

## Article

# Maneuvering of an Underwater Vehicle Using Bio-Inspired Pectoral Fins

Pedro C. Ormonde <sup>\*</sup>, Xiaowei He <sup>†</sup> and Kenneth Breuer 

School of Engineering, Brown University, Providence, RI 02912, USA; xiaowei.he@utah.edu (X.H.); kbreuer@brown.edu (K.B.)

<sup>\*</sup> Correspondence: pco@brown.edu

<sup>†</sup> Current address: Department of Mechanical Engineering, University of Utah, Salt Lake City, UT 84112, USA.

## Abstract

A cyber-physical underwater vehicle is equipped with bio-inspired flapping fins positioned on the sides of the vehicle's main body. The proposed control surfaces are inspired by fish pectoral fins, generating forces and moments that can potentially be harnessed for maneuvering, hovering and station keeping. The streamwise and cross-stream forces produced by the fins are characterized for a range of reduced frequencies and Strouhal numbers. The streamwise forces are shown to be predominantly a function of the fin's projected frontal area, while the lateral forces also depend on the Strouhal number. When operated simultaneously, different flapping synchronizations can be employed for specific goals; a symmetric motion suppresses the lateral forces, while an anti-symmetric motion decreases the peaks of the streamwise force produced. The cyber-physical vehicle demonstrates how the pair of fins can successfully maneuver the vehicle in the lateral direction.

**Keywords:** underwater locomotion; bio-inspired maneuvering; bio-locomotion; pectoral fins

## 1. Introduction

Fish are highly maneuverable while also being energetically efficient swimmers [1]. The majority of species employ body and caudal fin undulations as their primary source of thrust during high-speed forward swimming [2,3]. While accessory appendages such as the pectoral fin may also be used for steady rectilinear propulsion [4–7], they are often employed for maneuvering and breaking [8], station holding [9] and stabilization [1]. Control surfaces located posterior to the center of mass produce trimming forces [10–12] that can reorient the body and/or flow field passively, creating self-correcting forces that are stabilizing by nature, and are commonly used by fish swimming in steady and turbulent flows [13–16]. Conversely, control surfaces anterior to the center of mass produce self-amplifying disturbances that promote higher maneuverability instead of stability [1,17].

Unconventional underwater vehicle (UV) designs take inspiration in fish morphology and biomechanics to achieve high maneuverability. Flexible fish-like robots that can bend their bodies achieve high turning rates and small turning radii [18,19] while improving the hydrodynamic performance of rectilinear motion [20] compared to a rigid-body architecture [21]. At the same time, flexible-body UVs typically display a complex internal architecture and a high number of moving parts. Bio-inspired, rigid-body UVs, on the other hand, benefit from a reduced number of moving parts that yields high electromechanical efficiency, as well as higher payload capacities compared to flexible-body designs [22,23].



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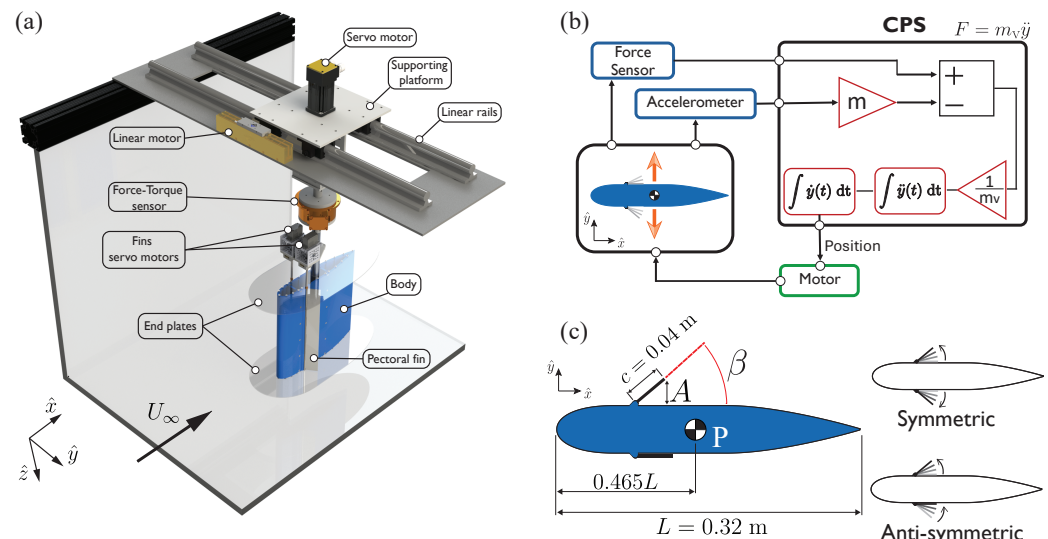
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The trend of reduced maneuverability of rigid-body UVs relative to flexible-body UVs can be addressed through the use of appendages as control surfaces [24–26], particularly at low forward speeds for small radius maneuvers and precise station holding, following a trend observed in nature —stiff-bodied fish tend to possess a greater number of appendages compared to flexible-bodied species [27] that are used for maneuvering. Due to the three-dimensional kinematics and wake structures being revealed for pectoral fins of real fish [28–30], computational simulations have been performed for finite aspect-ratio fins undergoing three-dimensional kinematics [31–33], and efforts in robotics have typically focused on the development of sophisticated bio-inspired mechanisms with rigid and flexible fins [25,34–36]. While greatly informative, it is difficult to generalize the hydrodynamic forces and flow structures from these results due to the high complexity of their fin shape, kinematics and in some cases the fin flexibility.

In an effort to understand the most salient parameters that dictate the force production of a flapping fin near a solid body and its ability to maneuver an underwater vehicle, we perform experiments on an idealized model with a bio-inspired flapping fin capable of a single degree-of-freedom. We use a model system to characterize the dynamics of a rigid underwater vehicle capable of lateral maneuvering and hovering through the use of control surfaces inspired by fish pectoral fins anterior to the vehicle's center of mass. For simplicity, our model is nominally two-dimensional with a fish-like profile and the fins are modeled as a pair of rigid flapping plates, attached to the sides of the vehicle as shown in Figure 1a,c. The flapping plates thus impart momentum to the surrounding flow and move independently from the vehicle's body, generating powered forces [10,11] to maneuver the vehicle. The choice of flapping fin geometry and kinematics is intentionally designed to be simple, yet is relevant to a wide range of bio-inspired control surfaces. Our goals here are to provide a detailed characterization of the forces produced by a single degree-of-freedom flapping plate and demonstrate its efficacy for the maneuvering and station keeping of an underwater vehicle. Our methodology thus contributes to the existing literature by identifying the key non-dimensional parameters associated with streamwise and cross-stream forces produced by such flapping fins. Due to its geometric simplicity, the results obtained from the current study should also be applicable to higher-complexity bio-inspired control surfaces, and should support the design of a broad range of underwater robotic designs.

The system of a thin pitching plate attached to a solid boundary draws a parallel to the canonical problem of oscillating flapping foils. Depending on their kinematics, they can add momentum to the flow, producing a net thrust for propulsion [37,38] or extract momentum from the flow, producing a net drag for energy harvesting [39–41], for which the thrust or drag force production is associated with reverse von-Kàrmàn and von-Kàrmàn wakes, respectively [42]. For the case of self-propelling oscillating foils, the presence of a nearby solid boundary deflects the wake advection trajectory away from the boundary and affects its hydrodynamic performance; under the ground-effect, the thrust is inversely proportional to the square of the distance to the boundary, increasing with smaller distances as the added-mass forces grow due to the presence of the boundary [43,44]. Unlike for those systems, however, the thin flapping plates studied here have their leading edge attached to the boundary, preventing the formation of leading edge vortices typically observed in canonical oscillating foils. Additionally, the pectoral fin motion is not symmetrical relative to the freestream direction. This asymmetry introduces the net lateral forces that will be explored for lateral maneuvering of the vehicle. Flow visualizations from [45,46] show the formation of trailing edge (TE) vortices during the fin's upstroke that eventually break-up during the downstroke. The advection trajectory of the TE vortex is shown to be affected by the fin kinematics. For low Strouhal, the TE vortex advects roughly along the

streamwise direction. As the Strouhal number increases, the TE vortex trajectory becomes oblique, being accelerated in lateral direction. This was associated with the increased lateral force production with increasing flapping frequency and amplitude, and the peak in the lift amplitude throughout the flapping cycle roughly coinciding with the TE vortex shedding event. For  $St \gtrsim 0.4$ , they observed an increased flow “suction” opposite to the streamwise direction in-between the fin and the body that produced vorticity with an opposite sign from the TE vortex that engulfs the TE vortex during the fin’s downstroke as it advects downstream.



**Figure 1.** Experimental setup. (a) Schematics of the experimental apparatus. (b) Cyber–physical system control loop. Orange arrows represent the lateral motion of the vehicle. (c) Left: Schematics of the vehicle profile and dimensions. Maximum fin amplitude  $\beta$  is defined relative to the lateral body line. Right: Symmetric and anti-symmetric modes of two-fin flapping motions.

Here we measure the streamwise and lateral forces produced by a single flapping plate and describe how it scales with the flapping kinematics. We then describe the forces produced by two plates flapping simultaneously for both a symmetric and anti-symmetric motion. Finally, a one-dimensional cyber–physical closed-loop control system [47–50] is employed to allow the vehicle to move freely in the cross-stream direction in response to the lateral forces produced by the flapping plates.

## 2. Materials and Methods

Experiments were carried in a recirculating water channel with test Section 4 m long, 0.8 m wide and 0.6 m deep. Figure 1a shows the experimental setup. The two-dimensional fish-inspired model [46] has a main body (in blue) of length  $L = 0.32$  m and is designed as a modified NACA 0025 airfoil with constant thickness section inserted between  $x/L = 0.19$  and  $0.56$ . End plates are placed at the top and bottom of the body and render the flow nominally two-dimensional, effectively suppressing three-dimensional features from affecting the flow. The “pectoral fins” consist of a pair of rectangular flapping plates of chord length  $c = 0.04$  m and thickness  $t_{\text{fin}} = 3.2$  mm, mounted on each side of the body. The fins pitch around their leading edge, located at  $0.26 L$ . The main body profile was designed to be streamlined, with its constant thickness section to be sufficiently long relative to the fin’s chord length. This is intended to minimize any undesirable effects of the body’s trailing edge onto the flapping fin forces and flow structures, such as adverse pressure gradients due to body tapering or body trailing edge vortices shed during lateral maneuvers. The fins are each independently controlled by a servo motor

with angular resolution of  $\pm 1^\circ$  (BLDC Planetary GearMotor, Anaheim Automation, model BLWRPG173D-24V-4000-R15-1000DN8).

A supporting shaft connects the fish model to a six-axis force and torque sensor (ATI Delta IP65 from ATI Industrial Automation, Apex, NC 27539, USA) that measures the instantaneous forces acting at point  $P = (0.465 L, 0)$  relative to the vehicle's leading edge, as depicted in Figure 1c. The model is mounted to a supporting platform that can move in the cross-stream,  $\hat{y}$ -direction commanded using a linear motor. The model motion is governed by a real-time, one degree of freedom closed-loop cyber-physical control system (CPS) [47] (Figure 1b) which mimics a swimming fish that is free to move in the cross-stream direction in response to the cross-stream hydrodynamic forces, and subject to user-defined *virtual* mass,  $m_v$ . In these experiments the ratio between the virtual mass and the mass of water displaced by the submerged vehicle  $m_f$  is  $m^* = m_v/m_f = 7.9$ . The fluid force acting on the vehicle is defined as  $\mathbf{F}_{\text{fluid}}(t) = \mathbf{F}_{\text{FT}}(t) - m_p a(t)$  where  $\mathbf{F}_{\text{FT}}(t)$  is the cross-stream force measured by the force sensor, and  $a(t)$  is the carriage acceleration measured by the accelerometer. Note that the force measured by the force sensor  $\mathbf{F}_{\text{FT}}$  is affected by the inertia of the carriage, such that the hydrodynamic force acting on the submerged model  $\mathbf{F}_{\text{fluid}}(t)$  is obtained by subtracting the carriage's inertial force  $m_p a(t)$  from  $\mathbf{F}_{\text{FT}}$  as it moves along the lateral degree-of-freedom. The fluid force signal is then divided by a user-specified mass  $m_v$  and integrated twice in time to determine the linear motor target position  $y(t) = \iint (F_{\text{fluid}}(\tau)/m_v) d\tau dt$ . The cyber-physical model thus defines the governing equation  $\mathbf{F}_{\text{fluid}}(\mathbf{t}) - m_v a(t) = 0$  in the cross-stream direction, where the virtual mass  $m_v$  is defined by the experimentalist and is independent from the physical mass of the model  $m_p$  by definition, as the closed-loop control defines the system's governing equation.

In this work, the pectoral fin kinematics are a prescribed sinusoidal pitching motion. For results of a single fin, one fin is flapping while the other is kept retracted close to the body. For the simultaneous flapping of the two fins, their phase synchrony is either symmetric or anti-symmetric as depicted in Figure 1c. For a symmetric motion, both fins extend simultaneously, reaching their highest angular amplitude at the same time and then retracting together. For an anti-symmetric motion, the fins move with a phase delay of half a cycle, i.e., one fin extends while the opposite side fin is retracting. The angle of the right and left fins are  $\theta_R(t) = \beta/2(\sin(2\pi ft) + 1)$  and  $\theta_L(t) = \beta/2(\sin(2\pi ft - \phi) + 1)$ , respectively, where  $f$  is the fin frequency (Hz),  $\beta$  is the maximum fin amplitude measured from the side of the body, and the phase synchrony is  $\phi = \pi$  for a symmetric motion or  $\phi = 0$  for an anti-symmetric motion. The reduced frequency is defined as  $k = fc/U$ , and the Strouhal number is  $St = fA/U$ , where  $A = c \sin \beta$  is the projected frontal area of the fin measured at the peak amplitude of the fin in the cross-stream direction as indicated in Figure 1c. A constant flow speed  $U = 0.2$  m/s with standard deviation of 3.5% of the mean is imposed throughout this study, with the Reynolds number based on the body length  $Re_L = 60 \times 10^3$  and a chord-based Reynolds  $Re_c = 7.6 \times 10^3$ .

A quasi-steady fin regime is also measured for comparison to the unsteady, flapping case defined above. In the quasi-steady regime, the fin moves slowly from its retracted position  $\theta_0 = 0^\circ$  to a maximum angle  $\theta_{\max} = 100^\circ$ , and then back to  $\theta = 0^\circ$  at a constant rate of  $\pi/90(\text{rad/s}) = 2^\circ/\text{s}$ , yielding a reduced frequency  $k = 0.002$ . At such a low frequency, any unsteadiness due to the fin motion is negligible, and the forces measured against the fin angle are treated as quasi-steady.

The non-dimensional forces and  $\hat{z}$ -moment measured at point  $P = (0.465 L, 0)$  of the vehicle are defined as

$$C_{F_x} = \frac{F_x}{q_{\text{dyn}} c}, \quad C_{F_y} = \frac{F_y}{q_{\text{dyn}} c} \quad \text{and} \quad C_{M_z} = \frac{M_z}{q_{\text{dyn}} c^2}, \quad (1)$$

respectively, where  $F_x$  and  $F_y$  are the forces in the streamwise and cross-stream directions, respectively,  $M_z$  is the moment,  $q_{dyn} = 0.5\rho U^2$  is the dynamic pressure based on the freestream speed  $U$ , and  $c$  is the fin cord-length. For two fins flapping simultaneously,  $F_x = F_x^R + F_x^L$ ,  $F_y = F_y^R + F_y^L$  and  $M_z = M_z^R + M_z^L$  are the net forces experienced by the body produced by the pair for a symmetric or anti-symmetric motion where the superscripts  $()^R$  and  $()^L$  denote the right- and left-side fins, respectively. The reported coefficients for forces and moment are averaged over three trials, each calculated from a minimum of 20 measured flapping cycles. The standard error,  $SE = \sigma/\sqrt{N}$ , is computed for all measurements, where  $\sigma$  is the sample standard deviation and  $N$  is the number of trials. It is visualized as shaded regions whenever it exceeds the symbol size. If no shaded region is visible, the standard error  $SE$  is smaller than the plotted symbols. Plots of the phase-averaged signals were constructed using a low-pass, 4th order Butterworth filter with cut-off frequency  $f_c = 4f_{fin}$  and down-sampled for easier visualization of the data, showing every 30th point of the resulting phase-averaged signal. No experimental trials were selectively removed from the dataset in order to influence the resulting scaling reported in the manuscript. As depicted in Figure 1a,b, the streamwise  $\hat{x}$ -axis is positive in the direction and sense of the free stream, thereby defining drag as a positive streamwise force (towards the rear of the model). Conversely, thrust is a negative streamwise force.

### 3. Results

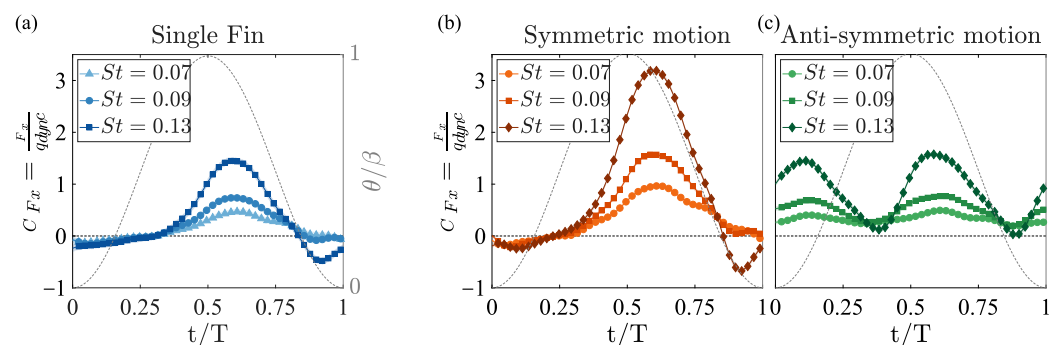
#### 3.1. Characterization of Fin Forces and z-Moment

The streamwise and lateral forces produced by a single fin flapping are measured for a vehicle subject to an oncoming flow  $U = 0.2$  m/s for a fixed flapping frequency  $f = 0.75$  Hz, reduced frequency  $k = 0.15$ , and varying amplitudes  $\beta = [30^\circ, 40^\circ, 60^\circ]$  and associated Strouhal numbers  $St = [0.07, 0.09, 0.13]$ . These parameters are similar to what is observed in real fish. As an example, the *Scarus quoyi* typically oscillates its pectoral fins at reduced frequencies  $0.2 < k < 0.6$  and Strouhal numbers  $0.05 < St < 0.12$  [51]. The phase-averaged, streamwise force coefficient  $C_{F_x}$  produced by a single flapping fin is shown in Figure 2a. A positive streamwise force  $C_{F_x} > 0$  denotes drag, whereas a negative streamwise force  $C_{F_x} < 0$  denotes thrust. A positive force  $C_{F_x} > 0$  is measured for all Strouhal cases during the interval  $0.25 < t/T < 0.75$ , for which the instantaneous angle of the fin  $\theta$  is large. The peak in the force magnitude increases monotonically with Strouhal. During the downstroke motion, a small thrust force is produced by the highest Strouhal case only. This is associated with flow ejected from the region between the fin and the side wall of the vehicle [46,52,53], imparting downstream momentum to the fluid, which produces a force with an upstream component and which is responsible for the thrust force observed at the end of the downstroke motion.

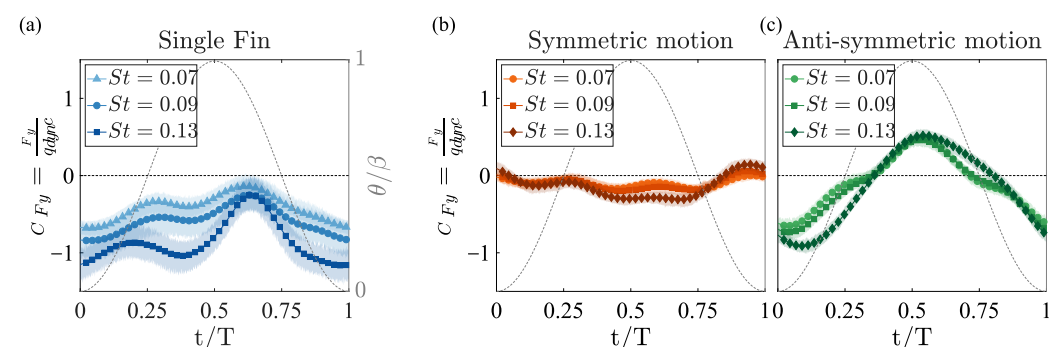
Figures 2b and 2c show the measured streamwise force from the simultaneous flapping of two fins for symmetric and anti-symmetric synchronization, respectively. For a symmetric motion with  $\phi = \pi$ , the up- and downstroke of both fins coincide, leading to the phase-averaged drag force profile to be equal to that of a single fin, but twice as strong. For an anti-symmetric motion  $\phi = 0$ , on the other hand, lower variations in the force are observed when compared to the symmetric motion case as shown in Figure 2c, as the peaks from each fin do not overlap since when one fin is fully open the opposite-side fin is fully retracted and produces close to zero force.

The phase-averaged lateral force produced by the right-side fin is shown in Figure 3a. A negative time-averaged lateral force is observed for all Strouhal numbers, i.e., the force points in the direction opposite to the side on which the flapping fin is located. Increasing the Strouhal number leads to higher amplitude oscillations of the lateral force over one flapping cycle, similarly to what is observed for the streamwise force. Figures 3b and 3c

show the streamwise force for two fins flapping with symmetric ( $\phi = \pi$ ) and anti-symmetric ( $\phi = 0$ ) motion, respectively. For both synchronies, the time-average lateral force is effectively zero, and the instantaneous forces are suppressed throughout the entire flapping cycle for the symmetric motion due to phase cancellation of the left- and right-side fins. Looking at the streamwise (drag) and cross-stream (lift) force profiles for a single flapping fin in Figures 2a and 3a, respectively, it can be observed that, for all three Strouhal numbers, the peak in the time-averaged drag magnitude coincides with the minimum in the lift force.



**Figure 2.** Phase-averaged, streamwise forces produced by (a) a single fin and (b,c) two fins flapping with phase synchronization  $\phi = \pi$  and  $\phi = 0$ , respectively. Fin amplitudes  $[30^\circ, 40^\circ, 60^\circ]$  and frequency are fixed at  $f = 0.75$  Hz and  $k = 0.15$ . The shaded region indicates the standard error interval for a total of three measurements. The gray dashed line shows the fin angle over one cycle  $\theta/\beta$  as indicated by the vertical axis on the right-hand side. Positive values correspond to drag, while negative values correspond to thrust.

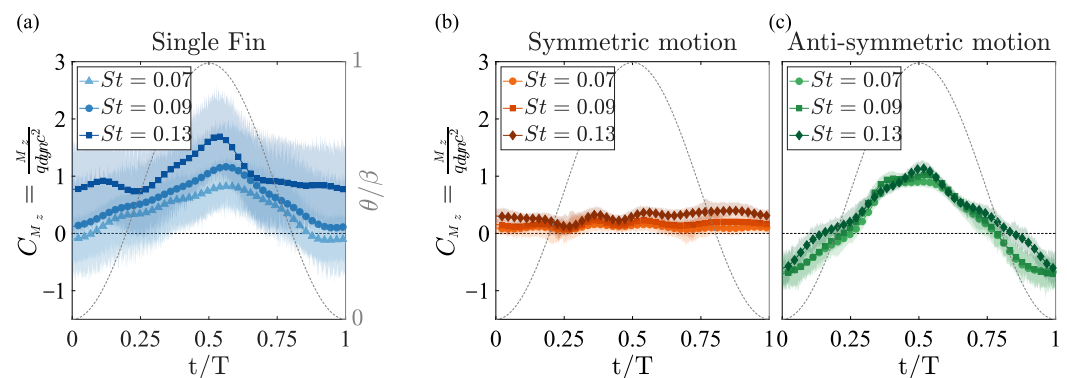


**Figure 3.** Phase-averaged, cross-stream forces produced by (a) a single fin and (b,c) two fins flapping with phase synchronization  $\phi = \pi$  and  $\phi = 0$ , respectively. Fin amplitudes  $\beta = [30^\circ, 40^\circ, 60^\circ]$  and frequency are fixed at  $f = 0.75$  Hz and  $k = 0.15$ . The shaded region indicates the standard error interval (SE) for a total of three measurements. The gray dashed line indicates the fin angle over one cycle  $\theta/\beta$  as indicated by the vertical axis on the right-hand side.

The force data shows how the fin generates forces in the horizontal  $x - y$  plane, and simultaneous flapping of two fins can be leveraged to amplify or suppress the streamwise and cross-stream components of the forces. The symmetric motion  $\phi = \pi$ , for example, effectively eliminates the lateral forces while preserving the profile of the phase-averaged streamwise force generation. It can thus be employed for fast deceleration of the vehicle, for example, without compromising its attitude relative to the swimming direction. An anti-symmetric motion, on the other hand, decreases the intra-cycle fluctuations of the streamwise force component while retaining the oscillatory behavior of the lateral force observed for a single fin. Since the fins are each located at one side of the vehicle, the flow structures produced by each fin do not interact among themselves, and their hydrodynamic forces are independent from one another. For symmetric fin motion, the streamwise forces

produced by the right and left side fins are the same  $F_x^R(t/T) = F_x^L(t/T)$ , while the lateral forces have opposite signs  $F_y^R(t/T) = -F_y^L(t/T)$ .

Figure 4a shows the measured moment coefficient for a single fin. Positive (clockwise) moments with peaks at instant  $t/T \approx 0.56$  are observed for all three frequencies. For the cases with  $St = 0.09$  and  $0.07$ , the moment force starts from zero, monotonically increasing until reaching its peak and then returning to zero at the end of the flapping cycle. For the higher amplitude case (square markers) on the other hand, the moment does not return to zero at the end of each cycle, achieving a minimum value of about 0.8, which is almost as high as the peak moment produced by the lower Strouhal cases (circle and triangle markers), and thus distinctly different. Figures 4b and 4c show the  $\hat{z}$ -moment for symmetric and anti-symmetric motion, respectively. The moment signal profile is suppressed throughout the entire flapping cycle for a symmetric motion, whereas the anti-symmetric case shows a similar behavior to the lift for the same synchronization. These results suggest that the main component that contributes to the  $\hat{z}$ -moment is the cross-stream (lift) component, and not the streamwise (drag) component of the force.



**Figure 4.** Phase-averaged  $\hat{z}$ -moment produced by (a) a single fin and (b,c) two fins flapping with phase synchronization  $\phi = \pi$  and  $\phi = 0$ , respectively. Amplitudes  $\beta = [30^\circ, 40^\circ, 60^\circ]$  and frequency  $f = 0.75$  Hz and  $k = 0.15$ . The shaded region indicates the standard error interval (SE) for a total of three measurements. The gray dashed line indicates the fin angle over one cycle  $\theta/\beta$  as indicated by the vertical axis on the right-hand side of figure (a).

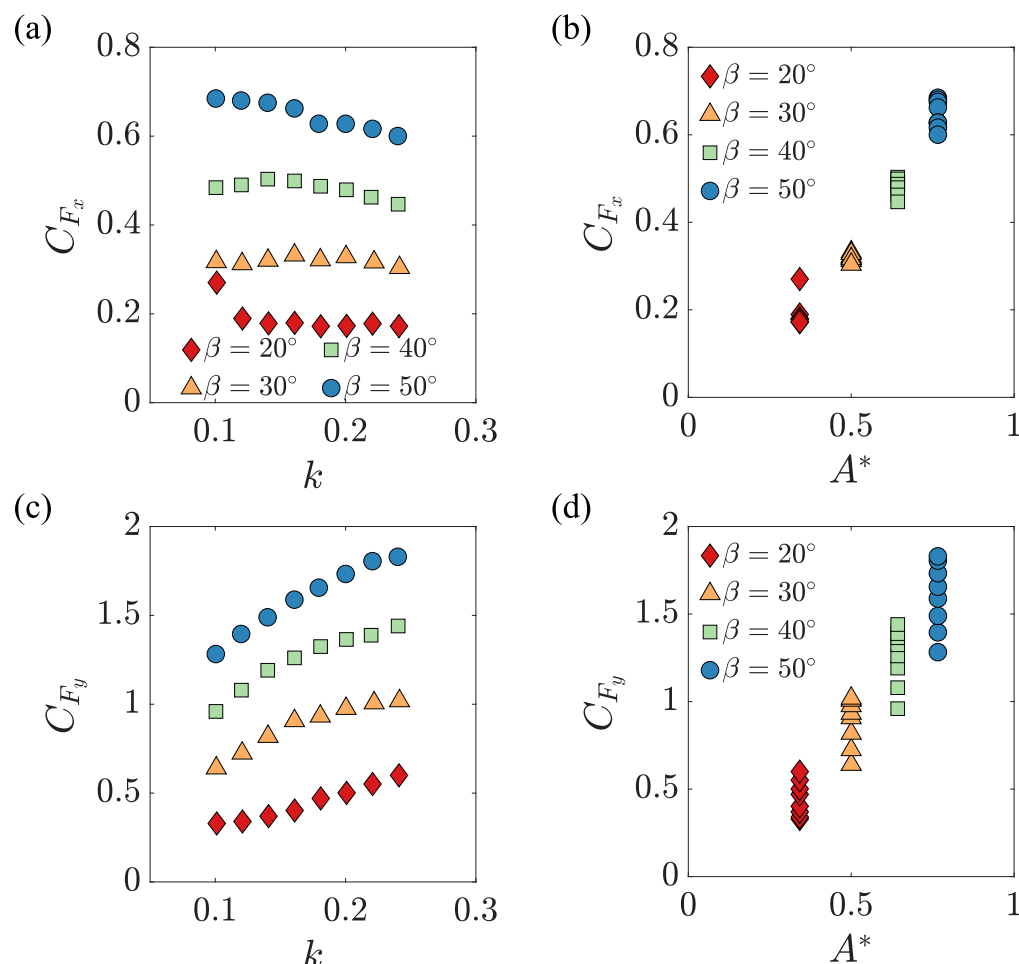
The measured standard error for the cross-stream force (Figure 3) and the  $\hat{z}$ -moment (Figure 4) signals is significantly larger than that of the streamwise force (Figure 2). We attribute this discrepancy to structural vibrations of the experimental apparatus. The streamlined profile of the vehicle leads to a small displacement of fluid in the streamwise direction in response to the oscillatory forces produced by the flapping fins. Conversely, the large projected area of the vehicle in the vertical  $\hat{x} - \hat{z}$  plane displaces a large volume of fluid, resulting in a large added mass force [54] that is captured by the force and torque transducer. That said, the structural oscillations occur at frequencies sufficiently higher than the commanded flapping fin frequencies. As a consequence, we are able to extract the most salient features of the fin forces and moment by evaluating the phase-averaged signals. A detailed evaluation of sample raw signals and their frequency content is reported in Appendix A.

### 3.2. Scaling of the Streamwise and Cross-Stream Forces

Experiments with a single flapping fin are performed for a range of frequencies  $f = [0.5, 0.6, \dots, 1.2]$  Hz and amplitudes  $\beta = [20^\circ, 30^\circ, 40^\circ, 50^\circ]$  at fixed free-stream speed  $U = 0.20$  m/s and chord-based Reynolds number  $Re_c = 7.6 \times 10^3$ .

The time-average, streamwise forces  $C_{Fx}$  are presented in Figures 5a and 5b as a function of reduced frequency  $k = fc/U$  and amplitude  $A^* = \sin \beta$ , respectively. To first

order, the streamwise (drag) force is insensitive to changes in the frequency  $k$  (Figure 5a), with a slight decrease in the force magnitude observed for the two highest amplitudes  $\beta = 40^\circ$  (green squares) and  $50^\circ$  (blue circles). The streamwise force is much more sensitive to the flapping amplitude  $A^* = \sin \beta$ . Figure 5b shows a large variation in the force coefficient for the amplitudes tested, going from an average  $\bar{C}_{F_x} = 0.19$  for  $\beta = 20^\circ$  (red diamonds) to  $\bar{C}_{F_x} = 0.65$  for the highest amplitude  $\beta = 50^\circ$ . The plot shows an approximately linear relationship between the streamwise (drag) force coefficient and the projected frontal area  $A^* = \sin \beta$ .

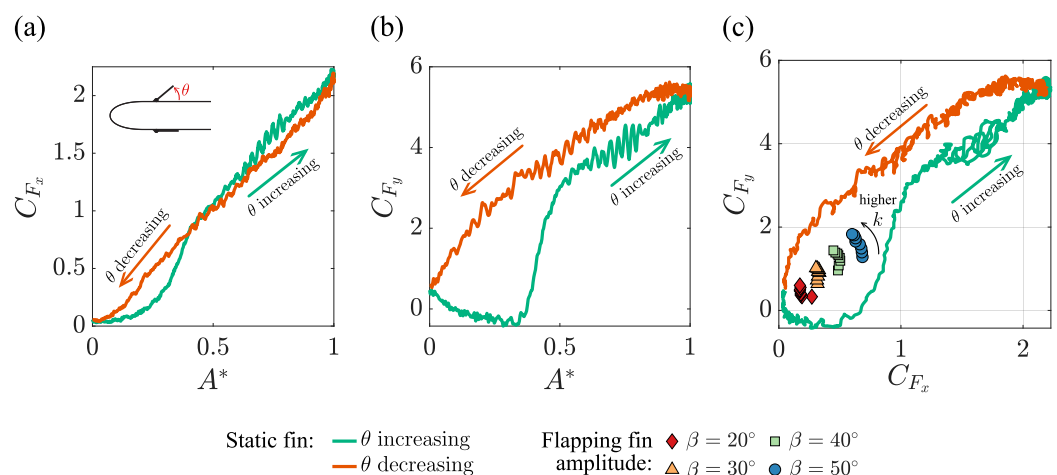


**Figure 5.** Time-average forces produced by a single flapping fin for a stationary vehicle for input parameters  $f = [0.5, 0.6, \dots, 1.2]$  Hz and  $\beta = [20^\circ, 30^\circ, 40^\circ, 50^\circ]$ . (a) Streamwise (drag) force coefficient vs. reduced frequency  $k = fc/U$  for different amplitudes  $\beta$ . The force is primarily a function of the pitching amplitude, with a weak dependence on the frequency. (b) Streamwise (drag) force coefficient vs. non-dimensional amplitude  $A^* = \sin \beta$ . The streamwise (drag) force is approximately a linear function of amplitude, with minor variations for the range of frequencies tested. (c) Cross-stream (lateral) force vs. reduced frequency  $k$  for different amplitudes  $\beta$ . (d) Cross-stream (lateral) force vs. non-dimensional amplitude  $A^*$ .

The streamwise force coefficient  $C_{F_x}$  presented in Figure 5b can be discussed in light of the forces measured for a quasi-steady case, for which the fin is slowly moved from its retracted position  $\theta_0 = 0^\circ$  to a maximum angle  $\theta_{max} = 100^\circ$ , and then back to  $\theta = 0^\circ$ . Due to the extremely small reduced frequency of the experiment  $k = 0.02$ , the unsteady effects from the fin motion are negligible. Figure 6a presents the quasi-steady, streamwise force data as a function of  $A^* = \sin \theta$ . The green line shows the force for an increasing fin angle up to  $\theta_{max}$ , whereas the orange line indicates the measured force for a decreasing

fin angle. The quasi-steady, streamwise (drag) force is a linear function of  $A^*$ , except for the region  $0 \leq A^* < 0.4$ , when the fin is moving away from the body starting from the retracted position  $\theta_0 = 0^\circ$ . This linear behavior indicates that the streamwise force can be interpreted as a pressure-based drag, typically observed for bluff bodies at sufficiently high Reynolds numbers [55]. It is important to note that the definition of  $A^* = \sin \theta$  for the quasi-steady case represents the instantaneous, projected length of the fin in the cross-stream direction, whereas for the flapping fin case  $A^*$  is defined by the maximum fin amplitude  $\beta$  as the fin angle varies  $0 \leq \theta \leq \beta$  throughout the flapping cycle. For that reason, different values of the streamwise force coefficient are expected between the two cases, and neither of the two datasets are universal definitions of the fin's drag coefficient since they represent two different regimes—the unsteady flapping case of Figure 5b and the quasi-steady case of Figure 6. That said, what *can* be inferred from both cases is that, to first order, the streamwise force is proportional to the projected frontal length (or area) of the fin, and is largely insensitive to the flapping frequency for the unsteady case within the range of reduced frequencies and Strouhal numbers studied here.

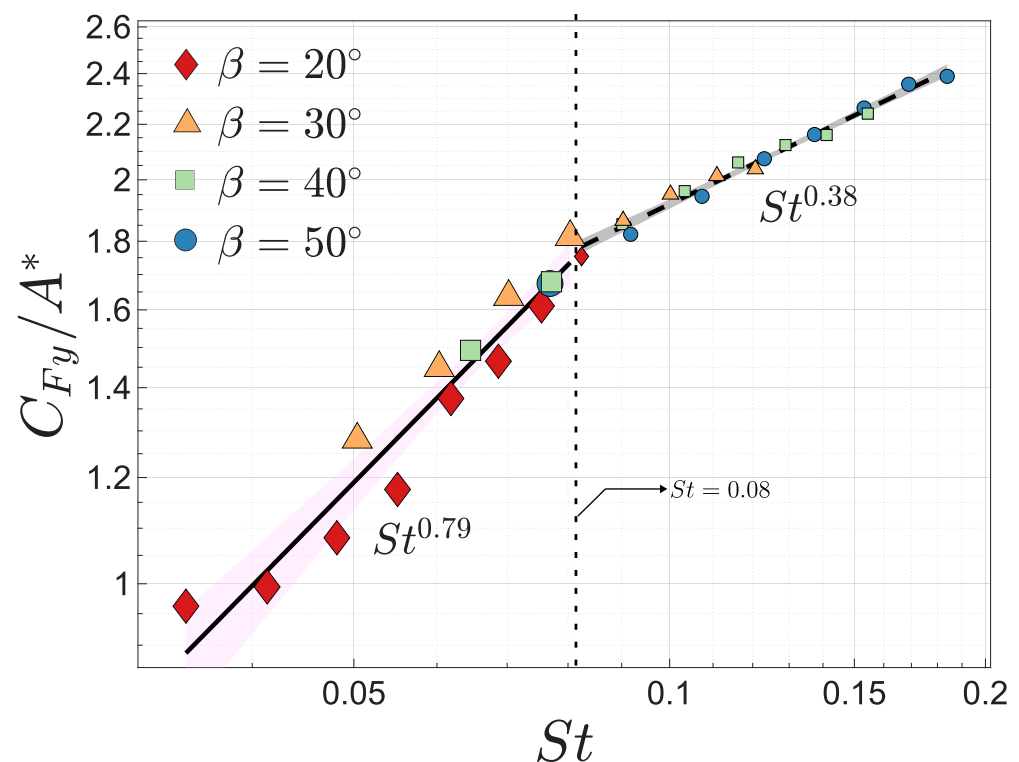
Figure 6b presents the cross-stream force coefficient  $C_{F_y}$  vs.  $A^*$  for the quasi-steady case. During the upstroke, as the fin angle increases from  $\theta_0 = 0$  (green line), the cross-stream force increases abruptly at  $A^* = 0.4$  (or  $\theta = 24^\circ$ ), then increases monotonically afterwards. During the downstroke (orange line), however, the force decreases monotonically for the entire range from  $\theta_{max}$  to  $\theta_0$ . This hysteretic behavior is attributed to the shear layer produced at the fin's leading edge, which, during the upstroke motion starting from  $\theta_0$ , stays continuously attached to the fin and the aft portion of vehicle, only “breaking up” at sufficiently high angles of attack. Once this attached shear layer breaks down, a pronounced vortex shedding occurs at the fin's trailing edge, and the flow is only able to reattach to the aft portion of the body when the fin is fully retracted again. This non-linear flow behavior explains the dramatic differences in the lateral force production between the upstroke (green line) and downstroke (orange line). A detailed description of this phenomenon, along with flow field visualizations, is found on [46].



**Figure 6.** Average forces produced by a quasi-steady fin during an upstroke motion (green line) starting from a retracted position  $\theta_0 = 0^\circ$  to the maximum angle  $\theta_{max} = 100^\circ$ , and a downstroke motion (orange line) back to  $\theta = 0^\circ$ . (a): Streamwise (drag) force vs. amplitude  $A^* = \sin \theta$ .  $C_{F_x}$  is a linear function of  $A^*$ , except at low amplitudes for an upstroke motion (green line). (b): Cross-stream force  $C_{F_y}$  vs. amplitude  $A^*$ . Large hysteresis in the lateral force is caused by non-linear flow attachment during the initial phase of the upstroke motion compared to a fully detached flow during the downstroke. (c)  $C_{F_y}$  vs.  $C_{F_x}$  for the quasi-steady fin (lines) compared to the time-averaged coefficients for a flapping fin (markers). The flapping fin forces seem bounded by the quasi-steady curve, and the lateral to streamwise force ratio increases with the flapping frequency  $k = fc/U$ .

Figure 6c shows the coefficients of cross-stream force vs. streamwise force for both the quasi-steady case (solid lines) and the flapping case (markers). For any given value of streamwise (drag) force  $C_{F_x}$ , the plot clearly shows that the lateral force  $C_{F_y}$  is always larger for the downstroke (orange line) than for the upstroke (green line). This indicates that the flow structures along the fin and body dramatically affect the forces produced by the fin, and the  $C_{F_y}/C_{F_x}$  ratio seems to be higher for the fully detached flow observed by [45,46] during the downstroke (orange line). The forces from a flapping fin are also plotted on Figure 6 for comparison (markers). The plot shows how both forces increase with larger flapping amplitudes  $\beta$ , as previously seen on Figure 5, and also helps visualize that higher flapping frequencies lead to an increase in the  $C_{F_y}/C_{F_x}$  ratio. Interestingly, all flapping cases are bounded by the two lines produced by the quasi-steady fin, suggesting that the maximum achievable  $C_{F_y}/C_{F_x}$  ratio occurs for a static fin when the flow field is fully detached downstream of the fin's trailing edge.

The time-averaged lateral force produced by a single flapping fin varies with the flapping frequency and amplitude. Figure 7 shows the cross-stream force coefficient divided by the projected cross-stream length  $C_{F_y}/A^*$  as a function of the Strouhal number in a log–log plot as colored symbols.



**Figure 7.** Lateral force coefficient divided by the projected cross-stream length  $C_{F_y}/A^*$  as a function of Strouhal number  $St$ . Data points are shown as colored symbols, and lines indicate power laws of the type  $St^\alpha$ . black line. The shaded areas represent the 95% confidence intervals for each linear regression. For the lower Strouhal range  $St < 0.08$ ,  $\alpha = 0.79$ , and for the higher Strouhal range,  $\alpha = 0.38$ , as indicated by the labels next to the lines.

Since the Strouhal number  $St$ , reduced frequency  $k$  and non-dimensional amplitude  $A^*$  have been shown to be key parameters for scaling laws of thrust-producing oscillating plates and foils [38,56], including for foils near a solid boundary [57], we use those same quantities as the building blocks for a data-driven scaling law. The data indicates that the lateral force follows not one, but two power laws of the form  $C_{F_y}/A^* \propto St^\alpha$ , where the value of the exponent  $\alpha$  depends on the range of Strouhal number of the flapping fin. The transition in the lateral force scaling from one regime to another occurs at the crossover Strouhal

number  $St_c = 0.08$ , indicated by the thin vertical dashed line. For  $St < St_c$ , the data scales with  $\alpha_1 = 0.79$  as indicated by the solid black line. For  $St > St_c$ , the data scales with  $\alpha_2 \approx \alpha_1/2 = 0.38$  as indicated by the dashed black line. The shaded areas represent the 95% confidence intervals for each linear regression. In order to determine  $St_c$  and the two proposed power laws, the corrected Akaike information criterion (AICc) for small samples [58] was combined with iterative piecewise linear regression computations. First, the dataset was sorted in ascending order based on the independent variable,  $St$ . Every data point was then evaluated as a potential critical threshold,  $St_c$ . At each iteration, the data was split into two subsets: one containing values below and the other containing values above the trial threshold. Linear regressions were then fitted to both subsets, and the Akaike Information Criterion was calculated for each candidate piecewise regression against a single linear regression of the entire dataset. After testing every possible split point across the dataset, the final critical threshold,  $St_c$ , and its corresponding piecewise model was selected from the configuration that yielded the lowest AIC score, representing the optimal goodness-of-fit regression. A Bayesian information criterion (BIC) [59] test was also performed on the optimal piecewise regression, resulting in a Bayes factor  $2 \ln B_{10} \approx \Delta BIC = 24.4$ , which is substantially higher than the  $2 \ln B_{10} > 10$  threshold proposed by [59], and provides very strong evidence in favor of the obtained piecewise regression. Finally, The 95% confidence intervals for the two linear regressions were computed via bootstrap resampling [60] with 1000 resamples.

The proposed scaling can also be defined in terms of the flapping frequency and amplitude as

$$C_{Fy} \propto A^* St^\alpha \quad (2)$$

$$C_{Fy} \propto (\sin \beta)^{1+\alpha} k^\alpha \quad (3)$$

Equations (2) and (3) are each comprised of two non-dimensional terms. The first term is a characteristic length, expressed as  $A^*$  in Equation (2), and  $\sin \beta$  in Equation (3). It can be interpreted as the projected frontal area of the fin at its maximum angle. The second term in Equation (2) is the Strouhal number  $St = fA/U$ , which is the ratio between two velocities:  $fA$  is a reference velocity of the fin's trailing edge in the cross-stream direction, and  $U$  is the reference velocity in the streamwise direction. In Equation (3), the second term is the reduced frequency  $k = fc/U$ , which is a ratio between two time scales:  $1/f$  is the period of the fin's oscillation, and  $c/U$  is the time it takes for a fluid particle to traverse a distance of one chord-length at the reference velocity  $U$ .

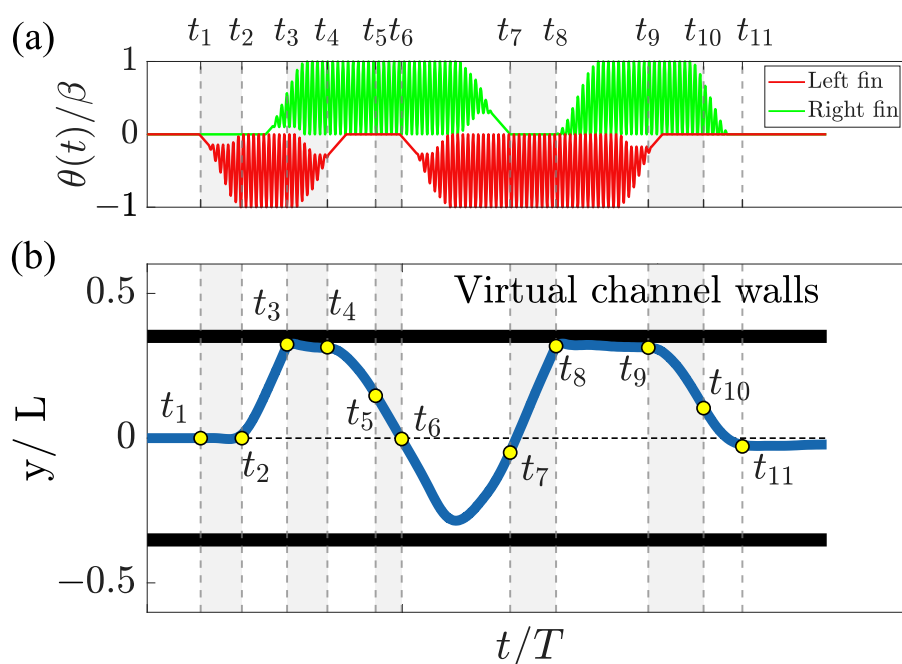
While the proposed scaling defined by Equations (2) and (3) collapses the data well (see Figure 7), it is worth noting that it is not derived from first principles, but rather through a physics-inspired data-driven procedure. That said, the scaling for the streamwise forces (see Figure 5b) and cross-stream forces (see Figure 7 and Equations (2) and (3)) are capable of predicting the fin forces as a function of its kinematics within the range of parameters tested, and have a practical application in the design and control of bio-inspired, rigid control surfaces flapping around their leading edge of underwater vehicles operating at Reynolds numbers of the same order of magnitude. The piecewise scaling law obtained for  $C_{Fy}$  indicates a change in the flapping fin regime, which we hypothesize is due to qualitative changes in the characteristics of the flow field and vortex dynamics shed at the fin's trailing edge [45]. Further flow visualization experimental data should help clarify the two regimes observed in the lateral force data described in Figure 7.

### 3.3. Lateral Maneuvering

Next, we demonstrate how the pectoral fins are used to maneuver the “fish” in a bio-inspired manner.

The cyber–physical system on which the test article is mounted allows the vehicle to translate in the cross-stream direction in response to the lateral forces generated by the fins, while maintaining the streamwise alignment, i.e., at a zero yaw angle.

In this demonstration, we operate the two fins in a Left–Right–Left–Right sequence, as shown in Figure 8a, which shows the angular position of the flapping fins as a function of time. Figure 8b shows the resulting position of the vehicle as a function of time. The cross-stream position of the vehicle is non-dimensionalized by its total length  $L$ . Virtual walls (black lines) are imposed by software to maintain the vehicle at a minimum distance of  $Y/L$  of the water channel walls, thus avoiding any near-boundary effects with the test facility.



**Figure 8.** Lateral maneuvering of the vehicle in response to lateral forces produced by two flapping fins using the cyber–physical system. (a) Angle  $\theta/\beta$  of the left (red) and right (green) fins as a function of time  $t/T$ . Flapping frequency  $f = 1.25$  Hz and amplitude  $\beta = 45^\circ$ . (b) Lateral position of the vehicle in time.

Initially at rest at  $t/T = 0$ , the left fin (red line) starts flapping at  $t_1$ , generating an increasingly positive force  $F_f > 0$ . At  $t/T = t_2$  the fin achieves its total prescribed flapping amplitude and the vehicle’s velocity increases quickly. From  $t_2$  to  $t_3$ , the vehicle moves in the  $+y$ -direction, briefly achieving its terminal velocity  $v_{eq}$  a few flapping cycles before instant  $t_3$ . At the terminal velocity, the vehicle’s acceleration is zero and the net fluid force  $F_f \approx 0$  since the total cross-stream force produced by the fin is balanced by opposing hydrodynamic forces due to its lateral velocity  $v_{eq}/U = 0.064 \pm 0.009$ . At  $t_3$ , the carriage is forced to stop abruptly at the virtual wall (black thick line in Figure 8b). From  $t_3$  to  $t_4$ , the left fin (red line, Figure 8a) is slowly retracted while the right fin (green line) starts flapping with increasing amplitude. At  $t_4$ , the fluid force becomes negative  $F_f < 0$  and the vehicle starts moving in the  $-y$ -direction. From  $t_4$  to  $t_5$ , the vehicle’s lateral speed increases. From  $t_5$  to  $t_6$  the vehicle travels at a roughly constant speed  $v_{eq}$ , as indicated by the straight line segment within those instants in Figure 8b. At instant  $t_6$  the left fin is commanded to flap again, decelerating the vehicle until it stops (without hitting the virtual wall). The vehicle then accelerates in the  $+y$ -direction as the right fin

(green line) is ramped down. From  $t_7$  to  $t_8$  only the left fin is flapping, and the vehicle travels at its terminal velocity again. The vehicle is allowed to hit the virtual wall at  $t_8$ . From  $t_8$  to  $t_9$  the right fin (green line) is commanded while the left fin (red line) is retracted. Once the net fluid force acting on the vehicle becomes negative ( $F_f < 0$ ) at  $t_9$ , the vehicle accelerates in the  $-y$ -direction. The right fin is slowly retracted once more, and at  $t_{10}$  the fin flapping amplitude is too small to overcome the hydrodynamic forces due to the vehicle's instantaneous lateral velocity and the vehicle starts to decelerate. Starting at  $t_{10}$ , as the right fin (green line) retracts the vehicle decelerates further due to the hydrodynamic force caused by its lateral velocity. At  $t_{11}$  the fins are fully retracted and the vehicle's velocity and acceleration are very small. At that point,  $F_f \approx 0$  and  $v \approx 0$ , i.e., the vehicle maintains a constant position until the end of the experiment.

#### 4. Discussion and Conclusions

In this study, flapping plates attached to the sides of an underwater vehicle are used as bio-inspired control surfaces. The forces produced by the fins were characterized as a function of its kinematics for a single fin and for the simultaneous flapping of both fins with symmetric and anti-symmetric synchronizations. A symmetric motion shows an almost perfect suppression of the instantaneous lateral force and  $\hat{z}$ -moment due to phase cancellation, while the instantaneous streamwise force production for two fins is just twice that of a single fin. A symmetric motion is thus suitable for fast deceleration maneuvers in the streamwise direction without disturbances in the lateral direction or yaw angle of a vehicle. Conversely, non-symmetric motions produce non-zero lateral forces and  $\hat{z}$ -moments within one flapping cycle. These forces and moment may be exploited for fine control of a vehicle's yaw angle, as well as to suppress transient flow disturbances that can disturb the vehicle's orientation relative to its traveling direction.

We find that the streamwise and cross-stream forces produced by the flapping fins are strongly dependent on the flapping amplitude and frequency. For all cases studied, the *net* streamwise force is opposite to the vehicle's forward motion, i.e., it is a drag or braking force. The streamwise force is, to first order, a linear function of the projected frontal area of the flapping amplitude  $A^* = \sin \beta$ , being generally insensitive to the flapping frequency. This linear relationship between the streamwise force and the amplitude  $A^*$  is also observed for a quasi-steady fin. These two results support our hypothesis that the streamwise force produced by the fin can be interpreted as a pressure-based drag acting on a bluff body at high Reynolds number. For high flapping amplitudes and Strouhal numbers, a small instantaneous thrust production is observed during the final portion of the flapping cycle. Future studies may be able to reduce the time-average drag force produced by the flapping plate through the use of non-sinusoidal, asymmetric flapping kinematics, for example. Our data shows that the cross-stream forces scale with the amplitude  $A^*$  and also with the Strouhal number  $St$ , unlike the streamwise forces that scale primarily with  $A^*$  alone. We propose a piecewise scaling model for the cross-stream force that collapses the data very well over the entire range of parameters measured, indicating a regime change that is a function of the Strouhal number, with a critical threshold around  $St_c = 0.08$  that qualitatively changes the power scaling coefficient to vary between below and above this threshold. We hypothesize that this deviation occurs due to the non-linear unsteady behavior of the flow over the fin at low angles. For sufficiently low fin angles during the upstroke phase of the motion, Ref. [46] showed that the flow remains attached to the fin and to the lateral of the vehicle instead of producing vortices at the fin's trailing edge. This non-linearity greatly affects the quasi-steady cross-stream forces in the quasi-steady case, and likely also creates the two regimes observed for the lowest Strouhal compared to the higher Strouhal cases.

Finally, we demonstrate how the flapping fins can be used to maneuver an underwater vehicle of large inertial mass using a closed-loop control cyber–physical system. Tests allowed the vehicle subject to an oncoming flow to maneuver in the lateral direction in response to the cross-stream forces produced by the flapping fins. The vehicle was constrained on all other degrees of freedom, providing an assessment of the cross-stream forces produced by the fins decoupled from other dynamic behaviors expected from a fully unconstrained vehicle or robot. The use of simultaneous flapping of the two fins located on opposite sides of the vehicle’s main body allows for a fine control of its lateral position.

In this study, the streamwise and cross-stream forces produced by a flapping fin were characterized for a stationary vehicle only. The scaling of the cross-stream forces as a function of the Strouhal number and frequency of the flapping fin is based on tests at a single flow speed throughout this study. For Reynolds numbers at different orders of magnitude, the dynamics of flow around the fins may change, thus leading to a different scaling of the fin force production. Also, the current experimental setup is not capable of power measurements. For that reason, it was not possible to quantify any of these metrics associated with energy expenditure and efficiency. Additional studies of bio-inspired flapping fins with higher-complexity geometries, multiple degrees-of-freedom and flexibilities will be required to test the current force scaling on other control surface designs found in the literature. Future investigations addressing the impact of the lateral velocity on the force production should help the development of vehicle control strategies, predicting its dynamical behavior based on the velocity and acceleration of the vehicle, and the fin kinematics. Additionally, allowing the vehicle to move in the streamwise and yaw degrees of freedom will provide a complete picture of the capabilities of the flapping plates as bio-inspired control surfaces for underwater vehicles and robots, introducing new challenges to the control strategies required to maneuver a vehicle that is free to move in all six degrees-of-freedom.

**Author Contributions:** Conceptualization, P.C.O. and K.B.; methodology, P.C.O., X.H. and K.B.; software, P.C.O.; validation, P.C.O. and K.B.; formal analysis, P.C.O. and K.B.; investigation, P.C.O., X.H. and K.B.; resources, K.B.; data curation, P.C.O., X.H. and K.B.; writing—original draft preparation, P.C.O.; writing—review and editing, P.C.O., X.H. and K.B.; visualization, P.C.O.; supervision, K.B.; project administration, K.B.; funding acquisition, K.B. All authors have read and agreed to the published version of the manuscript.

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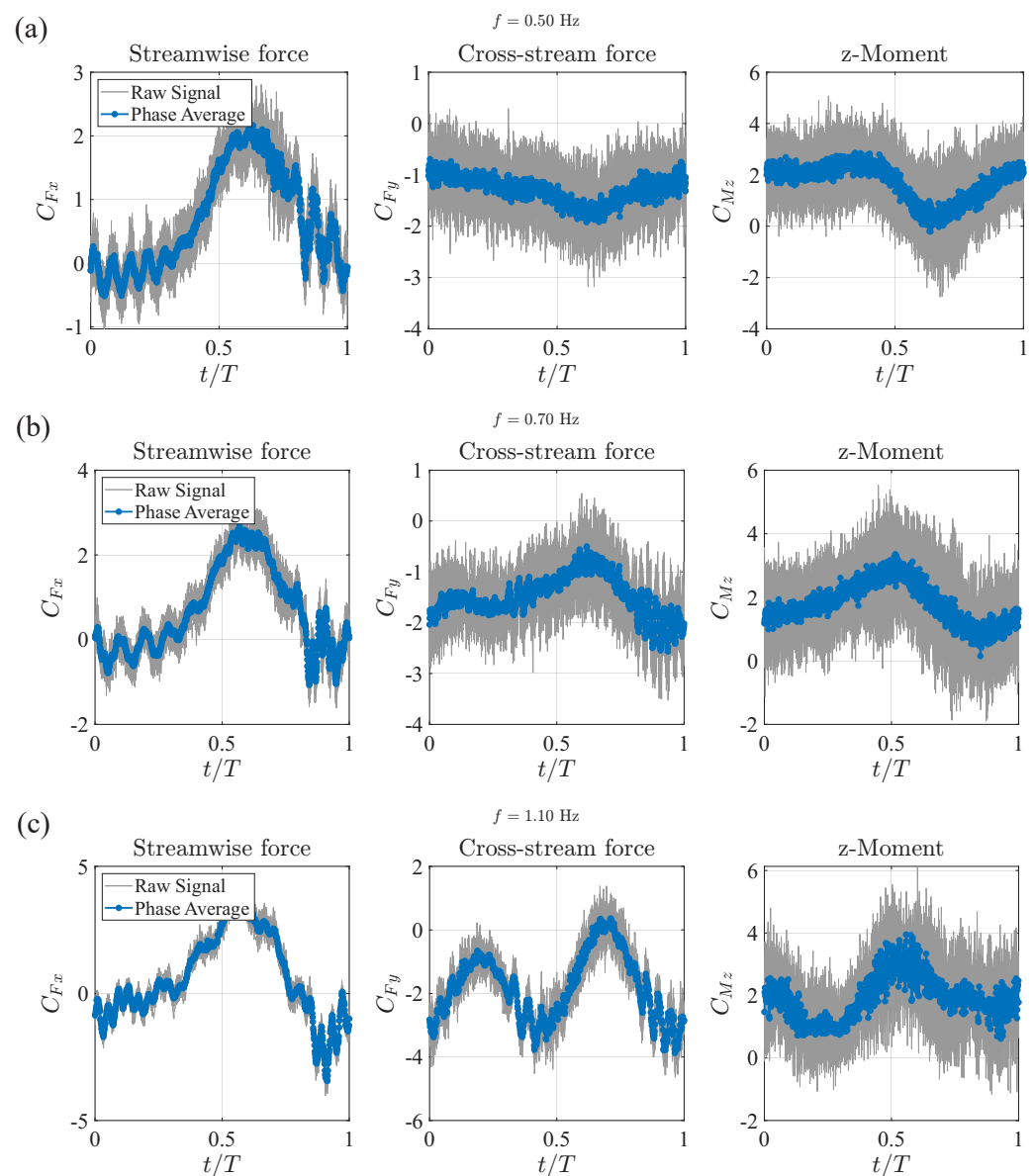
**Data Availability Statement:** The data presented in this study are openly available in Brown Digital Repository at <https://doi.org/10.26300/6mq5-qp27>.

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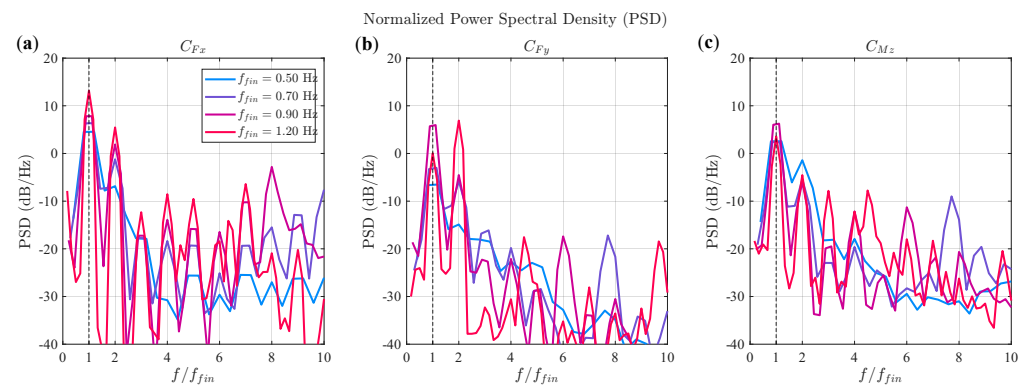
## Appendix A

Sample raw signals and phase-averaged signals of the force coefficients and z-moment produced by a single flapping fin are presented in Figure A1. The data shown was extracted from a single experimental trial comprised of a total of 20 cycles for three different frequencies and peak to peak amplitude  $\beta = 50^\circ$ . Figures A1a, A1b and A1c present the raw data (grey line) and unfiltered phase-averaged signal (blue line) for flapping frequencies  $f = 0.5$ , 0.7 and 1.1 Hz, respectively. It can be observed that, despite the presence of high-frequency

noise on all plots, the main features of the signals observed on Figures 2a, 3a and 4a are also found for the phase-averaged data. Figure A2 shows the power spectrum density of the raw signals collected from a single fin flapping with frequencies  $f = [0.5, 0.7, 0.9, 1.2]$  Hz. The spectra are normalized by their maximum amplitude of the flapping frequency  $f/f_{fin} = 1$ , and computed for (a) the streamwise force coefficient  $C_{Fx}$ , (b) cross-stream force coefficient  $C_{Fy}$  and (c) z-moment coefficient  $C_{Mz}$ . It can be observed that, for all cases, the dominant peak of the PSD occurs at the flapping frequency, with subsequently lower amplitude peaks occurring at multiples of  $f_{fin}$ , indicating that, despite the high-frequency noise observed on the raw data (see Figure A1), any extraneous vibrations due to the experimental apparatus occur at frequencies higher than the hydrodynamic forces experienced by the vehicle.



**Figure A1.** Sample signals of the raw (grey lines) and phase-averaged (blue lines) signals for the streamwise force coefficient  $C_{Fx}$ , cross-stream force coefficient  $C_{Fy}$  and z-Moment coefficient  $C_{Mz}$  for flapping frequencies (a)  $f = 0.5$  Hz, (b)  $f = 0.7$  Hz and (c)  $f = 1.1$  Hz.



**Figure A2.** Power spectral density plots of the (a) streamwise force coefficient  $C_{Fx}$ , (b) cross-stream force coefficient  $C_{Fy}$  and (c) z-Moment coefficient  $C_{Mz}$  from the raw measurements of a single fin flapping with amplitude  $\beta = 50^\circ$  and frequencies  $f = [0.5, 0.7, 0.9, 1.2]$  Hz.

## References

1. Lauder, G.; Drucker, E. Morphology and experimental hydrodynamics of fish fin control surfaces. *IEEE J. Ocean. Eng.* **2004**, *29*, 556–571. [\[CrossRef\]](#)
2. Lauder, G.V. Function of the caudal fin during locomotion in fishes: Kinematics, flow visualization, and evolutionary patterns. *Am. Zool.* **2000**, *40*, 101–122. [\[CrossRef\]](#)
3. Webb, P.W.; Maddock, L.; Bone, Q.; Rayner, J. The biology of fish swimming. In *Mechanics and Physiology of Animal Swimming*; Cambridge University Press: Cambridge, UK, 1994; p. 4562.
4. Drucker, E.G.; Jensen, J.S. Pectoral fin locomotion in the striped surfperch: I. Kinematic effects of swimming speed and body size. *J. Exp. Biol.* **1996**, *199*, 2235–2242. [\[CrossRef\]](#) [\[PubMed\]](#)
5. Lauder, G.V.; Jayne, B.C. Pectoral fin locomotion in fishes: Testing drag-based models using three-dimensional kinematics. *Am. Zool.* **1996**, *36*, 567–581. [\[CrossRef\]](#)
6. Webb, P. Kinematics of pectoral fin propulsion in *Cymatogaster aggregata*. *J. Exp. Biol.* **1973**, *59*, 697–710. [\[CrossRef\]](#)
7. Westneat, M.W. Functional morphology of aquatic flight in fishes: Kinematics, electromyography, and mechanical modeling of labriform locomotion. *Am. Zool.* **1996**, *36*, 582–598. [\[CrossRef\]](#)
8. Walker, J.A.; Westneat, M.W. Labriform Propulsion in Fishes: Kinematics of Flapping Aquatic Flight in the Bird Wrasse *Gomphosus varius* (Labridae). *J. Exp. Biol.* **1997**, *200*, 1549–1569. [\[CrossRef\]](#) [\[PubMed\]](#)
9. Arnold, G.; Webb, P.W.; Holford, B. The role of the pectoral fins in station-holding of Atlantic salmon parr (*Salmo salar* L.). *J. Exp. Biol.* **1991**, *156*, 625–629. [\[CrossRef\]](#)
10. Webb, P. Designs for stability and maneuverability in aquatic vertebrates: What can we learn. In *Proceedings of the Tenth International Symposium on Unmanned Untethered Submersible Technology: Proceedings of the Special Session on Bio-Engineering Research Related to Autonomous Underwater Vehicles*, Durham, NH, USA, 1–3 September 1997; Citeseer: Princeton, NJ, USA, 1997; pp. 86–103.
11. Webb, P.W. Kinematics of plaice, *Pleuronectes platessa*, and cod, *Gadus morhua*, swimming near the bottom. *J. Exp. Biol.* **2002**, *205*, 2125–2134. [\[CrossRef\]](#) [\[PubMed\]](#)
12. Wilga, C.D.; Lauder, G.V. Three-Dimensional Kinematics and Wake Structure of the Pectoral Fins During Locomotion in Leopard Sharks *Triakis semifasciata*. *J. Exp. Biol.* **2000**, *203*, 2261–2278. [\[CrossRef\]](#) [\[PubMed\]](#)
13. Liao, J.C. A review of fish swimming mechanics and behaviour in altered flows. *Philos. Trans. R. Soc. B Biol. Sci.* **2007**, *362*, 1973–1993. [\[CrossRef\]](#) [\[PubMed\]](#)
14. Webb, P.W. Entrainment by River Chub *Nocomis biguttatus* and Smallmouth Bass *Micropterus dolomieu* on Cylinders. *J. Exp. Biol.* **1998**, *201*, 2403–2412. [\[CrossRef\]](#) [\[PubMed\]](#)
15. Pavlov, D.; Kasumyan, A. Patterns and mechanisms of schooling behavior in fish: A review. *J. Ichthyol.* **2000**, *40*, S163.
16. Bartol, I.K.; Gharib, M.; Webb, P.W.; Weihs, D.; Gordon, M.S. Body-induced vortical flows: A common mechanism for self-corrective trimming control in boxfishes. *J. Exp. Biol.* **2005**, *208*, 327–344. [\[CrossRef\]](#) [\[PubMed\]](#)
17. Drucker, E.G.; Lauder, G.V. Function of pectoral fins in rainbow trout: Behavioral repertoire and hydrodynamic forces. *J. Exp. Biol.* **2003**, *206*, 813–826. [\[CrossRef\]](#) [\[PubMed\]](#)
18. Ay, M.; Korkmaz, D.; Ozmen Koca, G.; Bal, C.; Akpolat, Z.H.; Bingol, M.C. Mechatronic Design and Manufacturing of the Intelligent Robotic Fish for Bio-Inspired Swimming Modes. *Electronics* **2018**, *7*, 118. [\[CrossRef\]](#)
19. Hu, H.; Liu, J.; Dukes, I.; Francis, G. Design of 3D Swim Patterns for Autonomous Robotic Fish. In *Proceedings of the 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Beijing, China, 9–15 October 2006; pp. 2406–2411. [\[CrossRef\]](#)

20. White, C.H.; Lauder, G.V.; Bart-Smith, H. Tunabot Flex: A tuna-inspired robot with body flexibility improves high-performance swimming. *Bioinspir. Biomim.* **2021**, *16*, 026019. [[CrossRef](#)] [[PubMed](#)]
21. Zhu, J.; White, C.; Wainwright, D.K.; Di Santo, V.; Lauder, G.V.; Bart-Smith, H. Tuna robotics: A high-frequency experimental platform exploring the performance space of swimming fishes. *Sci. Robot.* **2019**, *4*, eaax4615. [[CrossRef](#)] [[PubMed](#)]
22. Liang, J.; Wang, T.; Wen, L. Development of a two-joint robotic fish for real-world exploration. *J. Field Robot.* **2011**, *28*, 70–79. [[CrossRef](#)]
23. Ko, H.; Saro-Cortes, V.; Mmari, B.; Ni, D.; Wissa, A.; Nagpal, R. BlueGuppy: Tunable kinematics enables maneuverability in a minimalist fish-like robot. *Bioinspir. Biomim.* **2025**, *20*, 056006. [[CrossRef](#)]
24. Bandyopadhyay, P.R. Maneuvering hydrodynamics of fish and small underwater vehicles. *Integr. Comp. Biol.* **2002**, *42*, 102–117. [[CrossRef](#)] [[PubMed](#)]
25. Kato, N.; Inaba, T. Control performance of fish robot with pectoral fins in horizontal plane. In Proceedings of the 1998 International Symposium on Underwater Technology, Tokyo, Japan, 15–17 April 1998; IEEE: Piscataway, NJ, USA, 1998; pp. 357–362.
26. Berlinger, F.; Dusek, J.; Gauci, M.; Nagpal, R. Robust maneuverability of a miniature, low-cost underwater robot using multiple fin actuation. *IEEE Robot. Autom. Lett.* **2017**, *3*, 140–147.
27. Webb, P.W. Stability and Maneuverability. In *Fish Biomechanics; Fish Physiology*; Academic Press: Cambridge, MA, USA, 2005; Volume 23, pp. 281–332. [[CrossRef](#)]
28. Drucker, E.G.; Lauder, G.V. Experimental hydrodynamics of fish locomotion: Functional insights from wake visualization. *Integr. Comp. Biol.* **2002**, *42*, 243–257. [[CrossRef](#)] [[PubMed](#)]
29. Drucker, E.G.; Lauder, G.V. Wake dynamics and locomotor function in fishes: Interpreting evolutionary patterns in pectoral fin design. *Integr. Comp. Biol.* **2002**, *42*, 997–1008. [[CrossRef](#)] [[PubMed](#)]
30. Lauder, G.V.; Madden, P.G.; Mittal, R.; Dong, H.; Bozkurtas, M. Locomotion with flexible propulsors: I. Experimental analysis of pectoral fin swimming in sunfish. *Bioinspir. Biomim.* **2006**, *1*, S25–S34. [[CrossRef](#)]
31. Beal, D.N.; Bandyopadhyay, P.R. A harmonic model of hydrodynamic forces produced by a flapping fin. *Exp. Fluids* **2007**, *43*, 675–682. [[CrossRef](#)]
32. Bozkurtas, M.; Mittal, R.; Dong, H.; Lauder, G.V.; Madden, P. Low-dimensional models and performance scaling of a highly deformable fish pectoral fin. *J. Fluid Mech.* **2009**, *631*, 311–342. [[CrossRef](#)]
33. Dong, H.; Bozkurtas, M.; Mittal, R.; Madden, P.; Lauder, G. Computational modelling and analysis of the hydrodynamics of a highly deformable fish pectoral fin. *J. Fluid Mech.* **2010**, *645*, 345–373. [[CrossRef](#)]
34. Kato, N. Control performance in the horizontal plane of a fish robot with mechanical pectoral fins. *IEEE J. Ocean. Eng.* **2000**, *25*, 121–129. [[CrossRef](#)]
35. Suzuki, H.; Kato, N.; Suzumori, K. Load characteristics of mechanical pectoral fin. *Exp. Fluids* **2008**, *44*, 759–771.
36. Tangorra, J.L.; Lauder, G.V.; Hunter, I.W.; Mittal, R.; Madden, P.G.A.; Bozkurtas, M. The effect of fin ray flexural rigidity on the propulsive forces generated by a biorobotic fish pectoral fin. *J. Exp. Biol.* **2010**, *213*, 4043–4054. [[CrossRef](#)] [[PubMed](#)]
37. Floryan, D.; Van Buren, T.; Rowley, C.W.; Smits, A.J. Scaling the propulsive performance of heaving and pitching foils. *J. Fluid Mech.* **2017**, *822*, 386–397. [[CrossRef](#)]
38. Moored, K.W.; Quinn, D.B. Inviscid scaling laws of a self-propelled pitching airfoil. *AIAA J.* **2019**, *57*, 3686–3700. [[CrossRef](#)]
39. Handy-Cardenas, E.E.; Zhu, Y.; Breuer, K.S. Optimal kinematics for energy harvesting using favourable wake–foil interactions in tandem oscillating hydrofoils. *J. Fluid Mech.* **2025**, *1012*, A23. [[CrossRef](#)]
40. Ribeiro, B.L.R.; Su, Y.; Guillaumin, Q.; Breuer, K.S.; Franck, J.A. Wake-foil interactions and energy harvesting efficiency in tandem oscillating foils. *Phys. Rev. Fluids* **2021**, *6*, 074703. [[CrossRef](#)]
41. Kim, D.; Strom, B.; Mandre, S.; Breuer, K. Energy harvesting performance and flow structure of an oscillating hydrofoil with finite span. *J. Fluids Struct.* **2017**, *70*, 314–326. [[CrossRef](#)]
42. Schnipper, T.; Andersen, A.; Bohr, T. Vortex wakes of a flapping foil. *J. Fluid Mech.* **2009**, *633*, 411–423. [[CrossRef](#)]
43. Mivehchi, A.; Zhong, Q.; Kurt, M.; Quinn, D.B.; Moored, K.W. Scaling laws for the propulsive performance of a purely pitching foil in ground effect. *J. Fluid Mech.* **2021**, *919*, R1. [[CrossRef](#)]
44. Han, T.; Zhong, Q.; Mivehchi, A.; Quinn, D.B.; Moored, K.W. Revealing the mechanism and scaling laws behind equilibrium altitudes of near-ground pitching hydrofoils. *arXiv* **2023**, arXiv:2304.14562.
45. He, X.; Breuer, K. Hydrodynamic loads and vortex evolution from a bio-inspired pectoral fin near a solid body. *arXiv* **2026**, arXiv:2604.21054.
46. He, X.; Breuer, K. Hydrodynamic loads and vortex evolution from a flapping fin near a solid body. *Bull. Am. Phys. Soc.* **2023**, *68*, T14.00001.
47. Erickson, E.; Handy-Cardenas, E.E.; Newbolt, J.W.; Murphy, C.; Breuer, K. Vibrissa inspired geometries enhance sensitivity of wake-induced vibrations. *arXiv* **2026**, arXiv:2603.22556.
48. Zhu, Y.; Mathai, V.; Breuer, K. Nonlinear fluid damping of elastically mounted pitching wings in quiescent water. *J. Fluid Mech.* **2021**, *923*, R2. [[CrossRef](#)]

49. Onoue, K.; Song, A.; Strom, B.; Breuer, K.S. Large amplitude flow-induced oscillations and energy harvesting using a cyber-physical pitching plate. *J. Fluids Struct.* **2015**, *55*, 262–275. [[CrossRef](#)]
50. Hover, F.S.; Techet, A.H.; Triantafyllou, M.S. Forces on oscillating uniform and tapered cylinders in cross flow. *J. Fluid Mech.* **1998**, *363*, 97–114. [[CrossRef](#)]
51. Aiello, B.R.; Olsen, A.M.; Mathis, C.E.; Westneat, M.W.; Hale, M.E. Pectoral fin kinematics and motor patterns are shaped by fin ray mechanosensation during steady swimming in *Scarus quoyi*. *J. Exp. Biol.* **2020**, *223*, jeb211466. [[CrossRef](#)] [[PubMed](#)]
52. Drucker, E.G.; Lauder, G.V. A hydrodynamic analysis of fish swimming speed: Wake structure and locomotor force in slow and fast labriform swimmers. *J. Exp. Biol.* **2000**, *203*, 2379–2393. [[CrossRef](#)] [[PubMed](#)]
53. Ormonde, P.; Handy-Cardenas, E.; Erickson, E.; He, X.; Breuer, K. Positional control of an underwater vehicle using pectoral fins. In *APS Division of Fluid Dynamics Meeting Abstracts*; American Physical Society: College Park, MD, USA, 2024; p. C09.010.
54. Brennen, C.E. *A Review of Added Mass and Fluid Inertial Forces*; Report CEL-CR-82.010; Defense Technical Information Center: Fort Belvoir, VA, USA, 1982.
55. Roshko, A. On the wake and drag of bluff bodies. *J. Aeronaut. Sci.* **1955**, *22*, 124–132. [[CrossRef](#)]
56. Smits, A.J. Undulatory and oscillatory swimming. *J. Fluid Mech.* **2019**, *874*, P1. [[CrossRef](#)]
57. Quinn, D.B.; Moored, K.W.; Dewey, P.A.; Smits, A.J. Unsteady propulsion near a solid boundary. *J. Fluid Mech.* **2014**, *742*, 152–170. [[CrossRef](#)]
58. Stoica, P.; Selen, Y. Model-order selection: A review of information criterion rules. *IEEE Signal Process. Mag.* **2004**, *21*, 36–47. [[CrossRef](#)]
59. Kass, R.E.; Raftery, A.E. Bayes Factors. *J. Am. Stat. Assoc.* **1995**, *90*, 773–795. [[CrossRef](#)]
60. James, G.; Witten, D.; Hastie, T.; Tibshirani, R.; Tibshirani, R. *An Introduction to Statistical Learning: With Applications in R*; Springer: Berlin/Heidelberg, Germany, 2013; Volume 103.

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