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New Natural Frequency Studies of Orthotropic Plates by Adopting a Two-Dimensional Modified Fourier Series Method

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Abstract: The free vibration behavior of orthotropic thin plates, which are clamped at three edges and free at one edge, is a matter of great concern in the engineering field. Various numerical/approximate approaches have been proposed for the present problem; however, lack precise analytic benchmark solutions are lacking in the literature. In the present study, we propose a modified two-dimensional Fourier series method to effectively handle free vibration problems of plates under various edge conditions. In the given solution, the adopted trial function automatically satisfies several boundary conditions. After imposing Stoke's transformation in the trial function and letting it satisfy the remaining boundary conditions, we can change the present plate problem into calculating several systems of linear algebra equations which are easily handled. The present method can be regarded as an easily implemented, rational, and rigorous approach, as it can exactly satisfy both the governing equation and the associated edge conditions. Another advantage of the present method over other analytical approaches is that it has general applicability to various boundary conditions through the utilization of different types of Fourier series, and it can be extended for the further dynamic/static analysis of plates under different shear deformation theories. Finally, all the novel analytical solutions are confirmed to be sufficiently accurate since they match well with the FEM results. The new analytic solution obtained may serve as a benchmark for validating other numerical and approximate methods.

Keywords: free vibration behaviors; orthotropic rectangular thin plates; two-dimensional modified Fourier series method



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1. Introduction

Rectangular orthotropic thin plates are increasingly being utilized in various engineering fields due to their outstanding mechanical performance such as the high stiffness-to-weight ratio and strength-to-weight [1–3]. Among the mechanical investigations of such structures, vibration problems have attracted continuous attention since they commonly cause early failure [4–6]. Within the framework of classical plate theory, the governing partial differential equation (PDE), as well as the boundary conditions, have been well established for a long time. Consequently, researchers have mainly focused on the solution methods of various plates. The solution procedure for plate problems falls into two categories: numerical/approximate approaches and analytical approaches. However, based on the authors' knowledge, precise mechanical analysis for plates subjected to different edge restraints, no matter numerical/approximate or analytic solutions, are far from complete.

Previous studies have indicated that numerous efforts have been devoted to developing various solution methods for plate problems. For instance, due to the development of computer technology, numerical/approximate methods, such as the finite element method [7–9], the bubble complex finite strip method [10], the discrete singular convolution

method [11,12], the finite difference method [13], the boundary element method [14], the classical Rayleigh–Ritz procedure [15], the differential cubature method [16], the discrete singular convolution–differential quadrature coupled methods [17], differential quadrature method [18], are becoming more and more popular in analyzing various plate problems. Several novel numerical approaches have also demonstrated enormous potential in analyzing plate mechanical problems [19–21]. It is true that the above mentioned methods can offer accurate results for free vibration behaviors of plates. However, numerical/approximate approaches also have some shortcomings, such as the huge input and output volume; the accuracy of the results heavily depends on the accuracy of the grid. Some other related studies have been conducted to investigate the properties of variable stiffness chain mail fabrics [22,23]. Additionally, experimental techniques have been employed to analyze a composite beam externally bonded with a carbon-fiber-reinforced plastic plate [24].

Due to the suitability of analytical approaches in optimizing structural design and providing precise theoretical data for examining numerical/approximate algorithms, it is still completely necessary to develop simple and effective analytical solution procedures. As for dynamic analysis of plate structures, obtaining analytic free vibration solutions is important for both theoretical understanding and engineering applications. The primary purpose of conducting free vibration analysis on plates is to acquire precise natural frequencies and the corresponding deformation shapes. These parameters usually serve as critical indicators for structural design. Due to the mathematical nature of plate problems, solving the BVPs (boundary value problems) of high-order PDEs (partial differential equations) is necessary. According to the history of solving plate problems, semi-inverse methods are the earliest and most conventional analytical solution procedures. Among these effective semi-inverse methods, the Navier’s method [25–28] and Lévy’s method [29,30] account for a large proportion. Similarly, Ray [31] established new governing PDEs for the static and dynamic problems of laminated plates on the basis of a new zeroth-order shear deformation theory (ZSDT). The author exactly predicted bending and free vibration solutions for both thin and thick laminate plates with all edges simply supported via taking the Navier’s method. By means of adopting the Lévy’s method, it is worth mentioning that the Lévy’s method exhibits better convergent performance over the Navier’s method. As for tapered plates subjected to SRSR (two opposite edges being simply supported and the other two edges being rotationally restrained) edge conditions, Kobayashi [32] also used the Lévy solutions procedure to provide new analytic solutions to determine the non-dimensional critical buckling load parameters and frequency coefficients of plates, in which the effect of boundary spring restraint and thickness variation on stability and dynamic performance of plates are well illustrated through numerical examples. Through the above studies, it can easily be seen that current semi-inverse free vibration solutions are primarily restricted to Navier-type plates whose edges are all simply supported, and Lévy-type plates whose two opposite edges are simply supported. Some other analytical methods have been developed for plate-related structures such as bridges [33–35]. However, in mainstream engineering practice, non-Lévy-type plates are the most commonly encountered. The absence of precise analytic benchmark solutions for other types of edge restraints substantially narrows the range of application of orthotropic plates. For this reason, several representative analytical approaches for non-Lévy-type plates have been proposed. For instance, Zhang developed a finite integral transform method [36–42] to solve the mechanical problems of plates subjected to classical or non-classical edge conditions; the method was proven to be rigorous and effective in analyzing plate mechanical performance. Rahbar employed a semi-analytical method [43] to study the forced vibration responses of plates with clamped and simply supported edges. Li invented a new analytical approach [44–46], which is the combination of the symplectic elastic method and the superposition method, to handle non-Lévy-type thin/thick plate problems. Chen achieved efficient benchmark random vibration solutions for thin plates by using the discrete analytical method [47], in which the edge conditions involved a clamped edge, simply supported edge, and free edge. Xing evaluated the natural frequencies of plates with all

combinations of simply supported and clamped edge restraints by employing an improved separation of variables method [48]. The dynamic formulation of a sandwich microshell considered modified couple stress and thickness–stretching [49]. With the consideration of the first-order shear deformation theory, Moghadam [50] provided analytic bending solutions for piezolaminated thick plates under simply supported/clamped edge conditions subjected to thermo-electro-mechanical loadings by applying the effective superposition solution procedure. All the given solutions demonstrated high accuracy and excellent convergence through comparison studies. Gorman [51] extended this method to investigate the dynamic and stability behaviors of plates subjected to rotational elastic edge support; accurate eigenvalues for both the critical buckling load and frequencies of squares are detailed for validating numerical/approximate approaches. Kiani [52] offered close-form solutions for the critical temperature of fully clamped FGM plates lying on elastic foundations by using three different types of approximate analytic methods. On the basis of the first shear deformation theory, Liew [53] formulated the governing vibration PDEs for Mindlin plates and chose boundary characteristic orthogonal polynomials as the trial function for three generalized displacements to solve dynamic problems of plates under several edge conditions, in which some deflection contour plots for plates under some special edge restraints were the first known. Focusing on rectangular orthotropic Mindlin plates under line elastic supports or point elastic supports and lying on non-homogeneous elastic foundations in a thermal environment, Zhou [54] developed an improved Fourier series approach to deal with free and forced vibration problems of plates. This method is believed to be an efficient tool for studying the mechanical performance of plates with non-classical boundary restraints due to the highly accurate solutions presented. By means of proposing a novel variational asymptotic approach, Peng [55] studied flexural vibration problems for sandwich plates with re-entrant honeycomb cores under some typical combinations of classical edge conditions. In this study, the effects of negative Poisson's ratios, core thickness ratio, and plate thickness on frequency parameters of plates are thoroughly discussed. Combining the Kantorovich method and the Galerkin method, Rostami [56] acquired precise analytical in-plane vibration solutions for rotating cantilever orthotropic plates, in which the obtained in-plane deformation shapes are heavily influenced when changing the plate's aspect ratio. It should be emphasized that despite the above methods providing precise analytical data for dynamic problems of non-Lévy-type plates, many of these approaches are only suitable for some specific non-Lévy-type plates. Consequently, to the best of the author's knowledge, there still very few available free vibration solutions for orthotropic plates with non-Lévy-type plates.

It is known that among the various analytic approaches for plate problems, Navier's method, based on Fourier series theory, is undoubtedly one of the most widely known of all. In the solution procedure of this classical method, the double Sine series is always treated as the trial function for plate deflection. However, such a trial function only automatically satisfies the Navier-type boundary condition. This solution procedure could be extended to deal with problems of plates subjected to more complex edge conditions by combining Stoke's transformation technique. For example, through the application of Stoke's transformation in the double Sine series form solution, Tang [57,58] developed a modified two-dimensional Fourier series method for conducting new thermal buckling investigations for plates under classical/non-classical edge conditions.

This method offers a significant advantage in overcoming boundary-continuous problems when employing Fourier series solutions for plate problems, making it promising for addressing plates subjected to diverse non-Lévy-type boundary restraints. Its primary benefit lies in providing a unified solution procedure similar to classical Navier's solution, simplifying the process compared to the finite integral transform method by avoiding complex transformation steps. Additionally, by extending this procedure appropriately, researchers can easily explore new exact analytical solutions for various plate behaviors, including bending, free vibration, and buckling, based on different shear deformation theories. Notably, compared with other analytical methods, this approach offers several

advantages: it simplifies precise plate free vibration analysis by avoiding complicated mathematical manipulations; it reduces the mathematical complexity by converting higher-order partial differential equations into linear algebra equations within the Fourier series framework; and it provides more precise solutions for moderately thick/thick plates under complex boundary conditions by utilizing different types of Fourier series. The trial function employed in this method precisely satisfies both the governing vibration formula and non-Levy-type boundaries after determining the unknown constants, which carry clear physical significance.

The primary objective of this study is to further extend the modified Fourier series method for new accurate free vibration analysis of orthotropic plates with one edge free and the other three edges clamped. These types of plate problems have the features of having both clamped edges and free edges in a plate, which increases the solving difficulties. In the present solution procedure, we first chose the two-dimensional Sine-half-sinusoidal series which automatically satisfies partial boundary conditions as the trial function for the deflection of plates. We then acquired new formulas for the first fourth-order partial derivatives by imposing Stoke's transformation on the deflection. We finally obtained four sets of easily solvable linear algebra equations after letting the deflection satisfy the governing PDE and the remaining edge conditions. In the present study, all the non-dimensional natural frequencies and the corresponding deformation shapes, which are accurately confirmed by FEM results and solutions available in the literature, are tabulated or plotted to serve as reference data for future studies.

2. Basic Equations

As demonstrated in Figure 1, the schematic figure of an orthotropic plate with one edge free and other three edges clamped is depicted. Employing the well-accepted classical Kirchhoff assumptions, the governing PDE for the free vibration of an orthotropic thin plate can be given as:

$$D_x \frac{\partial^4 W(x, y, t)}{\partial x^4} + 2H \frac{\partial^4 W(x, y, t)}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W(x, y, t)}{\partial y^4} + \rho h \frac{\partial^2 W(x, y, t)}{\partial t^2} = 0 \quad (1)$$

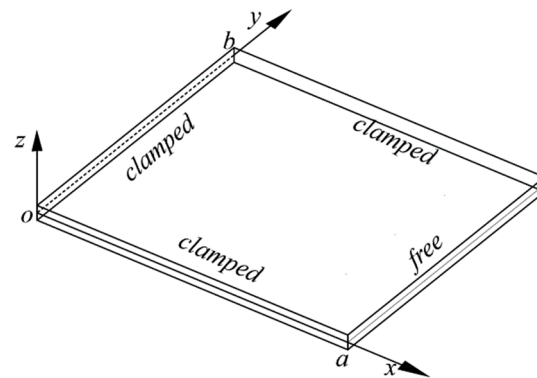


Figure 1. Orthotropic rectangular plates with one edge free and the other three edges clamped.

Here, ρ , h , and $W(x, y, t)$ are the density, thickness, and time-dependent deflection of plates, respectively. The flexural rigidities in Equation (1) are represented by the following formula:

$$D_x = \frac{E_x h^3}{12(1 - \mu_x \mu_y)}, D_y = \frac{E_y h^3}{12(1 - \mu_x \mu_y)}, D_{xy} = \frac{G_{xy} h^3}{12}, H = D_1 + 2D_{xy}, D_1 = \mu_y D_x = \mu_x D_y \quad (2)$$

where μ_x , μ_y , E_x , E_y and G_{xy} are the elastic constants of plates. The internal forces such as the bending moments M_x and M_y , and the equivalent shear forces V_x and V_y , can be expressed in terms of the above-mentioned constants as follows:

$$\begin{aligned} M_x &= -D_x \left(\frac{\partial^2 W}{\partial x^2} + \mu_y \frac{\partial^2 W}{\partial y^2} \right) \\ M_y &= -D_y \left(\frac{\partial^2 W}{\partial y^2} + \mu_x \frac{\partial^2 W}{\partial x^2} \right) \\ V_x &= - \left[D_x \frac{\partial^3 W}{\partial x^3} + (H + 2D_{xy}) \frac{\partial^3 W}{\partial y^2 \partial x} \right] \\ V_y &= - \left[D_y \frac{\partial^3 W}{\partial y^3} + (H + 2D_{xy}) \frac{\partial^3 W}{\partial x^2 \partial y} \right] \end{aligned} \quad (3)$$

Based on the vibration theory, taking $W(x, y, t) = W(x, y) \sin(\omega t)$, one can derive the following formula:

$$D_x \frac{\partial^4 W(x, y)}{\partial x^4} + 2H \frac{\partial^4 W(x, y)}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W(x, y)}{\partial y^4} - \rho h \omega^2 W(x, y) = 0 \quad (4)$$

When the orthotropic thin plate is clamped at $x = 0$, $y = 0$, $y = b$ and free at $x = a$, expressions for such type of boundary conditions are as follows:

$$W|_{x=0} = 0, W|_{y=0} = W|_{y=b} = 0, V_x|_{x=a} = 0 \quad (5)$$

$$\begin{aligned} \frac{\partial W}{\partial x} \Big|_{x=0} &= 0, -D_x \left(\frac{\partial^2 W}{\partial x^2} + \mu_y \frac{\partial^2 W}{\partial y^2} \right) \Big|_{x=a} = 0 \\ \frac{\partial W}{\partial y} \Big|_{y=0} &= 0, \frac{\partial W}{\partial y} \Big|_{y=b} = 0 \end{aligned} \quad (6)$$

Aiming to solve the present plate problem, a two-dimensional Sine-half-sinusoidal series was chosen as the trial function of the plate deflection $W(x, y)$:

$$W(x, y) = \sum_{m=1,3}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{\alpha_m x}{2} \sin(\beta_n y) \quad (7)$$

In which $\alpha_m = \frac{m\pi}{a}$, $\beta_n = \frac{n\pi}{b}$; $W_{mn} = \frac{4}{ab} \int_0^a \int_0^b W(x, y) \sin \frac{\alpha_m x}{2} \sin(\beta_n y) dx dy$ is an unknown constant.

Imposing Stoke's transformation [57–60] over the trial function in Equation (7), one can obtain new Fourier expansions for the higher-order partial derivatives of plate deflections, which are shown below:

$$\begin{aligned} \frac{\partial W}{\partial x} &= \sum_{m=1,3}^{\infty} \sum_{n=1}^{\infty} \left(-\frac{2}{a} I_{1n} + \frac{\alpha_m}{2} W_{mn} \right) \cos \frac{\alpha_m x}{2} \sin(\beta_n y) \\ \frac{\partial^2 W}{\partial x^2} &= \sum_{m=1,3}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{2}{a} \left[\frac{\alpha_m}{2} I_{1n} + (-1)^{\frac{m-1}{2}} I_{2n} \right] - \left(\frac{\alpha_m}{2} \right)^2 W_{mn} \right\} \sin \frac{\alpha_m x}{2} \sin(\beta_n y) \\ \frac{\partial^3 W}{\partial x^3} &= \sum_{m=1,3}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{2}{a} \left[\left(\frac{\alpha_m}{2} \right)^2 I_{1n} + \frac{\alpha_m}{2} (-1)^{\frac{m-1}{2}} I_{2n} - I_{3n} \right] - \left(\frac{\alpha_m}{2} \right)^3 W_{mn} \right\} \cos \frac{\alpha_m x}{2} \sin(\beta_n y) \\ \frac{\partial^4 W}{\partial x^4} &= \sum_{m=1,3}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{2}{a} \left[-\left(\frac{\alpha_m}{2} \right)^3 I_{1n} - \left(\frac{\alpha_m}{2} \right)^2 (-1)^{\frac{m-1}{2}} I_{2n} \right] + \left(\frac{\alpha_m}{2} \right)^4 W_{mn} \right\} \sin \frac{\alpha_m x}{2} \sin(\beta_n y) \end{aligned} \quad (8)$$

$$\begin{aligned}
\frac{\partial W}{\partial y} &= \sum_{m=1,3} \sum_{n=0} \left\{ \frac{\varepsilon_n}{b} [(-1)^n J_{2m} - J_{1m}] + \beta_n W_{mn} \right\} \sin \frac{\alpha_m x}{2} \cos(\beta_n y) \\
\frac{\partial^2 W}{\partial y^2} &= \sum_{m=1,3} \sum_{n=1} \left\{ -\frac{2}{b} \beta_n [(-1)^n J_{2m} - J_{1m}] - \beta_n^2 W_{mn} \right\} \sin \frac{\alpha_m x}{2} \sin(\beta_n y) \\
\frac{\partial^3 W}{\partial y^3} &= \sum_{m=1,3} \sum_{n=0} \left\{ \frac{\varepsilon_n}{b} \left\{ \begin{array}{l} (-1)^n J_{4m} - J_{3m} \\ -\beta_n^2 [(-1)^n J_{2m} - J_{1m}] \end{array} \right\} - \beta_n^3 W_{mn} \right\} \sin \frac{\alpha_m x}{2} \cos(\beta_n y) \\
\frac{\partial^4 W}{\partial y^4} &= \sum_{m=1,3} \sum_{n=1} \left\{ -\frac{2}{b} \beta_n \left\{ \begin{array}{l} (-1)^n J_{4m} - J_{3m} \\ -\beta_n^2 [(-1)^n J_{2m} - J_{1m}] \end{array} \right\} + \beta_n^4 W_{mn} \right\} \sin \frac{\alpha_m x}{2} \sin(\beta_n y)
\end{aligned} \tag{9}$$

$$\begin{aligned}
\frac{\partial^2 W}{\partial x \partial y} &= \sum_{m=1,3} \sum_{n=0,1} \left\{ \frac{\varepsilon_n}{b} [(-1)^n K_{2m} - K_{1m}] - \frac{2}{a} \beta_n I_{1n} + \frac{\alpha_m \beta_n}{2} W_{mn} \right\} \cos \frac{\alpha_m x}{2} \cos(\beta_n y) \\
\frac{\partial^3 W}{\partial x \partial y^2} &= \sum_{m=1,3} \sum_{n=1} \left\{ -\frac{2}{a} L_{1n} + \frac{\alpha_m \beta_n}{2} \frac{2}{b} [(-1)^n J_{2m} - J_{1m}] - \frac{\alpha_m \beta_n^2}{2} W_{mn} \right\} \cos \frac{\alpha_m x}{2} \sin(\beta_n y) \\
\frac{\partial^3 W}{\partial x^2 \partial y} &= \sum_{m=1,3} \sum_{n=0,1} \left\{ \begin{array}{l} \frac{2}{a} \left[\frac{\alpha_m \beta_n}{2} I_{1n} + \beta_n (-1)^{\frac{m-1}{2}} I_{2n} \right] \\ + \frac{\varepsilon_n}{b} [(-1)^n K_{4m} - K_{3m}] - \left(\frac{\alpha_m}{2} \right)^2 \beta_n W_{mn} \end{array} \right\} \sin \frac{\alpha_m x}{2} \cos(\beta_n y) \\
\frac{\partial^4 W}{\partial x^2 \partial y^2} &= \sum_{m=1,3} \sum_{n=1} \left\{ \begin{array}{l} -\frac{2}{a} \beta_n^2 \left[\frac{\alpha_m}{2} I_{1n} + (-1)^{\frac{m-1}{2}} I_{2n} \right] \\ -\frac{2}{b} \beta_n [(-1)^n K_{4m} - K_{3m}] + \left(\frac{\alpha_m}{2} \right)^2 \beta_n^2 W_{mn} \end{array} \right\} \sin \frac{\alpha_m x}{2} \sin(\beta_n y)
\end{aligned} \tag{10}$$

Denote

$$\begin{aligned}
I_{1n} &= \frac{2}{b} \int_0^b W|_{x=0} \sin(\beta_n y) dy, & I_{2n} &= \frac{2}{b} \int_0^b \frac{\partial W}{\partial x} \Big|_{x=a} \sin(\beta_n y) dy \\
I_{3n} &= \frac{2}{b} \int_0^b \frac{\partial^2 W}{\partial x^2} \Big|_{x=0} \sin(\beta_n y) dy, & I_{4n} &= \frac{2}{b} \int_0^b \frac{\partial^3 W}{\partial x^3} \Big|_{x=a} \sin(\beta_n y) dy \\
J_{1m} &= \frac{2}{a} \int_0^a W|_{y=0} \sin \frac{\alpha_m x}{2} dx, & J_{2m} &= \frac{2}{a} \int_0^a W|_{y=b} \sin \frac{\alpha_m x}{2} dx \\
J_{3m} &= \frac{2}{a} \int_0^a \frac{\partial^2 W}{\partial y^2} \Big|_{y=0} \sin \frac{\alpha_m x}{2} dx, & J_{4m} &= \frac{2}{a} \int_0^a \frac{\partial^2 W}{\partial y^2} \Big|_{y=b} \sin \frac{\alpha_m x}{2} dx \\
K_{1m} &= \frac{2}{a} \int_0^a \frac{\partial W}{\partial x} \Big|_{y=0} \cos \frac{\alpha_m x}{2} dx, & K_{2m} &= \frac{2}{a} \int_0^a \frac{\partial W}{\partial x} \Big|_{y=b} \cos \frac{\alpha_m x}{2} dx \\
K_{3m} &= \frac{2}{a} \int_0^a \frac{\partial^2 W}{\partial x^2} \Big|_{y=0} \sin \frac{\alpha_m x}{2} dx, & K_{4m} &= \frac{2}{a} \int_0^a \frac{\partial^2 W}{\partial x^2} \Big|_{y=b} \sin \frac{\alpha_m x}{2} dx \\
L_{1n} &= \frac{2}{b} \int_0^b \frac{\partial^2 W}{\partial y^2} \Big|_{x=0} \sin(\beta_n y) dy
\end{aligned} \tag{11}$$

According to the boundary conditions in Equation (5), we can clearly derive the following relationships:

$$I_{1n} = L_{1n} = 0, J_{1m} = J_{2m} = K_{1m} = K_{2m} = K_{3m} = K_{4m} = 0, D_x I_{4n} = (H + 2D_{xy}) \beta_n^2 I_{2n} \tag{12}$$

Substituting the plate deflection, new expressions of $\partial^4 W / \partial x^4$, $\partial^4 W / \partial y^4$ and $\partial^4 W / (\partial x^2 \partial y^2)$ of Equations (8)–(10), and constants relationships of Equation (12) into the governing PDE, we can derive Equation (13):

$$\sum_{m=1,3}^{\infty} \sum_{n=1}^{\infty} \left\{ \begin{array}{l} \left(\frac{D_x \alpha_m^4}{16} + \frac{H \alpha_m^2 \beta_n^2}{2} + D_y \beta_n^4 - \rho h \omega^2 \right) W_{mn} \\ - D_y \frac{2}{b} \beta_n [(-1)^n J_{4m} - J_{3m}] \\ - D_x \frac{2}{a} \left[(-1)^{\frac{m-1}{2}} \left(\frac{\alpha_m^2}{4} + \mu_y \beta_n^2 \right) I_{2n} - \frac{\alpha_m}{2} I_{3n} \right] \end{array} \right\} \sin \frac{\alpha_m x}{2} \sin(\beta_n y) = 0 \quad (13)$$

Simplifying Equation (13), we can obtain the relationship between the W_{mn} and constants I_{2n} , I_{3n} , J_{3m} , and J_{4m} , as follows:

$$W_{mn} = A_{mn} \left\{ \begin{array}{l} D_x \frac{2}{a} \left[(-1)^{\frac{m-1}{2}} \left(\frac{\alpha_m^2}{4} + \mu_y \beta_n^2 \right) I_{2n} - \frac{\alpha_m}{2} I_{3n} \right] \\ + D_y \frac{2}{b} \beta_n [(-1)^n J_{4m} - J_{3m}] \end{array} \right\} \quad (14)$$

in which $A_{mn} = 1 / \left(\frac{D_x \alpha_m^4}{16} + \frac{H \alpha_m^2 \beta_n^2}{2} + D_y \beta_n^4 - \rho h \omega^2 \right)$.

Utilizing the obtained W_{mn} , we finally derived the formula for $W(x, y)$ in terms of I_{2n} , I_{3n} , J_{3m} , and J_{4m} , which is given as follows:

$$W(x, y) = \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} A_{mn} \left\{ \begin{array}{l} D_y \frac{2}{b} \beta_n [(-1)^n J_{4m} - J_{3m}] \\ + D_x \frac{2}{a} \left[(-1)^{\frac{m-1}{2}} \left(\frac{\alpha_m^2}{4} + \mu_y \beta_n^2 \right) I_{2n} - \frac{\alpha_m}{2} I_{3n} \right] \end{array} \right\} \sin \frac{\alpha_m x}{2} \sin(\beta_n y) \quad (15)$$

Through the above derivations, the obtained plate deflection has satisfied zero deflection at clamped edges, and zero equivalent shear forces at the free edge. The unknown constants, I_{2n} , I_{3n} , J_{3m} , and J_{4m} , will be determined by the matching edge conditions described in Equation (6). In combination with the orthogonal properties of the Fourier series, we can derive the following four infinite systems of linear algebra equations after using the expressions of plate's slopes and bending moment in Equations (8)–(10) to satisfy the boundary conditions in Equation (6):

$$\sum_{m=1,3}^{\infty} \frac{\alpha_m}{2} A_{mn} \left\{ \begin{array}{l} D_y \beta_n \frac{2}{b} [(-1)^n J_{4m} - J_{3m}] \\ + D_x \frac{2}{a} \left[(-1)^{\frac{m-1}{2}} \left(\frac{\alpha_m^2}{4} + \mu_y \beta_n^2 \right) I_{2n} - \frac{\alpha_m}{2} I_{3n} \right] \end{array} \right\} n = 1, 2, 3 \dots \quad (16)$$

$$\begin{aligned} & \sum_{m=1,3}^{\infty} \left(\frac{\alpha_m^2}{4} + \mu_y \beta_n^2 \right) (-1)^{\frac{m-1}{2}} A_{mn} \beta_n D_y \frac{2}{b} [(-1)^n J_{4m} - J_{3m}] \\ & + \sum_{m=1,3}^{\infty} \frac{2}{a} (-1)^{m-1} \left[D_x A_{mn} \left(\frac{\alpha_m^2}{4} + \mu_y \beta_n^2 \right)^2 - 1 \right] I_{2n} \\ & - \sum_{m=1,3}^{\infty} \left(\frac{\alpha_m^2}{4} + \mu_y \beta_n^2 \right) A_{mn} (-1)^{\frac{m-1}{2}} \frac{\delta_m}{2} D_x \frac{2}{a} I_{3n} = 0 \quad n = 1, 2, 3 \dots \end{aligned} \quad (17)$$

$$\sum_{n=1}^{\infty} \beta_n A_{mn} \left\{ \begin{array}{l} D_y \beta_n \frac{2}{b} [(-1)^n J_{4m} - J_{3m}] \\ + D_x \frac{2}{a} \left[(-1)^{\frac{m-1}{2}} \left(\frac{\alpha_m^2}{4} + \mu_y \beta_n^2 \right) I_{2n} - \frac{\alpha_m}{2} I_{3n} \right] \end{array} \right\} m = 1, 3, 5 \dots \quad (18)$$

$$\sum_{n=1}^{\infty} \beta_n (-1)^n A_{mn} \left\{ \begin{array}{l} D_y \beta_n \frac{2}{b} [(-1)^n J_{4m} - J_{3m}] \\ + D_x \frac{2}{a} \left[(-1)^{\frac{m-1}{2}} \left(\frac{\alpha_m^2}{4} + \mu_y \beta_n^2 \right) I_{2n} - \frac{\alpha_m}{2} I_{3n} \right] \end{array} \right\} \quad m = 1, 3, 5 \dots \quad (19)$$

Through the above-mentioned derivation processes, we can only deal with the above systems of infinite linear algebra equations instead of solving complicated boundary value problems of free vibration governing PDE. Theoretically, we can only obtain an exact solution when n and m approach $+\infty$. Finite series terms can guarantee the precision of the present problem. In the present calculation process, the same terms, t , were selected for n and m , which implies that their upper limits are $N = t$ and $M = (2t - 1)/2$, respectively. To obtain non-trivial solutions for plates' natural frequencies, the determinant matrix formed by Equations (16)–(19) should equal zero, by which non-zero solutions for constants I_{2n} , I_{3n} , J_{3m} , and J_{4m} are also determined. The successive substitution of the constants substituted into Equations (16)–(19) finally yield the corresponding vibration shape solutions.

3. Results for Frequency Parameters and Deformation Shapes of Plates

To confirm the capacity of the present two-dimensional modified Fourier series method in predicting clear free vibration behaviors of orthotropic thin plates under the mentioned boundary conditions, plates with different material properties and aspect ratios were examined. Due to the shortage of available analytic data, a comparison of the natural frequencies with the results offered by FEM was performed using Abaqus 6.13. The constituent materials considered in this paper are as follows: as for the orthotropic plates, the elastic constants adopted were $H = 0.5D_x$, $D_y = 0.5D_x$, $\mu_x = 0.6$ and $\mu_y = 0.3$; as for the isotropic thin plate, the Poisson's ratio $\mu_y = \mu_x = \mu = 0.3$. The length-to-width ratio b/a was assumed to range from 0.5 to 5.

4. Discussion

All the non-dimensional natural frequencies obtained by the present method and the finite element method are listed in Tables 1 and 2. Analysis of these data reveals a consistent trend wherein increasing aspect ratios lead to a decrease in natural frequencies across both isotropic and orthotropic materials. Notably, a minimum in natural frequency is evident at an aspect ratio of 0.5. Moreover, a rapid decline in natural frequency is observed as the aspect ratios transition from 0.5 to 1 and 1.5. Subsequent increases in aspect ratios, particularly from 1 to 5, exert a minimal impact on the natural frequency. Interestingly, the results emphasize that an aspect ratio of 0.5 employs a more pronounced influence on natural frequency compared with ratios ranging from 1 to 5, irrespective of the material type. Furthermore, the comparative analysis between isotropic and orthotropic materials for identical boundary conditions indicated that isotropic materials consistently exhibited higher natural frequencies. This disparity sheds light on the inherent mechanical differences between isotropic and orthotropic materials, emphasizing the importance of material selection in engineering applications to achieve desired performance characteristics. Through Tables 1 and 2, it is also evident that the percentage errors between the present solutions with the FEM data are no more than 1%. The corresponding deformation shapes for isotropic/orthotropic square plates are given in Figures 2 and 3. Through Figures 2 and 3, it is evident that the obtained plate deformation shapes strictly satisfy the edge conditions. Through the above comparison, it was found that all the given numerical and graphical solutions were very close to the FEM solutions, which provides sufficient evidence to exhibit the validity of the present approach and the precision of the analytical results acquired.

Table 1. Non-dimensional natural frequencies of the isotropic rectangular thin plate with one free edge and three clamped edges.

b/a	Method	Mode									
		1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
0.5	Present FEM	90.573	104.12	135.07	186.88	247.59	260.38	263.77	296.88	348.49	355.27
		90.528	104.07	135.01	187.03	247.49	260.31	263.66	296.75	348.33	355.16
1	Present FEM	23.930	40.012	63.248	76.725	80.602	116.69	122.29	134.47	140.30	172.90
		23.918	39.991	63.216	76.704	80.562	116.64	122.23	134.43	140.23	172.83
1.5	Present FEM	11.838	29.283	29.305	47.419	55.498	67.735	74.141	84.933	90.430	109.37
		11.832	29.269	29.292	47.395	55.470	67.721	74.104	84.907	90.385	109.32
2	Present FEM	7.7778	17.544	25.860	32.231	36.036	51.210	51.835	64.926	71.104	74.341
		7.7748	17.536	26.232	32.215	36.018	51.184	51.809	64.915	71.069	74.322
2.5	Present FEM	6.0112	12.179	21.526	24.386	30.850	34.042	40.666	49.708	53.519	63.697
		6.0094	12.173	21.516	24.375	30.835	34.025	40.646	49.682	53.493	63.688
3	Present FEM	5.1190	9.3197	15.766	23.620	24.416	28.080	34.928	35.292	43.970	48.319
		5.1179	9.3158	15.759	23.609	24.404	28.066	34.911	35.275	43.949	48.294
3.5	Present FEM	4.6193	7.6354	12.329	18.653	23.175	26.425	26.597	31.506	36.158	38.206
		4.6185	7.6326	12.324	18.643	23.165	26.413	26.584	31.491	36.140	38.188
4	Present FEM	4.3164	6.5702	10.126	14.938	20.995	22.896	25.364	28.296	29.269	34.447
		4.3159	6.5681	10.121	14.931	20.985	22.886	25.352	28.282	29.255	34.431
4.5	Present FEM	4.1210	5.8595	8.6357	12.412	17.176	22.708	22.923	24.646	27.735	29.656
		4.1207	5.8579	8.6323	12.406	17.167	22.698	22.911	24.635	27.722	29.641
5	Present FEM	3.9884	5.3648	7.5855	10.621	14.460	19.098	22.575	24.135	24.533	26.643
		3.9881	5.3636	7.5827	10.616	14.453	19.089	22.565	24.124	24.521	26.630

Table 2. Non-dimensional natural frequencies of the orthotropic rectangular thin plate with one free edge and three clamped edges.

b/a	Method	Mode									
		1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
0.5	Present FEM	62.676	73.674	103.00	155.50	171.30	184.77	211.18	230.47	257.00	324.66
		62.642	73.634	102.96	155.43	171.22	184.68	211.08	230.40	256.86	324.50
1	Present FEM	16.330	31.326	43.269	57.299	68.858	84.012	91.278	98.204	127.30	129.56
		16.322	31.310	43.246	57.269	68.843	83.968	91.244	98.152	127.26	129.50
1.5	Present FEM	8.1418	19.810	25.375	35.630	37.770	52.846	61.836	64.447	73.038	76.747
		8.1384	19.800	25.363	35.612	37.749	52.819	61.803	64.434	73.018	76.707
2	Present FEM	5.5800	11.783	21.728	23.704	28.900	35.171	37.919	50.794	52.058	63.138
		5.5784	11.777	21.717	23.693	28.886	35.153	37.900	50.767	52.031	63.126
2.5	Present FEM	4.5790	8.2190	14.411	22.919	23.031	26.110	31.503	33.661	39.336	46.611
		4.5783	8.2156	14.403	22.907	23.020	26.098	31.487	33.644	39.315	46.586
3	Present FEM	4.1296	6.4037	10.527	16.335	22.697	23.730	24.720	28.258	32.674	33.433
		4.1292	6.4015	10.522	16.327	22.686	23.718	24.709	28.244	32.657	33.416
3.5	Present FEM	3.9037	5.3969	8.2637	12.426	17.793	22.508	23.935	24.318	26.419	30.054
		3.9035	5.3956	8.2603	12.421	17.784	22.497	23.924	24.305	26.407	30.039
4	Present FEM	3.7789	4.8034	6.8588	9.9434	13.984	18.932	22.390	23.451	24.763	25.286
		3.7789	4.8025	6.8563	9.9390	13.977	18.922	22.380	23.440	24.750	25.274
4.5	Present FEM	3.7043	4.4355	5.9467	8.2879	11.412	15.273	19.845	22.312	23.131	24.540
		3.7043	4.4349	5.9449	8.2846	11.407	15.266	19.835	22.302	23.121	24.529
5	Present FEM	3.6569	4.1972	5.3335	7.1437	9.6074	12.686	16.354	20.592	22.258	22.909
		3.6569	4.1968	5.3322	7.1411	9.6031	12.680	16.345	20.582	22.248	22.899

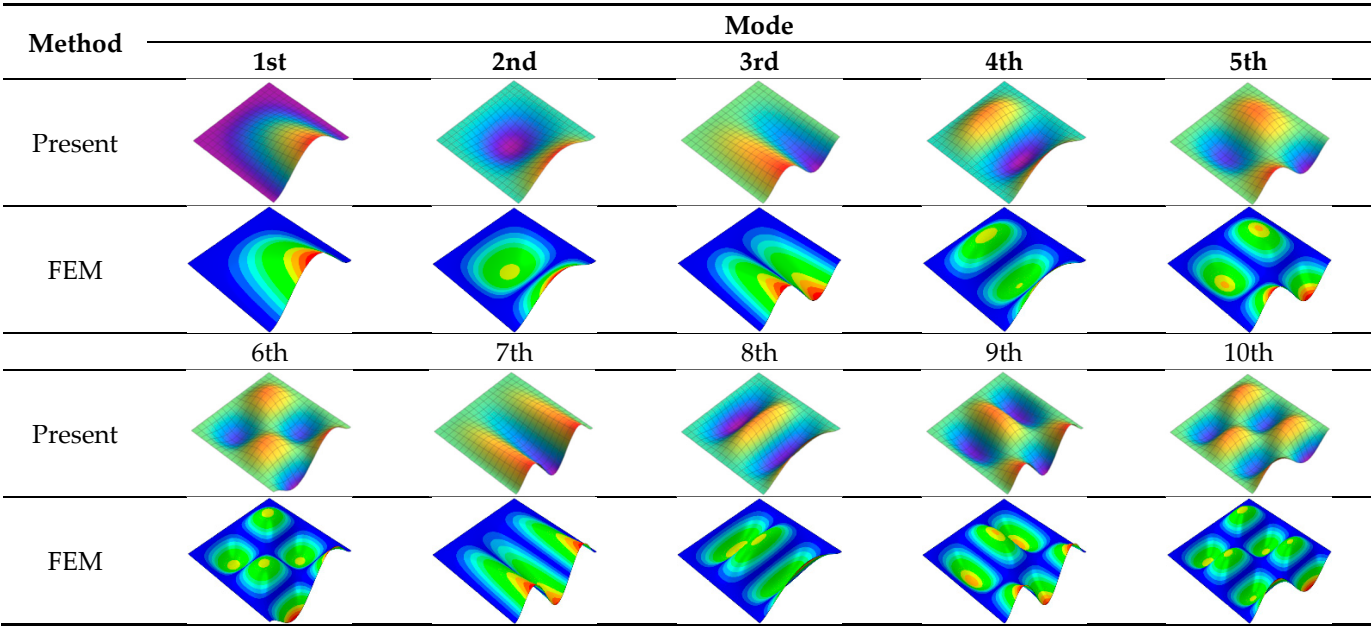


Figure 2. Mode shapes of an isotropic square thin plate with one edge free and three edges clamped.

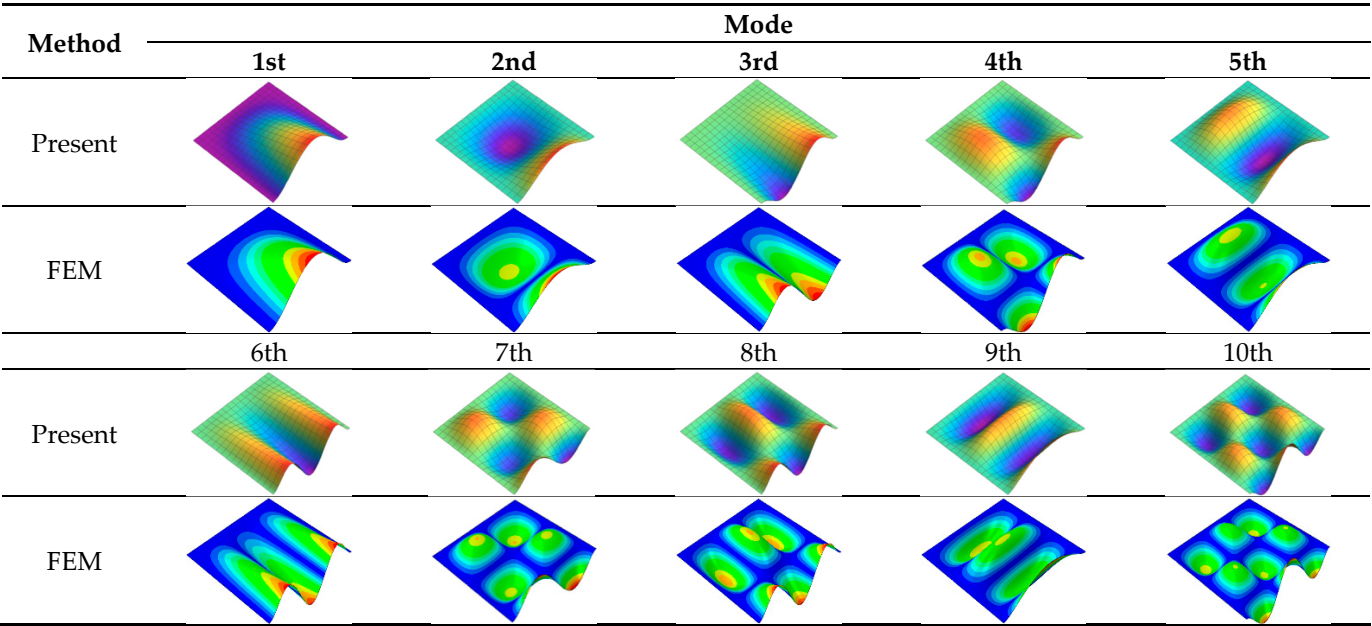


Figure 3. Mode shapes of an orthotropic square thin plate with one edge free and three edges clamped.

This study holds significant practical implications for engineering applications, particularly in structural design and material selection processes. The analysis of non-dimensional natural frequencies across various aspect ratios for both isotropic and orthotropic materials provides valuable insights into structural behavior. Engineers can utilize these findings to optimize the design of structures by adjusting aspect ratios to achieve desired natural frequencies, ensuring structural stability and performance. Moreover, the comparative analysis between isotropic and orthotropic materials highlights the importance of material selection in engineering applications, as isotropic materials consistently exhibit higher natural frequencies under identical boundary conditions. This understanding aids engineers in selecting the most suitable materials for specific applications, thereby enhancing the overall performance and durability of engineered systems. Additionally, the validation of analytical results against finite element method (FEM) data reinforces the accuracy and

reliability of the analytical approach, providing engineers with confidence in utilizing these methods for structural analysis and design. Overall, this study offers practical guidance for engineers in optimizing structural designs, informing material selection processes, and improving engineering practices across various industries.

5. Conclusions

In the present study, we developed a new two-dimensional modified Fourier series method for studying the free vibration behavior of orthotropic plates with three edges clamped and one edge free. These solutions provide comprehensive information on the natural frequencies and vibration mode analysis, considering various aspect ratios. These solutions can be used as valuable reference data for evaluating other approximate/numerical approaches. The present approach is advantageous as it enables linear algebra equations to be solved instead of handling difficult boundary value problems of PDE, providing an easy-to-implement approach for the mechanical problems of plates. More importantly, further developments of this method are expected for analyzing problems of plates under other non-Levy edge conditions, employing different types of Fourier series.

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