

## Article

# Experimental and Analytical Study on Shear Lag Effect of T-Shaped Reinforced Concrete Shear Walls

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**Abstract:** Because of the flanges on T-shaped shear walls (TSSWs), the shear force acting on such walls results in a shear lag effect, making it impossible to forecast with accuracy the normal stresses of the flanges using the Bernoulli–Euler assumption. Shear lag (SL) in flanged walls has, however, received less attention from researchers, particularly in experimental studies. Understanding the SL in T-shaped reinforced concrete shear walls under shear and axial force is the main goal of this work. First, a SL model is suggested for TSSWs. In this model, the SL deflection is considered to be the generalized displacement and the SL warping deformation, and it is assumed to be a quadratic nonlinear function. Then, experimental and numerical simulation studies are, respectively, conducted to investigate SL effect of TSSWs, and also to evaluate the accuracy of the SL method. Finally, the parameter analysis is conducted to investigate the influence of axial load, shear force, and flange length on the SL effect of TSSWs. The results show that the SL of the TSSW is significant, the normal stress distribution (NSD) of the flange is uneven, and the normal stresses near the web are higher, according to the results of the analytical, simulated, and experimental results. The SL model can accurately predict the normal stresses of the flange of TSSWs, and the quadratic parabola assumption of the SL warp displacement of TSSWs is reasonable. Parameter analysis shows that axial force has little effect on the SL effect of TSSWs. The TSSWs under larger shear force have the more obvious SL effect. A more obvious SL effect occurs in the TSSWs with longer flanges.

**Keywords:** T-shaped shear walls; principle of minimum potential energy; stress distribution; shear lag; variational principle



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## 1. Introduction

Reinforced concrete (RC) structures are used in high and super-high structures [1], and RC concrete walls are applied as often as the main lateral support members owing to their large stiffness and bearing capacity [2,3]. As for flanged walls under shear force, the NSD of the flanges is not caused by the influence of shear deformation (SD), and this SL phenomenon, different from the Bernoulli–Euler assumption, is called the SL effect [4–6]. The flanges of flanged members may crack earlier than expected under shear strain if the SL is not taken into account during design [7]. In practical engineering, shear walls (SW) are designed with a flanged cross-section, such as “T”, “L”, “I”, etc., to improve the stiffness and stability. The equations of shear lag deflection for the shear walls with different cross-sections are disparate and so the key parameters are different, and a unified calculation method cannot be established. The T-shaped shear wall is a common member form in shear wall structures because of its excellent seismic performance and strong stability. Therefore, this paper takes T-shaped shear walls as the research object and analyzes their shear lag effect in depth. Research on the SL impact of flanged SWs is currently lacking in comparison to studies on box girders, composite beams, etc. [8–12] This work seeks

to better understand the SL effect in TSSWs under shear force and axial load, as TS wall components are frequently utilized in constructions [13,14].

The model for determining the SL coefficient and deformation of the TS walls under shear force was developed by Zhang et al. [15] using the variational principle. While the boundary conditions of the SW employed for the calculation approach are different from the actual force situation of walls, research has concentrated on the compressive flange. To investigate the SL effect of TS short-leg walls, Li et al. [16] integrated thin plate bending and stress elements to create plate shell elements, and their correctness was also confirmed by the test results. In order to assess the SL of flanged walls, Shi et al. [17] employed the finite element approach. After conducting a parameter analysis, the value of the effective flange width under various operating circumstances was addressed. Ni et al. [18,19] established a model which can be used for SL effect calculation of I- and TS-reinforced members, and the accuracy was evaluated by the numerical simulation results. The energy vibrational technique and the analog rod method are two ways to describe the SL in reinforced concrete members, with the energy variational method being the more popular approach [20,21]. Zhang and Li [22] point out that shear lag effect occurs on shear walls with flanged cross-sections under the action of shear forces, and ignoring its influence will overestimate the strength and flexural stiffness of shear walls. Therefore, Zhang and Li et al. established a truss model for calculating the flexural stiffness. The comparative analysis of existing test data and finite element calculation results shows that the proposed calculation method has reasonable accuracy. Ke et al. [23] used experimental approaches to investigate the SL effect of composite TSSWs with steel plates through cyclic loading tests, and then they used finite element simulation technology to study the influence of many parameters on the SL effect of such a T-shaped wall. Hoult [24] used VecTor3 to analyze the SL effect of C-shaped shear walls and proposed a method to calculate the effective flange width of C-shaped shear walls by summarizing the FE data. Those constructing TSSWs should take into account how the SL effect would affect their bearing capacity, as they are the supporting parts of the constructions.

The present research on the SL effect focuses on RC beams; there is less research on the SL effect of the walls with flanges, and therefore this needs to be further studied. The TSSWs of the constructions must support both vertical and shear force. However, few studies have considered the effect of vertical load on shear hysteresis. Moreover, there has not been any experimental study on the SL of flanged walls when loaded with shear stress; the correctness of the current SL model is often assessed using finite element modeling.

In this study, the energy variational approach is used to describe the SL effect of the TSSW. The formulae for the normal stresses of the TSSWs under the combined influence of shear force and axial load are derived using the minimal potential energy principle. The SL deflection is taken as the generalized displacement. The experimental findings and the outcomes of the finite element simulation are used to assess the correctness of the proposed model.

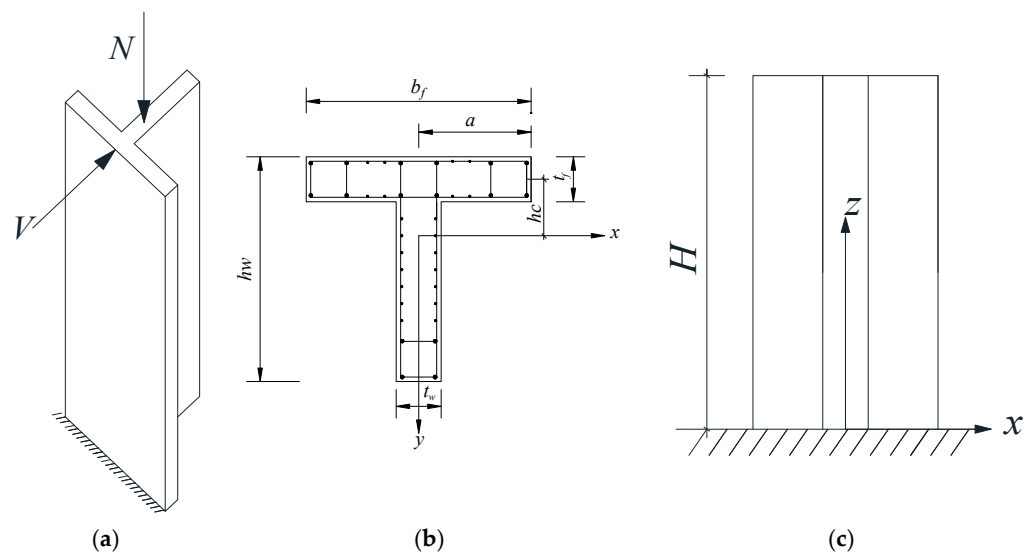
## 2. Analytical Model

As the supporting member of the overall structures, TS wall members are subjected to shear force ( $F$ ) and axial load ( $N$ ), and the bottom cross-section is fixed to the foundation. The force state of such walls can be simplified as Figure 1a.

$$h_c = \frac{\frac{1}{2}b_f t_f^2 + t_w(h_w - t_f)\left((h_w - t_f)/2 + t_f\right)}{b_f t_f + t_w(h_w - t_f)} - \frac{t_f}{2} \quad (1)$$

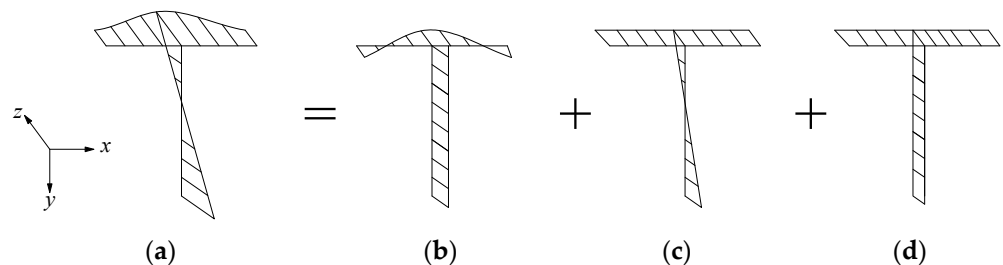
Figure 1b shows the cross-section and reinforcing of a TSSW as well as the plane-coordinate system used for the subsequent derivation. As for the size of the TS cross-section,  $h_w$  is the height,  $t_w$  is the web's thickness,  $b_f$  and  $t_f$  are, respectively, the width and thickness of the flange, and the coordinate origin is the centroid point of the cross-section.  $h_c$  is the distance from the center axis of the flange section and the coordinate axis  $x$ , shown as

Figure 1b, which can be obtained by Equation (1). The  $z$ -axis direction can be found in Figure 1c, and  $H$  is the wall's height.



**Figure 1.** Details of TSSWs: (a) simplified force state, (b) cross-section, (c) coordinate system.

According to the analysis of the mechanical properties of the TS wall under combined action of shear and axial force, the effect of the bending moment, axial load, and SL can induce the longitudinal displacement of the cross-section, so the longitudinal deformation is composed by SL warping displacement (Figure 2a), flexure displacement (Figure 2b), and the axial displacement (Figure 2c).



**Figure 2.** Longitudinal displacements: (a) longitudinal displacements of the cross section, (b) SL warping deformation, (c) flexure deformation, (d) axial deformation.

Many scholars have assumed the functions of SL warping displacement, such as parabolic function and trigonometric functions, and, in this paper, the quadratic parabolic function is taken as the function of SL warping displacement. As reported in Zhang et al. [10], the SL effect can lead to the additional deflection of reinforced members, so the deflection ( $d(z)$ ) of a TS wall is the sum of flexure deflection and SL deflection. The deflection of the wall can be expressed as follows:

$$d(z) = \omega(z) + f(z) \quad (2)$$

where  $\omega(z)$  is the flexure displacement, which can be calculated by the mechanics of materials, and  $f(z)$  is the SL deflection. In this model,  $f(z)$  is taken as the generalized displacement, so the expression of the longitudinal displacement of the cross-section can be expressed as follows:

$$u(x, y, z) = yd'(z) + \gamma w_{\xi}(x, y)f'(z) + u_n(z) \quad (3)$$

$$u_n(z) = \frac{qz}{E} = \frac{N}{EA}z \quad (4)$$

where  $u(x, y, z)$  is the total longitudinal deformation,  $\eta$  is the correction coefficient for the bending moment caused by the SL stresses,  $w_\xi(x, y)$  is the SL warping deformation function,  $E$  is the elastic modulus of the reinforced concrete,  $q$  is calculated according by  $q = N/A$ , and  $A$  is the cross-section area. So,  $u(x, y, z)$  can be simplified as follows:

$$u(x, y, z) = y\omega'(z) + w(x, y)f'(z) + \frac{N}{EA}z \quad (5)$$

with

$$w(x, y) = \gamma w_\xi(x, y) + y \quad (6)$$

According to existing tests and finite element studies [23,25], the normal stress on the flange of flanged RC members under shear force is larger near the web, and smaller and smaller away from the web, showing an approximate parabola form. Moreover, Li et al. [26] and Zhang et al. [10–12] assumed shear lag warp displacement as a quadratic parabola, and the theoretical calculation method has reasonable prediction accuracy.  $w_\xi(x, y)$  is the SL warping displacement, and it can be obtained by,

$$w_\xi(x, y) = \begin{cases} h_c[1 - \frac{(x-a)^2}{a^2}] + e & \text{For Flange} \\ e & \text{For Web} \end{cases} \quad (7)$$

where  $e$  is the correction coefficient for the normal stress (NS) caused by the SL effect, and other parameters can be found in Figure 1. The typical stress is best described as follows:

$$\sigma(x, y, z) = E \frac{\partial u(x, y, z)}{\partial z} = Ey\omega''(z) + Ew(x, y)f''(z) - \frac{N}{A} \quad (8)$$

The first term of Equation (8) is flexure stress, which can be obtained by Bernoulli–Euler assumption, and the second term is the SL stress  $\sigma_w$ , which can be obtained by,

$$\sigma_w = Ew(x, y)f''(z) \quad (9)$$

The bending moment acting on the TS cross-section should only be synthesized by the corresponding stress caused by the elementary beam bending moment, so the cross-section bending moment and resultant force induced by  $\sigma_w$  should be equal to 0, and we can obtain the following:

$$\iint Ew(x, y)f''(z)dA = 0 \quad (10)$$

$$\iint Ew(x, y)f''(z)y dA = 0 \quad (11)$$

Substituting Equations (3) and (5) into Equation (10), we can obtain the following:

$$e = -\frac{2h_c A_f}{3A} \quad (12)$$

where  $A_f$  is the cross-section area of flange. Substituting Equations (3) and (6) into Equation (8), we can obtain the following:

$$\gamma = -\frac{\int_A y^2 dA}{\int_A w_\xi(x, y)y dA} = -\frac{I_x}{I_{\xi x}} \quad (13)$$

According to Equation (7), we can obtain the following:

$$I_x = \frac{1}{12}b_f t_f^3 + (b_f t_f \left(\frac{t_f}{2}\right)^2 + \frac{1}{12}t_w(h_w - t_f)^3 + t_w(h_w - t_f)\left(\frac{(h_w - t_f)}{2} + t_f\right)^2) \quad (14)$$

$$I_{\xi x} = \int_A w_{\xi}(x, y) y dA = -\frac{2A_c h_c^2}{3} \quad (15)$$

The total potential energy of the TSSWs may be represented as follows:

$$\Pi = \int_0^{H_0} \int_A \left( \frac{\sigma^2}{2E} + \frac{\tau^2}{2G} \right) dA dz + V_p \quad (16)$$

$\tau(x, y, z)$  can be obtained by,

$$\tau(x, y, z) = G \frac{\partial u(x, y, z)}{\partial x} = -G \gamma \frac{2(x-a)}{a^2} h_c f'(z) \quad (17)$$

With the force state of TS walls, the external potential energy,  $V_p$ , can be expressed as follows:

$$V_p = \int_0^{H_0} M(z)[w''(z) + f''(z)] dz - qu(x, y, z)|_{z=H} \quad (18)$$

where the bending moment  $M(z)$  is generated by the shear force ( $V$ ), which can be calculated by,

$$M(z) = V(H - z) \quad (19)$$

The integration of Equation (16) can be simplified as follows:

$$\begin{aligned} \Pi = & \frac{1}{2} \int_0^H EI_x [\omega''(z)]^2 dz + \frac{1}{2} \int_0^H EI_w [f''(z)]^2 dz + \int_{V_0} \frac{q^2}{2E} dv + \frac{1}{2} \int_0^H GI_{\xi} [f'(z)]^2 dz \\ & + \int_0^{H_0} M(z)[w''(z) + f''(z)] dz - qu(x, y, z)|_{z=H} \end{aligned} \quad (20)$$

with,

$$I_w = \int_A [w(x, y)]^2 dA = \eta^2 \left[ \frac{8t_f b_f h_c^2}{15} + e^2 A + \frac{4}{3} e h_c A_c \right] - I_x \quad (21)$$

$$I_{\xi} = \int_A \left[ \frac{\partial w(x, y)}{\partial x} \right]^2 dA = \frac{16\eta^2 t_f h_c^2}{3b_f} \quad (22)$$

The first-order variation of  $\Pi(x, y, z)$  is as follows:

$$\begin{aligned} \delta \Pi = & \delta \omega''(z) \left[ \int_0^H EI_x \omega''(z) + M(z) \right] dz + \delta f'(z) \int_0^H [GI_{\xi} f'(z) - EI_w f'''(z) - V] dz \\ & + \delta f'(z) [EI_w f''(z) + M(z)]|_0^H \end{aligned} \quad (23)$$

It is possible to derive the governing differential equation using the idea of minimal potential energy  $\delta \Pi = 0$ :

$$EI_x \omega''(z) + M(z) = 0 \quad (24)$$

$$f'''(z) - k^2 f'(z) = \frac{V}{EI_w} \quad (25)$$

where  $k = \sqrt{\frac{GI_{\xi}}{EI_w}}$ .

The mandatory boundary conditions must be met, and they are as follows:

$$\delta f'(z) [EI_w f''(z) + M(z)]|_0^H = 0 \quad (26)$$

Based on the boundary conditions of TS wall, we can obtain,

$$\omega(L) = 0, \omega'(z)|_{z=0} = 0 \quad (27)$$

So, according to Equations (24) and (27),  $w(z)$  can be obtained.

$$\omega(z) = -\frac{V(H-z)^3}{6EI_x} + \frac{VH^3}{6EI_x} - \frac{VH^2z}{2EI_x} \quad (28)$$

Based on Equation (25),  $f(z)$  can be expressed as follows:

$$f(z) = C_1 \sinh(kz) + C_2 \cosh(kz) - \frac{Vz}{EI_w k^2} + C_3 \quad (29)$$

The bottom cross-section of TSSWs is fixed, so the bottom deflection of the walls induced by SL is 0.

$$f(z)|_{z=0} = 0 \quad (30)$$

Owing to fixed bottom cross-section, when  $z = 0$ , the first order differential of  $f(z)$  is 0, so we can obtain,

$$f'(z)|_{z=0} = 0 \quad (31)$$

According to Equations (28)–(30), the coefficients can be obtained:

$$\begin{cases} C_1 = \frac{V}{EI_w k^3 \cosh(kH)} \\ C_2 = 0 \\ C_3 = \frac{VH}{EI_w k^2} + \frac{V \sinh(kH)}{EI_w k^3 \cosh(kH)} \end{cases} \quad (32)$$

Substitute Equation (31) into Equation (8), the mathematical expression of normal stresses of the TS cross-section can be obtained.

$$\sigma(x, y, z) = Ey\omega''(z) - q + Ew(x, y)[k^2 C_1 \cosh(kz) + k^2 C_2 \sinh(kz)] \quad (33)$$

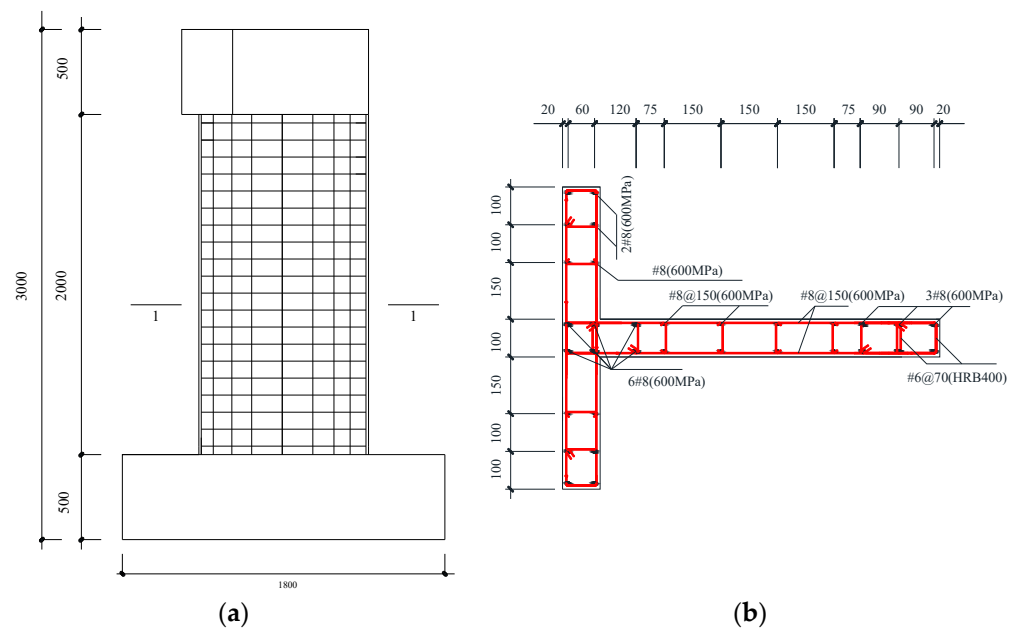
### 3. Experimental Study

#### 3.1. Details of the Specimen

Due to our limited funds, only one TSSW specimen was tested. Under two different loading conditions, the normal stresses of the cross-sections at different heights of the TSSW specimen were obtained. The test data were used for the subsequent evaluation of the accuracy of the theoretical and numerical models, and based on the verified analytical method and FE model, a great deal of parameter analysis was conducted, which can effectively make up for the lack of experimental research parameters. The wall specimen adopts the scale model of 1/2 of the prototype wall, and its elevation and reinforcement diagram are shown in Figure 3, where # represents the diameter of the reinforcing bars. A lower loading reinforced concrete plate was set up to simulate the fixed boundary between the TSSW and the foundation, with a size of 1300 mm (length)  $\times$  1400 mm (width)  $\times$  500 mm (height). A total of eight holes were reserved in this plate, and the steel anchors were used to anchor to the plate during the test loading. The wall height was 2000 mm, the section size of the flange was 100 mm (thickness)  $\times$  800 mm (length), and the size of the web was 100 mm (thickness)  $\times$  1000 mm (length). Eight steel bars of Grade 600 MPa with a diameter of 8 mm and a reinforcement ratio of 1.5 percent were used as the longitudinal reinforcing bars in the wall web's boundary restrictions. The 8 mm diameter HRB600 steel bars used for the distribution bars had a ratio of 0.67 percent. The reinforcement arrangement of the above wall specimen met the requirements of the specification [27].

C40 commercial concrete was used for the wall specimen. During pouring, six cuboid concrete specimens with sizes of 150 mm  $\times$  150 mm  $\times$  300 mm were made, and the cuboid concrete specimens were cured under the same environmental conditions as the SW specimens. The concrete cuboids were tested first to determine their axial compressive strength and elastic modulus before the wall specimens were evaluated. According to GB/T50081-2002 [28], concrete's elastic modulus and axial compressive strength ( $f_{ck}$ ) were

measured. The average measured value of its axial compressive strength was 34.9 MPa, and  $E_c$  was  $3.75 \times 10^4$  MPa. The measurement of the  $E_c$  of concrete is shown in Figure 4.



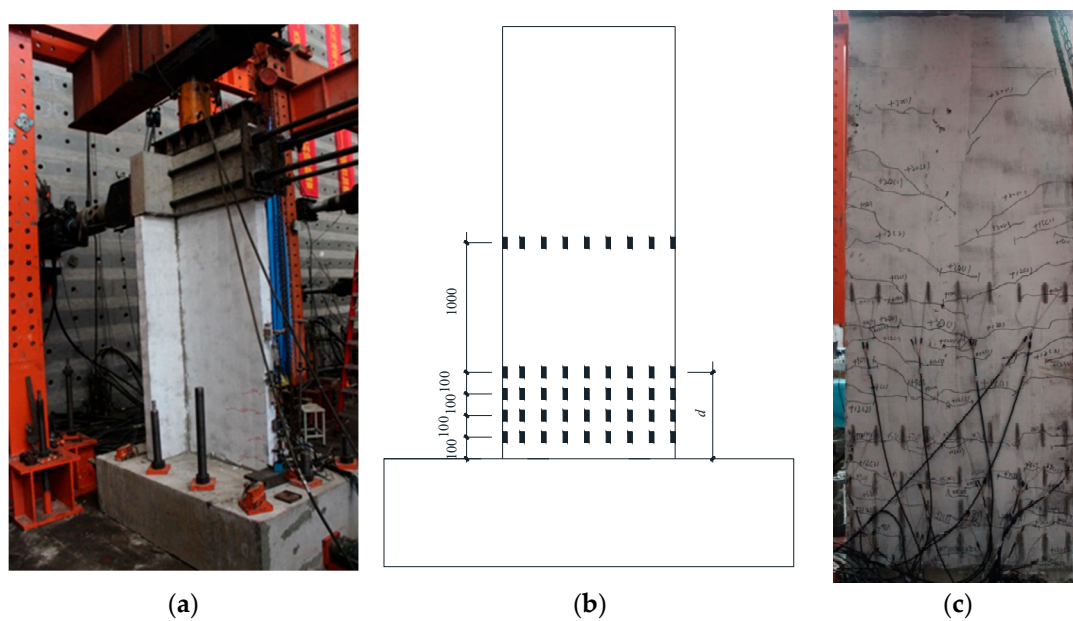
**Figure 3.** Specimen details: (a) elevation, (b) cross-section.



**Figure 4.** Test setup of elastic modulus of concrete.

### 3.2. Loading Setup and Instrumentations

The TSSW's flange experiences non-uniform stress distribution induced by the combined effects of axial load and shear force. Figure 5a displays the loading diagram for both horizontal and vertical loads. The centroid point of the TS cross-section is where the axial force was loaded, while the shear force was applied at the top beam's center point. Concrete strain gauges were positioned on the flange surface to monitor the flange strain distribution to determine the flange stress distribution of the TSSW.  $d$  is the vertical distance from the strain gauge location to the bottom cross-section of the wall specimen, shown as Figure 5a. The strain gauge configuration is seen in Figure 5b. The strain gauges were arranged in four rows, with nine strain gauges in each row, the horizontal spacing was 100 mm, and the vertical distance between the measuring points of each row and the foundation was equal.  $d$  is the distance from each row of strain gauges to the bottom of the flange. In the test, the flange strain gauge layout of TSSW is shown in Figure 5c. The loading setup used in this paper is the same as that in Abouzrar et al. [29].



**Figure 5.** Shear lag test: (a) test setup, (b) schematic diagram of flange strain gauge layout, (c) flange strain gauge layout diagram of specimen.

TSSW was loaded under constant axial and shear force to obtain the data from each strain gauge. The stress at each strain measurement point was obtained through the strain data of the measured point, combined with the elastic modulus of concrete ( $3.75 \times 10^4$  MPa), and then compared with the analytical values. The SL effect test was carried out in two loading cases: the first case was that the horizontal load was 20 kN and vertical load was 0 kN. In the second case, the horizontal load was 60 kN and the vertical load was 40 kN.

#### 4. Numerical Model

Three-dimensional finite element (FE) software VecTor3 [30] was used to establish the FE model of the above TS wall specimen. Many previous studies [31–36] have verified the excellent prediction accuracy of VecTor3 of RC members under different loading cases. The element size is  $50 \text{ mm} \times 50 \text{ mm} \times 50 \text{ mm}$ , there are 1600 Regular Hexahedral Elements in this model, and all reinforcement is modeled using the method of dispersion in concrete. The bottom nodes are fixed. The constitutive model of concrete is the Hognestad (Parabola) model [37] (shown as Figure 6), and it is relatively simple and suitable for normal strength concrete. Its stress–strain formula is as follows:

$$f_{ci} = -f_p \left\{ 2 \left( \frac{\varepsilon_{ci}}{\varepsilon_p} \right) - \left( \frac{\varepsilon_{ci}}{\varepsilon_p} \right)^2 \right\} < 0 \quad \text{for } \varepsilon_{ci} < 0 \quad (34)$$

where  $f_{ci}$  is the corresponding stress when the compressive strain of concrete is  $\varepsilon_{ci}$ , and  $\varepsilon_p$  and  $f_p$  are the peak strain and stress of concrete, respectively. The stress–strain relationship of concrete in the Hognestad (Parabola) model [38] is symmetric, the stress reaches its peak value at  $\varepsilon_p$ , and the corresponding stress is 0 at  $2\varepsilon_p$ . In this model, the initial tangent stiffness,  $E_c$ , can be calculated as follows:

$$E_c = 2f_p / |\varepsilon_p| \quad (35)$$

Post-peak stress,

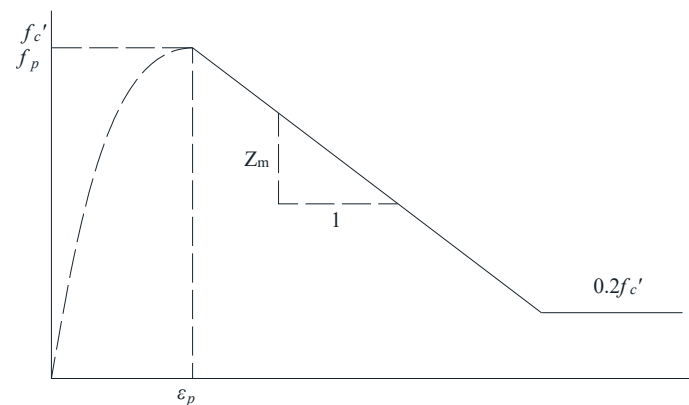
$$f_{ci} = -[f_p + Z_m f_p (\varepsilon_{ci} - \varepsilon_p)] < 0 \quad \text{for } \varepsilon_p < \varepsilon_{ci} < 0 \quad (36)$$

where

$$Z_m = -\frac{0.5}{\frac{3+0.29|f'_c|}{145|f'_c|-1000} \cdot \left(\frac{\varepsilon_o}{-0.002}\right) + \left(\frac{|f_{lat}|}{170}\right)^{0.9} + \varepsilon_p} < 0 \quad (f'_c \text{ and } f_{lat} \text{ in MPa}) \quad (37)$$

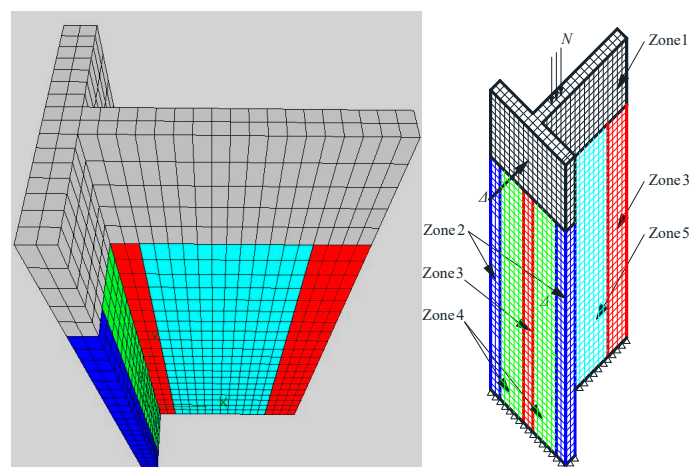
where  $f_{lat}$  is the sum of principal stresses,

$$f_{lat} = f_{c1} + f_{c2} + f_{c3} - f_{ci} \leq 0 \quad i = 1 \text{ or } 2 \quad (38)$$



**Figure 6.** Constitutive concrete model of modified Park–Kent.

Table 1 lists the concrete constitutive models for the numerical simulation. The formworks user's manual of VecTor3 can be found in ElMohandes [30]. The constitutive of reinforcing bars used linear strain-hardening (Trilinear), and all of them were modeled by dispersion in the concrete. The finite element model in Figure 7 is divided into five zones, and different zones have different reinforcing bar configuration. Zone 1 is the upper loading beam, which is used to load shear and axial force, so it is designed to be very strong with a reinforcement ratio of 4%. Zone 2 is the flange edge constrained members, and this zone has longitudinal reinforcement and stirrup. Zone 3 is the web edge constrained member, which also has longitudinal reinforcement and stirrup. Zone 4 is the unconstrained zone of flange, which only has distributed reinforcement. Zone 5 is the unconstrained zone of the web, which only has distributed reinforcement. The loading process of the finite element model is consistent with the loading process of the above test.



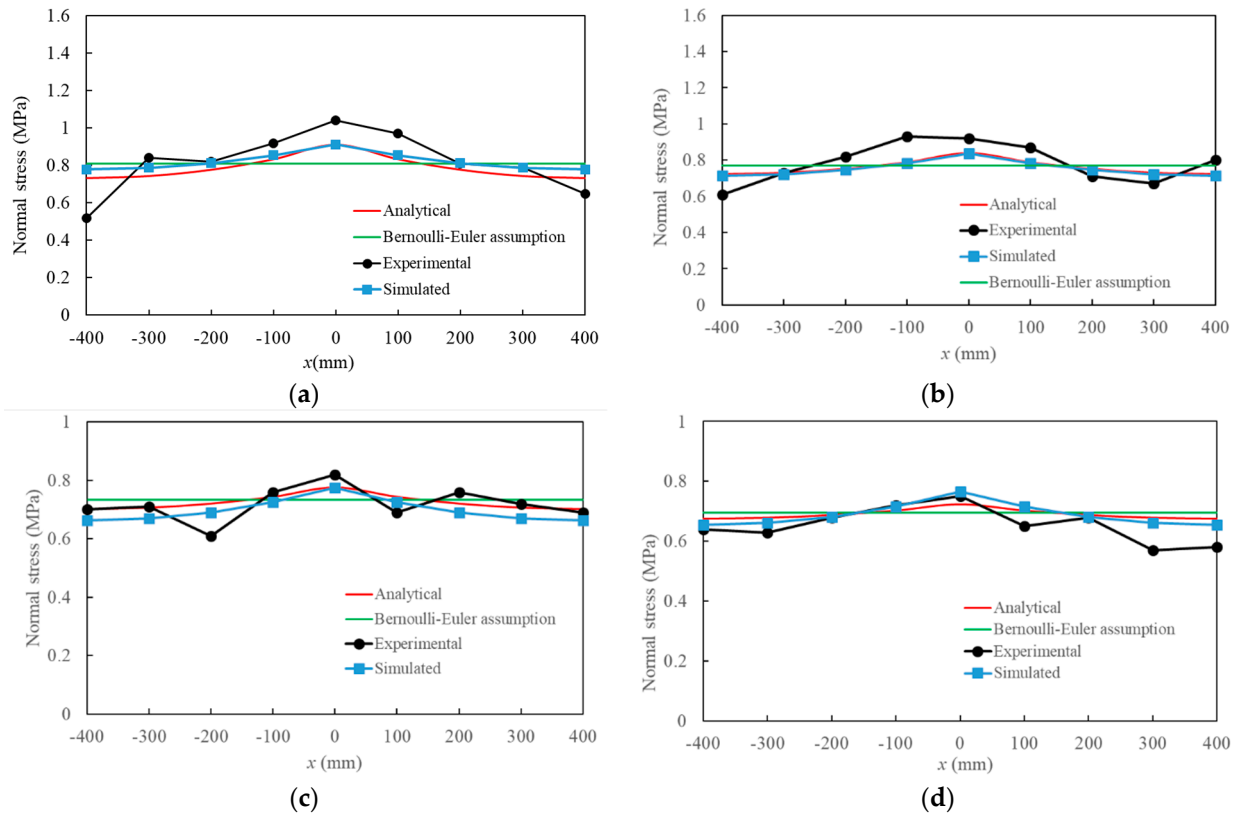
**Figure 7.** FE model of TSSWs.

**Table 1.** Concrete constitutive models.

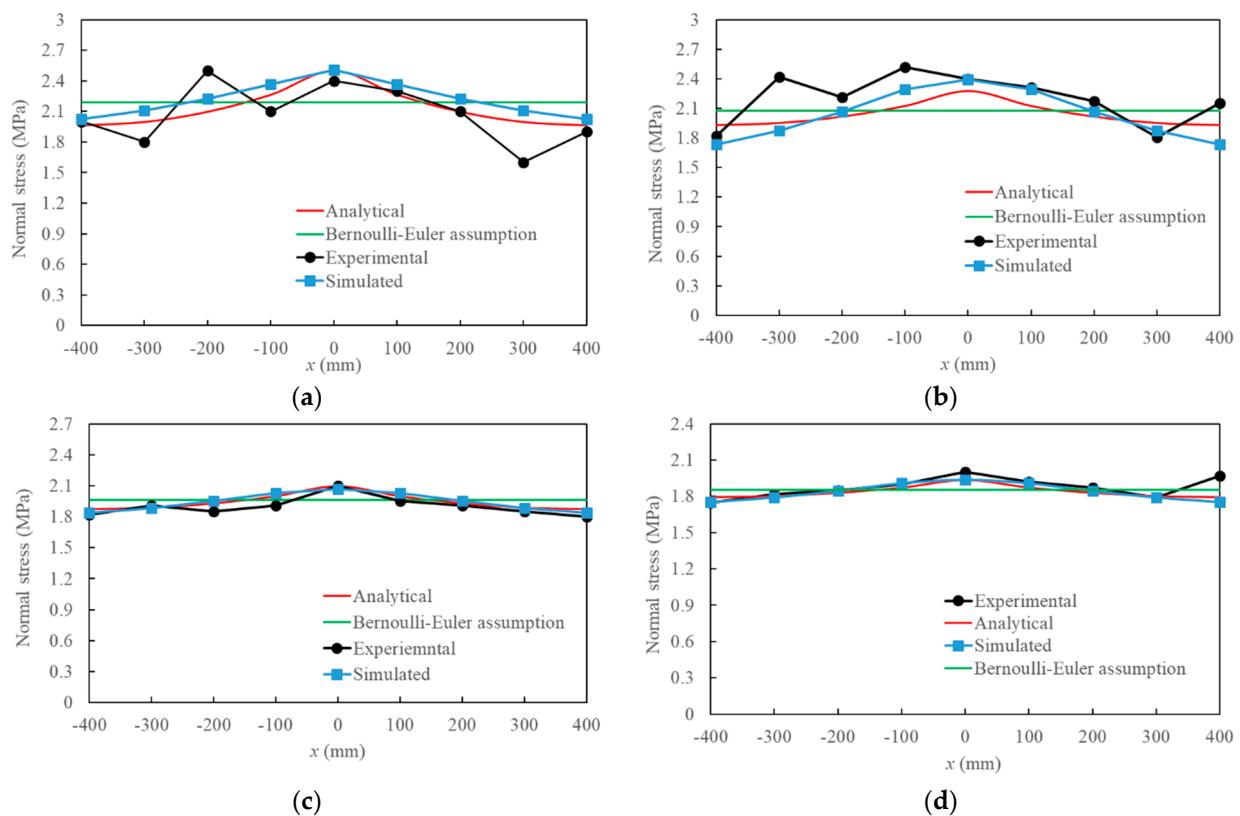
| Constitutive Behavior | Model                                            |
|-----------------------|--------------------------------------------------|
| Compression pre-peak  | Parabola [38]                                    |
| Compression post-peak | Modified Kent and Park [37]                      |
| Compression softening | Vecchio 1993-A [e1/e2-Form] [39]                 |
| Tension stiffening    | Modified Bentz Model [36]                        |
| Tension softening     | Linear                                           |
| Confined strength     | Kupfer/Richart [40]                              |
| Dilation              | Variable-Kupfer                                  |
| Cracking criterion    | Mohr–Coulomb [Stress]                            |
| Crack stress calc     | Basic [DSFM/MCFT]                                |
| Crack width           | Check Aggregate 2.5 mm max crack width [Default] |
| Crack slip calc       | Vecchio and Lai [41]                             |

### 5. Experimental and Numerical Validation for the Proposed Model

Figures 8 and 9 show the analytical and experimental comparison of the stresses, and the two figures also show the normal stresses obtained by the Bernoulli–Euler assumption. The proposed model and FE model are used, respectively, to obtain the normal stresses of the flange of the TSSW specimen under the shear force and axial load. The test findings show that under the influence of a shear force, the stress distribution is not uniform, and in such flanged walls, a clear SL phenomenon takes place. The usual strains on the flange are greater close to the web and less so farther away. The Bernoulli–Euler assumption results in an equal distribution of normal stress, which is very different from the experimental, simulated, and analytical normal stress distribution. As a result, while calculating the normal stress on the TSSW’s flange under shear force, the SL effect needs to be considered.



**Figure 8.** Flange normal stress distribution ( $F = 20$  kN,  $N = 0$  kN):  $d =$  (a) 100 mm, (b) 200 mm, (c) 300 mm, (d) 1000 mm.



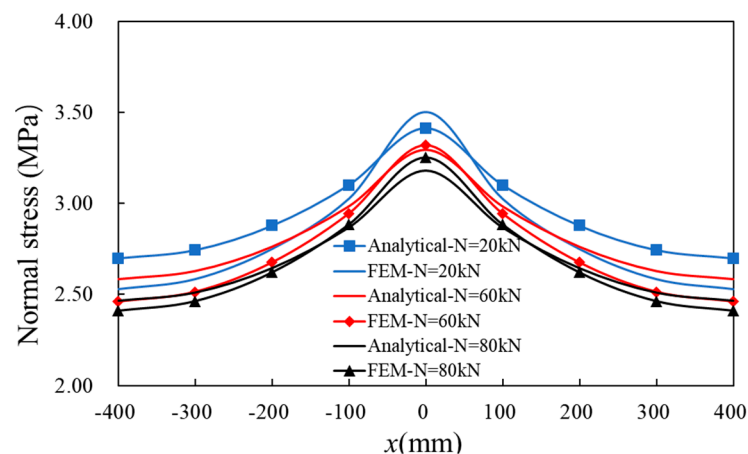
**Figure 9.** Flange normal stress distribution ( $F = 60$  kN,  $N = 40$  kN):  $d =$  (a) 100 mm, (b) 200 mm, (c) 300 mm, (d) 1000 mm.

Through analyzing the NSD of the flange, the average ratio of the analytical to experimental flange stresses obtained by the model proposed in this paper is 0.96, and the coefficient of variation (CoV) is 0.10. The average ratio of simulated to experimental flange stresses is 0.95, and the CoV is 0.104. The average ratio of the analytical to simulated flange stresses is 1.0, and the CoV is 0.05. As a result, there is a fair degree of agreement between the flange normal stresses predicted by the model described in this study and those predicted by tests and FE models. The quadratic parabola assumption of the SL warp deformation is realistic, and the suggested model in this study is capable of effectively predicting the normal stresses of the flange of TSSWs.

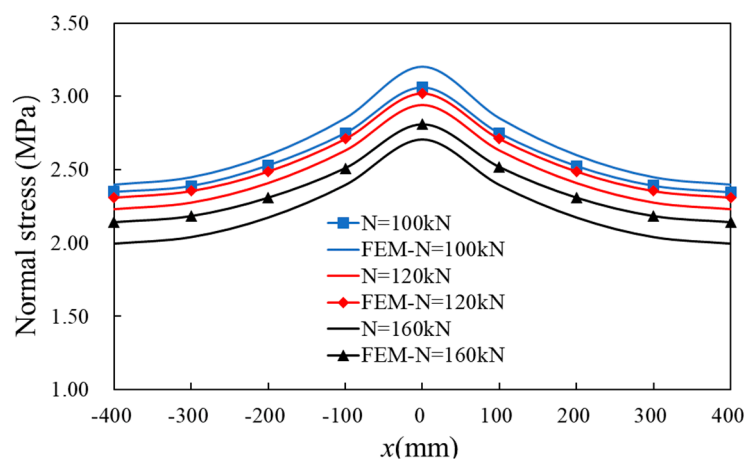
## 6. Parameter Analysis

### 6.1. Axial Load

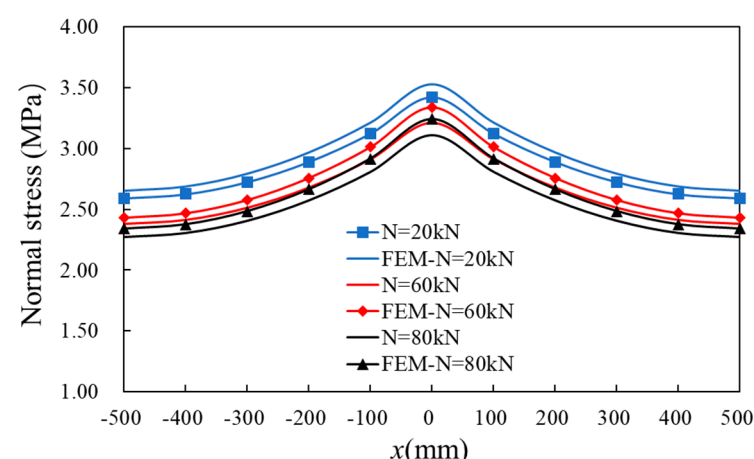
According to the FE model of TSSWs established above, the parameter analysis was conducted to investigate the effect of axial load on the normal stress distribution of the wall flange. By changing the axial load of the above FE model in Section 4, the NSD of the bottom cross-section of TSSWs is obtained, which is shown in Figures 10 and 11. In this model, the shear force is 80 kN and the flange length is 800 mm. The flange length of another wall numerical model was set as 1000 mm, the other details were the same as the model in Section 4, and the shear force is 80 kN. By changing the axial load of the model, the NSD of the bottom cross-section of TSSWs is obtained, which is shown as Figures 12 and 13. These figures present the normal stresses obtained by the proposed method and FE method and compare the normal stress distribution under different axial loads (20, 60, 80, 100, 120, and 160 kN). It can be found that the analytical values have the same trend as the values obtained by FE method, and the NS close to the web is larger, while the NS further away from the web is smaller. The analytical normal stresses agree well with those calculated by the finite element method.



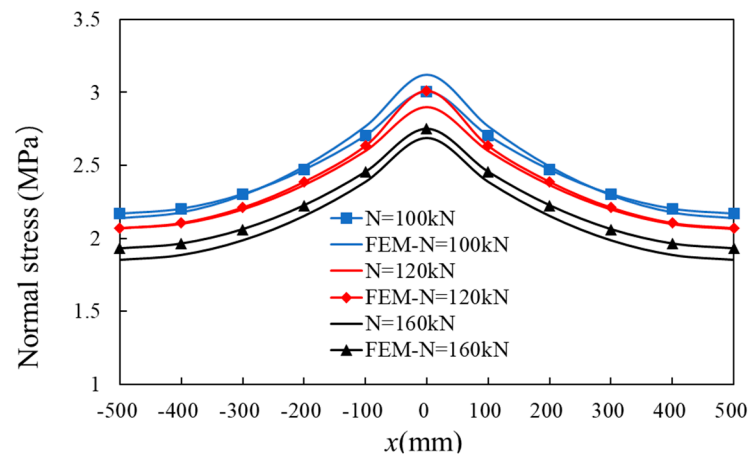
**Figure 10.** Flange normal stress distribution under different axial force (flange length = 800 mm,  $N = 20$  vs. 60 vs. 80 kN).



**Figure 11.** Flange normal stress distribution under different axial force (flange length = 800 mm,  $N = 100$  vs. 120 vs. 160 kN).



**Figure 12.** Flange normal stress distribution under different axial force (flange length = 1000 mm,  $N = 20$  vs. 60 vs. 80 kN).



**Figure 13.** Flange normal stress distribution under different axial force (flange length = 1000 mm,  $N = 100$  vs. 120 vs. 160 kN).

SL coefficient can be used to evaluate the SL effect of the members, which is calculated according to the following formula:

$$\lambda = \frac{\sigma}{\bar{\sigma}} \quad (39)$$

where  $\sigma$  is the maximum value of NS on the flange of shear walls, and  $\bar{\sigma}$  is the normal stresses obtained by the Bernoulli–Euler assumption.

Table 2 summarizes the SL coefficients of TSSW specimens under different axial forces. As for the model (flange = 800 mm,  $F = 80$  kN), the analytical shear lag coefficients ranged from 1.14 to 1.18 under different axial loads, and the simulated shear lag coefficient values had similar scopes, which ranged from 1.15 to 1.20. As for the model (flange = 1000 mm,  $F = 80$  kN), the analytical shear lag coefficients ranged from 1.18 to 1.24 under different axial loads, and the simulated shear lag coefficient values ranged from 1.22 to 1.27. It can be found that under the action of different coaxial forces, the SL coefficients are close, which indicates that the axial force has limited influence on the SL coefficient of the TSSWs. Moreover, Ke et al. [23] reached the same conclusion through test and finite element analysis.

**Table 2.** SL coefficient of TSSWs under different axial forces.

| N (kN)                               | $\bar{\sigma}$ | $\sigma$   |           | $\lambda$  |           |
|--------------------------------------|----------------|------------|-----------|------------|-----------|
|                                      |                | Analytical | Simulated | Analytical | Simulated |
| Flange length = 800 mm, $F = 80$ kN  |                |            |           |            |           |
| 20                                   | 3.00           | 3.41       | 3.50      | 1.14       | 1.17      |
| 60                                   | 2.88           | 3.30       | 3.32      | 1.14       | 1.15      |
| 80                                   | 2.76           | 3.18       | 3.25      | 1.15       | 1.18      |
| 100                                  | 2.65           | 3.06       | 3.20      | 1.15       | 1.21      |
| 120                                  | 2.53           | 2.94       | 3.02      | 1.16       | 1.19      |
| 160                                  | 2.29           | 2.71       | 2.75      | 1.18       | 1.20      |
| Flange length = 1000 mm, $F = 80$ kN |                |            |           |            |           |
| 20                                   | 2.90           | 3.42       | 3.53      | 1.18       | 1.22      |
| 60                                   | 2.69           | 3.21       | 3.34      | 1.19       | 1.24      |
| 80                                   | 2.59           | 3.11       | 3.24      | 1.20       | 1.25      |
| 100                                  | 2.48           | 3.00       | 3.12      | 1.21       | 1.26      |
| 120                                  | 2.38           | 2.90       | 3.01      | 1.22       | 1.27      |
| 160                                  | 2.16           | 2.69       | 2.75      | 1.24       | 1.27      |

## 6.2. Lateral Shear Force

According to the FE models of TSSWs established above, the effect of shear force on normal stress distribution of wall flanges is studied. Figure 14 presents the normal stress

distributions of the bottom cross-sections obtained by the proposed method and FEM when the shear force is 80 kN, 100 kN, and 150 kN, respectively. The analytical and simulated normal stresses are in good agreement and have the same change characteristics. With the increase of the shear load, the normal stress distribution is more uneven, and the SL effect is more obvious. Table 3 summarizes the SL coefficients of T-shaped shear wall specimens under different shear loads. It can be found that the SL coefficients of the TSSWs increase with the increase of the shear force, and this indicates that the shear load is one main factor affecting the SL effect of the TSSWs.

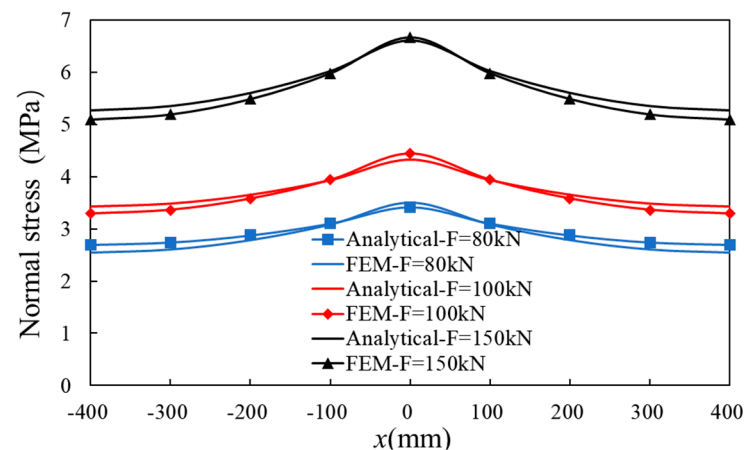


Figure 14. Flange normal stress distribution under different axial force ( $N = 40$  kN).

Table 3. SL coefficient of TSSWs under different shear forces.

| $F$ (kN) | $\bar{\sigma}$ | $\sigma$   |           | $\lambda$  |           |
|----------|----------------|------------|-----------|------------|-----------|
|          |                | Analytical | Simulated | Analytical | Simulated |
| 80       | 3.00           | 3.41       | 3.50      | 1.14       | 1.17      |
| 100      | 3.61           | 4.33       | 4.45      | 1.20       | 1.23      |
| 150      | 4.98           | 6.61       | 6.67      | 1.33       | 1.34      |

### 6.3. Flange Length

According to the FE models of TSSWs established above, the influence of the flange lengths on the NSD and the degree of SL effect are studied by changing the length of the flanges of the above FE models. Figure 15 shows the theoretical analysis and finite element calculation values of the flange NSD of the bottom cross-sections when the flange lengths are 800 mm, 1000 mm, and 1200 mm, respectively. In each finite element model,  $F = 80$  kN and  $N = 40$  kN. It can be found that the analytical normal stresses are in good agreement with those obtained by FEM, and they have the same change characteristics. With the increase in the flange length, the normal stress distribution is more uneven, and the SL effect is more obvious. Table 4 summarizes the SL coefficients of TSSW specimens with different flange lengths. With the increase of flange length, the shear lag coefficients of the specimen increase.

Table 4. SL coefficient of TSSWs with different flange lengths.

| bf (mm) | $\bar{\sigma}$ | $\sigma$   |           | $\lambda$  |           |
|---------|----------------|------------|-----------|------------|-----------|
|         |                | Analytical | Simulated | Analytical | Simulated |
| 800     | 3.00           | 3.41       | 3.50      | 1.14       | 1.17      |
| 1000    | 2.53           | 3.03       | 3.20      | 1.20       | 1.26      |
| 1200    | 2.20           | 2.77       | 2.83      | 1.26       | 1.29      |

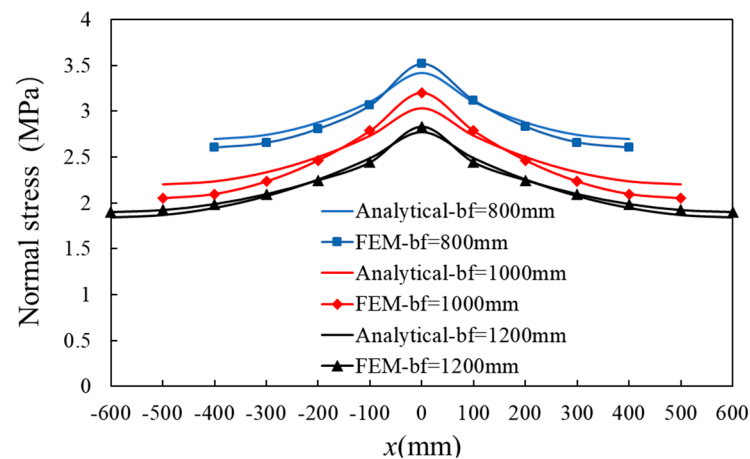


Figure 15. Flange normal stress distribution under different axial force.

## 7. Conclusions

In this study, the SL model of TSSWs is developed, and the correctness of the model is assessed using experimental data and findings from finite element simulations. The results lead to the following conclusions:

- (1) It is considered that SL warping, flexure, and axial displacement make up the longitudinal deformation of TSSWs. According to the concept of lowest potential energy, the SL warping deformation function is considered to be a quadratic parabola, and the formulae for normal stress and deflection are based on this assumption.
- (2) Three-dimensional FE software VecTor3 is used to create the FE model to examine the shear lag in such walls. Tests are conducted to assess the SL in TSSWs under the combined action of shear force and axial force. The test and simulation findings show that under the influence of a shear force, the flange normal stresses are greater close to the web and less farther away, and a noticeable shear lag phenomena takes place. The Bernoulli–Euler assumption results in an equal distribution of normal stress, which is very different from the experimental, simulated, and analytical normal stress distribution.
- (3) The model used in this paper’s analysis produces flange normal stresses that are reasonably consistent with stresses found in simulations and experiments. The comparison indicates that the quadratic parabola assumption of the SL warp displacement of TSSWs is plausible, and the suggested model can accurately estimate the flange normal stresses of the tested and simulated TSSW specimens under axial and shear force.
- (4) Based on the verified numerical models of TSSWs, parameter analysis is conducted, and the analysis shows that the wall specimens with various axial forces (20 vs. 60 vs. 80, vs. 100 vs. 120 vs. 160 kN) have a similar SL coefficient, which indicates axial force in these ranges has limited influence on the SL coefficient. The T-shaped wall specimens under larger shear force (80 vs. 100 vs. 150 kN) have the larger SL coefficients, which indicates a more obvious SL effect. A more obvious SL effect occurs in the TSSW specimens with longer flanges (800 mm vs. 1000 mm vs. 1200 mm).

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