Abstract: Anti-sliding stability safety is a critical issue that must be given sufficient and widespread attention during the entire lifecycle of gravity dams. The calculated anti-sliding stability safety factor (ASS-SF) is usually compared with the allowable value required by the standards in the traditional method, which ignores the influence of material parameter uncertainties and leads to unreasonable safety evaluation results. Therefore, the nonlinear functional relationship between the stability safety factor (SF) and the random variable parameters is constructed based on the response surface equations, and the distribution types of SF sequences calculated by the Monte Carlo sampling are determined, then a probabilistic stability evaluation method for concrete gravity dams is proposed. Engineering application shows that the calculated SF obeys the normal distribution; the minimum guaranteed rate of different sliding paths in a gravity dam is 86.66%, and the guaranteed rate for the overload safety factor (OSF) is 36.00%. The results imply that a guaranteed rate for the allowable value of the ASS-SF should be provided when making the stability safety evaluation of the dams, especially the OSF. The outcome of this research will advance the understanding of stability evaluation of concrete dams, and reduce the potential risk of sliding instability of concrete dams.

Keywords: concrete gravity dams; anti-sliding stability; safety factor; parameter uncertainties; guaranteed rate

1. Introduction

The failure of gravity dams can cause serious consequences, making dam safety a growing concern. Gravity dams are known to have multiple failure modes, including stress-controlled strength failure, overturning failure, and instability failure. The standards for controlling these failure modes vary depending on the load conditions, meaning that the control values differ under static and dynamic loading conditions. The instability failure is one of the most important failure modes for gravity dams, the control of which under the static loadings is discussed in this study. For the traditional determined analysis model of safety factor (SF) for dams, the calculated SF is a determined value, and the criterion for assessing whether the anti-sliding stability of the structure meets the requirements depends on whether it exceeds the allowable safety factor (ASF) specified in the standard \[1,2\]. However, the determined SF cannot reflect the inherent uncertainties of material properties or the loads acting on the dam system, thus proposing a probability evaluation method of SFs is of great importance for dam safety.
Safety risk analysis is an important part of dam design, construction, and operation, and has increasingly become a research focus and trend in the field of hydraulic and geological engineering, involving static and dynamic behavior analysis, risk analysis methods, and control standards of dams. For the behavior analysis of dams, Lu et al. [3] studied the impact of spatial variability of concrete material parameters (e.g., tensile strength) on the dynamic response of gravity dams. Ran et al. [4] investigated a risk analysis model for gravity dam safety considering material parameter spatial variability, and their case study showed that the instability failure risk is significantly affected by the parameter spatial variability. Khiavi et al. [5] proposed a new probabilistic model to study the behavior of dams under the action of hydrodynamic waves. Hariri-Ardebili et al. [6] investigated the influence of random field of materials on gravity dams using the linear elastic model under the action of the earthquake. For the risk analysis methods of dams, Li et al. [7] proposed a hybrid reliability assessment method for the arch dams, and then the dam’s probability reliability index and non-probability reliability index were investigated. Chen et al. [8] developed a seismic reliability assessment model considering the randomness and time-variability of materials. Wu et al. [9] discussed the suggested SF value for the dam slope stability with a dam height of over 200 m by the determined method. Zhou et al. [10] proposed the absolute and relative stress control indices for the arch dam under the condition of penetrated damage. Han et al. [11] proposed safety control indicators for the earth rock dam (height: 250 m) from the aspects of overall deformation, co-ordinated deformation of the dam body, seepage safety, seismic subsidence safety, structural and material durability, and structural safety. Wang et al. [12] studied a probabilistic stability analysis model for earth rock dams using multivariate adaptive regression splines’ soft calculation algorithm, which effectively evaluated the effect of parameter changes on the probability of slope failure. Siacara et al. [13] proposed a reliability analysis method by coupling the analysis software and the reliability solver, and the results indicate that the most dangerous sliding surface of the dam slope based on the minimum SF and reliability index is not consistent. Li and Su [14] put forward a determination method for the deformation safety threshold for gravity dams by discussing the relationship between the water pressure component and the reliability index of stability. However, lots of dams have been built, or are about to be built, in alpine valleys with diverse and discontinuous foundation conditions (i.e., joints, cracks, etc.), leading to uncertainties of material properties in the soil–rock foundation [15]. Neglecting material randomness will result in an unreasonable assessment of the stability safety of the gravity dams under complex geological and operating conditions. For the anti-sliding stability safety factor (ASS-SF), the role of the parameter uncertainties when evaluating the anti-sliding stability can be studied from the perspective of probability theory [16]. However, there is currently no effective research on how to quantify its impact, although some probabilistic methods have been used in the probabilistic and reliability analysis of concrete dams [17–19].

As the height of gravity dams increases, their importance becomes more and more significant. These high concrete gravity dams have complex climates, terrains, and geological environments, with high hydraulic loads, high seepage pressures, and complex combinations of weak rock structure. Gravity dam stability is particularly important under the influence of uncertainties such as material parameters, effects, models, and boundary conditions, and evaluating the stability remains a key scientific and technological challenge that has yet to be fully addressed. Therefore, the objective is to put forward a probabilistic assessment model for the ASS-SF in gravity dams. The research results will help quantify the impact of parameter uncertainty on the ASS-SF in the gravity dams, and assist us in truly evaluating the anti-sliding stability safety margin.

2. Overview of the ASF for Concrete Dams

The operation of dams often has enormous social benefits. Once a dam breaks, it will cause huge disasters, especially now that the dam height is getting higher and the storage
capacity is getting larger. The allowable safety factor (ASF) is still one of the important parameters used to judge whether the structure is stable, so a reasonable ASF is critical.

Nowadays, the determination of an ASF in the world is formed based on long-term engineering practice, and scholars around the world have conducted lots of relevant research based on these ASFs, involving the calculation method of the SF, the calibration of new dam-building materials’ SF, and the impact of non-linear parameters on the stability, etc., as shown in Table 1. The calculation method significantly affects the stability SF of dams or slopes, but the corresponding ASF is unchanged, which is related to the structural grade, working conditions, etc. Tables 2 and 3 show the standards for SFs in some countries. It can be found that the ASF varies from country to country, and whether there is material strength parameter test information has an important impact on the ASF, that is, the richness of strength parameter information helps to reduce the uncertainty in the determination of ASF. The current standard in China has no relevant provisions or descriptions on the ASF under the premise of strength parameter test information, so it may be unreasonable to adopt this method due to the existence of parameter uncertainty. However, it is difficult to determine the new ASF, so we can start from the impact of parameter uncertainty on the evaluation results, that is, the discriminating criterion that “As long as the calculated SF is bigger than the ASF, the structure is stable” needs to be adjusted considering the uncertainty of parameters. Therefore, it is necessary to propose a material parameter uncertainties-based probabilistic evaluation method for the ASS-SF in gravity dams, which can help to reasonably evaluate the anti-sliding stability safety margin and avoid potential instability failure hazards.

Table 1. A summary of the safety factors study of the dams and slopes.

<table>
<thead>
<tr>
<th>No.</th>
<th>Types</th>
<th>Methods</th>
<th>Allowable Safety Factor</th>
<th>Country</th>
<th>Ref.</th>
</tr>
</thead>
</table>
| 1   | GD    | RLEM    | Peak sliding factor: 3.0
      |        |         | Residual sliding factor: 1.5
      |        |         | Overturning factor: 1.2
      |        |         | Uplifting factor: 1.2
      |        |         | Usual, no material tests: 3.0
      |        |         | Usual, with material tests/ unusual, no material tests: 2.0 | Canada | [20] |
| 2   | GD    | RLEM    | Unusual, with material tests: 1.5
      |        |         | Extreme, no material tests: 1.3
      |        |         | Extreme, with material tests: 1.1
      |        |         | Structure grades I: 2.9
      |        |         | Structure grades II: 2.6
      |        |         | Structure grades III: 2.2 | Canada | [21] |
| 3   | CSGD  | RLEM    | Structure grades I: 2.9
      |        |         | Structure grades II: 2.6
      |        |         | Structure grades III: 2.2 | China | [22] |
| 4   | AD    | RLEM    | Structure grades I: 2.9
      |        |         | Structure grades II: 2.6
      |        |         | Structure grades III: 2.2
      |        | FEM     | 2.5 | China | [23] |
| 5   | GD    | RLEM    | Usual: 1.5
      |        |         | Extreme: 1.0
      |        |         | Unusual: 1.2 | Spain | [24] |
| 6   | ED    | RLEM    | At end of construction: 1.3
      |        |         | At steady state: 1.5
      |        |         | During sudden drawdown: 1.3 | Ethiopia | [25] |
| 7   | S     | RLEM    | 1.0 and 1.5 | Iran | [26] |
| 8   | S     | RLEM    | 1.3 | China | [27] |
| 9   | S     | SRE     | 1.0 and 1.5 | Indonesia | [28] |
| 10  | S     | /       | 1.0 | Korea | [29] |
| 11  | S     | SRE     | 1.35 | USA | [30] |
| 12  | S     | /       | 1.4 | India | [31] |

Note: GD, gravity dam; AD, arch dam; ED: embankment dam; S: slope; CSGD, cemented sand and gravel dam; RLEM, rigid limit equilibrium method; FEM, finite element method; SRE, strength reduction method.
Table 2. Allowable safety factors without considering the uncertainties in the related standards.

<table>
<thead>
<tr>
<th>Working Condition</th>
<th>Allowable Safety Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Usual</td>
<td>3.0</td>
</tr>
<tr>
<td>Unusual</td>
<td>2.0</td>
</tr>
<tr>
<td>Extreme</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 3. Allowable safety factors depending on the a priori information in the related standards.

<table>
<thead>
<tr>
<th>Working Condition</th>
<th>Allowable Safety Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CAD [36]</td>
</tr>
<tr>
<td>A Priori Information or Not</td>
<td>No</td>
</tr>
<tr>
<td>Usual</td>
<td>3.0</td>
</tr>
<tr>
<td>Unusual</td>
<td>2.0</td>
</tr>
<tr>
<td>Extreme</td>
<td>1.31</td>
</tr>
</tbody>
</table>

3. Probabilistic Anti-Sliding Stability Evaluation Methods

3.1. Definition of SF

In civil engineering, the SF $K$ is defined as

$$K = \frac{\sum R}{\sum S}$$

where $R$ and $S$ are the resistance and loading, respectively.

Usually, the ASS-SF $K_S$ in the gravity dams can be given by

$$K_S = \sum_{i=1}^{n} \left( f'_i \sigma_i + c'_i \right) l_i \div \sum_{i=1}^{n} \tau_i l_i$$

where $f'_i$ and $c'_i$ are the shear strength parameters of the foundation element $i$ on the sliding channel; $\sigma_i$ and $\tau_i$ are the normal, shear stresses of element $i$, respectively; $n$ is the element number along the sliding channel; $l_i$ is the length of element $i$ along the direction of the sliding channel.

The overload safety factor (OSF) is another important indicator of dam safety. When the loading increases to the $K_H$ multiple of the designed loading, the deep anti-sliding stability in the foundation reaches a critical state, and the corresponding value is the OSF. In this paper, the commonly used over-water level method is introduced to calculate $K_H$, as shown in Figure 1 and Equation (3).

$$k_H = \frac{\gamma H_0 \Delta H + 0.5 \gamma H_0^2}{0.5 \gamma H_0^2} = \frac{2 \Delta H + H_0}{H_0} = 2 \left( \Delta H / \frac{H_0}{H_0} \right) - 1$$

$K_H$ is closely related to the instability criteria of the gravity dams on complex foundations. The convergence criteria, feature point displacement mutation criteria, and the run-through of plastic zone criteria are the commonly used criteria in civil engineering. However, there is no unified standard for the specific criteria used to determine the critical instability in the gravity dams on complex foundations. The convergence criterion is selected as the instability criterion of the gravity dams in this paper due to its ability to distinguish the limit state of the gravity dams under the random fluctuation of parameters.
3.2. Probabilistic Evaluation Method for the Safety Factors in Dams

During the operation of a gravity dam, the loading acting on the system and the material parameters have certain fluctuation and randomness, which have a direct relationship with the safety state of the system. However, the functional relationship between the SF and the parameters is difficult to express explicitly. Taking the parameters with uncertainties as random variables (RVs), the functional relationship above can be expressed approximately using the response surface method \[39,40\], that is, taking the \( K_S \) and the \( K_H \) as the performance function values and constructing a quadratic response surface equation without cross-terms:

\[
K_S \text{ or } K_H = a_0 + \sum_{i=1}^{m} b_i x_i + \sum_{i=1}^{m} c_i x_i^2 \tag{4}
\]

where \( m \) represents the RVs’ number; \( x_1, x_2, \ldots, x_m \) represents the RV that affects the anti-sliding stability in the gravity dams, such as the elastic modulus, the shear strength index, the upstream water level, etc.; \( a_0, b_i, c_i \) are the parameters to be solved.

In this paper, the term “guaranteed rate” of the SF is defined as the probability of being greater than a certain determined value in the SF sequence, which can quantitatively reflect the influence of parameter uncertainties on the SF, thereby making a more reasonable evaluation of dam stability safety. It is usually represented by the cumulative frequency of the calculated value greater than the allowable value. Therefore, if the \( K_S \) or the \( K_H \) obeys a distribution characteristic with the probability density curve of \( f(x) \), the guaranteed rate of SF can be expressed by Equation (5). Figure 2 is the sketch of the guaranteed rate if \( K_S \) or \( K_H \) obeys normal distribution.

\[
P(K_S \text{ or } K_H > k) = \int_k^{+\infty} f(x) \, dx \tag{5}
\]

where \( P \) is the SF guaranteed rate; \( k \) is the ASF; and \( f(x) \) is the SF’s probability density distribution function.

Lastly, the flowchart is shown in Figure 3.
where \( P \) is the SF guaranteed rate; \( k \) is the ASF; and \( f(x) \) is the SF's probability density distribution function.

Lastly, the flowchart is shown in Figure 3.

### 4. Illustrative Example

#### 4.1. Project Specifications

A factual gravity dam with a maximum height of 200 m, crest width of 14 m, and normal storage level (NSL) of 1190 m was adopted in this study to be the case study. In the basalt foundation, there were several groups of joint cracks and two broken zones, A and B, which were detrimental to the dam’s stability (Figure 4). A variety of composite slip surfaces were formed due to the multiple joint cracks and broken zones in the rock mass, including four potential deep sliding channels composed of joint fissures and broken zones, and one sliding channel along the foundation surface, as shown in Table 4.
Buildings 2024, 14, x FOR PEER REVIEW 7 of 17

Figure 4. The Sectional drawing of the dam structure (including geological information) (unit: m).

Table 4. Potential sliding channels of the gravity dam.

<table>
<thead>
<tr>
<th>Code</th>
<th>Component</th>
<th>Safety Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path 1</td>
<td>①, A, ③, ④</td>
<td>Ks1</td>
</tr>
<tr>
<td>Path 2</td>
<td>①, ④</td>
<td>Ks2</td>
</tr>
<tr>
<td>Path 3</td>
<td>②, ⑤</td>
<td>Ks3</td>
</tr>
<tr>
<td>Path 4</td>
<td>②, B</td>
<td>Ks4</td>
</tr>
<tr>
<td>Path 5</td>
<td>Foundation surface</td>
<td>Ks5</td>
</tr>
</tbody>
</table>

4.2. Dam-Foundation System Model and Parameters

The model was meshed with the ANSYS software (19.0) with the SOLID45 isoparametric elements: the element number of the model was 34,503 and the number of nodes was 47,938. The two sides and the surfaces of the upstream, downstream, and bottom in the foundation were all normal constraints. The calculated loading included the weight of the dam and foundation, upstream water pressure, and uplift pressure.

Dam failure begins with local plastic deformation and gradually progresses to yield, forming a failure path. Considering the mechanical properties and potential failure modes, the Drucker–Prager criterion is introduced in this paper as the criterion for anti-sliding stability failure [41,42]. Relevant research indicates that the stability SF of dams is related to their corresponding failure probability, therefore determining the uncertainty characterization of parameters is crucial [9,43]. Based on the Drucker–Prager model, the test data in a similar gravity dam [44] and the monitoring data, the material parameters, and their statistical characteristics for this dam are shown in Tables 5 and 6.

Table 5. Material parameters of the gravity dam.

<table>
<thead>
<tr>
<th>Location</th>
<th>E (GPa)</th>
<th>Poisson Ratio</th>
<th>Density (kg/m³)</th>
<th>c’ (MPa)</th>
<th>f</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dam</td>
<td>28.00</td>
<td>0.20</td>
<td>2400</td>
<td>1.20</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>Foundation</td>
<td>20.00</td>
<td>0.22</td>
<td>2800</td>
<td>2.00</td>
<td>1.40</td>
<td></td>
</tr>
<tr>
<td>Fracture zone</td>
<td>0.50</td>
<td>0.35</td>
<td>2000</td>
<td>0.10</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>Joint</td>
<td>/</td>
<td>/</td>
<td>2800</td>
<td>0.20</td>
<td>0.80</td>
<td>Connectivity rate is 60%</td>
</tr>
</tbody>
</table>

Note: E is the elastic modulus; c’ is the cohesive force; and f is the friction coefficient.
Table 6. Statistical characteristics of Rvs.

<table>
<thead>
<tr>
<th>Location</th>
<th>Material Parameter</th>
<th>Distribution Type</th>
<th>Mean Value</th>
<th>Variation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dam</td>
<td>$E$ (GPa)</td>
<td>Lognormal</td>
<td>28.00</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>$c'$ (MPa)</td>
<td>Lognormal</td>
<td>1.20</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>$f'$</td>
<td>Normal</td>
<td>1.20</td>
<td>0.15</td>
</tr>
<tr>
<td>Foundation</td>
<td>$E$ (GPa)</td>
<td>Lognormal</td>
<td>20.00</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>$c'$ (MPa)</td>
<td>Lognormal</td>
<td>2.00</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>$f'$</td>
<td>Normal</td>
<td>1.40</td>
<td>0.20</td>
</tr>
<tr>
<td>Fracture zone</td>
<td>$E$ (GPa)</td>
<td>Lognormal</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>$c'$ (MPa)</td>
<td>Lognormal</td>
<td>0.10</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>$f'$</td>
<td>Normal</td>
<td>0.60</td>
<td>0.25</td>
</tr>
<tr>
<td>Joint</td>
<td>$c'$ (MPa)</td>
<td>Lognormal</td>
<td>0.20</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>$f'$</td>
<td>Normal</td>
<td>0.80</td>
<td>0.25</td>
</tr>
<tr>
<td>Upstream water level (m)</td>
<td>Normal</td>
<td>190.00</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

4.3. Analysis of Safety Factor

4.3.1. Safety Factor of Anti-Sliding Stability

Reliability is widely recognized as a comprehensive measure of the safety margin across various structures. It can be quantified using the reliability index (RI), which is typically obtained through the Monte Carlo (MC) method [45]. Figure 5 shows the SFs and the RIs of different paths (red line: safety factor; blue line: reliability index). According to the relevant standard in China [46], the ASF $K$ is 3.0. The results show that the five paths’ SFs are all greater than the ASF, indicating that the anti-sliding stability meets the safety requirements. Additionally, it can be seen that the SF and the RI of the same sliding channel are not completely equivalent. Taking Path 3 and Path 5 as examples, the SF of Path 3 is the largest with a value of 4.62, while its reliability index is not the largest. The reason may be that the influence of the randomness of material parameters is not considered in the calculation of SFs. Therefore, the guaranteed rate of the ASS-SF should be introduced to make a reasonable safety evaluation of the gravity dams.

![Figure 5. Comparison of the SF and RI in the gravity dams.](image-url)

4.3.2. Overload Safety Factor

Taking the normal storage level as the beginning of the overloading action, the water pressure of 0.2 times the dam height was selected as the step size, and the overload was carried out step by step until the calculation did not meet the convergence criteria. The development process of the plastic zone at the dam heel under different overloading is shown in Figure 6. The relationship curves among the OSF, the crest displacement, and the cumulative number of iterations are expressed in Figure 7. When $K_H = 1.0$, a small
local plastic zone appeared at the dam heel and the upstream side of the foundation joint ①. With the increase in upstream water loading, the area of the plastic zone increased gradually. In detail, the foundation plastic zone at the dam heel developed deeply, the plastic zone in joints ① and ② gradually developed downstream, and the plastic zone was mainly concentrated near joint ①. When $K_{uH} = 2.2$, the plastic strain of joint ① changed rapidly, and the displacement in the dam crest clearly changed (Figure 7a). When $K_{uH} = 3.4$, the numerical simulation did not meet the convergence criterion (Figure 7b), and the dam reached the ultimate bearing state with an OSF of 3.4.

4.4. Probability Analysis of Safety Factor

Because of the small variation coefficients of concrete and rock foundation density, Poisson’s ratio, structure size, etc., the randomness of these parameters is not considered in this paper. Therefore, the $E$, $c'$, $f'$, and upstream water level were chosen as the RVs, and their information is displayed in Table 6.

According to the calculation results, the response surface equation (RSE) sequences are established and solved using the Taguchi design method [47]. The results show that the RSEs’ multiple correlation coefficients are above 0.95, implying that the regression effect is of high accuracy.

Based on the RSEs, the MC sampling with a sampling frequency of $10^7$ was conducted, and then the distribution characteristics of the SFs were tested using the Kolmogorov–Smirnov (K–S) test, and the results are given in Table 7. The significance of the K–S test is greater than 0.05, indicating that the SF samples obey the normal distribution characteristics. The frequency distribution curve (PDF) and the cumulative distribution curve (CDF) of different SFs are given in Figure 8. Compared with the determined values of the ASS-SF in Figure 5 with the probability of 100%, the SFs calculated by the probabilistic methods

Figure 6. The development process of the plastic zone.

Figure 7. Relationship curves among the OSF, the crest displacement, and the cumulative iteration times.

(a) Dam crest displacement (b) Cumulative number of iterations
have obvious differences. As seen in Figure 8, the guaranteed rate for the SF of 3.36 on the foundation surface (Path 5) is 63.30%, and those of other four Paths 1–4 are between 52.16% and 55.11%. The guaranteed rate for the OSF (3.4) is 36.00%. As seen in Table 7, taking the ASF (3.0) as the evaluation index, the minimum guaranteed rate for the ASS-SF in paths 1–5 is over 86.66%. The results show that it is dangerous when using a determined SF to characterize the stability and safety of the dams. Therefore, a guaranteed rate for the allowable value of ASS-SF should be provided when evaluating the stability safety of the gravity dams, and a corresponding suggested value for the guaranteed rate is not less than 85%. It is noted that the guaranteed rate for the allowable value of the ASS-SF should be determined based on the characteristics of geology, loading, and environment of the gravity dam.

Table 7. Statistical characteristic values and the guaranteed rate of different channel safety factors and overload safety factors sequences.

<table>
<thead>
<tr>
<th>Statistical Parameters</th>
<th>$K_{S1}$</th>
<th>$K_{S2}$</th>
<th>$K_{S3}$</th>
<th>$K_{S4}$</th>
<th>$K_{S5}$</th>
<th>$K_{H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>3.53</td>
<td>3.96</td>
<td>4.68</td>
<td>3.70</td>
<td>3.56</td>
<td>2.03</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.41</td>
<td>0.51</td>
<td>0.61</td>
<td>0.41</td>
<td>0.53</td>
<td>0.7</td>
</tr>
<tr>
<td>Guaranteed rate</td>
<td>54.90%</td>
<td>53.90%</td>
<td>52.20%</td>
<td>55.10%</td>
<td>63.30%</td>
<td>36.00%</td>
</tr>
<tr>
<td>Guaranteed rate for $</td>
<td>89.99%</td>
<td>96.77%</td>
<td>99.72%</td>
<td>95.84%</td>
<td>86.66%</td>
<td>/</td>
</tr>
<tr>
<td>$K = 3.0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sig. (K-S inspection)</td>
<td>0.13</td>
<td>0.06</td>
<td>0.12</td>
<td>0.11</td>
<td>0.06</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Figure 8. Distribution of safety factors and overload safety factors of different channels.
In addition, the reason for the difference between the ASS-SF and the OSF may be that they are two different methods of evaluating stability safety, and the sensitivity to random variables is different, resulting in different distribution laws.

4.5. Sensitivity Analysis of Material Parameters

4.5.1. Influence of Shear Resistance Parameter Distribution Type on the Guaranteed Rate

Usually, the distribution type of shear strength parameters $f'$ and $c'$ generally follows the normal and log-normal distributions in civil engineering. Due to the large dispersion of foundation material parameters, the distribution type of shear strength parameters $f'$ and $c'$ in the dam-foundation system may have different combinations. Four combinations of distribution types for shear strength parameters $f'$ and $c'$ in the gravity dam are expressed in Table 8, and the guaranteed rate of SF is shown in Figure 9. Results show that the guaranteed rate in Case 4 is the lowest, and the anti-sliding stability reliability is also the lowest. Compared with Case 2, which is the suggested combination of shear strength parameters $f'$ and $c'$ in the standard [35], the reliability of Case 4 is more dangerous. Therefore, the determination of the distribution types for shear strength parameters can be carried out based on the shear parameter test values if there is enough geological prospecting data, otherwise it is suggested that the stability reliability of the dam be carried out with the parameter combination of Case 4, that is $f'$ and $c'$, that both follow the log-normal distributions.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c'$</td>
<td>Normal</td>
<td>Log-normal</td>
<td>Normal</td>
<td>Log-normal</td>
</tr>
<tr>
<td>$f'$</td>
<td>Normal</td>
<td>Normal</td>
<td>Log-normal</td>
<td>Log-normal</td>
</tr>
</tbody>
</table>

Figure 9. Relationship between combinations of $f'$ and $c'$ of each path and the safety guaranteed rate.

4.5.2. Influence of Correlation of Shear Resistance Parameters on the Guaranteed Rate

Studies have shown that the shear resistance parameters of $c'$ and $f'$ in rock mass have a certain correlation [48]. Based on the four combinations of distribution types in Table 6, the influence of correlation on the guaranteed rate for SFs in the dam-foundation system is discussed and the results are shown in Figure 10. It was found that the guaranteed rate for SFs in paths 1–5 decreases with the increase in correlation coefficient between $f'$ and $c'$. The reason may be that the increase in the correlation coefficient makes the sampling values more centralized (Figure 11), thereby making the SF sequences divergent. Taking the correlation coefficients 0.9 and $-0.9$ as examples, the differences among the guaranteed
rates under Cases 1–4 are between 3.69% and 18.78%, implying that the influence of the correlation between \( f' \) and \( c' \) on stability safety evaluation cannot be ignored.

**Figure 10.** The influence of the correlation coefficient on the guaranteed rate.

**Figure 11.** The sampling value with correlation coefficients (CC) of 0 and 0.5.

5. Discussion

5.1. Difference of Discrimination Criteria for Anti-Sliding Stability

From the above research, it can be seen that, under the influence of parameter uncertainty, if the traditional discrimination criteria are used to evaluate the stability safety, the probability level of evaluation results is not 100%. Taking Path 1 in Table 4 as an example, the determined value of the SF is 3.46. For the traditional method, the status of Path 1 is stable due to 3.46 being greater than 3.0, with a potential probability level of 100%. However, the SF is subject to the normal distribution with an average value of 3.53 and a standard deviation of 0.41 after considering the uncertainties of parameters (as shown in Table 7), and the probability level that the SF is bigger than 3.0 is 89.99%, not 100%. What is more, the OSF is a factor used to reflect the capacity of dams to bear overloads. As shown in Figure 7, the OSF is 3.4, but the actual guaranteed rate that can meet the overload capacity
is only 36%, indicating that the probability of this dam having an overload capacity of 3.4 times is only 36%, which is why traditional methods are not reasonable. Similarly, the arch dam abutment stability assessment based on SFs also has such problems, and the promotion and application results of an arch dam are displayed in Figure 12. It was found that the guaranteed rate that the SF of dam abutment stability on the banks is bigger than the ASF (3.5) was 99.99%, but the guaranteed rate of the OSF over 6.4 obtained by the traditional method was only 47.12%.

5. Discussion

5.1. Difference of Discrimination Criteria for Anti-Sliding Stability

From the above research, it can be seen that, under the influence of parameter uncertainty, if the traditional discrimination criteria are used to evaluate the stability safety, the probability level of evaluation results is not 100%. Taking Path 1 in Table 4 as an example, the determined value of the SF is 3.46. For the traditional method, the status of Path 1 is stable due to 3.46 being greater than 3.0, with a potential probability level of 100%. However, the SF is subject to the normal distribution with an average value of 3.53 and a standard deviation of 0.41 after considering the uncertainties of parameters (as shown in Table 7), and the probability level that the SF is bigger than 3.0 is 89.99%, not 100%. What is more, the OSF is a factor used to reflect the capacity of dams to bear overloads. As shown in Figure 7, the OSF is 3.4, but the actual guaranteed rate that can meet the overload capacity is only 36%, indicating that the probability of this dam having an overload capacity of 3.4 times is only 36%, which is why traditional methods are not reasonable. Similarly, the arch dam abutment stability assessment based on SFs also has such problems, and the promotion and application results of an arch dam are displayed in Figure 12. It was found that the guaranteed rate that the SF of dam abutment stability on the banks is bigger than the ASF (3.5) was 99.99%, but the guaranteed rate of the OSF over 6.4 obtained by the traditional method was only 47.12%.

Therefore, the method studied in this article can be extended to the stability problems of different types of dams, such as the stability of gravity dams, stability of arch dam abutments, and dam slope stability. The probabilistic methods for SF provide important theoretical support for setting reasonable stability thresholds and also help to clarify the ability of dams to resist extreme environments, especially with the significantly increased probability of extreme environments occurring globally.

Figure 12. Application results of abutment stability in an arch dam.
Therefore, the method studied in this article can be extended to the stability problems of different types of dams, such as the stability of gravity dams, stability of arch dam abutments, and dam slope stability. The probabilistic methods for SF provide important theoretical support for setting reasonable stability thresholds and also help to clarify the ability of dams to resist extreme environments, especially with the significantly increased probability of extreme environments occurring globally.

5.2. Future Research Directions

It should be noted that the guaranteed rate of using the ASF to evaluate the stability safety in gravity dams and the abutment stability in arch dams is affected by the parameter uncertainty. Maybe these guaranteed rates generally exceed 85%, although there is no significant impact on whether the structure is in a safe state. However, if a specific guaranteed rate can be indicated during the evaluation, it will make the structural safety assessment more reasonable. For the OSF, the guaranteed rate (e.g., 36% for the gravity dam, and 47.12% for the arch dam) is much lower than the stable SF. If the OSF is directly regarded as the overload capacity of the dam, it may cause dam failure in extreme cases. Combined with the research results above, a guaranteed rate of 85% is suggested for the overload capacity assessment of dams. As shown in Table 9, the OSF with a guaranteed rate of 85% is smaller than that obtained by traditional methods, with a decrease rate even approaching 50%, indicating that the OSF obtained by traditional methods cannot reflect the true overload capacity of the dam under extreme conditions. However, Related studies have shown that the ASF for dam slope stability should vary depending on the dam height [9]. Similarly, if we want to extend the content of this study to all gravity dams, a lot of research is still needed, such as the sequence law of ASS-SFs for gravity dams under different dam heights, working conditions, and foundation materials. The model proposed in this paper cannot be directly applied to failure modes such as dam overtopping that do not use SFs as safety indicators, and further research is needed.

Table 9. The overload safety factor for dams with a guaranteed rate of 85%.

<table>
<thead>
<tr>
<th>Terms</th>
<th>Gravity Dams</th>
<th>Arch Dams</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overload safety factor</td>
<td>3.4</td>
<td>6.4</td>
</tr>
<tr>
<td>Guaranteed rate</td>
<td>36%</td>
<td>47.12%</td>
</tr>
<tr>
<td>Overload safety factor with a guaranteed rate of 85%</td>
<td>1.82</td>
<td>5.32</td>
</tr>
</tbody>
</table>

6. Conclusions

(1) Taking the uncertain parameters as random variables, the sequences of safety factors and their distribution types were obtained using the RSEs and MC method, respectively, and a probabilistic evaluation method for the ASS-SF in the gravity dams was proposed.

(2) The case study shows that both the ASS-SF and the overload safety factor in gravity dams obeyed the normal distribution, considering the uncertainties of parameters. The guaranteed rates for the ASS-SF and the OSF were 86.66% and 36.00%, respectively. The distribution type combinations and the correlation between \( f' \) and \( c' \) had a certain impact on the guaranteed rate of the \( K_s \) in the gravity dams.

(3) It is indicated that the probability that a determined and unique safety factor that is greater than the ASF during the safety evaluation of concrete dams may not necessarily be 100%. The impact of material parameter uncertainty is not unique but rather depends on the type of dam, failure mode, and material characteristics.

Additionally, the guaranteed rate of the ASS-SF of gravity dams or arch dams is greatly affected by the randomness of parameters. This article only discusses one gravity dam and one arch dam, which is not sufficient to represent all concrete dams. Therefore, specific concrete gravity dam projects need to be discussed. In addition, the guaranteed
rate proposed in this article is only a suggested value, and further work is needed based on a large amount of in-depth research.

**Author Contributions:** Conceptualization, D.W. and X.L.; methodology, D.W., X.L., C.Z., K.G., L.P. (Litan Pan) and Z.Z.; software, C.Z., K.G., L.P. (Litan Pan), L.P. (Liang Pei) and Z.Z.; formal analysis, X.L., K.G., L.P. (Litan Pan) and Z.Z.; investigation, D.W.; resources, C.Z. and L.P. (Litan Pan); data curation, X.L., K.G. and Z.Z.; writing—original draft, D.W.; writing—review and editing, X.L. and L.P. (Liang Pei). All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was supported by the National Natural Science Foundation of China (No. 52309162) and the Fundamental Research Funds for the Central Universities (YJ202287).

**Data Availability Statement:** The data presented in this study are available on request from the corresponding author.

**Conflicts of Interest:** Authors Daquan Wang, Chengzhi Zheng, and Litan Pan were employed by the company Huadian Electric Power Research Institute Co., Ltd., Ke Gong was employed by the company Dagu Hydropower Branch of Huadian Xizang Energy Co., Ltd., and Zepeng Zhao was employed by the company Chengdu Municipal Engineering and Research Design Institute Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as potential conflicts of interest.

**References**


37. ANCOLD Guidelines on Design Criteria for Concrete Gravity Dams; Australian National Committee on Large Dams: Hobart, Australia, 2013.

38. USACE Stability Analysis of Concrete Structures; USACE: Washington, DC, USA, 2005.


