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Research on the Optimal Design of Retaining Piles of a Wide Metro Tunnel Foundation Pit Based on Deformation Control

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Abstract: Engineers pay more and more attention to the economic benefits of foundation pit engineering. At present, the optimal design of the foundation pit supporting structure mainly focuses on strength and functional design, and there is no mature theoretical design method for deformation control. In this paper, a method for calculating the overall deformation of a foundation pit supporting structure based on the principle of minimum potential energy is proposed. Based on this method, the optimal design of the foundation pit of Guangzhou Baiyun District Comprehensive Transportation Hub Metro Station is realized. The deformation calculation results and optimization design scheme are validated by finite element numerical simulation and field monitoring data. The results show that the proposed theoretical algorithm predicts the pile deformation curves better than the finite element method, suggesting the proposed theoretical method is reasonable and the optimization scheme of the retaining pile is feasible. In the optimized design, the deformation of the foundation pit retaining pile is controlled by its push-back effect. The proposed deformation calculation method can realize the overall deformation calculation of the foundation pit supporting structure.

Keywords: foundation pit retaining structure; structure optimization; principle of minimum potential energy; deformation calculation

Citation: Zhong, W.; Wan, Q.; Nie, N.; Ding, H.; Gao, F.; Xu, C. Research on the Optimal Design of Retaining Piles of a Wide Metro Tunnel Foundation Pit Based on Deformation Control. *Buildings* **2024**, *14*, 1906. https://doi.org/10.3390/buildings14071906

Academic Editor: Bingxiang Yuan

Received: 13 May 2024
Revised: 13 June 2024
Accepted: 20 June 2024
Published: 22 June 2024

1. Introduction

With the rapid advancement of infrastructure construction, the optimal design of foundation pits, particularly focusing on economic factors, has garnered increasing attention. A critical aspect of foundation pit design is balancing safety with economic efficiency, ensuring that the supporting structure not only maintains the safety and stability of the engineering project but also achieves greater economic benefits than conventional designs. Traditionally, the design standards for foundation pit supporting structures emphasized strength control; however, there has been a gradual shift towards deformation control. This shift has driven researchers to investigate whether the deformation of foundation pit supporting structures meets engineering requirements during the optimization process [1–5].

The deformation of foundation pit supporting structures is crucial as it directly influences the overall stability and safety of the construction. Excessive deformation can lead to structural failure, posing significant risks to both the project and its surroundings. Therefore, understanding and controlling deformation is not only a technical requirement but also a safety imperative. Most scholars have explored the deformation curve using branch simulation methods, optimizing design parameters by analyzing these curves

[1,2,4,6–14]. These methods focus on fine-tuning the design to achieve minimal deformation while maintaining structural integrity.

Current optimization designs are primarily applied in waterproof curtain and large-scale deep foundation pit engineering, focusing on structural strength and waterproof functionality [1,9,11,15–19]. There are also studies that have made significant contributions by using optimization techniques to improve soil quality (Yuan et al., 2024a, 2024b, 2023 [20–22]). Additionally, some scholars have addressed surface settlement control within the design parameter control of foundation pit supporting structures [7,23–29]. Surface settlement can significantly affect nearby structures and infrastructure, making its control a vital aspect of foundation pit design. In recent years, there has been significant progress in the prediction and calculation methods for foundation pit retaining structures, incorporating artificial intelligence techniques. These include meta-heuristic algorithms like the CO_2 algorithm (Arama et al., 2020 [30]), grey wolf algorithm (Kalemci et al., 2020 [31]), and particle swarm optimization [16,32], as well as machine learning algorithms [6,9,32–35]. However, these heuristic and machine learning algorithms primarily focus on identifying general patterns through large datasets, often lacking insights into the physical mechanisms underlying foundation pit engineering. While they offer potential improvements in design efficiency, they may not fully capture the complexities of real-world applications.

To address this, the Rayleigh–Ritz method in elastic mechanics offers a theoretical approach to calculating the flexible deformation curve of foundation pit supporting structures. This method examines the deformation behavior from an energy perspective, emphasizing the calculation of strain energy and external forces. The Rayleigh–Ritz method provides a systematic way to analyze the deformation characteristics by minimizing the total potential energy of the system. While some researchers have applied this method to foundation pit deformation calculations (Ou et al., 2019 [10]), it has generally been within unilateral design contexts.

This paper introduces a comprehensive method for calculating the overall deformation of foundation pit supporting structures based on energy principles, validated by field monitoring data. The proposed method facilitates the calculation of the overall deformation behavior of foundation pit supporting structures and was successfully applied in an actual project for optimizing foundation pit design based on deformation control. By integrating theoretical calculations with practical field data, the method ensures both accuracy and applicability. This method can serve as a reference for similar projects in the future, providing a robust framework for achieving optimal design in foundation pit engineering.

2. Project Overview

The Guangzhou Baiyun Integrated Transportation Hub, under construction in Baiyun District, Guangzhou, is set to be Asia's largest of its kind. This hub encompasses multiple functional areas including railway, high-speed rail, subway stations, a passenger terminal, and a logistics center. It features a complex design with four subway lines arranged side by side beneath the railway station within a broad, deep foundation pit. This setup results in a pit-in-pit structure, where the subway pit lies within the larger station building pit. The selected section for analysis shows that the pit is 46.9 m wide with symmetric excavation on both sides, reaching a depth of 12,385 m, which is shown in Figure 1. The surface around the pit is flat for 10 m on either side, beyond which there is a 2.25 m high slope. This design minimizes the overloading effect on the foundation pit's retaining piles, thereby enhancing structural safety. The soil overload on the pit sides is accounted for as 10 kPa in the design. The retaining structure consists of a 1 m thick underground diaphragm wall, designed symmetrically with a length of 16,385 m. The foundation pit is reinforced with a concrete support section measuring 1.2 by 1.0 m, incorporating horizontal braces connected to the retaining piles by crown beams. To counteract gravitational deformation, two lattice columns are positioned at intervals of 15.8 m and 31.6 m within the pit. Due to the large scale of the project and favorable geological conditions, the construction team, in collaboration with the design team, seeks to optimize the current design to reduce costs and shorten the construction period. The optimization efforts focus on enhancing the efficiency and economic benefits of the foundation pit retaining structure while ensuring safety and stability.

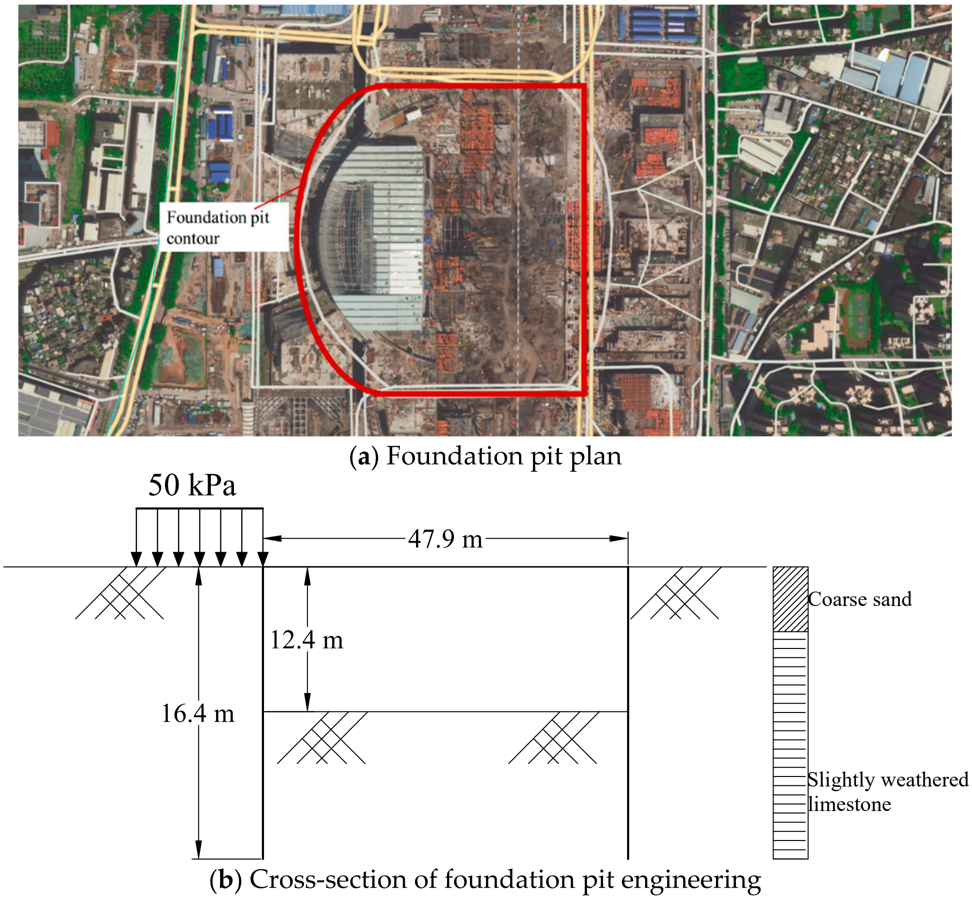

Figure 1. Schematic diagram of the calculation model of foundation pit supporting structure.

Due to being in the railway station square, the foundation pit has a deep excavation depth and good soil properties for its stratum. The soil layer at the excavation depth is the light-weathered limestone layer, whose stratum characteristic is excellent. In addition, due to the excellent characteristics of the stratum, the cost of constructing the diaphragm wall increases. Therefore, the minimum potential energy principle is introduced to analyze the foundation pit supporting structure according to the mechanical characteristics of the foundation pit supporting structure, and the optimization scheme of the foundation pit retaining piles under the predetermined deformation limitation is analyzed to further reduce the construction cost and improve the economic benefit and efficiency of the project.

3. Calculation Model

Based on the calculation diagram shown in Figure 1b, the foundation pit supporting structure needs to be further simplified into a theoretical supported calculation model as shown in Figure 2. Because the selected section is 40 m away from the toe of the outer pit, the soil squeezing effect of the pit can be ignored. The slope on each side of the foundation pit is simplified to an overloaded 10 kPa, and the top of the foundation pit retaining piles is assumed to flush with the ground level. Each of the foundation pit retaining piles is simplified as a beam model, and only the horizontal force and deformation of the supporting structure are considered. The horizontal bracing is simplified as an axial tension–compression bar, and there is a linear relationship between its overstress and strain. The connection between the horizontal brace and the foundation pit retaining piles is simplified as a hinged joint. The deformation of the two ends of the bracing is coordinated, but its effect on the bending moment curve of the retaining pile is ignored. The whole analysis model of the foundation pit supporting structure is established using the improved Rayleigh–Ritz Method. The total potential energy equation, the work carried out by external force, and the strain energy of the foundation pit supporting structure are defined accordingly. It is assumed that the single bracing satisfies the deformation coordination conditions on both sides of the horizontal bracing, and the deformation of the supporting structure on both sides of the pit is equal to the compression amount of the horizontal bracing. On such basis, the total potential energy equation of the foundation pit supporting structure is solved, the deformation curve and horizontal supporting axial force of the retaining piles are obtained, and the design of the foundation pit supporting structure is further optimized to improve economic benefits. The specific derivation process is as follows (Ding et al., 2024 [36]).

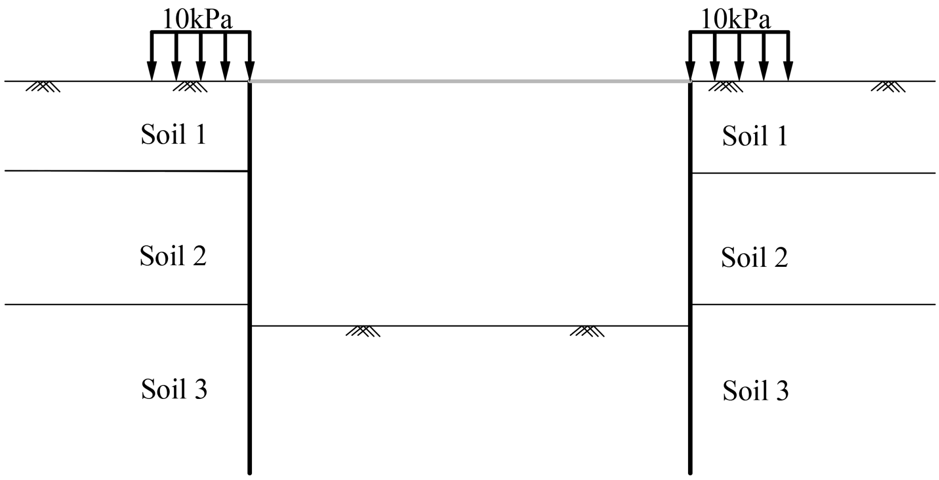

Figure 2. Simplified schematic diagram of the foundation pit supporting structure.

3.1. Total Potential Energy Equation

$\prod$ is defined as the total potential energy of the foundation pit support system, which consists of two main parts: the work carried out by the external force, W, and the strain energy received by the supporting structure, U. It is worth noting that the external force only covers the gravity to promote the deformation of the retaining pile, and the strain energy U of the supporting structure accounts for the bending strain energy of the retaining piles on both sides of the foundation pit and the compression strain energy of the horizontal support. Based on the above analysis, the strain energy equation of the retaining piles is defined in Equation (1). The equation describes the total potential energy equation, which combines the retaining piles on both sides of the foundation pit and the horizontal lateral support into the foundation pit supporting structure and examines the stress of the foundation pit supporting structure to make it better in line with the actual engineering situation.

$$\prod(w_l, w_r) = U - W \tag{1}$$

where w_l and w_r are the horizontal displacement curves of retaining piles on both sides, respectively. U is the total strain energy of supporting structure system, W is the work delivered by the external force acting on the supporting structure of the foundation pit. The strain energy and the work by external forces of the supporting structure will be explained in the following sections.

3.2. Work by the External Force

As clarified earlier, earth pressure induced by gravity is the only external force exerted on the retaining piles, and it is reflected by the horizontal displacement of the retaining piles. Many studies have pointed out that the earth pressure and the horizontal displacement of the retaining pile follow an S-shape relationship. The minimum and maximum earth pressures on the retaining pile are the ultimate active earth pressure and ultimate passive earth pressure, respectively. If the retaining piles are not subjected to any horizontal deformation, the earth pressure on the foundation pit supporting structure is static. The ultimate earth pressure of the retaining piles is calculated using the Coulomb earth pressure theory. The main soil layer in this engineering case is light-weathered limestone, whose cohesion is small and thereby negligible in the calculation. In this paper, the exponential curve model of earth pressure and displacement proposed by Professor Chen Yekai is used, as shown in Equations (2) and (3).

Active earth pressure:

$$P_a(s,z) = P_0(z) - (P_0(z) - P_{acr}(z))\frac{s}{s_a}e^{a'(1-\frac{s}{s_a})} \quad (2)$$

Passive earth pressure:

$$P_p(s,z) = P_0(z) + \left(P_{pcr}(z) - P_0(z)\right)\frac{s}{s_p}e^{a(1-\frac{s}{s_p})} \quad (3)$$

In the above equations, P_0 is the static earth pressure; P_{pcr} is the ultimate passive earth pressure; P_{acr} is the ultimate active earth pressure; s_a is the ultimate displacement induced by active earth pressure and has a default value of 5%L; s_p is the ultimate displacement induced by passive earth pressure and is default as 0.5% L, where L is the design length of the retaining piles; s is the horizontal displacement of the retaining pile; α is the correction coefficient of the earth pressure curve and can be taken as 0.9; P_a is the active earth pressure; and P_p is the passive earth pressure. Because of the exponential relationship between the earth pressure on the foundation pit retaining pile and the horizontal displacement of the retaining pile, it is difficult to achieve accurate integration when calculating the work applied by the external force. For simplification, a straight line is used to fit the relation curve between the horizontal displacement and the earth pressure. The fitting effect is shown in the Figure 3, and the fitting straight line is shown in the Equation (4).

$$p(x)/z = p_0 + (p_{pcr} - p_{acr}) * 1.6 * s \quad (4)$$

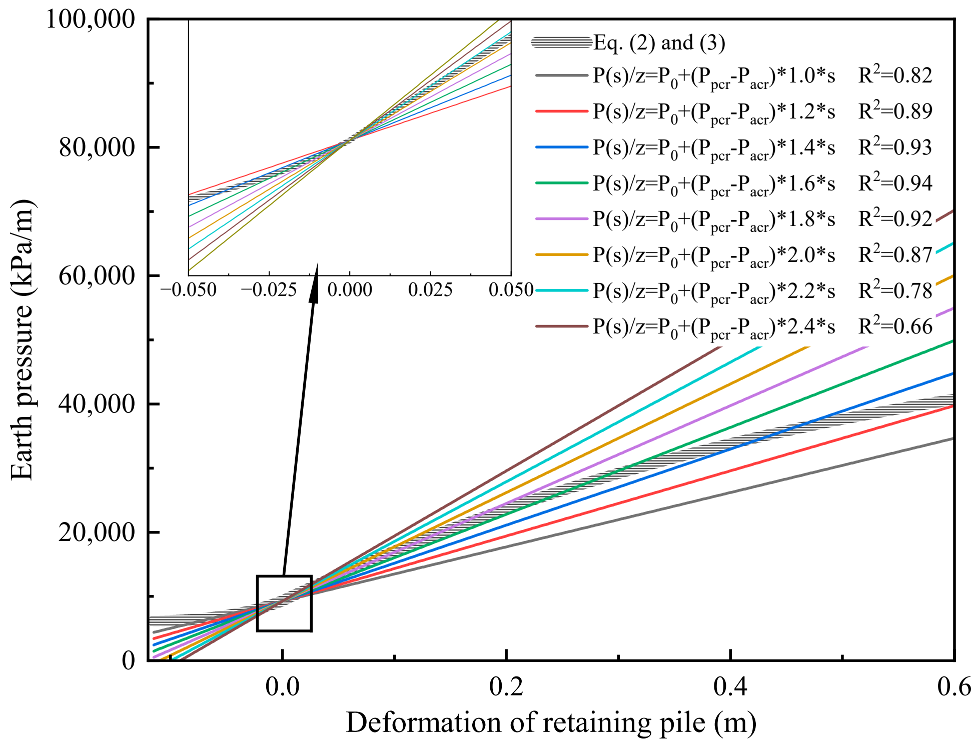

Figure 3. Fitting result of the relationship between earth pressure and deformation.

Based on the relation curve between horizontal displacement and earth pressure of the supporting structure derived above, the work equation of earth pressure is obtained, as shown in Equation (5).

$$W = \begin{aligned} &\int_0^{L_l}\int_0^{w_l(z)} P_a(s,z)\mathrm{d}s\mathrm{d}z + \int_0^{L_2}\int_0^{w_r(z)} P_a(s,z)\mathrm{d}s\mathrm{d}z \\ &-\int_{H_l}^{L_l}\int_0^{w_l(z)} P_p(s,z-H_l)\mathrm{d}s\mathrm{d}z - \int_{H_r}^{L_r}\int_0^{w_r(z)} P_p(s,z-H_r)\mathrm{d}s\mathrm{d}z \end{aligned} \quad (5)$$

where w_l and w_r are the deformation functions of retaining piles on the left and right sides, respectively; H_l and H_r are the excavation depths on the left and right sides, respectively.

3.3. Strain Energy of Supporting Structure

The foundation pit supporting structure consists of two parts, which are the retaining piles on both sides of the pit and the horizontal lateral support. Among them, the retaining piles on both sides are mainly subjected to bending deformation, and the horizontal bracing deforms mainly due to tension and compression. For the retaining piles on both sides, the beam model is used to investigate the deformation law. At present, the commonly used beam models are the Euler beam model and the Timoshenko beam model. Some studies have proved that these two models give very close deformation predictions for the rod structure whose length is much larger than its width, like beams. Therefore, the Euler beam model is used to derive the strain energy of the foundation pit retaining structure in this paper. The calculation method of integral strain energy of the foundation pit supporting structure is shown in Equation (6).

$$U_E = \frac{EI}{2}[\int_0^{L_l} (\frac{d^2 w_l}{dz^2})^2 dz + \int_0^{L_r} (\frac{d^2 w_r}{dz^2})^2 dz] + \frac{EA}{2}[\int_0^{L_l} (\frac{dw_l}{dz})^2 dz + \int_0^{L_r} (\frac{dw_r}{dz})^2 dz] + \frac{N^2 B}{2EA_N} \tag{6}$$

where EI is the bending stiffness of the foundation pit retaining piles; w_l and w_r are the deformation functions of the retaining piles on the left and right sides of the foundation pit, respectively. EA is the tension and compression stiffness of the retaining piles, N is an undetermined coefficient referred to the axial force of the horizontal brace, B is the width of the foundation pit, and EA_N is the tension and compression stiffness of the horizontal brace.

3.4. Boundary Conditions and Overall Structure Analysis

In this paper, high-order polynomials are used to fit the deformation of foundation pit piles on both sides, and high-order polynomials can be expressed as the product of an undetermined coefficient matrix and a complete function matrix. High-order polynomials can fit any form of function curve in theory, so the polynomial with a degree high enough can approach the actual stress state of the foundation pit retaining piles with high precision. By using a matrix form, the undetermined coefficient matrix and the displacement shape function matrix are separated. In this example, the deformation curve of the foundation pit retaining piles is fitted with a 10th-order polynomial, as shown in Equation (7).

$$w_i(z) = \phi_i f_i(z) \tag{7}$$

where $f_i(x) = [1, x, x^2, \ldots, x^{10}]^T$ is the vector of the n dimensions that represent the shape function of the retaining piles on both sides; $\phi_i = [\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{i10}]$ is the undetermined coefficient vector of the deformation function; and i is the number of retaining piles on the left or right side of the foundation. Based on Equation (7), combined with the embedded state of the foundation pit retaining piles and considering that the foundation pit retaining piles are embedded in the slightly weathered limestone layer, the foundation pit retaining piles are fixed at the bottom to zero the horizontal deformation and rotation angle there, as shown in the Equations (8) and (9).

$$\boldsymbol{G} = [f(L), f'(L)]^T \tag{8}$$

$$\boldsymbol{G}\phi_i^{\ T} = 0 \tag{9}$$

where L is the length of the retaining piles. To realize the overall analysis of the retaining piles on both sides of the foundation pit, the deformation coordination conditions at a depth of the horizontal bracing on both sides of the foundation pit need to be based on the deformation of the horizontal bracing so that the derived energy equation can more accurately describe the interaction between the retaining piles on both sides of the foundation pit. In the case of this project, the deformation coordination equation of the foundation pit retaining piles is shown in Equation (10).

$$[f_l(d_1)^T, f_r(d_1)^T][\phi_l, \phi_r]^T - \frac{NB}{EA_N} = 0 \tag{10}$$

Equation (10) shows that the sum of the retaining pile deformation is balanced by the compression effect of the horizontal brace. The deformation curve of the retaining piles on each side contains 11 undetermined coefficients, while the deformation coordination condition contains only one system of linear equations. Thus, solving Equation (10) gives an analytical solution space for the undetermined coefficients, as shown in Equation (11).

$$\phi^T = Z\varphi^T \tag{11}$$

where Z is the basic solution system and shown in the form of a set of numerical matrices of Equation (10), φ^T is the basis of the analytic correspondence and represents undetermined coefficients. Based on matrix theory, the number of undetermined coefficients in φ^T will be one less than that in ϕ^T. By substituting Equations (5), (6), and (11) into Equation (1), the total potential energy equation of the foundation pit supporting structure is obtained, which is a function of the undetermined coefficient φ^T, as shown in the Equation (12).

$$\prod(\varphi^T, N) = U(\varphi^T, N) - W(\varphi^T, N) \tag{12}$$

It is known that the extreme value of the total potential energy equation gives the minimum potential energy, and equations for obtaining the extreme value are shown in Equation (13).

$$\frac{\partial \Pi}{\partial \varphi_i} = 0, \frac{\partial \Pi}{\partial N} = 0 \tag{13}$$

By substituting the undetermined coefficients calculated from Equation (13) into Equation (7), the deformation curve of the foundation pit supporting structure can be obtained.

4. Calculation and Optimization

Based on the theoretical method derived above, the optimization design of the foundation pit retaining piles based on the deformation curve was carried out as follows: First, the calculation parameters of the actual project were brought into the equations mentioned above, and the deformation curve of the retaining piles was calculated. Then, according to the calculated stress form and deformation characteristics, the optimization scheme of the foundation pit retaining piles was analyzed. Last, the optimized foundation pit retaining piles were brought into the calculation theory to evaluate their deformation characteristics, and an adjustment was implemented to the optimization scheme if the deformation could not meet the engineering requirements. This iterative process can be seamlessly implemented in program design through the incorporation of a loop structure. A flowchart depicting the design optimization process is illustrated in Figure 4.

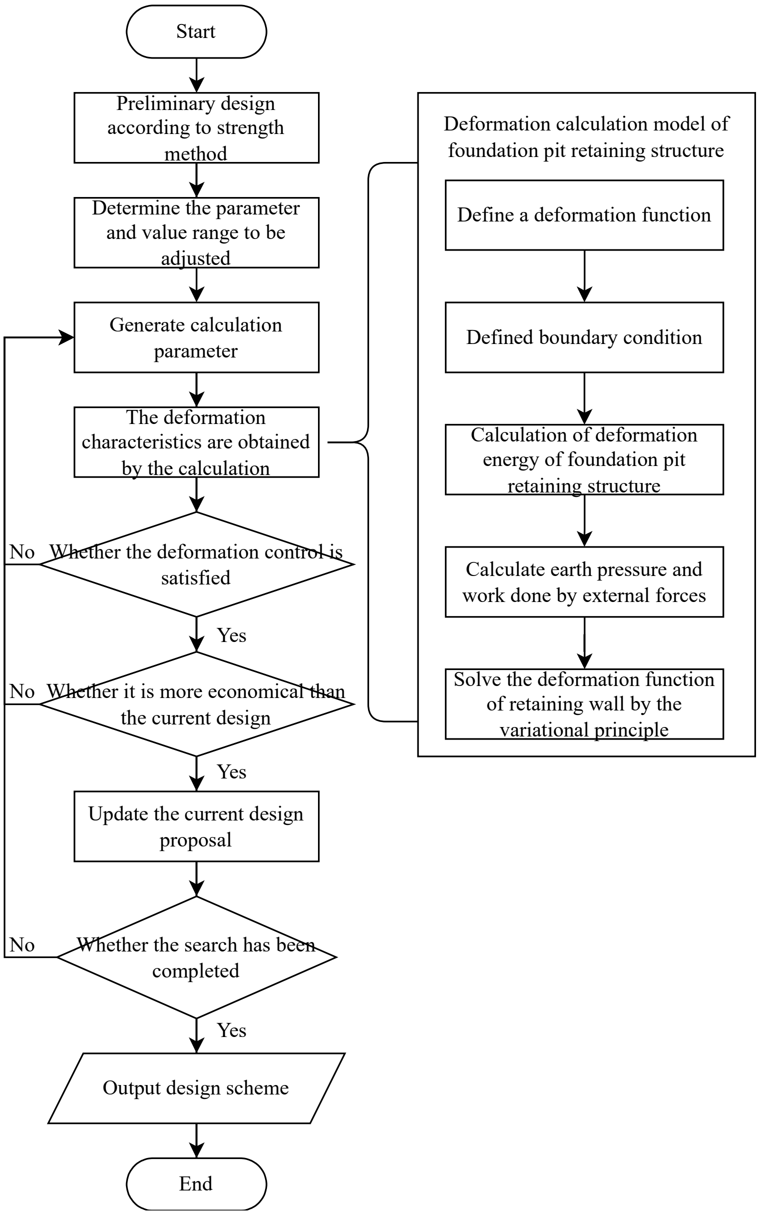

Figure 4. Flow chart of optimization process of foundation pit retaining structure.

4.1. Calculation Model and Parameters

According to the drilling geological data of the site, the whole construction site consists of five soil layers, whose specific soil parameters are shown in Table 1. The main type of engineering geological body involved in the analysis is slightly weathered limestone, so the parameters of slightly weathered limestone are mainly used in the actual design and calculation. The foundation pit retaining piles adopted an underground diaphragm wall with a thickness of 1 m. The diaphragm wall was continuous in the length direction of the foundation pit and was mainly responsible for resisting the bending effect of earth pressure. Therefore, in the actual design and calculation, the ground diaphragm wall structure was simplified to the beam model, and the force and deformation characteristics of the beam model were investigated. The ground diaphragm wall was a reinforced concrete structure, and the elastic modulus of concrete was used to describe the stiffness of the diaphragm wall. The horizontal brace was reinforced concrete support, and the present work only investigates its axial force and deformation. The main mechanical parameters of the foundation pit supporting structure are shown in Table 2.

Table 1. List of geological soil layer parameters.

NO.	Name	γ (kN/m³)	c (kPa)	φ (°)	Thickness (m)
(2)6-1	Coarse sand	19	0	30	1.3
(10)5-3	Slightly weathered limestone	25.9	300	30	26

Table 2. Sectional mechanical parameters of the foundation pit retaining structure.

	Bending Stiffness(kN/mm²)	Axial Stiffness(kN)
Retaining wall	3.00×10^7	3.60×10^7
Horizontal brace	/	3.60×10^6

The deformation curve of the foundation pit retaining pile was analyzed using the proposed calculation theory, the horizontal deformation of the foundation pit retaining structure is calculated using the theoretical calculation method according to the actual optimal design scheme, and compared with the measured data to ensure the feasibility of the subsequent parameter analysis, which is shown as Figure 5. The optimization scheme was put forward according to the deformation law of the retaining pile. The calculation diagram of the supporting structure of the foundation pit under the current design scheme is shown in the Figure 1. Since the rock-socketed soil in the excavation area of the foundation pit has strong properties, its bearing capacity at the pile end is high, leading to a relatively small maximum deformation of the pile. Considering that there is still a big gap between the calculated deformation and the limit value, the original design is proven very conservative. To achieve higher economic benefits, it is feasible to reduce the design and construction costs by cutting down the pile length on one side of the foundation pit. Figure 6 plots the deformation curves of the retaining piles on both sides of the foundation pit, in which schemes of cutting down pile length on one side by 0.5 m, 1 m, 1.5 m, 2 m, 2.5 m, and 3 m are demonstrated.

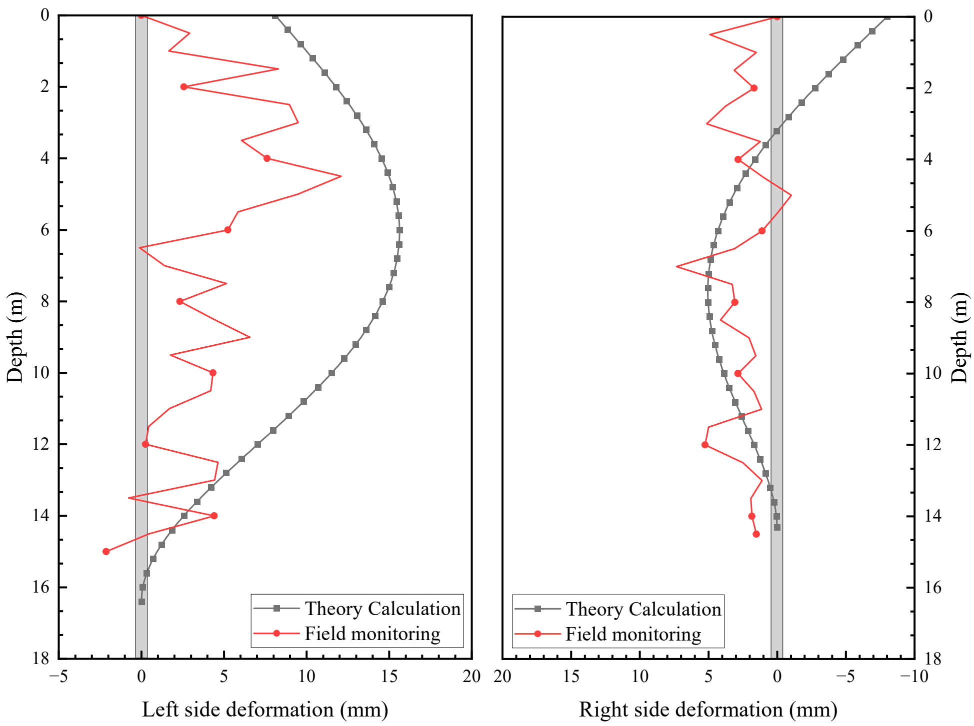

Figure 5. Comparison between in situ monitoring data and theoretical calculation of the deformation curve of the foundation pit retaining piles.

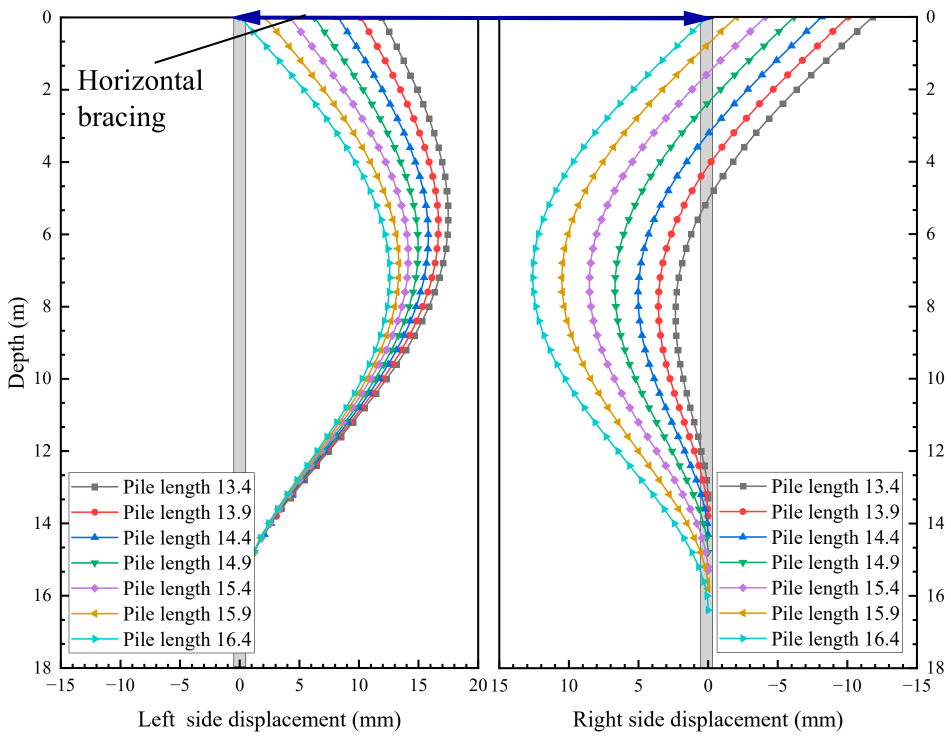

Figure 6. Deformation curves of retaining piles on two sides of the pit with different cutting-down schemes.

The results in Figure 6 show that when the design pile length of one side foundation pit supporting structure is reduced, the maximum deformation of the side retaining pile towards the pit decreases, but the maximum deformation outwards the pit increases. When the design pile length decreases by 2 m, the maximum deformation of the foundation pit retaining pile facing the inside of the foundation pit is almost the same as that facing the outside of the foundation pit. The main reason for this is that the large depth of the retaining pile causes the reduction in active earth pressure on the retaining pile to be larger than that of passive earth pressure, which leads to the push-back deformation of the retaining pile. The active earth pressure on the upper part of the pile is transformed into passive earth pressure to ensure the balance of horizontal earth pressure. Investigation of the retaining pile deformation shows that the deformation toward the pit tends to be more significant. The equilibrium position with equal maximum deformation in the two directions is selected as the design depth of the foundation pit retaining structure. In the case of this project, when the pile length of the foundation pit is reduced by 2 m unilaterally, the deformation law of the foundation pit retaining structure becomes more reasonable. In fact, the scheme of reducing the design length of retaining piles on both sides was also considered, but it led to the overall instability of foundation pit retaining piles. Therefore, in this study, the theoretical strategy for optimization is to reduce the pile length on one side of the pit by 2 m, which is shown as Figure 7.

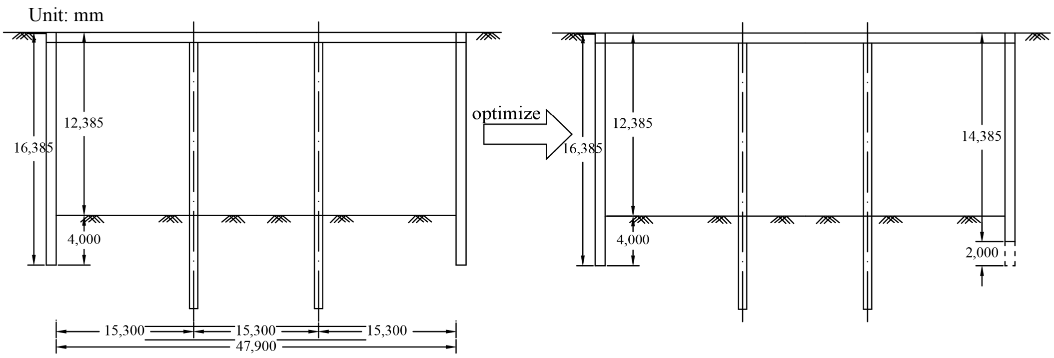

Figure 7. Optimization diagram of foundation pit retaining structure.

4.2. Finite Element Simulation Verification

Apparently, the results of theoretical calculation need validation. Thus, it is necessary to further investigate the stress and deformation characteristics of the optimized foundation pit supporting structure using the popular finite element analysis software Plaxis V21. A 2D finite element model was opted for because it sufficiently addresses the most unfavorable conditions for uniform section design, is computationally more efficient, and better represents actual site conditions compared to the more complex and less efficient 3D Plaxis modeling. The soil mechanics parameters involved in the foundation pit engineering were adjusted and obtained by comparing with the field monitoring data, as shown in Table 3, the soil constitutive model adopts the hardened soil model, and the calculation parameters of the foundation pit supporting structure are shown in Table 2. Compared with the derived theoretical calculation method, the role of groundwater can also be considered in the Plaxis software. Thus, a combination between the Plaxis modelling and theoretical calculation can better achieve the optimal design of the foundation pit supporting structure. The calculation model in Plaxis is shown in Figure 8. In the numerical model, the excavation depth of the foundation pit is simplified to 12.4 m, and the design length of the retaining pile is 16.4 m. In view of the actual excavation construction procedure, the whole foundation pit excavation is divided into five stages, as shown in Table 4. The simulated deformation curves of the retaining piles before and after optimization at each stage are compared in Figure 9.

Table 3. Soil layer parameters involved in the numerical calculation model.

Name	γ (kN/m3)	c (kPa)	φ'(°)	E_{50} (MPa)	E_{oed} (MPa)	E_{ur} (MPa)	G_0 (MPa)
Slightly weathered limestone	15	0	31.6	10	10	30	5.00×10^4

Table 4. Construction stages table for Plaxis analyses.

Stage	Description
1	Constructing ground diaphragm walls on both sides of foundation pit
2	Excavating foundation pit to 1.2 m depth
3	Constructing crown beam and the first horizontal support
4	Excavating foundation pit to 6 m depth
5	Excavating foundation pit to 12.4 m depth

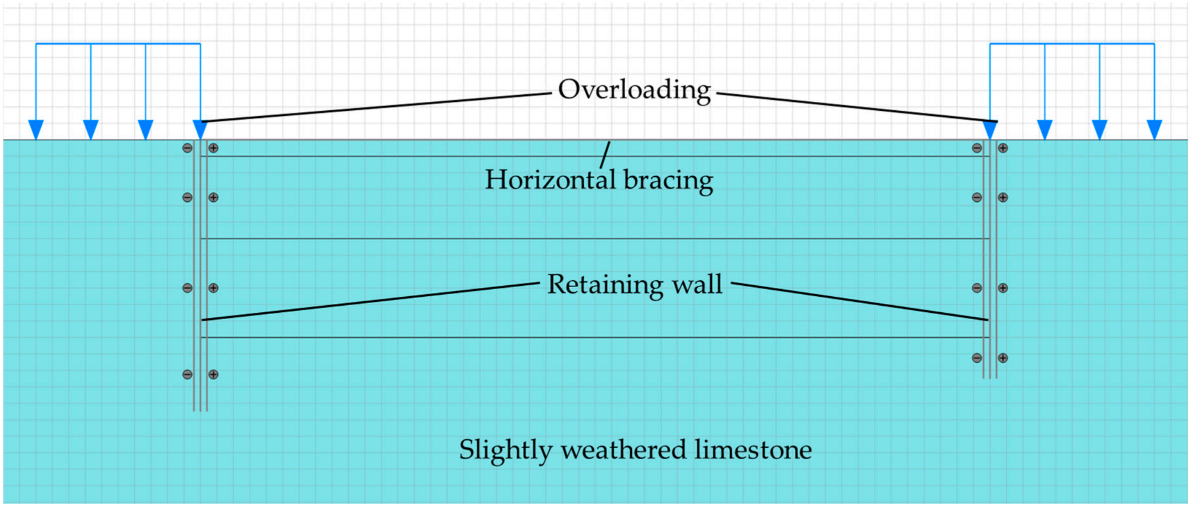

Figure 8. Calculation model of foundation pit supporting structure established by Plaxis.

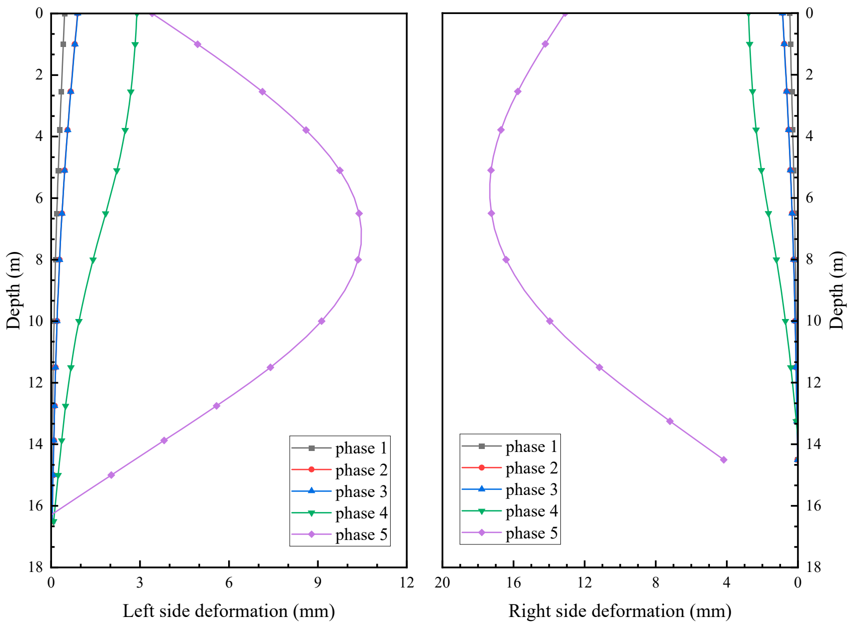

Figure 9. Deformation curves of retaining piles calculated by Plaxis under different construction stages.

The result of Figure 9 shows that the deformation of the foundation pit retaining pile increases as the pit construction progresses, and the deformation of the foundation pit retaining pile suddenly increases in the last two stages, which is caused by the rapid decrease of the passive earth pressure on the retaining pile after the foundation pit excavation is unloaded. Recalling the optimization scheme of the foundation retaining scheme mentioned above, the retaining pile on one side of the pit is shortened by 2 m. The deformation curve of the retaining pile before and after the design optimization in each stage is plotted in Figure 10. In the figure, it is noted that the deformation becomes smaller throughout the construction stages after the optimization, with an exception in the case where the excavation depth on both sides is large. Nevertheless, the increment of deformation is small, and the total deformation is still less than the deformation control value of foundation pit engineering. Therefore, the optimization scheme is feasible in the project.

In Figure 11, the deformation curves obtained from the finite element method are compared with those from the proposed theoretical algorithm. The figure shows an acceptable agreement between these two methods in the maximum deformations and deformation curve forms. The difference in the maximum pile deformation between the non-optimized side and the optimized side is 5 mm. From the theoretical calculation results, the shorter foundation pit retaining pile deforms outward at the top. The reason is that the shorter retaining pile of the foundation pit is subjected to less passive earth pressure at the top of the pile. In the finite element method results, the deformation at the top of the shorter pile is smaller because the calculated earth pressure at the top is greater than that in the theoretical calculation.

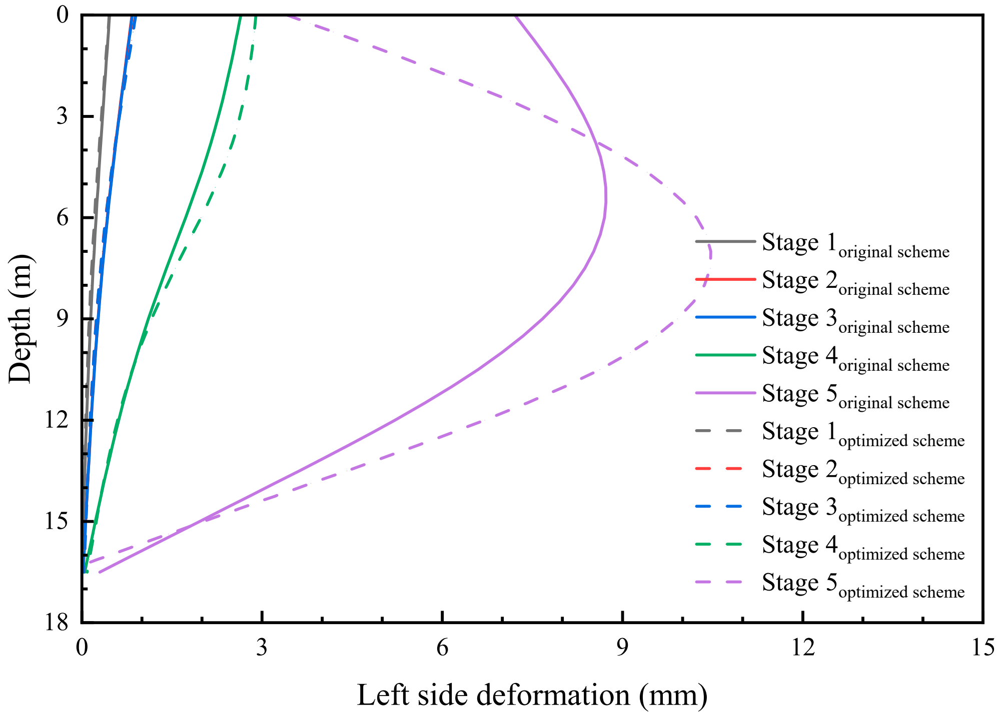

Figure 10. Deformation curve of foundation pit retaining pile before and after optimization in different construction stages.

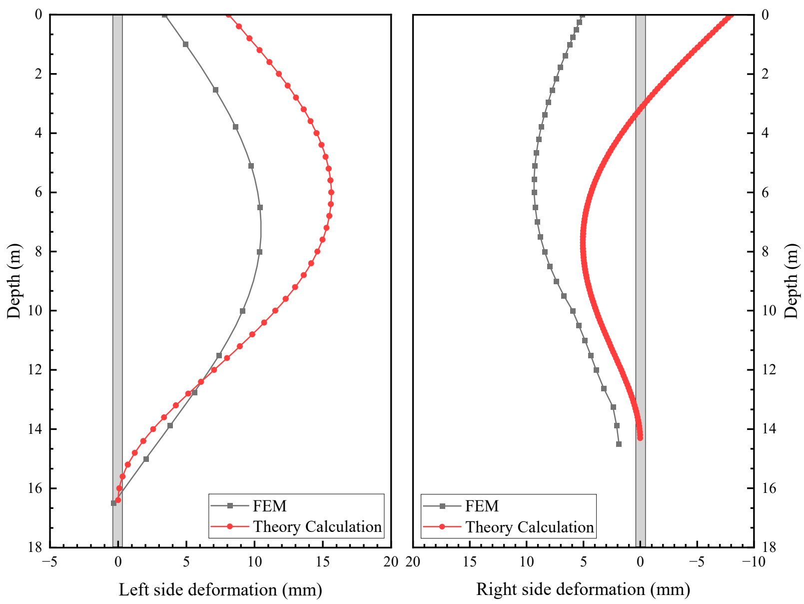

Figure 11. Comparison between finite element calculation and theoretical calculation.

5. Discussion

After optimizing the foundation pit retaining structure to achieve economic and time efficiency, it is crucial to perform a thorough stability verification to ensure that the optimized design maintains the necessary safety and stability standards. The stability verification involves several key steps: (1) global stability analysis, (2) compliance with minimum insertion ratio requirements.

The global stability analysis refers to verifying the overturning resistance of the foundation pit retaining structure under horizontal loads to ensure that the structure does not experience overturning failure. This primarily considers the earth pressure, water pressure, construction loads, and other external loads acting on the retaining structure. The overturning stability is mainly verified by calculating the overturning moment and the resisting moment of the foundation pit retaining structure. The calculation formula is as follows:

$$M_t \geq \gamma_t M_o \tag{14}$$

where M_t is the anti-overturning moment (kN·m), M_o is the overturning moment (kN·m), and γ_t is the safety factor against coefficient, usually 1.5. The calculation method of each variable can be referred to the Chinese standard "JGJ 120-2012 Technical specification for retaining and protection of building foundation excavations" [37].

To ensure that the foundation pit retaining structure can function properly during construction, the standards often require that the retaining piles meet a certain insertion ratio. The insertion ratio refers to the ratio of the length of the retaining structure embedded in the ground below the pit bottom to the depth of the foundation pit. The Chinese standard [37] specifies that the insertion ratio for diaphragm walls should not be less than 0.3. Additionally, the Chinese standard "JGJ 145-2004 Technical Code for Pile-Anchored Support" [38] specifies that the insertion ratio for piles should not be less than 0.5. Therefore, after completing the optimized design, it is necessary to verify the design scheme by combining the stability calculation methods and the minimum insertion ratio requirements.

6. Conclusions

In this paper, a flexible deformation elastic calculation method of foundation pit supporting structure is proposed, which is based on an improved Rayleigh–Ritz method and can be used for the cooperative deformation analysis of foundation pit retaining piles on both sides of the foundation pit. Based on the proposed method, an actual project case is calculated, and a design optimization is proposed, which successfully improves the economic benefit of the project. The main conclusions are as follows:

1. The proposed method for the deformation coordination condition of horizontal supports effectively enables cooperative deformation analysis of both sides of the foundation pit enclosure structure, ensuring bilateral coupling and improved stability.
2. By leveraging the push-back deformation of the asymmetrically loaded foundation pit enclosure, the design scheme's economic efficiency is significantly improved, demonstrating a cost-effective approach to foundation pit construction.
3. The improved Rayleigh–Ritz method for calculating stratum soil parameters and supporting structure design parameters shows a closer alignment with field monitoring data compared to traditional finite element methods, indicating higher predictive accuracy and reliability in practical engineering applications.

The accuracy of the model is influenced by the precision of the load-deformation prediction for the foundation pit retaining structure, with the maximum deformation difference being within 5 mm. The primary limitation lies in the 2D computational model's inability to accurately simulate 3D spatial problems. Despite these limitations, the computational model can effectively predict the deformation of similar foundation pit retaining structures. Consequently, further research on these issues is planned for future work.

Author Contributions: Methodology, Q.W. and F.G.; Software, Q.W. and F.G.; Formal analysis, W.Z.; Investigation, N.N.; Resources, H.D.; Data curation, W.Z.; Writing—original draft, Q.W.; Writing—review & editing, H.D. and C.X.; Project administration, C.X.; Funding acquisition, N.N. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by The National Key Technologies R&D Program of China grant number No. 2023YFC3009400, The National Natural Science Foundation of China grant number No. 52238009, Study on Long-term Performance Evolution and Big Date Analysis for Roadbed of High-speed Railways grant number No. U1934208 and The National Science Fund of Jiangxi Province grant number No. 20223BBG71018.

Data Availability Statement: Data is contained within the article.

Conflicts of Interest: Authors Wancai Zhong and Nan Nie were employed by the company China Railway Construction Engineering Group. Author Feng Gao was employed by the company Station Construction Headquarters of China Railway Guangzhou Bureau Group Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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