

Article

Stress Analysis of a Concrete Pipeline in a Semi-Infinite Seabed under the Action of Elliptical Cosine Waves Based on the Seepage Equation

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Abstract: This study aims to investigate the mechanical response of a submarine concrete pipeline under wave action in shallow waters, taking into account factors such as the compressibility of pores and the permeability of the seabed. The control equation of the elliptical cosine wave theory is adopted to simulate the action of waves. In order to simulate the interaction between the solid skeleton and pore fluid, the concept of a “porous medium” is used to establish the transient seepage control equation. Utilizing the stress and displacement conditions at the interface of the ideal fluid media, porous media, and concrete pipeline, the numerical solutions for the internal force and pore pressure of the concrete pipeline buried in a semi-infinite thickness seabed were obtained; meanwhile, the effects of changes in the gas content in pore water and changes in the seabed permeability coefficient on a concrete pipeline were analyzed. The numerical calculation results show that, with the increase in the gas content in the pore water, the amplitude of the pore pressure on the pipeline surface decreases, and both the horizontal and vertical forces acting on the pipeline decrease; the amplitude of the pore pressure on the pipeline surface increases with the increase in seabed permeability and decreases with the enhancement of seabed permeability anisotropy; the improvement of the seabed permeability or enhancement of the permeability anisotropy can increase the horizontal force acting on the pipeline. This study provides a reference for the stability evaluation of submarine concrete pipelines under wave action in shallow water areas.

Keywords: the concrete pipeline; the semi-infinite seabed; the transient seepage control equation; stress analysis



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1. Introduction

With the continuous development of ocean engineering technology, the interaction between ocean structures and seabed soil under wave loads has become an important research topic in the interdisciplinary field of ocean engineering and geotechnical engineering. In shallow offshore areas, waves not only directly act on marine structures, but also generate cyclic wave pressure on the seabed surface, which affects the excess pore water pressure and effective stress inside the seabed, thereby affecting the bearing capacity and stability of surrounding marine structures such as pipelines and single piles, etc.

A large number of engineering examples have shown that the instability and failure of offshore buildings are largely related to the instability of the seabed, and the main load to which these buildings are subjected is waves. The impact of wave action on the seabed and seabed structures has been of the attention of foreign governments and research

institutions, such as the American Petroleum Institute (API), the American Natural Gas Association (AGA), the American Bureau of Shipping (ABS), MIT, Cambridge University, etc. Mei and Foda [1] studied the interaction problem between pipelines, waves, and the semi-infinite seabed based on the boundary layer theory. Foda and Chang [2] studied the floating instability of pipelines partially buried in the seabed under wave action through physical simulation experiments and analyzed their mechanical mechanisms. Cheng and Liu [3] studied the pressure of seepage on pipelines buried in the seabed under wave action through flume experiments. Magda [4] proposed an analytical formula for the pore pressure of the soil around the buried pipe under wave action based on the diffusion equation and analyzed the influence of the gas content in the pore water on the stress of the pipe. Jeng and Cheng [5] used a differential method to analyze the response around the buried pipe according to the Mohr–Coulomb criterion. Kumar et al. [6] studied the effects of combined waves on submarine pipelines, which focused on the influence of cohesive soil on the results. Randolph et al. [7,8], Boylan et al. [9], and Liu et al. [10] simulated the effect of submarine landslides on pipelines. Shi and Zhang [11] established a mathematical model for describing the interaction between waves and porous structures based on an open-source program for computational fluid dynamics. Wu et al. [12] conducted a centrifuge test and numerical simulation of the wave–seabed–structure interaction using a wave generator in a beam centrifuge ZJU400 from Zhejiang University. Theodoros [13] studied the stability of reinforced concrete walls under compressive load strain through experiments. Liu [14] analyzed the seabed oscillatory response, accumulated response and the progressive liquefaction due to cnoidal waves based on a numerical calculation program for a wave-induced non-cohesive soil seabed response using C++ language and software for wave simulation. Duan [15] examined the wave-induced uplift force onto pipelines buried in sloping seabeds based on a two-dimensional numerical model for wave–seabed–pipeline interactions. Su [16] established a two-dimensional fully nonlinear time domain-coupled model to simulate and analyze the interaction between fully nonlinear waves and two-dimensional fixed structures. Liu [17] conducted a numerical analysis on the dynamic response and liquefaction phenomena in sandy seabed foundation around semi-circular breakwater under wave loading. However, because of limitations in theoretical derivation and calculation, many studies mainly focus on the seabed response under linear wave action. Research on the stability of submarine pipelines under wave action has not yet fully considered the influence of the compressibility of the pore and the permeability of the seabed on the forces acting on the pipeline in relevant theoretical studies.

This article proposes an improved algorithm for the numerical calculation of the elliptical cosine function based on the control equations of water wave dynamics and the elliptical cosine wave. The transient seepage equation is applied to study the effect of waves on the seabed and a submarine concrete pipeline in shallow water. The stress situation of a concrete pipeline buried in a semi-infinite thickness seabed is analyzed, and the influencing factors of pipeline stress are studied through a parameter analysis. This article provides theoretical support for the study of wave theory in coastal and shallow waters and provides a reference for the stability evaluation of submarine pipelines under wave action.

2. The Control Equation of Elliptic Cosine Wave Theory

As shown in Figure 1, it is assumed that the seabed is horizontal, smooth, and impermeable, and the static depth of the seawater is D . The z -axis is vertically inclined upward and the coordinate origin is located on the surface of the seabed, the waves propagate forward along the x -axis, and the equation for the free water surface is $z = \eta(x, t)$.

Assuming that the wave problem in shallow water can be simplified as a two-dimensional plane problem, and the total water depth can be a small quantity relative to the wavelength, the velocity of the water body along the horizontal direction does not

depend on the vertical coordinates; the velocity potential is expanded into a power series in references [18,19].

$$\Phi(x, z, t) = \sum_{n=0}^{\infty} z^n f_n(x, t) \quad (1)$$

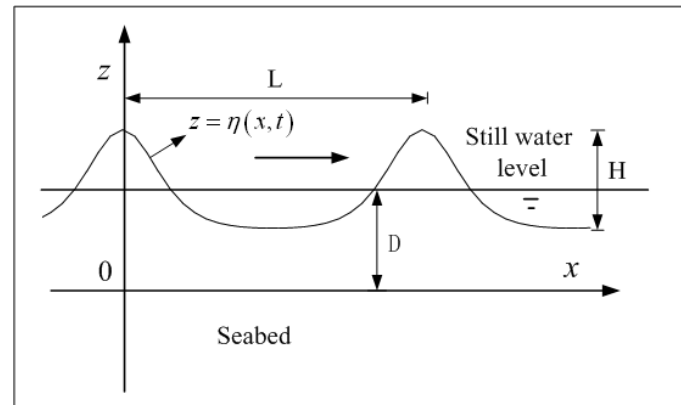


Figure 1. Sketch of the water wave problem.

Using the boundary conditions at the bottom of the water, Equation (1) is expanded and substituted into the control equation of the water wave problem to obtain the control equation of nonlinear waves in shallow water regions (for the specific derivation process in article [20]):

$$\eta_t + [(1 + \alpha\eta)f_x]_x - \left[\frac{1}{6}(1 + \alpha\eta^2)f_{xxxx} + \frac{1}{2}\alpha(1 + \alpha\eta^2)\eta_x f_{xxx} \right] \beta + O(\beta^2) = 0 \quad (2)$$

$$\eta + f_t + \frac{1}{2}\alpha f_x^2 - \frac{1}{2}(1 + \alpha\eta)^2 [f_{xxt} + \alpha f_x f_{xxx} - \alpha f_{xx}^2] \beta + O(\beta^2) = 0 \quad (3)$$

where the parameters α and β are dimensionless in Equations (2) and (3): $\alpha = \frac{H}{D}$, $\beta = \frac{D^2}{L^2}$.
If

$$w = f_x = \Phi_x(x, 0, t) \quad (4)$$

The second-order terms of the parameters are ignored, and Equations (2) and (3) can be written as

$$\eta_t + [(1 + \alpha\eta)w]_x - \frac{1}{6}\beta w_{xxx} = 0 \quad (5)$$

$$w_t + \alpha w w_x + \eta_x - \frac{1}{2}\beta w_{xt} = 0 \quad (6)$$

Formulas (2) and (3) are a variant which can be transformed into the KDV equation with a dimensional form through transformation.

$$\eta_t + c_0 \left(1 + \frac{3}{2} \frac{\eta}{D} \right) \eta_x + \vartheta \eta_{xxx} = 0 \quad (7)$$

where $c_0 = \sqrt{gD}$, $\vartheta = \frac{1}{6}c_0 D^2$.

Equation (7) has both non-periodic and periodic solutions, and the periodic solution is the solution of the first-order elliptic cosine wave.

Fixing the origin on a still-water plane, the wave surface of the first-order elliptical cosine wave can be described as follows:

$$\eta = H \left(\frac{1}{m^2} - 1 - \frac{E}{m^2 K} \right) + Hcn^2 \left[2K \left(\frac{x}{L} - \frac{t}{T} \right) \right] \quad (8)$$

In the equation, $cn(\cdot)$ is the elliptical cosine function, x is the horizontal coordinate, H is the wave height, L is the wavelength, T is the period, and m is the modulus. K and E are the first type and second type.

$$K = \int_0^{\pi/2} \frac{1}{\sqrt{1 - m^2 \sin^2 \varphi}} d\varphi$$

$$E = \int_0^{\pi/2} \sqrt{1 - m^2 \sin^2 \varphi} d\varphi$$

For the numerical calculation of the elliptic cosine functions, Taylor's expansions can be used to calculate elliptic cosine functions $cn(x, m)$. The Taylor expansions of elegant elliptic functions have the following form:

$$sn(x, m) = x - \frac{1 + m^2}{6} x^3 + o(x^5) \quad (9)$$

$$cn(x, m) = 1 - \frac{1}{2} x^2 + (1 + 4m^2) \frac{x^4}{4!} - o(x^6) \quad (10)$$

$$dn(x, m) = 1 - m^2 \frac{1}{2} x^2 + m^2 (4 + m^2) \frac{x^4}{4!} + o(x^6) \quad (11)$$

The double formula for elliptic functions is as follows:

$$sn(2x, m) = \frac{2sn(x, m)cn(x, m)dn(x, m)}{1 - m^2 sn^4(x, m)} \quad (12)$$

$$cn(2x, m) = \frac{1 - 2sn^2(x, m) + m^2 sn^4(x, m)}{1 - m^2 sn^4(x, m)} = 1 - \frac{2sn^2(x, m)[1 - m^2 sn^2(x, m)]}{1 - m^2 sn^4(x, m)} \quad (13)$$

$$dn(2x, m) = \frac{1 - 2m^2 sn^2(x, m) + m^2 sn^4(x, m)}{1 - m^2 sn^4(x, m)} = 1 - \frac{2m^2 sn^2(x, m)[1 - sn^2(x, m)]}{1 - m^2 sn^4(x, m)} \quad (14)$$

Assume

$$sn(2x, m) = S \quad (15)$$

$$\frac{2sn^2(x, m)[1 - m^2 sn^2(x, m)]}{1 - m^2 sn^4(x, m)} = C \quad (16)$$

$$\frac{2m^2 sn^2(x, m)[1 - sn^2(x, m)]}{1 - m^2 sn^4(x, m)} = D \quad (17)$$

Then, Equations (13) and (14) can be written as

$$cn(2x, m) = 1 - C \quad (18)$$

$$dn(2x, m) = 1 - D \quad (19)$$

However, when the distance x from the specified point x_0 is large, the number of Taylor expansions must be high, making calculation and error control inconvenient. Taking into account the periodicity of elliptic functions, the precision of the calculations can be controlled based on data pre-processing [21]. Due to the existence of a real period for $cn(x, m)$, for any x , x' can be found that satisfies $|x'| \leq 4K$, making $cn(x, m) = cn(x', m)$. In the theory of elliptical cosine waves, when calculating $cn[2K(\frac{x}{\lambda} - \frac{t}{T}), m]$, for any $\frac{x}{\lambda} - \frac{t}{T}$, ξ ($|\xi| \leq 2$) can be defined, satisfying the following equation.

$$cn\left[2K\left(\frac{x}{\lambda} - \frac{t}{T}\right), m\right] = cn(2K\xi, m) \quad (20)$$

When $m = 0.95$, the first type of complete elliptic integral $K \approx 2.59$; therefore, $2K\xi < 20$.

Based on the aforementioned theory, the following steps can be used to calculate $y = cn[2K(\frac{x}{\lambda} - \frac{t}{T})]$.

(1) Find the number ξ that satisfies Equation (20), assuming $\zeta = \frac{2K\xi}{1024}$;

(2) Use Formulas (9)–(11) to calculate $sn(\zeta, m)$, $cn(\zeta, m)$ and $dn(\zeta, m)$; because of $\zeta < 2.0 \times 10^{-2}$, the calculating error of $sn(\zeta, m)$ is 2.0×10^{-10} , and the calculating errors of $cn(\zeta, m)$ and $dn(\zeta, m)$ are 2.0×10^{-12} .

(3) Use Formulas (15)–(17) to calculate S_1 , C_1 and D_1 ,

Where $S_1 = sn(2\zeta, m)$.

$$C_1 = \frac{2sn^2(\zeta, m)[1 - m^2sn^2(\zeta, m)]}{1 - m^2sn^4(\zeta, m)}$$

$$D_1 = \frac{2m^2sn^2(\zeta, m)[1 - sn^2(\zeta, m)]}{1 - m^2sn^4(\zeta, m)}$$

Due to $2^{10} = 1024$, S_{10} , C_{10} and D_{10} can be obtained by repeating process (3) ten times, and finally, $y = cn(2K\xi, m) = 1 - C_{10}$.

3. Control Equation for Seabed Response under Wave Action

The shallow seabed is a mixture of fluid and solid particles with fluid distributed in chaotic, non-uniform, and irregular pores. Bear [22] cites the concept of “porous media” to describe the flow of fluids in rock and soil. Because of the extremely complex internal structure of porous media on the micro scale, it is very difficult to simulate the interaction between the solid skeleton and the pore fluid considering the microstructure of porous media. Bear introduced the concept of the REV (representative element volume) and introduced the research methods of continuum mechanics into the study of porous media.

If the density function of seabed pore water is ρ , and the porosity is n , then the mass of pore water in the control body V_t is

$$m_l = \int_{V_t} (\rho_l n) dV_t \quad (21)$$

According to the Reynolds transport equation, it can be obtained that

$$\int_{V_t} \left[\frac{\partial(\rho_l n)}{\partial t} + \nabla \cdot (\rho_l n v_l) \right] dV_t = 0 \quad (22)$$

In the equation, v_l is the velocity vector of the fluid, and v_s is denoted as the velocity of solid particles; according to the concept of seepage mechanics, the seepage velocity can be denoted as

$$v = n(v_l - v_s) \quad (23)$$

Substituting Equation (23) into Equation (22), the following can be obtained:

$$\frac{\partial(\rho_l n)}{\partial t} = -\nabla \cdot (\rho_l v) - \nabla \cdot (\rho_l n v_s) \quad (24)$$

Equation (24) is the differential form of the continuity equation for the seepage of the pore fluid.

For the left part of Equation (24),

$$\frac{\partial(\rho_l n)}{\partial t} = n \frac{\partial \rho_l}{\partial t} + \rho_l \frac{\partial n}{\partial t} = n \frac{\partial \rho_l}{\partial p} \frac{\partial p}{\partial t} + \rho_l \frac{\partial n}{\partial t} \quad (25)$$

Under the assumption of a continuous medium, the compressibility of the solid skeleton of the seabed can be defined, V_s is the volume of the solid skeleton, V is the total

volume of the soil, and then, the compression coefficient of the skeleton can be written as follows:

$$\alpha = -\frac{1}{V} \frac{\delta V}{\delta \sigma'} \quad (26)$$

where σ' is the effective stress between soil particles. Due to the very low compressibility of soil particles, their volume changes can be ignored, and V_s can be considered to remain constant, namely,

$$dV_s = d[(1-n)V_b] = 0 \quad (27)$$

From Equation (27), it can be obtained that

$$\frac{dV_b}{V_b} = \frac{dn}{1-n} \quad (28)$$

Substituting Equation (28) into Equation (26), it can be obtained that

$$\alpha = -\frac{1}{1-n} \frac{dn}{d\sigma'} \quad (29)$$

During the isothermal change process, assuming that the density of the pore fluid is ρ , β is defined as the compression coefficient and

$$\beta = -\frac{1}{V_f} \frac{dV_f}{dp} = \frac{1}{\rho} \frac{d\rho}{dp} \quad (30)$$

Substituting Equation (30) into Equation (25), it can be obtained that

$$\frac{\partial(\rho_l n)}{\partial t} = n\beta\rho_l \frac{\partial p}{\partial t} + \rho_l \frac{\partial n}{\partial t} \quad (31)$$

For the first item on the right side of Equation (24),

$$-\nabla \cdot (\rho_l v) = -\rho_l (\nabla \cdot v) - \nabla \rho_l \cdot v \quad (32)$$

For the second item on the right side of Equation (24),

$$-\nabla \cdot (\rho_l n v_s) = -\rho_l (\nabla \cdot n v_s) - \nabla \rho_l \cdot (n v_s) \quad (33)$$

In general, the change rate of the fluid density in space is much smaller than its change rate over time, and thus,

$$\begin{aligned} \nabla \rho_l \cdot v &<< \frac{\partial(\rho_l n)}{\partial t} \\ \nabla \rho_l \cdot (n v_s) &<< \frac{\partial(\rho_l n)}{\partial t} \end{aligned}$$

Equation (31) can be written as

$$-\nabla \cdot v - \nabla \cdot (n v_s) = \frac{\partial n}{\partial t} + n\beta \frac{\partial p}{\partial t} \quad (34)$$

The compressibility equation of a solid skeleton can be written as follows:

$$\frac{\partial n}{\partial t} = \frac{\partial n}{\partial \sigma'} \frac{\partial \sigma'}{\partial t} = -\alpha(1-n) \frac{\partial \sigma'}{\partial t} \quad (35)$$

When the permeability coefficient of the seabed is large, the pore pressure gradient in the soil is small, and the deformation of the solid skeleton can be ignored; the seepage equation in the seabed can be abbreviated as

$$\nabla \cdot v + n\beta \frac{\partial p}{\partial t} = 0 \quad (36)$$

Applying Darcy's law, Equation (36) can be written as

$$k_x \frac{\partial^2 p}{\partial x^2} + k_z \frac{\partial^2 p}{\partial y^2} = n\beta \frac{\partial p}{\partial t} \quad (37)$$

In the equation, k_x and k_z are the permeability coefficients of the seabed in the horizontal and vertical directions, and when the change in the solid volume is ignored, Equation (37) is the transient control equation for seabed seepage.

4. Analysis of Submarine Concrete Pipeline Forces under the Action of Elliptical Cosine Waves

When the oscillating wave pressure acts on the surface of the seabed, the pore pressure caused by it will exert additional force on the concrete pipeline buried in the seabed. If the additional force exceeds a certain limit, the pipeline will experience excessive displacement and failure. In the article, the seepage equation is used as the control equation for the seabed response to analyze the stress situation of the pipeline on the semi-wireless seabed under the action of elliptical cosine waves.

In the analysis process, the interaction between the pore water and pipeline is temporarily not considered, and the control equation is Equation (37). As shown in Figure 2, the waves propagate forward along the x -axis, with the seabed surface horizontal and the boundary conditions on the seabed surface being the same as before. The concrete pipeline is buried in the shallower part of the seabed and is impermeable. The boundary condition on its surface is

$$\frac{\partial p}{\partial n} = 0 \quad (38)$$

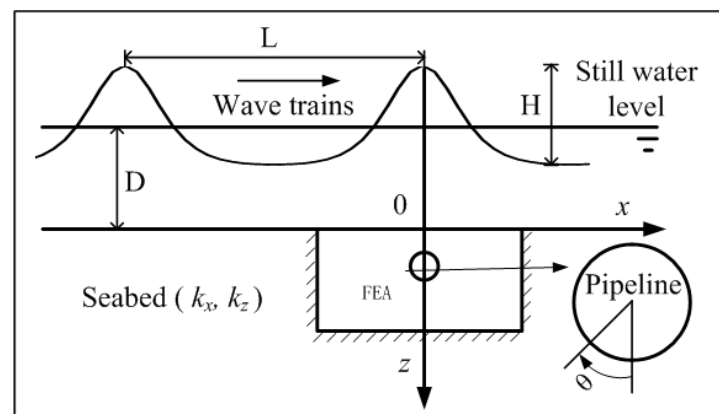


Figure 2. Diagram of subsea concrete pipeline under waves.

When conducting numerical calculations, a rectangular area with an impermeable boundary is selected; the permeability force acting on the pipeline can be obtained by integrating the pore pressure along the pipe wall, so that

$$F_h = \int_{\Gamma} p n_x dl \quad (39)$$

$$F_v = - \int_{\Gamma} p n_y dl \quad (40)$$

Here, F_h and F_v are the combined forces of the pore fluid acting on the concrete pipeline horizontally (positive to the right) and vertically (positive to the up), respectively. In order to analyze, the buoyancy force per unit length of pipeline in still water is as follows:

$$F_{buo} = \gamma_w \pi r^2 \quad (41)$$

The wave parameters used for the calculation are as follows: the wave period T is 10 s, wave height H is 2.0 m, water depth D is 6.5 m, wave modulus m is 0.97876, and wavelength L is 80.03 m. The permeability coefficients k_x and k_z of the seabed are both 2.0×10^{-4} m/s. The outer radius of the buried pipe is 1.0 m, and the buried depth of the concrete pipeline (distance from the center of the pipe to the surface of the seabed) is 2.0 m. The calculation was completed using the large-scale general finite element software COMSOL 5.4. The size of the calculation model is 50×30 m².

Figure 3 shows the distribution of the pore pressure on the surface of the pipeline, and Figure 3 (1) and (2) correspond to the pore pressure when the wave peak and trough pass through, respectively. As shown in the figure, when the wave peak passes above the pipeline, the water pressure at the top of the pipeline is about $0.38 H\gamma_w$, and the water pressure at the bottom of the pipeline is $-0.02 H\gamma_w$. When the trough passes over the pipeline, the water pressure at the top of the pipeline is about $-0.24 H\gamma_w$, and the water pressure at the bottom of the pipeline is $0.02 H\gamma_w$.

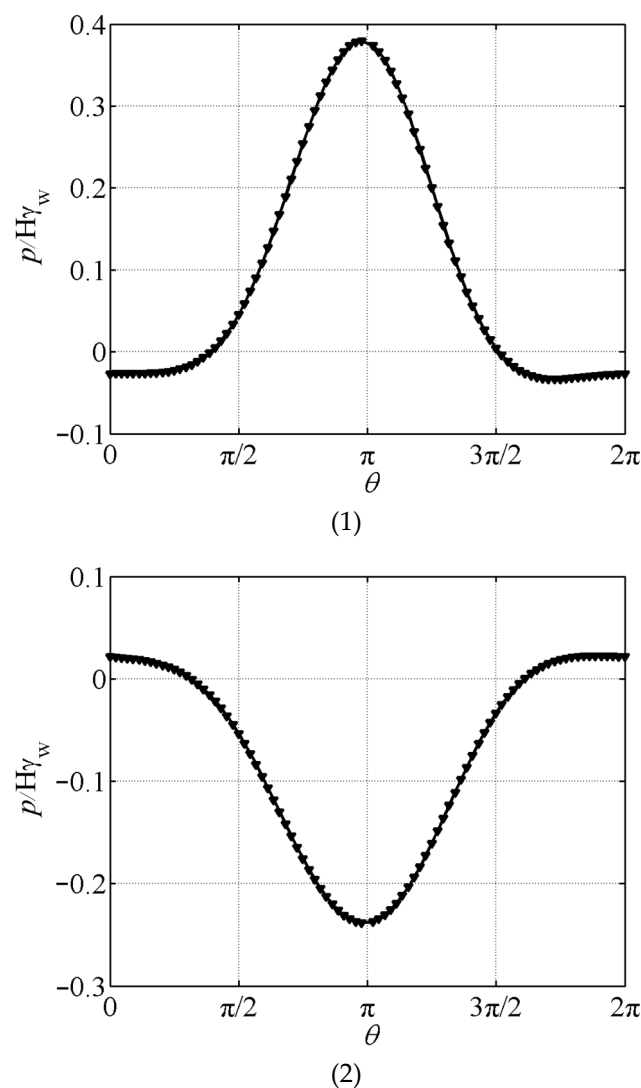


Figure 3. Pore pressure on the pipe surface. (1) the wave peak, (2) the wave trough.

Figure 4 shows the horizontal and vertical forces acting on the submarine pipeline when waves propagate on the sea surface. It can be seen from the figure that the amplitude of the horizontal force acting on the pipeline is very small, but the vertical force is relatively large and the maximum vertical force is about $0.25F_{buo}$.

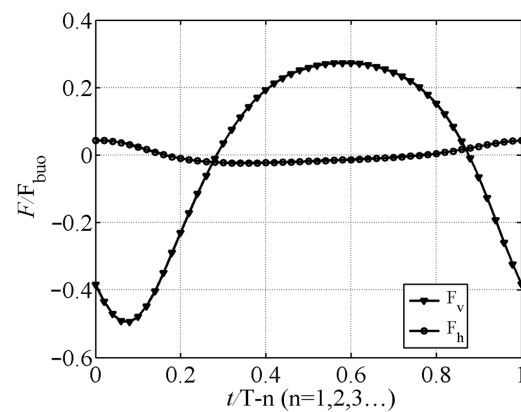


Figure 4. Horizontal and vertical force acting on the concrete pipeline.

5. The Influence of Parameter Changes on the Forces in Submarine Concrete Pipeline

When waves are large, pore water can exert a significant force on the concrete pipeline, affecting its stability. To analyze the pipeline stress situation, the influence of the gas content in the pore water and the seabed permeability on the pipeline stress is studied through a parameter analysis.

5.1. The Influence of Gas Content in Pore Water

Figure 5 shows the distribution of the pore pressure on the pipeline surface when the gas content in the pore water changes. Figure 5 (1) and (2), respectively, show the situation of the wave peaks and valleys passing by. From the figure, it can be seen that changes in the gas content have a significant impact on the pore pressure amplitude on the pipeline surface: as the gas content increases, the pore pressure amplitude decreases. Meanwhile, it can be seen from the figure that, as the gas content increases, the amplitude of the pore pressure in the lower part of the pipeline gradually approaches zero. +

Figure 6 shows the variation in the horizontal and vertical forces acting on the concrete pipeline during a cycle over time when the gas content in the pore water changes. From Figure 6 (1), it can be seen that the amplitude of the force acting on the pipeline decreases with the increase in the gas content; when the gas content changes from 0.5% to 2%, the amplitude of the horizontal force acting on the pipeline changes from about $0.07F_{bu0}$ to about $0.02F_{bu0}$. From Figure 6 (2), it can be seen that the gas content has little effect on the upward permeability of the pipeline. This is because as the gas content increases, the water content in the seabed will decrease, and the seepage force will also change accordingly, which will affect the reduction in the pore pressure on the pipeline surface. The horizontal and vertical forces acting on the pipeline will also decrease.

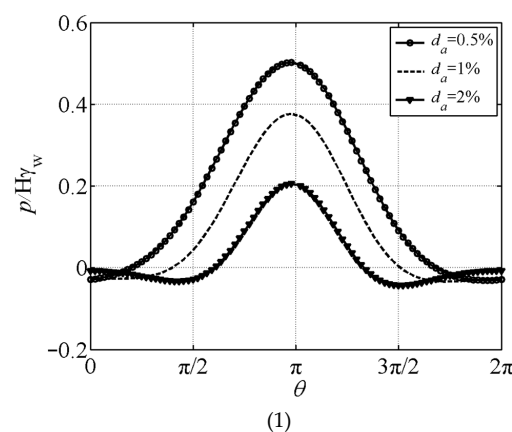


Figure 5. Cont.

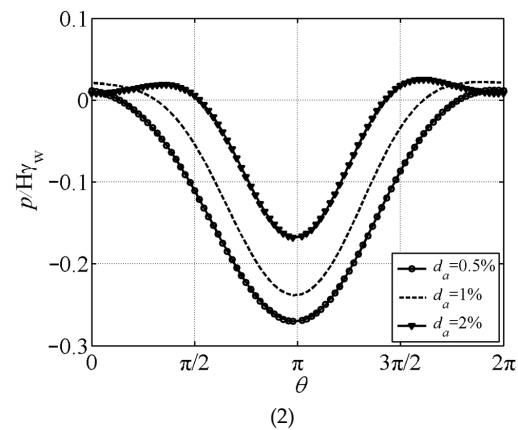


Figure 5. Pore pressure on the pipe surface with variable air content in pore water. (1) the wave peak, (2) the wave trough.

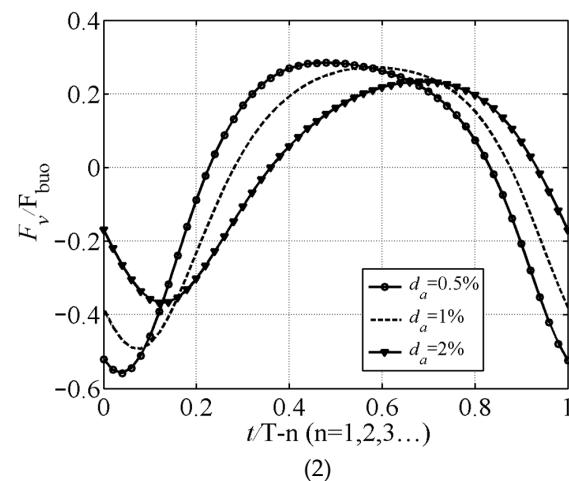
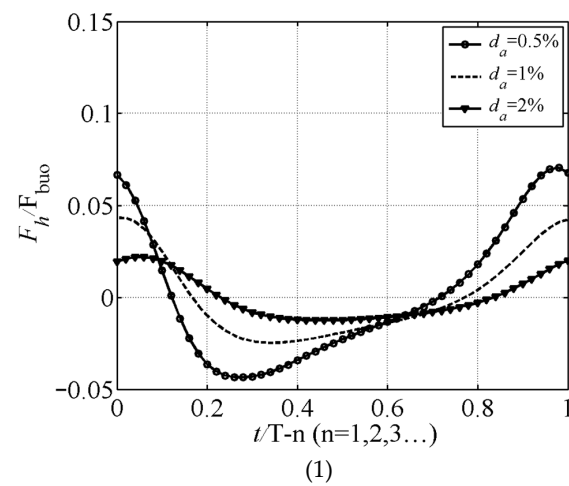


Figure 6. The influence of gas content changes in pore water on pipeline stress. (1) horizontal force, (2) vertical force.

5.2. The Influence of Seabed Permeability

Figure 7 shows the effect of changes in the seabed permeability coefficients on the distribution of pore pressure on the concrete pipeline surface. Figure 7 (1) and (2) correspond to the pore pressure on the pipeline surface when the wave crest and trough pass through, respectively. From the figure, it can be seen that the permeability coefficient of the seabed has a very significant impact on the pore pressure, and as the permeability coefficient

increases, the amplitude of the pore pressure gradually increases. When the permeability coefficient is 1.0×10^{-4} m/s ($k_z = k_x$), the pore pressure amplitude in the lower part of the pipeline is close to zero, because the depth of the wave influence is shallower when the permeability coefficient is small, and the lower part of the pipeline is less affected by waves.

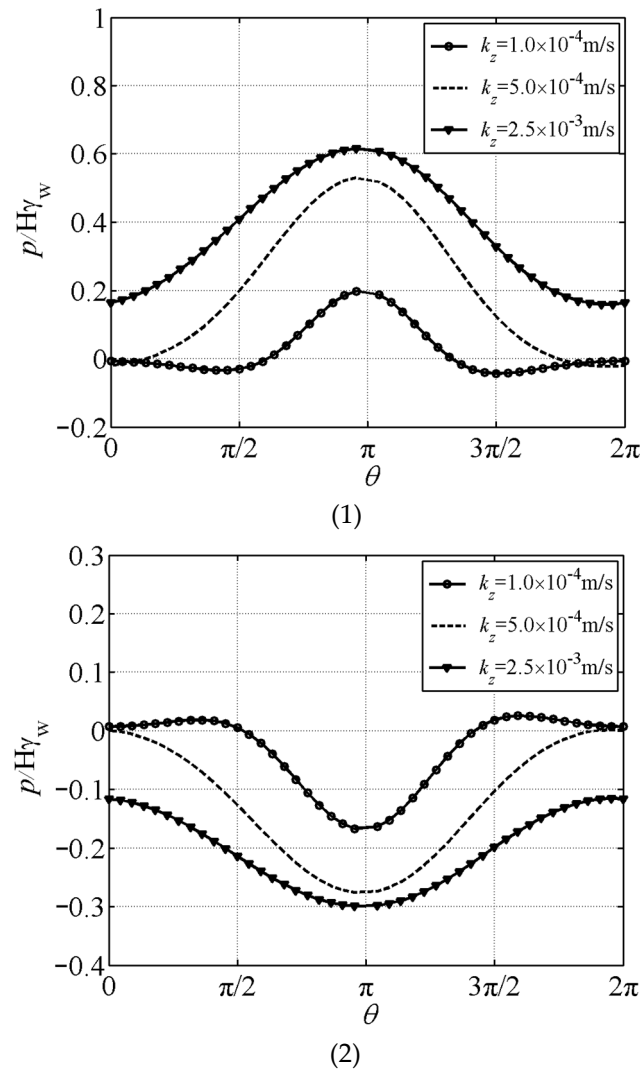


Figure 7. Pore pressure on the pipeline surface with variable seabed permeability. (1) the wave peak, (2) the wave trough.

Figure 8 shows the influence of changes in the seabed permeability coefficient on the horizontal and vertical forces acting on the concrete pipeline. It can be seen in the figure that the permeability coefficient has a significant impact on the horizontal force acting on the pipeline. When k_z is 1.0×10^{-4} m/s ($k_z = k_x$), the magnitude of the horizontal force is close to zero. When k_z is 2.5×10^{-3} m/s ($k_z = k_x$), the magnitude of the horizontal force can reach about $0.15F_{bu0}$. From Figure 8 (2), it can be seen that the permeability coefficient has little effect on the maximum upward force acting on the pipeline. This is because waves propagate horizontally, and as the permeability coefficient increases, the horizontal seepage velocity of water in the seabed increases due to the action of the waves, and the permeability coefficient has a significant impact on the horizontal force acting on the pipeline. As the permeability coefficient increases, the pore pressure on the surface of the pipeline also increases, and the horizontal force acting on the pipeline shows an increasing trend. However, the vertical seepage velocity of water in the seabed has a relatively small impact; therefore, the change in the permeability coefficient has little effect on the maximum upward force acting on the pipeline.

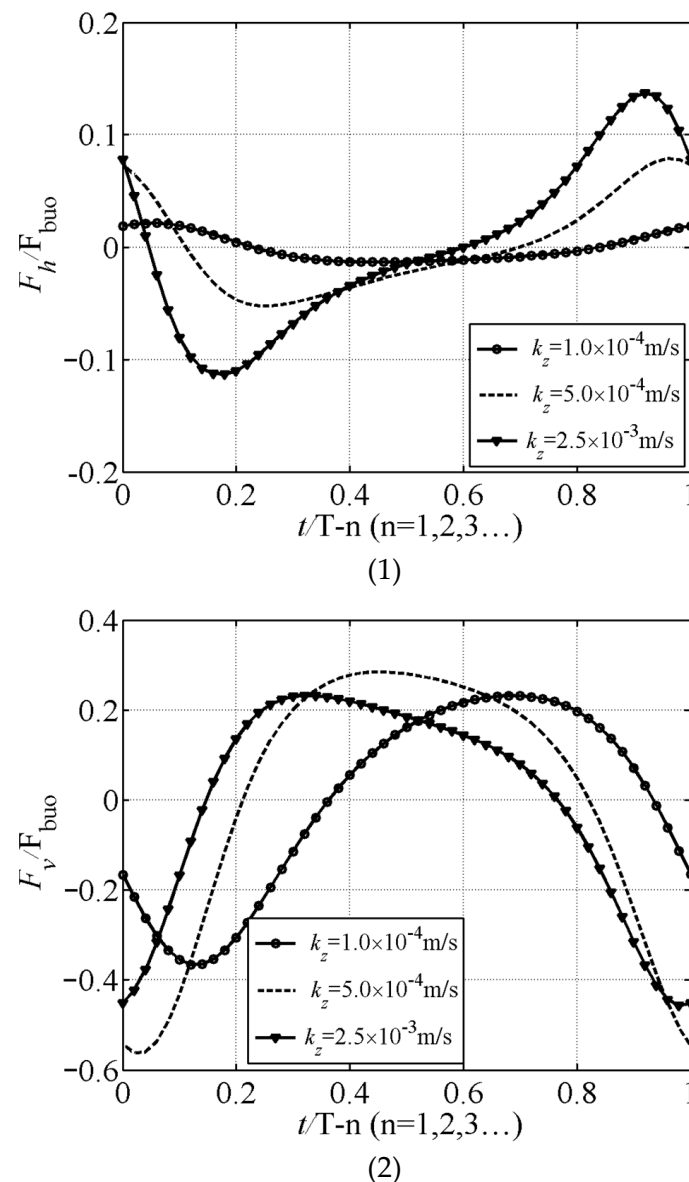
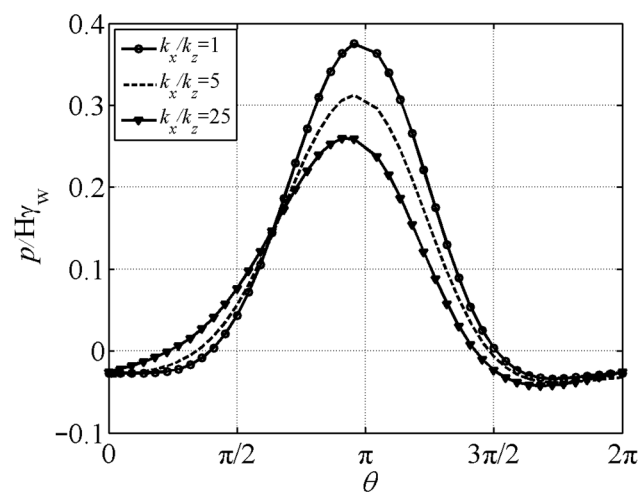


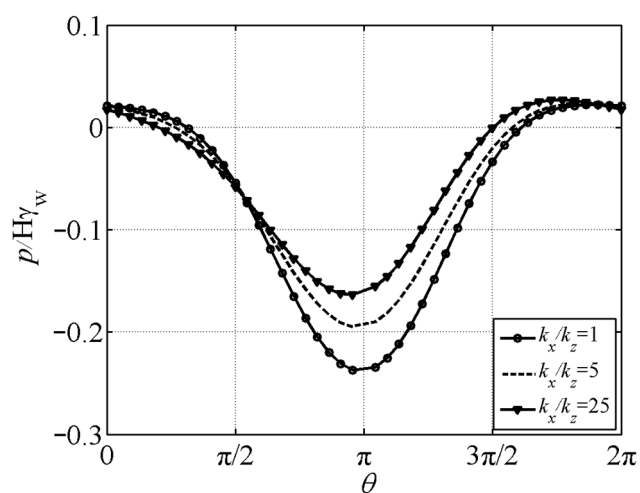
Figure 8. The influence of changes in seabed permeability on pipeline stress. (1) horizontal force, (2) vertical force.

Figure 9 shows the variation in the pore pressure on the concrete pipeline surface with the change in the k_x/k_z ratio. Figure 9 (1) and (2) correspond to the peak and trough of the wave passing through the pipeline surface. From the figure, it can be seen that the change in the ratio of k_x/k_z has a significant impact on the extreme values of the pore pressure distribution on the pipeline surface. When the ratio increases, the amplitude of the pore pressure at the top of the pipeline decreases, while the ratio change of k_x/k_z has a smaller impact on the pore pressure at the bottom of the concrete pipeline.

Figure 10 shows the changes in the horizontal and vertical forces acting on the concrete pipeline with the change in the ratio of k_x/k_z . It can be seen from the figure that the change in k_x/k has a certain impact on the amplitude of the horizontal force. When the ratio of k_x/k is 1, the horizontal force amplitude is about $0.05F_{buo}$, and when the ratio of k_x/k_z is 25, the horizontal force amplitude can reach 0.1. The amplitude of the vertical force acting on the pipeline decreases with increasing k_x/k_z , but the decrease is relatively small. When k_x/k_z varies between 1 and 25, the maximum vertical force acting on the pipeline is about $0.22 F_{buo}$.

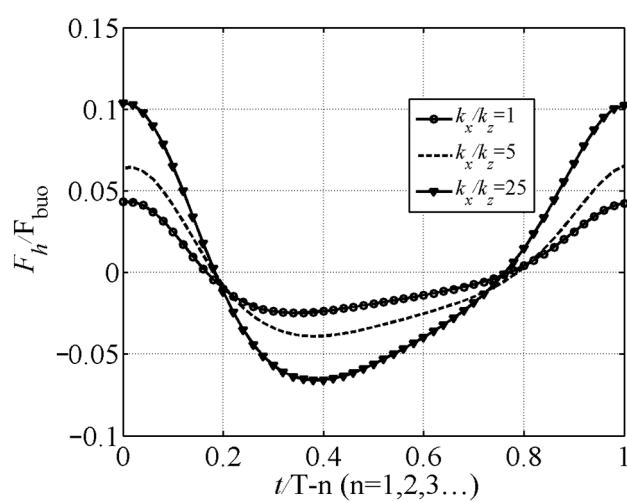


(1)



(2)

Figure 9. Pore pressure on the pipe surface with variable k_x/k_z . (1) the wave peak, (2) the wave trough.



(1)

Figure 10. Cont.

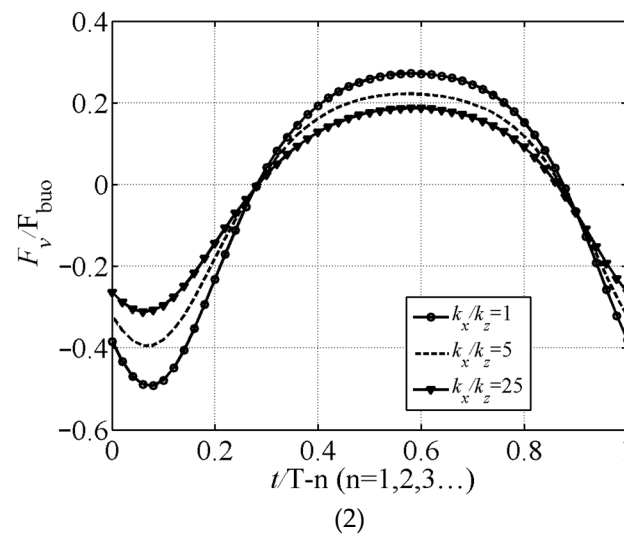


Figure 10. The influence of variable k_x/k_z on pipeline stress. (1) horizontal force, (2) vertical force.

6. Conclusions

In this paper, the mechanical response of a submarine concrete pipeline under wave action was studied based on the control equations of water wave dynamics and elliptical cosine waves. The analytical solutions for the internal forces and pore pressures of pipelines buried in a semi-infinite seabed were derived, and the effects of changes in the gas content in pore water and changes in the seabed permeability coefficient on a concrete pipeline were discussed. This provides a theoretical reference and a basis for wave theory research in shallow coastal waters; meanwhile, the outcome of this study provides assistance for the design of marine structures. Based on the numerical results, the following conclusions can be drawn:

- (1) The gas content in pore water has a significant impact on the force acting on the concrete pipeline under wave action; the higher the gas content, the smaller the amplitude of the pore pressure on the surface of the pipeline, and the smaller the horizontal and vertical forces acting on the pipeline.
- (2) The permeability coefficient of the seabed soil also has a significant impact on the force acting on the concrete pipeline; the higher the permeability coefficient of the seabed, the greater the amplitude of the pore pressure on the surface of the pipeline and the greater the amplitude of the horizontal force acting on the pipeline.
- (3) The anisotropy of the seabed permeability also has a significant impact on the force acting on the concrete pipeline; the larger the ratio of k_x/k_z , the smaller the amplitude of the pore pressure on the surface of the pipeline, and the greater the horizontal force acting on the pipeline.

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