Structural Estimates of Supply and Demand Elasticity for Houses in Sydney

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Abstract: We report estimates of supply and demand elasticities for houses (i.e., non-strata properties) in three geographic locations of Sydney. In the Inner Ring of Local Government Areas (LGAs)—those closest to the Central Business District (CBD)—our estimates indicate that the supply curve for houses is perfectly inelastic. This finding allows us to condition on the stock of houses and estimate the corresponding Inner Ring demand curve using ordinary least squares. In the Middle and Outer Rings—where the supply curve for houses has positive elasticity—we use instrumental variables to estimate the demand curve for houses. For all three locations, we obtain theoretically reasonable point estimates of standard demand elasticities, although the degree of uncertainty surrounding the Outer Ring estimates is relatively large. Averaging across the three regions of Sydney, the price elasticity of demand for houses is \( -1.3 \), cross-price elasticity with units is 1.1, and income elasticity is 2.1. Based on our elasticity estimates, only in the Outer Ring are any of the direct burdens of stamp duty born by buyers (about 40 percent).

Keywords: housing supply; housing price bubbles; supply elasticity

1. Introduction

Policy analysis of the housing market—both formal and informal—typically requires estimates of supply and demand elasticities. Reliable estimates are important for undertaking quantitative analysis of policy interventions. Absent estimates, Ableson and Joyeux (2007) [1] are forced to use the assumed values for supply and demand elasticities in examining the effects of different taxes and subsidies on the Australian housing market. Davidoff and Leigh (2013) [2] investigate the effective incidence of stamp duty in Australia. Lacking suitable elasticity estimates, they sought to identify exogenous or policy-induced variation in stamp duty thresholds to directly estimate their impact on house prices. While this methodology is effective, the analysis would be simpler if estimates of price elasticities for the demand and supply of houses were available.

Conceptually, the econometric issues of endogeneity and identification that arise with estimating structural supply and demand curves are well understood [3]. In practice, the key difficulty with estimating supply and demand elasticities for housing lies in obtaining suitable data for key variables. For Australia, a comprehensive set of data is not readily available for the standard variables that theory predicts would enter supply and demand curves for housing. While a lack of data on important variables has the usual negative consequences associated with model misspecification, the potential endogeneity of house prices and quantities means that some exogenous variables are likely to be required as instruments to obtain consistent estimates of own price elasticities of supply and demand.

Typically, it is easier to obtain data on variables that are thought to cause exogenous variation in the demand curve for housing, namely, income, population, and interest rates [4–6]. Consequently, these variables—to the extent they are exogenous—can be used as
instruments for estimating housing supply elasticity [7]. In contrast, potential exogenous shifters of the housing supply curve—planning regulations, industry productivity, and production costs—are less readily available. It is harder to obtain suitable instruments to estimate the price elasticity of housing demand.

In this paper, we make use of a characteristic of the market for houses (specifically non-strata title residential properties) located in the Inner Ring of Local Government Areas (LGAs) in Sydney (LGAs in Sydney are classified into three rings, Inner, Middle, and Outer based on (approximate) distance from the CBD). During the period 1991–2016, the number of houses in the Inner Ring increased by 6.5 percent, equating to an increase of about 8700 houses. Over the same period, population in Inner Ring LGAs increased by about 245,000, while the real median house prices rose by about 310 percent. These statistics suggest the supply of houses in the Inner Ring is relatively unresponsive to increased demand. While the raw figures are not conclusive, evidencing zero price elasticity of supply, we report estimates that are numerically and statistically close to zero. From an econometric perspective, the benefit of a zero-supply elasticity is that it implies a recursive structural ordering for the Inner Ring supply and demand curve for houses and allows the demand curve to be estimated by ordinary least squares (OLS).

A limitation of the above identification strategy is that it cannot be used to estimate a demand curve for houses in Sydney LGAs that are outside of the Inner Ring. In these LGAs, the assumption of an inelastic supply curve for houses is unlikely to be a good approximation [7,8]. For the Middle and Outer Ring LGAs, we have enough instruments to estimate the supply elasticity, but we lack suitable data on exogenous supply curve shifters to estimate the demand curve. Our solution to the identification problem is to impose a linear restriction on the coefficients of two variables that enter the demand curve. Specifically, we restrict the elasticity of demand for houses with respect to population to equal unity. In the absence of additional restrictions, our identifying restriction is untestable for the Middle and Outer Rings. However, we can test for a unit population elasticity of demand in the Inner Ring. Non-rejection of the restriction for the Inner Ring demand curve would provide some support for its imposition in the other two locations.

Following the approach outlined above, we use quasi-panel data for LGAs in metropolitan Sydney to estimate supply and demand curves for the stock of houses in three broad geographic areas, i.e., Inner Ring, Middle Ring, and Outer Ring LGAs using the IV approach. The demand curve for houses in the Inner Ring is estimated using OLS, and in the Middle and Outer Rings using instrumental variables (IV).

The contribution of this paper is twofold. First, this is the first paper that obtains both the supply and demand elasticity estimates for Australian housing markets. Our approach yields plausible estimates of supply and demand elasticities for houses in Sydney. The supply of houses is effectively inelastic in areas of Sydney closest to the CBD, but supply elasticity does increase with distance for the CBD, and approaches a unit elasticity for LGAs in the Outer Ring. Demand for houses is price elastic in all three regions, and is also quite responsive to changes in the price of units (a substitute). Income elasticity varies across the regions, ranging from 0.5 in the Middle Ring to 3.0 in the Outer Ring. In terms of uncertainty, it is probably the highest for the Outer Ring estimates. Second, using the elasticity estimates and the standard formula for the incidence of a tax, we examine the likely incidence of taxes on houses in Sydney regions to find out who ultimately bears the burden of a tax, which provides evidence about the distribution of stamp duty tax between buyers and sellers and sheds a light on answers to the recent policy debate about if the stamp duty should be abolished and replaced by the annual property taxes for reducing the housing affordability problem.

This paper has the following structure: Section II provides a brief review of the methods and results of previous Australian studies that are related to our analysis. A formal description of our econometric methodology and assumptions is provided in Section III. The data sources and some descriptive analysis are reported in Section IV. Section V
contains our results and discusses their implications for tax incidence. Section VI concludes this paper.

2. Previous Studies

Empirical studies of the supply and demand for residential housing can be usefully classified by whether the empirical specification models a stock or flow elasticity. We can think about stock specifications as relating the level (or log-level) of house prices to a measure of the housing stock. This model describes a situation where the price of housing is determined by the demand and supply of the housing stock [9–12]. In contrast, flow specifications typically relate measures of the flow of new housing (i.e., investment or building approvals) to either changes in house prices or to a Tobin’s q ratio, such as house prices relative to construction costs. In such specifications, changes in the growth rate of house prices or in Tobin’s q ratio are associated with changes in the supply of new houses [13–17].

Previous empirical work on the Australian housing market falls into these two categories. Papers that estimate demand elasticities include [18–20]. These studies report estimates of a static (or long-run) inverse demand curve for the stock of housing (By an inverse demand curve, we mean the price of housing—rather than the quantity—is used as the dependent variable). (At least some) variables included in the demand curve are assumed to be non-stationary, but combine to form a cointegrating relationship.

Based on the quarterly data for the sample period 1975–2003, the estimates in Abelson et al. imply a long-run elasticity of demand for (detached) houses of −0.3, an income elasticity of demand of 0.5, and a real interest rate (semi) elasticity of −0.015 (i.e., a 1 percentage point increase in the real rate causes a 1.5 percent fall in the quantity of houses demanded). Using a data sample from 1982–2009, Caldera and Johansson’s estimates imply a long-run elasticity of demand for all types of housing of −0.1, and income elasticity of 0.2, and a real interest rate (semi) elasticity of zero. Liu (2019) [20] pools data for 144 local housing markets in the state of New South Wales over the years from 1991 to 2015, and estimates an income elasticity of housing demand of around unity.

Aside from the somewhat different sample periods, there are three likely sources for the differences in estimated elasticities. Abelson et al. include seven explanatory variables in their model, while Caldera and Johansson only include four. Secondly, Caldera and Johansson’s use of all types of housing (houses and units) rather than just detached houses may result in a relatively more inelastic estimate. Finally, Liu (2019) [20] pools data across a range of geographic housing markets and uses panel cointegration techniques in their estimation.

A larger number of papers examine housing supply in Australia, including [7,8,19,21–25]. With models of supply, the distinction between stock and flow elasticity is important. Depending on the exact specification, the estimated supply elasticity from these two approaches can have different interpretations, and estimates may not be directly comparable. Most of the studies adopt a flow measure of new housing supply.

Ball et al. (2010) [21] and Caldera et al. (2013) [19] use broadly similar specifications. Ball et al. link private housing starts to a measure of house prices and construction costs, while Caldera and Johansson use real residential investment as their quantity variable. Both studies use aggregate data for Australia, log-linear models, and one can interpret the coefficient on house prices as measuring the percentage change in the flow of new housing in response to a one percent increase in the price of housing relative to the cost of new housing construction. In effect, the studies estimate the elasticity of housing investment with respect to Tobin’s q for housing. Both papers report similar estimates of about 0.50 for this elasticity using quarterly data for similar sample periods (1982–2009 and 1983–2008).

McLaughlin (2011, 2012) [22,23], McLaughlin et al. (2016) [24], and Ong et al. (2017) [25] use a specification for supply that relates the log of building approvals to the first-difference in the log of real prices (The approach of these authors follows that used by
In effect, this model measures the percentage change in approvals to a one percentage point change in the growth rate of house prices.

McLaughlin (2011, 2012) [22,23] report estimates for Australian capital cities, and McLaughlin et al. (2016) [24] report estimates for Local Government Areas (LGAs) in Adelaide. For the capital cities, the estimates suggest average long-run marginal responses in the range of 4 to 6 for single-family dwellings. A 1 percentage point increase in the growth rate of house prices leads to about a 5 percent increase in building approvals (in aggregate) over roughly 1.5 years. The marginal effect estimates for Adelaide using LGA data are about three times larger than this figure.

Ong et al. (2017) [25] develop a model of housing supply in Australia. They model monthly building approvals (a proxy for new residential construction) in Australian LGAs as a function of lagged growth in prices, plus several other controls. Data for the period July 2005 to June 2014 are used, and the authors estimate separate models for houses and units. Ong et al.’s estimates imply that a 1 percentage point increase in the growth rate of house prices will cause about a 5 percent increase in building approvals after 15 months, which is virtually identical to McLaughlin’s estimate. With an average monthly building approvals per LGA of 27, this will only add about one additional building approval to the average LGA. The authors argue that this figure implies a conventional elasticity of the house stock of about 0.09, that is, that a 1 percent increase in house prices leads to a 0.09 percent increase in the housing stock. This figure suggests that the economy-wide supply curve for houses in Australia is highly inelastic. In a recent study, Melser, et al. (2022) [26] estimate the individual supply elasticity for each of the 341 Australian LGAs for houses and units, respectively, using the IV methods and the annual data from 2001 to 2019. The supply elasticity for houses averages at 0.34 and ranges from 0.17 at the 25th percentile to 0.44 at the 75th percentile, while for units, it averages at 0.96 and varies between 0.56 and 1.17.

In their analysis of the housing supply in Sydney [7,8] relate the number of residential properties (units and/or houses) to a relevant level of real prices. With a double-log specification, this type of model recovers the percentage change in the stock of housing in response to a one percent change in real housing prices. The estimated supply elasticity is consistent with the stock specifications used by [18,19] in estimating various (indirect) demand elasticities. The results in [7,8] imply that supply elasticity for housing geographically varies across Sydney—supply becomes relatively more elastic with distance from the CBD—and the supply of units is relatively more elastic than the supply of houses. Across all types of residential property, their models suggest an average housing supply elasticity for Sydney in the range of 0.3 to 0.5.

It can be difficult to compare the supply elasticity estimates obtained from flow and stock models. While there are arguments for both specifications, in this paper, we use the stock model for the housing supply curve. One advantage of this specification is that it provides estimates of supply elasticity that correspond to the demand curve elasticities.

3. Methodology

Following the housing supply and demand model in literature [7,8,26], we use a simple example of the supply and demand model to outline our identification and estimation strategy based on the assumption that the housing supply in the market can be cleared in the same period with housing demand. The general model for the housing market takes the following form:

\[ h^d = \gamma_d p + z^d \theta_d + v^d \]  \hspace{1cm} (1)

\[ h^s = \gamma_s p + z^s \theta_s + v^s \]  \hspace{1cm} (2)

\[ h^d = h^s \]  \hspace{1cm} (3)
Equation (1) represents the demand curve for the stock of houses, (2) is the supply curve, and (3) is the condition for market equilibrium. The vectors $\mathbf{z}^d$ and $\mathbf{z}^s$ include exogenous variables that affect the demand and supply curves for houses, respectively, and $v^d$ and $v^s$ are structural demand and supply shocks. At this stage, we make the minimal assumption that both shocks are stationary (i.e., no unit roots).

In equilibrium, house stocks and prices are jointly determined, and some type of simultaneous equations estimator is required for consistency. We propose using an instrumental variables (IV) estimator. If the variables in $\mathbf{z}^d$ and $\mathbf{z}^s$ are uncorrelated with the error terms in (1) and (2), they can be used as instruments in the estimation of the supply and demand curves. A necessary condition for the identification of both curves is that at least one variable in each of $\mathbf{z}^d$ and $\mathbf{z}^s$ is unique to each curve.

In our application to the housing market, we have data on three variables (income, population, and real interest rate) that might enter $\mathbf{z}^d$, but not $\mathbf{z}^s$, and be available as instruments in the estimation of the supply curve (It is conceivable that real interest rates might directly affect the supply curve for houses. However, we expect this effect to be much smaller than their effect on the demand curve). In the case of $\mathbf{z}^s$, measures of relevant variables are less readily available, and this limits our ability to estimate the demand curve. As a practical matter, the available data on potential instruments allow us to estimate the supply curve (supply elasticity), but not the demand curve (demand elasticity).

### 3.1. Recursive Model

A special case of the above model arises if either the demand curve or the supply curve is perfectly inelastic. Under those conditions, the model would be recursive, and both equations could be consistently estimated using OLS (avoiding the need for (additional) instruments).

In most circumstances, the assumption of a recursive structure for a supply and demand model of the housing market is unlikely to be valid. However, there may be some situations where natural or regulatory constraints effectively prevent the supply of the housing stock from increasing in response to an increase in its price. Our conjecture is that a plausible assumption for houses in locations near the Sydney CBD is that supply is perfectly inelastic.

Under this assumption, we have the following recursive structure for the house market. With $\gamma_s = 0$, we can write:

$$h^s = \mathbf{z}^s \theta_s + v^s$$  \hspace{1cm} (4)

Since variations in the housing stock are exogenous, we can re-write the demand curve (1) to obtain an inverse demand curve for houses, as follows:

$$p = \frac{1}{\gamma_d} h - \mathbf{z}^d \theta_d - \frac{v^d}{\gamma_d}$$  \hspace{1cm} (5)

or

$$p = \pi_d h + \mathbf{z}^d \pi_d + \eta^d$$  \hspace{1cm} (6)

where we can now simply condition on the stock of houses. Equation (6) can be estimated using OLS and the parameters (i.e., elasticities) of the demand curve recovered from the coefficient estimates.

Equation (6) reflects the basic form of the long-run demand curves for Australian housing used by [18,19]. Interestingly, the additional regressors included in $\mathbf{z}^d$ by Caldera and Johansson are real income, the real interest rate, and the population. Abelson et al. exclude population, but also add stock prices, the exchange rate, and unemployment. Both studies rely on the existence of at least one cointegrating relationship among the variables in their models.

While the recursive structure allows us to estimate a demand curve using data for one region of Sydney, it will not be useful for other regions where the supply elasticity of
houses is not zero. For these areas, we need a different approach to estimating the demand curve. We propose the use of a coefficient restriction on the demand curve (We experimented with assuming the structural shocks to the housing demand and supply curves are uncorrelated [27,28]. However, where it was possible to test this restriction, it was rejected by the data).

3.2. Unit Population Elasticity of Demand

Consistent with [19], we assume that the level of population is a potential influence on the demand curve for houses. We re-write Equation (1) to make this explicit:

\[ h^d = \gamma_d p + \theta_n n + \delta^d \theta_d + \nu^d \tag{7} \]

where \( n \) represents population, and \( \theta_n \) is the population elasticity of demand. Our identification strategy is to impose the restriction \( \theta_n = 1 \), i.e., the demand for houses has a unit population elasticity. Under this restriction, the inverse demand curve is as follows:

\[ p = \frac{1}{\gamma_d} (h - n) - \frac{\delta^d \theta_d}{\gamma_d} + \frac{\nu^d}{\gamma_d} \tag{8} \]

Equation (8) can be estimated by IV using population, \( n \), as an instrument for houses per capita \( h - n \). Since (8) is exactly identified, we cannot directly test the validity of the unit elasticity restriction when estimating it by IV. However, at least for the Inner Ring—where (8) can be estimated using OLS—we can test whether the unrestricted coefficients on the stock of houses and population sum to zero.

4. Data

We use annual data (measured on a financial year basis) from 1991 to 2016 for 35 LGAs in metropolitan Sydney. Based on their distance from the Sydney CBD, the LGAs are classified into three broad geographic areas: Inner Ring (9 LGAs), Middle Ring (12 LGAs), and Outer Ring (14 LGAs) (See Table A1 of the Appendix A for LGA classification and boundaries). For an individual LGA, we have measures of median prices for non-strata properties (houses) and strata properties (units) and the number of each type of property (i.e., non-strata and strata). Available data on variables that typically affect the demand curve for houses include real income, population, and the real interest rate. We have some data on one variable that might affect the supply curve for houses, which is the median time required for a local council to decide on a development application. However, data for this variable are only available from 1995 to 2015, so we used this data mainly as a robustness check on the baseline specification for supply. Comprehensive variable definitions and sources are provided in the Data Appendix A.

Table 1 reports the change in the number of houses (i.e., non-strata properties) and in the real median house price from 1991 to 2016. During this period, the stock of houses in metropolitan Sydney increased by about 30 percent (from approximately 990,000 to 1.3 million). The largest increase occurred in the Outer Ring, where the house stock increased by about 47 percent (or about 250,000 additional houses). The smallest increase in houses occurred in the Inner Ring, where the stock of houses increased by about 6.5 percent (or approximately 8700 houses). The percentage increase in the Middle Ring was around 12 percent, or about 38,000 additional houses.


<table>
<thead>
<tr>
<th></th>
<th>Houses Numbers</th>
<th>Growth %</th>
<th>Real Median Price 2011–2012, Thousands (AUD)</th>
<th>Growth %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner Ring</td>
<td>134,127</td>
<td>142,827</td>
<td>440.3</td>
<td>1809.40</td>
</tr>
</tbody>
</table>
| Middle Ring | 328,694        | 366,402  | 339                                         | 1500.60  | 342.7
Notes: Houses are the number of private (non-strata) residential properties. Median house prices are deflated using the CPI for Sydney (2011–2012 = 100).

We convert the nominal median house prices into real values using the CPI for Sydney (2011–2012 = 100). The growth in real median house prices was greatest in the Middle Ring at approximately 343 percent, and the Inner Ring at about 311 percent. Growth in real house prices was substantially less in the Outer Ring, at about 194 percent. Figures 1 and 2 show the levels and growth rates of real median house prices in each of the three geographic regions of Sydney. Visually, there is some suggestion that house prices in the Outer Ring co-move less closely than those in the Inner and Middle Rings.

![Figure 1. Real median house prices in Sydney.](image1)

![Figure 2. Annual growth rate of real median house prices in Sydney.](image2)

Table 2 reports the data for population and real income over the sample period by region. Growth in population is similar in the Inner and Outer Rings at about 40 percent, and a little lower in the Middle Ring at 30 percent. Real income growth of 88 percent was highest in the Inner Ring, and a little over double the growth rate in the Outer Ring.

<table>
<thead>
<tr>
<th></th>
<th>Population (Persons)</th>
<th>Growth %</th>
<th>Real Income (AUD 2006–2007)</th>
<th>Growth %</th>
</tr>
</thead>
</table>

In summary, there was a relatively small increase in the stock of houses in the Inner Ring. If we assume that all the changes in house stocks and prices from 1991 to 2016 could be roughly approximated as a movement along a given supply curve, then we get supply elasticity estimates of 0.02 for the Inner Ring (compared with 0.03 (Middle Ring) and 0.24 (Outer Ring)). While the assumption is unlikely to be an adequate approximation, it does suggest that supply elasticity in the Inner Ring and the Middle Ring is relatively inelastic. We now investigate this issue using formal techniques.

5. Results

We begin our empirical analysis using the data for the nine LGAs in the Inner Ring. Our first step is to estimate a supply curve for houses, allowing for the possibility of a non-zero supply elasticity. This is feasible, since we have data on a number of exogenous demand-curve shifters. In the second step, we estimate the demand curve for Inner Ring houses. We use two strategies. One approach is to estimate a demand curve using OLS. The consistency of the OLS estimates is contingent on the supply curve being perfectly inelastic. The second approach does not require an inelastic supply curve; instead, it assumes a unit population elasticity of demand for houses.

5.1. Inner Ring

5.1.1. Supply Curve

The initial step is to estimate a supply curve and the elasticity of supply for houses. The basic fixed-effects (FE) specification has the form:

$$ h_{it} = \mu_i + \gamma_y p_{it} + \theta_d a_{it} + D_t \beta + \epsilon_{it} \tag{9} $$

where \( i \) is an LGA and \( t \) is a year and \( h_{it} \) is the (natural) log of the house stock, \( p_{it} \) is the log of the real median house price and \( a_{it} \) is the log of the median time taken by a local council to decide on a development application. Since data for the \( da \) variable are only available for a sub-sample (1995–2015), we also report results for the full sample (1991–2016) with \( da \) excluded. \( \mu_i \) is a fixed-effect dummy for each of the nine LGAs, and controls for any time-invariant heterogeneity across the nine LGAs.

The final variable, \( D_t \), is a vector of time-dummy variables that seek to control the behavior of Sydney house prices during the period 2002–2004. There is evidence that house prices in Sydney exhibited a period of temporary explosive behavior in the early 2000s and became disconnected from economic fundamentals [29,30]. We recognize that the inclusion of the dummies is only an approximate solution and would not be strictly appropriate in the case of an explosive bubble. However, this is an unresolved issue in modeling the data.

We are interested in whether \( \gamma_y = 0 \). If this is not the case, then we expect \( E(p_{it} \epsilon_{it}^2) \neq 0 \), and so need to estimate (9) using IV. We allow for relatively general forms of heteroscedasticity and serial correlation in \( \epsilon_{it}^2 \) [31]. Furthermore, if the variables in (9) have stochastic trends but are not cointegrated, a first-difference specification may provide more reliable estimates, or, more generally, if there is relatively persistent first-order serial correlation in the error term for the fixed-effects model. For comparison, we estimate a first-differences (FD) version of (9).

IV estimates of the FE and FD specifications are reported in Table 3 (fixed effects and dummy variable parameter estimates are not reported). The specification, including the
da variable, is based on a shorter sample (1995–2015 vs. 1991–2016). Excluded instruments are real income, population, and real interest rate. Instrument quality is measured using the scaled version of the non-robust F-statistic for the excluded instruments [32]. Called the “effective” F-statistic and denoted \( F_{eff} \), the statistic allows for heteroscedasticity and serial correlation in the standard instrument quality test regression [33]. Instrument quality looks to be very strong for the FE specification, but weaker for the FD specifications, although the \( F_{eff} \) statistic is still above the Stock and Yogo rule-of-thumb value of 10. The Sargan–Hansen test of overidentifying restrictions is given by the J statistic, and does not reject the FD specification, but is marginal for the FE models [34,35].

Table 3. Supply elasticity for Inner Ring LGAs.

<table>
<thead>
<tr>
<th></th>
<th>Fixed-Effects</th>
<th>First-Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(i)</td>
<td>(ii)</td>
</tr>
<tr>
<td>( \gamma_0 )</td>
<td>0.0148</td>
<td>0.0363</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>( F_{eff} )</td>
<td>94.84</td>
<td>85.72</td>
</tr>
<tr>
<td>( J ) statistic</td>
<td>0.052</td>
<td>0.078</td>
</tr>
<tr>
<td>( N )</td>
<td>234</td>
<td>189</td>
</tr>
</tbody>
</table>

Notes: \( N \) is the total number of usable observations. The variance–covariance matrix is heteroscedasticity robust, includes three lagged autocovariances, and uses a flat window. The \( F_{eff} \) statistic is an indicator of instrument relevance [32]. The J statistic is the \( p \)-value for the Sargan–Hansen test of overidentifying restrictions [34,35].

As anticipated, point estimates of supply elasticity for houses are relatively small across all specifications. Estimates are slightly larger for the FD specification, and are also statistically significant. For the FE models, supply elasticity is small and insignificant in model 3(i). For model 3(ii), with the da variable, this variable has a small but statistically significant negative effect on the housing stock. The estimated supply elasticity is marginally significant in this model, but is very small in magnitude (0.04). The supply curve for houses in the Inner Ring LGAs looks to be perfectly inelastic or a close approximation. We view the results in Table 3 as providing a reasonable basis for the use of OLS to estimate the demand curve for houses in the Inner Ring.

5.1.2. Demand Curve

We now consider the estimation of the demand curve and various demand elasticities. Our econometric model is an inverse demand curve like Equation (8), but it is generalized in two ways. We allow for a partial adjustment mechanism for the price of houses. In addition, we include the real price of units (i.e., strata properties) in Equation (10) to account for their role as a possible substitute for houses.

\[
p_{lt} = \mu_l^d + \lambda p_{lt-1} + \left( \frac{1 - \lambda}{\gamma_d} \right) \left[ (h_{lt} - n_{lt}) - \bar{z}d \bar{\beta}_d - \theta_u p_{lt}^u - D_l \beta_b \right] + \epsilon_{lt}^d
\]

The exogenous regressors included in \( \bar{z}d \) are real income, population, and real interest rate. Can we treat unit price as an exogenous variable? Probably not, but we have no obvious instrument. Initially, we will treat \( p_{lt}^u \) as exogenous and then examine the effect of using its lagged value as an instrument.

Re-writing (10) with unrestricted coefficients yields the following:

\[
p_{lt} = \mu_l^d + \lambda p_{lt-1} + \bar{\pi}_d h_{lt} + \bar{\pi}_y y_{lt} + \bar{\pi}_r r_t + \bar{\pi}_n n_{lt} + \bar{\pi}_u p_{lt}^u + D_l \beta_b + \epsilon_{lt}^d
\]

Our prior expectations for the signs of the estimated coefficients in (11) are: \( \bar{\pi}_d < 0, \bar{\pi}_y < 0, \bar{\pi}_r > 0, \bar{\pi}_n > 0 \). The sign of \( \bar{\pi}_u \) depends on whether units are substitutes or
complements for houses. We expect \( \pi_u > 0 \), indicating that houses and units are substitutes. Finally, we expect \( \pi_u + \pi_n = 0 \).

Provided that the regressors in (11) are uncorrelated with the error term, we can consistently estimate the model using OLS (In using OLS (or IV) to estimate a dynamic panel model with fixed effects we are relying on the fact that we have a large T relative to the number of cross-section units N). However, conditional on the assumption of a unit population elasticity, we can allow for a possible correlation between \( h_{it} \) and \( \epsilon_{it} \) and obtain the consistent estimates using IV.

5.1.3. Estimates

Table 4 reports estimates of the inverse demand curve for the Inner Ring. From the demand curve estimates, we can recover the following direct elasticities of demand for houses: own price, cross-price (with units), interest rate, income, and population (if unrestricted). The economically relevant elasticity estimates along with their standard errors—obtained using the delta method—are reported in the lower half of Table 4.

Table 4. Inverse demand curve and elasticity estimates for houses in the Inner Ring.

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>OLS</th>
<th>IV</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>0.2678</td>
<td>0.2720</td>
<td>0.2712</td>
<td>0.338</td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.083)</td>
</tr>
<tr>
<td>( \pi_u )</td>
<td>-0.2428</td>
<td>-0.3195</td>
<td>-0.3259</td>
<td>-0.3372</td>
</tr>
<tr>
<td></td>
<td>(0.236)</td>
<td>(0.062)</td>
<td>(0.063)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>( \pi_n )</td>
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<td>0.5452</td>
<td>0.5449</td>
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<td>(0.054)</td>
<td>(0.053)</td>
<td>(0.083)</td>
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<tr>
<td>( \pi_y )</td>
<td>0.5018</td>
<td>0.4952</td>
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<tr>
<td></td>
<td>(0.077)</td>
<td>(0.074)</td>
<td>(0.075)</td>
<td>(0.069)</td>
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<tr>
<td>( \pi_r )</td>
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<td>0.0159</td>
<td>0.0162</td>
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<td>(0.009)</td>
<td>(0.009)</td>
<td>(0.009)</td>
<td>(0.01)</td>
</tr>
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</table>

Test of \( \pi_u = \pi_n \)

\( F_{\text{statistic}} \) 2165.02

\( R^2_p \) 0.917 0.918 0.494

Elasticity

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Own Price</td>
<td>-3.02</td>
<td>-2.28</td>
<td>-2.24</td>
<td>-1.96</td>
</tr>
<tr>
<td></td>
<td>(3.035)</td>
<td>(0.529)</td>
<td>(0.512)</td>
<td>(0.47)</td>
</tr>
<tr>
<td>Cross Price</td>
<td>2.25</td>
<td>1.71</td>
<td>1.67</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td>(2.234)</td>
<td>(0.404)</td>
<td>(0.391)</td>
<td>(0.355)</td>
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<tr>
<td>Income</td>
<td>2.07</td>
<td>1.55</td>
<td>1.53</td>
<td>1.48</td>
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<td>(2.126)</td>
<td>(0.407)</td>
<td>(0.397)</td>
<td>(0.375)</td>
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<td>Population</td>
<td>1.3</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
<td></td>
<td>(1.159)</td>
<td>na</td>
<td>na</td>
<td>na</td>
</tr>
</tbody>
</table>

Notes: Model (i) is the OLS estimates of Equation (11). Model (ii) is the OLS estimates of Equation (12). Model (iii) is the IV estimates of Equation (12), using \( n_{it} \) as an instrument for \( (h_{it} - n_{it}) \). Model (iv) is the same as (iii), except that \( p_{it}^{n-1} \) is used to instrument \( p_{it}^{n} \). The variance–covariance matrix is heteroscedasticity robust, includes three lagged autocovariances, and uses a flat window. The \( F_{\text{statistic}} \) statistic is an indicator of instrument relevance [32]. \( R^2_p \) is Shea’s (1997) [35] (adjusted) partial R-squared indicator of instrument relevance. Standard errors for the elasticity estimates are obtained using the delta method.
Column (i) presents the results from estimating the unrestricted inverse demand curve (11) using OLS. In this specification, we are conditioning on (i.e., assuming weakly exogenous) both the current value of the house stock and the price of units. Except for the real interest rate, the estimated coefficients in Equation (11) have the expected signs. Consistent with identifying a demand curve, the coefficient estimate on the stock of houses is negative (−0.2). In addition, the estimated coefficients on income, population, and unit prices are positive. In the case of unit prices, the positive value is consistent with our expectation that houses and units are substitutes. Only the estimated coefficient on the real interest rate has an unexpected (positive) sign. However, the economic size of the coefficient is small, and its effect is only marginally statistically significant (p-value ≈ 0.10) (The influence of the real interest rate on the demand curve is affected somewhat by the inclusion of unit prices).

One problem that is evident from the estimates of the unrestricted model (11) is the relatively large standard error for the coefficient estimate on the house stock. Formally, the size of the standard error implies that there is no statistically significant relationship between house prices and quantities. In contrast, the coefficient estimates on unit prices, income, and population are all statistically (and economically) significant. However, when the delta method is used to calculate standard errors for the various demand elasticities, the large standard error for the coefficient estimate on the stock of houses causes all of the elasticities to be statistically insignificant. In effect, the large size of the standard errors means that the resulting estimates are very uninformative, e.g., the 95% confidence interval for own price elasticity ranges from −9.1 to 3.1.

Our response to this problem is to impose the unit population elasticity restriction. Given the unrestricted point estimate for the population elasticity of demand is 1.3, imposing a unit elasticity does not appear to be strongly at odds with the data. The unit restriction implies \( \bar{\pi}_d + \bar{\pi}_n = 0 \), and from the model (11) we obtain:

\[
p_{it} = \mu_t + \lambda p_{i,t-1} + \bar{\pi}_d (h_{it} - n_{it}) + \bar{\pi}_y y_{it} + \bar{\pi}_r r_t + \bar{\pi}_u p_{i,t} + D_{it} \theta_b + \epsilon_{it} \tag{12}
\]

The results from estimating the restricted model, along with a test of the restriction, are reported in column (ii). The restriction is not rejected by the data (p-value of 0.7). Furthermore, it is evident that imposing the restriction substantially reduces the magnitude of the standard error on the house stock coefficient (from 0.24 to 0.06), but produces only minor changes to the other model estimates (A potential source of the relatively large standard error on the house stock coefficient is a high degree of collinearity between the number of houses and the population in an LGA).

Comparing the implied demand elasticity estimates in columns (i) and (ii), we see the point estimates are smaller in (absolute) size for the restricted model, but are all statistically significant. For the restricted model, the 95% confidence interval for the own price elasticity is −3.3 to −1.2. The estimated cross-price elasticity of 1.7 indicates a high degree of substitutability between houses and units in the Inner Ring. The relatively large income elasticity of 1.6 indicates that Inner Ring houses would be categorized as luxury goods.

OLS estimates of the restricted version of the inverse demand curve produce plausible elasticities. However, it is possible to investigate the robustness of the OLS estimates in a couple of directions. How do the estimates change if we relax the assumptions that the house stock and unit prices are both weakly exogenous and can be used as their own instruments? We do this in two steps.

If we focus on the restricted version of the inverse demand curve and maintain the weak exogeneity assumption for unit prices, then it is possible to estimate this model by IV. The restriction on the coefficients of the house stock and population allows the population—an exogenous variable—to be used as an instrument for \( (h_{it} - n_{it}) \). The inverse demand curve (12) is just identified.

The results for the IV estimation of Equation (12) are reported in column (iii). Both the \( F \)-statistic and Shea’s (1997) partial R² (adjusted for degrees of freedom) indicate the population is a very strong instrument for \( (h_{it} - n_{it}) \). In fact, the IV estimates for the
restricted model are very similar to those obtained using OLS. These results provide some confirmation that it is reasonable to treat the house stock as an exogenous variable.

The estimates in column (iii) assume that unit prices are weakly exogenous. If we allow \( p_{t-1} \) to be correlated with the error term in (12), it needs to be removed from the list of instruments. Our strategy is to instrument unit prices by their one-period lagged value. The results are reported in column (iv). Since the model contains two endogenous variables, we examine instrument quality for both variables using Shea’s (1997) partial \( R^2 \) (adjusted for degrees of freedom). For the house stock, the partial \( R^2 \) is 0.9, but even for unit prices, it is relatively high at 0.49. The only notable effect of instrumenting unit prices is to somewhat reduce the absolute value of both the own and cross-price elasticity estimates.

Across models (ii) to (iv), the elasticity estimates are broadly similar and imply the following points. Demand for houses in the Inner Ring is elastic. This is consistent with the fact that there are several substitutes for Inner Ring houses, including units in the same location, but also houses and units in the Middle and Outer Rings. The positive cross-price elasticity for units is greater than unity, and implies that there is a degree of substitutability between houses and units in the Inner Ring LGAs. Finally, the income elasticity of demand for houses is greater than one. Due to the inelastic supply of Inner Ring houses, growth in real income will have a relatively large effect on house prices.

5.2. Middle and Outer Rings

We now consider the estimation of the supply and demand elasticities for LGAs in the Middle and Outer Ring. Table 5 reports the estimates of the supply elasticity of houses for both regions, while Tables 6 and 7 report estimates of the inverse demand curve for houses in the Middle and Outer Rings, respectively. With one exception, we report estimates for the same models of the demand curve as for the Inner Ring (Table 3).

### Table 5. Supply elasticity for Middle and Outer Ring LGAs.

<table>
<thead>
<tr>
<th></th>
<th>Middle Ring Fixed-Effects</th>
<th>Outer Ring Fixed-Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(i)</td>
<td>(ii)</td>
</tr>
<tr>
<td>( \hat{\gamma} )</td>
<td>0.0888</td>
<td>0.1109</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>( \hat{\theta}_{DA} )</td>
<td>-0.0011</td>
<td>-0.0024</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.025)</td>
</tr>
<tr>
<td>( F \text{-}statistic )</td>
<td>113.06</td>
<td>95.43</td>
</tr>
<tr>
<td>( J \text{-statistic} )</td>
<td>0.142</td>
<td>0.288</td>
</tr>
<tr>
<td>( N )</td>
<td>312</td>
<td>252</td>
</tr>
</tbody>
</table>

Notes: \( N \) is total number of usable observations. The variance–covariance matrix is heteroscedasticity robust, includes there lagged autocovariances, and uses a flat window. The \( F \text{-}statistic \) is an indicator of instrument relevance [32]. The \( J \text{-statistic} \) is the \( p \)-value for the Sargan–Hansen test of overidentifying restrictions [34,35].

### Table 6. Inverse demand curve for houses in Middle Ring LGAs.

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>OLS</th>
<th>IV</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(i)</td>
<td>(ii)</td>
<td>(iii)</td>
<td>(iv)</td>
</tr>
<tr>
<td>( \hat{\lambda} )</td>
<td>0.5223</td>
<td>0.5197</td>
<td>0.5157</td>
<td>0.5507</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.044)</td>
<td>(0.043)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>( \hat{\pi}_d )</td>
<td>-0.1529</td>
<td>-0.4724</td>
<td>-0.4957</td>
<td>-0.4927</td>
</tr>
<tr>
<td></td>
<td>(0.369)</td>
<td>(0.068)</td>
<td>(0.059)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>( \hat{\pi}_u )</td>
<td>0.4064</td>
<td>0.4169</td>
<td>0.4181</td>
<td>0.3574</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td>(0.062)</td>
<td>(0.062)</td>
<td>(0.07)</td>
</tr>
</tbody>
</table>
\begin{align*}
\hat{\pi}_y &= 0.2134, 
\hat{\pi}_n &= 0.4923, 
\hat{\pi}_r &= 0.0014 \\
\text{Test of } \hat{\pi}_d = \hat{\pi}_n &= 0.338 \\
F^{df} \text{ statistic} &= 884.95 \\
R^2_p &= 0.927, 
0.987, 
0.341 \\
\text{Elasticity} \\
\text{Own Price} &= -3.13, 
-1.02, 
-0.98, 
-0.91 \\
\text{Cross Price} &= 2.66, 
0.88, 
0.84, 
0.73 \\
\text{Income} &= 1.4, 
0.54, 
0.53, 
0.54 \\
\text{Population} &= 3.22, 
1, 
1, 
1 \\
\text{Notes: Model (i) is OLS estimates of Equation (11). Model (ii) is OLS estimates of Equation (12). Model (iii) is IV estimates of Equation (12), using } n_{it} \text{ as an instrument for } (h_{it} - n_{it}). \text{ Model (iv) is the same as (iii), except that } p_{it-i} \text{ is used to instrument } p_{it}. \text{ The variance-covariance matrix is heteroscedasticity robust, includes three lagged autocovariances, and uses a flat window. The } F^{df} \text{ statistic is an indicator of instrument relevance [32]. } R^2_p \text{ is Shea’s (1997) [36] (adjusted) partial } R^2 \text{ indicator of instrument relevance. Standard errors for the elasticity estimates are obtained using the delta method.}
\end{align*}

| Table 7. Inverse demand curve for houses in Outer Ring LGAs. |
|---------------------|-----|-----|-----|-----|-----|
|                     | OLS (i) | OLS (ii) | IV (iii) | IV (iv) | IV (v) |
| \( \hat{\lambda} \) | 0.5542 | 0.5693 | 0.5873 | 0.5848 | 0.8355 |
|                     | (0.05) | (0.046) | (0.05) | (0.046) | (0.077) |
| \( \hat{\pi}_d \) | 0.0237 | -0.0126 | -0.3211 | -0.2781 | -0.1596 |
|                     | (0.104) | (0.096) | (0.353) | (0.215) | (0.206) |
| \( \hat{\pi}_u \) | 0.4575 | 0.4515 | 0.4423 | 0.4436 | 0.1279 |
|                     | (0.041) | (0.042) | (0.041) | (0.082) | (0.082) |
| \( \hat{\pi}_y \) | 0.5809 | 0.6151 | 0.6363 | 0.6333 | 0.6122 |
|                     | (0.103) | (0.097) | (0.094) | (0.094) | (0.1) |
| \( \hat{\pi}_n \) | 0.0168 | 0.0126 | 0.3211 | 0.2781 | 0.1596 |
|                     | (0.096) | (0.353) | (0.215) | (0.206) | (0.206) |
| \( \hat{\pi}_r \) | 0.0075 | 0.0078 | 0.0088 | 0.0087 | 0.0039 |
|                     | (0.006) | (0.007) | (0.007) | (0.054) | (0.054) |
| Test of \( \hat{\pi}_d = \hat{\pi}_n \) & 0.238 |
| \( F^{df} \) statistic | 6.78 | 16.62 |
| \( R^2_p \) | 0.051 | 0.207 | 0.17 |
|                     | 0.439 |
| \( J \)-statistic | 0.871 | 0.225 |
| \text{Elasticity} |
| \text{Own Price} | -1.883 | -34.2 | -1.29 | -1.49 | -1.03 |
|                     | (81.864) | (1.181) | (262.851) | (1.47) | (1.563) |
| \text{Cross Price} | -19.32 | 35.85 | 1.38 | 1.6 | 0.8 |
|                     | (84.826) | (1.216) | (274.144) | (1.519) | (1.206) |
Income | -24.535 | 48.84 | 1.98 | 2.2 | 3.84  
|-------|--------|--------|------|-----|------|
|        | (107.273) | (375.862) | (2.145) | (1.745) | (4.864)  

Population | -0.71 | 1 | 1 | 1 | 1  

Notes: Model (i) is the OLS estimates of Equation (11). Model (ii) is the OLS estimates of Equation (12). Model (iii) is the IV estimates of Equation (12) using $n_{it}$ as an instrument for $(h_{it} - n_{it})$. Model (iv) is the same as (iii), but adds the square of $n_{it}$ as an additional instrument. Model (v) is the same as (iv), except that $p_{t1}^{w}$ is used to instrument $p_{it}^{w}$. The variance–covariance matrix is heteroscedasticity robust, includes three lagged autocovariances, and uses a flat window. The $F_{i}$ statistic is an indicator of instrument relevance [32]. $R^2_p$ is Shea’s (1997) [36] (adjusted) partial R-squared indicator of instrument relevance. The J statistic is the $p$-value for the Sargan–Hansen test of overidentifying restrictions [34,35]. Standard errors for the elasticity estimates are obtained using the delta-method.

5.2.1. Supply Curves

As with the Inner Ring, we can estimate the supply elasticity using the available instruments. In Table 5, we report results for the FE model (Results from the FD specification imply a similar supply elasticity for the Middle Ring (0.08) and a slightly lower estimate for the Outer Ring (0.58)). For both regions, the large $F_{i}$ statistics are indicative of strong instruments. For the Middle Ring, the J statistic does not suggest any misspecification, although this is not the case for the Outer Ring, where the $p$-values are about 1 percent (Dropping population as an instrument for the Outer Ring increases the $p$-value for the J statistic to about 0.07 and reduces the point estimate of the supply elasticity slightly to 0.66).

For the Middle Ring, the estimates imply a relatively inelastic supply curve for houses; however, the estimated elasticity (approximately 0.09) is highly significant, and suggests that conditioning on the stock of houses in estimating the demand curve is likely to be inappropriate. In the case of the Outer Ring, the estimated supply elasticity is larger (around 0.8), and a unit elastic supply curve for houses cannot be rejected.

5.2.2. Middle Ring Demand Curve

As an initial reference point, we estimate the unrestricted and restricted demand curves for the Middle Ring LGAs using OLS. The results are reported in columns (i) and (ii) in Table 6. It is notable that the estimates for the unrestricted model are affected by a similar problem as those in the Inner Ring. In column (i), the coefficient on the stock of houses ($-0.15$) has a relatively large standard error (0.37), and this causes the implied demand elasticities to have implausibly large standard errors. As with the Inner Ring, the restricted model solves the problem, and the imposition of the restriction is not rejected by the data ($p$-value = 0.34). Comparing column (ii) with (i), we see that the restricted model produces lower (absolute) elasticity estimates, and all are now statistically significant. The implied elasticities from the OLS estimates of the restricted model have the anticipated signs and are plausible in magnitude, but may suffer from endogeneity bias. To check for possible bias columns, (iii) and (iv) report IV estimates of the restricted model.

Column (iii) contains the IV results for the exactly identified model with houses per capita instrumented by population. Both indicators for instrument quality indicate high relevance. Comparing the OLS and the IV estimates of the restricted model does not indicate any large quantitative differences. This suggests that the supply curve for houses may be sufficiently close to vertical and that there is relatively little bias in OLS estimates of the demand curve.

Results that allow for the potential endogeneity of unit prices are reported in column (iv), and are very similar to the column (iii) estimates. To a first approximation, we could summarize the results in the following way. For Middle Ring houses, the own price elasticity is around $-1.0$, while the cross-price elasticity for units is about 0.85. Income elasticity is around 0.5, and suggests that houses in the Middle Ring fall into the necessity category.
5.2.3. Outer Ring Demand Curve

Table 7 reports the demand curve estimates for the Outer Ring LGAs. For the OLS estimates of the unrestricted and restricted models—see columns (i) and (ii), respectively—the coefficient on the house stock is economically close to zero and statistically insignificant. This suggests that the endogeneity of the stock of houses is a serious issue for this region and needs to be accounted for by the use of IV.

IV estimates of the restricted model are reported in column (iii), and we now obtain a sizable negative coefficient on the house stock (−0.32). However, the standard error is also large (0.35), and, while the IV coefficient estimates produce plausible elasticities, they also have large standard errors, and standard confidence intervals would include zero. The problem that apparently arises with the exactly identified IV estimation of the restricted model for the Outer Ring (but not for the Inner or Middle Rings) is one of weak instruments. Both the $F_{id}$ statistic and the partial $R^2$ imply that $n_\mu$ is a weak instrument for $(h_\mu - n_\mu)$ in the Outer Ring data.

One strategy is to examine why the population is such a weak instrument for houses per capita in the Outer Ring region. We can look at the simple bivariate correlation between these two variables. Figure 3 presents a scatter-plot of houses per capita against the population for both the Middle Ring and the Outer Ring. For the Middle Ring, we find a negative but essentially linear relationship, while in the Outer Ring, the relationship appears to be nonlinear (i.e., initially rising and then falling). Accounting for this nonlinear relationship may increase instrument quality. Based on the pattern of the scatter plot, we include the square of population ($n^2$) as an additional instrument in estimating the restricted model.

![Figure 3. Scatter-plot of houses per capita and population in the Middle and Outer Rings.](image)

Results from this model are reported under column (iv). Comparing the results under columns (iii) and (iv), we see that including the square of the population improves instrument quality, and the J statistic does not provide any evidence against the overidentifying restriction. Comparison of the elasticity estimates indicates broadly similar point estimates, and the standard errors—while still relatively large—are smaller for the overidentified model.

Finally, in column (v), lagged unit prices are used as an instrument for unit prices. Doing so gives smaller (absolute) own and cross-price elasticities and a larger income elasticity. The partial $R^2$ suggests that instrument quality is moderate, and based on the J statistic, the overidentifying restriction is not rejected.

The results in Table 7 suggest that demand elasticities are harder to identify and estimate precisely in the Outer Ring LGAs. Our best (point) estimates for the Outer Ring demand elasticities are about −1.0 to −1.5 for own price and about 0.8 to 1.6 for cross-price. The range for income elasticity is around 2 to 4.
5.3. Summary

Table 8 presents a summary of the elasticity estimates for each region of Sydney. The supply elasticity for houses is effectively inelastic in the Inner Ring LGAs, but approaches unit supply elasticity for LGAs in the Outer Ring. The demand for houses is most elastic in the Inner and Outer Rings. The estimated cross-price elasticities for units are of broadly similar magnitudes to the own price elasticities. The relatively high positive value on the price of units (cross-price elasticity) suggests that houses and units are quite substitutable across all regions of Sydney. Income elasticity is highest in the Inner and Outer Rings. A house in these two locations is closer to a luxury item than a necessity. In contrast in the Middle Ring, the demand for houses finds inelastic.

Table 8. Supply and demand elasticity estimates for houses in Sydney regions.

<table>
<thead>
<tr>
<th></th>
<th>Inner</th>
<th>Middle</th>
<th>Outer</th>
<th>Sydney</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Supply</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Own Price</td>
<td>0</td>
<td>0.1</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>Demand</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Own Price</td>
<td>-2.2</td>
<td>-1</td>
<td>-1.3</td>
<td>-1.3</td>
</tr>
<tr>
<td>Cross Price</td>
<td>1.6</td>
<td>0.8</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td>Income</td>
<td>1.5</td>
<td>0.5</td>
<td>3</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Notes: Demand elasticities are the average of columns (ii) to (iv) are for the Inner Ring, columns (iii) to (iv) are for the Middle Ring, and columns (iv) and (v) are for the Outer Ring. The results for metropolitan Sydney are obtained using a share-weighted sum of the three regions, where the weights are the share of houses in each region.

Estimates of supply and demand elasticities for metropolitan Sydney are constructed using the share of houses in each region to weight the relevant elasticity. At -1.3 and 2.1, respectively, our estimates of own price and income elasticity of demand are larger in (absolute) value than the economy-wide estimates of [18, 19]. This is not unreasonable, given that we focus on only a subset of the Australian (and Sydney) housing market.

5.4. Long-Run Effects

House prices are frequently modeled using a reduced-form approach with income, population, and user costs [4, 5, 12, 20, 37, 38]. Reduced-form models provide direct estimates of short and long-run multipliers, which—in principle—measure the causal effects on house prices of change in exogenous variables. Given our results, we can obtain estimates of the multipliers in two ways. One approach is to directly estimate the reduced-form models for house prices and quantities using OLS. Alternatively, we can use our structural estimates to calculate the implied multipliers.

In Table 9, we report the two estimates of the long-run multipliers. The OLS estimates of the multipliers are obtained from a static (or long-run) model, while the multipliers for the structural model are derived from the elasticity estimates reported in Table 8. For house prices, the reduced-form estimates are generally consistent with those obtained from the structural model. Population, real income, and unit prices are all estimated to be economically important influences on house prices in the long run.

Table 9. Long-run multipliers: reduced-form and structural estimates.

<table>
<thead>
<tr>
<th>Exogenous Variable</th>
<th>Inner Ring</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reduced-Form</td>
<td>Structural</td>
<td></td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>h</td>
<td>p</td>
</tr>
<tr>
<td>Population</td>
<td>0.29 (5.3)</td>
<td>0.12 (6.1)</td>
<td>0.45</td>
</tr>
<tr>
<td>Income</td>
<td>0.48 (8.3)</td>
<td>0.02 (-0.8)</td>
<td>0.68</td>
</tr>
<tr>
<td>Unit Price</td>
<td>0.62 (10.8)</td>
<td>0.02 (0.0)</td>
<td>0.73</td>
</tr>
</tbody>
</table>
For the stock of houses, there are a couple of notable differences between the two sets of estimates.

In the Inner Ring, we find a small but statistically significant effect of population on the house stock in the estimated reduced-form equation. This result points to a direct role of the population in the supply curve. One way this could arise is if the effective supply of new houses is constrained by the availability of planning permits and planners use population forecasts to decide on the number of new houses allowed in a region.

In the Outer Ring results, the reduced-form estimates point to a large effect from population and zero effect from unit prices on house stocks. The estimates indicate a big range for the implied supply elasticity for houses in this region.

5.5. Tax Incidence

In motivating this analysis, we firstly quote the policy debate from Peter Mares on the Sydney Morning Herald (Monday 19 September 2018), “Why homeowners should pay the rent”: “The first step is to abolish stamp duty and replace it with a broad-based property tax as the ACT is doing. Why? First, stamp duty is levied at the point of purchase, when buyers can least afford it, whereas a broad-based property tax spreads the costs over time. The major beneficiaries of a transition from stamp duty to property taxes would be younger Australians trying to buy their first home”.

As the quotation indicates, there is a debate over whether the lump-sum stamp duty tax payment upon the purchase should be removed and replaced by an annual property tax payment. To provide an argument for this debate, we may try to figure out how much the buyers bear the stamp duty eventually before we can answer how effective the policy can be.

The tax incidence depicts the distribution of tax obligations between the buyer and the seller upon purchase. As a result, there is an important use for own price elasticities of supply and demand in estimating the economic incidence of a tax (or subsidy). We can use the elasticities in Table 8 and the standard formula for the incidence of a tax to examine the likely incidence of taxes on houses in regions in Sydney and to find out who ultimately bears the burden of tax. We can write the housing sellers’ tax incidence formula as below:

\[ T_{inc} = \frac{\hat{\pi}_d}{\hat{\pi}_s + \hat{\pi}_d} \]  

where \( \hat{\pi}_d \) and \( \hat{\pi}_s \) are the own price elasticities of the supply and the demand of houses in Table 8.
Equation (13) shows that for LGAs in the Inner Ring, our results are clear-cut. Since the supply of houses is inelastic, and therefore despite removing the stamp duty, house prices will not fall and will remain constant, implying that the tax obligation all falls onto the seller, because if the stamp duty tax is removed, the seller will be paid the full price, while with the stamp duty tax, the payment to the seller will be the full price minus the tax. To understand this, we will use an example. In the case of a house that sells for AUD 5 m and where the stamp duty is 5%, the buyer will pay AUD 5.25 m (AUD 5 m + AUD 0.25 m). But in the absence of the stamp duty, the buyer would still have been willing to pay a total price of AUD 5.25 m because the supply elasticity in the area is perfectly inelastic, and in that case, the full price of AUD 5.25 m would have been paid to the seller. In one word, with zero stamp duty, the seller gets AUD 5.25 m, but with 5% stamp duty despite the price of AUD 5.25 m, he receives only AUD 5 m, which means the seller bears the 100% of the stamp duty.

In the Middle Ring, the tax incidence implies that about 90 percent of the stamp duty is paid by the house seller, and removing the stamp duty will drive the house prices to decrease by 10 percent of the stamp duty value. This is because when the stamp duty is removed, the house price is supposed to fall by the stamp duty value. However, because the demand is elastic, the lower price will attract more demand, yet because the supply elasticity is quite low, the supply can only satisfy a small portion of the demand, and buyers will still need to pay a price that is only slightly less than the full original price (equal to about 10 percent of the stamp duty value).

In the Outer Ring, this figure falls to about 60 percent, implying that removing the stamp duty will drive the house prices to drop by 40 percent of the stamp duty value. Because the supply of houses in the Outer Ring is reasonably elastic, in the case of having the stamp duty, a house buyer will end up paying about 40 percent of the value of any stamp duty. The finding suggests that removing stamp duty will provide some partial benefit to first-home buyers in the Outer Ring in the form of lower prices (equal to 40% of the stamp duty value.)

We then can come back to discuss the policy debate mentioned at the start of this section. We can find the policy decisions on the amendment of the stamp duty tax on 1 July 2017. The NSW State Government eliminated stamp duty for first home buyers on residential properties with a sale price of up to AUD 650,000, and for properties with a sale price between AUD 650,000 and AUD 800,000, a concessional rate of stamp duty is payable. It is known that houses with the value at this range are located in the Outer Ring of Sydney, so the stamp duty concessions can effectively provide benefits to first-home buyers for houses in the Outer Ring, consistent with our empirical findings about the tax distribution between buyers and sellers (40 and 60 percent, respectively) for the Outer Ring home buyers. This analysis verifies that the study of the tax incidence of sellers using our own-price elasticities of supply and demand estimates can provide reasonable predictions on the distribution of the stamp duty tax between the sellers and buyers in the Sydney housing market.

6. Conclusions

This paper reports estimates of conventional demand elasticities for houses using LGA-level data for metropolitan Sydney. The LGAs are grouped into three geographic regions, being the Inner, Middle, and Outer Rings. For the Iner Ring, we establish that the supply curve for houses is perfectly inelastic, and this allows us to condition on the stock of houses in estimating the demand curve. Our estimates indicate that the demand for houses in LGAs nearest the CBD is elastic with respect to house prices, the price of units (a substitute), and real income. In contrast, the demand for houses in the Inner Ring is inelastic with respect to the population level in its LGAs.

For Middle Ring LGAs, the supply of houses—which not perfectly inelastic—is highly inelastic. Our demand curve estimates indicate that own and cross-price elasticities are about half the size of those for the Inner Ring and are a little below unit value. Income
elasticity for the Middle Ring is about 0.5, and implies that houses in this region are treated as necessities rather than luxuries. The largest demand elasticity is for population, which is around 1.0 and is about double the size of the Inner Ring.

In the Outer Ring, the elasticity of supply for houses is found to be a little below unity. As a consequence, OLS produces highly implausible estimates of the demand curve for LGAs in this region. While the use of IV is warranted, neither of our two identifying restrictions is without problems, and there is greater uncertainty about the Outer Ring estimates than for the other two regions. Our “best” estimates indicate elastic own price, cross-price, and income elasticities. The magnitudes are broadly like those for the Inner Ring. Population elasticity is less than one, and lies about halfway between those of the Inner and Middle Rings.

Using the elasticity estimates, we examine the likely incidence of taxes on houses, which explains the distribution of stamp duty between sellers and buyers. The results indicate that, in the Inner Ring, house sellers bear 100 percent of the stamp duty payment, in the Middle Ring, house sellers bear 90 percent of the stamp duty payment, and in the Outer Ring, house sellers bear 60 percent of the stamp duty payment. If the stamp duty is removed, the purchase costs of buyers in the Inner Ring will not be affected, in the Middle Ring, it will be reduced by 10 percent of the stamp duty payment, and in the Outer Ring, it will be reduced by 40 percent of the stamp duty value. This finding explains the benefits of the current NSW stamp duty tax concession policy, which can help reduce the first-home buyers’ purchase cost on houses in the Outer Ring.

Finally, as a reference for future study, we briefly discuss that the estimates of demand and supply elasticities in the study could be biased because there is a lack of important exogenous variables that can be used to resolve the potential endogeneity problem between housing prices and supply in the housing demand function. In particular, when we estimate the housing demand elasticity, we have to restrict the elasticity of demand for houses with respect to the population equal to unity because we lack important exogenous supply-side shifters to relieve the endogeneity problem in the housing demand function. Despite the fact that we provide plausible solutions to deal with this issue, we recommend that future studies improve this by finding data for appropriate supply-side exogenous variables.

**Author Contributions:** Conceptualization, G.O. and X.L.; methodology, G.O.; software, X.L.; formal analysis, X.L.; investigation, G.O.; resources, X.L.; data curation, X.L.; writing—original draft preparation, G.O. and X.L.; writing—review and editing, G.O. and X.L.; supervision, G.O.; project administration, X.L. All authors have read and agreed to the published version of the manuscript.

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**Data Availability Statement:** The data is stored in my personal computer. The data is available on request.

**Conflicts of Interest:** The authors declare no conflict of interest.

**Appendix A**

**Appendix A.1. House Prices**

Median sales prices for houses (i.e., non-strata residences) and units/apartments (i.e., strata residences) in LGAs are sourced from the quarterly *Sales Reports* published by FACS NSW. A few LGAs have missing observations due to a small number of sales in some quarters. In these cases, we use the surrounding quarterly observations for the LGA to interpolate the missing observations. Occasionally when there are missing observations for several sequential quarters, we use the data for a neighboring LGA to interpolate the missing sequence. The annual observations used in the analysis represent financial years, and are calculated using a simple average of relevant quarterly sales prices. For example, the median price for 1992 is the average of September 1991 to June 1992 quarter prices.

Appendix A.2. House Stocks

The quantity (i.e., number) of private houses in an LGA is the sum of separate houses and semi-detached, row, or terrace houses or townhouses obtained from Census data for the years 1991, 1996, 2001, 2006, 2011, and 2016. Estimates of house stocks for inter-censal years are obtained using interpolation using annual data for building approvals for non-strata residences.


Appendix A.3. Real Income

We construct a series for per-capita real income in an LGA using data from two sources. For the period 1990–1991 to 2005–2006, we use a measure of average real income per taxpayer (in 2007–2008 prices) for an LGA calculated by the Bureau of Infrastructure, Transport, and Regional Economics (BITRE). The original figures for nominal taxable income are deflated using the CPI for Australia.

For the period 2005–2006 to 2014–2015, we use ABS data in Estimates of Personal Income for Small Areas. The data for LGAs is converted to constant 2007–2008 prices using the CPI for Australia.

The BITRE and ABS measures of average real income for an LGA are not directly comparable. Thus, we use the growth rates implied by the two measures to link the two series. Specifically, the estimate of mean total income in an LGA for 2015–2016 (in 2007–2008 prices) is used as a reference value, and this is adjusted backwards to 1990–1991 using growth rates.


Appendix A.4. Population

Estimates of annual resident population (end-June) for an LGA are available from the ABS for the period 1990–1991 to 2015–2016.


Appendix A.5. Real Interest Rate

Data on the real interest rate is obtained from the RBA Capital Market Yields (F2). It is the yield on the Australian Government inflation-indexed bond with the longest maturity.

Appendix A.6. Consumer Price Index

The consumer price index for Sydney is obtained from ABS release 6401.0—Consumer Price Index, Australia.


Appendix A.7. Development Approvals

The median gross time for a Local Council to decide on a DA are obtained from two sources. Data from 1994–1995 to 2011–2012 are reported in Comparative Information on NSW Local Government. Data from 2012–2013 to 2015–2016 are from the NSW Department of Planning and Environment Local Development Performance Monitoring.


Appendix A.8. Dummy Variables (D)

$D_t$ is a vector of three dummy variables. $D_{0216}$ is a dummy equal to 0 (1991–2001) and 1 (2002–2016), $D_{0316}$ is a dummy equal to 0 (1991–2002) and 1 (2003–2016), and $D_{0416}$ is a dummy equal to 0 (1991–2003) and 1 (2004–2016).

Table A1. List of LGAs by Location.

<table>
<thead>
<tr>
<th>Inner Ring</th>
<th>Middle Ring</th>
<th>Outer Ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Botany Bay</td>
<td>Burwood</td>
<td>Blacktown</td>
</tr>
<tr>
<td>Inner West</td>
<td>Canada Bay</td>
<td>Blue Mountains</td>
</tr>
<tr>
<td>Lane Cove</td>
<td>Canterbury-Bankstown</td>
<td>Camden</td>
</tr>
<tr>
<td>Mosman</td>
<td>Hunters Hill</td>
<td>Campbelltown</td>
</tr>
<tr>
<td>North Sydney</td>
<td>Georges River</td>
<td>Cumberland</td>
</tr>
<tr>
<td>Randwick</td>
<td>Ku-ring-gai</td>
<td>Central Coast</td>
</tr>
<tr>
<td>Sydney</td>
<td>Northern Beaches</td>
<td>Fairfield</td>
</tr>
<tr>
<td>Waverley</td>
<td>Parramatta</td>
<td>Hawkesbury</td>
</tr>
<tr>
<td>Woollahra</td>
<td>Rockdale</td>
<td>Hornsby</td>
</tr>
<tr>
<td></td>
<td>Ryde</td>
<td>Liverpool</td>
</tr>
<tr>
<td></td>
<td>Strathfield</td>
<td>Penrith</td>
</tr>
<tr>
<td></td>
<td>Willoughby</td>
<td>Sutherland</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The Hills</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Wollondilly</td>
</tr>
</tbody>
</table>

Notes: LGAs reflect June 2016 boundaries except for Parramatta, The Hills, Hornsby, and Cumberland, which use 2011 boundaries. For these LGAs, it was not possible to construct consistent series for population and the housing stock based on the 2016 boundaries. For these LGAs, the growth rates for these series based on the 2016 boundaries are used to extrapolate the levels based on the 2011 boundaries.

References


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