

Article

Evaluation of Critical Thresholds for Surrounding Rock Stability in TBM Tunnels Utilizing Limit State Analysis

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Abstract: The safety and stability of tunnel construction are paramount concerns in tunnel engineering. Traditionally, the assessment of tunnel stability involves evaluating whether tunnel displacement and deformation velocities exceed acceptable thresholds, using specifications and analogical reasoning. However, standardized cautionary indicators often lack adaptability and can result in unnecessary reinforcement and increase economic costs. This study presents a method for determining limit displacement and deformation velocities based on tunnel deformation limit states. Based on the finite difference method, which considers the quality of surrounding rock mass and tunnel geometric configuration, a parametric analysis of these velocities is presented. The vault settlement, relative spandrel displacement, relative hance displacement, and deformation velocity are investigated to establish the thresholds for displacement and deformation velocities. Furthermore, a sensitivity analysis using a multivariate adaptive spline regression model is performed to measure the parameter effects on the early allowance values. Finally, a response surface method is developed to estimate the indicators in order to provide criteria for assessing and warning about surrounding rock stability. The obtained results show that the proposed limit state analysis method is effective for providing a standardized criterion for tunnel excavation, and the sensitivity analysis indicates tunnel depth as the most influential factor on tunnel reliability.



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Keywords: pre-warning index; multivariate adaptive sample regression; mathematical statistics; limit displacement; tunnel stability

1. Introduction

With the progression of societal development, there has been a surge in the proliferation of large-scale underground water conveyance projects. These significant underground construction projects often confront formidable obstacles while navigating through complex terrains. The New Austrian Tunneling Method (NATM) stands out for its exceptional ability to adjust to challenging geological conditions, establishing itself as the preferred construction technique for such endeavors. As NATM gains widespread adoption, the imperative to monitor surrounding rock deformation in tunneling projects intensifies [1,2]. The deformation of surrounding rock serves as a crucial and comprehensive indicator of tunnel stability [3], facilitating meticulous assessment against displacement control criteria.

The assessment and control of tunnel stability have garnered considerable attention from both the academic and engineering communities [4–6]. National standards in China dictate allowable displacements for tunnel crown settlement and peripheral convergence based on rock categories and burial depths [7]. Warning control standards for moderate-sized tunnel crown settlements have been reported, taking into account burial depths and rock categories based on construction experience [8]. For instance, in the case of the Alberg Tunnel in Austria, the allowable net displacement was specified as 10% of the tunnel radius

or the length of the anchor, preferably not exceeding 30 cm. Tan et al. [9] suggested that the maximum allowable displacement varied with burial depth and was approximately 1/1000 of the burial depth. Skempton and MacDonald [10] suggested limits on structural deformations based on studies of permissible structural deformations. Ning [11] investigated the relationship between measured convergence displacement in tunnels, rock mass categories, and the time taken to establish safety judgment criteria. This included characteristic values, such as the stability time of various rock masses, stable moment displacement, final convergence displacement, and deformation velocities, achieved through the construction of a database of measured displacements and the application of mathematical statistics. Yu et al. [12] compiled a significant amount of measured displacement data from tunnel engineering to construct a displacement database. They utilized the confidence interval method to determine the stable mean and standard deviation of the tunnel crown settlement, considering both as the warning standard value. Liu et al. [13] analyzed the characteristics of monitoring displacement curves during tunnel excavation. Furthermore, based on a database of measured tunnel deformation and a usage of confidence intervals, they established warning criteria for tunnel collapse deformation. Wu et al. [14] conducted a three-dimensional finite element analysis on tunnels with weak interlayers under different working conditions. They proposed quantitative indicators for controlling tunnel excavation deformation. However, these existing warning indicators, both domestic and overseas, are derived from statistical analyses of excavation deformation and deformation rates of specific tunnels. The warning indicators obtained from this method may not be applicable for the target projects of interest. Moreover, the abovementioned measured deformation and deformation velocities are based on convergent deformation at the stable stage of tunnels. This may lead to conservative warning indicators and further cause excessive support usage during a tunnel excavation. Limited studies focus on standard warning indicators of tunnel excavations with wide applicability, which are obtained from the limit states of tunnels.

The overall objective of this study is to evaluate the threshold value of displacement and deformation velocities based on the tunnel deformation limit states. The finite difference method is adopted to investigate the deformation and deformation velocities of excavated tunnels, considering the impacts of rock mass property (e.g., the rock mass quality index *BQ* value) and geometric parameters (including burial depth and radius) on tunnel excavation. Validations of the established numerical model with the analytical methods and a case study are subsequently conducted. A standardized criterion is developed to recognize the limit state of tunnel excavation. Moreover, a sensitivity analysis on parameters influencing the tunnel predictive indicator values is carried out using the multivariate adaptive spline regression model. Furthermore, response surfaces are presented to illustrate how tunnel rock mass quality, burial depth, and diameter impact the ultimate displacement and deformation rates, thus complementing the established prediction models.

2. Numerical Model Establishment and Verification

The finite difference software FLAC3D 5.0 is adopted to simulate the tunnels in this study. Figure 1 illustrates the dimensions and geometry of a quarter of the numerical calculation model. The model dimensions are 120 m along the transverse axis of the tunnel (X-direction), 120 m along the vertical axis (Z-direction), and 0.5 m along the longitudinal axis (Y-direction), simplifying the tunnel excavation into a two-dimensional plane strain problem. The model boundaries are fully constrained, and the tunnel burial depth and excavation radius vary based on the working conditions. The rock mass follows an ideal elastic–plastic yielding criterion, with parameters determined using the Hoek–Brown formula and considering rock mass quality classification. Initial ground stress is represented by a static stress field influenced solely by self-weight stress. The horizontal stress is calculated using the well-known Jaky's equation [15] based on the internal friction angle of the rock mass ($K_0 = 1 - \sin\varphi$), where K_0 is the coefficient of static earth pressure

and φ is the internal friction angle. The lining system includes steel arches, steel mesh, shotcrete, and grouted rock bolts.

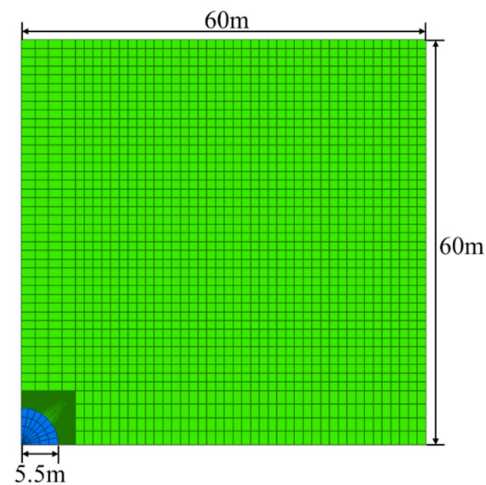


Figure 1. The configuration of the numerical model.

To validate the accuracy of the established model using the finite different method, comparisons with the analytical methods and a case study are carried out, respectively. Figure 2 presents a comparison of radial displacements obtained from numerical simulations conducted in this study with the analytical solution proposed by Carranza-Torres [16]. The physical parameters of the rock mass are obtained from the Engineering Rock Mass Classification Standard [17], where the elastic modulus of the rock mass is 0.66 GPa, the cohesion is 0.26 MPa, the internal friction angle is 27.9° , the Poisson's ratio is 0.45, and the unit weight is 19.8 kN/m^3 . The comparison encompasses tunnel burial depths ranging from 300 m to 1100 m and diameters ranging from 7 m to 11 m. The comparisons depicted in Figure 2 reveal a good agreement between the outcomes derived from the numerical model in this study and those proposed by Carranza-Torres [16]. This alignment indicates that the established model employed for estimating the deformation of fully excavated tunnels with varying burial depths and diameters demonstrates sufficient accuracy.

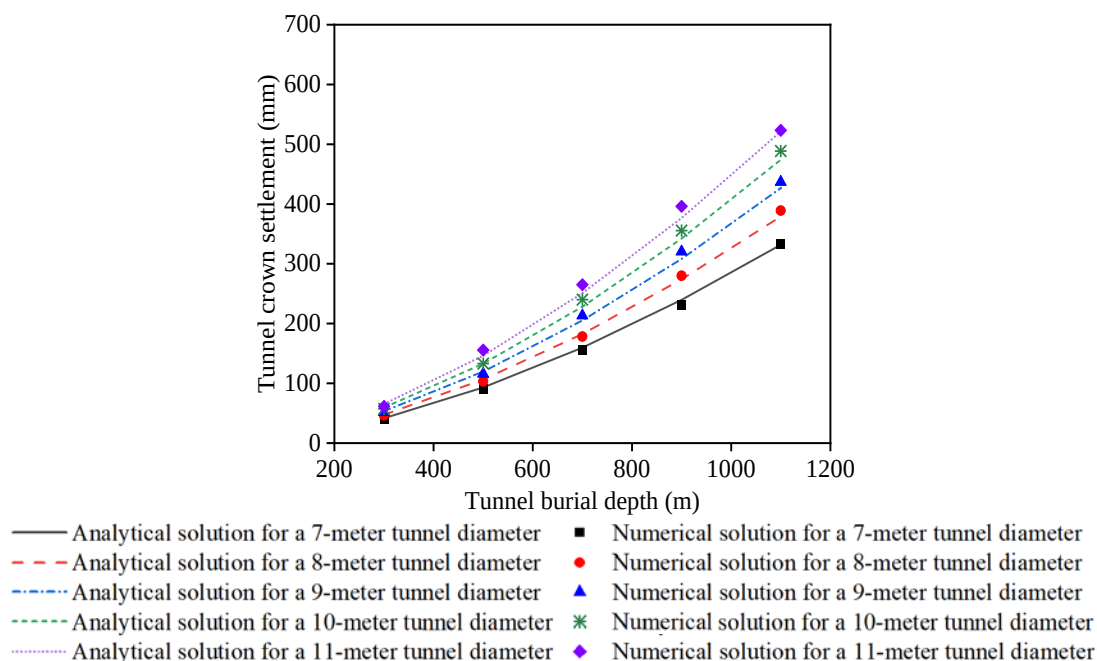


Figure 2. A comparison of the radial displacements derived from the numerical simulations in this investigation and the analytical solution proposed by Carranza-Torres [16].

Figure 3 presents a comparison of the vault settlement of the tunnel obtained from the established model in this study with the measured results from a case study by Qiao et al. [18]. The geometric configuration and the property parameters of the tunnel in the case are as follows: the buried depth of the tunnel is 400 m, the elastic modulus of the rock mass is 17.1 GPa, the cohesion is 2.5 MPa, the internal friction angle is 46° , the Poisson's ratio is 0.19, and the unit weight is 28 kN/m^3 . The variation in the vault settlement of the tunnel versus the distance from the excavation face is compared in Figure 3, indicating a good agreement of the results obtained from this study and those from the in-situ measurement, both in values and the variation trend. This demonstrates that the established model has sufficient accuracy for the following evaluation in this study.

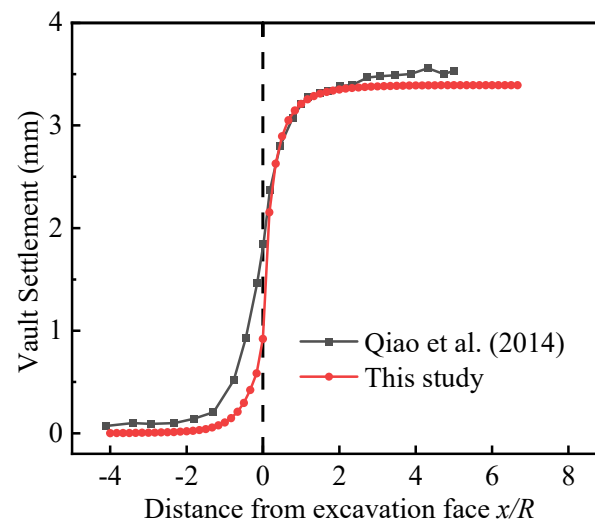


Figure 3. A comparison of the vault settlement derived from the numerical simulations in this investigation and the measured results from a case study [18].

3. Critical Thresholds

The tunnel excavation sets off a redistribution of initial rock stress, which is particularly noticeable in unlined tunnels where the surrounding rock undergoes plastic yield as stress surpasses its strength. This leads to a gradual expansion of the plastic zone and subsequent progressive rock displacement. The stable state of the tunnel precisely before the collapse signifies its ultimate condition.

As for tunnels equipped with support structures, both the surrounding rock and the support system can afford an increase in pressure caused by the redistribution of ground stress. The ultimate condition of the surrounding rock and support system is indicated by the phenomena of their synchronized deformation without collapse. However, defining the ultimate condition of equipped tunnels is challenging due to various factors such as the rock mass quality, excavation method, and support form. Previous studies lack standardized criteria for determining tunnel ultimate conditions. By examining the tunnel excavation displacement versus stress variations, this study introduces a quantitative representation of the tunnel's ultimate condition to determine the ultimate displacement and deformation velocity of tunnels.

3.1. Displacement

The determination of the displacement warning value involves evaluating displacements at various monitoring points under specific limit states. These assessments are influenced by factors such as rock mass quality, tunnel burial depth, diameter, excavation method, and support structure. Currently, the assessment of ultimate displacement relies on various approaches, including statistical analysis of field data, laboratory experiments, theoretical analysis, and normative recommendations. It is usually supplemented by engineering analogies. However, the limitations in field conditions and monitoring capabilities

may result in insufficient data, particularly lacking data near instability or during unstable states. Additionally, it is difficult to replicate field geological conditions in laboratory settings because of the complication from the heterogeneous composition of engineering rock masses. Also, large-scale field experiments may face challenges due to complicated site conditions. Therefore, this study develops a limit state analysis method to determine the ultimate displacement of tunnel surrounding rock based on the numerical simulation. The proposed methodology involves the following steps:

- (1) Construct a two-dimensional plane strain numerical model of tunnel excavation with the release of surrounding rock stress. After the stage of the equilibrium of initial ground stress field, remove the inner part of the tunnel and replace with a decreased support stress along edge of the tunnel compared with the original earth stress, and evaluate the model again. This replaced support stress is set to be uniformly released, and the deformation values of tunnel monitoring points are then extracted at each stress release stage [19].
- (2) Plot the curve of the deformation of tunnel monitoring points versus the stress release rate. Find out the threshold value where the displacement shows a sharp increase when the stress release rate exceeds this value (e.g., 0.9 as shown in Figure 4). This condition is considered to reach the limit state.
- (3) Extract the displacement corresponding to the stress release stage at the limit state of instability. This serves as the warning value of displacement (Figure 4).

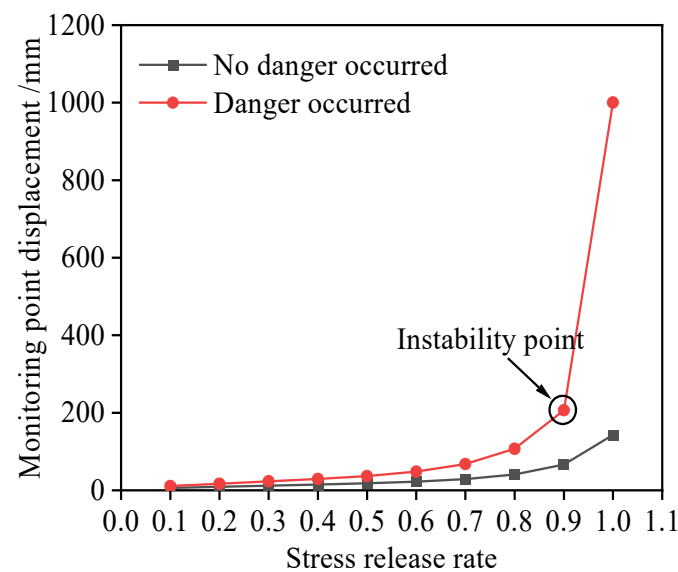


Figure 4. Variation of tunnel monitoring point displacement with stress release rate.

3.2. Deformation Velocity

Tunnel excavation poses spatial challenges due to three-dimensional constraints, with semi-circular vault-shaped limitations longitudinally and annular constraints laterally. These constraints hinder the immediate release of elastic–plastic deformations near the excavation face. A virtual support force is added to represent this spatial constraint effect and to simplify tunnel excavation analysis as a plane strain problem. It is observed when a specific tunnel section is excavated, resulting in immediate and continuous convergence displacement. This deformation is arrested by applying a radial force against the tunnel wall, known as the virtual support force. As the excavation face moves away from the section under scrutiny, the virtual support force gradually diminishes until it reaches zero. Previous investigations showed that tunnel excavation can be simplified as a continuous decrease in virtual support force alongside the continuous increase in rock displacement. Upon reaching the limit state of the surrounding rock, the tunnel experiences a significant increase in displacement following a minor stress release. Subsequent displacements

at monitoring points can be assessed based on the variation of displacement with the virtual support force. In this study, the slope represents the deformation velocity of the tunnel, indicating the ratio of displacement at the monitoring point to the released virtual support force. Sun [20] derived a comprehensive expression for the virtual support force, allowing for the determination of its variation curve with excavation distance under different working conditions. Based on Sun's method [20], various variation curves for V-grade rock masses ($BQ = 150\sim 250$) at burial depths ranging from 300 m to 1100 m are illustrated in Figure 5, providing detailed data for researchers to analyze. Furthermore, the deformation velocity of the tunnel curves versus the distance from excavation face are showed in Figure 6, with various values of V-grade rock masses at burial depths from 300 m to 1100 m. It can be observed from the curves that the maximum deformation velocity shows a significant increase with the crucial depth. By using these curves, researchers can determine the virtual support force and deformation velocity at any point during tunnel excavation under varying burial depths. The results from these curves are crucial for assessing rock mass stability and support design during tunnel excavation, offering valuable guidance for tunnel design and construction in practical engineering projects.

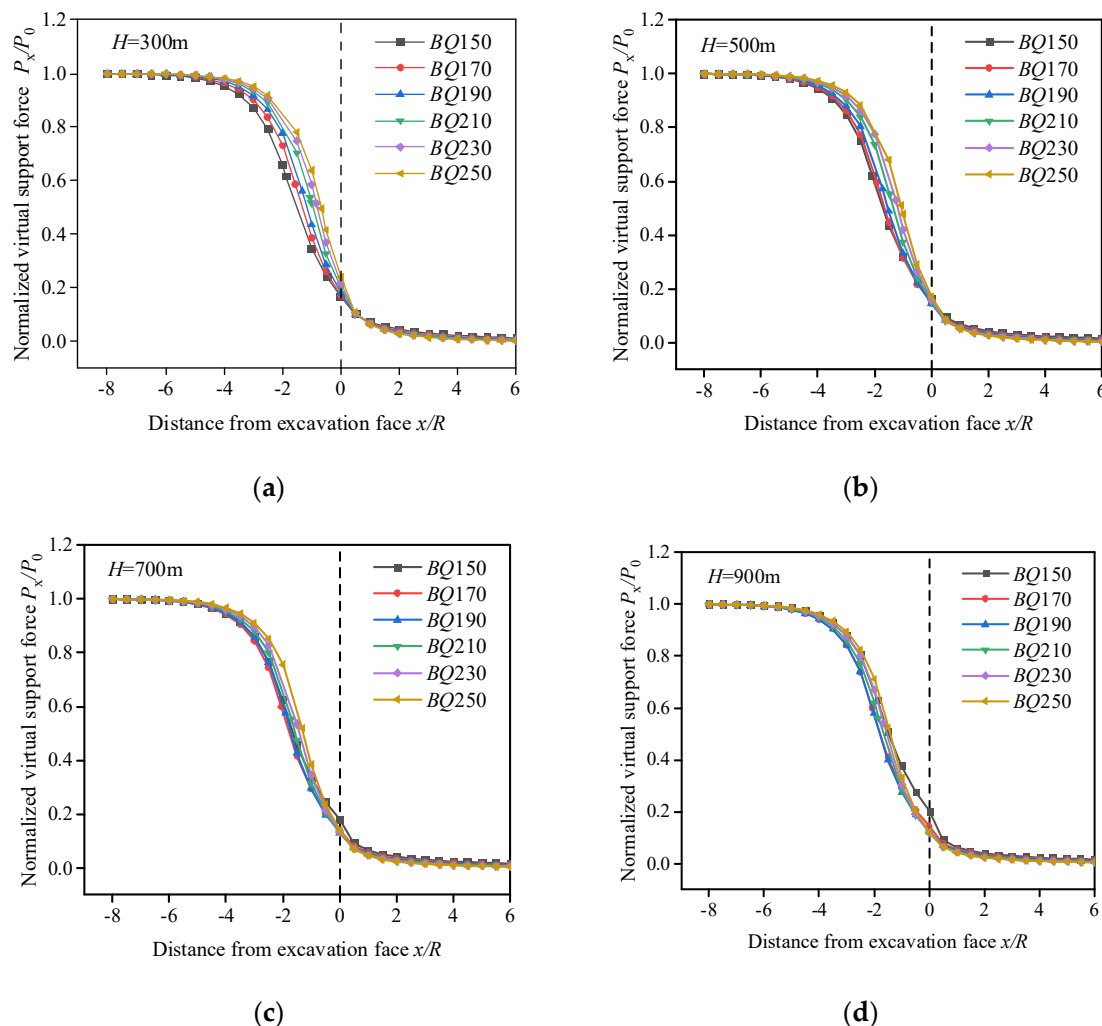
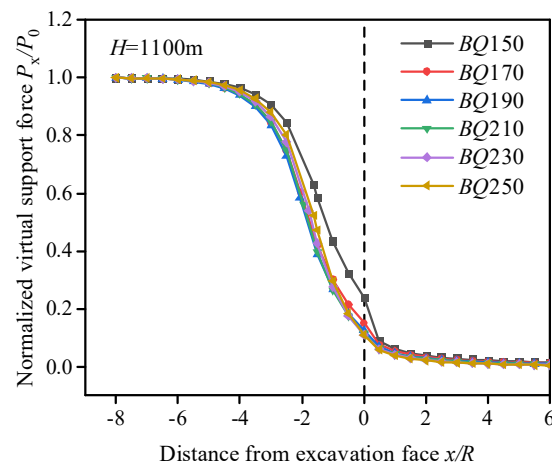
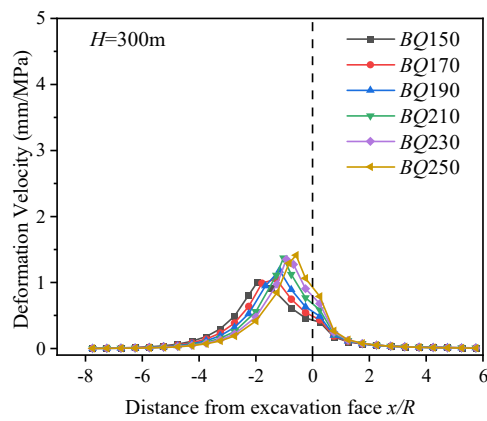


Figure 5. Cont.

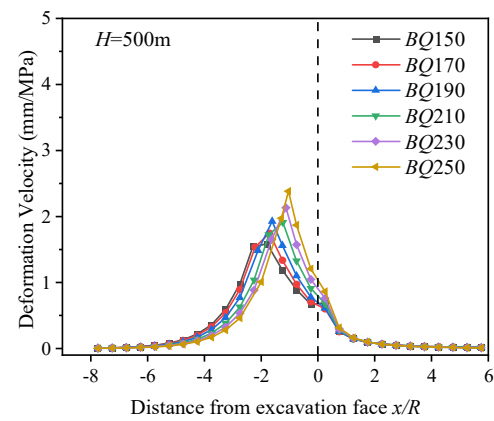


(e)

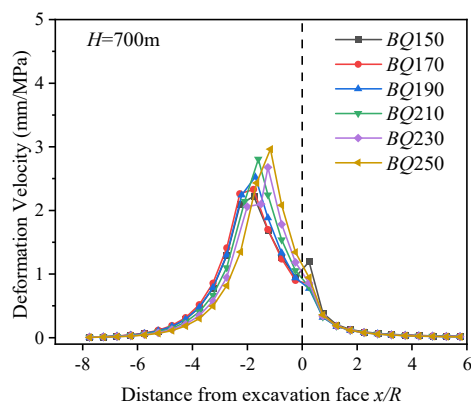
Figure 5. Longitudinal distribution curve of virtual support force: (a–e) represent the time histories of virtual support forces for V-class surrounding rock at burial depths of 300 m, 500 m, 700 m, 900 m, and 1100 m, respectively.



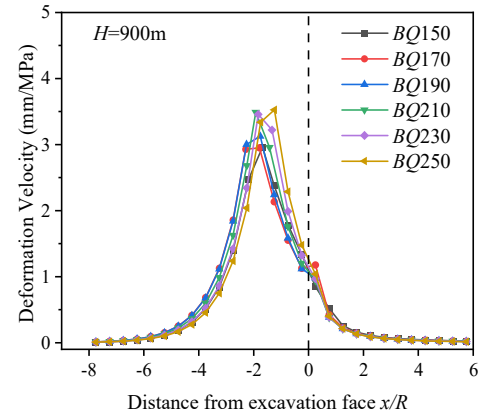
(a)



(b)

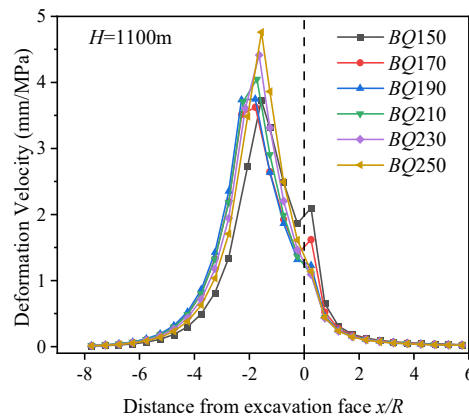


(c)



(d)

Figure 6. Cont.



(e)

Figure 6. Longitudinal distribution curve of deformation velocity: (a–e) represent deformation velocity values for V-class surrounding rock at burial depths of 300 m, 500 m, 700 m, 900 m, and 1100 m, respectively.

4. Numerical Simulations of Ultimate Displacement

The numerical model in this study adopts a circular cross-section and assesses rock mass quality using the *BQ* value recommended by the Engineering Rock Mass Classification Standard [17]. Considering the impact of rock mass quality, tunnel burial depth (ranging from 300 m to 1100 m), and diameter (ranging from 7 m to 11 m) on tunnel ultimate displacement, various scenarios' databases for the ultimate displacement of tunnels are established. The physical parameters corresponding to each rock mass quality level are chosen based on conversion methods proposed in the literature [21,22] by the Engineering Rock Mass Classification Standard [17]. Details of the parameter selections are provided in Table 1. The support design includes shotcrete, steel arches, steel mesh, and fully bonded grouted rock bolts. Shotcrete, steel arches, and steel mesh are uniformly simulated using the equivalent modulus method, while rock bolts are simulated using cable structural elements.

Table 1. Rock mass quality classification and corresponding mechanical parameters.

Rock Mass Classification	<i>BQ</i>	<i>c</i> /MPa	φ /°	<i>E</i> /GPa	μ	γ /kN/m ³
II	451~550	1.22~2.54	51.4~57.9	10.2~32.7	0.20~0.25	26
III	351~450	0.68~1.22	42.6~51.4	3.17~10.2	0.25~0.30	25.5
IV	251~350	0.26~0.68	27.9~42.6	0.66~3.17	0.30~0.35	23.5
V	150~250	0.14~0.26	21.4~27.9	0.26~0.66	0.35~0.45	19.8

Note: *c* = cohesion; φ = angle of internal friction; *E* = elastic modulus; μ = Poisson's ratio; γ = unit weight.

Due to space constraints, this study focuses solely on discussing the simulation results for rock mass grade V, as the distribution forms and variation patterns of simulation results for rock mass grades II–IV are similar. The summarized simulation results for rock mass grades II–IV can be referenced in the literature [17]. Table 2 displays the selected parameters for rock mass grade V, considering the variability of rock mass parameters.

Table 2. Determination of physical mechanical parameters for grade V rock mass.

Rock Mass Classification	<i>BQ</i>	<i>c</i> /MPa	φ /°	<i>E</i> /GPa	μ
V	150	0.14	21.4	0.26	0.45
	170	0.17	22.9	0.31	0.43
	190	0.19	24.2	0.37	0.41

Table 2. Cont.

Rock Mass Classification	BQ	c/MPa	$\varphi/^\circ$	E/GPa	μ
V	210	0.22	25.5	0.45	0.39
	230	0.24	26.7	0.55	0.37
	250	0.26	27.9	0.66	0.35

4.1. Results

In this study, the effects of rock mass quality, burial depth, and tunnel diameter are investigated. According to the serviceability requirement of China Railway Corporation [7], the ultimate values of crown settlement (y_1), relative displacement at the crown shoulder (y_2), relative displacement at the crown waist (y_3), and the deformation velocity (y_4) are statistically analyzed as the four parameters of interest. Particularly, the deformation velocity (y_4) is computed based on the crown settlement value. That is, the released virtual support force at the monitoring point is firstly obtained from the corresponding crown settlement using Sun's method [20], and the deformation velocity (y_4) is defined as the ratio of the crown settlement to the virtual support force. These data are obtained at the monitoring points, which are depicted in Figure 7. For a grade V rock mass, a minimum of 150 simulations are required to cover various scenarios. Statistical analysis of the simulation results allows for the determination of data distribution forms, mean values (E), standard deviations (D), and confidence intervals for each statistical indicator value concerning each rock mass grade.

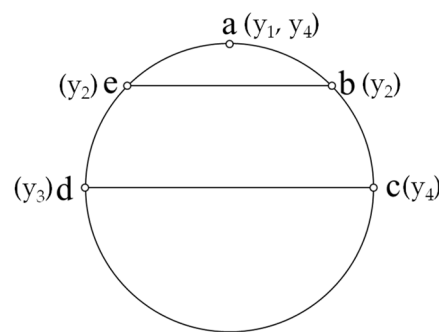


Figure 7. Layout of measurement lines in the numerical model.

Based on the normal distribution forms of ultimate displacement and deformation velocity, along with the introduction of confidence intervals, mean values, variances, and confidence intervals were determined, as detailed in Table 3. The ultimate displacement values of each statistical indicator form a distribution region, showing a gradual increase as the burial depth increases. Notably, for rock mass grade V, characterized by poor rock mass quality, the impact of excavation disturbance on rock mass stress and displacement redistribution is more severe. This effect intensifies with increasing burial depth, leading to larger ultimate displacements and observable macroscopic phenomena, such as the spalling and peeling of shotcrete.

The maximum and minimum values of displacement distribution, located at the interval boundaries, lack typical significance as indicators of potential issues. Statistical methods, employed to construct confidence intervals, assume that 95% of ultimate displacements fall within this interval range for each rock mass grade at each burial depth. Utilizing the upper limit of the confidence interval as the warning indicator value is thus deemed more reasonable than solely relying on the calculated maximum and minimum values [23,24].

Table 3. Ultimate displacement and deformation velocity in V-rock tunnels at varied depths.

Depth/m	Monitoring Item	Mean	Variance	Confidence Interval
300	Vault Settlement (mm)	78.970	733.936	(68.86845, 89.07155)
	Shoulder Relative Displacement	0.0103	0.00001	(0.009355, 0.011292)
	Waist Relative Displacement	0.0174	0.00002	(0.015665, 0.019117)
	Deformation Velocity (mm/MPa)	1.6529	0.30502	(27.49294, 35.31830)
500	Vault Settlement (mm)	136.45	2109.66	(119.3236, 153.5764)
	Shoulder Relative Displacement	0.0175	0.00002	(0.015863, 0.019069)
	Waist Relative Displacement	0.0301	0.00006	(0.027154, 0.032961)
	Deformation Velocity (mm/MPa)	3.0193	0.92337	(29.27040, 37.15301)
700	Vault Settlement (mm)	190.44	4299.62	(165.9903, 214.8897)
	Shoulder Relative Displacement	0.0246	0.00004	(0.022368, 0.026845)
	Waist Relative Displacement	0.0419	0.00012	(0.037762, 0.046058)
	Deformation Velocity (mm/MPa)	4.4745	1.93899	(29.94677, 37.80916)
900	Vault Settlement (mm)	246.66	7235.75	(214.9424, 278.3776)
	Shoulder Relative Displacement	0.0318	0.00006	(0.028912, 0.034703)
	Waist Relative Displacement	0.0542	0.00020	(0.048924, 0.059536)
	Deformation Velocity (mm/MPa)	6.0042	2.91517	(30.41619, 37.63139)
1100	Vault Settlement (mm)	301.93	10970.9	(262.8746, 340.9854)
	Shoulder Relative Displacement	0.0389	0.00009	(0.035368, 0.042442)
	Waist Relative Displacement	0.0664	0.00031	(0.059824, 0.072911)
	Deformation Velocity (mm/MPa)	7.7081	4.96734	(30.63403, 38.03783)

4.2. Response Surface

The Multivariate Adaptive Regression Splines (MARS) method, introduced by Friedman [25], is a widely adopted non-parametric and non-linear statistical regression approach extensively utilized across diverse geotechnical engineering domains. Its applications encompass the prediction of liquefaction settlement during shallow foundation earthquakes [26], the estimation of displacements in multi-level support structures within excavations [27], and the assessment of spatially varying soil slope stability reliability [28]. The MARS model is constructed by a set of piecewise linear segments (splines) of different gradients, which are known as the basis functions (BFs) connected at knots. The MARS method is employed to conduct a sensitivity analysis, quantifying the impact of tunnel geometry and physical parameters on rock mass displacement and deformation velocity. Furthermore, leveraging the MARS model, this study elucidates intricate response relationships between tunnel rock mass displacement and deformation velocity and tunnel geometry and physical parameters. Analytical expressions for crown settlement, relative displacement at the crown shoulder, relative displacement at the crown waist, and deformation velocity are established, as depicted in Tables 4–7.

Figure 8 illustrates the results of the sensitivity analysis performed on the influencing factors of the selected warning indicators. Among the input variables, tunnel burial depth emerges as the most critical parameter for crown settlement, deformation velocity, relative displacement at the crown shoulder, and relative convergence at the crown waist. Conversely, the influence of tunnel diameter is relatively minor. Specifically, for relative displacement at the crown shoulder and waist, the relative importance coefficients are only 0.15 and 0.16, respectively. The *BQ* value, representing the quality of the tunnel rock mass, holds a central position. Its relative importance coefficients are 0.62, 0.55, 0.58, and 0.63 for crown settlement, deformation velocity, relative displacement at the crown shoulder, and relative convergence at the crown waist, respectively.

Table 4. Basis functions and expressions of the MARS model for vault settlement.

BF	Equation	BF	Equation
BF1	$\text{Max}(0, H - 500)$	BF11	$\text{BF4} \times \text{max}(0, H - 500)$
BF2	$\text{Max}(0, 500 - H)$	BF12	$\text{BF4} \times \text{max}(0, 500 - H)$
BF3	$\text{Max}(0, BQ - 210)$	BF13	$\text{BF4} \times \text{max}(0, D - 8)$
BF4	$\text{Max}(0, 210 - BQ)$	BF14	$\text{BF4} \times \text{max}(0, 8 - D)$
BF5	$\text{BF1} \times \text{max}(0, D - 8)$	BF15	$\text{BF3} \times \text{max}(0, H - 500)$
BF6	$\text{BF1} \times \text{max}(0, 8 - D)$	BF16	$\text{BF3} \times \text{max}(0, 500 - H)$
BF7	$\text{BF5} \times \text{max}(0, BQ - 230)$	BF17	$\text{BF2} \times \text{max}(0, 11 - D)$
BF8	$\text{BF5} \times \text{max}(0, 230 - BQ)$	BF18	$\text{BF3} \times \text{max}(0, D - 8)$
BF9	$\text{Max}(0, D - 8)$	BF19	$\text{BF3} \times \text{max}(0, 8 - D)$
BF10	$\text{Max}(0, 8 - D)$	BF20	$\text{BF12} \times \text{max}(0, D - 8)$
		BF21	$\text{BF12} \times \text{max}(0, 8 - D)$
$y_1 = 105.23 + 0.20816 \times \text{BF1} - 0.31824 \times \text{BF2} - 0.7668 \times \text{BF3} + 0.927 \times \text{BF4} + 0.033446 \times \text{BF5} - 0.040306 \times \text{BF6} - 0.00031569 \times \text{BF7} + 0.00044958 \times \text{BF8} + 17.316 \times \text{BF9} - 17.019 \times \text{BF10} + 0.0016186 \times \text{BF11} - 0.001903 \times \text{BF12} + 0.18176 \times \text{BF13} - 0.29323 \times \text{BF14} - 0.0012991 \times \text{BF15} + 0.0016717 \times \text{BF16} + 0.030553 \times \text{BF17} - 0.092463 \times \text{BF18} + 0.20584 \times \text{BF19} - 0.00044172 \times \text{BF20} + 0.0010429 \times \text{BF21}$			

Table 5. MARS model basis functions and expressions for shoulder relative displacement.

BF	Equation	BF	Equation
BF1	$\text{max}(0, H - 900)$	BF11	$\text{BF1} \times \text{max}(0, 230 - BQ)$
BF2	$\text{max}(0, 900 - H)$	BF12	$\text{BF4} \times \text{max}(0, D - 10)$
BF3	$\text{max}(0, BQ - 210)$	BF13	$\text{BF4} \times \text{max}(0, 10 - D)$
BF4	$\text{max}(0, 210 - BQ)$	BF14	$\text{BF13} \times \text{max}(0, H - 900)$
BF5	$\text{BF2} \times \text{max}(0, BQ - 210)$	BF15	$\text{BF13} \times \text{max}(0, 900 - H)$
BF6	$\text{BF2} \times \text{max}(0, 210 - BQ)$	BF16	$\text{BF3} \times \text{max}(0, D - 8)$
BF7	$\text{max}(0, D - 9)$		
BF8	$\text{max}(0, 9 - D)$		
BF9	$\text{BF2} \times \text{max}(0, D - 9)$		
BF10	$\text{BF2} \times \text{max}(0, 9 - D)$		
$y_2 = 0.029201 + 2.6087 \times 10^{-5} \times \text{BF1} - 3.2885 \times 10^{-5} \times \text{BF2} - 0.00018047 \times \text{BF3} + 0.00024545 \times \text{BF4} + 2.1551 \times 10^{-7} \times \text{BF5} - 2.7606 \times 10^{-7} \times \text{BF6} + 0.0011782 \times \text{BF7} - 0.0012724 \times \text{BF8} - 1.4056 \times 10^{-6} \times \text{BF9} + 1.3462 \times 10^{-6} \times \text{BF10} + 3.2639 \times 10^{-7} \times \text{BF11} + 5.9642 \times 10^{-6} \times \text{BF12} - 1.5261 \times 10^{-5} \times \text{BF13} - 5.7722 \times 10^{-8} \times \text{BF14} + 2.0827 \times 10^{-8} \times \text{BF15} - 9.7931 \times 10^{-6} \times \text{BF16}$			

Table 6. MARS model basis functions and expressions for arch waist relative displacement.

BF	Equation	BF	Equation
BF1	$\text{max}(0, H - 500)$	BF11	$\text{BF2} \times \text{max}(0, BQ - 190)$
BF2	$\text{max}(0, 500 - H)$	BF12	$\text{BF2} \times \text{max}(0, 190 - BQ)$
BF3	$\text{max}(0, BQ - 210)$	BF13	$\text{max}(0, D - 9)$
BF4	$\text{max}(0, 210 - BQ)$	BF14	$\text{max}(0, 9 - D)$
BF5	$\text{BF1} \times \text{max}(0, BQ - 230)$	BF15	$\text{BF8} \times \text{max}(0, BQ - 170)$
BF6	$\text{BF1} \times \text{max}(0, 230 - BQ)$		
BF7	$\text{BF1} \times \text{max}(0, D - 10)$		
BF8	$\text{BF1} \times \text{max}(0, 10 - D)$		
BF9	$\text{BF4} \times \text{max}(0, D - 9)$		
BF10	$\text{BF4} \times \text{max}(0, 9 - D)$		
$y_3 = 0.027423 + 4.9268 \times 10^{-5} \times \text{BF1} - 6.6151 \times 10^{-5} \times \text{BF2} - 0.0001895 \times \text{BF3} + 0.00023807 \times \text{BF4} - 3.4841 \times 10^{-7} \times \text{BF5} + 5.164 \times 10^{-7} \times \text{BF6} + 2.2504 \times 10^{-6} \times \text{BF7} - 5.9589 \times 10^{-6} \times \text{BF8} + 1.5585 \times 10^{-5} \times \text{BF9} - 2.1042 \times 10^{-5} \times \text{BF10} + 4.152 \times 10^{-7} \times \text{BF11} - 5.2569 \times 10^{-7} \times \text{BF12} + 0.00041925 \times \text{BF13} - 0.00086936 \times \text{BF14} + 5.0145 \times 10^{-8} \times \text{BF15}$			

The interactive effects of tunnel rock mass quality index BQ , tunnel burial depth H , and diameter D are considered. These effects on crown settlement, relative displacement at the crown shoulder, relative displacement at the crown waist, and deformation velocity are analyzed separately, as depicted in Figures 9–12. The results indicate that for small burial depths, large BQ values, and small tunnel diameters, the ultimate displacement and deformation velocity show minimal variation. With decreasing rock mass quality,

tunnel burial depth and diameter increase, leading to continuous increases in ultimate displacement and deformation velocity. Conversely, improvements in rock mass quality, accompanied by decreases in tunnel burial depth and diameter, result in steady decreases in ultimate displacement and deformation velocity. Utilizing the response surfaces of ultimate displacement enables the efficient evaluation of these parameters under specific conditions of rock mass quality, tunnel burial depth, and tunnel diameter.

Table 7. MARS model basis functions and expressions for deformation velocity.

BF	Equation	BF	Equation
BF1	$\max(0, D - 10)$	BF11	$\max(0, 230 - BQ) \times \max(0, D - 8)$
BF2	$\max(0, 10 - D)$	BF12	$\max(0, 230 - BQ) \times \max(0, 8 - D)$
BF3	$\max(0, BQ - 230)$	BF13	$BF4 \times \max(0, 700 - H)$
BF4	$\max(0, 230 - BQ) \times \max(0, D - 10)$	BF14	$\max(0, H - 500) \times \max(0, BQ - 170)$
BF5	$\max(0, 230 - BQ) \times \max(0, 10 - D)$	BF15	$BF14 \times \max(0, D - 8)$
BF6	$\max(0, 500 - H)$		
BF7	$\max(0, 230 - BQ) \times \max(0, H - 500)$		
BF8	$BF7 \times \max(0, D - 10)$		
BF9	$\max(0, H - 500) \times \max(0, D - 8) \times \max(0, 190 - BQ)$		
BF10	$BF4 \times \max(0, 900 - H)$		

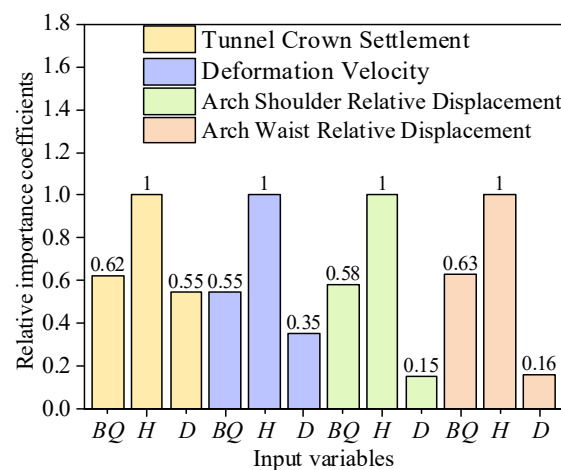
$$y_4 = 33.245 + 3.7842 \times BF1 - 4.841 \times BF2 - 0.13539 \times BF3 - 0.14439 \times BF4 + 0.062196 \times BF5 - 0.0095833 \times BF6 + 3.2462 \times 10^{-5} \times BF7 + 0.00019886 \times BF8 - 7.801 \times 10^{-5} \times BF9 + 0.00033859 \times BF10 + 0.10592 \times BF11 - 0.10536 \times BF12 - 0.00032374 \times BF13 + 5.8542 \times 10^{-5} \times BF14 - 1.6349 \times 10^{-5} \times BF15$$


Figure 8. Sensitivity analysis of tunnel rock mass quality index (BQ), burial depth (H), and diameter (D).

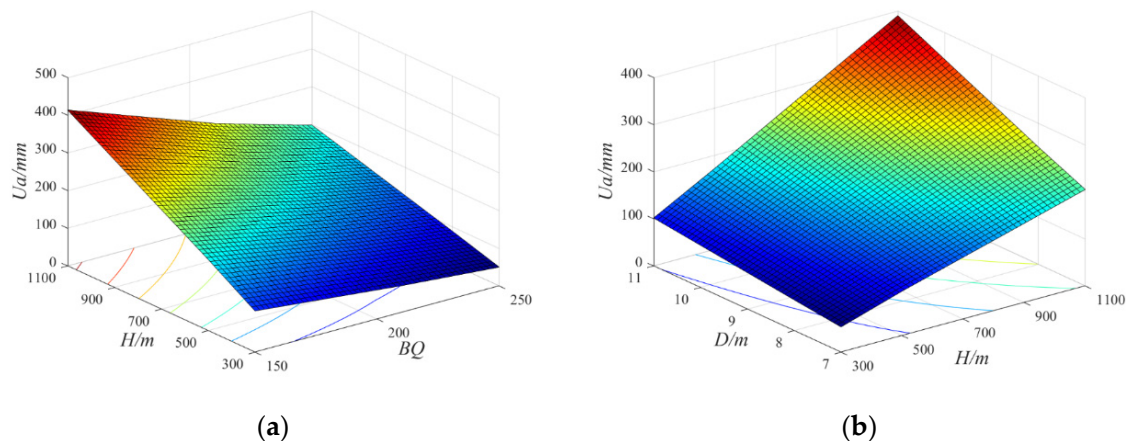
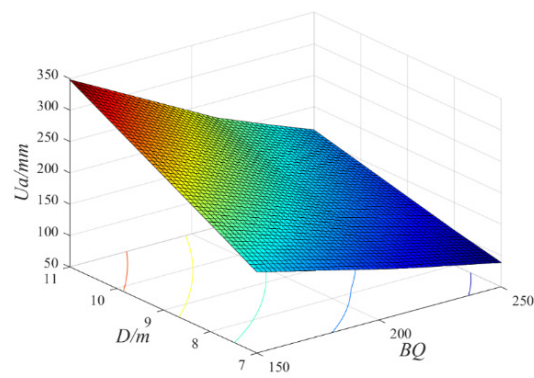
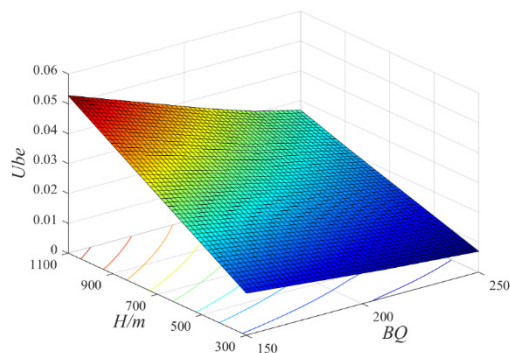


Figure 9. Cont.

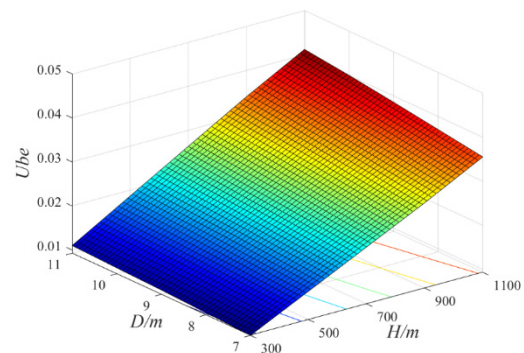


(c)

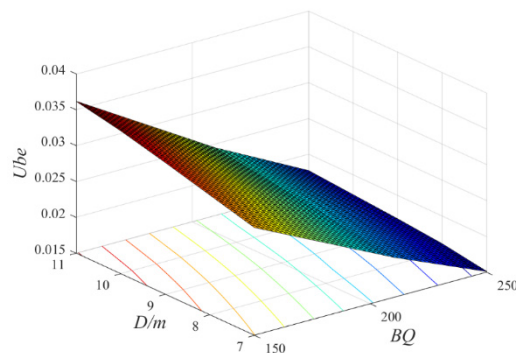
Figure 9. Response surface of maximum settlement at the crown: (a–c) are the response surfaces of arch crown settlement with respect to variations in H and BQ , D and H , and D and BQ , respectively.



(a)



(b)



(c)

Figure 10. Response surface of relative maximum displacement at the shoulder: (a–c) are the response surfaces of shoulder relative displacement with respect to variations in H and BQ , D and H , and D and BQ , respectively.

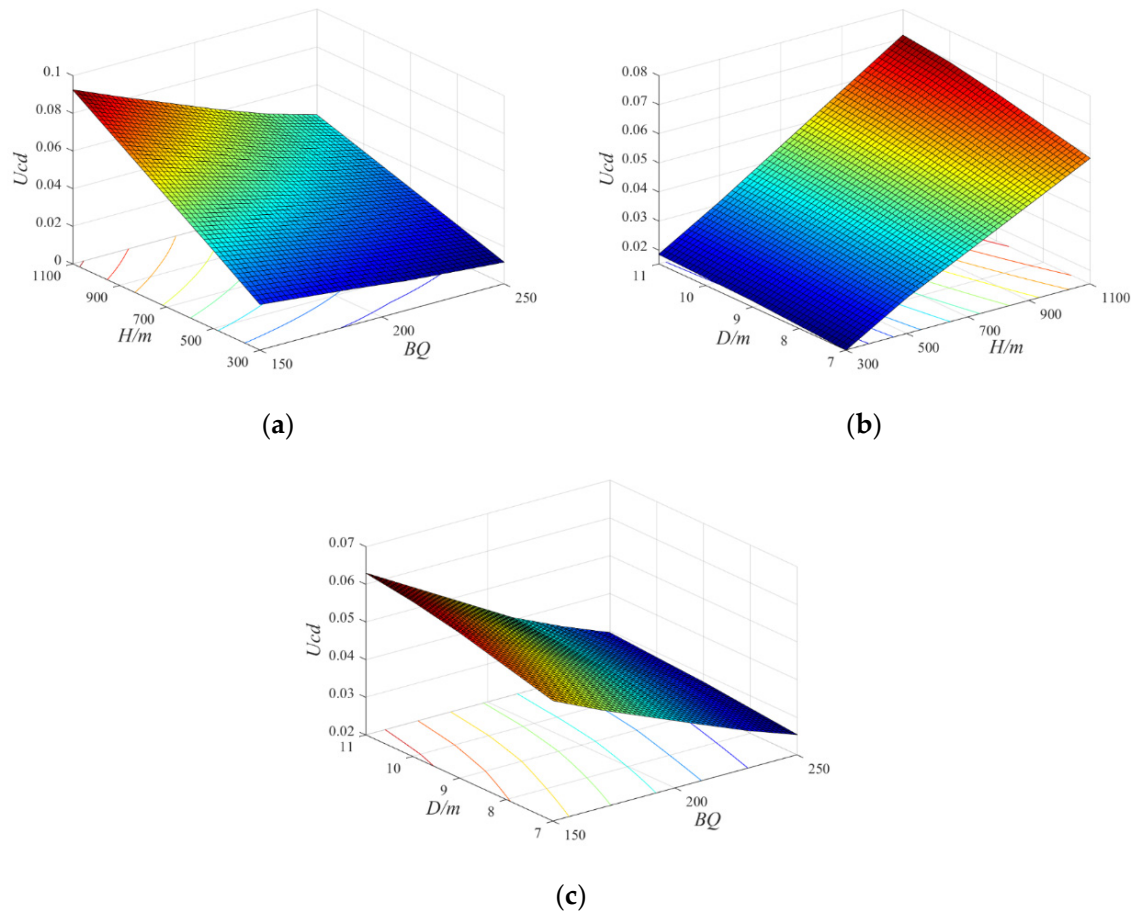


Figure 11. Response surface of relative maximum displacement at the arch waist: (a–c) are the response surfaces of waist relative displacement with respect to variations in H and BQ , D and H , and D and BQ , respectively.

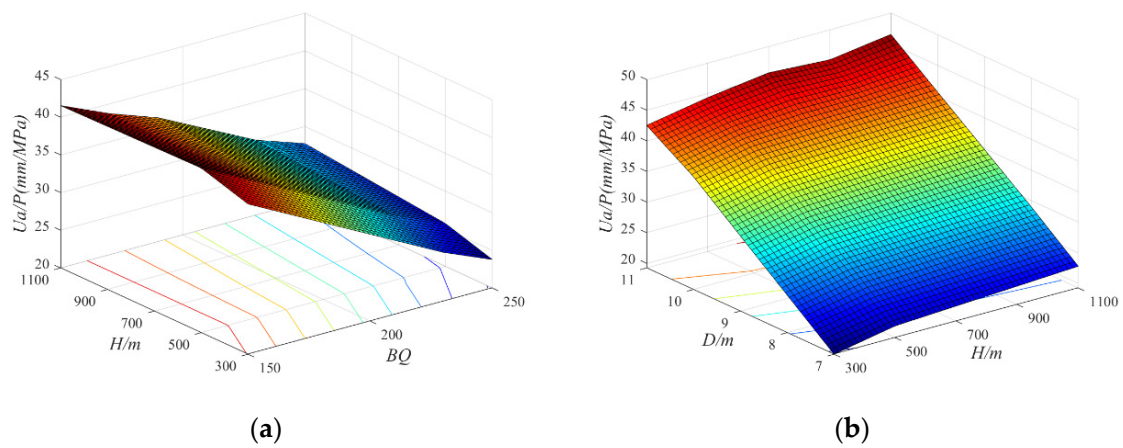


Figure 12. Cont.

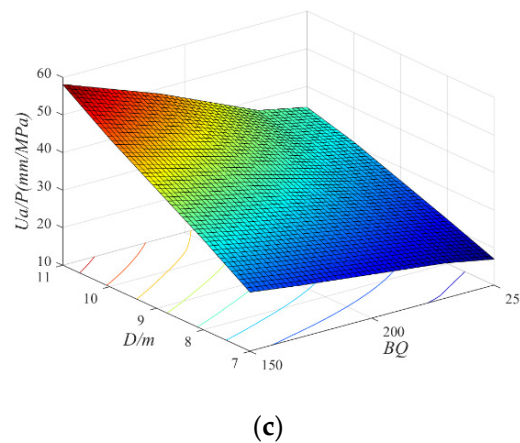


Figure 12. Response surface of maximum deformation velocity: (a–c) are the response surfaces of deformation velocity with respect to variations in H and BQ , D and H , and D and BQ , respectively.

5. Conclusions

This study expands upon the significance of limit states and the methodologies for determining limit displacements and deformation velocities. It employs a continuum model to simulate tunnel excavation under diverse conditions, considering parameters such as rock mass quality (BQ), tunnel depth, and diameter. Statistical methods are utilized to analyze simulation data, thereby establishing predictive models for rock mass warning levels. Additionally, sensitivity analyses and quantifications of the coupled effects of influential parameters on warning indicators are performed using the MARS model. Response surfaces for limit displacement and deformation velocity, accounting for rock mass quality, tunnel depth, and diameter, are developed to complement the warning models. The corresponding conclusions are presented as follows:

- (1) A finite different model for tunnel excavation is established based on the stress release process. Using this model, a standardized criterion of the point where a notable surge in displacement with the stress release process is observed is developed for assessing tunnel instability. This point is defined as the limit state of the tunnel. The observed displacement at this point is regarded as the limit displacement, and the slope is regarded as the deformation velocity.
- (2) As tunnel excavation depths and diameters increase, there is a corresponding gradual increase in both the limit displacement and deformation velocity. This trend is particularly pronounced in scenarios involving deeper excavations and larger diameters, where the rock mass quality is lower. These observations emphasize the complex interplay between rock mass characteristics, tunnel geometry, and stability considerations, underscoring the need for meticulous planning in tunneling projects.
- (3) The sensitivity analysis conducted by the MARS model reveals that tunnel depth exerts the most substantial influence on warning indicator values, followed closely by rock mass quality (BQ). Conversely, tunnel diameter exhibits the least impact on these indicators. This comprehensive evaluation provides valuable insights into the relative importance of each parameter at the design stage of tunnel, offering pertinent information for engineering decision making and design optimization.
- (4) A series of response surfaces is developed to enhance the prediction of limit displacement and deformation velocity for tunnels under diverse scenarios. The rock mass quality, tunnel depth, and diameter are considered in these surfaces. Their utilization helps engineers with the evaluation of tunnel stability, and allows for a better understanding of the parameters and more accurate predictions for displacement and deformation velocity.

Note that the derived values for limit displacement, while based on the existing theoretical framework and comprehension level, only partially characterize and guide the limit

displacement and deformation velocity of tunnel excavation. It is essential to acknowledge that the stability analysis of tunnel excavation presents a multifaceted, multidimensional challenge, requiring continuous theoretical advancements and deeper insights into tunnel excavation. These efforts are necessary for refining warning models with greater precision and enhancing our understanding of tunnel stability.

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