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Prediction of the Yield Performance and Failure Mode of RC Columns under Cyclic-Load by PSO-BP Neural Network

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Abstract: The yield performances and failure mode of reinforced concrete (RC) columns, which are critical structural performances to the design and research of engineering structures, have a significant impact on the dynamic response, the performance level, and the design of seismic ductility. The traditional empirical theoretical method used to predict the yield performances and failure mode leads to large dispersions in most cases. To better estimate the yield performances and failure mode of RC columns, this paper developed a novel neural network method. Empirical theoretical models are used to determine the input parameters of the neural network by analyzing the factors that affect the yield performance and failure mode of RC columns, and the rationality of these parameters is verified by sensitivity analysis. The back-propagation (BP) neural network method was adopted. The influence of the number of hidden neurons was studied to improve the model accuracy. Comparative analysis revealed that the prediction results of the neural network are in good agreement with the experimental results and are more accurate than other traditional empirical theoretical models. The initial parameters were optimized using particle swarm optimization (PSO), which has been proven to be superior to the genetic algorithm (GA) and sparrow search algorithm (SSA) optimization methods in terms of effectiveness and computation time. The high generalization ability of the prediction model was calibrated using the test and validation sets and another eight additional sets of experimental data. The proposed method provides a new way to predict the structural performance under seismic actions when experimental data are insufficient.

Keywords: PSO-BP neural network; RC columns; yield performance; failure mode; prediction model

1. Introduction

Due to the gradual increase in government, society, and public awareness of disaster prevention and mitigation, performance-based seismic design and seismic damage prediction of existing buildings are hot issues and frontier development directions in earthquake engineering [1]. It is of great significance to evaluate the yield performance and failure mode of RC Columns as a critical member bearing vertical loads and resisting horizontal loads in the actual structure [2]. In the process of structural design, the yield performance and failure mode of RC columns are reasonably evaluated to meet the following prevention objectives: (1) no damage under frequent earthquakes; (2) no unrepairable damage under design-based earthquakes; and (3) no collapse under maximum considered earthquakes [3]. For structural seismic damage prediction, understanding the yield performance and failure mode of RC Columns can help guide retrofitting and strengthening [4].

Quantitative calculation methods of RC columns flexural yield point without a significant turning point of mechanical behavior are still not widely recognized [5]. RC columns are composite materials consisting of steel and concrete with very different mechanical properties. The ratio of the two materials and the form of construction cause the different
mechanical properties of RC columns [6]. The structural design requires that the RC column yields when the longitudinal reinforcement yields [7–9]. According to mechanical analysis, Zheng He and Jinping Ou [10] simplified the force process of RC columns into a three-fold line theory by using cracking point, yield point, and limit point as control points and gave a formula of yield strength based on section analysis. For the yield displacement, Priestley et al. [11] proposed a computational model to estimate the yield displacement of circular bridge columns, which contains flexural deformation and shear deformation. However, the results calculated by these methods only agree well with experimental results under specific conditions. Based on the force–deformation relationship of the columns, the relationship generally has no obvious turning point in the actual test [12]. Methods for determining the yield point have been proposed in many kinds of literature, including the graphical method, the isoenergetic method, and the farthest point method [13–16].

The failure mode of RC Columns under earthquake loads can be divided into three categories: flexural failure, flexural shear failure, and shear failure [17]. In seismic investigation and seismic experiments, the observation method is the primary basis for discriminating the seismic failure mode of columns [18]. The different failure mode corresponds to different stress states of the reinforcement and different forms of concrete crack development. The flexural failure corresponds to the flexural yield of the longitudinal reinforcement and flexural cracking of the concrete. The shear failure corresponds to the rapid yield of the transverse reinforcement and diagonal cracking of the core concrete. While the flexural and shear failure corresponds to the successive appearance of flexural and shear cracks and yield of the transverse reinforcement [19]. Although the observation method is practical and straightforward, it is highly subjective, and the failure mode cannot be judged without an earthquake [20]. Then, the failure mode of RC columns is related to their deformation capacity and shear bearing capacity. Ang [21] conducted pseudo-static tests on 25 RC circular columns with different shear-to-span ratios, axial compression ratios, transverse reinforcement ratios, and classified the failure mode of the columns using displacement ductility factors. The method of judgment based on bearing capacity was first proposed in ATC-6 [22]. The failure mode of RC columns, whether flexural or shear, is determined by comparing the amplitude between shear demand $V_m$ (calculated by flexural theory) and shear capacity $V_u$. Various literature works [23–25] and codes [8,26,27] give modified models based on the method. Considering a single performance index to discriminate the seismic failure mode of RC columns is not comprehensive enough [28]. By analyzing the relationship between the main parameters such as shear bearing capacity, shear-to-span ratio, hoop ratio, axial compression ratio, and column damage mode, some scholars [29–31] proposed a discriminative method considering the influence of multiple parameters, i.e., multi-parameter discriminatory method.

Neural Network (NN) [32] is a new technique that has been rapidly developed since the 1940s to simulate the neural activity of the human brain. It is a computational model suitable for dealing with complex nonlinear relationships, especially for solving problems with inherently complex internal mechanisms [33] and is widely used in civil engineering [34,35]. The BP neural network algorithm [36] is a multilayer feedforward network that can continuously adjust the network weights and thresholds in reverse, with good adaptive capability, distributed storage capability, and large-scale data processing capability. The principle is simple, easy to implement, and able to approximate arbitrary nonlinear curves. However, BP (Back-propagation) neural networks have unavoidable defects: they easily to fall into local minima, have slow convergence speed, and inconsistent network structure [37]. The PSO (Particle Swarm Optimization) [38] algorithm is a highly adaptive algorithm, which is based on the principle of finding the optimal solution based on the collaboration and information sharing among individuals of the population itself and individuals. This algorithm has the advantages of simple principles, few adjustment parameters required, fast convergence and high accuracy [39].
2. Method
2.1. Yield Strength Empirical Models

According to the plane-section assumption[10], the RC column yields when the tensile reinforcement begins to yield. As shown in Figure 1, the force and moment of the cross-section can be obtained by integration. The equilibrium equations of the internal forces and moments of the section for the neutral axis are established by integration:

\[
N' \approx b \sum \sigma_c \Delta x + \varepsilon_s E_s A_s' - f_y A_s
\]  

\[
M_y' \approx b \sum \sigma_c x_i \Delta x + \varepsilon_s E_s A_s' (x - a_s') - f_y A_s (h_0 - x) + N (h - y - x)
\]

Figure 1. RC column yield force diagram.

The yield moment and yield curvature cannot be solved directly and need to be calculated iteratively. However, an approximate method can derive the yield moment and yield curvature expressions directly, assuming a triangular shape of the concrete stresses in the compression zone:

\[
M_y = A_s f_y \left( h_0 - a_s' \right) + n b h_0 f_c \left( \frac{h}{2} - a_s' \right) - 0.5 \eta b h_0 f_{c0} \left( \frac{1}{3} \eta h_0 - a_s' \right)
\]

where, \( f_{c0} \) is the maximum compressive stress of concrete during section yielding and \( \eta \) is the height coefficient of the concrete compressed zone. They are determined by the following equation:

\[
f_{c0} = \frac{\eta \cdot f_y}{1 - \eta \cdot a_E}
\]

\[
\eta = \left[ \left( \rho + \rho' + \rho f_c f_y \right) a_E^2 + 2 a_E \left( \rho' a_s' h_0 + \rho + \rho f_c f_y \right) \right]^{1/2} - \left( \rho + \rho' + \rho f_c f_y \right) a_E
\]

\[
a_E = E_s / E_c
\]

\[
a_f = f_y / f_c
\]

where, \( f_c \) is the axial compressive strength of concrete, \( f_y \) is the yield strength of reinforcement, \( E_c \) is the modulus of elasticity of concrete, \( E_s \) is the modulus of elasticity of
reinforcement, $\rho$ is the ratio of tensile reinforcement, $\rho'$ is the ratio of compressive reinforcement, and $\rho_l$ is the ratio of longitudinal reinforcement.

In order to simplify the calculation, current seismic codes usually make relevant provisions for yield strength [7–9]. The formulations given in ACI-318 and GB50010, as shown in Equation (8), are the same, although they are given in different forms. In BSL, the expression of yield strength is shown in Equation (9):

$$M_y = A_s f_y \left( h_0 - a_s \right) + 0.5N \left(1 - N/bh f_c\right)$$

$$M_y = 0.8A_s f_y h + 0.5(1 - n)nbh f_c$$

In the static seismic test, the horizontal load applied at the top of the column is generally utilized to simulate the flexural load, so the equivalent horizontal load $V_y = M_y/L$.

2.2. Yield Displacement Empirical Models

The yield displacement of RC columns comprises contributions: flexural deformation, shear deformation, and bond slip deformation [40]. Meanwhile, because the stiffness of the specimen base in the lateral loading direction is large and well anchored, the effect of base rotation on the displacement of the top of the column is neglected.

The flexural deformation component of the column top displacement can be determined by curvature distribution along with the column height. In practical applications, it is generally assumed that the curvature is distributed linearly along with the column height when the column bottom section yields:

$$\Delta_{\text{flex}} = \int_0^L x \cdot \phi(x) dx = \frac{L^2}{3} \phi_y$$

$$\phi_y = \frac{\varepsilon_y}{h_0 - \eta h_0}$$

where, $\Delta_{\text{flex}}$ is the flexural deformation component of the yield displacement, $L$ is the height of the RC column, $\phi_y$ is yield curvature, and $\varepsilon_y$ is yield strain of reinforcement.

Before the column bottom section yields, the shear effect is mainly elastic shear. Therefore, the resulting plastic shear deformation can be ignored in the calculation of shear deformation. Assuming that the column is a non-cracking elastic homogeneous material and that the shear strains at different locations of the column are the same, the shear deformation displacement of the column can be determined by the following equation:

$$\Delta_{\text{shear}} = \frac{V_y}{K_v A G_{\text{eff}}} L$$

where, $V_y$ is the shear yield strength, $K_v$ is the shape factor (0.85 for circular columns and 5/6 for rectangular columns), $A$ is the cross-sectional area, and $G_{\text{eff}}$ is the shear modulus, which can be approximated as 0.5 times the modulus of elasticity.

The bond-slip deformation is caused by the elongation of the longitudinal reinforcement with respect to the base, and the yield stress of the reinforcement is equilibrated by the bond stress between the reinforcement embedded into the concrete and the concrete. In addition, the strain of the reinforcement in the base is assumed to be linearly elastic. Based on the equilibrium equation and integral calculation, the following equation for bond-slip deformation is obtained:

$$\Delta_{\text{slip}} = \frac{d_b f_y \phi_y}{8u_b} L$$

where, $d_b$ is the diameter of the longitudinal reinforcement; $u_b$ is the uniform bond stress produced by the elastic deformation of the longitudinal reinforcement, taken as $0.5 \sqrt{f_{c'}} - 1.0 \sqrt{f_{c'}}$. 


2.3. Failure Empirical Models

There are many factors affecting the failure mode of RC columns, such as shear-to-span ratio, axial compression ratio, vol transverse reinforcement ratio, longitudinal reinforcement ratio, compressive strength of concrete and reinforcement strength, etc. At the same time, the factors have a coupling effect on each other. Ma et al. [30] analyzed 315 groups of experimental data obtained from her own experiments and Peer-Structural Performance Database. The factors influencing the failure mode of RC columns are summarized, and a probabilistic method to identify the failure mode of RC is proposed:

$$\omega = \frac{10(n + 0.11) \cdot (\rho_l \cdot f_y / f_c)}{\lambda^3 (\rho_{sv} \cdot f_{yv} / f_t)^2}$$

(14)

where, $\omega$ is a failure determination factor. The flexural failure occurs when $0 < \omega \leq 0.13$, flexural shear failure occurs when $0.13 < \omega \leq 0.20$, and shear failure occurs when $0.20 < \omega$.

The failure of the specimen develops from flexural ductile failure to flexural shear failure or shear brittle failure as the shear-to-span ratio $\lambda$ decreases, the axial compression ratio $n$ increases, the longitudinal reinforcement parameter $\rho_l \cdot f_y / f_c$ increases, and the vol transverse reinforcement parameter $\rho_{sv} \cdot f_{yv} / f_t$ decreases. $\rho_{sv}$ is the vol transverse reinforcement ratio, $f_{yv}$ is the yield strength of transverse reinforcement, and $f_t$ is the tensile strength of concrete.

3. Neural Networks (NN)

3.1. Structure and Composition of BP Neural Network

BP neural network is a multilayer feed-forward neural network. The input signal is processed from the input layer through the implicit layer to the output layer in the forward signal transmission. The state of neurons in the next layer is only influenced by the state of neurons in the previous layer. The error is propagated backward if the output layer does not obtain the desired output. The network weight and threshold are adjusted according to the prediction error so that the output keeps approaching the desired value. The BP neural network structure is shown in Figure 2.

![Figure 2. Principle diagram of BP networks.](image)

The most common three-layer structure of BP neural networks consists of the input layer, the hidden layer, and the output layer. There is a nonlinear mapping relationship between each network layer. The input values of the BP neural network are $x_1, x_2, ..., x_n$, and the predicted values are $y_1, y_2, ..., y_m$; then, $w_{ij}$ and $w_{jk}$ are connection weight. Assume that the number of nodes in the hidden layer is $l$, and $a$ is the initial threshold of the hidden layer, then $b$ is the initial threshold of the output layer. The output of the hidden layer is $H$:

$$H_j = f \left( \sum_{i=1}^{n} w_{ij} x_i - a_j \right) \quad j = 1, 2, ..., l$$

(15)
The predicted output of the BP neural network is $O$:

$$O_k = \sum_{j=1}^{l} w_{jk} H_j - b_k \quad k = 1, 2, \ldots, m$$  \hspace{1cm} (16)$$

The network prediction error is $e_k$

$$e_k = Y_k - O_k \quad k = 1, 2, \ldots, m$$  \hspace{1cm} (17)$$

Then, update the weights and thresholds of the network. $\eta$ is learning rate:

$$w_{ij} = w_{ij} + \eta H_j (1 - H_j) x(i) \sum_{k=1}^{m} w_{jk} e_k \quad i = 1, 2, \ldots, n, j = 1, 2, \ldots, l$$  \hspace{1cm} (18)$$

$$w_{jk} = w_{jk} + \eta H_j e_k \quad j = 1, 2, \ldots, l, k = 1, 2, \ldots, m$$  \hspace{1cm} (19)$$

$$a_j = a_j + \eta H_j (1 - H_j) \sum_{k=1}^{m} w_{jk} e_k \quad j = 1, 2, \ldots, l$$  \hspace{1cm} (20)$$

$$b_k = b_k + e_k \quad k = 1, 2, \ldots, m$$  \hspace{1cm} (21)$$

Based on the above analysis, the description of the BP neural network algorithm is obtained in Figure 3.

![Flowchart of BP networks](image-url)

**Figure 3.** Flowchart of BP networks.

### 3.2. PSO-BP Algorithm Process and Model Construction

The above three-layer standard BP neural network can arbitrarily approximate non-linear continuous functions, but the algorithm still has drawbacks: it can easily fall into
local minima and has a slow convergence rate. To solve the above problems, the PSO algorithm, called particle swarm optimization, is utilized to simulate the swarming behavior of organisms such as birds and fish. These organisms search for food by cooperating in the survival process, and each member of the swarm constantly changes its search pattern by learning from experiences of its own and other members. Assuming that there are $M$ particles forming a particle population in a D-dimensional search space, the primary computational derivation of the PSO algorithm is as follows:

$$v_{id}^{t+1} = u \times v_{id}^t + c_1 r_1 (p_{id} - x_{id}^t) + c_2 r_2 (p_{gd} - x_{id}^t)$$

where $v_{id} = (v_{i1}, v_{i2}, \ldots, v_{id})$ denotes the flight speed of the $i$th particle; $x_i = (x_{i1}, x_{i2}, \ldots, x_{id})$ denotes the position of $M$ particles in space; $p_i = (p_{i1}, p_{i2}, \ldots, p_{id})$ denotes the historical best position passed by the $i$th particle in space; $p_g = (p_{g1}, p_{g2}, \ldots, p_{gd})$ denotes the historical best position experienced by the whole population in space, $c_1$, $c_2$ denote the learning acceleration coefficients; $r_1$, $r_2$ are random numbers varying between $[0,1]$; and $u$ denotes the inertia weight.

Because of the shortcomings of standard BP networks, this paper uses the PSO algorithm to optimize BP neural networks. The weights and thresholds of the BP neural network are regarded as particles, and the training process of the system is completed by mutual learning among the particles. At the same time, self-adaptive variation is introduced to enhance the ability of the particle swarm optimization algorithm to jump out of the optimal local solution by the random variation of the current best particles, and then the weights change like the following formula. The change of the weights is given in the following equation:

$$\Delta w_{ij} = c_1 r_1 (w_{ij}(p) - w_{ij}) + c_2 r_2 (w_{ij}(g) - w_{ij})$$

The flow chart of PSO-BP algorithm is as follows in Figure 4.

Figure 4. Implementation process of the PSO-BP algorithm.

4. Performance of Neural Networks

4.1. Dataset Processing

The data set samples of this paper were obtained from the PEER structural performance database, and the selection conditions were (1) rectangular cross-section, (2) re-
ciprocal action until failure, and (3) complete data. In addition, 166 sets of samples were selected as the data set for this paper. According to the previous paper, the following eight design parameters were found to have a strong influence on the yield performance and failure mode, which are (1) Compressive Strength of Concrete \( f_c \) (MPa), (2) Axial Compression Ratio \( n = F/(Af_c) \), (3) Cross-sectional area \( A \) (mm²), (4) Length \( L \) (mm), (5) Longitudinal Reinforcement Ratio \( \rho_l \), (6) Yield Strength of Longitudinal Reinforcement \( f_y(l) \) (MPa), (7) Yield Strength of Transverse Reinforcement \( f_y(t) \) (MPa), and (8) Vol Transverse Reinforcement Ratio \( \rho_v \). The values of design parameters are shown in Figure 5.

**Figure 5.** The values of parameters.

For the processing of yield force and yield displacement, the simplified method for determining the yield point proposed by Peng Feng is applied [16]. This method is named the Farthest Point Method. The point on the curve farthest from the line connecting the origin and the peak response point is the yield point. If there is more than one point, the yield point can be obtained by averaging the load values of these points and corresponding to the curve. As in Figure 6, in the force–deformation curve, the parallel line between the origin and the peak line and the tangent point of the force–deformation curve is the yield point. The required translation distance \( d \) must not be too small, usually taking the point with the most significant \( d \) value as the yield point when there are multiple tangent points:

\[
(F_y, D_y) = \max_{(F_y, D_y) = (F, D)} d = \frac{|F_p \cdot D - D_p \cdot F|}{\sqrt{F_p^2 + D_p^2}}
\]

(25)

where, \( (F, D) \) are the coordinates of any point on the force–deformation curve of the component; \( (F_y, D_y) \) are the coordinates of the yield point determined by the farthest point method; \( (F_p, D_p) \) are the coordinates of the peak point, and there are \( 0 \leq D \leq D_p \). The yield displacement and yield strength of the 166 sets of samples were calculated according to the above method.
Figure 6. Farthest Point Method to determine yield point: (a) bilinear curve; (b) without obvious turning curve; (c) traditional elastic–plastic curve.

After calculation, 166 groups of complete rectangular column experimental data in the database were selected for to analyze yield strength, yield displacement, and failure mode. The dataset was divided into three parts: training set, validation set, and test set, with the number ratio of 7:1.5:1.5, which means 116:25:25. The training set is the data sample applied for model fitting. The validation set is used to check the state of the model during the training process, convergence. The validation set is usually used to adjust the hyperparameters. In addition, the validation set can be employed to monitor the model for overfitting during the training process. The test set is utilized to evaluate the generalization ability of the model.

4.2. The Number of the Hidden Neurons

In this paper, the number of PSO-BP algorithm particles is chosen to be 30, the number of evolutions is 100, the learning acceleration coefficients $c_1$, $c_2$ are 1.49445, the number of nodes in the input layer $n = 8$, and the maximum number of learning is 1000 times. The number of hidden nodes has a significant impact on the learning quality of the neural network. Using too few neurons in the hidden layer will result in underfitting. Conversely, using too many neurons can also lead to several problems. Firstly, when a neural network has too many nodes, the limited amount of information in the training set is not enough to train all the neurons in the hidden layer, thus leading to overfitting. Even if the training data contain enough information, too many neurons in the hidden layer will increase the training time, thus making it difficult to achieve the desired results. Choosing an appropriate number of neurons in the hidden layer is crucial. The following equation is used to estimate the number of neurons in the hidden layer [41]:

$$l = \sqrt{m + n + a_0} \quad (26)$$

where, the number of nodes in the input layer, the hidden layer, and output layer are $m$, $l$, and $n$; $a_0$ is an arbitrary constant between 0 and 10.

The three models of the yield strength, the yield displacement, and the failure mode include 8 input layer nodes, 3–13 hidden layer nodes, and 1 output layer node, respectively. After training, the main parameters MSE (Mean Square Error) and correlation coefficients (predicted and experimental values) of the network model are given in Tables 1–3. For the yield strength model, the yield displacement model, and the failure mode model, the hidden layer nodes that achieve the best performance values are 12, 11, and 10, respectively. Among them, the relatively large value of mse for the failure mode is because this neural network belongs to the classification algorithm, and the expected output value is only three. However, it does not affect the accuracy of the classification results. N8-12-1 (Yield Strength),
N8-11-1 (Yield Displacement), and N8-10-1 (Failure Mode) were selected for further study. The MSE of their training, validation, and test sets have small differences, proving that these network stars have strong generalization ability. Meanwhile, the correlation coefficients of their network predictions and experimental values were 0.978523, 0.953280, and 0.944785, demonstrating a strong linear correlation and indicating a good match between the network predictions and experimental values.

Table 1. Performance value of Yield Strength.

<table>
<thead>
<tr>
<th>NN Model</th>
<th>MSE Training Set</th>
<th>MSE Validation Set</th>
<th>MSE Test Set</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>N8-3-1</td>
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<td>0.008878</td>
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Table 2. Performance value of Yield Displacement.

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Table 3. Performance value of Failure Mode.

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<th>Correlation Coefficient</th>
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<td>0.36126</td>
<td>0.466991</td>
<td>0.328764</td>
<td>0.610306</td>
</tr>
<tr>
<td>N8-4-1</td>
<td>0.027117</td>
<td>0.382329</td>
<td>0.394654</td>
<td>0.877963</td>
</tr>
<tr>
<td>N8-5-1</td>
<td>0.104077</td>
<td>0.366758</td>
<td>0.016119</td>
<td>0.883297</td>
</tr>
<tr>
<td>N8-6-1</td>
<td>0.054855</td>
<td>0.028622</td>
<td>0.429218</td>
<td>0.905252</td>
</tr>
<tr>
<td>N8-7-1</td>
<td>0.007015</td>
<td>0.46228</td>
<td>0.285102</td>
<td>0.895492</td>
</tr>
<tr>
<td>N8-8-1</td>
<td>0.128677</td>
<td>0.286993</td>
<td>0.041658</td>
<td>0.874645</td>
</tr>
<tr>
<td>N8-9-1</td>
<td>0.117975</td>
<td>0.163588</td>
<td>0.182011</td>
<td>0.879383</td>
</tr>
<tr>
<td>N8-10-1</td>
<td>0.071244</td>
<td>0.05678</td>
<td>0.311265</td>
<td>0.907002</td>
</tr>
<tr>
<td>N8-11-1</td>
<td>0.034483</td>
<td>0.20156</td>
<td>0.388967</td>
<td>0.899709</td>
</tr>
<tr>
<td>N8-12-1</td>
<td>0.119895</td>
<td>0.267908</td>
<td>0.106535</td>
<td>0.873913</td>
</tr>
<tr>
<td>N8-13-1</td>
<td>0.182866</td>
<td>0.277581</td>
<td>0.412712</td>
<td>0.780623</td>
</tr>
</tbody>
</table>

As the number of evolution increases, the particle position and velocity keep changing with declining fitness function values, as shown in Figure 7. When the number of evolution is 1, a BP neural network is utilized for prediction, and the value of the fitness function...
is maximum. The value decreases step by step as the number of evolution increases. By introducing the PSO algorithm, the network is effectively prevented from falling into the local optimum, and the global optimum is reached easier.

![Graphs showing fitness function optimization process.](image)

Figure 7. Fitness function optimization process.

At the same time, there are various optimization algorithms for BP neural networks [42], such as Particle swarm optimization (PSO), Genetic algorithm (GA) [43], Sparrow search algorithm (SSA) [44], etc. In order to compare the performance of the three optimization algorithms, in this paper, the predicted yield displacement, 11 hidden layer nodes, 30 population sizes, and evolution number of 100 are chosen as uniform computational parameters for prediction using each of these optimization methods. PSO, GA, and SSA calculation time in the same computer is 232.47 s, 240.70 s, and 501.34 s, respectively. The correlation coefficients between network predictions and experimental values of PSO, GA, and SSA are shown in Table 4. It can be seen that the PSO optimization method is better than the other two methods in terms of effectiveness and computation time.

![Table 4](image)

Table 4. Correlation coefficient of PSO, GA, and SSA.

<table>
<thead>
<tr>
<th>Optimization Algorithm</th>
<th>Yield Strength</th>
<th>Yield Displacement</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO</td>
<td>0.971779</td>
<td>0.956099</td>
<td>0.907002</td>
</tr>
<tr>
<td>GA</td>
<td>0.951574</td>
<td>0.938407</td>
<td>0.898371</td>
</tr>
<tr>
<td>SSA</td>
<td>0.923419</td>
<td>0.932279</td>
<td>0.845981</td>
</tr>
</tbody>
</table>

5. Predicted Results and Analysis

5.1. Model Validation

For the yield strength, the normalized results of the PSO-BP model and the theoretical model, ACI-318, GB50010, and BSL codes were compared with the experimental results in Figure 8. Their correlation coefficients with the experimental results are shown in Table 5. Most of the results calculated by the theoretical model are larger than the experimental values. Eight values of the theoretical model are more than three times that of the experimental values. After analyzing the data, it was found that the axial compression ratios of these eight groups of data were all greater than 0.5. Therefore, in the high axial compression ratio, the theoretical model is not recommended. The values are closer after calculating the method specified in the modern codes. The yield strength determined by the PSO-BP neural
network shows reasonably close values to the measured Yield Strength of RC columns. Results presented in Figure 8 and Table 5 illustrate that PSO-BP neural network results are closer to experimental results than the theoretical model and codes.

![Comparison of PSO-BP with Theoretical Model and Code Model for Yield Strength](image)

**Figure 8.** Comparison of PSO-BP with Theoretical Model and Code Model for Yield Strength.

**Table 5.** Correlation Coefficient of Yield Strength.

<table>
<thead>
<tr>
<th></th>
<th>Theoretical Model</th>
<th>ACI-318 and GB50010</th>
<th>BSL</th>
<th>PSO-BP</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0.789843</td>
<td>0.881884</td>
<td>0.876644</td>
<td>0.971779</td>
</tr>
</tbody>
</table>

For the yield displacement, the normalized results of the PSO-BP and the theoretical models were compared with the experimental results in Figure 9. Their correlation coefficients with the experimental results are shown in Table 6. The yield displacement values calculated by the theoretical model are generally smaller than the experimental values. The yield displacements predicted by the PSO-BP neural network are in better agreement with the experimental values.

**Table 6.** Correlation Coefficient of Yield Displacement.

<table>
<thead>
<tr>
<th></th>
<th>Theoretical Model</th>
<th>PSO-BP</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0.642322</td>
<td>0.956099</td>
</tr>
</tbody>
</table>

For the failure mode, the normalized results of the PSO-BP model and the theoretical model were compared with the experimental results in Figure 10. Their rate of accuracy with the experimental results are shown in Table 7. The results above suggest that the developed PSO-BP model can determine the failure mode of RC columns better than the theoretical model.
Figure 9. Comparison of PSO-BP with Theoretical Model for Yield Displacement.

Figure 10. Comparison of PSO-BP with Theoretical Model for Failure Mode.

Table 7. Rate of Accuracy of Failure Mode.

<table>
<thead>
<tr>
<th></th>
<th>Theoretical Model</th>
<th>PSO-BP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Accuracy</td>
<td>79.52%</td>
<td>94.58%</td>
</tr>
</tbody>
</table>

To further demonstrate the effectiveness of the trained model, the test data on yield performance and failure mode of RC columns, which were conducted by the authors [45], are used in Figure 11. The data are employed for validation purposes only and are not used for neural network training. The detailed designed parameters of the RC columns are given in Table 8. The tested force–displacement hysteresis curves of the specimens are plotted in Figure 12. Result comparisons between the tests and the PSO-BP models on key parameters, including yield strength, yield displacement, and failure mode, are shown in Tables 9–11. Most of the prediction errors are within 15%, The average prediction errors for
yield strength and yield displacement are 16.74% and 16.39%, respectively. In addition, the failure mode was correctly predicted for 7 of the 8 RC columns. Generally, the three trained PSO-BP models show reasonable results for predicting the yield performance and failure mode of RC columns.

Figure 11. Test setup of the eight RC columns.

Figure 12. Force–displacement hysteresis curves diagram. Images (a–h) correspond to the eight RC columns used in the test.
Table 8. Design parameters of eight RC columns.

<table>
<thead>
<tr>
<th>No.</th>
<th>( f_c ) (MPa)</th>
<th>( n )</th>
<th>( A ) (mm(^2))</th>
<th>( L ) (mm)</th>
<th>( \rho_l )</th>
<th>( f_{iy} ) (MPa)</th>
<th>( f_{yt} ) (MPa)</th>
<th>( \rho_v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z1</td>
<td>42.27</td>
<td>0.6</td>
<td>90,000</td>
<td>1750</td>
<td>0.0205</td>
<td>480</td>
<td>417.5</td>
<td>0.018</td>
</tr>
<tr>
<td>Z2</td>
<td>42.27</td>
<td>0.3</td>
<td>90,000</td>
<td>1750</td>
<td>0.0205</td>
<td>480</td>
<td>417.5</td>
<td>0.018</td>
</tr>
<tr>
<td>Z3</td>
<td>42.27</td>
<td>0.9</td>
<td>90,000</td>
<td>1750</td>
<td>0.0205</td>
<td>480</td>
<td>417.5</td>
<td>0.018</td>
</tr>
<tr>
<td>Z4</td>
<td>42.27</td>
<td>0.9</td>
<td>90,000</td>
<td>1750</td>
<td>0.0205</td>
<td>480</td>
<td>417.5</td>
<td>0.0029</td>
</tr>
<tr>
<td>Z5</td>
<td>42.27</td>
<td>0.9</td>
<td>90,000</td>
<td>1750</td>
<td>0.0419</td>
<td>480</td>
<td>417.5</td>
<td>0.018</td>
</tr>
<tr>
<td>Z6</td>
<td>42.27</td>
<td>0.6</td>
<td>90,000</td>
<td>1080</td>
<td>0.0205</td>
<td>480</td>
<td>417.5</td>
<td>0.018</td>
</tr>
<tr>
<td>Z7</td>
<td>42.27</td>
<td>0.6</td>
<td>90,000</td>
<td>600</td>
<td>0.0205</td>
<td>480</td>
<td>417.5</td>
<td>0.018</td>
</tr>
<tr>
<td>Z8</td>
<td>42.27</td>
<td>0.6</td>
<td>90,000</td>
<td>600</td>
<td>0.0205</td>
<td>480</td>
<td>417.5</td>
<td>0.0029</td>
</tr>
</tbody>
</table>

Table 9. Comparison of experimental data and predicted data for yield strength.

<table>
<thead>
<tr>
<th>No.</th>
<th>Experimental Data (KN)</th>
<th>Predicted Data</th>
<th>Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z1</td>
<td>176.14</td>
<td>147.03</td>
<td>16.52%</td>
</tr>
<tr>
<td>Z2</td>
<td>177.42</td>
<td>100.19</td>
<td>43.53%</td>
</tr>
<tr>
<td>Z3</td>
<td>243.60</td>
<td>226.59</td>
<td>6.98%</td>
</tr>
<tr>
<td>Z4</td>
<td>212.19</td>
<td>256.24</td>
<td>20.76%</td>
</tr>
<tr>
<td>Z5</td>
<td>297.95</td>
<td>245.07</td>
<td>17.75%</td>
</tr>
<tr>
<td>Z6</td>
<td>288.29</td>
<td>319.35</td>
<td>10.78%</td>
</tr>
<tr>
<td>Z7</td>
<td>421.05</td>
<td>359.85</td>
<td>14.54%</td>
</tr>
<tr>
<td>Z8</td>
<td>270.66</td>
<td>262.18</td>
<td>3.13%</td>
</tr>
</tbody>
</table>

Table 10. Comparison of experimental data and predicted data for yield displacement.

<table>
<thead>
<tr>
<th>No.</th>
<th>Experimental Data (mm)</th>
<th>Predicted Data</th>
<th>Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z1</td>
<td>12.22</td>
<td>10.33</td>
<td>15.44%</td>
</tr>
<tr>
<td>Z2</td>
<td>10.02</td>
<td>7.47</td>
<td>25.41%</td>
</tr>
<tr>
<td>Z3</td>
<td>9.34</td>
<td>10.42</td>
<td>11.51%</td>
</tr>
<tr>
<td>Z4</td>
<td>5.91</td>
<td>4.86</td>
<td>17.79%</td>
</tr>
<tr>
<td>Z5</td>
<td>12.52</td>
<td>16.58</td>
<td>32.43%</td>
</tr>
<tr>
<td>Z6</td>
<td>4.26</td>
<td>3.69</td>
<td>13.38%</td>
</tr>
<tr>
<td>Z7</td>
<td>2.28</td>
<td>1.94</td>
<td>15.13%</td>
</tr>
<tr>
<td>Z8</td>
<td>1.62</td>
<td>1.62</td>
<td>0.08%</td>
</tr>
</tbody>
</table>

Table 11. Comparison of experimental data and predicted data for failure mode.

<table>
<thead>
<tr>
<th>No.</th>
<th>Experimental Data</th>
<th>Predicted Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z1</td>
<td>Flexural Failure</td>
<td>Flexural Failure</td>
</tr>
<tr>
<td>Z2</td>
<td>Flexural Failure</td>
<td>Flexural Failure</td>
</tr>
<tr>
<td>Z3</td>
<td>Flexural Failure</td>
<td>Flexural Failure</td>
</tr>
<tr>
<td>Z4</td>
<td>Flexural Shear Failure</td>
<td>Flexural Failure</td>
</tr>
<tr>
<td>Z5</td>
<td>Flexural Shear Failure</td>
<td>Flexural Shear Failure</td>
</tr>
<tr>
<td>Z6</td>
<td>Flexural Shear Failure</td>
<td>Flexural Shear Failure</td>
</tr>
<tr>
<td>Z7</td>
<td>Shear Failure</td>
<td>Shear Failure</td>
</tr>
<tr>
<td>Z8</td>
<td>Shear Failure</td>
<td>Shear Failure</td>
</tr>
</tbody>
</table>

5.2. Sensitivity Analysis

Sensitivity analysis [46] is a method to study the degree of importance of the input variables of a numerical model on the output variables. It measures the importance of the input by a sensitivity coefficient, and the larger the sensitivity coefficient, the greater the degree of influence of that input factor of the model output. The Garson algorithm [47] is representative of the connection-weight-based sensitivity analysis method, which uses the product of the connection weights of a neural network to calculate the degree of
contribution to the input to the output. For a neural network of N-n-l-m, the computational formulation is given in the following equation:

\[ Q_i = \sum_{j=1}^{l} \left( \frac{|\omega_{ij}^1 \omega_{j}^2|}{\sum_{k=1}^{n} |\omega_{kj}^1 \omega_{j}^2|} \right) \]

\[ \sum_{i=1}^{n} \sum_{j=1}^{l} \left( \frac{|\omega_{ij}^1 \omega_{j}^2|}{\sum_{k=1}^{n} |\omega_{kj}^1 \omega_{j}^2|} \right) \] (27)

where, \( \omega^1 \) and \( \omega^2 \) represent the weights from the input layer to the hidden layer and from the hidden layer to the output layer, respectively. The above sensitivity analysis is performed, and the results are shown in Figure 13. For Yield Strength (N8-12-1), the order of sensitivity of the effect is Cross-sectional area, Vol Transverse Reinforcement Ratio, Yield Strength of Longitudinal Reinforcement, Axial Compression Ratio, Compressive Strength of Concrete, Yield Strength of Transverse Reinforcement, Length, Longitudinal Reinforcement Ratio. For Yield Displacement (N8-11-1), the order of sensitivity of the effect is Yield Strength of Transverse Reinforcement, Vol Transverse Reinforcement Ratio, Yield Strength of Longitudinal Reinforcement, Axial Compression Ratio, Longitudinal Reinforcement Ratio, Length, Cross-sectional area, Compressive Strength of Concrete. For Failure Mode (N8-10-1), the order of sensitivity of the effect is Cross-sectional area, Compressive Strength of Concrete, Longitudinal Reinforcement Ratio, Length, Yield Strength of Longitudinal Reinforcement, Yield Strength of Transverse Reinforcement, Vol Transverse Reinforcement Ratio, and Axial Compression Ratio.

Figure 13. Sensitivity Coefficient of PSO-BP.

6. Discussion

1. In processing the data set, using the furthest point method has a wide range of practicality compared to the traditional isoenergetic method and graphical method, and it is convenient to turn into calculations to reduce the data processing time significantly.

2. Our proposed experimental results showed that the PSO-BP neural network provides better fitness function results than the BP neural network in Figure 7. Each evolution is equivalent to the training of a BP neural network. After 100 evolutions, the fitness function values decreased from 5.734 to 4.478 for yield strength, from 9.118 to 6.755 for yield displacement, and from 5.846 to 3.038 for failure mode. Comparing the different
3. Compared with the empirical theoretical formulation, the prediction of the PSO-BP neural network has an absolute advantage in terms of accuracy, which can be seen in Figures 8–10 and Tables 5–7. Firstly, in the theoretical formulation, because reinforced concrete columns are essentially complex composite structures, many simplifications are carried out, resulting in a large gap between the calculated results and the experimental results. In the data set of this paper, the correlation coefficients of yield strength and yield displacement are 0.789843 and 0.642322. Secondly, the empirical formula and the formula of the codes require relatively simple calculations. The interpolation methods such as Lagrange, Newton, and Hermite and the fitting methods such as linear regression, least squares, and polynomial are mostly used in the data processing. The effectiveness of these methods in dealing with complex nonlinear problems is not significant. The neural network method has the absolute advantage in dealing with nonlinear fitting. In addition, 25 validation sets and 25 test sets out of 166 data sets were used to prevent getting into overfitting. In addition, the prediction model was shown to have a high generalization capability by analyzing eight experimental datasets that were not in the dataset.

4. After sensitivity analysis, once again, it proves that the effect of each input parameter in the neural network model on the column yield displacement is non-negligible, indicating that the input parameters of the neural network selected in the previous paper by using previous empirical model analysis are reasonable.

7. Conclusions

In this work, a method based on PSO-BP neural network, used to predict the yield performance and failure mode of RC columns, is proposed and researched. Results demonstrate that this method shows good accuracy and is time-efficient. Detailed conclusions are given below.

1. Factors affecting the yielding performance and failure mode of RC columns were carried out using traditional empirical theoretical models. Eight critical parameters are selected: the concrete compressive strength, axial compression ratio, cross-sectional area, length, longitudinal reinforcement yield stress, longitudinal reinforcement ratio, transverse reinforcement yield stress, and vol transverse reinforcement ratio. These parameters are then utilized as input parameters of the PSO-BP neural network to predict the yield performance and failure mode of RC columns.

2. The influence of the number of hidden neurons in the PSO-BP neural network was studied. Results show that the hidden layer nodes that achieve the best performance values are 12, 11, and 10 for the yield strength, yield displacement, and failure mode models. The PSO-BP-based prediction model shows outstanding accuracy compared with the empirical theoretical models. The correlation coefficients between test and prediction results for yield strength and yield displacement are 0.97 and 0.96, respectively. The accuracy for the failure mode is 94.58.

3. Optimization performance among different optimization methods, including GA, SSA, and PSO, are compared. The PSO method is the most efficient among these three methods and requires the shortest computation time.

4. Eight sets of experimental data, which are not included in the dataset, are used further to calibrate the effectiveness and robustness of the proposed method. Their performance is accurately predicted. The average prediction errors for yield strength and displacement are 16.74% and 16.39%, and the failure mode is correctly predicted for 7 of the 8 RC columns.

5. In addition, the rationality of the input parameters and their corresponding degree of influence are researched using the Garson sensitivity analysis method. For Yield Strength (N8-12-1), Yield Displacement (N8-11-1), and Failure Mode (N8-10-1), the largest impacts are Cross-Sectional Area, Yield Strength of Transverse Reinforcement,
and Cross-Sectional Area, and the smallest impacts are Longitudinal Reinforcement, Compressive Strength of Concrete, and Axial Compression Ratio, respectively.

**Author Contributions:** Data curation, G.Z. and H.Z.; Formal analysis, B.S.; Funding acquisition, B.S.; Investigation, G.Z.; Methodology, G.Z.; Project administration, G.Z. and B.S.; Resources, W.B.; Software, G.Z.; Supervision, W.B.; Validation, B.S. and H.Z.; Visualization, G.Z.; Writing—original draft, G.Z. and B.S.; Writing—review & editing, G.Z. and W.B. All authors have read and agreed to the published version of the manuscript.

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