Behavioral Strategies between Government and Real Estate Developers on Prefabricated Buildings Based on Triangular Fuzzy Matrix Game

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Abstract: The subsidies provided by the local government to real estate developers are an important and common practice to promote the development of prefabricated buildings worldwide. However, there is a lack of current research on how government subsidies affect the decision-making of real estate developers with respect to adopting prefabricated construction methods. This study developed a fuzzy game model integrating classical game theory with the triangular fuzzy number approach between the local government and a real estate developer, with different behavioral strategies regarding whether or not to develop prefabricated buildings. It analyzed the Nash equilibriums under the circumstances of pure and mixed strategies and probed the influencing factors of the game equilibrium results via numerical simulation. The research conclusions are as follows: (1) the government should encourage real estate developers to actively participate in prefabricated building by using financial subsidies, fund rewards, process supervision and economic penalties comprehensively; (2) the pure strategy (no manufacturing process supervision, active development) between the government and real estate developers is feasible when the benefit of “active development” for real estate developers is larger than that of “passive development”; (3) positive incentives, such as offering financial subsidies, reducing the cost of prefabricated building development and improving the popularity of prefabricated construction in the building market, should be taken to motivate real estate developers; and (4) negative incentives, such as increasing economic punishment and enhancing manufacturing process supervision, can also facilitate real estate developers to actively participate in the development of prefabricated buildings. Incentives are more effective when the economic penalties are greater than the incremental costs of developing prefabricated buildings. Some policy implications are given to guide prefabricated building market development. Finally, the main problems that need further study in the future are highlighted.

Keywords: fuzzy game; prefabricated building; government subsidy; real estate developers; decision-making

1. Introduction

Traditional cast-in situ construction techniques have a negative impact on the environment, are a waste of resources, lack a labor force, have low production efficiency, etc. [1]. Prefabricated buildings serve as a sustainable alternative to traditional construction methods and have been recognized as a new development trend worldwide [2]. Due to the huge demand for buildings, serious constraints on resource supply and concerns for environment protection, prefabricated buildings have gradually become a new green construction mode in China [3]. Meanwhile, In Japan, Europe, the United States and some other developed countries, prefabricated building technology has been widely adopted in the construction
industry [4]. In Hong Kong, the use of prefabrication techniques has led to the large-scale production of public housing [5].

However, prefabricated buildings in many countries (e.g., in China) still have much room for further development on account of weak industrial foundations and a lack of experience, which means large investment costs, long return periods and strong externality. This presents a hard process of replacing traditional cast-in-place construction with prefabricated construction [6]. Accordingly, a series of incentive policies by the government have been introduced to advocate for the development of prefabricated buildings. Specifically, the subsidies provided by the government to real estate developers are an important and common practice to promote the development of the prefabricated building industry. For example, in Shanghai, the government stated that the proportion of prefabricated buildings cannot be less than 50% in all newly built buildings required in the land transfer stage, thus guaranteeing the stable growth of prefabricated building projects. In addition, a financial subsidy of CNY 100 per square meter and a maximum subsidy of CNY 10 million for a single project with a gross floor area of more than 30,000 square meters and an assembly rate of 45% or above are provided to real estate developers developing prefabricated housing. As the producers and suppliers of prefabricated building products, for real estate developers’ decision-making on whether to manufacture prefabricated buildings or not, higher cost is the main factor that affects the fast growth of prefabricated building. Their decision-making process is influenced by both internal factors, such as the economic interests of the enterprise itself, and external factors, such as incentives related to the incentive policies. Therefore, the strategy for developing prefabricated buildings selected by real estate developers plays a leading role in the promotion of prefabricated building. In summary, how real estate developers react to government policy guidance based on their own economic considerations can exert a direct impact on prefabricated building decisions.

The above analysis shows that the government and real estate developers are the key participants in prefabricated building technology promotion in the real estate market and they have different goals for the development of prefabricated buildings. When faced with prefabricated buildings, the former pursues public interest such as environmental protection and industrial upgrading, while the latter pursues individual economic profits. Obviously, there exists a game of behavioral decisions between the government and real estate developers. Although previous studies revealed that government policies can effectively affect the decision-making of prefabricated building developers [7], there is a lack of research on how government policies affect the decision-making of real estate developers with respect to adopting the prefabricated building method. Therefore, in order to facilitate the growth of the prefabricated building market, this study will try to establish a game model to answer the following research questions from the perspective of the game between the government and a real estate developer. (1) How does the government formulate an effective guidance policy based on the actual situations of the prefabricated building? (2) How does the real estate developer respond to policies and make decisions? (3) How do the real estate developer and the government interact with each other effectively? The following sections start with an overview of the related literature in Section 2. Section 3 presents the research methodology, including the game model and the model analysis. Section 4 discusses the numerical simulation and suggests the policy implications for promoting the development of prefabricated buildings. Finally, Section 5 concludes the paper.

2. Literature Review


Researchers have conducted some research on the questions of the relationship between government subsidies and enterprise decisions, mainly focusing on enterprise R&D and the low-carbon supply chain. The research on government financial subsidies originated from Pigou’s “Welfare Economics”. He proposed that social welfare was not able to achieve maximization due to external diseconomies and that the government should
adopt a way of introducing “extra rewards” (subsidies) or “extra restrictions” (Tax) to intervene in market activities. Two types of subsidies, including R&D subsidies and production subsidies, were studied in 1997, and they were divided into four kinds of games according to the government’s commitment [8]. By studying the effect of Irish government subsidies, Girma et al. [9] found that government subsidies were effective in overcoming the financial crises of enterprises and promoting the adoption of new technologies. In order to explore the important influence of government subsidies on the performance of the remanufacturing supply chain, Mitra et al. [10] established a two-stage game model between a manufacturer and a remanufacturer. Sheu and Chen [11] studied the impact of a government subsidy on the profits of a green supply chain based on a three-stage game model. Raz et al. [12] analyzed the government’s discounts and subsidies to coordinate the manufacturer’s pricing and supplying abilities through a numerical analysis using data from the electric vehicle industry. Ma et al. [13] explored the impact of carbon tax policies on suppliers’ wholesale prices, production quantities, manufacturers’ purchasing decisions and sales prices.

2.2. Government Policy in Prefabricated Building Promotion

The studies conducted by Tarn [14], Jaillon and Poon [15] indicated that initial construction costs in the primary stage of the development of prefabricated buildings might increase compared with conventional construction, although long-term construction costs were reduced in terms of the entire life cycle. The increased cost decreases the enthusiasm of real estate developers participating in prefabricated building. Therefore, the government should create enough relevant incentive policies to facilitate prefabricated construction methods by mitigating real estate developers’ cost pressure. Different kinds of policies have been established to promote prefabricated building worldwide [16,17]. The global spread of prefabrication can be verified in the “Guidelines for the Government’s Policy and Measures for the Progressive Implementation of Building Industrialization” issued by the United Nations in 1974. In 1945, the British Government first issued a white paper that put forward an industrialized construction method in the AEC industry to satisfy the large-scale demand for housing [18]. In 1966, the Ministry of Construction in Japan released the “Basic Conception of the Industrialization of Residential Building Construction” to promote the manufacture of components in factories, displacing traditional cast-in situ behaviors [19]. Swedish Parliament launched the “Million Dwellings Programme” between 1965 and 1974, in which an off-site construction method was encouraged to resolve the housing shortage [20]. The Singaporean HDB issued various policies, respectively, in 1963, 1972 and 1981 for applying prefabrication in the public and private sectors [21]. The Construction Industry Council in Hong Kong [22] formulated and published the comprehensive criteria for implementing prefabricated buildings [22]; the Building and Construction Authority in Singapore [23] issued a similar policy document for carrying out assembled building projects [23]. A new round of policies have been developed to help extend and adopt the prefabricated construction methods by both central and local governments of China since 2010 [24]. Furthermore, the prefabricated building-related policy effect is explored and discussed. To the best of our knowledge, Park et al. [21] simulated the performance of the Singapore construction industry and examined the effectiveness of alternative prefabrication policies using system dynamics. Li et al. [25] investigated how construction enterprises perceive the industrialized residential building policy. Gao and Tian [17] looked into the effects of China’s prefabrication policies on the performance of the construction industry.

2.3. Knowledge Gaps

From Section 2.1, we can see that model establishment based on game theory has become the mainstream research method for studying the mutual relationships between government subsidies and enterprise decisions in general. However, these studies on how government subsidies influence enterprise decision-making mainly fall in the scope of the manufacturing industry, and the existing research results do not apply to the scope
of prefabricated building. From Section 2.2, the government role in the development of prefabricated buildings has been qualitatively emphasized and discussed, mainly from a macro point of view. Meanwhile, a small amount of research is more focused on the impact of policy on the performance of construction enterprises, suppliers and the construction industry. It is obvious that the impact of prefabricated incentive policies on real estate developers has been not studied fully. Hence, how the government and real estate developers interact with each other and how the policies influence the decision-making of developers urgently need further research. In summary, this study will apply game theory to explore the impact of government subsidies on a real estate developer’s decisions to adopt prefabricated building.

3. Methodology

3.1. Model Assumption

The game participants involved in this study include the government and a real estate developer. The real estate developer faces two kinds of behavioral strategy choices. One is to actively develop prefabricated buildings (active development), and the other is to passively develop prefabricated building (passive development). There are also two kinds of behavioral strategy choices by the government. One is to carry out process supervision on the development of prefabricated buildings (with process supervision), and the other is not to carry out process supervision (without process supervision).

In classical game theory, the deterministic formula is used to express the profit and loss value of each strategy for comparison and analysis to obtain the optimal strategy. However, it is difficult for people to make accurate judgments about the future due to the influence of various factors, including the limitations of public cognitive level, incomplete information, and structural and random fluctuations of the system. On the other hand, the judgment information from both academic researchers and practical experts in the field of prefabricated building is not completely accurate due to their lack of experience and deviated opinions on factors such as the total cost of and income from the active and passive development of prefabricated buildings by the real estate developer, and the total amount of subsidy from the government. [26]. Therefore, triangular fuzzy numbers can be used to express the profit and loss values, which are difficult to confirm. A profit and loss value matrix composed of triangular fuzzy numbers is called a triangular fuzzy profit and loss value matrix, and a game determined by such a triangular fuzzy profit and loss matrix is called a triangular fuzzy matrix game. Obviously, the behavioral game between the government and the real estate developer in this study is a fuzzy game problem which can be resolved by the triangular fuzzy matrix game. Without changing the nature of the problem, some complex conditions are simplified. The following assumptions are made for the model.

**Hypothesis 1 (H1).** “Government” refers to the local government management and its related departments, mainly for the development of prefabricated buildings, to provide policy support, improvements in standards and other supporting work. “Real estate developers” refers to the building development units engaged in housing development and sales, especially the enterprises capable of organizing prefabricated building development, which have the basic will and ability to meet environmental protection and dedicate their work to enterprise transformation.

**Hypothesis 2 (H2).** The game information between the government and the real estate developer is incomplete, and both participants are entirely rational.

**Hypothesis 3 (H3).** The total cost of the active and passive development of prefabricated buildings by the real estate developer is $\tilde{c}_a$ and $\tilde{c}_p$ respectively. Including land cost, construction cost, equipment cost, etc., in general, $\tilde{c}_a > \tilde{c}_p$. The total income from the active and passive development of prefabricated buildings by the real estate developer is $\tilde{r}_a$ and $\tilde{r}_p$, respectively. The total income includes income from sales of prefabricated building products, government subsidies, etc.
Hypothesis 4 (H4). The government exercises the examination content such as investigating and evaluating the project assembly rate level of the real estate developer, and meets the subsidy commitments, including fiscal subsidies, plot rate awards and other supporting policies according to the results of the investigation. The total amount of subsidy from the government is \( r_g \).

Hypothesis 5 (H5). In addition to actively supporting the real estate developer, the government should also supervise the assembly rate and other work items for the real estate developer. All assessment and supervision costs are \( \tilde{c}_s \). If the real estate developer does not conduct the prefabricated building development, or fails to reach the assembly rate prescribed by the government, the economic penalty to the real estate developer is \( t_p \). The passive development of prefabricated buildings by the real estate developer will have a negative impact on the environment, society and industry. \( \tilde{l}_p \) is used to indicate the social welfare losses paid by the government.

Hypothesis 6 (H6). Both the government and the real estate developer are economic agents. The real estate developer aims to maximize their own economic interests, while the government pursues the greatest social benefits (the prefabricated building proportion in all newly built buildings and the assembly rate of one single building).

3.2. Triangular Fuzzy Game Matrix

The triangular fuzzy game problem between the government and the real estate developer is recorded as \( \varepsilon = \{s_1, s_2, A\} \), in which \( s_1 = \{a_1, a_2\} \) is the strategy set of the government, respectively expressed as “with process supervision” and “without process supervision” of the prefabricated construction process of the real estate developer. \( s_2 = \{\beta_1, \beta_2\} \) is the strategy set of the real estate developer. It is expressed as active development and passive development of prefabricated buildings for the real estate developer, abbreviated as “active development” and “passive development”. \( \tilde{A} \) is the triangular fuzzy profit and loss value matrix represented by a triangular fuzzy number, which is recorded as

\[
\tilde{A} = \left\{ \tilde{a}_{ij} = (\tilde{a}_{ij}^m, \tilde{a}_{ij}^l, \tilde{a}_{ij}^a) \right\}_{m \times n}.
\]

\( \tilde{a}_{ij} \) is the triangular fuzzy profit and loss value gained by the government, and \( \tilde{b}_{ij} = (\tilde{b}_{ij}^m, \tilde{b}_{ij}^l, \tilde{b}_{ij}^a) \) represents the triangular fuzzy profit and loss value gained by the real estate developer, when the situation is \( \{a_i, \beta_j\} \).

Based on the above assumptions, an income matrix of the government and the real estate developer is constructed, as shown in Table 1.

Table 1. Triangular fuzzy profit and loss value matrix of game between government and real estate developer.

<table>
<thead>
<tr>
<th>Government</th>
<th>Real Estate Developer</th>
<th>Active Development</th>
<th>Passive Development</th>
</tr>
</thead>
<tbody>
<tr>
<td>With process supervision</td>
<td>( -\tilde{c}_s - \tilde{r}_g, \tilde{r}_a - \tilde{c}_a + \tilde{r}_x )</td>
<td>( -\tilde{r}_a - \tilde{l}_p + \tilde{l}_p, \tilde{r}_p - \tilde{c}_p - \tilde{l}_p )</td>
<td></td>
</tr>
<tr>
<td>Without process supervision</td>
<td>( 0, \tilde{r}_a - \tilde{c}_a )</td>
<td>( -\tilde{l}_p, \tilde{r}_p - \tilde{c}_p )</td>
<td></td>
</tr>
</tbody>
</table>

3.3. Triangular Fuzzy Game Equilibrium Analysis

3.3.1. Nash Equilibrium Analysis of Pure Strategy

(1) If \( \tilde{r}_a - \tilde{c}_a + \tilde{r}_g < \tilde{r}_p - \tilde{c}_p - \tilde{l}_p \), then the income of a real estate developer using the active development strategy is less than that of a real estate developer using the passive development strategy. This inequation is equivalent to \( \tilde{l}_p < (\tilde{c}_a - \tilde{c}_p) - (\tilde{r}_a - \tilde{r}_p) - \tilde{r}_g \), indicating that the increased cost of the active development strategy by the real estate developer is greater than the increased income and government subsidy, and the difference...
between the increased cost and income is greater than the economic penalty for a real estate developer adopting the passive development strategy.

If \(-c_s - l_p + t_p < -l_p\), then \(t_p < c_s\), indicating that the cost of the government’s assessment and supervision is less than the government’s economic penalty. At this time, there is a unique Nash equilibrium solution, and the strategy combination is (with process supervision, passive development). It can be understood that if the increased cost of the active development strategy by the real estate developer is greater than thepunishment imposed by the government, the real estate developer would rather pay the penalty than take the “active development” strategy.

If \(-c_s - l_p + t_p > -l_p\), then \(t_p > c_s\), indicating that the cost of government assessment and supervision is greater than the government’s economic penalty; from this, it can be understood that if the increased cost of the “active development” strategy by the real estate developer is less than the punishment imposed by the government, the government will adopt the strategy of “without process supervision”, regarding the government as a rational economic agent. At this point, there are two more cases in which the discussion can continue.

(a) If \(\tilde{r}_p - \tilde{c}_p > \tilde{r}_a - \tilde{c}_a\), then \(\tilde{c}_a - \tilde{c}_p > \tilde{r}_a - \tilde{r}_p\). This means that the profit of the “active development” strategy adopted by the real estate developer is less than the increased cost, and the government adopts the strategy of “without process supervision”, so the game has a unique Nash equilibrium solution, and the strategy combination is (without process supervision, passive development).

(b) If \(\tilde{r}_p - \tilde{c}_p < \tilde{r}_a - \tilde{c}_a\), then \(\tilde{c}_a - \tilde{c}_p < \tilde{r}_a - \tilde{r}_p\). This indicates that a real estate developer taking the “active development” strategy can gain more than increased cost. At this time, the government adopts the strategy of “without process supervision”, so the game has the only Nash equilibrium solution, and the strategy combination is (without process supervision, active development). This situation is the most ideal case, in which the real estate developer will adopt the strategy of “active development” driven by benefit pursuit, and the government does not need to carry out process supervision, with no payment of the corresponding supervision cost and the lowest total social cost.

(2) If \(\tilde{r}_a - \tilde{c}_a + \tilde{r}_p - \tilde{c}_p < -l_p\), then the income of the “active development” strategy is greater than that of the “passive development” strategy by the real estate developer. This inequation is equivalent to \(l_p > (\tilde{c}_a - \tilde{c}_p) - (\tilde{r}_a - \tilde{r}_p)\), indicating that the increased income and government subsidy for the real estate developer can compensate for the increased cost of adopting an “active development” strategy (the right side of the inequation is less than zero), or that it cannot be fully compensated (the right side of the inequation is greater than zero), whereas the uncompensated part is less than the penalty by the government. At this time, the real estate developer will take the “active development” strategy, while the government will adopt the strategy of “without process supervision”. At present, the strategy combination (without process supervision, active development) is no longer a pure strategy Nash equilibrium solution. We continue the discussion for following three cases.

(a) If \(\tilde{r}_p - \tilde{c}_p < \tilde{r}_a - \tilde{c}_a\), the game has a unique Nash equilibrium solution. The strategy combination is (without process supervision, active development).

(b) If \(\tilde{r}_p - \tilde{c}_p > \tilde{r}_a - \tilde{c}_a\) and \(-\tilde{c}_s - \tilde{l}_p + \tilde{t}_p > -\tilde{l}_p\), then \(\tilde{t}_p > \tilde{c}_s\). The game has a unique Nash equilibrium solution, and the strategy combination is (without process supervision, passive development).

(c) If \(\tilde{r}_p - \tilde{c}_p > \tilde{r}_a - \tilde{c}_a\) and \(-\tilde{c}_s - \tilde{l}_p + \tilde{t}_p < -\tilde{l}_p\), then \(\tilde{t}_p > \tilde{c}_s\). At this time, the game has no unique Nash equilibrium solution, and the two parties will adopt a mixed strategy.
3.3.2. Nash Equilibrium Analysis of Mixed Strategy

Assume that the probability of the government adopting the strategy of “with process supervision” is \( \hat{p} \), and the probability of “without process supervision” is \( 1 - \hat{p} \); the probability of the adoption of the strategy of “active development” by the real estate developer is \( \hat{q} \), and the probability of them adopting “passive development” is \( 1 - \hat{q} \). Both \( \hat{p} \) and \( \hat{q} \) are triangular fuzzy numbers. Table 2 shows the triangular fuzzy profit and loss value matrix under the mixed game strategy between the government and the real estate developer. According to the existence theorem of the Nash equilibrium, there is at least one Nash equilibrium for any finite game, so there must be a mixed strategy equilibrium solution.

**Table 2.** Triangular fuzzy profit and loss value matrix of mixed game strategy between government and real estate developer.

<table>
<thead>
<tr>
<th>Government</th>
<th>Real Estate Developer</th>
<th>Active Development (( \hat{q} ))</th>
<th>Passive Development (( 1 - \hat{q} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>With process supervision (( \hat{p} ))</td>
<td>( -\tilde{c}_s - \tilde{r}_g, \tilde{r}_a - \tilde{c}_a + \tilde{r}_g )</td>
<td>( -\tilde{c}_s - \tilde{l}_p + \tilde{r}_p, \tilde{r}_p - \tilde{c}_p - \tilde{l}_p )</td>
<td></td>
</tr>
<tr>
<td>Without process supervision (( 1 - \hat{p} ))</td>
<td>0, ( \tilde{r}_a - \tilde{c}_a )</td>
<td>( -\tilde{l}_p, \tilde{r}_p - \tilde{c}_p )</td>
<td></td>
</tr>
</tbody>
</table>

The expected income functions of the government and real estate developer, respectively, are:

\[
\tilde{E}_g(\hat{p}, \hat{q}) = \hat{p} \left[ \hat{q} \left( -\tilde{c}_s - \tilde{r}_g \right) + \left( 1 - \hat{q} \right) \left( -\tilde{c}_s - \tilde{r}_p + \tilde{r}_p \right) \right] + \left( 1 - \hat{p} \right) \left( 1 - \hat{q} \right) \left( -\tilde{l}_p \right) \tag{1}
\]

\[
\tilde{E}_d(\hat{p}, \hat{q}) = \hat{q} \left[ \hat{p} \left( \tilde{r}_a - \tilde{c}_a + \tilde{r}_g \right) + \left( 1 - \hat{p} \right) \left( \tilde{r}_a - \tilde{c}_a \right) \right] + \left( 1 - \hat{q} \right) \left[ \hat{p} \left( \tilde{r}_p - \tilde{c}_p + \tilde{r}_p \right) + \left( 1 - \hat{p} \right) \left( \tilde{r}_p - \tilde{c}_p \right) \right] \tag{2}
\]

By finding the first derivative of the expected income function of the government and the real estate developer, the first-order condition of the optimal government strategy and the optimal real estate developer strategy can be obtained:

\[
\frac{d\tilde{E}_g(\hat{p}, \hat{q})}{d\hat{p}} = \hat{q} \left( -\tilde{c}_s - \tilde{r}_g \right) + \left( 1 - \hat{q} \right) \left( -\tilde{c}_s - \tilde{l}_p + \tilde{l}_p \right) + \left( 1 - \hat{q} \right) \tilde{l}_p = 0 \tag{3}
\]

\[
\frac{d\tilde{E}_d(\hat{p}, \hat{q})}{d\hat{q}} = \hat{p} \left( \tilde{r}_a - \tilde{c}_a + \tilde{r}_g \right) + \left( 1 - \hat{p} \right) \left( \tilde{r}_a - \tilde{c}_a \right) - \hat{p} \left( \tilde{r}_p - \tilde{c}_p + \tilde{r}_p \right) - \left( 1 - \hat{p} \right) \left( \tilde{r}_p - \tilde{c}_p \right) = 0 \tag{4}
\]

The following solution is obtained:

\[
\hat{q}^* = \frac{\tilde{l}_p - \tilde{c}_s}{\tilde{l}_p + \tilde{r}_g} = 1 - \frac{\tilde{r}_g + \tilde{c}_s}{\tilde{l}_p + \tilde{r}_g} \tag{5}
\]

\[
\hat{p}^* = \frac{\left( \tilde{r}_p - \tilde{r}_a \right) - \left( \tilde{c}_p - \tilde{c}_a \right)}{\tilde{l}_p + \tilde{r}_g} \tag{6}
\]

That is to say, under the optimal mixed strategy, the real estate developer adopts the strategy of “active development” with a probability of \( \hat{q}^* \) and “passive development” with a probability of \( 1 - \hat{q}^* \); the government adopts the strategy of “with process supervision” with a probability of \( \hat{p}^* \) and “without process supervision” with a probability of \( 1 - \hat{p}^* \).
3.4. Model Analysis

3.4.1. Analysis on the Influencing Factor $\tilde{q}^*$

Conclusion 1: $\tilde{q}^*$ is an increasing function of the government’s economic penalty according to Equation (5). Namely, it will be more willing to adopt the “process supervision” strategy if the government takes into account the economic benefit and $t_p$ gradually increases. At this time, the real estate developer is also more inclined to adopt the “active development” strategy. Of course, the government will strengthen its process supervision from the perspective of promoting the enthusiasm of the real estate developer regarding prefabricated building.

Conclusion 2: $\tilde{q}^*$ is a decreasing function of the government’s evaluation and regulation cost $\tilde{c}_s$ according to Equation (5). Namely, when the cost $\tilde{c}_s$ of the government’s assessment and regulation increases, the strength of government’s “process supervision” will decrease. Therefore, the willingness of the real estate developer to adopt the “active development” strategy is reduced.

3.4.2. Analysis on the Influencing Factor $\tilde{p}^*$

Conclusion 3: $\tilde{p}^*$ is an increasing function of $\tilde{r}_p - \tilde{r}_a$ according to Equation (6). $\tilde{r}_p - \tilde{r}_a$ represents the difference in incomes between the real estate developer’s implementation of the “passive development” and “active development” strategies. When the difference increases gradually, it indicates that the real estate developer has more income from adopting the “passive development” strategy, and the willingness to adopt the “active development” strategy is reduced. At this time, the government needs to strengthen the “with process supervision” strategy.

Conclusion 4: $\tilde{p}^*$ is a decreasing function of $\tilde{c}_p - \tilde{c}_a$, $\tilde{t}_p$, and $\tilde{r}_g$. $\tilde{c}_p - \tilde{c}_a$ represents the cost difference between the real estate developer’s adoption of the “passive development” and “active development” strategies for prefabricated building. The increase in $\tilde{c}_p - \tilde{c}_a$ indicates that the development cost of prefabricated buildings is lower, which will enhance the willingness of the real estate developer to adopt “active development” of prefabricated building. When $\tilde{t}_p$ is gradually increasing, the willingness to adopt “active development” will be improved in order to be protected from economic punishment. When $\tilde{r}_g$ is gradually increasing, the willingness to adopt “active development” is also improved in order to obtain more economic support from the government. Therefore, the probability of the government adopting “process supervision” is reduced as the willingness of the real estate developer to adopt “active development” increases.

3.4.3. Analysis of Equilibrium Results under Two Strategies

Conclusion 5: From the game equilibrium between the government and the real estate developer, it can be seen that the Nash equilibrium pure strategy (without process supervision, active development) can occur conditionally. The realization condition of the equilibrium is that the income of a real estate developer adopting “active development” is more than that of a real estate developer adopting “passive development” of prefabricated buildings. In the early development stage of the prefabricated building industry, the cost to the real estate developer of adopting “active development” of prefabricated buildings is higher, which hinders the popularization of prefabricated building. Therefore, the government should provide perfect technical and institutional support for the real estate developer to reduce prefabricated construction costs.

Conclusion 6: In the case of the mixed strategy, if the government adopts the “with process supervision” strategy, the real estate developer tends to favor the “active development” strategy; when the real estate developer implements the “active development” strategy, the probability of the government adopting the “with process supervision” strategy will cut down. Although there are no realization conditions for real estate developer to implement
the “active development” strategy, the impact of the government’s supervision on the real estate developer’s choice of strategy is obvious. At the same time, the economic punishment from the government plays a key role in the game between the government and the real estate developer. If the economic punishment is strong enough, it can effectively promote the real estate developer to develop prefabricated buildings actively. In the short and long term, the government plays a critical role in the behavioral strategy choice of the real estate developer and the development of prefabricated buildings. The establishment of specific measures such as financial subsidies, tax concessions, land assignment and transfer, economic punishment should be carried out by the government for different real estate developers.

4. Numerical Simulation and Discussion

4.1. Equilibrium Solution of Pure Strategy Game

Set the total cost of the real estate developer actively developing prefabricated buildings at $\tilde{c}_a = (2, 5, 7)$, the income at $\tilde{r}_a = (7, 9, 12)$, and the total amount of subsidy from the government at $\tilde{r}_g = (2, 3, 4)$. If the real estate developer passively develops prefabricated buildings, the total cost $\tilde{c}_p = (1, 3, 5)$, and the income $\tilde{r}_p = (2, 4, 6)$. Due to the passive attitude of the real estate developer, they accept the economic penalties from the government $\tilde{t}_p = (1, 3, 4)$; all assessment and supervision costs of government supervision on the real estate developer $\tilde{c}_s = (1, 2, 3)$, and the loss of social welfare paid by the government $\tilde{l}_p = (2, 3, 5)$. According to the calculation rules of triangular fuzzy numbers, the triangular fuzzy profit and loss value matrix of the pure strategy game between the government and the real estate developer can be obtained, as shown in Table 3.

### Table 3. Triangular fuzzy profit and loss value matrix of pure strategy game between government and real estate developer.

<table>
<thead>
<tr>
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<th>Real Estate Developer</th>
<th>Active Development</th>
<th>Passive Development</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With process supervision</td>
<td>(-3, -5, -7), (7, 7, 9)</td>
<td>(-8, -9, -13), (0, -2, -3)</td>
</tr>
<tr>
<td></td>
<td>Without process supervision</td>
<td>(0, 0, 0), (5, 2, 5)</td>
<td>(-2, -3, -5), (1, 1, 1)</td>
</tr>
</tbody>
</table>

According to the fuzzy matrix game solution based on the structural element method, the government and real estate developer can realize the Nash equilibrium of the pure strategy (without process supervision, active development) at this time. Under this pure strategy, the fuzzy income of the government and the real estate developer, respectively, is $(0, 0, 0)$ and $(5, 2, 5)$. Now, the result is the best. This is mainly due to the real estate developer adopting the active development of prefabricated buildings, which can lead to income maximization, and the government does not need to conduct “process supervision”, which can decrease the assessment and supervision costs.

4.2. Equilibrium Solution of Mixed Strategy Game

The data from Section 4.1 on the real estate developer’s income from the active development of prefabricated buildings are changed to $\tilde{r}_a = (5, 7, 9)$. When real estate developer adopts the strategy of the “passive development” of prefabricated building, it will incur the government’s economic penalty $\tilde{t}_p = (1, 2, 3)$. The other data remain the same as in Section 4.1. The triangular fuzzy profit and loss value matrix of the mixed strategy game between the government and the real estate developer can be obtained, as shown in Table 4.
Similarly, according to the fuzzy matrix game solution based on the structural element method, there is no Nash equilibrium of the pure strategy between the government and the real estate developer at this time, and they will adopt a mixed strategy. Assume that $x$ and $y$ separately represent the probability of the government adopting the strategy of “with process supervision” and the probability of the real estate developer adopting the strategy of “active development”. Then, the expected income functions of the government and the real estate developer are, respectively:

$$E_{Gov.}(x, y) = (x, 1 - x) \left( \begin{array} { c c c } { 17 } & { -103 } & { 0 } \\ { 0 } & { 12 } & { -12 } \\ { -39 } & { 12 } & { 39 } \\ \end{array} \right) \left( \begin{array} { c } { y } \\ { 1 - y } \\ \end{array} \right) = \frac{47}{12} xy - \frac{16}{3} x + \frac{39}{12} y - \frac{39}{12}$$ (7)

$$E_{Dev.}(x, y) = (y, 1 - y) \left( \begin{array} { c c c } { 37 } & { -7 } & { 1 } \\ { 0 } & { 3 } & { 12 } \\ { 7 } & { 12 } & { x } \\ \end{array} \right) = \frac{1}{3} xy - \frac{10}{3} xy + \frac{7}{12} x + 1$$ (8)

The first-order conditions of Equations (7) and (8) can be obtained, respectively, as follows:

$$\frac{\partial E_{Gov.}(x, y)}{\partial x} = \frac{\partial \left( -\frac{1}{3} xy - \frac{16}{3} x + \frac{39}{4} y - \frac{13}{4} \right)}{\partial x} = -\frac{1}{3} y + \frac{16}{3} = 0$$ (9)

$$\frac{\partial E_{Dev.}(x, y)}{\partial y} = \frac{\partial \left( \frac{61}{12} xy - \frac{10}{3} y - \frac{7}{12} x + 1 \right)}{\partial y} = \frac{61}{12} x - \frac{10}{3} = 0$$ (10)

So: $x = \frac{20}{31}, \ y = \frac{34}{61}$.

Therefore, the optimal hybrid strategy between the government and the developer is:

$$x^* = (x, 1 - x) = \left( \frac{20}{31}, \frac{11}{31} \right)$$

$$y^* = (y, 1 - y) = \left( \frac{34}{61}, \frac{27}{61} \right)$$

That is, the government adopts the strategy of “with process supervision” with a probability of $\frac{20}{31}$, and adopts the strategy of “without process supervision” with a probability of $\frac{11}{31}$; the real estate developer adopts the strategy of “active development” with a probability of $\frac{34}{61}$, and adopts the strategy of “passive development” with a probability of $\frac{27}{61}$. At this time, the expected income of both parties reaches the maximum.

Under this mixed strategy, the fuzzy incomes of the government and the real estate developer are, respectively:

$$\tilde{r}_{Gov.} = \left( \frac{20}{31}, \frac{11}{31} \right) \left( \begin{array} { c c c } { -3 } & { -5 } & { -7 } \\ { 0 } & { -6 } & { -8 } \\ { -2 } & { -3 } & { -5 } \\ \end{array} \right) \left( \begin{array} { c } { 34 } \\ { 61 } \\ \end{array} \right) = \left( \frac{10702}{2945}, \frac{15173}{2945}, \frac{21538}{2945} \right)$$ (11)

$$\tilde{r}_{Dev.} = \left( \frac{20}{31}, \frac{11}{31} \right) \left( \begin{array} { c c c } { 5 } & { 5 } & { 6 } \\ { 3 } & { 2 } & { 2 } \\ { 0 } & { -1 } & { -2 } \\ \end{array} \right) \left( \begin{array} { c } { 34 } \\ { 61 } \\ \end{array} \right) = \left( \frac{1793}{2945}, \frac{1419}{2945}, \frac{1419}{2945} \right)$$

4.3. Analysis of Equilibrium Solution under Two Strategies

In the above two cases, the equilibrium solution of the game is a pure strategy (without process supervision, active development) and mixed strategy ($x^* = \left( \frac{20}{31}, \frac{11}{31} \right), y^* = \left( \frac{34}{61}, \frac{27}{61} \right)$, respectively). By analyzing the numerical simulation results, we note that there are three factors that affect the equilibrium realization of the game. One is the income of the real estate

<table>
<thead>
<tr>
<th>Government</th>
<th>Real Estate Developer</th>
<th>Active Development ($\hat{q}$)</th>
<th>Passive Development ($1 - \hat{q}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>With process supervision ($p$)</td>
<td>(-3, -5, -7), (5, 5, 6)</td>
<td>(-6, -8, -11), (0, -1, -2)</td>
<td></td>
</tr>
<tr>
<td>Without process supervision ($1 - p$)</td>
<td>(0, 0, 0), (1, 1, 1)</td>
<td>(-2, -3, -5), (1, 1, 1)</td>
<td></td>
</tr>
</tbody>
</table>
developer adopting the strategy of the “active development” of prefabricated buildings. Second, the economic penalty should be put forward by the government to the real estate developer adopting the strategy of the “passive development” of prefabricated buildings. Third, the financial subsidy should be put forward by the government to the real estate developer adopting the strategy of the “active development” of prefabricated buildings. Meanwhile, when the income of the real estate developer adopting the strategy of “active development” is higher, the economic penalty imposed by the government on the real estate developer adopting the strategy of “passive development” is also higher. At this time, the equilibrium result is more inclined toward the pure strategy (without process supervision, active development). When the income of the real estate developer adopting the strategy of “active development” is lower, the economic penalty imposed by the government on the real estate developer adopting the strategy of “passive development” is also lower. At this time, the equilibrium result is more inclined toward a mixed strategy.

4.4. Prospects, Challenges and Limitations

The hypothesis proposed in this study, in terms of economic aspects, can represent a main-interest game relationship between the government and a real estate developer in the development of prefabricated building, which can really promote the participation of real estate developers in the development of prefabricated building. The research findings suggest that the situation of “without process supervision, active development” can be achieved, whereby real estate developers cannot react passively to rigid government policies. In practice, the high cost of developing prefabricated building is still a major impediment to the development of prefabricated buildings. In the future, the cost of prefabricated building development can be reduced from the perspectives of multi-participants and multi-factors, so as to create conditions for realizing the above ideal situation. However, there may be some limitations. For example, it is assumed that the government and the real estate developer are complete economic agents. The government still plays an important role in facilitating the transformation of the construction industry under conditions of resource constraints and environmental pressure, despite considering the supervision costs in the development of prefabricated buildings. This may be somewhat inconsistent with the hypothesis. In addition, this paper also assumes the relevant parameters, such as the benefits and costs to the government and real estate developers, without considering the impact of their specific compositions on the development of prefabricated buildings based on the applicability of game theory, which also suggests a future research direction.

4.5. Policy Implications

This paper proved the reality that the government does not need to supervise the prefabricated building development process of real estate developers, while real estate developers can also treat prefabricated building with a positive attitude. Nevertheless, the essential condition of realizing this reality is that the income of a real estate developer adopting the strategy of the “active development” of prefabricated buildings is higher than that of a real estate developer adopting the strategy of the “passive development” of prefabricated buildings. Some managerial implications are also found.

• The government should make comprehensive use of financial subsidies, capital rewards, process supervision, economic punishment and other positive and negative incentive measures to effectively improve the actual income of real estate developers, in order to offset the cost increment of prefabricated building and to increase the cost of the violation of mandatory regulations by real estate developers. Otherwise, real estate developers could passively deal with the government’s mandatory policy on prefabricated building implementation.

• When the government carries on with the positive incentives, it should consider how to expand the income of real estate developers developing prefabricated buildings actively, and take measures such as increasing financial subsidies, reducing tax and
widely publicizing the advantages of prefabricated building to obtain consumers’ market recognition.

- When the government carries on with negative incentives, increasing economic punishment and strengthening process supervision will prompt real estate developers to actively develop prefabricated buildings. In particular, when the economic penalty is greater than the cost increment of developing prefabricated buildings, the policy effect is more obvious.

5. Conclusions

5.1. Summary

Considering the important roles of the government and real estate developers in promoting the development of prefabricated buildings, this study establishes a fuzzy game model between the government and a real estate developer using different behavioral strategies. Combining the advantages of triangular fuzzy numbers with classical game methods, it analyzes the influencing factors of the game equilibrium results and equilibrium solutions under the pure strategy, mixed strategy and these two strategies via numerical simulation. Prospects, challenges and limitations in terms of model assumptions are also discussed. Based on the research results, some policy implications are put forward in order to enable prefabricated building to be carried out smoothly.

5.2. Contribution

This study mainly discusses the game relationship between the government and real estate developers from an economic point of view, so as to promote the participation enthusiasm of real estate developers in developing prefabricated buildings; this contributes to the body of knowledge in the following ways. First, this study proposes a fuzzy game model that integrates triangular fuzzy numbers and classical game method to analyze the behavioral strategies of the government and real estate developer, while the existing research about government policies affecting the behaviors of real estate developers mainly focuses on qualitative discussions. It can overcome some problems including vagueness of expert judgments in traditional game models and inaccurate information caused by the lack of experience in the initial stage of the development of prefabricated buildings. Second, this study analyzes the Nash equilibriums under the two situations of pure and mixed strategies, and explores the conditions that need to be met for realizing equilibrium results. Finally, the results demonstrate the important role of the government’s positive incentives (e.g., subsidies) and negative incentives (e.g., penalties) in the promotion of prefabricated building, which has never appeared in existing studies.

5.3. Future Research Directions

In addition to the future research directions proposed in Section 4.4, the government is still cautious about the means of economic punishment against real estate developers, and the policy effect needs to be further tested. Furthermore, information asymmetry exists between the government and real estate developers. How the government obtains real market information, such as assembly rate level and cost structure, is crucial to the formulation of subsidy policies. The situation under information asymmetry is an issue to be further studied in the future.

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