Numerical Modeling of Concrete Deep Beams Made with Recycled Aggregates and Steel Fibers

Nancy Kachouh, Tamer El-Maaddawy *©, Hilal El-Hassan © and Bilal El-Ariss ©

Department of Civil and Environmental Engineering, College of Engineering, Al Ain Campus, United Arab Emirates University, Al Ain P.O. Box 15551, United Arab Emirates; nancy.kachouh@uaeu.ac.ae (N.K.); helhassan@uaeu.ac.ae (H.E.-H.); bilal.elariss@uaeu.ac.ae (B.E.-A.)
* Correspondence: tamer.maaddawy@uaeu.ac.ae

Abstract: A bilinear tensile softening law that can describe the post-cracking behavior of concrete made with recycled concrete aggregates (RCAs) and steel fibers was developed based on an inverse analysis of characterization test data. Numerical simulation models were developed for large-scale concrete deep beams. The tensile softening laws along with characterization test results were used as input data in the analysis. The numerical deep beam models were validated through a comparative analysis with published experimental results. A parametric study was conducted to investigate the effect of varying the shear span-to-depth (a/h) ratio, steel fiber volume fraction (v_f), and the presence of a web opening on the shear response. Results of the parametric study indicated that the shear strength gain caused by the addition of steel fibers at v_f of 1 and 2% was higher in the deep beam models with a lower a/h of 0.8, relative to that of their counterparts with a/h of 1.6. The effect of a/h on the shear strength gain of the solid deep beam models diminished at the higher v_f of 3%. The solid deep beam models with a/h of 0.8 exhibited a shear strength gain of 78 to 108% due to the addition of steel fibers, whereas their counterparts with the web opening experienced a reduced shear strength gain of 45 to 70%.

Keywords: numerical modeling; steel fibers; recycled concrete aggregates; deep beam; tensile softening; shear behavior; web openings

1. Introduction

Construction and demolition wastes (CDW) resulting from the demolition of aged concrete structures are mostly disposed of in landfills, creating serious environmental hazards [1]. Recycling wasted construction material in the form of recycled concrete aggregates (RCAs) is considered a sustainable solution to the problem of disposal of the CDW and to reduce the demand on natural aggregates (NAs). Despite their significant environmental benefits, the use of RCAs in the concrete industry is still limited to non-structural applications due to their inferior properties compared with those of NAs [2–4]. The addition of steel fibers to cement-based mixtures improves the tensile strength, post-cracking tensile resistance, flexural strength, and energy absorption capacity [5,6]. Steel fibers have a potential to transform the mode of failure of cement-based mixtures from quasi-brittle to pseudo-ductile through bridging micro- and macro-cracks caused by an applied load [6]. As such, one of the solutions proposed in the literature to improve various properties of concrete with RCAs is the addition of steel fibers [7–10]. Steel fibers should, however, be used with caution in the case of concrete structures in an aggressive environment [11].

Reinforced concrete (RC) deep beams (i.e., a/h ≤ 2 [12]) are typically used as pre-cast structural walls, bent caps in concrete bridges, or transfer girders in high-rise buildings. Openings are typically installed in RC deep beams to create windows or provide passage for electrical and mechanical ducts. Web openings produce discontinuity in the normal
flow of stresses which reduces the beam shear capacity and stiffness [13–17]. Minimum amounts of transverse reinforcement are typically required in RC structural elements to ensure adequate crack control and sufficient ductility [12,18]. The use of steel fibers instead of conventional steel stirrups could compensate for the initial cost of steel fibers, eliminate congestion of steel, and reduce the risk of defects and cracks [19–22]. Previous experimental studies verified the potential of steel fibers to improve the shear behavior of RC deep beams with and without openings [23–28]. Results of these studies are, however, limited to specific properties of the materials used, dimensions and geometry of the tested beams, and reinforcement details. Published analytical formulas for shear strength prediction of steel fiber-reinforced concrete deep beams [23,24,29–32] are not universally applicable and not capable of predicting the complete structural response.

Finite element (FE) analysis is considered a powerful approach for an in-depth analysis of the non-linear behavior of complex concrete members, such as deep beams with and without openings made with RCAs and steel fibers. One of the key parameters needed for modeling the response of steel-fiber reinforced concrete deep beams made of RCAs is the tensile softening law (i.e., tension function). In principle, a uniaxial tensile test is used to directly determine the tensile softening law of steel fiber-reinforce concrete [33]. Steel fibers remarkably enhanced the concrete post-cracking behavior through achieving higher ductility, residual strength, and toughness [34]. However, due to the complexity of this test, alternative indirect methods involving an inverse analysis have been proposed. These methods require testing of concrete prisms, setting up parameters of a tension function, and iterations until the difference between the predicted response of the tested prisms and that obtained from the experimental test becomes negligible [35–38].

Previous studies indicated that the tensile softening law of concrete reinforced with a single type of fiber can be approximated with a bilinear law, whereas for concrete made with the addition of different types of micro and macro steel fibers, a trilinear law may be suggested [35,36]. A comparison between the predicted load-deflection response and that of the tested prisms verified the validity of employing a bilinear softening curve in the analysis [36]. Employing a trilinear softening curve also reasonably predicted the load-crack mouth opening displacement (CMOD) response of the tested prisms [36]. The predicted tensile stress-crack opening relationship employing the concept of the inverse analysis of flexural test data was in good agreement with that recorded experimentally [37]. The accuracy of the response predicted based on the inverse analysis conducted by [37] was even better than that obtained by adopting the procedure of the fib 2010 Model Code [38]. Efforts on the development of FE models for large-scale steel fiber-reinforced concrete beams are continuously reported in the literature [39,40]. Numerical FE models of large-scale concrete elements reinforced with steel fibers employed a concrete tensile softening law derived from flexural tests on concrete prisms combined with the inverse analysis [39,40]. Predictions of such FE models compared well with experimental data of tested deep beams with \( v_f \) of 0.75% and 1% [39] and slender beams having \( v_f \) of 0.5% [40]. To the best knowledge of the authors, tensile softening laws necessary for FE modeling of concrete structures made with RCAs and steel fibers are not available in the literature.

2. Research Objectives

International codes and standards do not offer a closed form solution to predict the shear behavior of steel fiber-reinforced recycled aggregates concrete deep beams. Published analytical formulas are largely empirical and not capable of predicting the complete structural response. This research aims to fill this gap through development of numerical simulation deep beam models and conducting a parametric study to examine the influence of key parameters on the shear behavior of RC deep beams made with RCAs and steel fibers. The specific objectives of the work are:

- Establish tensile softening laws (tension functions) that can describe the post-cracking behavior of concrete made with RCAs and different steel fiber volume fractions.
- Develop numerical simulation models capable of predicting the shear behavior of steel fiber-reinforced recycled aggregates concrete deep beams.
- Validate the prediction of the numerical models through a comparative analysis with published experimental data.
- Investigate the effect of varying \(a/h\), \(v_f\), and presence of a web opening on the shear capacity and strength gain caused by the addition of steel fibers.

3. Methodology and Material Characteristics

Previous investigations [40–48] verified the validity of the software ATENA 3D [49] to predict and simulate the behavior of different structural elements with good accuracy, and hence, was adopted in the current study. Although the software has built-in constitutive laws for conventional concrete, such laws are not available for concrete made with RCAs and steel fibers. These constitutive laws are developed by the users based on material characterization test data.

The uniaxial constitutive laws of the concrete in compression and tension start with a linear part with a slope value equal to the concrete modulus of elasticity \((E_c)\). The compressive plastic behavior and the tensile softening response are then represented by a nonlinear function as shown in Figure 1a,b, respectively. The compressive plastic response starts at a compressive stress ratio \((\sigma_c/f'_c)\) of 0.25, where \(f'_c\) is the concrete compressive strength. The following change in slope occurs at \(\sigma_c/f'_c = 0.8\) with a corresponding plastic compressive strain of 0.5\(\varepsilon_{cp}\), where \(\varepsilon_{cp}\) is the plastic concrete strain at compressive strength generated by the software based on the concrete cube strength \((f_{cu})\). The peak compressive stress-strain law is linearly descending. The value of the concrete plastic strain at a complete release of stress \((\varepsilon_d)\) is generated by the software. For steel-fiber reinforced concrete, the software manual recommends adopting a value of \(\kappa\varepsilon_d\) for the concrete plastic strain at zero stress, where \(\kappa\) is a magnification factor equal to 100 [49]. It should be noted that adopting a magnification factor of 50 rather than 100 had no effect on numerical results, whereas deactivating the magnification factor reduced the shear capacity by less than 4%.

![Figure 1. Details of the user concrete material model: (a) compressive hardening-softening behavior; (b) tensile softening behavior.](image)

As shown in Figure 1b, the post-peak response of plain concrete in tension is bilinear. The first part is linearly descending until the tensile stress reaches a tensile stress ratio \((\sigma_t/f_t)\) of 0.25 at a plastic tensile strain of 0.15\(\varepsilon_{tp,max}\), where \(f_t\) is tensile strength, and \(\varepsilon_{tp,max}\) is maximum plastic fracture strain at zero stress. The second part of the post-peak response is linearly descending but at a slightly lower slope until a complete release of stress occurs at a plastic strain value of \(\varepsilon_{tp,max}\). The user-defined model allows users to account for the tension stiffening effect in heavily reinforced concrete by adopting a limiting minimum value of a reduced tensile strength \(\leq0.4 f_t\) on the tensile softening branch. In such cases, the tensile strength cannot drop below the pre-specified limiting tensile strength taken as 0.25\(f_t\) in the current study for the models without steel fibers. Steel fiber-reinforced concrete is characterized by a significant post-peak residual tensile stress–strain response.
The typical post-peak response of this concrete is linearly descending until it reaches a stress value of \( C_{sf} f_r \), where \( C_{sf} \) is coefficient determined from the inverse analysis, and then the response continues to decrease, linearly or multilinearly at slighter slopes, until it reaches a maximum plastic fracture strain of \( \epsilon_{\text{fr}} \) determined from the inverse analysis. The steel bars used in the large-scale RC deep beam models included No. 25 (25 mm diameter) deformed bars as flexural reinforcement and No. 5 (5 mm diameter) smooth bars as shear reinforcement. Based on testing of three replicate steel samples, the average measured yield strengths of the No. 25 and No. 5 bars were 539 and 505 MPa, respectively, whereas their ultimate tensile strengths were 649 and 543 MPa, respectively. The elastic modulus of the reinforcing steel bars was 200 GPa. The reinforcing steel bars were modeled using a bilinear stress–strain relationship with a post-yield strain hardening of 1%.

3.1. Properties of Concrete Mixtures

Characterization tests were conducted on five concrete mixtures adopted in the current study. The mixes are labeled RX-SFY, where X denotes the percentage of RCAs and Y indicates the value of \( v_f \) (Table 1). The control mix R0-SF0 was designed to develop \( f'_c \) of 30 MPa, based on the American Concrete Institute (ACI) 211.1 [50]. The cement was ordinary Portland cement (OPC), while the water–cement ratio (w/c) was 0.49. The natural coarse aggregates were crushed dolomitic limestone with a nominal maximum particle size (NMS) of 19 mm, whereas the recycled coarse ratio (w/c) was 0.49. The natural coarse aggregates were crushed dolomitic limestone with a nominal maximum particle size (NMS) of 19 mm, whereas the recycled coarse aggregates were collected from a local concrete recycling plant with an NMS of 25 mm. The concrete recycling plant crushes old concrete structures with an unknown compressive strength. The fine aggregates used were dune sand. The concrete mix proportions by weight were as follows (cement:fine aggregates:coarse aggregates:w/c; 1:1.21:2.4:0.49). The steel fibers used in this study, Dramix®3D 65/35BG, were end-hooked with a tensile strength of 1345 N/mm², Young’s modulus of 210 GPa, mean diameter \((d_f)\) of 0.55 mm, mean length \((l_f)\) of 35 mm, and an aspect ratio \((l_f/d_f)\) of 65 [51]. A photograph of the steel fibers is presented in Figure 2.

Mechanical properties of the concrete mixtures, based on data of three replicate specimens, are reported in Table 1. The addition of steel fibers insignificantly affected the compressive strength and modulus of elasticity of the concrete, but remarkably improved the modulus of rupture \( f_r \). An increase of 71% in \( f_r \) was recorded at \( v_f \) of 1%. Higher values of \( v_f \) resulted in more than a 2-fold increase \( f_r \).

Figure 2. Steel fibers used in the current study.
Table 1. Mechanical properties of concrete mixtures.

<table>
<thead>
<tr>
<th>Property</th>
<th>Standard</th>
<th>Sample Type and Dimensions (mm)</th>
<th>Characterization Test Results *</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$ (MPa)</td>
<td>ASTM C39 [52]</td>
<td>Cylinder, 150 x 300</td>
<td>R0-SF0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>36.4 (1.5)</td>
</tr>
<tr>
<td>$f_{cu}$ (MPa)</td>
<td>BS 12390-3 [53]</td>
<td>Cube, 150</td>
<td>40.5 (1.8)</td>
</tr>
<tr>
<td>$f_t$ (MPa)</td>
<td>ASTM C1609 [54]</td>
<td>Prism, 100 x 100 x 500</td>
<td>3.4 (0.2)</td>
</tr>
<tr>
<td>$E_c$ (GPa)</td>
<td>ASTM C469 [55]</td>
<td>Cylinder, 150 x 300</td>
<td>34.7 (1.8)</td>
</tr>
</tbody>
</table>

* Values between parentheses represent the standard deviation.

3.2. Tensile Softening Laws of Concrete Mixtures

Four-point bending tests were conducted on concrete prisms (100 x 100 x 500 mm) made of RCAs and steel fibers following the ASTM C1609 [54]. Figure 3a shows the four-point bending test setup, while Figure 3b shows a photograph of a test in progress. The inverse analysis technique started by developing FE models for the tested prisms and setting up specific tensile parameters in a user-defined tensile softening law of the concrete. The value of $f_t$ was kept constant at 0.6$f_c$ [56]. The other input parameters of the post-cracking tension function were modified, and several iterations were considered until the difference between the numerical and experimental load–displacement curves of the prisms became negligible.

![Figure 3. Four-point test setup.](image)

Figure 4a shows the experimental load–deflection curves of three replicate prisms for each concrete mix along with the corresponding predicted response, whereas Figure 4b shows the tension functions implemented in the analysis. Figure 4a shows that the flexural load capacity and toughness increased with an increase in the steel fiber volume fraction. Figure 4b indicates that the tension function of the concrete with steel fibers is approximated with a bilinear law. When the concrete reaches its tensile strength, $f_t$, the tensile stress drops to a value of approximately 0.6$f_c$, then decreases linearly with an increase in the tensile strain. The tension function of the concrete with $\nu_f$ of 2 and 3% exhibited a slightly reduced rate of degradation in the tensile stress after cracking relative to that of the concrete with $\nu_f$ of 1%. The agreement between the predicted and experimental load–deflection responses of the prisms verified the validity of the tension function developed from the inverse analysis.
Table 2. Details of the deep beam models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Type</th>
<th>Reinforcement</th>
<th>Concrete Mix</th>
<th>Steel Fiber</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-1</td>
<td>Solid</td>
<td>2 No. 25 (25 mm in diameter) at the tension side</td>
<td>40-250</td>
<td>1%</td>
<td></td>
</tr>
<tr>
<td>S-2</td>
<td>Solid</td>
<td>4 No. 25 (25 mm in diameter) at the tension side</td>
<td>40-250</td>
<td>3%</td>
<td></td>
</tr>
<tr>
<td>N-1</td>
<td>Steel</td>
<td>4 No. 25 (25 mm in diameter) at the tension side</td>
<td>40-250</td>
<td>1%</td>
<td></td>
</tr>
<tr>
<td>N-2</td>
<td>Steel</td>
<td>4 No. 25 (25 mm in diameter) at the tension side</td>
<td>40-250</td>
<td>3%</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4. Four-point bending test data: (a) load–deflection response; (b) tension functions.

3.3. Large-Scale Deep Beam Numerical Models

Fourteen FE models were developed for large-scale RC deep beams tested previously by the authors [27,28]. A summary of the deep beam models is given in Table 2. The models were divided into two groups, S and N, where S refers to solid beams, and N refers to beams with a circular web opening in the middle of each shear span with an opening height-to-beam depth ratio (h₀/h) of 0.3. The deep beam models of group S had a/h of 1.6, whereas those of group N had a/h of 0.8. The variables in each group included the type of aggregates (NAs and RCAs), steel fiber volume fraction (1%, 2%, and 3%), and presence of minimum steel stirrups recommended by provisions of the American Concrete Institute (ACI) 318-14 [12] for deep beams.

A quarter FE model was built for each beam to take advantage of the planes of symmetry and reduce the overall computational time. Figures 5 and 6 show the geometry, boundary condition, details of reinforcement, and location of monitoring points of typical deep beam models of groups S and N, respectively. The deep beam models were reinforced with 4 No. 25 (25 mm in diameter) steel bars at the tension side and 2 No. 25 (25 mm in diameter) at the compression side. The concrete cover to the center of the steel reinforcement was 50 mm, rendering an effective depth (d) of 450 mm. The steel stirrups, when employed, consisted of two curtains of No. 5 (5 mm in diameter) bars spaced at 80 mm in both vertical and horizontal directions. Steel plates (150 × 150 × 20 mm) were placed at the load and support points. The concrete and steel plates were modeled as solid 3D tetrahedron macro-elements with a mesh size of 15 mm. The steel bars were modeled as one-dimensional discrete elements embedded in the concrete macro-element.
Table 2. Summary of deep beam models.

<table>
<thead>
<tr>
<th>Group</th>
<th>Model Designation</th>
<th>a/h</th>
<th>RCA (%)</th>
<th>vf (%)</th>
<th>Presence of Traditional Shear Reinforcement</th>
<th>Presence of Openings</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>SR0-SF0</td>
<td>1.6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>SR0-SF0-S</td>
<td>1.6</td>
<td>-</td>
<td>-</td>
<td>√</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>SR100-SF0</td>
<td>1.6</td>
<td>100</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>SR100-SF0-S</td>
<td>1.6</td>
<td>100</td>
<td>-</td>
<td>√</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>SR100-SF1</td>
<td>1.6</td>
<td>100</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>SR100-SF2</td>
<td>1.6</td>
<td>100</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>SR100-SF3</td>
<td>1.6</td>
<td>100</td>
<td>3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>N</td>
<td>NR0-SF0</td>
<td>0.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>NR0-SF0-S</td>
<td>0.8</td>
<td>-</td>
<td>-</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>NR100-SF0</td>
<td>0.8</td>
<td>100</td>
<td>-</td>
<td>-</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>NR100-SF0-S</td>
<td>0.8</td>
<td>100</td>
<td>-</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>NR100-SF1</td>
<td>0.8</td>
<td>100</td>
<td>1</td>
<td>-</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>NR100-SF2</td>
<td>0.8</td>
<td>100</td>
<td>2</td>
<td>-</td>
<td>√</td>
</tr>
<tr>
<td></td>
<td>NR100-SF3</td>
<td>0.8</td>
<td>100</td>
<td>3</td>
<td>-</td>
<td>√</td>
</tr>
</tbody>
</table>

Figure 5. A typical deep beam model of group S (dimensions are in mm): (a) beam geometry configuration and boundary conditions; (b) reinforcement details and monitoring points.
Figure 6. A typical deep beam model of group N (dimensions are in mm): (a) beam geometry configuration and boundary conditions; (b) reinforcement details and monitoring points.

A line support was placed at the middle of the bottom surface of the support steel plate to restrict its movement in the vertical and transverse directions. The surfaces of the planes of symmetry were restrained from movement in a direction perpendicular to the other symmetrical part of the beam through surface supports. A displacement-controlled applied load was induced at the middle of the top steel plate at a rate of 0.1 mm per step. The standard Newton–Raphson iterative solution method implemented in ATENA [49] was adopted. The iteration had to satisfy a tolerance limit of convergence criteria of 1%.

4. Numerical Results and Validation of FE Deep Beam Models

Predictions of the FE deep beam models are compared with those obtained from experimental tests conducted previously by the authors [27,28] for validation.

4.1. Solid Deep Beam Models

4.1.1. Shear Load–Deflection Response

The shear load–deflection curves of the solid RC deep beam models are illustrated in Figure 7. The response of the deep beam models made with 100% RCAs without steel fibers was inferior to that of their counterparts made with NAs (Figure 7a). The addition of steel fibers greatly improved the shear capacity and stiffness of the RCA-based models (Figure 7b). The stiffness of model SR100-SF2, with $v_f = 2\%$, coincided with that of SR100-SF0-S having steel stirrups, while that of model SR100-SF3, with $v_f = 3\%$, was...
superior to that of SR100-SF0-S. These findings are in agreement with those obtained from experimental tests [27,28].

The experimental and numerical shear capacities, $V_{\text{EXP}}$ and $V_{\text{FE}}$, respectively, along with the deflections at failure, $\Delta_{\text{EXP}}$ and $\Delta_{\text{FE}}$, are compared in Table 3. The difference between the numerical and experimental shear capacities was within 10% only. The ratio of $V_{\text{FE}}/V_{\text{EXP}}$ was on average 0.98 with a corresponding standard deviation and coefficient of variation of 0.07% and 7%, respectively. The FE models tended to underestimate the deflections at failure. The ratio of $\Delta_{\text{FE}}/\Delta_{\text{EXP}}$ was on average 0.83 with a standard deviation of 0.10 and a coefficient of variation of 12%. Generally, the difference between numerical and experimental results could be due to a variation between the actual properties of materials of the tested large-scale deep beams and those obtained from characterization test samples used as input data in the analysis. The difference between numerical and experimental results can be considered within the expected margin of error.

Figure 7. Numerical shear load–deflection response of group S: (a) beam models without steel fibers; (b) beam models with RCAs/steel fibers.

The shear load–deflection responses predicted numerically are compared with those obtained from the tests in Figure 8. The response of the deep beam models SR0-SF0 and SR100-SF0 exhibited a drop in load at the onset of the initiation of shear cracking. Then, the deflection continued to increase but at a higher rate (Figure 8a,c). Another drop in load was observed prior to failure due to the initiation of another shear crack (Figure 8a,c). The deep beam models with steel stirrups (Figure 8b,d) or steel fibers (Figure 8e–g) exhibited a quasilinear load–deflection response until the shear capacity was reached. The stiffness of the numerical models was insignificantly higher than that obtained from the tests. Actual beams tested in the laboratory were vulnerable to microcracking due to shrinkage, which would reduce the actual stiffness of the tested beams.

The experimental and numerical shear capacities, $V_{\text{EXP}}$ and $V_{\text{FE}}$, respectively, along with the deflections at failure, $\Delta_{\text{EXP}}$ and $\Delta_{\text{FE}}$, are compared in Table 3. The difference between the numerical and experimental shear capacities was within 10% only. The ratio of $V_{\text{FE}}/V_{\text{EXP}}$ was on average 0.98 with a corresponding standard deviation and coefficient of variation of 0.07% and 7%, respectively. The FE models tended to underestimate the deflections at failure. The ratio of $\Delta_{\text{FE}}/\Delta_{\text{EXP}}$ was on average 0.83 with a standard deviation of 0.10 and a coefficient of variation of 12%. Generally, the difference between numerical and experimental results could be due to a variation between the actual properties of materials of the tested large-scale deep beams and those obtained from characterization test samples used as input data in the analysis. The difference between numerical and experimental results can be considered within the expected margin of error.

Figure 8. Cont.
Figure 8. Numerical and experimental responses of group S: (a) SR0-SF0; (b) SR0-SF0-S; (c) SR100-SF0; (d) SR100-SF0-S; (e) SR100-SF1; (f) SR100-SF2; and (g) SR100-SF3.

Table 3. Comparison between numerical and experimental results of group S.

<table>
<thead>
<tr>
<th>Model</th>
<th>Shear Capacity</th>
<th>Deflection Capacity</th>
<th>$V_{FE}/V_{EXP}$</th>
<th>$\Delta FE/\Delta EXP$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_{EXP}$ (kN)</td>
<td>$V_{FE}$ (kN)</td>
<td>$\Delta EXP$ (mm)</td>
<td>$\Delta FE$ (mm)</td>
</tr>
<tr>
<td>SR0-SF0</td>
<td>203</td>
<td>198</td>
<td>7.3</td>
<td>7.1</td>
</tr>
<tr>
<td>SR0-SF0-S</td>
<td>309</td>
<td>280</td>
<td>10.3</td>
<td>8.0</td>
</tr>
<tr>
<td>SR100-SF0</td>
<td>193</td>
<td>181</td>
<td>9.0</td>
<td>6.2</td>
</tr>
<tr>
<td>SR100-SF0-S</td>
<td>300</td>
<td>270</td>
<td>10.7</td>
<td>8.2</td>
</tr>
<tr>
<td>SR100-SF1</td>
<td>235</td>
<td>251</td>
<td>8.3</td>
<td>7.9</td>
</tr>
<tr>
<td>SR100-SF2</td>
<td>271</td>
<td>296</td>
<td>10.4</td>
<td>9.5</td>
</tr>
<tr>
<td>SR100-SF3</td>
<td>401</td>
<td>387</td>
<td>15.8</td>
<td>12.0</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std Dev</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COV (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.1.2. Crack Patterns

The crack patterns of the solid deep beams captured numerically are shown in Figure 9a. The deep beam models SR0-SF0 and SR100-SF0 exhibited initially an inclined shear crack in the shear span. At failure, the models exhibited an additional shear crack in the shear span in addition to longitudinal splitting cracks parallel to the tension steel. The deep beam models with stirrups, SR0-SF0-S and SR100-SF0-S, showed a band of shear cracks in the shear span prior to failure. The steel fibers played a role similar to that of the steel stirrups and prevented the formation of longitudinal splitting cracks. The deep beam models with steel fibers failed in a diagonal compression model of failure.

![Figure 9](image.png)

Figure 9. Crack patterns and principal strains of group S: (a) numerical crack patterns; (b) minimum principal strains; and (c) experimental crack patterns.

The minimum principal concrete strain of the deep beam models prior to failure are shown in Figure 9b. The profile of the minimum principal concrete strain indicated the formation of a bottle-shaped strut in the shear span of the deep beam models with steel
stirrups and a prismatic strut for the models with steel fibers. Photographs of the tested beams at failure are shown in Figure 9c. The crack patterns obtained from the tests, shown in Figure 9c, verified the formation of a band of shear cracks in the shear span of the deep beams with steel stirrups and concentration of inclined shear cracks along the natural load path of the beams with steel fibers.

It should be noted that the deep beam models without steel fibers exhibited similar values of minimum principal concrete strain of approximately $-0.006$ to $-0.008$ prior to failure. The models with steel stirrups failed, however, at a higher shear capacity than that of their counterparts without stirrups, signifying the role of stirrups in reducing the rate of increase in the minimum principal concrete strain. The deep beam models with steel fibers exhibited minimum principal concrete strain values of approximately $-0.006$ to $-0.01$ at peak load but failed at a higher load relative to that of their counterparts without stirrups. The increased shear capacity of the steel-fiber reinforced models signified the effectiveness of the steel fibers in reducing the rate of increase in the minimum principal concrete strains.

### 4.1.3. Stirrup Strains

The numerical stirrup strain responses of the model SR0-SF0-S are compared with those measured experimentally in the west and east shear spans in Figure 10a,b, respectively. Readings of SG-V2 were not captured experimentally in the east shear span due to malfunction of the SG. The locations of the stirrups monitoring points are shown in Figure 5. It should be noted that predicted strains of SG-V1 and SG-V2 were identical. Similarly, the rate of increase in the stirrup strain in the vertical stirrups recorded experimentally was insignificantly different, as shown in Figure 10a. Numerical data of the model SR0-SF0-S indicated failure of the beam without yielding stirrups. This behavior was in alignment with the experimental data, except in one location (SG-V1) in the west shear span. Numerical results indicated that no strains were recorded in the stirrups in the pre-cracking stage. Following shear cracking, the stirrup strains increased almost linearly until the model reached its shear capacity. This behavior was in agreement with the stirrup strains measured in the west and east shear spans as shown in Figure 10a,b, respectively.

![Figure 10. Numerical and experimental stirrup strain response of SR0-SF0-S: (a) west shear span; (b) east shear span.](image)

The numerical stirrup strain responses of the model SR100-SF0-S are compared with those measured experimentally in the west and east shear spans in Figure 11a,b, respectively. The vertical stirrups in the model SR100-SF0-S made with RCAs exhibited higher strains than those exhibited by the horizontal stirrups, indicating more contribution to the shear resistance. Similarly, the vertical stirrup strains obtained from the tests tended to be higher than those of the horizontal stirrups in the west and east shear spans as shown in Figure 11a,b, respectively. The strain in the vertical stirrup of model SR100-SF0-S predicted numerically at peak load exceeded the yielding strain. Experimental results verified yielding of SG-V1 and SG-V2 in the west shear span (Figure 11a) and SG-V2 in the east shear span (Figure 11b) prior to failure. The numerical results show that the vertical stirrup
strains of model SR100-SF0-S increased at a higher rate than that of SR0-SF0-S, indicating less contribution of RCAs to the shear resistance. This behavior was less evident in the experimental results that showed no significant difference in the rate of increase in the stirrup strains of the two tested beams, SR0-SF0-S and SR100-SF0-S.

![Figure 11](image1.png)

**Figure 11.** Numerical and experimental stirrup strain response of SR100-SF0-S: (a) west shear span; (b) east shear span.

4.1.4. Tensile Steel Strains

The numerical and experimental tensile steel strain responses of the solid RC deep beam models are presented in Figure 12. The monitoring point of SG1 was located at a distance 125 mm away from the face of the support plate, whereas that of SG4 was under the load point. Other monitoring points, SG2 and SG3, were located in between SG1 and SG4 at a spacing of 200 mm (refer to Figure 5). The numerical and experimental steel strain responses showed a similar trend. The steel strain responses of the deep beam models SR0-SF0 and SR100-SF0 exhibited load decays due to development of shear cracks (Figure 12a,c). No load decays occurred in the deep beam models with steel stirrups (Figure 12b,d) or steel fibers (Figure 12e–g). The strains predicted numerically at SG1, which was close to the support, were lower than those recorded at other locations in all models. In agreement with the experimental results, the steel strains in other locations within the shear span were insignificantly different, verifying the arch action effect in all models. The tensile steel reinforcement did not reach the yielding strain in any of the models, which was verified experimentally.

![Figure 12](image2.png)

**Figure 12.** Cont.
Figure 12. Numerical and experimental steel strain responses of group S: (a) SR0-SF0; (b) SR0-SF0-S; (c) SR100-SF0; (d) SR100-SF0-S; (e) SR100-SF1; (f) SR100-SF2; and (g) SR100-SF3.

4.2. Deep Beam Models with Openings

4.2.1. Shear Load–deflection Response

The predicted shear load–deflection curves of the deep beam models containing openings are illustrated in Figure 13. The use of RCAs rather than NAs reduced the stiffness and shear capacity of the models, as shown in Figure 13a. The addition of steel fibers improved the stiffness and remarkably increased the shear capacity of the deep beam models, as shown in Figure 13b. The beam models having steel fibers exhibited an
improved stiffness even better than that of NR100-SF0 with NAs and conventional steel stirrups. It is worth noting that when the steel fiber-reinforced concrete models reached approximately 90% of the shear capacity, they started to exhibit an increased midspan deflection without a significant increase in load until failure. This plastic response can be attributed to the tension stiffening effect caused by steel fibers, which improved the deformability of the models.

Figure 13. Numerical shear load–deflection responses of group N: (a) beam models without steel fibers; (b) beam models with RCAs/steel fibers.

The shear load–deflection responses of the models without steel fibers predicted numerically are compared with those obtained experimentally in Figure 14a–d. The predicted response of the deep beam models without steel fibers tended to be stiffer than that obtained from the experiment. This behavior may be attributed to the existence of microcracks in the tested beams prior to testing caused by shrinkage or occurred during handling. Such microcracks would reduce the actual stiffness of the tested beams. The numerical shear load–deflection responses of the models with steel fibers are compared with those obtained from the tests in Figure 14e–g. The difference between the numerical and experimental stiffnesses of the deep beam models with steel fibers was less significant relative to that of the models without steel fibers.

The experimental and numerical shear capacities, $V_{\text{EXP}}$ and $V_{\text{FE}}$, respectively, along with the deflections at failure, $\Delta_{\text{EXP}}$ and $\Delta_{\text{FE}}$, are compared in Table 4. The predicted shear capacities were within the 11% error band, except for model NR100-SF3 with $V_{\text{FE}}/V_{\text{EXP}}$ of 0.82. The ratio of $V_{\text{FE}}/V_{\text{EXP}}$ was on average 0.92 with a corresponding standard deviation and coefficient of variation of 0.05% and 6%, respectively. The predicted deflections at failure for the models with steel fibers were generally in good agreement with those measured experimentally with $\Delta_{\text{FE}}/\Delta_{\text{EXP}}$ in the range of 0.81 to 0.98. The models tended, however, to underestimate the deflection at the failure of the beams without steel fibers. It is worth noting that the measured deflections at the failure of the models without steel fibers were very small which made the ratio of $\Delta_{\text{FE}}/\Delta_{\text{EXP}}$ very sensitive to any small difference (within 2 mm) between predicted and measured deflections. The ratio of $\Delta_{\text{FE}}/\Delta_{\text{EXP}}$ of all models was on average 0.72 with a standard deviation of 0.18 and a coefficient of variation of 25%.
Figure 14. Numerical and experimental responses of group N: (a) NR0-SF0; (b) NR0-SF0-S; (c) NR100-SF0; (d) NR100-SF0-S; (e) NR100-SF1; (f) NR100-SF2; and (g) NR100-SF3.
4.2.2. Crack Patterns

The crack patterns of the deep beam models with openings predicted numerically are shown in Figure 15a. All models initially exhibited a diagonal shear crack crossing the center of the opening. This crack did not propagate further as the load progressed. At higher loads, inclined shear cracks developed in the upper and lower chords in the direction of the corresponding load paths. The models without steel stirrups nor steel fibers failed due to the formation of independent diagonal splitting cracks in the direction of upper and lower load paths. The models with stirrups exhibited multiple parallel inclined cracks along in the direction of the upper and lower natural load paths prior to failure.

The minimum principal concrete strain of the deep beam models prior to failure are shown in Figure 15b. The profile of the minimum principal concrete strains of the models with steel stirrups or steel fibers verified the formation of diagonal struts in the direction of the upper and lower load paths. The deep beam models without steel fibers exhibited an average minimum principal concrete strain value of approximately \(-0.008\) at peak load. The models with steel stirrups failed, however, a higher shear capacity than that of their counterparts without stirrups, indicating a lower rate of increase in the minimum principal concrete strains relative to that of their counterparts without stirrups. The deep beam models with steel fibers at \(v_f\) of 1\%, 2\%, and 3\% reached their shear capacity at minimum principal strain values of approximately \(-0.014\), \(-0.018\), and \(-0.020\), respectively. The increased minimum principal strain capacities exhibited by the models with steel fibers enabled the models to sustain higher load and deflection capacities prior to failure than those of their counterparts without steel fibers.

Figure 15c shows photographs of the tested beams with openings at failure. The crack patterns obtained from the tests (Figure 15c) are in good agreement with those predicted numerically (Figure 15a). Experimental results verified the frame-type mode of failure in the deep beams without shear reinforcement and the shear compression mode of failure along the upper and lower load paths in the deep beams with stirrups or steel fibers.

Table 4. Comparison between numerical and experimental results of group N.

<table>
<thead>
<tr>
<th>Model</th>
<th>Shear Capacity</th>
<th>Deflection Capacity</th>
<th>(V_{FE}/V_{EXP})</th>
<th>(\Delta_{FE}/\Delta_{EXP})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(V_{EXP}) (kN)</td>
<td>(V_{FE}) (kN)</td>
<td>(\Delta_{EXP}) (mm)</td>
<td>(\Delta_{FE}) (mm)</td>
</tr>
<tr>
<td>NR0-SF0</td>
<td>273</td>
<td>273</td>
<td>4.7</td>
<td>2.9</td>
</tr>
<tr>
<td>NR0-SF0-S</td>
<td>341</td>
<td>327</td>
<td>4.8</td>
<td>3.0</td>
</tr>
<tr>
<td>NR100-SF0</td>
<td>237</td>
<td>211</td>
<td>4.9</td>
<td>2.6</td>
</tr>
<tr>
<td>NR100-SF0-S</td>
<td>279</td>
<td>255</td>
<td>5.3</td>
<td>2.7</td>
</tr>
<tr>
<td>NR100-SF1</td>
<td>329</td>
<td>306</td>
<td>5.6</td>
<td>5.4</td>
</tr>
<tr>
<td>NR100-SF2</td>
<td>362</td>
<td>326</td>
<td>6.0</td>
<td>5.9</td>
</tr>
<tr>
<td>NR100-SF3</td>
<td>435</td>
<td>358</td>
<td>7.4</td>
<td>6.0</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std dev</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COV (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 15. Cont.
4.2.3. Stirrup Strains

The stirrup strain responses predicted numerically of the deep beam models NR0-SF0-S and NR100-SF0-S are presented in Figure 16a,b respectively. The locations of the monitoring points are shown in Figure 6. All predicted stirrup strains were below the yielding value indicating that failure was controlled by concrete due to the presence of the openings. Although experimental stirrup strain data at locations of most of the monitoring points were not available, the few stirrup strain data obtained from the experimental tests showed no yielding of stirrups [28]. The full-depth vertical stirrups located at the left and right sides of the opening in NR0-SF0-S and NR100-SF0-S exhibited minimal strains as shown in Figure 16a,b, respectively. These reduced strains occurred because the major shear cracks were developed in the upper and lower chords rather than at the sides of the opening. Upper and lower chord stirrup strains indicate that shear cracks occurred in NR0-SF0-S at a load value of approximately 170 kN (Figure 16a). Following cracking, there was no significant difference in the rate of increase in the horizontal and vertical stirrup strains in the upper and lower chords. The strains in the vertical and horizontal stirrups at peak load were insignificantly different. The stirrups in the lower chord tended, however, to exhibit slightly higher strains at peak load than those exhibited by the stirrups in the upper chord. Upper and lower chord stirrup strains of model NR100-SF0-S, shown in Figure 16b, indicate the initiation of shear cracks at a load value of approximately 130 kN. Figure 16c shows the horizontal and vertical stirrup strain responses in the upper chord for models NR0-SF0-S and NR100-SF0-S. It can be seen that the deep beam model NR0-SF0-S made with RCAs exhibited shear cracks at a lower load than that of NR0-SF0-S made with NAs. Following shear cracking, the stirrup strains of NR100-SF0-S increased at a slightly higher rate than that of model NR0-SF0-S. The reduced shear cracking load and the increased stirrup strains of model NR100-SF0-S compared with those of model NR0-SF0-S indicated less contribution of RCAs to the shear resistance than NAs.
Figure 16. Numerical stirrup strain response of group N: (a) NR0-SF0-S; (b) NR100-SF0-S; and (c) horizontal versus vertical stirrup strain responses in the upper chord.

4.2.4. Tensile Steel Strains

The numerical and experimental tensile steel strain responses of the deep beam models containing openings without steel fibers are presented in Figure 17a–d, whereas those with steel fibers are shown in Figure 17e–g. The monitoring point of SG1 was located at a distance 85 mm away from the face of the support plate, whereas that of SG4 was under the load point. Other monitoring points, SG2 and SG3, were located in between SG1 and SG4 at a spacing of 80 mm (refer to Figure 6). The numerical and experimental steel strain responses showed a similar trend. The steel strains predicted numerically in all models at different locations within the shear span were insignificantly different, verifying the deep beam action. This behavior was verified experimentally despite being less evident in the steel strain data measured experimentally in beam NR100-SF0 (Figure 17c). The strains recorded numerically at SG1, which was close to the support, were slightly lower than those recorded at other locations. The tensile steel reinforcement did not reach the yielding strain in any of the models, which has been verified experimentally, except at one location (SG2) in beam NR100-SF3 (Figure 17g).

Figure 17. Cont.
5. Parametric Study and Discussion

It was not possible to test solid RC deep beams with $a/h$ of 0.8 due to the limited capacity of the laboratory facility. The models developed and verified in the current study are considered a valid alternative to laboratory testing. As such, an additional seven solid deep models with $a/h$ of 0.8 were developed. The performance of these models was predicted numerically and compared with that of other models presented earlier with and without openings.

5.1. Effect of Shear Span-to-Depth Ratio

Figure 18 presents the shear load–deflection responses of RCA-based solid deep beam models with different $a/h$ ratios. The models with $a/h$ of 0.8 (Figure 18a) were significantly stiffer than their counterparts with $a/h$ of 1.6 (Figure 18b). Figure 18a shows that at $a/h$ of 0.8, the use of steel fibers with $v_f$ of 1% (model R100-SF1) was sufficient to restore the stiffness of the model with RCAs and minimum steel stirrups (model R100-SF0-S). However, at $a/h$ of 1.6, the use of steel fibers at $v_f$ of 2% (model SR100-SF2) was necessary to restore the stiffness of the counterpart model with RCAs and minimum steel stirrups (model SR100-SF0-S), as shown in Figure 18b. At both $a/h$ values, the stiffness of the models with $v_f$ of 3% coincided with that of the model with NAs and conventional steel stirrups. Figure 19 shows the effect of $a/h$ on the shear capacity of the RCA-based deep beam models. The shear capacity of the deep beam models with steel fibers increased almost linearly with an increase in the steel fiber volume fraction. Obviously, the shear capacity of the deep beam models with $a/h$ of 0.8 was higher than that of their counterparts with $a/h$ of 1.6. For the deep beam models without steel fibers, the increase in the shear capacity due to the decrease in the $a/h$ from 1.6 to 0.8 ranged from 31 to 66% with an average of 51%, whereas for those with steel fibers, the increase in shear capacity ranged from 42 to 88% with an average of 67%.
The shear strength gains exhibited by the models with NAs and those of their counterparts made with RCAs without steel stirrups nor steel fibers served as a benchmark. The shear strength gain was calculated relative to the shear capacity ratios. The strength gains caused by the steel stirrups are also included in Figure 21 to serve as a benchmark. The shear strength gain was calculated relative to the shear capacity of RCA-based models. For the deep beam models without steel fibers, the increase in the shear capacity of 1.6. For the deep beam models with steel fibers, the increase in shear capacity ranged from 42 to 88% with an average of 67%.

Figure 18. Shear load–deflection responses of solid deep beam models: (a) $a/h = 0.8$; (b) $a/h = 1.6$.

Figure 19. Effect of $a/h$ on the shear capacity of RCA-based models.

Figure 20 illustrates the effect of the inclusion of minimum steel stirrups on the shear strength gain of the beam models with different types of aggregates at different $a/h$ ratios. The shear strength gains exhibited by the models with NAs and those of their counterparts made with RCAs were insignificantly different, regardless of $a/h$. Figure 21 shows the effect of steel fibers on the shear strength gain of the deep beam models with RCAs at different $a/h$ ratios. The shear strength gains caused by the steel stirrups are also included in Figure 21 to serve as a benchmark. The shear strength gain was calculated relative to the shear capacity of the corresponding control model made with RCAs without steel stirrups nor steel fibers. At $a/h$ of 1.6, the addition of steel fibers at $v_f$ of 1%, 2%, and 3% resulted in respective shear strength gains of 39%, 64%, and 114%. For the models with $a/h$ of 0.8, the addition of steel fibers at $v_f$ of 1%, 2%, and 3% resulted in shear strength gains of 78%, 91%, and 108%, respectively. At $v_f$ of 1 and 2%, the shear strength gain was higher for the models with the lower $a/h$ of 0.8. The improved effectiveness of steel fibers in increasing the shear capacity at the lower $a/h$ has also been reported in previous studies for concrete beams made with NAs. Ashour et al. [23] reported shear strength gains of 97% and 32% at shear span-to-effective depth ($a/d$) ratios of 1 and 6, respectively, due to the addition of steel fibers at $v_f$ of 1.5%. Kwak et al. [24] indicated that the shear strength gain caused by the addition of steel fibers at $v_f$ of 0.75% was in the range of 69 to 80% at $a/d = 2$, whereas for larger $a/d$ of 3 and 4, a reduced shear strength gain of 22 to 38% was reported.
5.2. Effect of Web Openings

Figure 20. Shear strength gain of the models with minimum stirrups at different a/h ratios.

Figure 21. Shear strength gain of the models with RCAs and steel fibers at different a/h ratios.

The presence of steel fibers reduced the severity of cracks within the shear span, improved the tension stiffening effect, and hence, increased the strut capacity. The shear capacity (V) of RC deep beams is sensitive to the strut capacity (Fs) and its angle of inclination in the shear span (θ) through the relationship V = Fs \sin θ. The impact of increasing the strut capacity on the shear strength is more pronounced for struts with a greater angle of inclination θ. This relationship could explain why the shear strength gain caused by steel fibers was higher at the lower a/h of 0.8. The effect of a/h on the shear strength gain caused by steel fibers diminished at the higher vf of 3%. It seems that, at vf of 3%, the increase in the shear capacity was too high to show an effect for the strut angle of inclination. From Figure 21, it can be seen that at a/h of 0.8, the use of vf of 1% was sufficient to provide a shear strength gain higher than that provided by the minimum steel stirrups. At a/h of 1.6, it was necessary to use vf of 2% to substitute the minimum steel stirrups.

5.2. Effect of Web Openings

Figure 22a,b present the shear load–deflection responses of the deep beam models having a/h of 0.8 without and with web openings, respectively. The deep beam models with the web opening (Figure 22b) exhibited a reduced stiffness and significant reduction in the shear capacity relative to those of their counterparts without opening (Figure 22a). At both a/h values, the inclusion of steel fibers at vf of 1% was sufficient to restore the stiffness of the corresponding counterpart model with RCAs and minimum steel stirrups. This behavior is manifested in the responses of counterpart models R100-SF1 and R100-SF0-S in Figure 22a and those of models SR100-SF1 and SR100-SF0-S in Figure 22b.
with web openings, irrespective of the type of aggregates. This occurred because of the reduction in shear capacity due to the presence of the web opening ranged from 17 to 28%, with an average of 23%. For the models with steel fibers, the reduction in shear capacity caused by the web opening ranged from 35 to 36%, with an average of 35%. It is worth mentioning that the shear capacity of the deep beam model having $v_f$ of 3% and the web opening was equal to that of the solid beam model with steel stirrups. Numerical results of the deep beam models with a web opening are consistent with those published in the literature for RC deep beams with openings made with NAs [25,26]. Al Sarraf et al. [25] reported a shear strength reduction of 33% due to the presence of a square opening with $h_0/h$ of 0.2 in RC deep beams ($a/h = 0.7$) without steel fibers, whereas their counterparts with $v_f$ of 1% exhibited a shear strength reduction of 40% due to the web opening. Zewair et al. [26] reported that the existence of a circular web opening ($h_0/h$ of 0.31) reduced the shear capacity of RC deep beams ($a/h = 1$) without steel fibers by 27%, whereas it caused a shear strength reduction of 43 to 58% in their counterparts with $v_f$ of 1%.

![Figure 22](image-url)  
Figure 22. Shear load–deflection responses: (a) solid models; (b) models with web openings.

Figure 23 shows the effect of the web opening on the shear capacity of the RCA-based deep models at different steel fiber volume fractions. The shear capacity of the models with and without openings increased almost linearly with an increase in $v_f$. For the models without steel fibers, the reduction in shear capacity due to the presence of the web opening ranged from 17 to 28%, with an average of 23%. For the models with steel fibers, the reduction in shear capacity caused by the web opening ranged from 35 to 36%, with an average of 35%. It is worth mentioning that the shear capacity of the deep beam model having $v_f$ of 3% and the web opening was equal to that of the solid beam model with steel stirrups. Numerical results of the deep beam models with a web opening are consistent with those published in the literature for RC deep beams with openings made with NAs [25,26]. Al Sarraf et al. [25] reported a shear strength reduction of 33% due to the presence of a square opening with $h_0/h$ of 0.2 in RC deep beams ($a/h = 0.7$) without steel fibers, whereas their counterparts with $v_f$ of 1% exhibited a shear strength reduction of 40% due to the web opening. Zewair et al. [26] reported that the existence of a circular web opening ($h_0/h$ of 0.31) reduced the shear capacity of RC deep beams ($a/h = 1$) without steel fibers by 27%, whereas it caused a shear strength reduction of 43 to 58% in their counterparts with $v_f$ of 1%.

![Figure 23](image-url)  
Figure 23. Effect of web opening on the shear capacity of RCA-based models.

Figure 24 illustrates the effect of the minimum steel stirrups on the shear strength gain of the deep beam models having $a/h$ of 0.8 with and without openings. The inclusion of minimum steel stirrups was less effective in improving the shear capacity of the models with web openings, irrespective of the type of aggregates. This occurred because of the failure mode of these models that was controlled by properties of concrete since no yielding of stirrups took place. Figure 25 shows the shear strength gain caused by the addition of steel fibers to RCA deep beams having $a/h$ of 0.8 with and without web openings. The strength gains caused by the steel stirrups are also provided in Figure 25 as a benchmark. The strength gain was calculated relative to the shear capacity of the corresponding control
models made with RCAs without steel stirrups nor steel fibers. The shear strength gain caused by steel fibers for the deep beam models with the web opening was in the range of 45 to 70%, whereas their solid counterparts exhibited a shear strength gain of 78 to 108%. Nevertheless, for the models with and without openings, the addition of steel fibers at $v_f$ of 1% was sufficient to achieve a shear strength gain higher than that provided by the minimum conventional shear reinforcement. The reduced shear strength gain in the presence of a web opening has also been reported in the literature for deep beams made with NAs. Zewair et al. [26] reported a shear strength gain due to the addition of steel fibers in the range of 60 to 110% for solid RC deep beams with $a/d$ of 1, whereas their counterparts with a web opening showed a reduced shear strength gain in the range of 20 to 29%.

![Figure 24](image)

**Figure 24.** Shear strength gain of models with minimum stirrups with and without openings.

![Figure 25](image)

**Figure 25.** Shear strength gain of RCA steel fiber-reinforced models with and without openings.

6. Conclusions

Tensile softening laws of concrete made with RCAs and steel fibers were established based on an inverse analysis of flexural test data of concrete prisms. Numerical deep beam models were developed and validated through a comparative analysis with published experimental results. A parametric study was conducted to investigate the effect of $a/d$ and presence of a web opening on the shear response of RC deep beam models at different steel fiber volume fractions. Main conclusions of the work are summarized hereafter.

The tensile softening law of the concrete made with RCAs and steel fibers can be approximated by a bilinear relationship. The modulus of rupture of the mixture with $v_f$ of 1% was 71% higher than that of a benchmark mixture without fibers. The addition of steel fibers at $v_f \geq 2\%$ resulted in more than a two-fold increase in the modulus of rupture. The tension function of the concrete with $v_f$ of 2 and 3% exhibited a slightly reduced rate of degradation in the tensile stress after cracking relative to that of the concrete with $v_f$ of 1%.

The numerical deep beam models developed in the current study predicted the shear behavior of previously tested RC deep beams with good accuracy. The numerical results of
the solid RC deep beam models indicated that the shear capacity increased almost linearly with an increase in the steel fiber volume fraction. At $a/h$ of 1.6, the addition of steel fibers at $v_f$ of 1%, 2%, and 3% increased the shear capacity by 39%, 64%, and 114%, respectively. At $a/h$ of 0.8, the respective shear strength gains were 78%, 91%, and 108%. At $a/h$ of 0.8, a steel fiber volume fraction of $v_f = 1\%$ was sufficient to substitute the minimum steel stirrups, whereas at $a/h$ of 1.6, it was necessary to use $v_f$ of 2% to substitute the steel stirrups.

The addition of steel fibers was more effective in improving the shear capacity of the solid deep beam models rather than those with the web opening at the same $a/h$ of 0.8. The shear strength gain caused by the addition of steel fibers for the deep beam models with the web opening was in the range of 45 to 70%, whereas their solid counterparts exhibited a shear strength of 78 to 108%.

For the deep beam models without steel fibers, the reduction in shear capacity due to the presence of the web opening ranged from 17 to 28%, with an average of 23%. For the deep beam models with steel fibers, a more pronounced reduction in the shear capacity of approximately 35% was reported because of the web opening.

The addition of steel fibers to the deep beam models with the web opening at $v_f$ of 1% was sufficient to achieve a shear strength gain higher than that provided by the minimum steel stirrups. The shear capacity of the deep beam model with the web opening and $v_f$ of 3% was equal to that of a similar solid deep beam model with steel stirrups.

This research provided insight into the shear behavior of steel-fiber reinforced recycled aggregates concrete deep beams with and without web openings. Steel fibers are vulnerable to corrosion under harsh environmental conditions. In such cases, appropriate measures should be adopted to protect the steel fibers from corrosion. Future work should focus on studying the behavior of conventional and geopolymer concrete beams reinforced with nonmetallic fibers.


**Funding:** This project is supported by the United Arab Emirates University (UAEU) (grant number 12N004).

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** The data presented in this study are available on request from the corresponding author. The data are not publicly available due to privacy issues.

**Conflicts of Interest:** The authors declare no conflict of interest.

**References**


11. Lehner, P.; Horňáková, M. Effect of amount of fibre and damage level on service life of sfr recycled concrete in aggressive environment. Buildings 2021, 11, 489. [CrossRef]

12. American Concrete Institute ACI. Building Code Requirements for Structural Concrete and Commentary on Building Code Requirements for Structural Concrete (ACI 318R-14); American Concrete Institute (ACI): Farmington Hills, MI, USA, 2014.


19. ACI Committee 544.6R. Report on Design and Construction of Steel Fiber-Reinforced Concrete Elevated Slabs; American Concrete Institute: Farmington Hills, MI, USA, 2015.


28. Kachouh, N.; El-Maaddawy, T.; El-Hassan, H.; El-Ariess, B. Shear response of recycled aggregates concrete deep beams containing steel fibers and web openings. Sustainability 2022, 14, 945. [CrossRef]


45. Mansour, M.; El-Ariss, B.; El-Maaddawy, T. Effect of support conditions on performance of continuous reinforced concrete deep beams. Buildings 2020, 10, 212. [CrossRef]


