

Article

Dynamic Response Analysis Method of a High-Strength RC Beam Subjected to Long-Duration Blast Loading

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Abstract: An analysis method of normalized pressure–impulse (*P-I*) diagrams related to the ductility ratio of structural components is proposed, to quickly estimate the dynamic response of high-strength reinforcement concrete (RC) beams subjected to long-duration blast loading. Firstly, the overall bending deformation mode of RC beams is uncovered via explosion tests in a closed chamber, where the durations of the near-planar blast loadings are varied within 80–105 ms. Then, a single-degree-of-freedom (SDOF) model is established based on the bending deformation mode. The resistance function for the uniform pressure loading is developed using a novel approach, consisting of (1) developing and benchmarking a three-dimensional (3D) improved steel–concrete separated finite-element (FE) model; (2) using the benchmarked FE model to conduct numerical simulations for uniform pressure loading; and (3) idealizing the resistance function for uniform pressure using a bilinear relationship. Finally, the SDOF model is used to conduct parametric analyses and develop a normalized *P-I* diagram that can be used to analyze or design RC beams for far-field blast effects. This *P-I* diagram is verified using results from blast load tests that are primarily in the dynamic region. A total of 188 additional 3D nonlinear FE analyses of RC beams are conducted to expand the database in the impulse and quasi-static regions. Considering the limitations of the proposed method in predicting the shear-dominated deformation and the fracture behavior of members, the *P-I* diagram is applicable to the dynamic response of the bending deformation of members under far-field explosion, which can provide an important reference for the blast-resistant design and analysis of high-strength RC beams.

Keywords: long duration blast loading; bilinear resistance model; *P-I* diagram



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1. Introduction

In recent years, the number of underground explosion accidents is increasing. For example, in March 2010, the “Lubyanka” subway station and “Cultural Park” subway station in the center of Moscow exploded one after another, resulting in 40 deaths and nearly 100 injuries. In September 2017, five people were killed and more than 30 injured in a bombing at Parsons Green Underground station in west London, as well as a subway station bombing in Belarus. The positive pressure region of the shock wave generated by these explosion events can be as high as $10^2\sim 10^3$ ms, belonging to the long-pulse-width explosion load [1–3]. Long-duration blast waves are typically defined by positive pressure durations over 100 ms, which develop in later stages of shock wave propagation, i.e., in the ‘far field’ from the source of detonation.

As one of the load-bearing structures of underground buildings, high-strength reinforced concrete (RC) beams are also important components under the action of long-pulse-width shock waves. Studying their dynamic response and blast-resistant performance

under the action of a long-pulse-width shock wave has important theoretical significance and engineering value for improving the blast-resistant safety of RC structures.

As a common component of major blast-resistant structures, high-strength beams are generally improved from the aspects of materials and section properties, etc., and the influences of concrete strength, steel fiber content, steel strength, and reinforcement ratio on the blast resistance and deformation degree of beams are mainly studied [4–14].

In terms of experiments, several explosive experimental research studies were carried out based on special experimental equipment. For example, LIAL Z et al. [9] used the explosion pressure simulator to study the blast-resistant behavior of high-reinforcement-ratio and high-strength steel bars against beams, respectively, and the results showed that both high-reinforcement-ratio and high-strength steel bars could effectively reduce the peak displacement of members and reduce the length and width of cracks. Zhou Xiwu et al. [10] adopted the domestic advanced ultra-high weight hammer impact testing machine system and found that the overall stiffness of the beam was effectively improved with the reinforcement ratio increasing. Magnusson et al. [11] used a shock tube device to study the effects of high-strength concrete and steel fiber content on the deformation degree and failure mode of beams. Wang, Y.D et al. [15] investigated the reinforcing effect of an expanded section on the ductility and ultimate shear strength of concrete beams. Li, C.J et al. [16] investigated the effects of ultra-high-performance fiber-reinforced concrete jacketing on the static and blast behavior of reinforced beams. Though the displacement response of the beam was derived from the explosion experiment accurately, several limitations were still reserved. For instance, concerning the loading condition of planar wave, it is difficult to realize. For example, the loading condition of plane shock wave is difficult to achieve; for the newly designed high-strength beams, it is necessary to carry out blast-resistant performance experiments in batches, and the experiment cost is too high. Due to the limitations of explosion experiments, the research on the dynamic response of high-strength beams under a long pulse width explosion load is insufficient.

In terms of numerical calculation, the single-degree-of-freedom (SDOF) model has been widely used in dynamic analysis in the explosion resistance of structure [17,18] due to its high computational efficiency [19]. When the SDOF model is used to analyze the displacement of high-strength beams, a corresponding resistance model [20] should be established to determine the parameters such as equivalent stiffness and yield resistance. For the far-field explosion effect, the blast wave acts uniformly on the target surface, so the resistance model must be applicable to the uniformly distributed load environment [21]. At present, the resistance model used to analyze the response and displacement of beam members under an explosion load is relatively simple. For example, Xu Juechun et al. [22] used elastic and plastic resistance models to obtain the dynamic response equation of the direct shear failure of components under an explosion load, and Nassr et al. [23] used the rigid–plastic resistance model to study the influence of the axial force effect on the strength and stability of an H-shaped steel beam. The ideal elastic–plastic or rigid–plastic models ignore the effect of resistance on the SDOF system and have not studied the hardening effect of resistance deeply.

In this paper, based on the dynamic response analysis of a high-strength beam subjected to long-pulse blast loading, the normalization P - I diagram related to the ductility ratio of high-strength beam is constructed by the fusion of numerical values and a few experiments, so as to realize the rapid calculation of its dynamic response. First of all, we select a typical ordinary strength beam; based on this, by improving the strength of concrete and steel bars and increasing the ratio of reinforcement, we design a different bearing capacity of the beam. And then, the deformation mode of the reference beam is studied through the underground closed reinforced concrete explosion chamber [24]. Combined with the deformation pattern, the SDOF model based on bilinear resistance model is established, and the calculation accuracy of the SDOF model is verified by a small number of explosion experiments. Among them, the results of resistance and mid-span displacement simulated by the finite-element model [25] of uniformly loaded beams are summarized, and a bilinear

resistance model is formed by reasonable simplification. Finally, the verified SDOF model is used to obtain the normalized P - I diagram related to the ductility ratio of the high-strength beam. The validity of the P - I diagram is verified by the explosion experiment results of a few reference beams and a large number of finite-element simulation results.

2. Design of High-Strength Beams

In this paper, the commonly used RC beam of ordinary strength is selected as a reference. Four kinds of high-strength beams are established, improving their bearing capacity by increasing the concrete strength, number of rebars, and reinforcement ratio based on the reference beam (see Figure 1). The reference beam is numbered B1-C40-HRB400-D14, with a geometric size of 200 mm $\times$ 200 mm $\times$ 2500 mm, made of C40 concrete, reinforced with HRB400 steel bars with a diameter of 14 mm at the top and bottom, and HRB400 steel bars with a diameter of 8 mm at the hoop. The spacing is 150 mm (see Table 1). The geometry of the beam section, the area, and the reinforcement ratio of the tensile reinforcement, as well as the reinforcement ratio and the spacing of the hoop reinforcement, have been taken into account in the design code [26]. The longitudinal reinforcement consisted of diameters of 14 mm or 20 mm (steel areas equal to 307 mm² and 626 mm²), resulting in reinforcement ratios of 0.85% and 1.74%, not exceeding the 2.5% required in the design code [26].

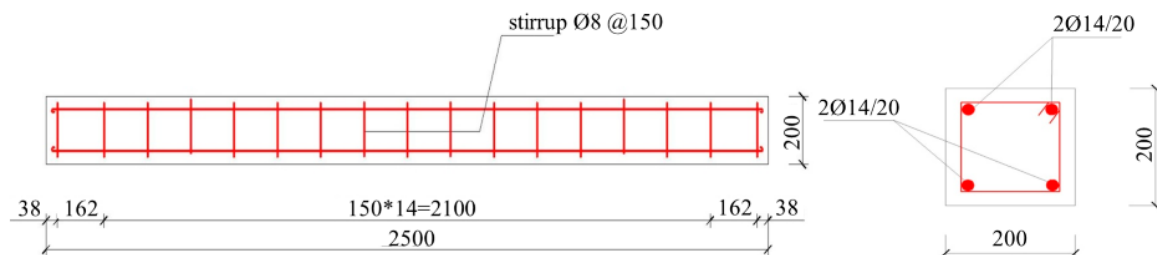


Figure 1. Dimensions and reinforcement of a reference beam (unit: mm).

Table 1. Design parameters of high-strength RC beams.

Beam Sample	Grade of Concrete	Longitudinal Bar Grade	Diameter of Longitudinal bar/mm	Reinforcement Ratio
B1-C40-HRB400-D14	C40	HRB400	14	0.85%
B2-C110-HRB400-D14	C110	HRB400	14	0.85%
B3-C40-HRB400-D20	C40	HRB400	20	1.74%
B4-C40-HRB700-D14	C40	HRB700	14	0.85%
B5-C110-HRB700-D20	C110	HRB700	20	1.74%

3. Reference Beam Explosion Test

In order to generate the long-duration explosion loadings, an underground confined RC shot cavity was designed and excavated [24], as shown in Figure 2. The internal dimensions of the cavity were 3800 mm $\times$ 3500 mm $\times$ 3000 mm. An equipment mounting well with pre-drilled holes was prefabricated on the side of the blast pit to connect the sensor wires inside the cavity to the data acquisition room (Figure 2a). During experiments, the specimen and related supporting facilities were first placed in the cavity, and steel cover plates were placed on both sides of the specimen and the steel plates of the upper steel cover plate. The specimen and concrete support at the bottom were combined and connected by bolts to realize the simply supported constrained boundary conditions (Figure 2b). The flexural stiffness of the support was calculated to be about 2.5×10^9 N/m², which is much larger than that of RC beams. Hence, it is reasonable that the end supports of the beam have been assumed to be rigid and simple during experiments. In order to attenuate the rigid reflection of the blast wave, a layer of rubber was attached to the inner wall of the crater.

In experiments, the multi-point charge initiation technology was used to realize the plane explosion wave acting on the specimen by adjusting the blast height, mass, and plane position of the charge. Five charges C1 to C5, adopting high explosive 8701, were tied on the suspended iron wires in the shot cavity, and the detonating cords converged to ensure simultaneous detonation, as shown in Figure 2c.

In order to evaluate the failure pattern and explosion wave distribution of the specimen, three pressure sensors (with a measuring range of 0~1 MPa), P1~P3, were placed flush with the upper surface of the specimen under each working condition, as shown in Figure 2d. Meanwhile, displacement sensors with a measuring range of 100 mm and an accuracy of 2‰ were installed on the bottom surface of the specimen at the positions of 1/4, 1/2, and 3/4 of the net span of the beams.

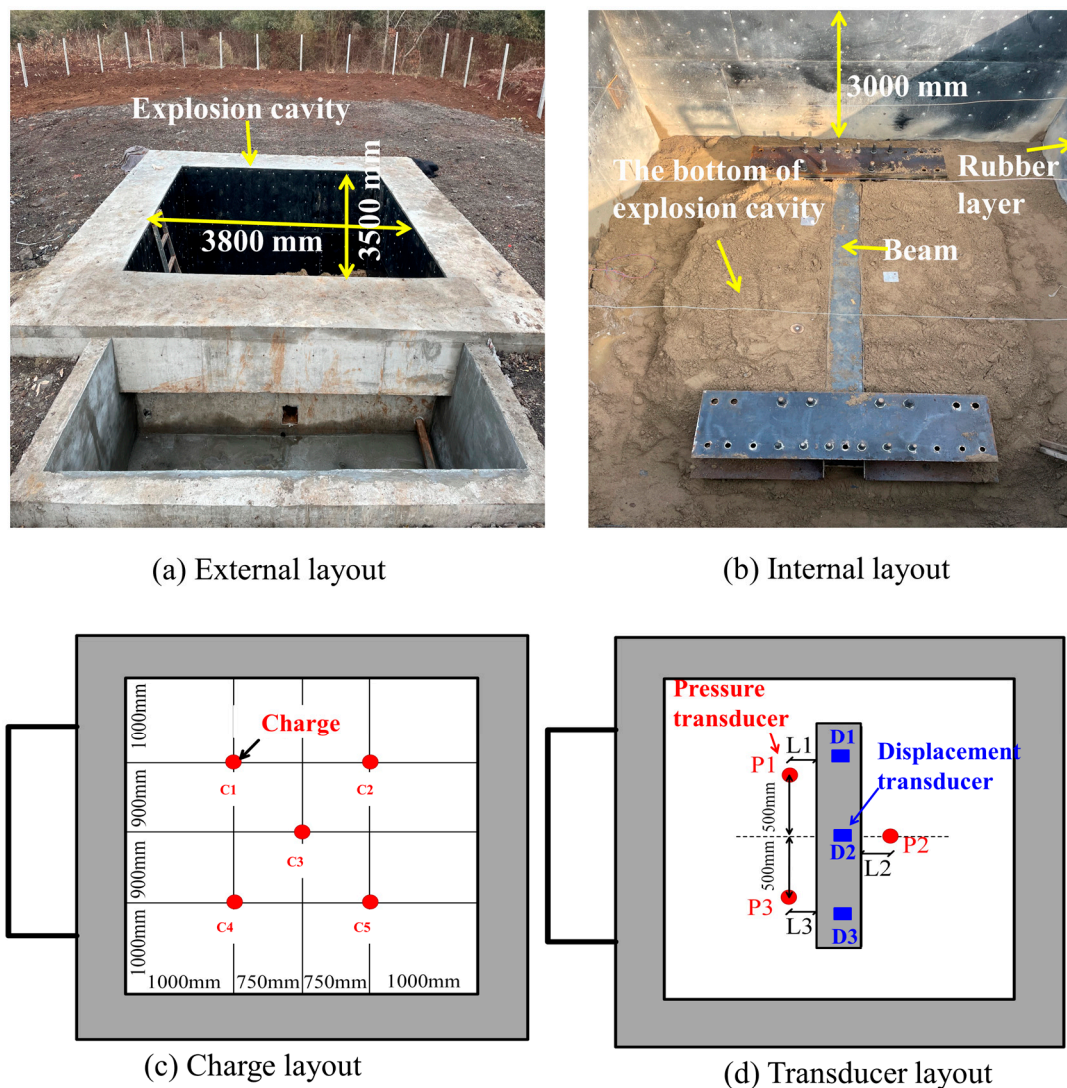


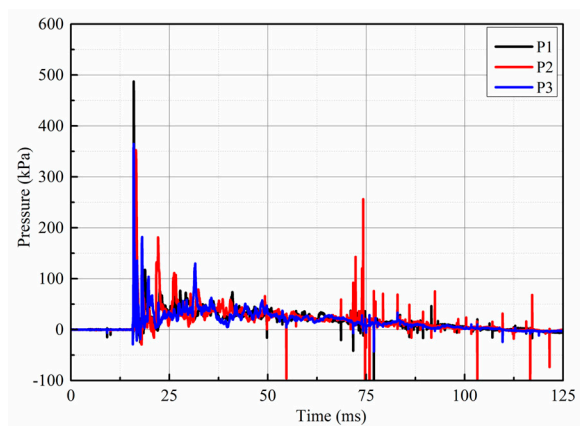
Figure 2. Various main views and parts of the experimental setup: (a) External layout; (b) Internal layout; (c) Charge layout; (d) Transducer layout.

The reference beam numbered B1-C40-HRB400-D14 was selected as the experimental object. The vertical distance between the 5-point distributed explosive center and the test piece was about 1 m, and a total of three explosion experiments were carried out. The explosive yield and experimental conditions are shown in Table 2.

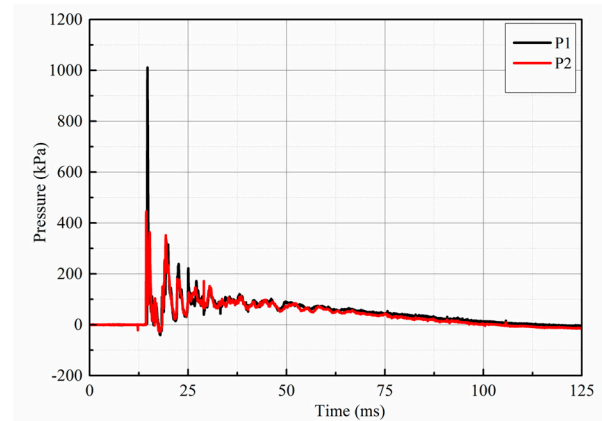
Table 2. Experimental conditions.

Experimental Conditions	P1 Explosion Height (mm)	P2 Explosion Height (mm)	P3 Explosion Height (mm)	P4 Explosion Height (mm)	P5 Explosion Height (mm)	Charge Mass (kg)
Exp 1	990	990	990	980	980	0.5
Exp 2	950	930	945	900	910	0.9
Exp 3	950	940	920	900	900	1.15

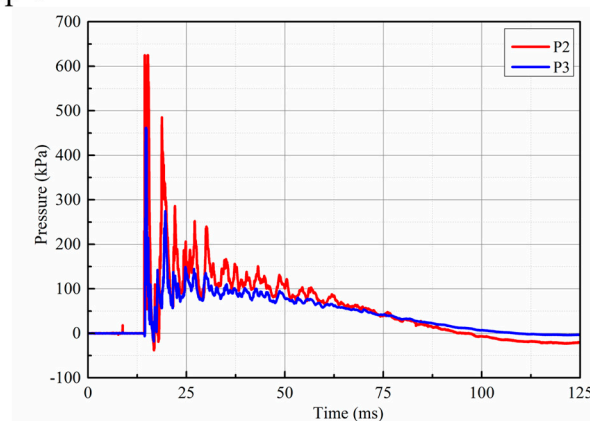
Figures 3 and 4 show the measured results and corresponding impulse time-history curves of the P1~P3 pressure sensors under different explosion conditions. The variation tendencies of pressure time-history curves at different points match well with each other, which indicates that the blast shock wave load acting on the beam surface can be approximated as a uniform distribution load. At the same time, the positive pressure region time of the shock wave is about 80~105 ms. In addition, with the increase in charge, the overpressure and impulse of blast loading acting on beam members increase significantly.



(a) Exp 1



(b) Exp 2



(c) Exp 3

Figure 3. Time-history plots of the pressure for three experiments: (a) Exp 1, (b) Exp 2, and (c) Exp 3.

Figure 5 shows the damage morphology of the beam. As for Exp 1, the whole specimen is in elastic deformation except for a small number of transverse cracks in the middle span of the back of the specimen. With the increase in charge, such as in Exp 2 and Exp 3, crack initiation positions gradually develop from the middle span to the two ends, and the number, width, and length of cracks increase significantly. The deformation degree of the specimen was gradually intensified, and obvious residual deformation appeared.

The damage mode of the overall bending of members was obtained, which was basically consistent with the other results [4,5,9].

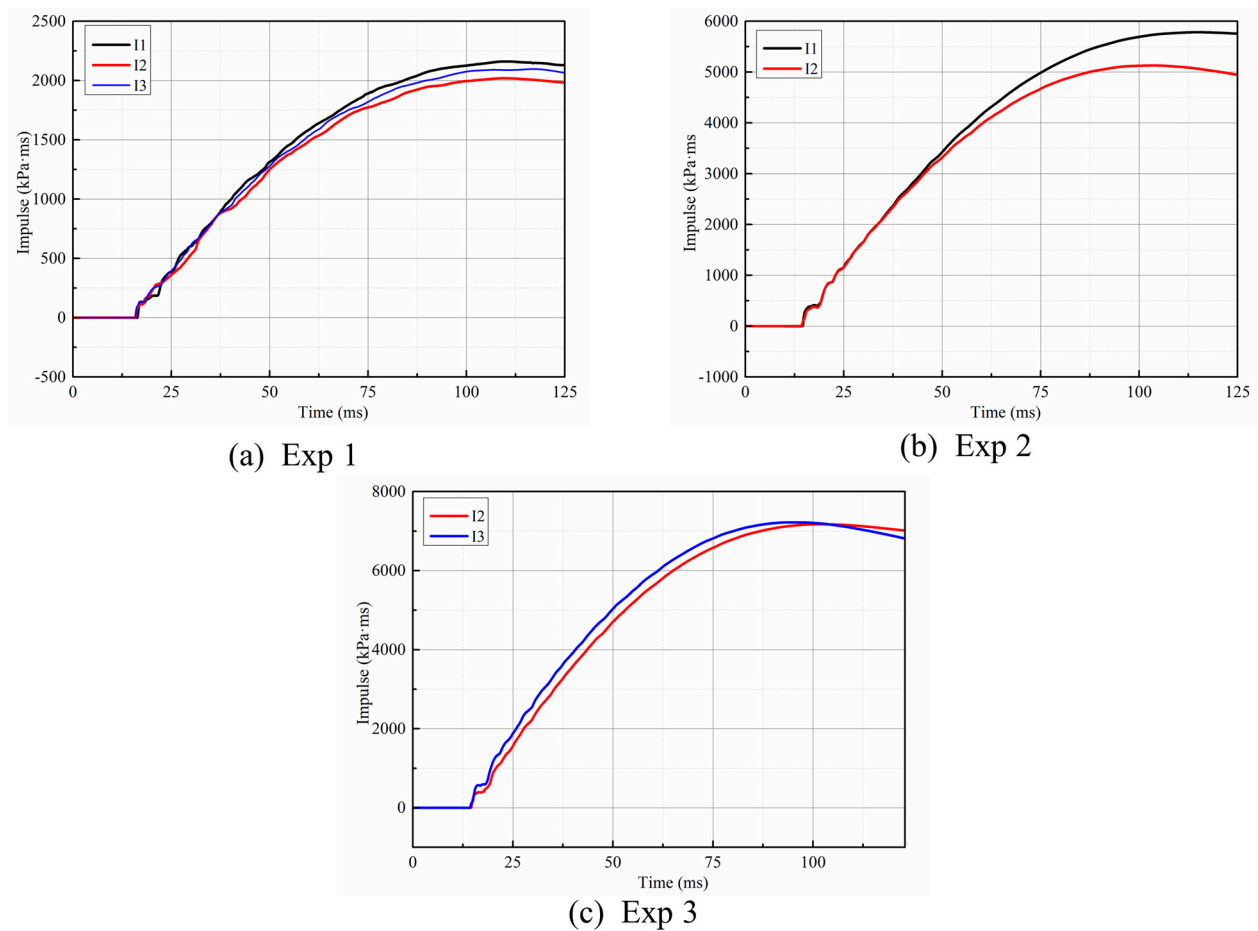


Figure 4. Time-history plots of the impulse for three experiments: (a) Exp 1, (b) Exp 2, and (c) Exp 3.

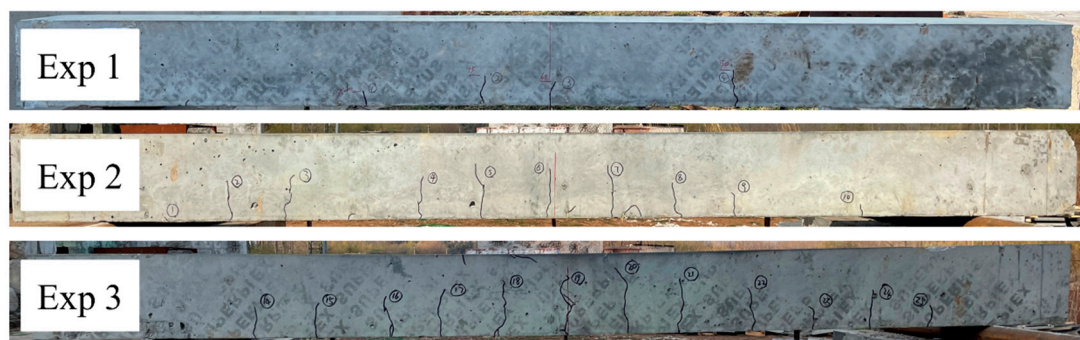


Figure 5. Failure morphology of various beam specimens.

Figure 6 shows the measurement results of the D1~D3 displacement time-history of the displacement sensor located at the measurement point on the back of the beam. As for Exp 1, the maximum deflection of D1, D2, and D3 of the three displacement meters of the specimen was 4.08 mm, 6.10 mm, and 4.30 mm, respectively. And the residual deflection was 1.02 mm, 1.61 mm, and 0.59 mm, respectively. As for Exp 2, the maximum deflection of the three displacement meters D1, D2, and D3 of the specimen was 9.91 mm, 13.60 mm, and 10.93 mm, respectively. And the residual deflection was 3.30 mm, 2.37 mm, and 2.30 mm, respectively. As for Exp 3, the maximum deflection of D1, D2, and D3 of the three displacement meters of the specimen was 16.28 mm, 24.94 mm, and 16.07 mm,

respectively. And the residual deflection was 2.66 mm, 7.82 mm, and 5.31 mm, respectively. According to measurable damage and displacement of beams, it can be found that the beam presents an overall bending deformation mode subjected the planar blast wave at a magnitude of 100 milliseconds.

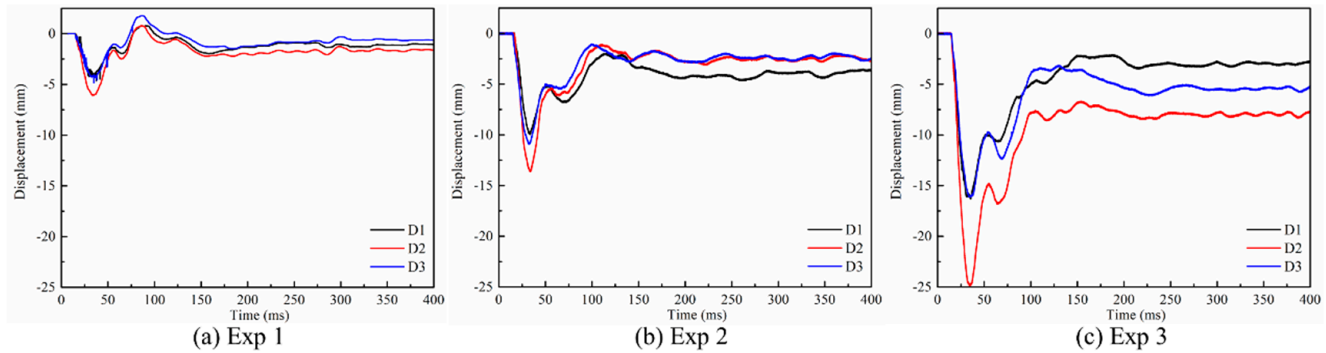


Figure 6. Time-history plots of the maximum deflection of three specimens: (a) Exp 1, (b) Exp 2, and (c) Exp 3.

4. Numerical Modeling

Since the beam presents an overall bending failure subjected the planar blast wave at a magnitude of 100 milliseconds, the single-degree-of-freedom model of the equivalent bending system can be used for simplified calculation, as shown in Figure 7, and its motion equation is as follows:

$$K_{LM}m\ddot{x}(t) + C\dot{x}(t) + R(x) = F(t) \quad (1)$$

where m is the system mass, x is the mid-span vertical displacement of the beam member, C is the damping coefficient, $R(x)$ is the resistance function of the bending deformation mode of the beam member, $F(t)$ is the equivalent external force of the uniform load, and K_{LM} is the load mass transformation coefficient.

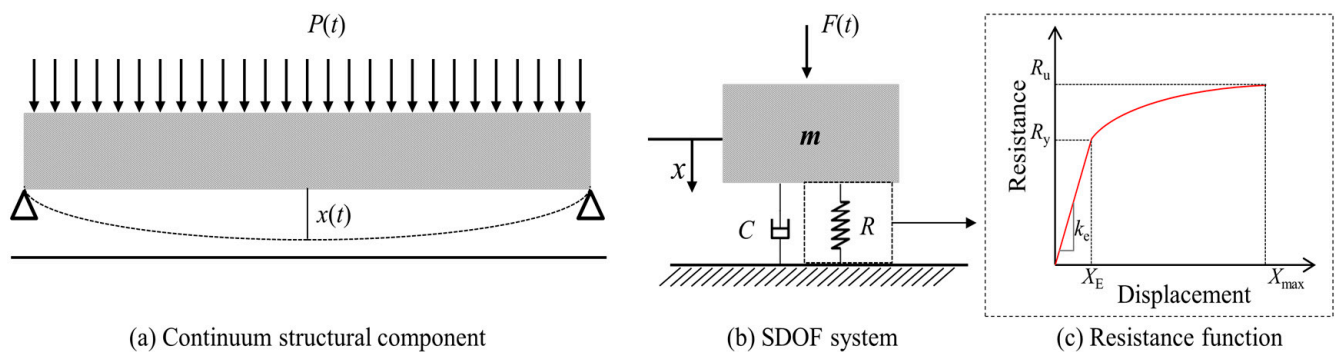


Figure 7. Constructing the SDOF system for the loaded beam.

Selecting the yield resistance R_y , mid-span elastic limit displacement X_E , and mass m to be the basic physical quantities, the dimensionless form of the single-degree-of-freedom motion equation can be written as follows:

$$\ddot{x}^* + 2\zeta\dot{x}^* + R^*(x^*) = F^*(t^*) \quad (2)$$

In this formula, the dimensionless quantity $x^* = x/X_E$, $R^* = R(x)/R_y$, $F^* = F(t)/R_y$, $t^* = tw$, elastic stiffness $k_e = R_y/X_E$, frequency $w = \sqrt{k_e/mK_{LM}}$, damping rate satisfies $C = 2\zeta k_e/w$. ζ is the flexural damping ratio and the value is 0.1. m is the mass of the beam member. The values of the load–mass equivalence factor K_{LM} are 0.78 in the elastic stage of the beam and 0.66 in the elastic–plastic stages of the beam, respectively.

The resistance of the beam is the internal force tending to restore the element to its unloaded static position. There are several functional forms of resistance, and for the bilinear resistance function, the following relation is satisfied:

$$R(x) = \begin{cases} k_e x & x \leq X_E \\ R_y + k_p(x - X_E) & X_E \leq x \leq X_{\max} \end{cases} \quad (3)$$

where k_e is the effective stiffness of the elastic stage of the beam member, and k_p is the effective stiffness of the plastic stage of the beam member.

According to the dimensionless motion equation, under the condition of a uniform distribution load, the main controlling factor of the displacement response is the resistance model of the member. There are several resistance models frequently used, including the ideal elastic–plastic model, rigid–plastic model, elastic–plastic-hardening model, and elastic–plastic-softening model. In order to obtain the resistance model of high-strength beams under a uniform load, the improved separated finite-element model of reinforced concrete proposed by our research group [25] was applied, using it on the above five kinds of beams. And then, we analyzed the displacement response using evaluation software of the damage grade of the strong shock wave structure [27]. Next, we carried out analysis of the key physical parameters such as yield resistance, elastic limit displacement, and stiffness from the beam resistance model to establish an idealized bilinear resistance model.

Figure 8 shows the improved separated finite-element model of the RC beam with a uniform load. The model consists of two types of grids: one is the grid containing the steel skeleton lines, which is defined as the reinforced concrete grid; the other type is the grid that does not contain the reinforcement skeleton line, which is defined as plain concrete grid. The classical Holmquist–Johnson–Cook (HJC) model is adopted for a plain concrete mesh, and the reinforced concrete composite constitutive model based on modified mixture theory is adopted for a reinforced concrete mesh.

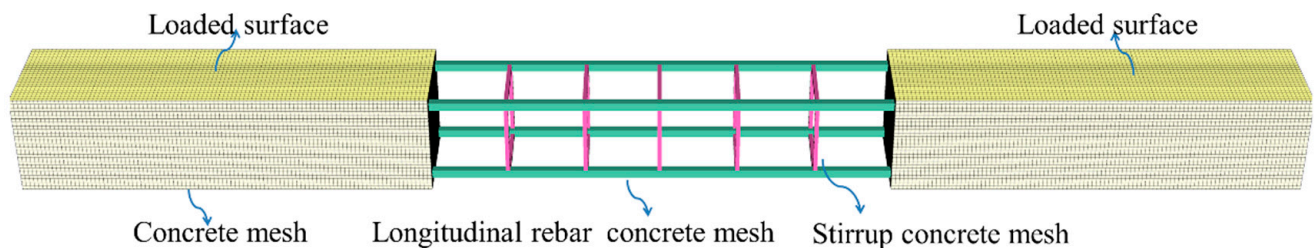


Figure 8. Improved discrete numerical model for RC beams.

The plain concrete grid adopts the HJC constitutive model, and its stress–strain relationship is as follows:

$$\sigma_m = f_m \left[A_m(1 - D) + B_m(P/f_m)^{N_m} \right] \left[1 + C_m \ln(\dot{\epsilon}/\dot{\epsilon}_0) \right] \quad (4)$$

where the subscript m represents the physical quantity related to the concrete matrix; σ_m represents the stress of concrete; f_m represents the concrete compressive strength; A_m represents the cohesion parameter; B_m represents the pressure hardening coefficient; N_m represents the pressure hardening index; C_m represents the strain rate parameter; and $\dot{\epsilon}_0$ represents the reference strain rate.

The reinforced concrete grid adopts the composite material constitutive model, and its stress–strain relationship is as follows:

$$\sigma = V_m \sigma_m + V_f \sigma_f^{\text{eq}} \quad (5)$$

where σ_m is established by the nondestructive concrete matrix constitutive Equation (3) which does not consider the pressure sensitivity. And σ_f^{eq} is the equivalent stress of the steel bar considering the interface bond slip effect, and its expression is as follows:

$$\sigma_f^{eq} = (f_y^{eq} + H_f^{eq} \varepsilon_p) [1 + C_f \ln(\dot{\varepsilon}_p / \dot{\varepsilon}_0)] \quad (6)$$

where f_y^{eq} represents the actual yield strength of steel bars embedded in concrete, which satisfies $f_y^{eq} = f_y [0.93 - 2(f_{mr}/f_y)^{1.5}/\rho]$; ρ represents the effective reinforcement ratio; f_{mr} represents the tensile fracture stress of concrete; H_f^{eq} represents the equivalent plastic modulus, which satisfies $H_f^{eq} = \int_{\delta_y}^{\delta_c} (\sigma_f - f_y) / (\varepsilon_p + \delta/l) d\delta / (\delta_c - \delta_y)$; δ represents the interface slip displacement of reinforced concrete; δ_y and δ_c represent the displacements at different stages of interface slip; l represents the length of the interfacial bonding zone; ε_p represents the plastic strain; and C_f represents the strain ratio coefficient.

Taking B3-C40-HRB400-D20 as an example, the material parameters in numerical simulation are briefly given. The mesh size was 10 mm. The concrete mesh adopted the HJC concrete constitutive model, and the parameters were as follows: the initial density was 2.4 g/cm³, the shear modulus was 14.4 GPa, Poisson's ratio was 0.20, f_m was 50.16 MPa, A_m was 0.28, B_m was 1.84, N_m was 0.84, and C_m was 0.007.

The parameters of the longitudinal reinforcement were as follows: the tensile yield strength was 466.7 MPa; the ultimate strength was 601.3 MPa; the elastic modulus was 200 GPa; and the plastic hardening modulus was 1.12 GPa. And the parameters of the stirrup were as follows: the tensile yield strength was 483.5 MPa; the ultimate strength was 582.4 MPa; the elastic modulus was 200 GPa; and the plastic hardening modulus was 0.94 GPa.

For reinforced concrete mesh, in the numerical model considering interface bond slip, the length of the interface bond zone was 300 mm, and the equivalent material parameters of longitudinal reinforcement and stirrup were calculated as follows: The longitudinal bar's equivalent strength f_y^{eq} , equivalent elastic modulus E_f^{eq} , and equivalent hardening modulus H_f^{eq} were 425.0 MPa, 162.2 GPa, and 788.8 MPa, respectively, and the stirrup's equivalent strength f_y^{eq} , equivalent elastic modulus E_f^{eq} , and equivalent hardening modulus H_f^{eq} were 368.1 MPa, 150.2 GPa, and 656.5 MPa, respectively. Then, according to the concrete's parameters, combined with Formula (4), the composite material model parameters in the reinforced concrete grid were obtained.

The influence of finite-element mesh size on numerical simulation results was studied. Taking the B3-C40-HRB400-D20 beam as an example, the pressure time history measured by Exp 3 was used as the pressure load on the surface of the beam. Three mesh sizes, 20 mm, 10 mm, and 5 mm, were selected and the corresponding grid scales were 12,500, 100,000, and 800,000, respectively. Figure 9 shows the time history of the mid-span displacement of the RC beam with different element sizes. The numerical simulation results of the three mesh sizes could be summarized as follows. The numerical simulation results are related to the mesh size; for example, the simulation results of 20 mm and 10 mm are slightly different. As the mesh size decreases gradually, the difference in the time history of the mid-span displacement curve decreases and converges gradually (10 mm and 5 mm). Further refinement of the mesh size can improve the calculation accuracy, but the calculation cost is doubled, so the mesh size of 10 mm is used to balance the computational accuracy and cost.

Figure 10a shows the bending moment–angle curve results of five kinds of beams, respectively. According to the conversion relationship between bending moment and resistance, the resistance–displacement curve of the beam is further given, as shown in Figure 10b. By increasing the strength of concrete, the strength of rebar, and the diameter of rebar, the bending mechanical properties of the member are significantly improved. Under a uniform load, the beam presents two stages of elastic and plastic hardening. Further, the yield resistance R_y and elastic limit displacement X_E of five beam samples

were extracted, and the dimensionless resistance–displacement curves were obtained, as shown in Figure 11. It is found that the resistance model has a good normalization property and can be further simplified into a bilinear model. Table 3 shows the yield resistance R_y , stiffness k_e in the elastic stage, stiffness k_p in the plastic stage, and the ratio of the two-stage stiffness of the five kinds of beams. By comparison, it is found that the stiffness ratio of k_p/k_e in the plastic stage and elastic stage of the five beam samples is small, and its average value is 0.017. A more conservative ideal elastoplastic resistance model, $k_p = 0$, can be used in the design and blast-resistant analysis of high-strength RC beams. The key physical parameters in the bilinear resistance model, such as yield resistance, elastic stiffness, and plastic stiffness, can quantify the resistance model of beam members and, as important input parameters of the SDOF model, effectively analyze the dynamic response of RC beams under a uniform load of support.

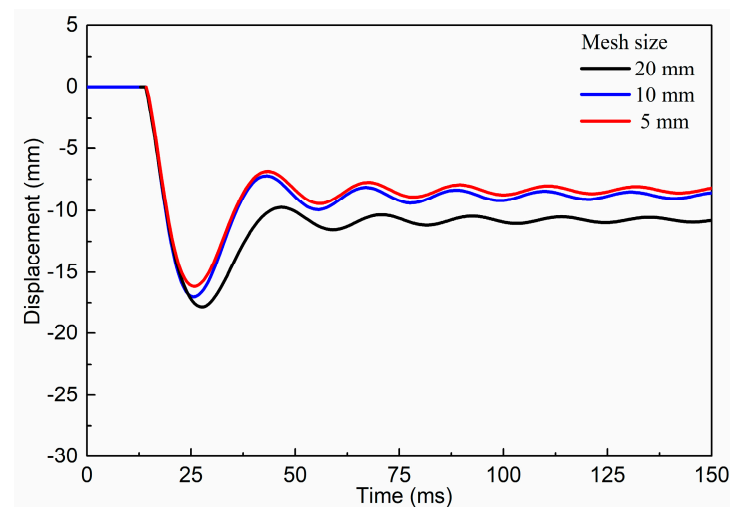


Figure 9. Mid-span displacement–time histories of RC beams with different mesh sizes.

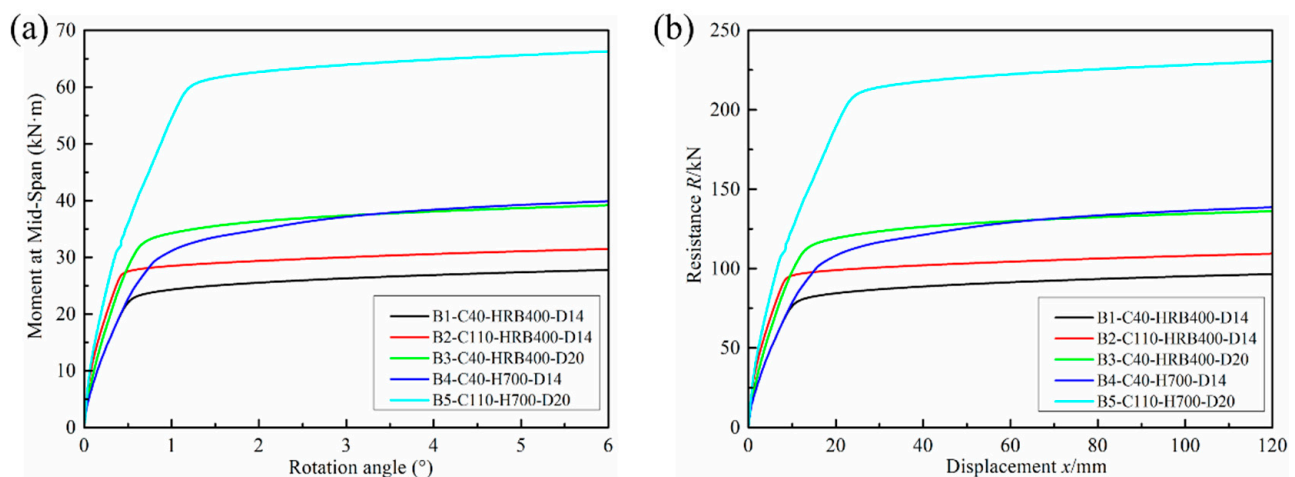


Figure 10. Results from uniform pressure loaded FE models: (a) moment–angle; and (b) resistance–displacement.

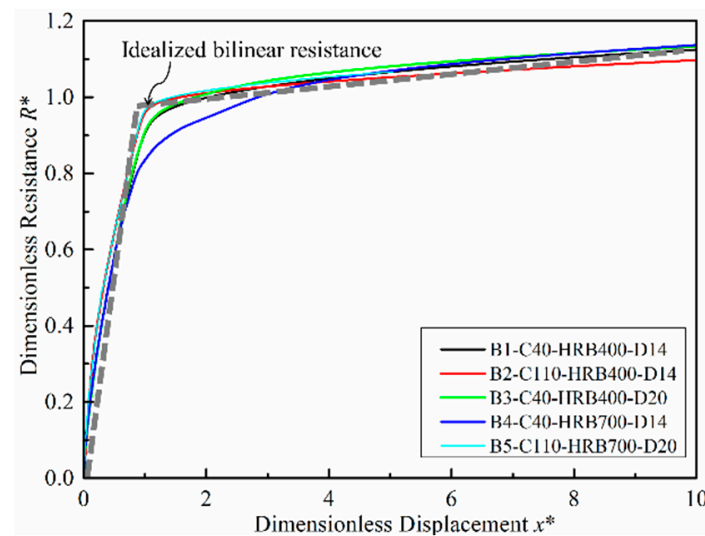


Figure 11. Results of nondimensional resistance–displacement curves.

Table 3. Bilinear resistance model.

Beam Sample	Yield Strength R_y (kN)	Elastic Stiffness k_e (kN/m)	Plastic Stiffness k_p (kN/m)	k_p/k_e
B1-C40-HRB400-D14	77.3	8.63×10^3	123.5	0.015
B2-C110-HRB400-D14	88.9	11.33×10^3	154.7	0.014
B3-C40-HRB400-D20	109.8	9.79×10^3	203.1	0.021
B4-C40-HRB700-D14	112.2	8.58×10^3	167.1	0.019
B5-C110-HRB700-D20	197.5	13.23×10^3	242.1	0.018

5. Numerical Model Validation

The accuracy of the SDOF model and the finite-element model is verified by the measured results of the displacement time history of the midpoint D1 of the beam member. In the process of numerical calculation, the pressure time history curve measured by experiment is taken as the input of explosion load. In the SDOF analysis, the Newmark–Beta method is used to solve the differential equation of the single-degree-of-freedom model, and the time step is 0.1% of the natural vibration period. The finite-element model integrated into the evaluation software of the damage grade of the strong shock wave structure [27,28] is used to analyze the dynamic response of an RC beam subjected to blast loading, and the data of the mid-span displacement are outputted.

Figure 12 shows the comparison of the SDOF calculation results with the finite-element calculation results and experimental results. For the working condition of Exp 1, the maximum displacement of D1 calculated by the SDOF is 6.13 mm, and the maximum displacement of D1 calculated by the finite-element model is 6.25 mm. The deviation between the SDOF and finite-element model is 0.5% and 2.4% compared with the experimental results. For the working condition of Exp 2, the maximum displacement of D1 calculated by the SDOF model is 14.6 mm, and that by the finite-element model is 14.9 mm. Compared with the experimental results, the deviations of D1 and the SDOF are 7.3% and 9.5%, respectively. For the working condition of Exp 3, the maximum displacement of D1 calculated by the SDOF model is 23.7 mm, and that by the finite-element model is 27.6 mm. Compared with the experimental results, the deviations between the two are −4.9% and 10.8%. These results verified the simulation accuracy of the SDOF model and finite-element model by the comparison of the results of three working conditions. Additionally, in the plastic rebound stage, the simulation results of the SDOF model and finite-element model differ greatly from the experimental results, leading to the discrepancy between the simulated residual displacement and the experimental results. The numerical model used can simulate

the dynamic response of beams under long-duration blast loading. However, there are still some differences between the numerical model and experiments, as follows: (1) the numerical model lacks an elaborate method to simulate the interaction process between the reinforcement layer and the concrete; (2) the propagation, refraction, and reflection of the stress wave in structures lead to a complex state of stress due to the non-uniform medium characteristics of reinforced concrete, which is difficult to be reasonably described by mathematical means; (3) the load-blast-enhanced strategy to simulate the shock wave loading did not take into account the diffraction effect of the explosion wave and wave-structure interaction, and it also ignored the thermal effect of high-temperature gas acting on the structure.

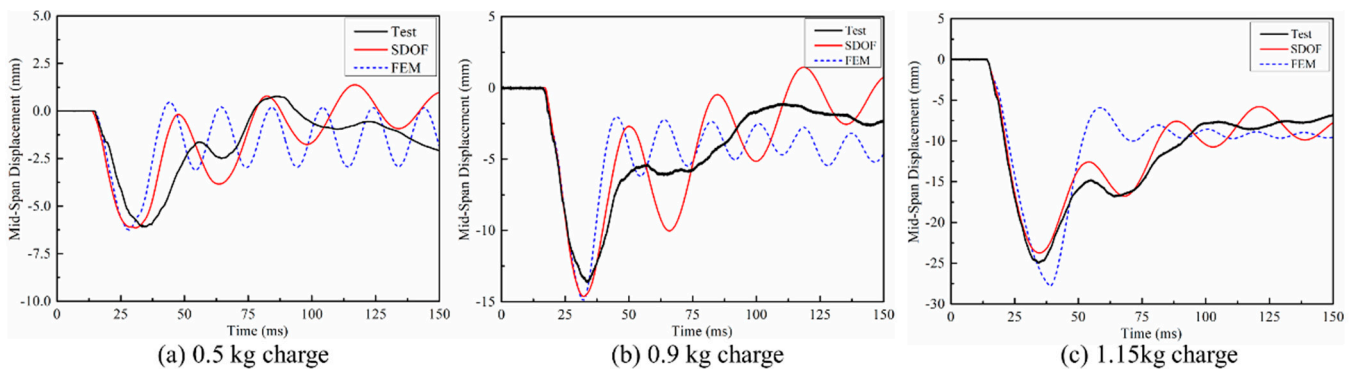


Figure 12. Comparison of mid-span displacement of beam specimens.

In the *P-I* curve analysis in the following section, the maximum displacement at the mid-span position is used as a physical quantity to evaluate the dynamic response degree of the component.

6. Pressure–Impulse Curve

6.1. Reference Beam *P-I* Curve

The pressure–impulse curve is an important analysis method for dynamic response analysis and the blast-resistant design of components. At the same time, the displacement ductility ratio μ (the ratio of the maximum displacement x_{max} to the elastic limit displacement X_E during deformation) can quantify the degree of deformation or damage of the component. Therefore, *P-I* curves based on the ductility ratio can provide an effective means for the blast-resistant design and dynamic response analysis of high-strength beams.

The displacement analysis of reference beams numbered B1-C40-HRB400-D14 was carried out using the SDOF model verified by experiments, and *P-I* curves related to ductility ratio were drawn. According to the classification of explosion load, the pressure–impulse space can be divided into three load regions: impulse load, quasi-static load, and dynamic load. The validity of *P-I* curves based on the SDOF model in three load zones is evaluated by combining the experimental results of shock wave explosion and finite-element simulation. Figure 13 shows the *P-I* curves corresponding to the ductility ratio of the reference beam when $\mu = 1, 2$, and 4, and the comparison with the experiment. The displacement ductility ratios of the components corresponding to the three explosion experiments are 0.62, 1.34, and 2.46, respectively, which correspond to the numbers close to the red mark symbol. The experimental explosion load belongs to the dynamic load, and the corresponding ductility ratio points of the three experimental load spaces all fall within the corresponding interval, which verifies the validity of the *P-I* curve in the dynamic load interval. Furthermore, 44 finite-element calculation examples are carried out in the quasi-static load and pulse load regions combined with finite-element model. Figure 14 shows the comparison between the ductility ratio results obtained based on the finite-element model and the ductility ratio contour lines obtained based on the SDOF model. The finite-element

simulation results basically fall within the corresponding ductility ratio interval, which further verifies the validity of the curve obtained based on the SDOF model.

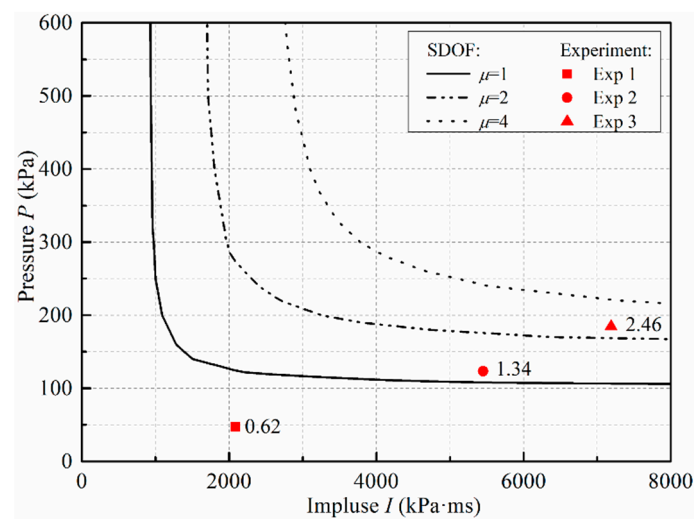


Figure 13. *P-I* diagram for the reference beam with experimental results.

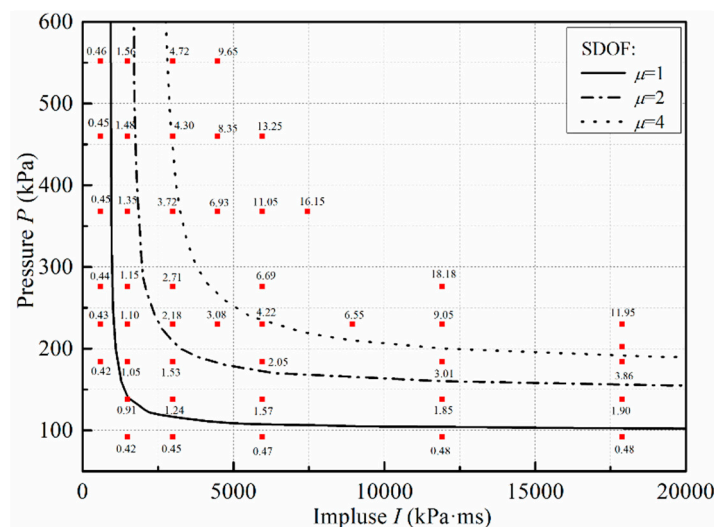


Figure 14. *P-I* diagram for the reference beam based on the finite-element method.

6.2. *P-I* Curve of the High-Strength Beam

The SDOF model was used to analyze the dynamic response of four kinds of high-strength beam samples and then draw *P-I* curves. Figure 15 shows the *P-I* curve corresponding to the ductility ratio $\mu = 1$ of a high-strength beam. It is found that compared with the reference beam, the pressure asymptote and impulse asymptote of the high-strength beam can be improved by increasing the strength of concrete, the strength of reinforcement, and the diameter of reinforcement. Further, the normalized *P-I* curve is obtained by the yield resistance R_y and frequency w . Figure 16 shows the dimensionless pressure–impulse curves (P^*-I^*) corresponding to the ductility ratio $\mu = 1, 4$, and 10 , where the dimensionless pressure $P^* = PA/R_y$ and dimensionless impulse $I^* = IA\omega/R_y$, and A is the stress area of the member. As can be seen from the figure, although the yield resistance and stiffness of beams with different strengths vary greatly (see Table 3 for details), the P^*-I^* based on the ductility ratio has a normalized property; that is, through the dimensionless pressure–impulse curve of the reference beam, combined with the yield resistance and circular frequency parameters of the high-strength beam sample, the results show that the pressure–impulse curve of a high-strength beam under a plane shock wave load can be obtained quickly and accurately,

which is convenient for the explosion response analysis and blast-resistant design of beam members. According to the damage criterion for ductility ratios exceeding 10, given by the UFC 3-340-02 design code [29], when the peak overpressure of blast loading is 202 kPa, an ordinary RC beam such as B1 is close to yield failure, while a high-strength RC beam, such as B5, begins to yield failure when the peak overpressure value is 500 kPa. These results are basically consistent with the other results [9].

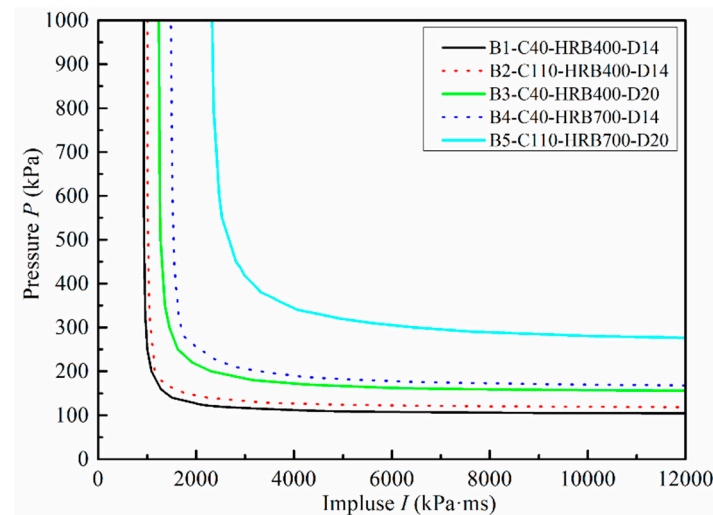


Figure 15. P - I diagram for high-strength beams with a ductility ratio of 1.

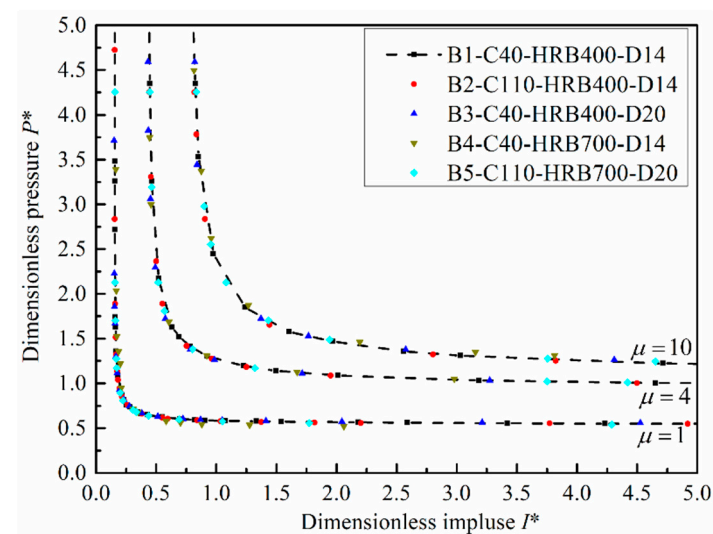


Figure 16. P^* - I^* diagram for five types of beams with ductility ratios of 1, 4, and 10.

In order to further verify the normalized properties of dimensionless pressure–impulse curves, the finite-element model was used to analyze the ductility of three kinds of high-strength beams, including B3-C40-HRB400-D20, B4-C40-HRB700-D14, and B5-C110-HRB700-D204, under a uniform explosive load, and the results were compared with the dimensionless pressure–impulse diagram. Figure 17 shows the comparison between the P^* - I^* diagrams and 144 finite-element simulation results corresponding to ductility ratios $\mu = 1, 2$, and 4. The following trends are shown in Figure 17: (1) Most of the finite-element simulation results fall within the ductility ratio interval of the corresponding P^* - I^* graph. (2) In the three load areas of pulse load, quasi-static load, and dynamic load, the ductility ratio of the beam under the same (P^*, I^*) load is consistent. For example, the pulse load $(P^*, I^*) = (5, 0.75)$, and the ductility ratios of the three beams are 7.70, 7.94, and 7.71, respectively. The dynamic load $(P^*, I^*) = (5, 0.75)$, and the ductility ratios of the three beams are

6.61, 6.51, and 6.68, respectively. The quasi-static loads (P^* , I^*) = (0.75, 5), and the ductility ratios of the three beams are 1.24, 1.19, and 1.22, respectively. (3) In some areas, there are certain differences in beam ductility ratios; for example, when (P^* , I^*) = (1.1, 5), the ductility ratios of the three beam samples are 8.79, 7.84, and 9.47, respectively. (4) In the quasi-static load area, the finite-element results are more conservative than the SDOF results. For example, if the load point (P^* , I^*) = (1, 5), the ductility ratios of the B3-C40-HRB400-D20 and B5-C110-HRB700-D204 beams are 4.42 and 4.44, which are higher than the isoline of the SDOF ductility ratio.

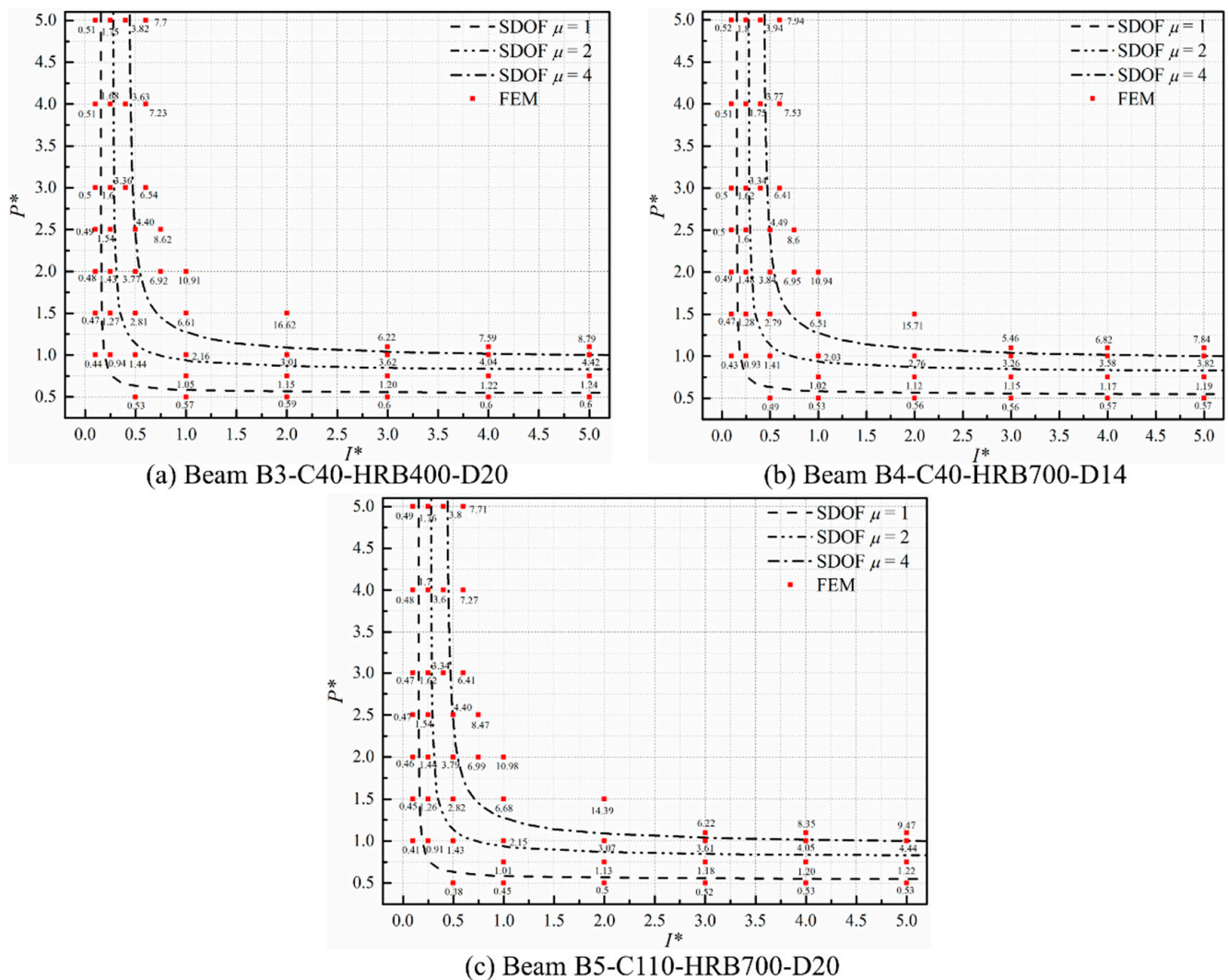


Figure 17. P^* - I^* diagram with finite-element results.

By correlating the P - I diagram with industry design standards, Figure 17 shows the P - I damage contours for the damage levels for the high-strength RC beams defined in UFC 3-340-02 [29] and ASCE 2010 [30], which can be used to analyze the far-field explosion in specified explosion scenarios such as long-range large equivalent chemical explosions and industrial gas explosions. For example, for high-strength RC beams (B5-C110-HRB700-D20 sample), a critical overpressure peak of 500 kPa is obtained, agreeing with experimental results [9].

P - I diagrams for high-strength beams have been partially verified via some experimental results (see Figure 18). However, they also have certain limitations. First of all, the experimental verification is limited to three test points, which mainly belong to the dynamic load region in the figure. For the pulse load and quasi-static load region, the

calibrated finite-element model is used to study the effectiveness of the two load regions. Then, the second limitation is that the pressure–impulse diagram is valid only for far-field explosions, assuming that the explosion pressure is evenly distributed on the surface of the member and that the pressure time history curve is reasonably approximated as a triangular pulse. This figure is not suitable for analyzing the near-field explosion effect on high-strength beams. The third limitation is derived from the deformation mode of the beam, which means the SDOF model used to generate the pressure–impulse diagram assumes the deformation mode of the beam. For pulsed loads, this assumed deformation shape may not be accurate because of the risk of shear failure near the beam end support.

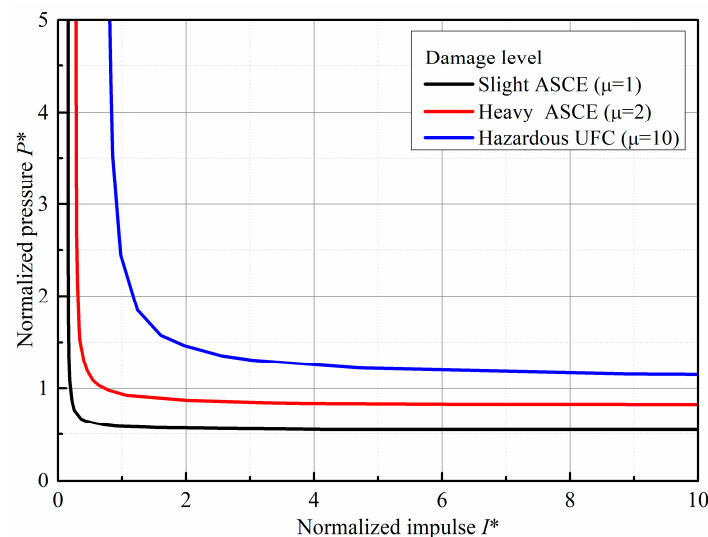


Figure 18. P^* – I^* diagram for design of high-strength RC beams.

7. Conclusions

An experimentally validated P – I diagram for reinforced concrete beams for far-field blast effects was developed and proposed. The ductility contours for P – I diagrams were developed using an experimentally validated SDOF model that used, as input, an idealized bilinear resistance function for reinforced concrete beams subjected to uniform load. P – I diagrams can quickly predict the critical overpressure peak corresponding to the hazardous level of reinforced concrete beams in specified explosion scenarios such as large equivalent chemical explosions and industrial gas explosions.

- (1) The overall bending of reinforced concrete beams is obtained via explosive chamber experiments with a long-duration pulse of 80–105 ms.
- (2) A single-degree-of-freedom model with an idealized bilinear resistance function is proposed based on the overall bending mode. The resistance function is developed using a novel approach combined with a three-dimensional improved steel–concrete separated finite-element model.
- (3) Through the normalized P – I analysis method with the ductility criterion given by the UFC 3-340-02 design code, the peak overpressure 202 kPa, corresponding to the hazardous state of the ordinary beams, is obtained, while the peak overpressure thresholds for high-strength beams reaches 500 kPa, significantly higher than that of ordinary beams.
- (4) The analytical method does not accurately predict the shear-dominated deformation and fracture behavior of each member. Meanwhile, the validation of damage overpressure thresholds for high-strength beams is insufficient.
- (5) In the next work, long-duration blast loading experiments of high-strength concrete beams will be carried out to explore the damage mode, especially for the end-supported shear failure. Meanwhile, the pulse of the shock wave will be broadened

from 100 ms to the order of seconds, supporting the validation of P - I diagrams in the quasi-static region.

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