

Article

Time-Dependent Reliability Analysis of Anhydrite Rock Tunnels under Swelling Conditions: A Study on Stress, Deformation, and Engineering Solutions

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Abstract: This study presents an analytical approach for evaluating the reliability of anhydrite rock tunnels, focusing on their characteristic swelling behavior. Anhydrite rocks, prone to significant expansion upon moisture exposure, pose a challenge in tunnel construction, potentially leading to structural issues such as floor heave and lining damage. To address this, this research develops an elastic swelling analytical solution based on humidity stress field theory, enabling the assessment of time-dependent stress and deformation changes in anhydrite tunnels. The solution's applicability is demonstrated through its application to the Lirang tunnel. The investigation into the effects of support pressure, swelling time, and reserved deformation on tunnel reliability reveals that circumferential stress at the tunnel wall increases by 13.94% and 21.86% for swelling periods of 30 and 365 days, respectively. Similarly, radial displacement escalates by 22.97% and 35.93% over these periods, highlighting the significant impact of swelling behavior. Using a spreadsheet-based First Order Reliability Method (FORM) for analysis, this study finds that the original design of the Lirang tunnel did not meet the desired reliability standards under swelling conditions. However, strategic adjustments in construction variables, such as increasing support pressure to 1.2 MPa or enhancing reserved deformation to 59 mm, elevated the tunnel's reliability to meet safety requirements. This research provides a vital framework for assessing and enhancing the reliability of anhydrite rock tunnels, considering the long-term effects of swelling. It underscores the importance of incorporating swelling behavior in the design and construction of tunnels in anhydrite rock formations, offering valuable insights for optimizing tunnel stability in such challenging geological conditions.



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Keywords: anhydrite rock; swelling behavior; elastic swelling analytical solution; tunnel reliability

1. Introduction

Anhydrite rock, composed mainly of CaSO_4 , possesses unique engineering properties, particularly its significant expansion behavior upon exposure to water. This expansion tendency profoundly impacts the stability of tunnel openings and slope engineering. Upon contact with water, anhydrite rock undergoes hydration reactions, leading to a theoretical maximum volume increase of approximately 61%. This expansion poses a considerable challenge to slope and tunnel stability, often resulting in instability in large slopes and damage to tunnel tops and bottoms. The uneven distribution of groundwater exacerbates this challenge, potentially causing localized or widespread instability in slopes and tunnels due to uneven expansion.

In tunnels traversing anhydrite rock formations, swelling behavior can lead to significant structural challenges, including tunnel floor heave, lining destruction, and even the

uplifting of entire tunnel sections and the land surface [1–3]. This swelling is primarily driven by the transformation of anhydrite (CaSO_4) into gypsum ($\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$), a process involving indirect anhydrite dissolution and gypsum precipitation, leading to a substantial increase in the volume of sulfate minerals [4,5]. The swelling pressure, generated when volumetric swelling strain is constrained, can reach up to 60 MPa at the crystal level, 10 MPa in rock specimens, and 5 MPa at the tunnel scale [6]. Although in situ swelling pressures are generally lower than laboratory measurements, they can still challenge or exceed tunnel support pressures, significantly affecting stress and displacement distribution within the tunnel. Ignoring the swelling behavior of anhydrite rock in tunnel design can result in frequent structural damage [7,8].

After excavation, the surrounding rock deforms due to hydrostatic far-field stress and swelling behavior. To leverage the self-supporting effect of the surrounding rock, initial support structures are designed to allow a certain degree of deformation, with excavation lines planned to accommodate this deformation. Exceeding the reserved deformation value can lead to intrusion phenomena and lining structure failure. Tunnel reliability is assessed by comparing the radial displacement at the tunnel wall against the reserved deformation value using the convergence constraint method.

Recent research has developed performance functions based on tunnel analytical solutions to analyze tunnel reliability [9–13]. These solutions, based on criteria such as Hoek–Brown or Mohr–Coulomb, include elastic–plastic, elastic–brittle–plastic, and elastic strain softening models [14–17]. However, these models do not account for the impact of swelling behavior on stress and displacement. Miao et al. [18,19] introduced the humidity stress field theory for clay-sulfate rocks, providing a method to analyze stress and displacement in rocks exposed to water inflow. This theory equates the effects of humidity changes to those of temperature changes in materials, suggesting that stress fields generated by humidity changes can be analyzed in a manner similar to temperature-induced stress fields. This approach, typically employed for rocks with high water-absorbing minerals such as mudstone and shale, is expanded upon in this study to include anhydrite rocks. Anhydrite rocks exhibit swelling behavior driven by sulfate hydration over prolonged periods. The resulting analytical solution captures the ongoing influence of swelling behavior on stress and displacement distribution within anhydrite tunnels. Consequently, this paper addresses two crucial objectives: developing an elastic swelling analytical solution tailored for anhydrite rock tunnels while considering swelling behavior and assessing the reliability of such tunnels.

2. Analytical Solution for Elastic Swelling in Anhydrite Rock Tunnels

Miao et al. and Wu et al. [18–20] extensively derived the mathematical model of humidity stress field theory in the analysis of expansive rock engineering. Cheng et al. [21,22] conducted experimental research on the above theory. Chen and Wu et al. [19,21,23] further validated this theory through engineering practice. The theory of humidity stress field has been proven to be an effective theoretical method for studying the stress–strain relationship of the expansive surrounding rock. Therefore, this study draws on this theory to establish an elastic analytical model for hard gypsum rock tunnels. The validation process of the adopted theory is as follows:

Figure 1 depicts a circular tunnel excavation within a uniform, homogeneous, and isotropic anhydrite rock mass. This mass, with a radius denoted as R_0 , is under hydrostatic far-field stress (P_0) and uniform support pressure (P_s). The prolonged swelling behavior of the anhydrite rock continually modifies the stress and deformation patterns of the surrounding rock over time.

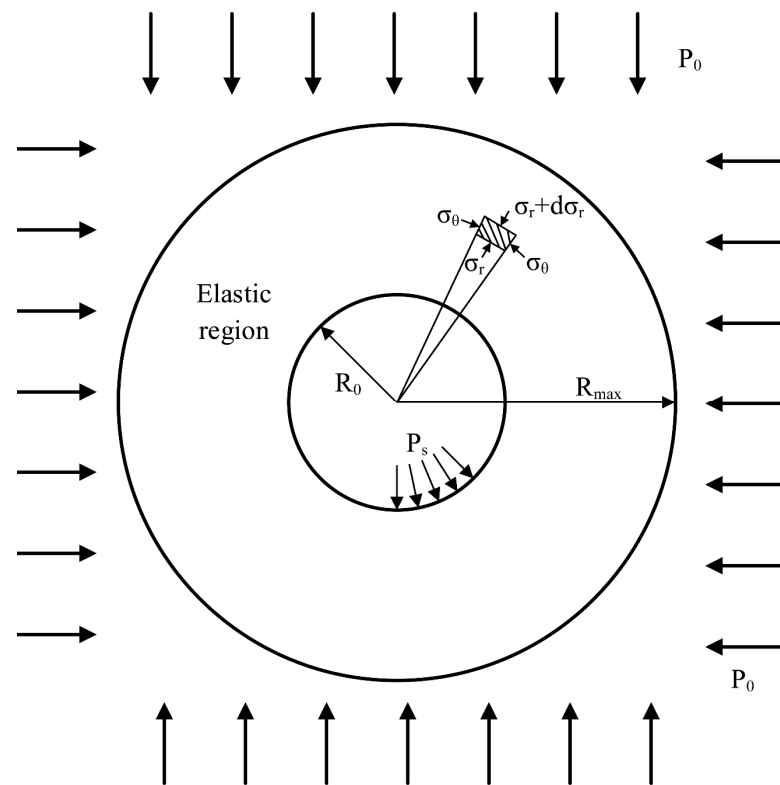


Figure 1. A circular anhydrite rock tunnel in a hydrostatic in situ stress field.

It is essential to recognize that the stress equilibrium equation remains unaffected by the root causes of stress, and likewise, the relationship between displacement and strain remains independent of causation. Therefore, in this context, both the equilibrium differential equation and the geometric equation align with those of a standard elastic problem encountered in circular axisymmetric tunnel analysis. These equations serve as the cornerstone for comprehending and forecasting the behavior of the tunnel structure under the provided conditions. Specifically, as shown in the following formula:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (1)$$

$$\begin{cases} \varepsilon_\theta - \frac{u_r}{r} = 0 \\ \varepsilon_r - \frac{du_r}{dr} = 0 \end{cases} \quad (2)$$

where σ_r and σ_θ are the radial stress and circumferential stress, respectively, ε_r and ε_θ are the radial strain and annular strain, respectively, and u_r is the radial displacement.

This research integrates the humidity stress field theory to tackle the swelling behavior observed in anhydrite rock. This theory suggests a quasi-coupled relationship between the humidity and stress fields, following Hooke's law for the stress–strain relationship. In this analysis, it is assumed that the mechanical properties of the surrounding rock, notably the elastic modulus and Poisson's ratio, remain constant within a specified range of moisture content. While this assumption is essential for deriving analytical solutions, it is crucial to acknowledge the nonlinear effects of these factors, necessitating thorough consideration in more detailed numerical assessments.

Within the scope of this study, complexities such as the seepage movement of groundwater and its resulting influence on swelling pressure are not considered. This simplification is grounded in the assumption that anhydrite rock can completely absorb water if the initial humidity level is below the saturation threshold. Additionally, it is hypothesized that the

initial humidity distribution within the surrounding rock follows a linear increase relative to the radial distance from the tunnel center.

$$W = w_0 r / R_0 \quad (3)$$

where w_0 is the initial humidity when $r = R_0$.

In the context of plane stress, the strain component in polar coordinates is as described by Miao et al. and Wu et al. [18,19]:

$$\begin{cases} \varepsilon_r = \frac{1}{E}(\sigma_r - \nu\sigma_\theta) + \alpha\Delta W \\ \varepsilon_\theta = \frac{1}{E}(\sigma_\theta - \nu\sigma_r) + \alpha\Delta W \\ \gamma_{r\theta} = \frac{2(1+\nu)}{E}\tau_{r\theta} \end{cases} \quad (4)$$

where E is the elastic modulus, ν is Poisson's ratio, α is the linear swelling coefficient, ΔW is the change in humidity, $\gamma_{r\theta}$ is the shear strain, and $\tau_{r\theta}$ is the shear stress.

The equilibrium differential equation is expressed as follows:

$$\begin{cases} \frac{\partial\sigma_r}{\partial r} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{\sigma_r - \sigma_\theta}{r} - \frac{E\alpha}{1-\nu}\frac{\partial\Delta W}{\partial r} = 0 \\ \frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta} + \frac{\partial\tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} - \frac{E\alpha}{1-\nu}\frac{1}{r}\frac{\partial\Delta W}{\partial\theta} = 0 \end{cases} \quad (5)$$

To address the plane strain problem within this context, a transformation of the material parameters in the constitutive equation is necessary. Specifically, the elastic modulus (E) is modified to $E/(1-\nu^2)$, where ν is Poisson's ratio, which in turn is adjusted to $\nu/(1-\nu)$. Additionally, the linear swelling coefficient (α) is recalibrated to $(1+\nu)\alpha$. This adjustment is crucial as the tunnel traverses through the anhydrite formation, which is closely intertwined with adjacent geological strata, leading to an axial strain that effectively approaches zero. Therefore, the constitutive equation, when expressed in polar coordinates for the plane strain scenario, is modified to account for these variations, thereby ensuring a more precise portrayal of the mechanical behavior of the anhydrite rock under tunneling conditions.

$$\begin{cases} \sigma_r = \frac{(1-\nu)^2 E}{(1-2\nu)(1-\nu^2)}(\varepsilon_r + \frac{\nu}{1-\nu}\varepsilon_\theta) - \frac{E\alpha\Delta W}{1-2\nu} \\ \sigma_\theta = \frac{(1-\nu)^2 E}{(1-2\nu)(1-\nu^2)}(\varepsilon_\theta + \frac{\nu}{1-\nu}\varepsilon_r) - \frac{E\alpha\Delta W}{1-2\nu} \end{cases} \quad (6)$$

In employing the humidity stress field theory to simulate the swelling behavior of anhydrite rock, a prevalent assumption is that the deformation modulus during swelling aligns with the elastic modulus. However, experimental observations on anhydrite rock swelling reveal a significant discrepancy; the deformation modulus in the swelling stage is notably lower than the elastic modulus. This discrepancy is illustrated in Figure 2, where equating the deformation modulus with the elastic modulus results in an overestimation of stress levels.

In this study, the swelling behavior is likened to the effects of pore water pressure, effectively reducing the effective stress. Here, effective stress is conceptualized as the difference between the elastic stress and the swelling-induced stress. To accurately characterize the stress–strain relationship during swelling, a distinct swelling modulus, denoted as E_s , is introduced. Simultaneously, the elastic modulus is specifically denoted as E_e (instead of the generic E) to differentiate it from the swelling modulus. Consequently, the constitutive equation, when formulated in polar coordinates, incorporates these adjustments to offer a more precise representation of the stress–strain dynamics during the swelling process of anhydrite rocks.

$$\begin{cases} \sigma_r = \frac{(1-\nu)^2 E_e}{(1-2\nu)(1-\nu^2)}(\varepsilon_r + \frac{\nu}{1-\nu}\varepsilon_\theta) - \frac{E_s\alpha\Delta W}{1-2\nu} \\ \sigma_\theta = \frac{(1-\nu)^2 E_e}{(1-2\nu)(1-\nu^2)}(\varepsilon_\theta + \frac{\nu}{1-\nu}\varepsilon_r) - \frac{E_s\alpha\Delta W}{1-2\nu} \end{cases} \quad (7)$$

Consistent with the tenets of the humidity stress field theory, a dedicated swelling experiment was conducted on anhydrite rock. This experiment played a crucial role in establishing a quantitative relationship between water absorption and time, a fundamental aspect in comprehending the swelling dynamics of anhydrite rock. As outlined in the study by Chen et al. [21], this relationship is essential for understanding the temporal aspects of anhydrite rock's response to moisture exposure. The experiment's findings establish a foundational understanding of how anhydrite rock's water absorption rate evolves over time, providing essential insights into the swelling behavior under different humidity conditions. This relationship is integral to the development of more accurate models for predicting the swelling-induced changes in anhydrite rock formations.

$$A_t = A_{\max}(1 - e^{-at}) \quad (8)$$

where A_t is the water absorption rate at time t , $A_{\max}$ is the water absorption rate at the end of swelling, and a is the water absorption coefficient, which is used to describe the speed of water absorption.

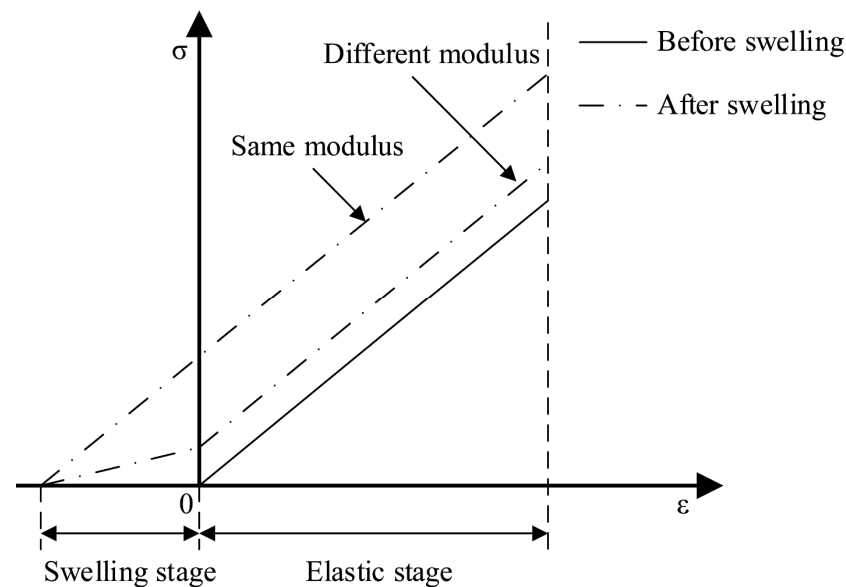


Figure 2. The elastic swelling constitutive model.

The constitutive equation, essential for analyzing the swelling behavior of anhydrite rock, incorporates the established relationship between water absorption and time. This incorporation is crucial for understanding the dynamics of swelling, especially in scenarios where the water content within a unit of surrounding rock is insufficient for the complete hydration of CaSO_4 in that unit. In such scenarios, the water is fully absorbed by the rock, resulting in distinct swelling characteristics. The resulting constitutive equation encapsulates the essence of elastic swelling evolution, integrating the temporal aspect. This equation stands as a fundamental tool in forecasting and comprehending the swelling behavior of anhydrite rock over time, offering a more nuanced and comprehensive perspective on the rock's response to fluctuating water content. It is expressed as follows:

$$\begin{cases} \sigma_r = \frac{(1-\nu)^2 E_e}{(1-2\nu)(1-\nu^2)} (\epsilon_r + \frac{\nu}{1-\nu} \epsilon_\theta) - \frac{E_s a W (1-e^{-at})}{1-2\nu} \\ \sigma_\theta = \frac{(1-\nu)^2 E_e}{(1-2\nu)(1-\nu^2)} (\epsilon_\theta + \frac{\nu}{1-\nu} \epsilon_r) - \frac{E_s a W (1-e^{-at})}{1-2\nu} \end{cases} \quad (9)$$

In tunnel engineering, the radial stress encountered by the tunnel wall after excavation corresponds to the applied support pressure, a critical factor in comprehending tunnel stability mechanics. The outer boundary of the elastic swelling rock mass, pivotal in

this context, is influenced by its neighboring rock mass and the prevailing ground stress conditions. This study explores a scenario where radial stress comprises both swelling stress and hydrostatic far-field stress, a situation commonly encountered in practical engineering. The boundary condition, crucial in this analysis, is mathematically represented by an equation that succinctly captures the relationship between these stress components. This equation plays a key role in modeling the stress dynamics around the tunnel, offering insights essential for effective tunnel design and stability assessment. The boundary condition equation can be expressed as:

$$\begin{cases} \sigma_r(r = R_0) = P_s \\ \sigma_\theta(r = R_{\max}) = P_0 + E_s \varepsilon_s(r = R_{\max}) \end{cases} \quad (10)$$

Drawing upon the principles of elastic mechanics, this study integrates an equilibrium differential equation, a geometric equation, and an elastic swelling evolution constitutive equation. This comprehensive approach facilitates the accurate determination of both radial and circumferential stresses in the surrounding rock. Such calculations are indispensable for comprehending stress dynamics in the context of tunnel stability and design.

$$\sigma_r = -\frac{E_e \alpha}{(1 - \nu^2)r^2} \left(\int_{R_0}^r \frac{E_s}{E_e} W(1 - e^{-at}) r dr + C_1 \right) + C_2 \quad (11)$$

$$\sigma_\theta = \frac{E_e \alpha}{(1 - \nu^2)r^2} \left(\int_{R_0}^r \frac{E_s}{E_e} W(1 - e^{-at}) r dr + C_1 - \frac{E_s}{E_e} W(1 - e^{-at}) r^2 \right) + C_2 \quad (12)$$

$$C_1 = \frac{R_{\max}^2 R_0^2 (1 - \nu^2) [\sigma_\theta(r = R_{\max}) - P_s - \frac{E_e \alpha}{(1 - \nu^2) R_{\max}^2} (\int_{R_0}^{R_{\max}} \frac{E_s}{E_e} W(1 - e^{-at}) r dr - \frac{E_s}{E_e} W(1 - e^{-at}) R_{\max}^2)]}{E_e \alpha (R_0^2 + R_{\max}^2)} \quad (13)$$

$$C_2 = \frac{E_e \alpha}{(1 - \nu^2) R_0^2} C_1 + P_s \quad (14)$$

where C_1 and C_2 are general solution parameters, which can be determined by boundary conditions.

Utilizing Equations (9), (11) and (12), the circumferential strain can be calculated, and the radial displacement of the surrounding rock can be derived using the geometric equation. Post-excavation displacement occurs due to stress incrementation. By employing this increment, particularly the radial and annular stresses minus the hydrostatic far-field stress, the post-excavation radial displacement can be articulated as follows:

$$u_r = \frac{1 - \nu^2}{E_e} \left[(\sigma_\theta - P_0) - \frac{\nu}{1 - \nu} (\sigma_r - P_0) \right] r + \alpha (1 + \nu) \frac{E_s}{E_e} W(1 - e^{-at}) r \quad (15)$$

3. Application

The Lirang tunnel, located in Chongqing, China, traverses an anhydrite rock formation and spans approximately 300 m (refer to Figure 3). This formation is primarily composed of 94% CaSO_4 and 6% other minerals. Above the anhydrite layer lies a limestone stratum averaging 120 m in thickness. The area encompasses two significant reservoirs and multiple streams. The surrounding rock is classified as Grade V, characterized by its relatively fragmented nature. The main lining of the tunnel, designed to accommodate building boundary and auxiliary structure requirements, adopts a triple-arched sidewall configuration with an arch height of 7.05 m and a radius of 5.45 m (as depicted in Figure 4).

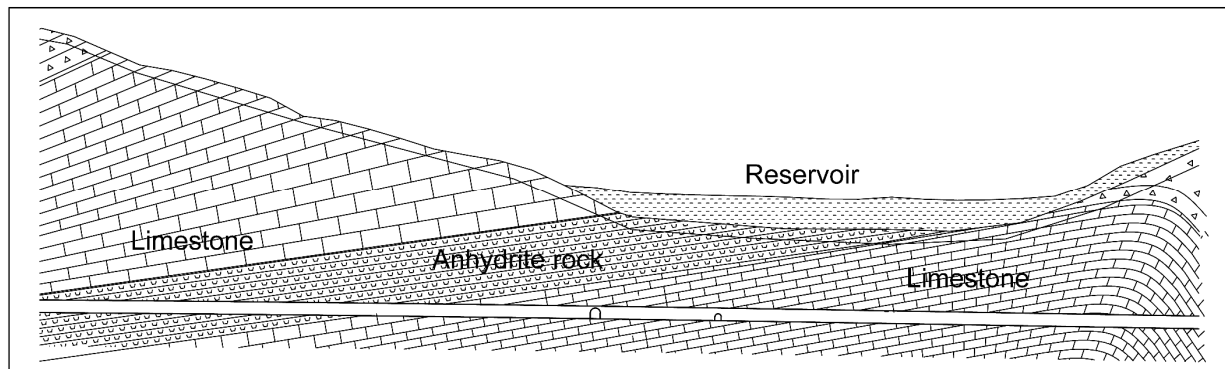


Figure 3. Geological section of the Lirang tunnel.

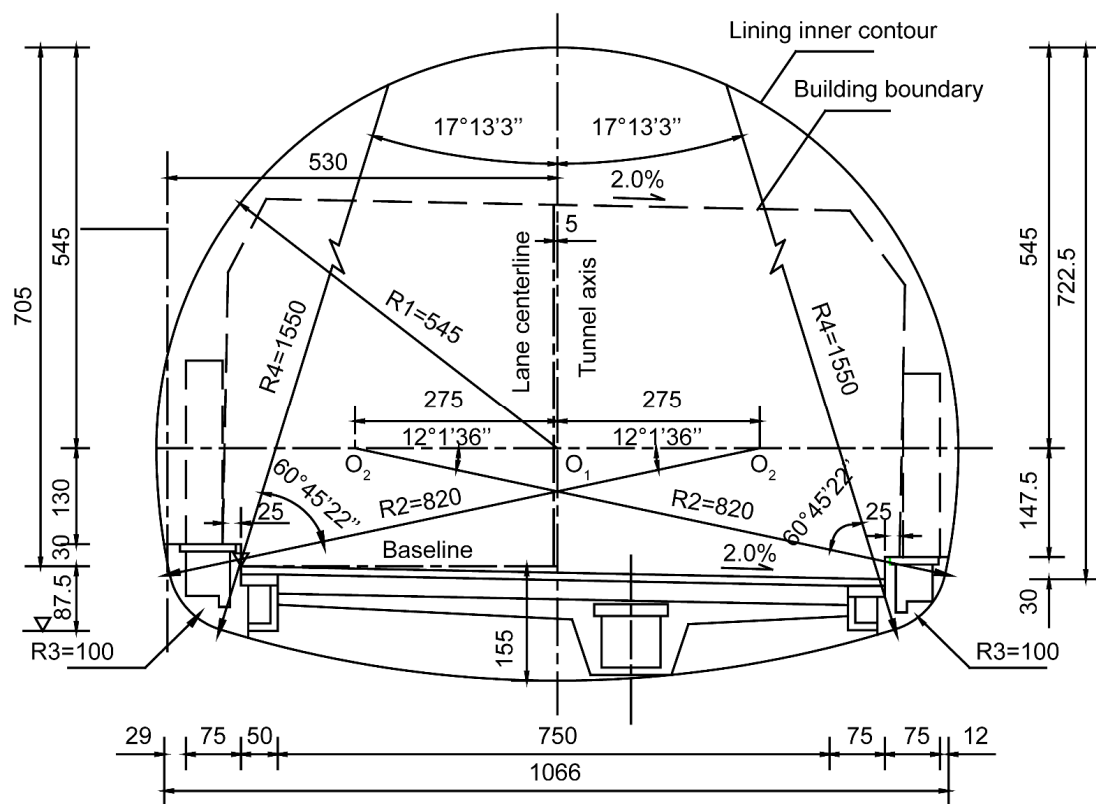


Figure 4. Outline design of the Lirang tunnel (length unit: cm).

To validate the analytical solution and evaluate the impact of anhydrite rock's swelling behavior on stress and displacement distribution, parameters were selected based on laboratory experiments and specific tunnel design criteria. The elastic modulus and Poisson's ratio were determined through uniaxial testing of the anhydrite rock. The initial humidity at $r = R_0$ represents the average natural moisture content of the anhydrite rock and was from the natural water content test. Swelling modulus, linear swelling coefficient, and water absorption coefficient were derived from swelling experiments on anhydrite rock [21,24]. The tunnel's radius, the maximum thickness of the anhydrite formation, and the hydrostatic far-field stress were determined from the tunnel design. Considering the grade and integrity of the surrounding rock, a reduction coefficient of 0.15 was applied between the rock mass's elastic modulus and the experimentally measured rock elastic modulus [25]. These parameters are detailed in Table 1.

Table 1. The values of parameters in the analytical solution.

Parameters	E_e/MPa	ν	$w_0/(\%)$	E_s/MPa	α	a	R_0/m	$R_{\max}/\text{m}$	P_0/MPa
Reference value	735.45	0.311	1.5	94.383	0.001	0.034	5.45	30	3

Figure 5 depicts the stress and displacement distribution under unsupported conditions. At zero swelling time, the radial stress at the tunnel wall measures 0 MPa, while the circumferential stress peaks at 5.81 MPa due to stress concentration. Subsequently, the radial stress significantly rises, gradually converging with the hydrostatic far-field stress. In contrast, the circumferential stress diminishes as the radius increases, eventually approaching the hydrostatic far-field stress. Comparable patterns are observed under varying swelling time conditions. At swelling times of 30 and 365 days, the radial stress at the tunnel wall remains at 0 MPa, while circumferential stresses measure 6.62 MPa and 7.08 MPa, respectively. This pattern suggests that swelling deformation in circumferential directions at the tunnel wall is mutually constrained, resulting in swelling stress. As swelling time advances, both radial and circumferential stresses intensify at the same location, indicating the swelling behavior of the surrounding rock. However, the stress increment decreases with increasing distance from the tunnel wall.

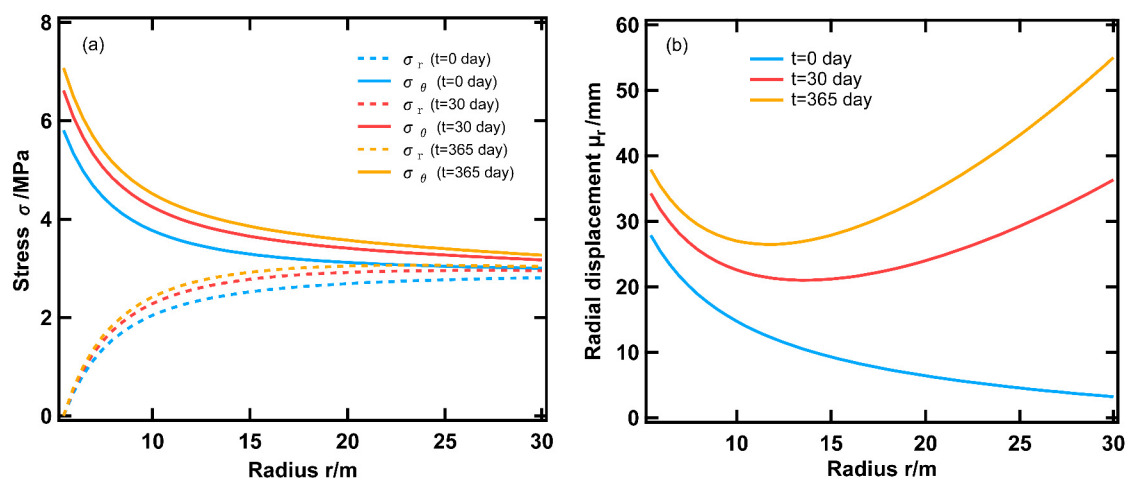


Figure 5. The stress and radial displacement distributions of different swelling times under unsupported conditions: (a) the stress distribution; and (b) the radial displacement distribution.

Additionally, as stated in the original text [20], we conducted a quantitative and qualitative comparative analysis of the data in Figure 5 through finite element simulation. This is why we have validated the results shown in Figure 5 for the stress distribution and radial displacement within the tested periods of 0, 30, and 365 days.

According to Figure 5b, radial displacement at the tunnel wall measures 27.86 mm, 34.26 mm, and 37.87 mm for swelling times of 0, 30, and 365 days, respectively. Initially, radial displacement decreases with radius, reflecting the limited influence range of tunnel excavation on the surrounding rock. However, for swelling times of 30 and 365 days, radial displacement first decreases and then increases with radius. This phenomenon is attributed to the higher initial humidity and resultant swelling deformation in the anhydrite rock at larger radii. Beyond a certain radius, swelling behavior predominantly dictates radial displacement.

Post-excavation, the displacement of tunnel walls significantly impacts tunnel design and construction. As illustrated in Figure 6, radial displacement at the tunnel wall stabilizes after 150 days of swelling, corresponding to the reduction in moisture content in the anhydrite rock and the subsequent decrease in swelling behavior. It is noteworthy that

radial displacement at the tunnel wall decreases significantly with increased support pressure for the same duration of swelling.

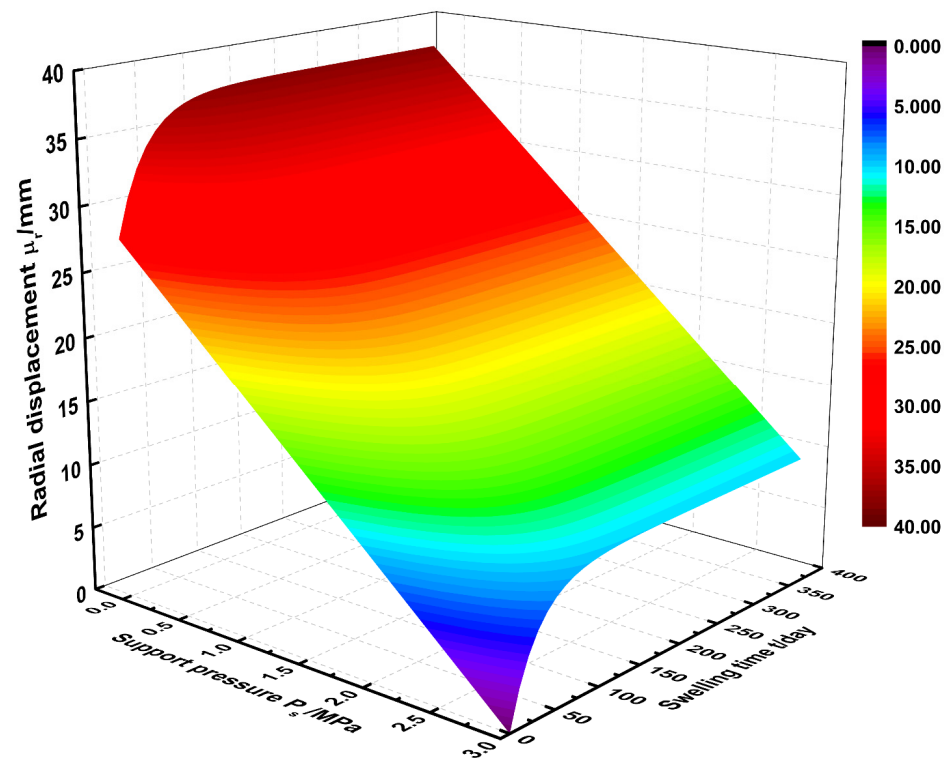


Figure 6. Synergistic effect of support pressure and swelling time on radial displacement at the tunnel wall.

4. Reliability Analysis

4.1. The Spreadsheet for the Reliability Solution

This study utilizes the spreadsheet solution method based on the First Order Reliability Method (FORM) developed by Low and Tang [26,27]. This approach is noteworthy for its straightforward input process and suitability for handling complex functions. To ascertain the reliability of the anhydrite rock tunnel, the following steps were undertaken:

- (1) A performance function was formulated for reliability analysis, defined as follows:

$$g(t) = u_{cv} - u_{R_0}(t) \quad (16)$$

where u_{cv} represents the tunnel's reserved deformation. If the displacement at the tunnel wall exceeds this critical threshold, it leads to the intrusion of the lining. The magnitude of the reserved deformation is influenced by various factors, such as the quality of the rock mass and the active span of the tunnel. $u_{R_0}(t)$ is the result of displacement at the free face, i.e., $r = R_0$ changing with expansion time, which is a time-dependent function. Therefore, $g(t)$ is a time-dependent functional function.

- (2) This study involves categorizing variables for reliability analysis and determining their distribution types and parameters. In this context, variables such as u_{cv} , P_0 , P_s , R_0 , R_{max} , and t are treated as deterministic variables. The values of u_{cv} and P_s are determined within specified constraints by tunnel engineers. For the Lirang tunnel, u_{cv} and P_s are set at 50 mm and 0.5 MPa, respectively. Given the inherent uncertainties in geotechnical parameters, variables such as E_s , E_e , v , α , a , and w_0 are categorized as random variables. The types and parameters of their distributions are outlined in Figure 7 [13,28]. As shown in Figure 7, for a normally distributed random variable, para1 represents the mean, and para2 represents the variance. The solution in Column x*(cells G7:G12) of Figure 7 denotes

the design point. The x^* values shown in cells G7:G12 render Equation (16)—performance function $g1(x)$ —equal to zero.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N														
1				P_o/Mpa	P_s/Mpa	R_o/m	R_{max}/m	t/day	C_1	C_2																		
2				3	0.5	5.45	30	0	69.14958	2.920129																		
3																												
4																												
5	Mean value			Standard deviation			Correlation matrix																					
6	variable	Distribution	para1	para2	para3	para4	x^*	R						n_i														
7	E_s	Normal	94.383	6.621			94.38373	1	0	0	0	0	0	0.000111														
8	E_e	Normal	735.45	50			459.3828	0	1	-0.5	0	0	0	-5.52134														
9	ν	Normal	0.311	0.09			0.716869	0	-0.5	1	0	0	0	4.509654														
10	α	Normal	0.0011	0.00019			0.0011	0	0	0	1	0	0	-9×10^{-6}														
11	a	Normal	0.034	0.007			0.034	0	0	0	0	1	0	-5.8×10^{-5}														
12	w_o	Normal	1.5	0.203			1.500001	0	0	0	0	0	1	-7.04×10^{-6}														
13	Performance function $g(t)=0.05-U \text{ (r=R}_0\text{)}$																											
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Figure 7. The spreadsheet for the reliability solution.

(3) Assessing correlations among random variables is a crucial aspect of this analysis. Particularly, the variable E_e demonstrates a negative correlation with v , indicated by a correlation coefficient of -0.5 [13]. This suggests an inverse relationship between these two parameters. All other random variables in this study are regarded as independent, with no interdependencies influencing their behavior or impact.

(4) The optimization process was carried out using the constraint optimization feature integrated within Excel. This method involved leveraging Excel's built-in optimization solver to minimize the reliability index β . The optimization was constrained by the condition $g(t) = 0$, ensuring that the solution aligns with the specified performance function. During the optimization process, the initial values assigned to the variable x^* were set as their respective mean values, establishing a balanced starting point for the optimization procedure.

4.2. The Influence of Variables on Reliability

Aligned with the “Uniform Standard for Reliability Design of Building Structures”, this study adopts a target reliability index of 4.2. As illustrated in Figure 8a, the reliability of the Lirang tunnel fluctuates with varying swelling durations and support pressures. Particularly noteworthy is that with a designed support pressure of 0.5 MPa, the tunnel's reliability index progressively diminishes with increasing swelling time, falling below the target threshold after 30 days. Conversely, increasing the support pressure enhances the tunnel's reliability index. A support pressure of 1.2 MPa ensures that the tunnel's reliability index remains compliant with the target requirements, even amidst swelling effects. Figure 8b depicts the tunnel's reliability concerning swelling time for different levels of reserved deformation. Achieving a reserved deformation of 59 mm enables the tunnel's reliability index to satisfy the target requirements under swelling conditions.

This analysis reveals that the initial design of the Lirang tunnel does not meet the target reliability criteria when accounting for swelling effects. However, by adjusting design variables such as support pressure and reserved deformation, the desired reliability target can be achieved. Figure 9 illustrates the combined impact of support pressure and reserved deformation on tunnel reliability over a swelling period of 150 days.

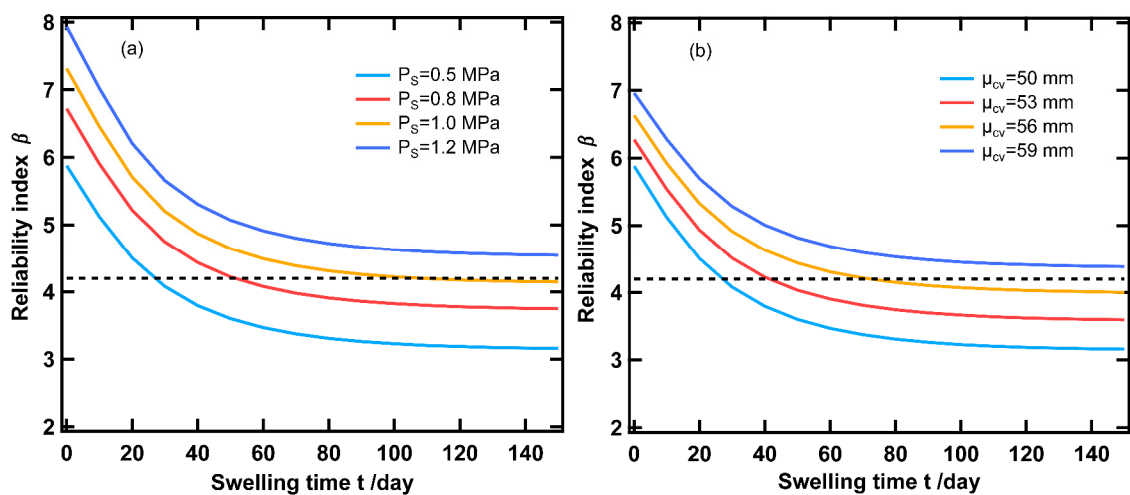


Figure 8. The reliability of the Lirang tunnel varies with swelling time under different support pressures and reserved deformation: (a) the reserved deformation remains at 50 mm; and (b) the support pressure remains at 0.5 MPa.

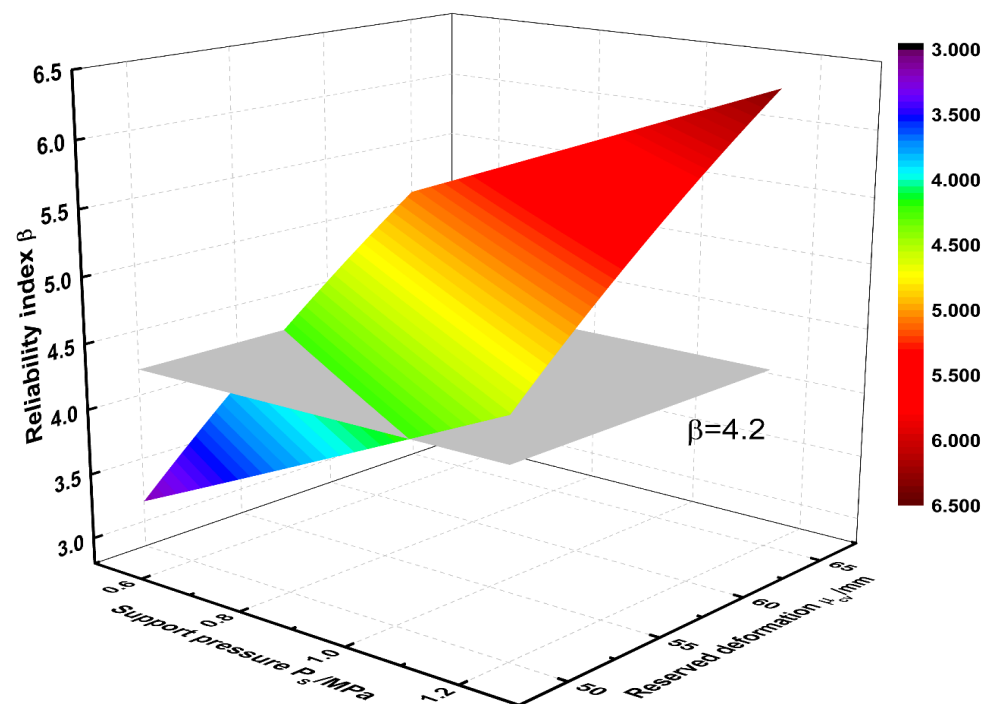


Figure 9. Synergistic effect of support pressure and reserved deformation on the reliability of the Lirang tunnel.

In summary, the reliability of the anhydrite tunnel is intricately tied to factors such as support pressure, swelling duration, and reserved deformation. Through careful adjustment and optimization of these controllable variables during the construction phase, the tunnel's reliability under swelling conditions can be enhanced to meet predefined reliability standards.

5. Discussion

5.1. Elastic Swelling Boundary Condition

When formulating the elastic swelling analytical solution, the external boundary condition for the anhydrite surrounding rock is set with the circumferential stress accounting

for both the ground stress and swelling stress. However, in practical scenarios, these outer boundary conditions are closely tied to the characteristics of the adjacent rock mass.

To precisely and scientifically determine these external boundary conditions and better represent the stress and displacement distribution of the surrounding rock, the integration of an ideal elastic rock mass surrounding the elastic swelling region is proposed. This approach is depicted in Figure 10. Here, the radial stress at the outer boundary of this idealized elastic region is equated to the hydrostatic far-field stress. Simultaneously, the radial stress at the inner boundary matches that observed in the elastic swelling region. Consequently, the boundary condition equation can be expressed as follows:

$$\begin{cases} \sigma_r(r = R_0) = P_s \\ \sigma_r^{\text{es}}(r = R_{\text{max}}) = \sigma_r^e(r = R_{\text{max}}) \\ \sigma_\theta(r = R_e) = P_0 \end{cases} \quad (17)$$

where σ_r^{es} and σ_r^e are the radial stresses in the elastic-swelling region and the elastic region, respectively.

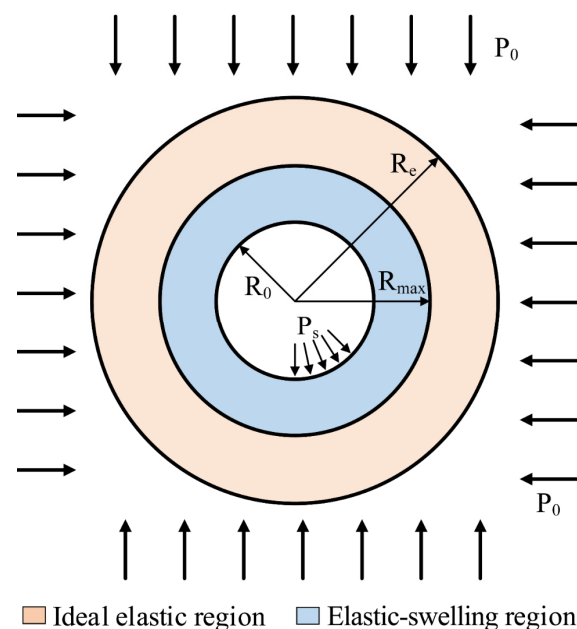


Figure 10. The circular anhydrite rock tunnel with an ideal elastic region and an elastic swelling region.

This methodology guarantees a more realistic and comprehensive comprehension of the stress distribution around the tunnel, considering the intricate interactions between the swelling anhydrite rock and its adjacent geological structures.

5.2. Application in Noncircular Tunnels

In Section 2, the analytical solution was tailored specifically for circular tunnels. However, it is crucial to acknowledge that tunnels are frequently constructed in diverse sizes and shapes, customized to their intended purpose. For example, horseshoe-shaped tunnels, as emphasized by Zhang [29], are renowned for their efficient use of underground space and their capacity to minimize soil excavation volume compared to circular tunnels.

When encountering noncircular tunnel configurations, such as semicircular or horseshoe-shaped designs (as depicted in Figure 11), a pragmatic approach is to approximate these structures as equivalent circular tunnels. This method enables the application of the circular tunnel analytical solution to a wider range of tunnel geometries, thereby expanding its utility and relevance across diverse tunneling projects. This conversion to

an equivalent circular format is a pragmatic solution that streamlines the analysis and design of various tunnel shapes within the framework of the established circular tunnel model [30,31].

$$R_0 = (h + b)/4 \quad (18)$$

$$R_0 = \sqrt{h^2 + (b/2)^2} / 2 \cos[\tan^{-1}(2h/b)] \quad (19)$$

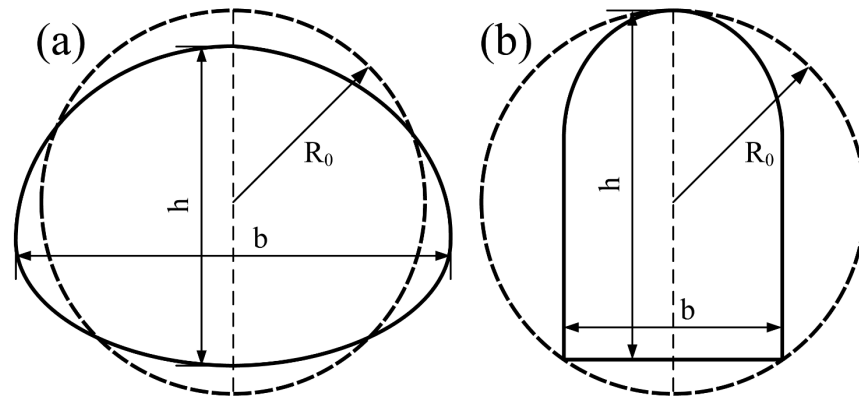


Figure 11. Conversions of noncircular tunnels into circular tunnels: (a) semicircular tunnel; (b) horseshoe-shaped tunnel.

Equations (18) and (19) represent the transformation formulas for semicircular and horseshoe-shaped tunnels, respectively. In these equations, ‘*h*’ denotes the height of the tunnel, while ‘*b*’ signifies its span. These formulas establish a mathematical framework for converting the dimensions of noncircular tunnel geometries into their circular equivalents, thereby facilitating the application of circular tunnel analysis methods to a broader spectrum of tunnel shapes. This conversion is pivotal for aligning established analytical solutions with diverse tunnel configurations, ensuring their relevance and applicability across various engineering scenarios.

6. Conclusions

This study presents an elastic swelling analytical solution for anhydrite rock tunnels rooted in the principles of humidity stress field theory. The solution’s validity is affirmed through its application to the Lirang tunnel. Additionally, this research explores the effects of support pressure, swelling duration, and reserved deformation on tunnel reliability. Key findings include:

1. Initially, radial stress intensifies while circumferential stress diminishes as the radius increases, both converging towards the hydrostatic far-field stress. Radial displacement similarly decreases. With prolonged swelling, all three parameters—radial stress, circumferential stress, and radial displacement—escalate at the same radial distance, mirroring the swelling behavior of anhydrite rock. Notably, at swelling durations of 30 and 365 days, radial displacement first decreases and then surges with increasing radius. This pattern is attributed to the swelling behavior becoming the dominant factor in radial displacement beyond a certain radius, where initial humidity is higher. After 150 days, radial displacement at the tunnel wall stabilizes, correlating with the diminishing swelling behavior as the rock’s internal moisture is absorbed.

2. The original design of the Lirang tunnel falls short of meeting the desired reliability standards when considering the swelling effect. However, increasing the support pressure to 1.2 MPa or enhancing the reserved deformation to 59 mm can guarantee that the tunnel meets the target reliability criteria.

3. The reliability of an anhydrite tunnel is intricately linked to factors such as support pressure, swelling time, and reserved deformation. Through careful adjustment and

optimization of these controllable variables during construction, such as modifying support pressure and reserved deformation, the tunnel's reliability under swelling conditions can be significantly enhanced to meet predetermined reliability requirements.

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References

- Alonso, E.E.; Berdugo, I.R.; Ramon, A. Extreme expansive phenomena in anhydritic-gypsiferous claystone: The case of Lilla tunnel. *Géotechnique* **2013**, *63*, 584. [\[CrossRef\]](#)
- Einstein, H.H. Tunnelling in difficult ground—Swelling behaviour and identification of swelling rocks. *Rock Mech. Rock Eng.* **1996**, *29*, 113–124. [\[CrossRef\]](#)
- Wu, J.; Ren, S.; Ouyang, X.; Fan, J.; Deng, G.; Li, Z. Research on Swelling of Gypsiferous Rock and Its Influence on Invert Arch of Tunnel. *J. Hunan Univ. (Nat. Sci.)* **2018**, *45*, 142–149.
- Jeschke, A.A.; Vosbeck, K.; Dreybrodt, W. Surface controlled dissolution rates of gypsum in aqueous solutions exhibit nonlinear dissolution kinetics. *Geochim. Cosmochim. Acta* **2001**, *65*, 27–34. [\[CrossRef\]](#)
- Taherdangkoo, R.; Meng, T.; Amar, M.N.; Sun, Y.; Sadighi, A.; Butscher, C. Modeling Solubility of Anhydrite and Gypsum in Aqueous Solutions: Implications for Swelling of Clay-Sulfate Rocks. *Rock Mech. Rock Eng.* **2022**, *55*, 4391–4402. [\[CrossRef\]](#)
- Konstantinos, S.; Anagnostou, G.; Vrakas, A. Scale effects in relation to swelling pressure in anhydritic claystones. *Geomech. Micro Macro* **2015**, *48*, 795–800.
- Butscher, C.; Einstein, H.H.; Huggenberger, P. Effects of tunneling on groundwater flow and swelling of clay-sulfate rocks. *Water Resour. Res.* **2011**, *47*, 198–206. [\[CrossRef\]](#)
- Steiner, W. Swelling rock in tunnels: Rock characterization, effect of horizontal stresses and construction procedures. *Int. J. Rock Mech. Min. Sci. Geomech. Abstr.* **1993**, *30*, 361–380. [\[CrossRef\]](#)
- Hooman, G.S.; Nowak, A.S. Reliability analysis of circular tunnel with consideration of the strength limit state. *Geomech. Eng.* **2018**, *15*, 879–888.
- Qing, L.; Chan, C.L.; Low, B.K. System reliability assessment for a rock tunnel with multiple failure modes. *Rock Mech. Rock Eng.* **2013**, *46*, 821–833.
- Song, L.; Li, H.Z.; Chan, C.L.; Low, B.K. Reliability analysis of underground excavation in elastic-strain-softening rock mass. *Tunn. Undergr. Space Technol.* **2016**, *60*, 66–79. [\[CrossRef\]](#)
- Yang, X.L.; Li, W.T. Reliability analysis of shallow tunnel with surface settlement. *Geomech. Eng.* **2017**, *12*, 313–326. [\[CrossRef\]](#)
- Li, H.-Z.; Low, B.K. Reliability analysis of circular tunnel under hydrostatic stress field. *Comput. Geotech.* **2010**, *37*, 50–58. [\[CrossRef\]](#)
- Wang, Y. Ground Response of Circular Tunnel in Poorly Consolidated Rock. *J. Geotech. Eng.* **1996**, *122*, 703–708. [\[CrossRef\]](#)
- Carranza-Torres, C. Elasto-plastic solution of tunnel problems using the generalized form of the Hoek-Brown failure criterion. *Int. J. Rock Mech. Min. Sci.* **2004**, *41* (Suppl. S1), 629–639. [\[CrossRef\]](#)
- Hou, Z.; Liu, Y.; Han, Z.; Tang, M.; Gong, X.; Su, D.; Wang, L. Experimental Study of the Bearing Characteristics of a Novel Energy-Saving and Environmentally Friendly Pile: Drilling with Prestressed Concrete Pipe Cased Piles. *Int. J. Geomech.* **2024**, *24*, 04024035. [\[CrossRef\]](#)
- Alonso, E.; Alejano, L.R.; Varas, F.; Fdez-Manin, G.; Carranza-Torres, C. Ground response curves for rock masses exhibiting strain-softening behaviour. *Int. J. Numer. Anal. Methods Geomech.* **2003**, *27*, 1153–1185. [\[CrossRef\]](#)

18. Miao, X.X.; Yang, C.Y.; Chen, Z.D. Humidity stress field theory in swelling rock mass. *Rock Soil Mech.* **1993**, *14*, 49–55.
19. Wu, J.; Hou, Z.; Chen, L.; Du, X.; Fan, J.; Xu, X.; Sun, C. A time-dependent swelling constitutive model for reliability analysis of anhydrite tunnel. *Int. J. Mater. Prod. Technol.* **2022**, *65*, 277–298. [[CrossRef](#)]
20. Wu, J. Study on the Watering-Swelling Evolution of Anhydrite and Its Application in the Reliability of Tunnelling. Ph.D. Dissertation, Chongqing University, Chongqing, China, 2018.
21. Chen, F.; Wu, J.; Ouyang, X.; Ren, S. The swelling tests of anhydrite based on the humidity stress field theory. *Rock Soil Mech.* **2018**, *39*, 1–9.
22. Cheng, S.; Zeng, W.; Ye, Y.; Gao, R.; Li, H. Analysis Model of Geotechnical Materials Based on Relative Humidity Field and Variable Modulus Constitutive Modell. *Adv. Eng. Sci.* **2023**, *55*, 99–108.
23. Chen, X. Analysis on Deformation of Expansive Soil Tunnel and Influence on Adjacent Buildings under Humidification Condition. Ph.D. Thesis, Hefei University of Technology, Hefei, China, 2021.
24. Ren, S.; Wu, J.X.; Ou, Y.X.; Liu, R.; Wang, L. Influence of pressured water on the swelling of anhydrite rock. *Rock Soil Mech.* **2018**, *39*, 4351–4359.
25. Institute Changjiang River Scientific Research; Northeastern University. *GB/T 50218-2014*; 2014 Standard for Engineering Classification of Rock Mass. Ministry of Housing and Urban-Rural Development of the People's Republic of China; General Administration of Quality Supervision, Inspection and Quarantine of the People's Republic of China: Beijing, China, 2014.
26. Low, B.K.; Tang, W.H. Reliability analysis using object-oriented constrained optimization. *Struct. Saf.* **2004**, *26*, 69–89. [[CrossRef](#)]
27. Low, B.K.; Tang, W.H. Reliability analysis of reinforced embankments on soft ground. *Can. Geotech. J.* **1997**, *34*, 672–685. [[CrossRef](#)]
28. Xiang, L.; Wang, J.-T.; Wei, H. Reliability analysis of rock tunnel stability based on interval non-probability under multiple failure modes. *Rock Soil Mech.* **2023**, *44*, 8.
29. Zhang, J.; Feng, T.; Yang, J.; Yang, F.; Gao, Y. Upper-bound stability analysis of dual unlined horseshoe-shaped tunnels subjected to gravity. *Comput. Geotech.* **2018**, *97*, 103–110. [[CrossRef](#)]
30. Carranza-Torres, C.; Fairhurst, C. The elasto-plastic response of underground excavations in rock masses that satisfy the Hoek–Brown failure criterion. *Int. J. Rock Mech. Min. Sci.* **1999**, *36*, 777–809. [[CrossRef](#)]
31. Liu, S.-F.; Su, Y.-H.; Li, S. Calculation of safety factor of tunnel structure based on Duncan-Fama convergence curve. *Chin. J. Geotech. Eng.* **2016**, *38*, 1307–1315.

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