrow$ Operadic composition	
Operational account	$\leftrightarrow$ Wiring diagram	

This correspondence is intended only as a guide to interpretation and does not identify semiotic and categorical concepts.

Definition 6. A symmetric monoidal category is a category $(\mathcal{C}, \otimes, I)$ equipped with:

- a bifunctor

$$\otimes : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$$

called the tensor product,

- a distinguished object $I \in \mathcal{C}$ called the unit,
- natural isomorphisms (the associators and unitors)

$$\alpha_{A,B,C} : (A \otimes B) \otimes C \rightarrow A \otimes (B \otimes C), \quad \lambda_A : I \otimes A \rightarrow A, \quad \rho_A : A \otimes I \rightarrow A$$

- and a natural symmetry (the swap map)

$$\sigma_{A,B} : A \otimes B \rightarrow B \otimes A,$$

such that the usual coherence conditions hold.

Remark 6. We may interpret the operadic terms of $\mathcal{O}_\Sigma$ as abstract wiring diagrams, which admit realisation as morphisms in a symmetric monoidal category $\mathcal{S}$. Under this interpretation, diagrammatic types $d \in \mathcal{D}$ are treated as objects, and derivations—previously represented as proof

trees—are realised as morphisms constructed through sequential and parallel composition. This perspective allows us to pass from the tree-based syntax of Section 2 to a graphical semantics in which composition is represented explicitly as the flow of information along wires. This places the semantics within the general framework of string diagrammatic reasoning for symmetric monoidal categories, in which algebraic composition is represented topologically. In particular, the substitution operation in the operad corresponds to the plugging together of outputs and inputs, while the tensor product accounts for the parallel combination of independent derivations. We therefore represent operadic terms as wiring diagrams, in which:

- boxes correspond to generators in Σ (i.e., instances in $\mathcal{R}$), wires correspond to diagrammatic types $d \in \mathcal{D}$, and
- connectivity encodes the compositional structure of derivations.

This representation makes explicit the intermediate stages of semantic construction identified in the van Wolde [12] operational account of the semiotic square, and provides a visual realisation of how complex terms arise from the composition of simpler transformations. As Lagopoulos and Boklund-Lagopoulou [15], p. 100 argue, there is a generative trajectory inherent in the initial operation on semes which “establishes a first hierarchy, because the logical types governing the relations between terms are hierarchically superior to the terms themselves.” This in turn becomes the basis for a second generation of categorical terms (meta-terms) that emerge through the composition of morphisms.

We begin with the basic generators corresponding to diagrammatic negation. Consider the wiring diagrams we give in Figure 2. Here, we have two instances of the negation operator

$$\nu_d : d_s \rightarrow d_{\neg s}. \quad (15)$$

In our wiring diagrams, wire-branching (denoted by a filled node on a type) represents structural reuse of a diagrammatic type. Formally, this is modelled by equipping each colour $d \in \mathcal{D}$ with a commutative comonoid structure

$$\Delta_d : d \rightarrow d \otimes d, \quad (16)$$

corresponding to the COPYSPIDER rule in D_{GS} . This reflects the non-linear character of the semantics, in which inputs may be used multiple times within a computation. Such duplication is not available in arbitrary symmetric monoidal categories, but is accommodated in cartesian or comonoidal settings. In this way, the wiring diagram semantics faithfully captures the reuse of semantic information present in the proof-theoretic formulation. For clarity, we also leave the spider diagrams unlabelled.

In Figure 2a, we introduce negation on the first seme of the square as the derivation

$$\nu_{s_1} : d_{s_1} \rightarrow d_{\neg s_1},$$

and in Figure 2b, negation on the second seme of the square as

$$\nu_{s_2} : d_{s_2} \rightarrow d_{\neg s_2}.$$

The operator ν_d is thus uniform at the level of its generating rule sequence, but its concrete realisations depend on the typing of its inputs and outputs, yielding distinct instances for s_1 and s_2 .

We have a similar case for our wiring diagrams in Figure 3, where we again have two instances of the implication operator:

$$\iota_d : d_{\neg s} \rightarrow d_s, \quad (17)$$

such that in both Figure 3a and Figure 3b, the diagrams track the collapse of the OR condition that defines the negated seme instances as:

$$(S_1 \cap S_2 = \emptyset) \wedge \exists x(x \in (S_j \setminus S_i) \vee x \in (X \setminus (S_1 \cup S_2 \cup M)))$$

into

$$(S_1 \cap S_2 = \emptyset) \wedge \exists x(x \in (S_j \setminus S_i)),$$

where $i, j \in \{1, 2\}$. As Lagopoulos and Boklund-Lagopoulou [15], p. 99 note, this completes the transformation of s_i logico-semantically, so that “the passage from one term to the other is defined as a transformation, due to the operations of negation and assertion [implication].” In effect, the wiring diagrams of ν and ι provide visualisations and an accompanying compositional semantics that tracks how the negated terms valorise their diagrammatically implicated positive (s_1) or negative (s_2) semes, and thus establish the conditions by which the Positive and Negative Deixes are established axiologically. The implication operations $\iota_{\neg s_1}$ and $\iota_{\neg s_2}$ make explicit the sequential dependency encoded in the operational account of the square: negated terms are mapped into their corresponding implicated terms through typed composition. The wiring-diagram representation exposes the intermediate stages of these transformations and provides a compositional account of how semantic configurations are assembled from elementary operations. This interpretation is compatible with broader semiotic applications of the square, although such applications are not formalised in the present work.

In Figures 4 and 5, the compositional structure underlying the construction of the Greimasian meta-terms is made explicit. Each diagram realises an instance of the ‘+’ operation as a composite of generators drawn from the signature Σ , thereby refining the proof-theoretic observation that all meta-terms arise from a uniform derivation schema. In particular, the diagrams for the complex term **S** (Figure 4a) and the neutral term $\bar{\mathbf{S}}$ (Figure 4b), as well as the Positive and Negative Deixis (Figure 5), exhibit a common structural pattern: a conjunctive input is decomposed, duplicated, and recombined through a fixed sequence of operations.

A notable feature of these constructions is the essential role of the COPYSPIDER operation, which enables controlled reuse of diagrammatic inputs within the derivation. As seen in the proof trees (Appendixes A.1–A.8), meta-term formation requires repeated use of constituent semes across multiple branches. This repeated participation bears a structural resemblance to Greimasian notions of semantic recurrence and coherence (“isotopy”), although no formal equivalence is assumed. In the wiring-diagram representation, this reuse is realised by wire duplication and branching, reflecting the non-linear character of the underlying proof system.

Remark 7. Although the construction of the Greimas square yields four distinguished meta-term outputs—namely the complex term (**S**), neutral term ($\bar{\mathbf{S}}$), Positive Deixis, and Negative Deixis—we package these collectively as a single aggregate diagrammatic type

$$d_{\square} := (d_{\mathbf{S}}, d_{\bar{\mathbf{S}}}, d_P, d_N) \in \mathcal{D}.$$

The notation is purely descriptive and does not introduce additional operadic or monoidal structure on $\mathcal{D}$. The wiring diagrams in Figures 4–6 expose the internal organisation of this aggregate output by displaying its constituent semantic components separately. Consequently,

$$\Omega : (d_{s_1}, d_{s_2}) \rightarrow d_{\square}$$

remains a single-output operation in the coloured operad $\mathcal{O}_\Sigma$, while admitting a graphical realisation whose internal structure may be interpreted componentwise. Accordingly, the complete square is represented by the operadic term

$$\Omega \in \mathcal{O}_\Sigma(d_{s_1}, d_{s_2}; d_\square).$$

Proposition 2 (Operadic realisation of the semiotic square). *The wiring diagram Ω in Figure 6 corresponds to a composite operation*

$$\Omega : (d_{s_1}, d_{s_2}) \rightarrow d_\square, \quad (18)$$

in the free coloured operad $\mathcal{O}_\Sigma$ obtained by operadic substitution of the generators,

$$\nu_{s_1}, \nu_{s_2}, \iota_{\neg s_1}, \iota_{\neg s_2}, \mu, \gamma, \psi, \delta.$$

Moreover, Ω is a well-typed operadic term whose underlying tree corresponds to the hierarchical composition of the derivations given in Figures 2–5, and therefore realises the construction of the Greimas semiotic square as a single composite operation in $\mathcal{O}_\Sigma$.

Sketch of Proof of Proposition 2. Each box in Figure 6 corresponds to a typed generator in the signature Σ (Definition 2). By Definition 3, these generators determine elements of the free coloured operad $\mathcal{O}_\Sigma$ and compose via operadic substitution. The wiring diagram Ω specifies a directed acyclic graph whose edges are typed by diagrammatic types $d \in \mathcal{D}$, and whose connections match output types to input types. This structure determines a well-typed operadic tree obtained by reading the diagram from inputs to outputs, with substitution corresponding to the plugging of outputs into inputs. By the universal property of the free coloured operad $\mathcal{O}_\Sigma$, this yields a unique composite operation:

$$\Omega \in \mathcal{O}_\Sigma(d_{s_1}, d_{s_2}; d_\square). \quad (19)$$

The resulting term encodes precisely the hierarchical composition of the derivations exhibited in Figures 2–5. $\square$

The wiring diagram Ω given in Figure 6 provides a global synthesis of the preceding constructions by assembling the generators for negation, implication, and meta-term formation into a single wiring diagram. It directly models the operational account of a typical square’s construction given by van Wolde [12], Lagopoulos and Boklund-Lagopoulou [15] as well as by Greimas [13], p. 69 and Greimas and Courtés [6], pp. 308–311. It begins with an identification of a seme pair s_1 and s_2 satisfying the relation $d \#_{\text{diag}} d'$ (cf. Figure 1, and [8]) of contrariety for which the outputs of the negation operators ν_{s_1} and ν_{s_2} feed into the implication operators $\iota_{\neg s_1}$ and $\iota_{\neg s_2}$. Traditionally, these syntactic operations are considered as generative trajectories that reinforce the homologation between the initial s_1/s_2 and its negation $\neg s_1/\neg s_2$, thereby establishing syntagmatic units via these hypotactic axes. The establishment of these units provides the basis for defining the meta-terms, which in Ω , are the generators: γ (complex term **S**), μ (neutral term $\tilde{\mathbf{S}}$), ψ (Positive Deixis), δ (Negative Deixis). The resulting diagram thus exhibits a layered structure, in which successive stages of semantic transformation are composed into a coherent whole.

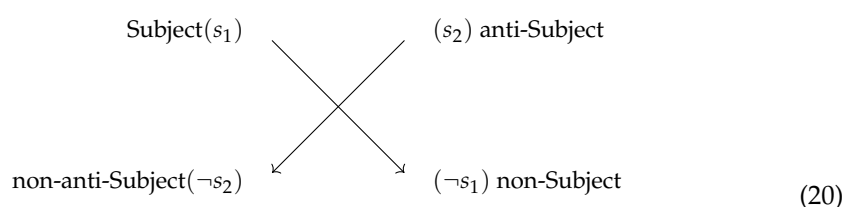
From the perspective of operadic semantics, Figure 6 realises the construction of the square as a single composite operation whose internal structure reflects the substitution of the wiring diagrams introduced in Figures 2–5. Each box in the diagram corresponds to a generator in the signature Σ , and the connections between boxes encode the flow of intermediate diagrams through the composite. In this sense, the wiring diagram Ω provides a concrete representation of an operadic term in $\mathcal{O}_\Sigma$, in which the hierarchical structure

of the proof trees is translated into a network of interconnected operations. The semiotic square thus emerges not as a static configuration of relations, but as the output of a compositional process governed by the systematic interaction of its generating operations.

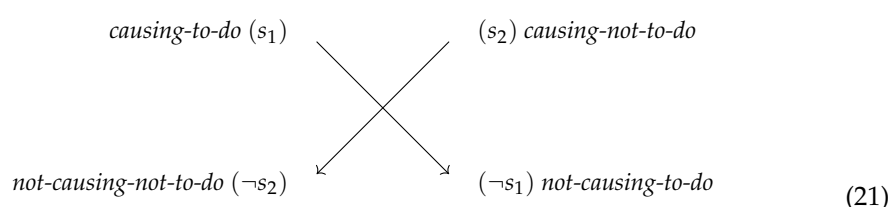
4. Semantic Interpretations as Algebras

In terms of the traditional interpretive semantics of the square, the wiring diagram Ω directly addresses what Parret identifies as one intended function of the Greimas square as a “constitutional model” that “reconstructs the structure and production/understanding of meaning” [16], p. xiv. From the perspective of traditional semiotic interpretation, the wiring diagram Ω may be viewed as making explicit the compositional organisation implicit in the generative use of the square. In particular, it provides a formal representation of how successive semantic transformations may be assembled into a single composite construction.

For Greimas and Courtés [6], p. 294, the square serves as a utility in the domain of narratology. Here, it is used to model the dynamics of actantial systems of story logic and identify how polemical structures of a narrative, such as the presence of a Subject (s_1) and anti-Subject (s_2) map onto the square as:



The square can similarly express the notion of a Positive Object (s_1) vs. Negative Object (s_2), etc., in order to describe paradigmatic disjunctions within a narrative. This then led to using the syntagmatic grammar of the square to assign a set of accompanying modalities to actants and their implicated narrative utterances. For example, factitive modalities could be intrinsically modelled as [13], p. 126:



As such, Greimas subsequently argued for using the square to analyse narrative structures given that “narrative syntax consists of operations carried out on terms that can be invested with content values; the syntax thus transforms and manipulates these terms by negating or affirming them, or, and it amounts to the same thing, *disjoining* and *conjoining* them.’ [13], p. 70. The wiring diagram Ω thus provides a specification for these “syntactic operators” in the square (i.e., implication, negation, meta-term formation), in effect acting as a meta-description of possible narrative structures as interpretations of the operad $\mathcal{O}_\Sigma$.

Definition 7 (Algebra of $\mathcal{O}_\Sigma$). Let $\mathcal{O}_\Sigma$ be the free coloured operad generated by Σ , and let $\mathfrak{S}$ be a symmetric monoidal category. An algebra (or interpretation) of $\mathcal{O}_\Sigma$ in $\mathfrak{S}$ consists of:

- for each diagrammatic type $d \in D$, an object

$$A(d) \in \mathfrak{S},$$

- for each operation

$$T \in \mathcal{O}_\Sigma(d_1, \dots, d_n; d'),$$

a morphism

$$A(T) : A(d_1) \otimes \dots \otimes A(d_n) \rightarrow A(d'),$$

such that:

1. composition is preserved:

$$A(T \circ (T_1, \dots, T_n)) = A(T) \circ (A(T_1), \dots, A(T_n)),$$

2. and identities are preserved.

Equivalently, and more structurally, such an algebra may be viewed as a morphism of coloured operads, as described in Remark 8 (below). In what follows, we adopt this perspective.

Remark 8. Equivalently, an algebra of $\mathcal{O}_\Sigma$ in a symmetric monoidal category $\mathfrak{S}$ may be defined as a morphism of coloured operads

$$A : \mathcal{O}_\Sigma \rightarrow \text{End}_\mathfrak{S}, \quad (22)$$

where $\text{End}_\mathfrak{S}$ is the endomorphism operad whose operations are morphisms:

$$\text{End}_\mathfrak{S}(X_1, \dots, X_n; Y) = \text{Hom}_\mathfrak{S}(X_1 \otimes \dots \otimes X_n, Y). \quad (23)$$

Thus, the operad $\mathcal{O}_\Sigma$ encodes the compositional form of the semiotic square, while its algebras realise this form as concrete semantic structures.

Example 2 (A set-valued algebra). We now give a concrete algebra of $\mathcal{O}_\Sigma$ in **Set**. Let $A : \mathcal{O}_\Sigma \rightarrow \mathbf{Set}$ be defined on the diagrammatic types associated with the Greimas square by

$$\begin{aligned} A(d_{s_1}) &= /being/, & A(d_{s_2}) &= /seeming/, \\ A(d_{\neg s_1}) &= /non-being/, & A(d_{\neg s_2}) &= /non-seeming/, \end{aligned}$$

and

$$\begin{aligned} A(d_S) &= /truth/, & A(d_{\bar{S}}) &= /falsehood/, \\ A(d_P) &= /secret/, & A(d_N) &= /lie/. \end{aligned}$$

For the aggregate output colour we set

$$A(d_\square) = A(d_S) \times A(d_{\bar{S}}) \times A(d_P) \times A(d_N).$$

The generating operations are interpreted as functions. Thus, diagrammatic negation is interpreted by

$$A(v_{s_1})(/being/) = /non-being/, \quad A(v_{s_2})(/seeming/) = /non-seeming/,$$

and diagrammatic implication by

$$A(\iota_{\neg s_1})(/non-being/) = /seeming/, \quad A(\iota_{\neg s_2})(/non-seeming/) = /being/.$$

The meta-term operations are interpreted as

$$A(\gamma)(/being/, /seeming/) = /truth/,$$

$$A(\mu)(/non-being/, /non-seeming/) = /falsehood/,$$

$$A(\psi)(/being/, /non-seeming/) = /secret/,$$

and

$$A(\delta)(/non-being/, /seeming/) = /lie/.$$

The composite operation Ω is therefore interpreted as the function

$$A(\Omega) : A(d_{s_1}) \times A(d_{s_2}) \rightarrow A(d_{\square}),$$

given by

$$A(\Omega) = (/being/, /seeming/)(/truth/, /falsehood/, /secret/, /lie/).$$

Thus, the operadic composition preserves the Greimasian generative relations: constructing meta-terms after semantic transformation yields the same result as first composing the corresponding operations in $\mathcal{O}_{\Sigma}$ and then interpreting them in **Set**. This assignment extends uniquely to all of $\mathcal{O}_{\Sigma}$, since $\mathcal{O}_{\Sigma}$ is the free coloured operad generated by Σ . In particular, for any operadic substitution

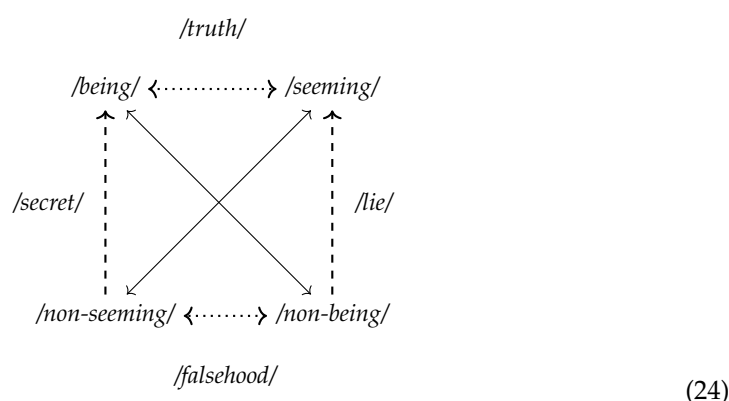
$$f \circ_i g$$

we have

$$A(f \circ_i g) = A(f) \circ_i A(g),$$

where the right-hand side denotes substitution of the corresponding functions in **Set**. Identities are preserved because each identity operation $\text{id}_d \in \mathcal{O}_{\Sigma}(d; d)$ is interpreted as the identity function on $A(d)$. Hence, A is a concrete set-valued algebra of $\mathcal{O}_{\Sigma}$.

Remark 9. The interpretation above corresponds to the Greimasian veridictory square which we give in its traditional visualisation:



Although the present construction does not extend the derivational power of D_{GS} introduced in [8], it contributes additional structure that is not available in the proof-theoretic formulation alone. First, derivations are elevated from witnesses of admissibility to compositional objects in their own right. In D_{GS} , proof trees certify that a given semantic configuration can be derived through rule application. In contrast, the free coloured operad $\mathcal{O}_{\Sigma}$ internalises substitution itself as an algebraic operation, allowing derivations to be composed, compared, and manipulated independently of any particular proof instance. Second, the operadic formulation separates syntax from interpretation. While D_{GS} provides a single proof-theoretic semantics, the operadic account organises semantic realisations as algebras in $\mathbf{Alg}(\mathcal{O}_{\Sigma})$ (the category of semantic interpretations), thus allowing structurally distinct interpretations to be treated as instances of a common compositional schema. Third, this shift permits the same compositional construction to be transported across

domains without modifying the underlying derivation system. The seme-level, veridictory, modal, and actantial variants of the Greimas square may therefore be understood as distinct algebras of the same operadic object rather than independent diagrammatic constructions.

The wiring diagram Ω thus determines, under each such algebra, a composite transformation whose form is invariant across textualisations or axiologies. In this sense, the contribution of the operadic layer is not additional inferential strength, but the explicit representation of compositional invariants that remain implicit in the proof-theoretic account. For example, given our main result as:

$$\Omega : (d_{s_1}, d_{s_2}) \rightarrow d_{\square}, \quad (25)$$

we could use the same wiring diagram to produce a semantic computation in another domain (for which in Cartesian settings, $\otimes = \times$) as:

$$A(\Omega) : A(d_{s_1}) \otimes A(d_{s_2}) \rightarrow A(d_{\square}), \quad (26)$$

where, for instance $A(d_{s_1}) = \text{causing-to-do}$, $A(d_{s_2}) = \text{not-causing-to-do}$, $A(v)$ realises the corresponding negation operation, etc. Moreover, distinct semantic realisations may be related by a structure-preserving family of morphisms between the corresponding operad algebras. Given two algebras

$$A, B : \mathcal{O}_{\Sigma} \rightarrow \text{End}_{\mathfrak{S}},$$

such translations, together with the corresponding operad algebras, naturally assemble into a category of semantic interpretations of the square.

Definition 8 (Category of semantic interpretations of the Greimas square). *Let $\text{Alg}(\mathcal{O}_{\Sigma})$ denote the category of semantic interpretations of the Greimas square, whose:*

- *objects are algebras $A : \mathcal{O}_{\Sigma} \rightarrow \text{End}_{\mathfrak{S}}$ of the coloured operad $\mathcal{O}_{\Sigma}$ in a symmetric monoidal category $\mathfrak{S}$,*
- *morphisms are coherent families of morphisms*

$$\eta_d : A(d) \rightarrow B(d),$$

indexed by diagrammatic types $d \in \mathcal{D}$, and satisfying the coherence condition that for every operation

$$T \in \mathcal{O}_{\Sigma}(d_1, \dots, d_n; d'),$$

the following diagram commutes:

$$\begin{array}{ccc} A(d_1) \otimes \dots \otimes A(d_n) & \xrightarrow{A(T)} & A(d') \\ \eta_{d_1} \otimes \dots \otimes \eta_{d_n} \downarrow & & \downarrow \eta_{d'} \\ B(d_1) \otimes \dots \otimes B(d_n) & \xrightarrow{B(T)} & B(d') \end{array} \quad (27)$$

- *Composition and identities are inherited componentwise from the ambient category $\mathfrak{S}$.*

The category $\text{Alg}(\mathcal{O}_{\Sigma})$ collects the various semantic realisations of the square as instances of a common compositional schema, together with the structure-preserving transformations between them. In particular, it directly describes the approach of Greimas [7,13] towards narrative analysis, in which the square is adapted to describe signification not

only at the seme level, but also at modal and actantial levels. In this setting, each algebra realises the same underlying compositional schema within a particular semantic domain. Concrete analyses of texts then arise as instances of these algebras, obtained by applying the composite operation $A(\Omega)$ within a given semantic domain.

5. Conclusions

In this paper, we have shown that the diagrammatic proof system D_{GS} developed in [8] admits a natural operadic semantics, in which proof trees correspond to elements of a free coloured operad $\mathcal{O}_\Sigma$, and the construction of the Greimas semiotic square is realised as a single composite operation Ω . This perspective makes explicit the compositional structure underlying the square: rather than treating derivations as syntactic witnesses, we interpret them as algebraic objects whose combination is governed by operadic substitution. The operadic layer therefore contributes not additional derivational power but a separation of compositional syntax from semantic interpretation, allowing distinct domains to be represented as algebras of a common operadic object. The corresponding representation of these operations as wiring diagrams in a symmetric monoidal setting provides a visual and formal account of the flow of semantic information through successive transformations.

This shift from a proof-theoretic to an operadic and diagrammatic semantics clarifies the generative character of the semiotic square. In particular, the uniform derivation schema for meta-term construction identified in the diagrammatic system is recast as an instance of a general compositional principle, in which complex semantic configurations arise from the structured interaction of simpler operations. The wiring diagram Ω thus establishes that the square may be understood not as a static configuration of oppositions, but as the output of a compositional process, whose internal organisation is fully determined by the algebraic structure of $\mathcal{O}_\Sigma$.

Beyond this structural reconstruction, the operadic perspective provides a framework for understanding the extension of the semiotic square across different domains of application. By interpreting $\mathcal{O}_\Sigma$ in a semantic category, the same compositional schema may be realised as an algebra whose elements correspond to semes, modalities, or actantial roles. Under such interpretations, the wiring diagram Ω determines a composite transformation whose form is invariant across these domains, thereby capturing the square's syntagmatic relations. Crucially, these interpretations do not stand in isolation, but are organised within the category $\mathbf{Alg}(\mathcal{O}_\Sigma)$, whose objects are semantic realisations of the square and whose morphisms are structure-preserving transformations between them. In this way, modality-based descriptions, actantial role systems as well as veridictory or seme-level analyses may be understood not merely as parallel uses of the square, but as interconnected interpretations of a shared compositional structure.

In this way, the operadic semantics developed here provides both a formal foundation and a conceptual generalisation of the Greimas semiotic square. It not only reconstructs the combinatorial logic of seme-wise transformations, but also offers a mathematically precise account of how such transformations may be composed, interpreted, and transported across domains of meaning. This suggests further directions for research, including the comparison of semiotic systems via operadic morphisms, the study of alternative diagrammatic calculi within the same framework, and the application of compositional methods to broader questions in semiotics and narrative theory.

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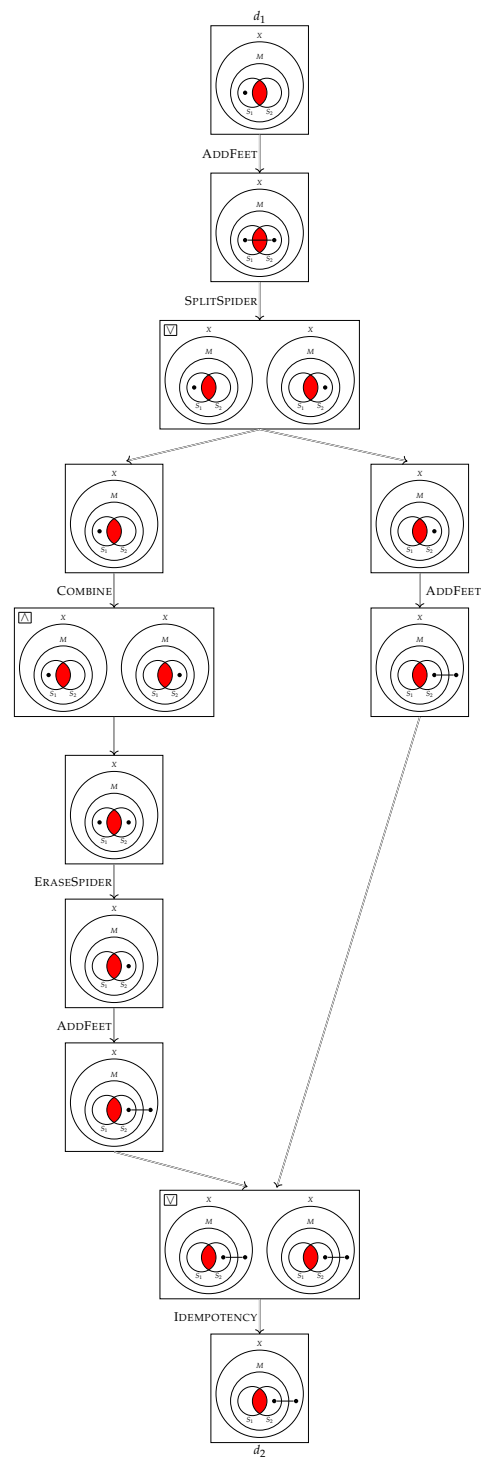
Informed Consent Statement: Not applicable.

Data Availability Statement: Data is contained within the article.

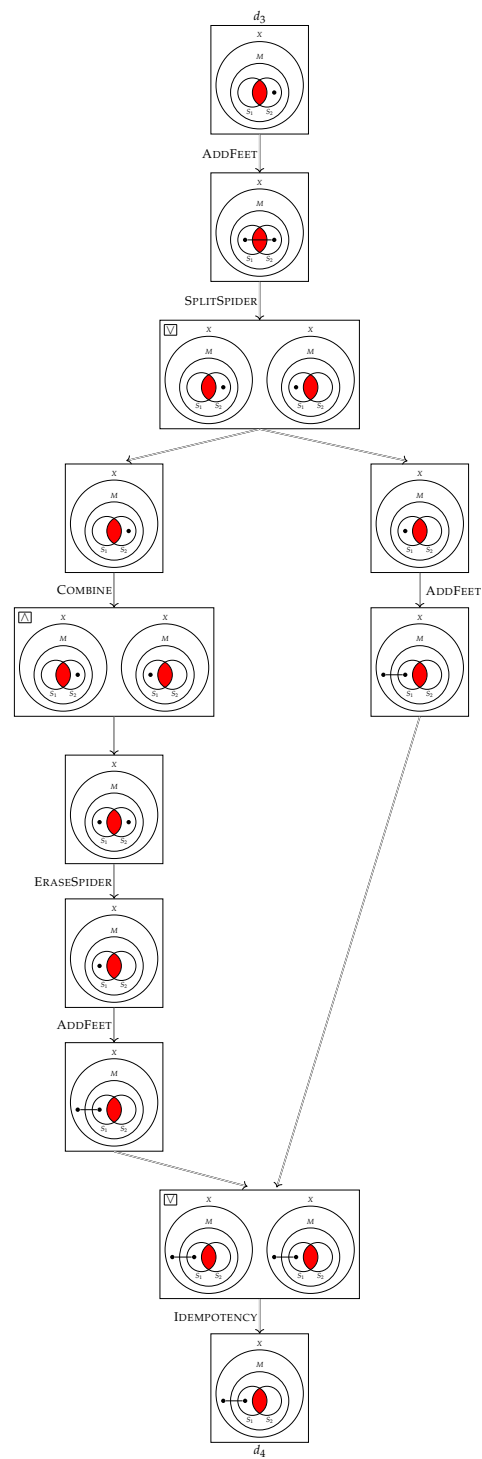
Conflicts of Interest: The author declares no conflicts of interest.

Appendix A

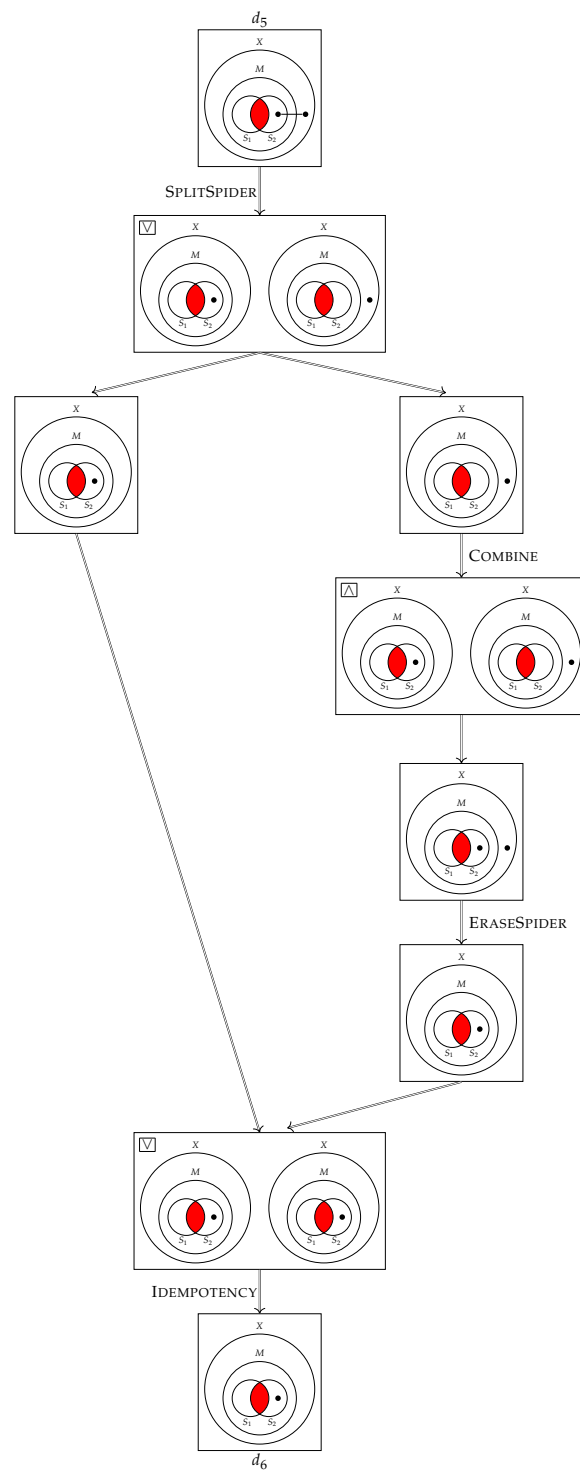
Appendix A.1. Proof Tree $T_1 : d_1 \vdash d_2$



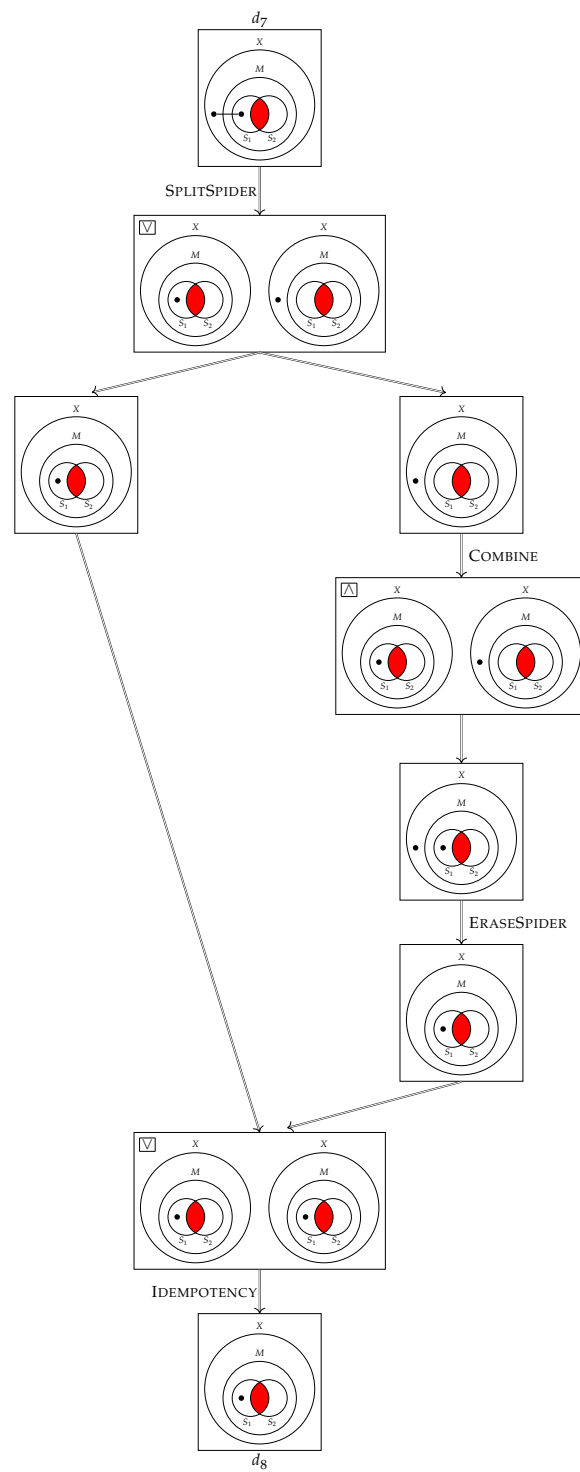
Appendix A.2. Proof Tree $T_2 : d_3 \vdash d_4$



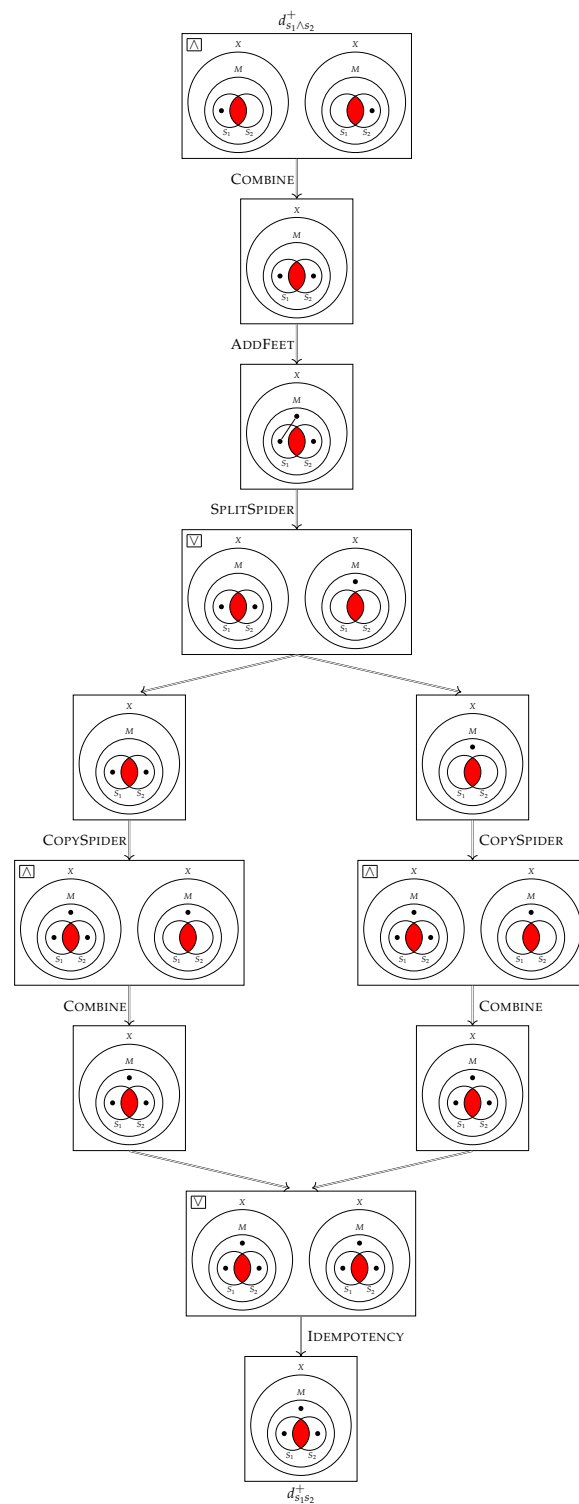
Appendix A.3. Proof Tree $T_3 : d_5 \vdash_{\mathcal{R}} d_6$



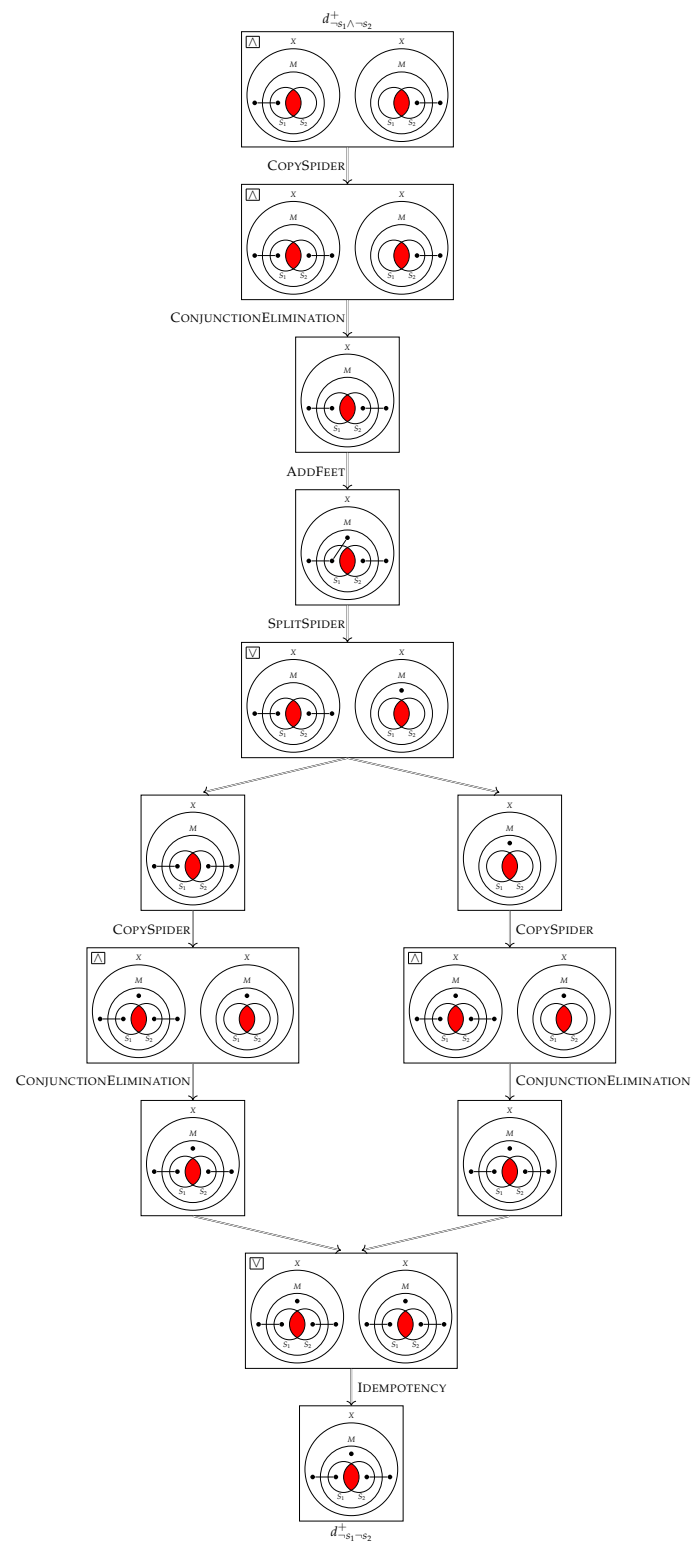
Appendix A.4. Proof Tree $T_4 : d_7 \vdash_{\mathcal{R}} d_8$



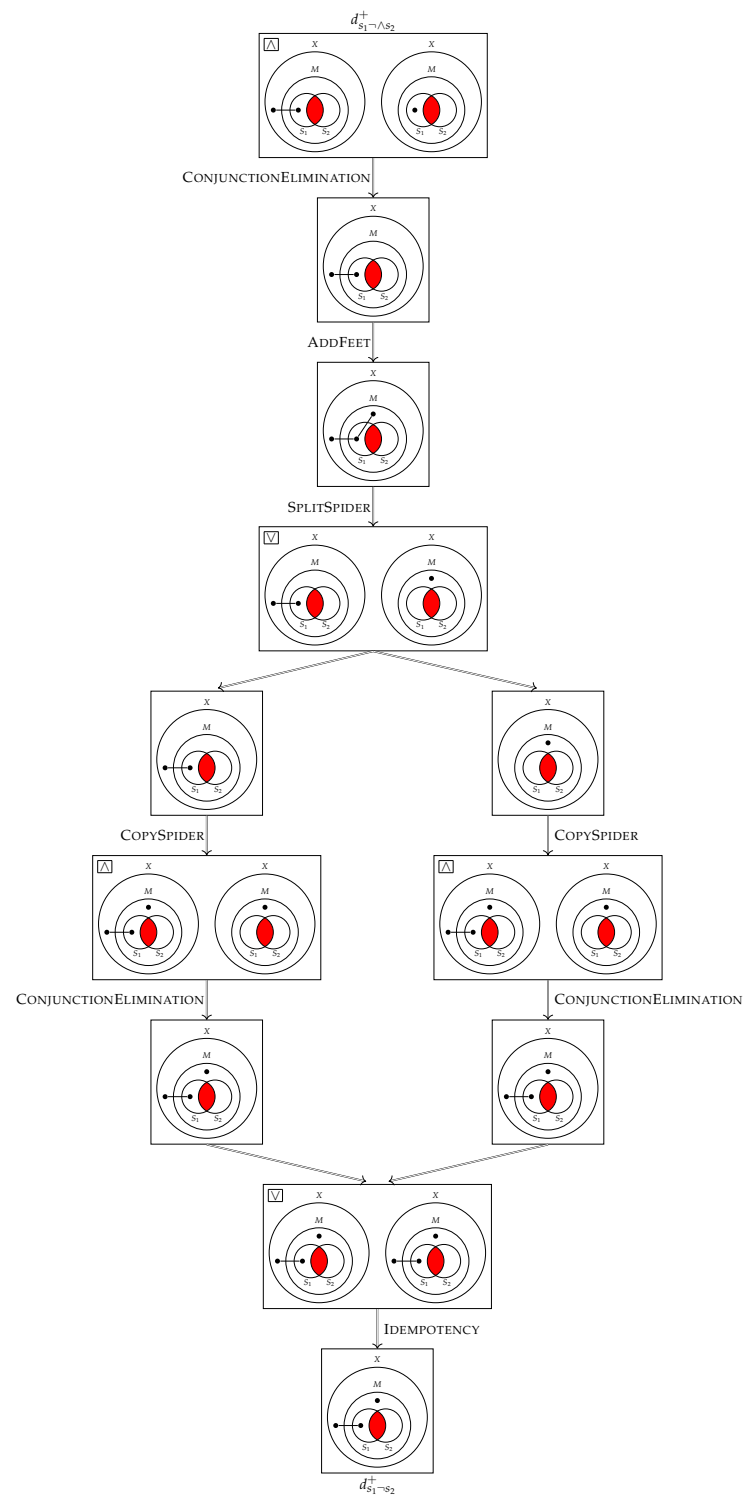
Appendix A.5. Proof Tree $T_5 : d_{s_1 \wedge s_2} \vdash_{\mathcal{R}} d_{s_1 s_2}^+$



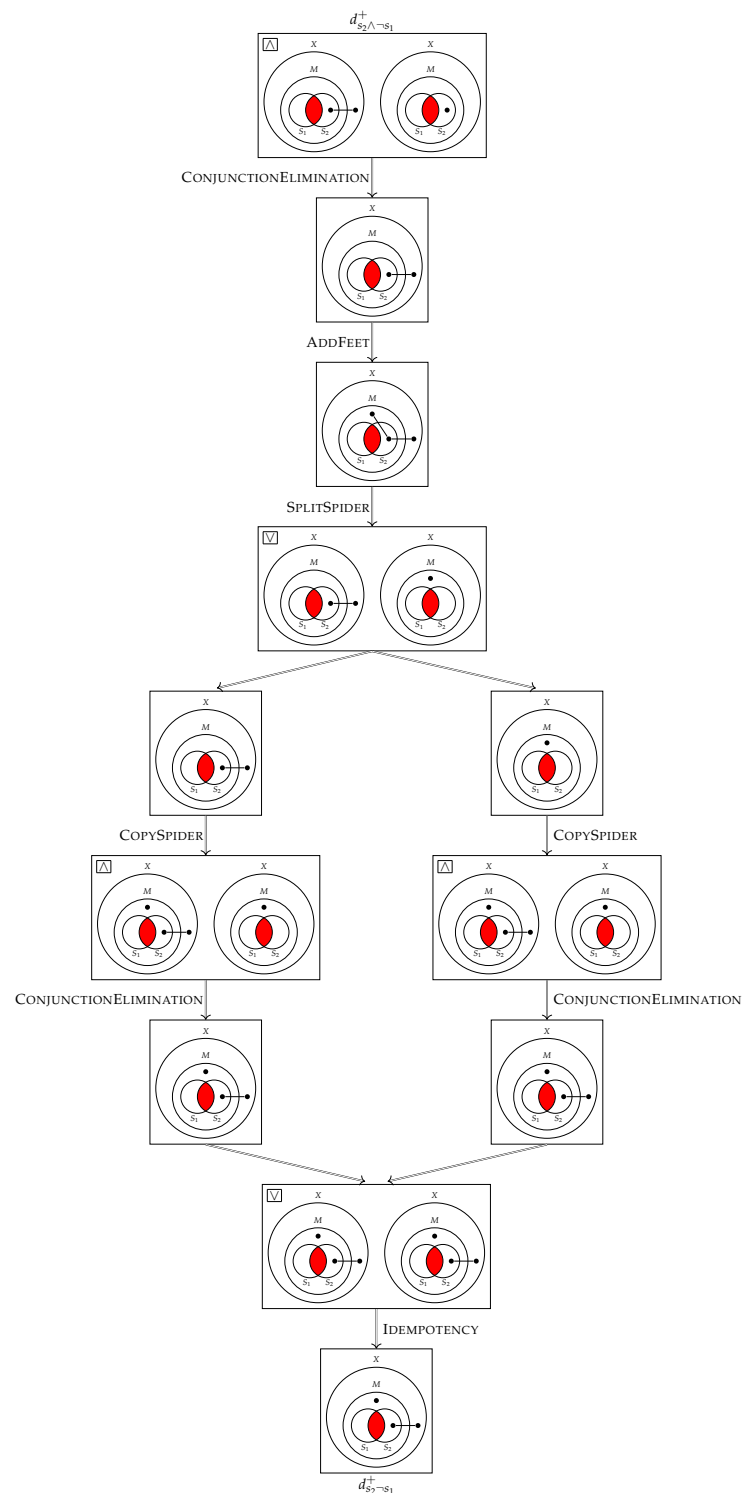
Appendix A.6. Proof Tree $T_6 : d_{\neg s_1 \wedge \neg s_2}^+ \vdash_{\mathcal{R}} d_{\neg s_1 \neg s_2}^+$



Appendix A.7. Proof Tree $T_7 : d_{s_1 \wedge \neg s_2} \vdash_{\mathcal{R}} d_{s_1 \neg s_2}^+$



Appendix A.8. Proof Tree $T_8 : d_{s_2} \wedge \neg s_1 \vdash_{\mathcal{R}} d_{\neg s_1 s_2}^+$



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