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A New Proposal for the Shear Strength Prediction of Beams Longitudinally Reinforced with Fiber-Reinforced Polymer Bars

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Abstract: This paper investigates composite reinforcement with regard to its use as longitudinal reinforcement. The methods used to calculate the shear strength of concrete members reinforced with fibre-reinforced polymer (FRP) bars are analysed. The main parameters having a bearing on the shear strength of beams reinforced with composite bars are defined. A comparative analysis of the shear strength calculating algorithms provided in the available design recommendations concerning FRP reinforcement and formulas derived by others researchers is carried out. A synthesis of the research to date on sheared concrete members reinforced longitudinally with FRP bars is made. The results of the studies relating to shear strength are compared with the theoretical results yielded by the considered algorithms. A new approach for estimating the shear capacity of support zones reinforced longitudinally with FRP bars without shear reinforcement was proposed and verified. A satisfactory level of model fit was obtained—the best among the available proposals. Taking into account the extended base of destructive testing results, the estimation of the shear strength in accordance with the proposed model can be used as an accompanying (non-destructive) method for the empirical determination of shear resistance of longitudinally reinforced FRP bars.

Keywords: FRP reinforcement; shear; capacity; reinforced concrete beams; analytical method

1. Introduction

Steel-reinforced concrete members exposed to extremely adverse environmental conditions relatively quickly fail to meet the facility use requirements concerning durability and reliability [1–4]. A structural member's corrosion resistance (among other things) can be increased by applying non-metallic glass fibre-reinforced polymer (GFRP), carbon-fibre-reinforced polymer (CFRP), basalt fibre-reinforced polymer (BFRP) or aramid fibre-reinforced polymer (AFRP) reinforcements to it [1]. Fibre-reinforced polymer (FRP) bars are characterized by good mechanical (high tensile strength) and physical (density much lower than that of reinforcing steel) properties [1,2,5]. FRP rebars have been applied to structural members incorporated in structures highly exposed to an aggressive environment and in facilities whose proper operation is contingent on the electromagnetic neutrality of, among other things, its structural members [5]. Since FRP rebars are electromagnetically neutral, they are used in facilities requiring particularly high operating precision (and so no disturbance to equipment operation) and in infrastructure facilities (no corrosion causing stray currents).

The shear failure of steel-reinforced concrete (steel-RC) beams without shear reinforcement is often abrupt and has a brittle character. Therefore, the design shear capacity of a structural member should be sufficient to ensure the transfer of interactions not weaker than the ones corresponding to the design flexural capacity. The load capacity loss due to bending is indicated by a large increment in deflections, whereby the failure is much less sudden. The situation is different in the case of FRP reinforcement, which is characterized by a linear-elastic behaviour in the whole strength range until failure [6]. Since composite rebars do not exhibit a plastic behavior, they break rapidly without any warning, which is their drawback. Nevertheless, mainly owing to their corrosion resistance and electromagnetic neutrality, FRP rebars have found application as reinforcement for members in flexure [1]. In the case of both bending and shearing, the failure of beams reinforced with composite rebars is much more rapid than that of steel-RC beams [7]. Therefore, it is important to design flexural members reinforced with composite rebars with regard to both bending and shearing.

This paper presents composite bar reinforcement design recommendations from the available codes and formulas derived by other researchers. The available design recommendations concerning the shear strength of beams without shear reinforcement are verified on the basis of the experimental studies to date of support zones reinforced with FRP rebars. Moreover, the paper presents a proposal for a new approach for estimating the shear capacity of support zones reinforced longitudinally with FRP bars without shear reinforcement.

2. Available Models and Design Recommendations

The very good properties of composite bars provided an incentive to investigate the possibility of using polymer reinforcement as the primary reinforcement in concrete members [1]. The recommendations concerning composite reinforcement are modifications of the standards for designing steel-RC units, which are mostly based on the semiprobabilistic method of limit states. Among the design recommendations concerning concrete elements reinforced with FRP bars one can distinguish: American ACI 440.1R-15 [5], Canadian CSA-S806-12 [8], Japanese JSCE 1997 [9] and Italian CNR-DT 203/2006 [10]. The chronology of design recommendations concerning concrete members reinforced with FRP bars is presented by Bywalski et al. [1] and Drzazga [11]. Recommendations [5,8–10] include information on: the experimental strength characteristics of FRP rebars, the available rebar diameters, the available types of reinforcement, the mechanical and physical properties of FRP reinforcement, etc. Moreover, all the recommendations concerning composite bar reinforcement introduce safety factors (for the member and/or the material) appropriately higher than the ones specified in the standards for designing RC members.

As a result of experimental research in the field of beams reinforced longitudinally with composite bars, many authors gave their own proposals for estimating the shear capacity of such elements. Table 1 presents the algorithms for estimating shear capacity of beams without transverse reinforcement. The algorithms are valid if no axial force is present. Similarly, as in the standards for designing steel-RC members, in the available models and design recommendations concerning composite bars, the shear strength of members without shear reinforcement is determined mainly from empirical formulas. This is due to the high complexity of the shear transfer mechanisms, the different types of failure and the internally interdependent forces in the beam. Kosior-Kazberuk [12] indicated that shear strength (V_c) consists of: the strength resulting from aggregate interlock, the shear strength of the concrete in the compression zone, the dowel action of the longitudinal reinforcement and residual tensile strength of concrete across the crack. The stiffness of composite rebars is much lower than that of steel reinforcement. In comparison with a steel-RC member, after cracking, the distance from the compressed fibres to the neutral axis in a concrete member longitudinally reinforced with FRP bars is smaller (the compression region of the cross section is reduced). This is due to the lower axial stiffness of FRP reinforcement. Since the compression zone extent is smaller, the shear strength of the concrete in the compression zone is also smaller [5,7]. Moreover, the crack width is larger in the case of FRP reinforcement. Hence, the component associated with aggregate interlock is smaller [5,7]. The low transverse stiffness of FRP rebars significantly reduces the component stemming from dowel action [5,7]. As a result, at the same longitudinal reinforcement area, the concrete member reinforced with FRP bars has a lower shear strength than the corresponding steel-RC member [13,14].

Table 1. Algorithms for determining the shear resistance of the shear zones longitudinally reinforced with fibre-reinforced polymer (FRP) bars.

Algorithm	Shear Strength of Member without Shear Reinforcement	
Tottori et al. [15]	$V_c = 0.2\left(100\rho_f f_c \frac{E_{fl}}{E_s}\right)^{1/3}\left(\frac{d}{1000}\right)^{-1/4}\left(0.75 + \frac{1.4}{\frac{a}{d}}\right)b_w d$	(1)
JSCE-97 [9]	$V_c = \beta_d \beta_p \beta_n f_{vcd} b_w d$ $\beta_d = \sqrt[4]{\frac{1}{d}} \le 1.5$; d in (m); $\beta_p = \sqrt[3]{100\rho_f \frac{E_{fl}}{E_s}} \le 1.5$; $\beta_n = 1.0$ when there is no axial force; $f_{vcd} = 0.2\sqrt[3]{f_c} \le 0.72$ MPa	(2)
Michaluk et al. [16]	$V_c = \frac{E_{fl}}{E_s}\left(\frac{1}{6}\sqrt{f_c} b_w d\right)$	(3)
Deitz et al. [17]	$V_c = 3\frac{E_{fl}}{E_s}\left(\frac{1}{6}\sqrt{f_c} b_w d\right)$	(4)
El-Sayed et al. [18]	$V_c = \left(\frac{\mathit{TheNetherlands}\rho_f E_{fl}}{90\beta_1 f_c}\right)^{1/3}\left(\frac{\sqrt{f_c} b_w d}{6}\right) \le \frac{\sqrt{f_c} b_w d}{6}$ $\beta_1 = \begin{cases} 0.85 & \text{for} \quad f_c \le 28 \text{ Mpa} \\ 0.85 - 0.05\frac{f_c - 28}{7} & \text{for} \quad f_c = 28 \div 56 \text{ Mpa} \\ 0.65 & \text{for} \quad f_c \ge 56 \text{ Mpa} \end{cases}$	(5)
Wegian et al. [19]	$V_c = 2\left(f_c \frac{\mathit{TheNetherlands}\rho_f E_{fl}}{E_s}\frac{d}{a}\right)^{1/3} b_w d$	(6)
CNR DT 203/2006 [10]	$V_c = 1.3\left(\frac{E_{fl}}{E_s}\right)^{\frac{1}{2}}\tau_{Rd} k_d (1.2 + 40\rho_f) b_w d \le V_{Rd,max} = 0.5\upsilon_1 f_c b_w \cdot 0.9d$ $\tau_{Rd} = 0.25 f_{ct}$; $k_d = 1.6 - d[m] \ge 1.0$; $1.3\left(\frac{E_{fl}}{E_s}\right)^{1/2} \le 1.0$ $\upsilon_1 = \begin{cases} 0.6 & \text{for} \quad f_c \le 60 \text{ Mpa} \\ 0.9 - \frac{f_c}{200} \ge 0.5 & \text{for} \quad f_c > 60 \text{ Mpa} \end{cases}$	(7)
Nehdi et al. [20]	$V_c = 2.1\left(\frac{f_c \mathit{TheNetherlands}\rho_f d}{a}\frac{E_{fl}}{E_s}\right)^{0.3} b_w d \cdot \frac{2.5d}{a}$ $\frac{2.5d}{a} \ge 1.0$	(8)
Hoult et al. [21]	$V_c = \beta\sqrt{f_c} b_w \cdot 0.9d$ $\beta = \frac{0.3}{0.5 + (1000\varepsilon_x + 0.15)^{0.7}}\frac{1300}{1000 + s_{ze}}$ $\varepsilon_x = \frac{\frac{M}{0.9d} + V}{2E_{fl}A_f}$; $s_{ze} = \frac{31.5d}{16 + a_g} \ge 0.77d$; $a_g = \begin{cases} a_g & \text{for} \quad f_c < 60 \text{ Mpa} \\ a_g - \frac{a_g}{10}(f_c - 60) & \text{for} \quad 60 \le f_c < 70 \text{ Mpa} \\ 0 & \text{for} \quad f_c \ge 70 \text{ Mpa} \end{cases}$	(9)
Razaqpur et al. [22]	$V_c = 0.045 k_m k_a k_r \sqrt[3]{f_c} b_w d$ $k_m = \left(\frac{Vd}{M}\right)^{1/2}$; $k_r = 1 + (\mathit{TheNetherlands}\rho_f E_{fl})^{1/3}$; $k_a = \begin{cases} 1.0 & \text{for} \quad \frac{M}{Vd} \ge 2.5 \\ \frac{2.5Vd}{M} & \text{for} \quad \frac{M}{Vd} < 2.5 \end{cases}$	(10)
Alam [23]	$V_c = \frac{0.2\lambda}{\left(\frac{a}{d}\right)^{2/3}}\left(\frac{\mathit{TheNetherlands}\rho_f E_{fl}}{d}\right)^{1/3}\sqrt{f_c} b_w d$ $\frac{0.1\lambda d}{a}\sqrt{f_c} b_w d \le V_c \le 0.2\lambda\sqrt{f_c} b_w d$	(11)
Kara [24]	$V_c = 0.997 b_w d\left(6.837\sqrt[3]{\frac{d}{a}} f_c \mathit{TheNetherlands}\rho_f \frac{E_{fl}}{E_s}\right)^{1/3}$	(12)
CSA S806-12 [8]	$V_c = 0.05\lambda k_m k_a k_s k_r \sqrt[3]{f_c} b_w d_v$ $k_m = \sqrt{\frac{Vd}{M}} \le 1.0$; $k_a = \frac{2.5Vd}{M}$; $1.0 \le k_a \le 2.5$; $k_s = \frac{750}{450 + d} \le 1.0$; $k_r = 1 + (\mathit{TheNetherlands}\rho_f E_{fl})^{1/3}$; $d_v = \max(0.9d; 0.72h)$; $0.11\sqrt{f_c} b_w d_v \le V_c \le 0.22\sqrt{f_c} b_w d_v$; $f_c < 60$ MPa	(13)
Kurth [25]	$V_c = \beta\frac{1}{313}\kappa(100\rho_f E_{fl} f_c)^{1/3} b_w d$ $\beta = 3\frac{d}{a} \ge 1.0$; $\kappa = 1 + \sqrt{\frac{200}{d}} \le 2.0$; d in (mm)	(14)
Jang et al. [26]	$V_c = \frac{1}{6}\beta_f\sqrt{f_c} b_w d$ $\beta_f = 0.716 + 0.466\frac{E_{fl}}{E_s} - 0.095\frac{a}{d} + 32.101\rho_f$	(15)

Table 1. *Cont.*

Algorithm	Shear Strength of Member without Shear Reinforcement	
Lignola et al. [27]	$V_c = 1.65\left(\frac{E_{fl}}{E_s}\right)^{0.6} C_{Rd,c}k(100\rho_f f_c)^{1/3} b_w d$ $k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$; d in (mm); $C_{Rd,c} = \begin{cases} 0.18 \text{ for normal concrete} \\ 0.12 \text{ for lightweight concrete} \end{cases}$	(16)
ACI 440.1R-15 [5]	$V_c = \frac{2}{5}k\sqrt{f_c}b_w d$ $k = \sqrt{2\rho_f n_f + (TheNetherlands\rho_f n_f)^2} - TheNetherlands\rho_f n_f;\ n_f = \frac{E_{fl}}{E_c}$	(17)
Valivonis et al. [28]	$V_c = \frac{2\varphi_f f_{ct} b_w d^2}{a} \geq 0.45\varphi_f f_{ct} b_w d$ $TheNetherlands\varphi_f = 0.4\left(\frac{E_{fl}}{E_s}\right)^{TheNetherlands\rho_f}; a \leq 3.33d$	(18)
Thomas et al. [29]	$V_c = k_1 k_2 \tau_c b_w d$ $\tau_c = \frac{0.85\sqrt{f_c}\left(\sqrt{1+5\beta}-1\right)}{6\beta};\ \beta = \frac{f_c}{45.55p_t};$ $k_1 = \begin{cases} 2.2\frac{d}{a} + 0.12 & \text{for } \frac{a}{d} \leq 2.5 \\ 1.0 & \text{for } \frac{a}{d} > 2.5 \end{cases};$ $k_2 = \begin{cases} \frac{750}{450+d} & \text{for } d > 300 \text{ mm} \\ 1.0 & \text{for } d \leq 300 \text{ mm} \end{cases};\ p_t = TheNetherlands\rho_f \frac{E_{fl}}{E_s}$	(19)
Hamid et al. [30]	$V_c = f_c b_w d\left[0.00203(TheNetherlands\rho_f E_{fl} f_c)^{1/3} + 0.153\frac{d}{a}\right]$	(20)

where: $TheNetherlands\rho_f$—longitudinal FRP reinforcement ratio of beam; f_c—compressive strength of concrete (MPa); f_{ct}—tensile strength of concrete (MPa); E_{fl}—elastic modulus of FRP rebars (MPa); E_s—elastic modulus of steel rebars (MPa); E_c—elasticity modulus of concrete (MPa); d—effective depth of cross section (mm); a—length of the shear zone—distance of concentrated force from the support (mm); $a/$—shear slenderness; b_w—web width (mm); λ—modification factor related to density of concrete; V—shear force (N); M—bending moment (Nmm); a_g—maximum size of coarse aggregate [mm]; h—height of cross section (mm); p_t—equivalent longitudinal FRP reinforcement ratio of beam regarding to steel.

3. Experimental Database

The dimensioning algorithms included in design recommendations should assure a specific level of reliability by describing the real behaviour of the considered members. In the present study, algorithms for shear member dimensioning, as applied to beams longitudinally reinforced with FRP bars without shear reinforcement were verified. As part of the evaluation of the algorithms for beams longitudinally reinforced with FRP bars, but without shear reinforcement, 310 support zones described in 53 research papers (and briefly presented by Drzazga in [11]) were investigated. In Table 2, the characteristics of shear design parameters for beams used in the database that have been the subject of previous research in the field of shearing of concrete beams longitudinally reinforced with FRP bars are presented.

Table 2. Shear design parameters for beams used in the database.

Number of Support Zones					310
Properties		**Min**	**Max**	**Average**	***COV*** [1] **(%)**
b_w	(mm)	89	1000	251	68
h	(mm)	100	1000	318	51
d	(mm)	73	937	270	54
a	(mm)	200	3055	907	53
a/d	(–)	0.8	12.5	3.7	43
f_c	(MPa)	20	93	44	39
$TheNetherlands\rho_f$	(%)	0.12	11.57	1.35	134
E_{fl}	(MPa)	29,400	192,000	73,408	59
Longitudinal reinforcement material	(–)	AFRP, BFRP, CFRP, GFRP			
V_{test}	(N)	9000	291,300	62,490	85

[1] Coefficient of variation.

4. Verification of the Available Models and Design Recommendations

For each of the support zones, theoretical shear strength was calculated in accordance with the procedures presented in Table 1 and was compared with the experimental ultimate shear strength. Theoretical strength V_n was calculated without reduction factors taken into account. One should note that the values of strength V_{test} were obtained for the short-term loading of the beams. This way of loading precludes any study of the long-term processes taking place in concrete and in FRP rebars and of the effect of an aggressive environment.

The comparative analysis included the model fit indicators in the form of the average of the ratio of the experimental and theoretical shear force, V_{test}/V_n, parameter *X* (inverse of regression curve slope), coefficient of variation (*COV*), mean absolute percentage error (*MAPE*) and the percentage of support zones with overestimated strength. The results of the comparative analyses of procedures included in the design recommendations ACI 440.1R-15 [5], Canadian CSA-S806-12 [8], Japanese JSCE-97 [9] and Italian CNR-DT 203/2006 [10] are presented in Table 3. The results of the comparative analyses of the algorithms proposed by Tottori et al. [15], Michaluk et al. [16], Deitz et al. [17], El-Sayed et al. [18], Wegian et al. [19], Nehdi et al. [20], Hoult et al. [21], Razaqpur et al. [22], Alam [23], Kara [24], Kurth [25], Jang et al. [26], Lignola et al. [27], Valivonis et al. [28], Thomas et al. [29] and Hamid et al. [30] are presented in Table 4.

Table 3. Comparison of experimental (V_{test}) and theoretical (V_n) values of the concrete shear strength for available design recommendations. Results according to Equations (2), (7), (13) and (17).

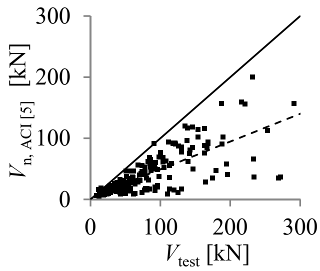
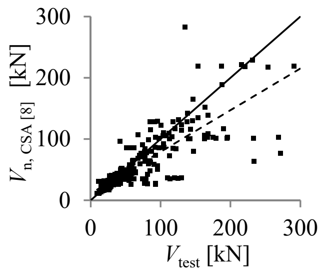
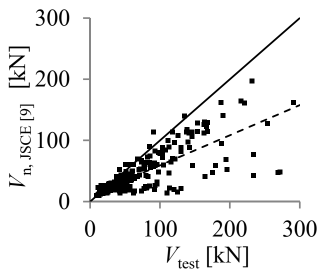
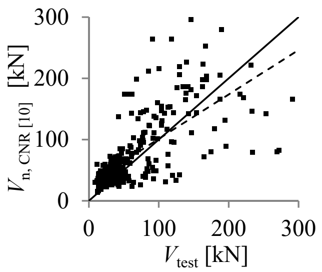
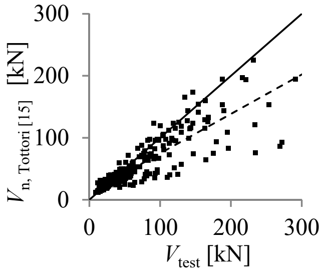
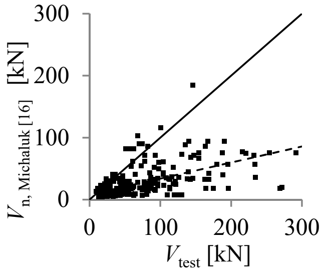
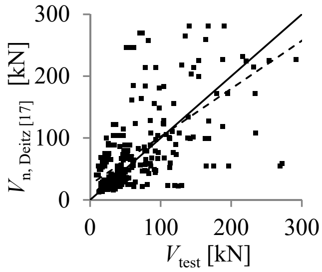
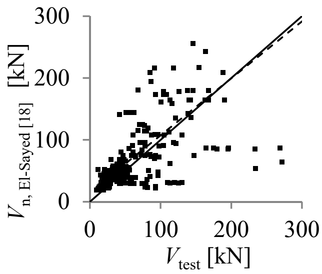
$\frac{V_{test}}{V_n}$	**ACI 440.1R-15 [5]**	**CSA S806-12 [8]**
Arithmetic mean	2.20	1.18
Parameter *X* (inverse of regression curve slope)	2.20	1.47
Coefficient of variation (*COV*) (%)	63.38	43.18
Mean absolute percentage error (*MAPE*) (%)	45.88	20.61
Percentage of beams with overestimated strength (%)	0.97	39.03
Theoretical concrete shear strength versus experimental concrete shear strength		
$\frac{V_{test}}{V_n}$	**JSCE-97 [9]**	**CNR-DT 203/2006 [10]**
Arithmetic mean	1.61	0.89
Parameter *X* (inverse of regression curve slope)	2.03	1.38
Coefficient of variation (*COV*) (%)	62.86	62.39
Mean absolute percentage error (*MAPE*) (%)	28.29	56.32
Percentage of beams with overestimated strength (%)	10.32	77.74
Theoretical concrete shear strength versus experimental concrete shear strength		

Table 4. Comparison of experimental (V_{test}) and theoretical (V_n) values of the concrete shear strength for algorithms proposed by other authors. Results according to Equations (1), (3)–(6), (8)–(12), (14)–(16) and (18)–(20).

$\frac{V_{test}}{V_n}$	**Tottori et al.** [15]	**Michaluk et al.** [16]
Arithmetic mean	1.20	3.26
Parameter *X* (inverse of regression curve slope)	1.57	3.87
Coefficient of variation (*COV*) (%)	36.98	73.26
Mean absolute percentage error (*MAPE*) (%)	19.38	59.09
Percentage of beams with overestimated strength (%)	30.00	10.97
Theoretical concrete shear strength versus experimental concrete shear strength		

$\frac{V_{test}}{V_n}$	**Deitz et al.** [17]	**El-Sayed et al.** [18]
Arithmetic mean	1.09	0.97
Parameter *X* (inverse of regression curve slope)	1.29	1.09
Coefficient of variation (*COV*) (%)	73.26	72.67
Mean absolute percentage error (*MAPE*) (%)	64.15	49.49
Percentage of beams with overestimated strength (%)	51.29	75.48
Theoretical concrete shear strength versus experimental concrete shear strength		

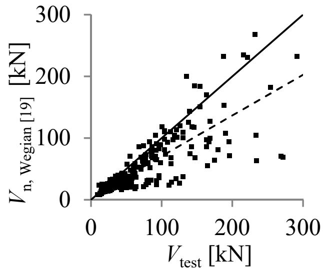
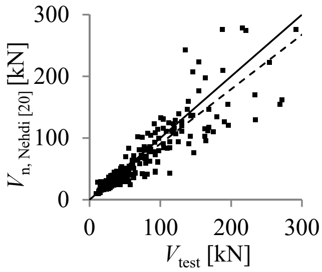
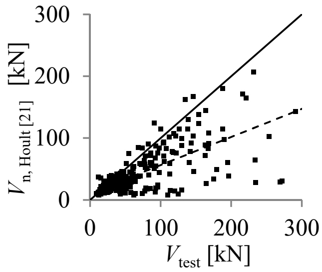
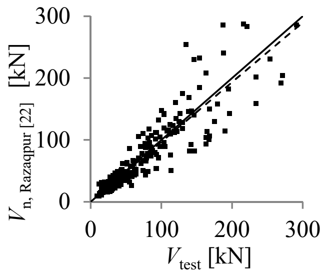
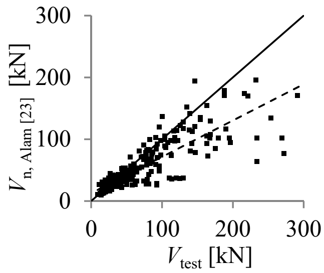
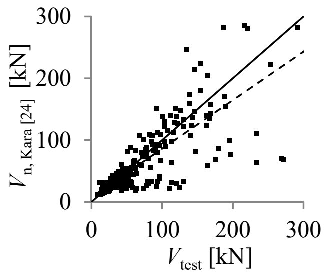
$\frac{V_{test}}{V_n}$	**Wegian et al.** [19]	**Nehdi et al.** [20]
Arithmetic mean	1.44	1.12
Parameter *X* (inverse of regression curve slope)	1.52	1.13
Coefficient of variation (*COV*) (%)	43.55	25.50
Mean absolute percentage error (*MAPE*) (%)	26.32	18.96
Percentage of beams with overestimated strength (%)	12.58	34.52
Theoretical concrete shear strength versus experimental concrete shear strength		

Table 4. *Cont.*

$\frac{V_{test}}{V_n}$	**Hoult et al. [21]**	**Razaqpur et al. [22].**
Arithmetic mean	1.88	1.09
Parameter *X* (inverse of regression curve slope)	2.21	1.04
Coefficient of variation (*COV*) (%)	91.29	24.59
Mean absolute percentage error (*MAPE*) (%)	34.15	18.97
Percentage of beams with overestimated strength (%)	13.23	39.97
Theoretical concrete shear strength versus experimental concrete shear strength		

$\frac{V_{test}}{V_n}$	**Alam [23]**	**Kara [24]**
Arithmetic mean	1.25	1.20
Parameter *X* (inverse of regression curve slope)	1.70	1.27
Coefficient of variation (*COV*) (%)	40.54	56.75
Mean absolute percentage error (*MAPE*) (%)	21.89	22.05
Percentage of beams with overestimated strength (%)	28.39	50.00
Theoretical concrete shear strength versus experimental concrete shear strength		

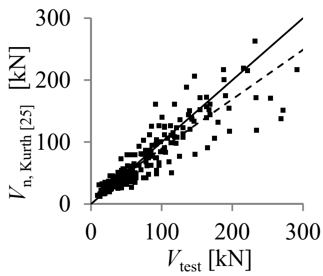
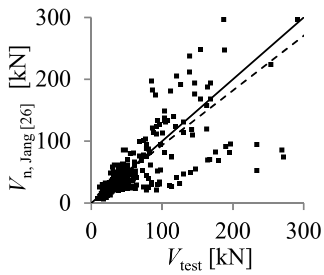
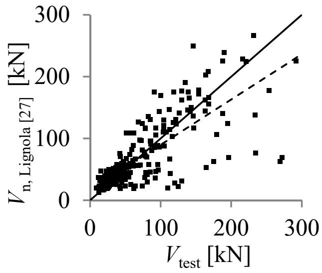
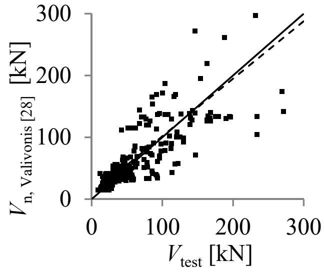
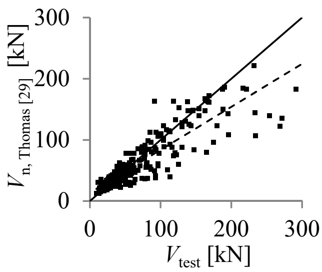
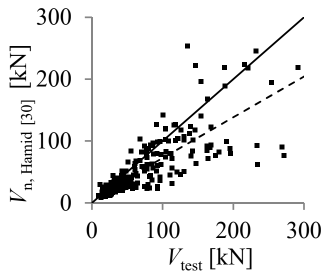
$\frac{V_{test}}{V_n}$	**Kurth [25]**	**Jang et al. [26]**
Arithmetic mean	1.03	1.25
Parameter *X* (inverse of regression curve slope)	1.26	1.13
Coefficient of variation (*COV*) (%)	27.11	57.86
Mean absolute percentage error (*MAPE*) (%)	19.09	33.80
Percentage of beams with overestimated strength (%)	54.52	46.45
Theoretical concrete shear strength versus experimental concrete shear strength		

Table 4. *Cont.*

$\frac{V_{test}}{V_n}$	**Lignola et al. [27]**	**Valivonis et al. [28]**
Arithmetic mean	1.07	0.98
Parameter X (inverse of regression curve slope)	1.37	1.08
Coefficient of variation (*COV*) (%)	64.64	39.97
Mean absolute percentage error (*MAPE*) (%)	31.91	32.81
Percentage of beams with overestimated strength (%)	64.84	65.48
Theoretical concrete shear strength versus experimental concrete shear strength		
$\frac{V_{test}}{V_n}$	**Thomas et al. [29]**	**Hamid et al. [30]**
Arithmetic mean	1.06	1.38
Parameter X (inverse of regression curve slope)	1.42	1.52
Coefficient of variation (*COV*) (%)	33.86	40.66
Mean absolute percentage error (*MAPE*) (%)	55.16	27.87
Percentage of beams with overestimated strength (%)	21.18	23.87
Theoretical concrete shear strength versus experimental concrete shear strength		

On the basis of the comparative analyses, it was found that some of the current proposals estimate the shear capacity of the support zones of beams longitudinally reinforced with FRP bars relatively well. Among the design recommendations, the American ACI 440.1R-15 [5] provides the most conservative approach for beams without shear reinforcement. The smallest margin of strength was obtained when the algorithms given in the Italian standard CNR-DT 203/2006 [10] were used. Moreover, the number of overestimated strength values is then the largest ($V_{test} < V_n$), which adversely affects the level of safety. The load capacities of more than 77% of the analysed support zones were overestimated when the procedure was used. In addition, a high value of the coefficient of variation and the mean absolute percentage error were obtained, which indicates a relatively large spread of results and an inappropriate adjustment of the model. Taking into account the criterion of conservativeness and the scatter of results, the best-fit model is included in the Canadian CSA S806-12 standard [8]. Using the procedures given in [8], the smallest values of the coefficient of variation and the mean absolute percentage error were obtained. Nevertheless, a relatively high percentage of support zones with an overestimated load capacity, with the parameter X of 1.47, indicates that the model is not adjusted properly.

A better model fit was observed for proposals not included in any design standards. The procedures proposed by Nehdi et al. [20], Razaqpur et al. [22] and Kurth [25], in particular, show a relatively good fit of the model. This is evidenced by, among other factors, being close to the unity of the arithmetic mean value of the ratio V_{test}/V_n, and the value of parameter X. In addition, using the procedures proposed by Nehdi et al. [20], Razaqpur et al. [22] and Kurth [25], a relatively low value of the coefficient of variation and the mean absolute percentage error were obtained.

5. Proposed Model for Estimating the Shear Capacity of Support Zones Reinforced Longitudinally with FRP Bars without Shear Reinforcement

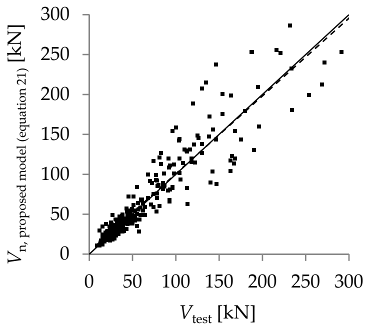
Based on the conducted analyses, it was found that it is reasonable to propose modifications to the current procedures to better match the model determining the shear strength of support zones reinforced longitudinally with FRP bars. The Razaqpur et al. [22] proposal—for which the best fit was found in terms of the coefficient of variation and the mean absolute percentage error and parameter *X*—was selected as the basis of the modification. The original form of the model (10) proposed by Razaqpur et al. [22] in 2010 is presented Table 1.

Wegian et al. [19], Kurth [25] and Hamid et al. [30] describe that the shear capacity of concrete support zones reinforced longitudinally with FRP bars without shear reinforcement depends on the cube root of the axial stiffness of the longitudinal reinforcement and the compressive strength of the concrete. Formula (10), in contrast to, for example, Kurth's [25] proposal, does not include the size effect, which, in the support zones of concrete beams, is important, as described by El-Sayed et al. [18] and Alam [23]. In addition, the boundary shear slenderness a/d = 2.5 was taken as the criterion for taking into account the arch action. Among others, Kurth [25] indicates that the effects of the arch action are observed for support zones with higher values of the a/d ratio. Based on the regression analysis and the evaluation of the impact of particular parameters, the following Formula (21) was determined for the shear capacity of support zones reinforced longitudinally with FRP bars (except the case of pure shear, where the bending moment, M = 0):

$$V_{\mathrm{c}} = 0.028 k_{\mathrm{m}} k_{\mathrm{a}} k_{\mathrm{r}} k \sqrt[3]{f_{\mathrm{c}}} b_{\mathrm{w}} d.$$
$$k = 1 + \sqrt[3]{\frac{200}{d}} \leq 2.0;\ k_{\mathrm{m}} = \left(\frac{Vd}{M}\right)^{\frac{1}{2}};\ k_{\mathrm{r}} = (TheNetherlands \rho_{\mathrm{f}} E_{\mathrm{fl}})^{\frac{1}{3}};\ k_{\mathrm{a}} = \begin{cases} 1.0 & \text{for} \quad \frac{M}{Vd} \geq 2.7 \\ \frac{2.7Vd}{M} & \text{for} \quad \frac{M}{Vd} < 2.7 \end{cases} \tag{21}$$

The size effect was taken into account by introducing the coefficient k, the value of which is determined based on the formula, the form of which is similar to the proposed in Eurocode 2 [31]. Alam [23], in his research, observed that the limit tangential stresses are proportional to $\frac{1}{\sqrt[3]{d}}$. Moreover, it was assumed that the shear capacity is proportional to the cube root of the stiffness of the longitudinal reinforcement. In addition, a higher limit value of the shear slenderness was introduced. Taking into account the arch action of the support zone is possible only in the case of appropriate anchoring of the longitudinal reinforcement. Table 5 presents the results of the verification analysis of the proposed model.

Table 5. Comparison of experimental (V_{test}) and theoretical (V_{n}) values of the concrete shear strength for the proposed model.

$\frac{V_{test}}{V_n}$	**Proposed Model (21)**
Arithmetic mean	1.00
Parameter *X* (inverse of regression curve slope)	1.03 (R^2 = 0.87)
Coefficient of variation (*COV*) (%)	22.50
Mean absolute percentage error (*MAPE*) (%)	18.62
Percentage of beams with overestimated strength (%)	52.90
Theoretical concrete shear strength versus experimental concrete shear strength	

The parameter value X = 1.03 was determined based on linear regression with a relatively high coefficient of determination, R^2 = 0.87, which is a measure of the model's fit. In addition, the smallest coefficient of variation and the mean absolute percentage error were obtained, which indicates the smallest distribution of results among the described models. The percentage of beams with overestimated strength is 52.90%, which is a result of obtaining the average value of the ratio of the experimental and theoretical shear force $V_{\mathrm{test}}/V_{\mathrm{n}}$ close to 1.00.

Figure 1 shows the distribution of values of the experimental and theoretical shear force ratio, $V_{\mathrm{test}}/V_{\mathrm{n}}$, for the particular ranges. The distribution is close to the normal distribution, and the lower endpoint of the 95% confidence interval of the average value is 0.977.

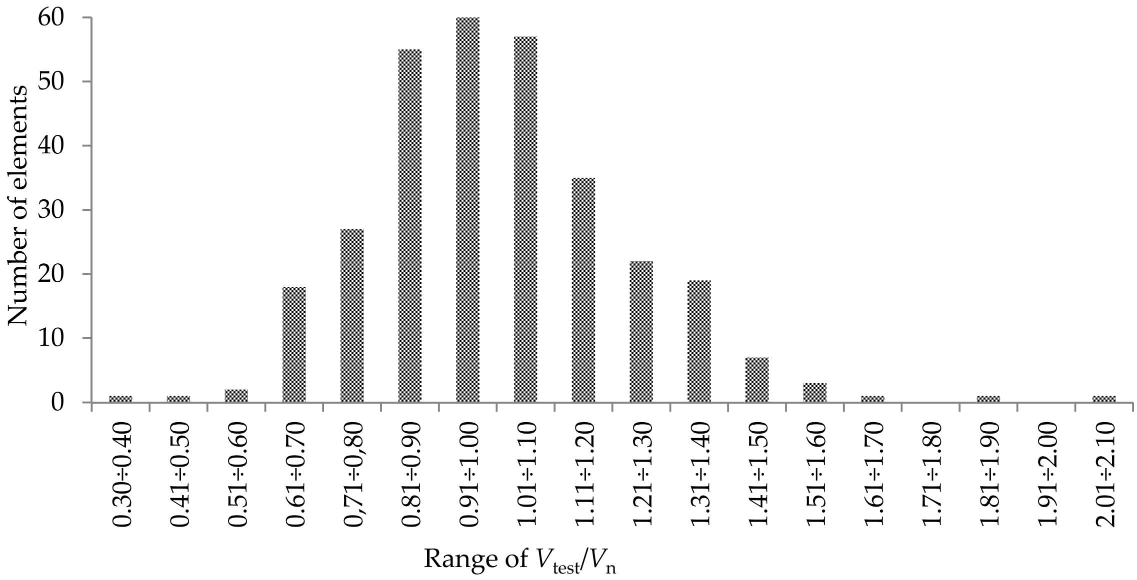


Figure 1. Distribution of values of the ratio of the shear resistance attained experimentally, V_{test}, to the corresponding analytical, V_{n} for particular ranges.

One of the verification criteria for the proposed procedure was the analysis of the impact of particular parameters on the ratio of the shear resistance attained experimentally, V_{test}, to the corresponding analytical, V_{n}. Figure 2 shows the value of $V_{\mathrm{test}}/V_{\mathrm{n}}$ in relation to shear slenderness, a/d, compressive concrete strength, f_{c}, the axial stiffness of longitudinal reinforcement, $TheNetherlands\rho_{\mathrm{f}}E_{\mathrm{fl}}$ and the effective depth of cross section, d.

Based on Figure 2, it is concluded that the proposed Formula (21) appropriately takes into account the influence of particular parameters on the shear capacity of support zones of beams longitudinally with composite bars.

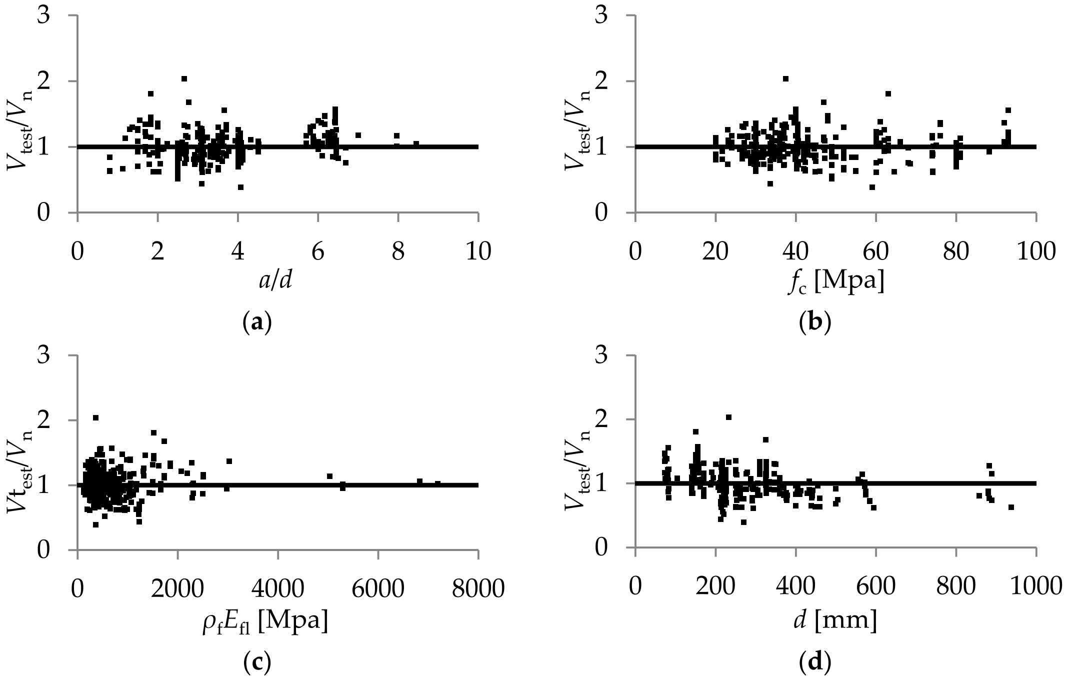


Figure 2. The ratio of the experimental and theoretical shear force $V_{\text{test}}/V_{\text{n}}$ as a function of: (**a**) shear slenderness, *a*/*d*; (**b**) the compressive strength of concrete, f_{c}; (**c**) the stiffness of the longitudinal reinforcement, *TheNetherlands*$\rho_{\text{f}}E_{\text{fl}}$; (**d**) the effective depth of cross section, *d*.

6. Conclusions

The available design recommendations and formulas derived by other researchers introduce various algorithms for determining the shear strength of beams longitudinally reinforced with FRP bars, taking into account the characteristic features of the composite bars—their relatively low elastic modulus and low strength in the perpendicular direction. This paper presents verification of the current procedures implemented in available codes and the formulas derived by other researchers in the field of estimating the shear strength of concrete support zones longitudinally reinforced with FRP bars. In this paper, a new approach for estimating shear capacity of support zones reinforced longitudinally with FRP bars without shear reinforcement was proposed and verified. The following conclusions have been drawn from the analyses:

(1) In the case of support zones reinforced longitudinally with FRP bars without transverse reinforcement, the best model among the available design standards is given in CSA S806-12 [8]. Nevertheless, the values of the verification parameters—especially the parameter $X = 1.47$, the coefficient of variation, $COV = 43.18\%$, and the mean absolute percentage error, $MAPE = 20.61\%$—indicate an unsatisfactory adjustment of the model.

(2) Better model fit was observed for proposals not included in any standards. The procedures proposed by Nehdi et al. [20], Razaqpur et al. [22] and Kurth [25], in particular, show a relatively good fit of the model.

(3) The developed empirical Formula (21) is a modification of the Razaqpur et al. [22], which was supplemented with, among others, size effect and changing the criterion of taking into account the arch action. Based on the verification analysis, the arithmetic mean ratio close to 1.00 was obtained. The parameter value $X = 1.03$ (inverse of linear regression curve slope) was determined with a relatively high coefficient of determination $R^2 = 0.87$. A satisfactory level of value of $COV = 22.50\%$ and $MAPE = 18.62\%$ was obtained. The proposed model appropriately takes into account the influence of particular parameters (such as shear slenderness, the compressive strength of concrete, the stiffness of the longitudinal reinforcement and the effective depth of cross section) on the shear capacity of

support zones of beams longitudinally reinforced with composite bars. Based on the verification analysis, a satisfactory level of model fit was obtained—the best among the available proposals.

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