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Piezoelectric Gauge of Small Dynamic Bending Strains

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Abstract: This paper is devoted to a new gauge of small dynamic bending deformations of structures. Unlike previously existing strain gauges that measure elongation or compression at a certain point on the surface of a deformable body, the proposed gauge measures the change in curvature at a point on the surface of a deformable body and does not respond to elongation–compression strains. The gauge is a layered bar made of piezoelectric and elastic materials. It functions using the direct piezoelectric effect. In order to competently study the deformed state of a structure at points on a surface, it is necessary to determine all components of the strain tensor. The gauges currently used measure only elongational or compressive strains, which does not provide a complete picture of the strain state. It is very important to complement these deformations with bending strains measured by the new gauge.

Keywords: strain gauge; bending strain gauge; elasticity; electroelastic composite; energy conversion efficiency coefficient



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1. Introduction

For reliable operation of a structure, it is important not only to perform a qualitative calculation of its stress–strain state (SSS) but also to study the SSS of the structure using experimental methods. Experimental analysis of the stress–strain state of a body as a rule consists of measuring deformations on the surface of the object under study. To measure strains on the surface of a deformable body, various methods are used: mechanical, acoustic, optical, holographic interference, piezoelectric, etc. Currently, the vast majority of measurements are made using strain gauge resistors. The idea of strain gauges is still in use today. Their first experimental application was by W. Thomson (Lord Kelvin) [1] and dates back to the middle of the 19th century. A Kelvin strain gauge is a conductor placed on the surface of the body under study. It is assumed that the conditions for ideal contact between the conductor and the deformable body are met. The conductor through which electrical current is passed is deformed along with the body under study. As a result of longitudinal deformation of the conductor (elongation or compression), its cross-sectional area changes and the resistance of the conductor changes. Deformation of the conductor is calculated from the measured change in resistance.

At the end of the 20th century, piezoelectric gauges became widespread. The theory of the piezoelectric gauge was constructed and experiments were performed [2,3]. Thin-walled elements made of pre-polarized piezoelectric ceramics with a strong piezoelectric effect were used as a gauge. The surfaces of the piezoelectric element were covered with electrodes. The piezoelectric gauge was glued to the surface of the deformable body and deformed along with it. The operation of the gauge is based on the use of the direct piezoelectric effect. The direct piezoelectric effect can be described as follows. When a piezoelectric element is mechanically loaded, an electric charge appears on its electrodes. For a material with a strong piezoelectric effect, a significant part of the mechanical energy of the deformable piezoelectric element is converted into electrical energy. Tangential strains are calculated from the measured electric potential difference across the gauge electrodes.

Currently, a lot of papers are devoted to piezoelectric gauges [4–15]. It should be noted that piezoelectric materials are widely used in modern electronics, robotics, sound emitters and receivers, various measuring devices, and all kinds of actuators, etc. Piezoelectric materials are characterized by stable properties over wide temperature and time ranges and low cost.

Let us analyze the differences between a classical Kelvin gauge and a piezoelectric gauge. The Kelvin gauge measures a small value—the change in resistance of a conductor due to its deformation. The change in the resistance of a conductor is proportional to the change in its cross-sectional area. Note that the change in resistance as a result of deformation of the conductor is very small (it is a fraction of 1 percent). Its measurement requires additional equipment. The Kelvin gauge requires external sources of electrical energy.

A piezoelectric gauge measures the main quantity: the difference in electric potential across the electrodes of the piezoelectric element, which is proportional to the tangential deformation of the gauge. For piezoceramics with a strong piezoelectric effect, the measured electrical signal represents more than 30% of the total energy of the gauge. Another advantage of the piezoelectric gauge is the ease of determining deformations: to determine the deformations, we need only a piezoelectric gauge and a voltmeter. The piezoelectric gauge does not require external energy sources.

It should be noted that the piezoelectric gauge measures only dynamic strains. If a piezoelectric element is loaded with a static load, then a static SSS will arise in it. At the initial moment after loading, an electric charge will appear on the electrodes of the piezoelectric element, but the electric potential difference across the electrodes will quickly decrease to zero due to the current in the electrical circuit connecting the electrodes and the voltmeter, as well as due to the deposition of air ions on the electrodes.

Using the above gauges, it is possible to measure tangential strains in the vicinity of the point under study. Using a rosette of three gauges, it is possible to determine all components of the longitudinal strain deformation at a point on the surface of the body under study. However, this is not enough to determine the shape of a deformed element: in addition to elongations and shear, the surface of the element may become bent as a result of deformation. Bending deformations of a surface element can be characterized by the change in its curvature and torsion. Surface curvature is not measured by Kelvin strain gauges or piezoelectric gauges. The change in curvature can only be measured in the special case of a thin-walled structure. In this case, we should use two gauges located parallel on the faces of the thin structure at the points of intersection of the normal to the middle surface with the faces. From the difference in longitudinal strains on the faces, the bending strain can be determined, but in practice, this technique is often not feasible. For example, it is impossible to measure strains on the inside surface of a nuclear boiler or on the outside surface of a building dome covered with a layer of snow or ice, etc.

For elements of building structures, however, as well as for parts of machines and mechanisms, a load that causes their bending is very dangerous. According to the equations of state, the bending moments that arise in structures are proportional to the curvatures, so it is important to measure them. This is the topic of our paper.

In this paper, a new gauge is proposed for the first time—a small dynamic bending strain gauge.

2. Initial Equations

The proposed flexural strain gauge is a three-layer bar structure symmetric along the thickness. The outer layers are made of piezoelectric material with a strong piezoelectric effect. The middle elastic layer has a thickness of h_e , and each piezoelectric layer has a thickness of h_p . The bar is referred to using Cartesian coordinates. The x_1 axis is directed along the length of the bar, the x_2 axis is directed along the width of the bar, and the x_3 axis is orthogonal to them.

It is assumed that one of the piezoelectric layers is pre-polarized in the direction of the x_3 axis and the second layer is polarized in the opposite direction. There are electrodes on

the faces of the piezoelectric layers. They are shown with thicker lines in Figures 1 and 2. The geometry of the layered electroelastic bar and the direction of the preliminary polarization vectors of each of the piezoelectric layers are schematically depicted in Figure 1. The values of the electric potential on the electrodes for the case of tangential deformation are also indicated. From Figure 1, it can be seen that the difference in electric potential in the case of tangential deformations is equal to zero, i.e., the gauge does not respond to tangential deformations.

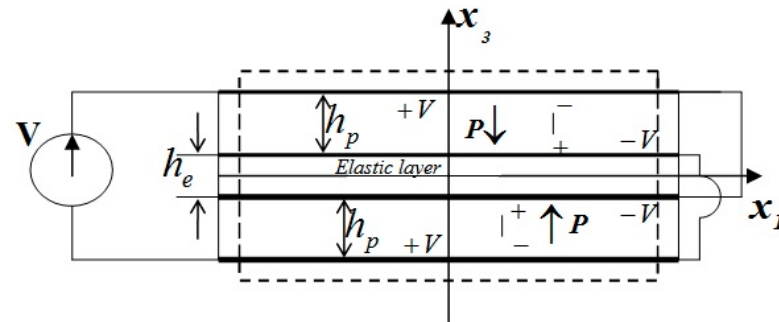


Figure 1. Scheme of operation of the gauge under longitudinal deformations.

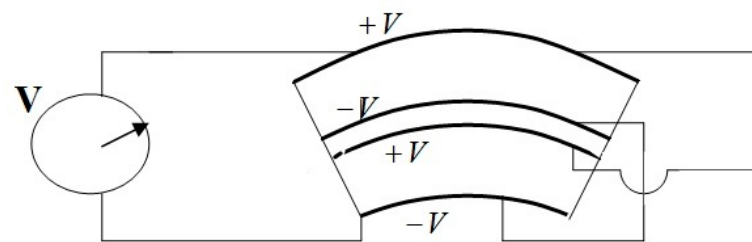


Figure 2. Scheme of operation of the gauge under bending deformations.

Figure 2 shows that in the case of bending deformation of the gauge, the potential difference is not zero.

To construct the theory of a piezoelectric gauge, we will use the theory of layered piezoelectric bars obtained in [16]. As in the theory of elastic bars, for piezoelectric bars in equations of state, the stresses σ_{22} and σ_{33} can be neglected in comparison with the main stress σ_{11} and it can be assumed that the electroelastic state does not depend on the x_2 coordinate.

The equations for elastic and electroelastic layers can be written in the following forms. Equations of motion:

$$\frac{\partial \sigma_i^{(k)}}{\partial x_i} + \frac{\partial \sigma_{ij}^{(k)}}{\partial x_j} = \rho_k \frac{\partial^2 u_i^{(k)}}{\partial t^2}, \quad i \neq j = 1, 3, \quad k = -2, 1, 2 \quad (1)$$

Equations of state for piezoelectric layers:

$$\sigma_1^{(\pm 2)} = \frac{1}{s_{11}^{(\pm 2)}} e_1^{(\pm 2)} \mp \frac{d_{31}^{(\pm 2)}}{s_{11}^{(\pm 2)}} E_3^{(\pm 2)} \quad (2)$$

$$D_3^{(\pm 2)} = \epsilon_{33}^{T(\pm 2)} E_3^{(\pm 2)} \pm d_{31}^{(\pm 2)} \sigma_{11}^{(\pm 2)}$$

Equations of state for an elastic layer:

$$\sigma_1^{(1)} = E e_1^{(1)} \quad (3)$$

Strain–displacement formula:

$$e_1 = \frac{\partial u_1}{\partial x_1} \quad (4)$$

Formula relating the electric field component to the electric potential:

$$E_3^{(\pm 2)} = -\frac{\partial \varphi^{(\pm 2)}}{\partial x_3} \quad (5)$$

The superscript in parentheses indicates the layer number. Here and below, each formula containing $\pm$ signs combines two formulas: we obtain one equation by leaving only the upper signs, and in the other only the lower ones. The piezoelectric layers have numbers (± 2) , and the elastic layer has number (1) .

In Formulas (1)–(5) $u_1^{(k)}$ and $e_1^{(k)}$ are the displacement and deformation in the x_1 direction, $E_3^{(\pm 2)}$ and $D_3^{(\pm 2)}$ are the components of the electric field vector and the electric induction vector in the x_3 direction, s_{11}^E is the elastic compliance at zero electric field, d_{31} is the piezoelectric constant, ε_{33}^T is the dielectric constant at zero voltage, $\varphi^{(\pm 2)}$ is the electric potential, and E is the modulus of elasticity of the elastic layer. The notation used here coincides with the notation adopted in [16].

Below are the boundary conditions for the electrical quantities that will be used in constructing the gauge theory.

If the electrodes are closed by an electrical circuit with a given complex conductivity $Y = Y_0 + iY_1$, then the current strength I in the electrical circuit is determined by the formula:

$$I = \int_{\Omega} \frac{dD_3}{dt} d\Omega = 2VY \quad (6)$$

When the electrodes are open, the electric current is zero:

$$I = \int_{\Omega} \frac{dD_3}{dt} d\Omega = 0 \quad (7)$$

If there is an electric potential V on the electrodes of the piezoelectric layers, then the conditions on the electrodes have the form:

$$\varphi^{(\pm 2)} \Big|_{x_3=\pm h} = \pm V, \quad \varphi^{(\pm 2)} \Big|_{x_3=\pm h_e} = \mp V \quad (8)$$

At short-circuited electrodes, the electric potential is zero:

$$\varphi^{(k)} \Big|_{x_3=z_k} = \varphi^{(k)} \Big|_{x_3=z_{k-1}} = 0 \quad (9)$$

3. Bending Strain Gauge Equations

It is shown in [3,16] that when the bar is bent, the electric potential changes along the x_3 coordinate of each piezoelectric layer according to a quadratic law:

$$\varphi^{(\pm 2)} = \varphi_{,0}^{(\pm 2)} + x_3 \varphi_{,1}^{(\pm 2)} + x_3^2 \varphi_{,2}^{(\pm 2)} \quad (10)$$

Here and below, the subscript after the decimal comma for the coefficients of a polynomial in the variable x_3 is equal to the power of x_3 for this coefficient.

Satisfying the electrical conditions (8), we obtain:

$$\varphi_{,0}^{(\pm 2)} = \mp \frac{h + h_e}{h_p} V + h \cdot h_e \varphi_{,2}^{(\pm 2)}, \quad \varphi_{,1}^{(\pm 2)} = \frac{2V}{h_p} \mp (h + h_e) \varphi_{,2}^{(\pm 2)}$$

Using Formula (5), we find that:

$$E_3^{(\pm 2)} = E_{3,0}^{(\pm 2)} + x_3 E_{3,1}^{(\pm 2)}, E_{3,0}^{(\pm 2)} = -\varphi_{,1}^{(\pm 2)}, E_{3,1}^{(\pm 2)} = -2\varphi_{,2}^{(\pm 2)} \quad (11)$$

For the bending strain gauge under consideration, the bending problem should be considered. The mechanical displacements and strains of any layer, from the linear theory of elastic bars, are written in the form:

$$u_3 = u_{3,0} = -w, u_1 = x_3 u_{1,1} = x_3 \frac{\partial w}{\partial x_1}, e_1 = x_3 \kappa, \kappa = \frac{\partial^2 w}{\partial x_1^2} \quad (12)$$

where w is the deflection and κ is the bending strain of the middle line of the bar $x_3 = 0$.

Substituting Formulas (11) and (12) into equations of state (1)–(3) and equating to zero the coefficients at the same powers of the variable x_3 , we obtain:

$$\sigma_{1,0}^{(1)} = 0, \sigma_{1,1}^{(1)} = E\kappa, \sigma_{1,0}^{(\pm 2)} = \mp \frac{d_{31}}{s_{11}^E} E_{3,0}^{(\pm 2)}, \sigma_{1,1}^{(\pm 2)} = \frac{1}{s_{11}^E} \kappa \mp \frac{d_{31}}{s_{11}^E} E_{3,1}^{(\pm 2)} \quad (13)$$

As shown in [3], the electrical induction vector in each piezoelectric layer covered with electrodes does not depend on the variable x_3 :

$$D_3^{(\pm 2)} = D_{3,0}^{(\pm 2)} = \varepsilon_{33}^T E_{3,0}^{(\pm 2)} \pm d_{31} \sigma_{1,0}^{(\pm 2)} \quad (14)$$

Using Formula (13), we obtain:

$$\begin{aligned} D_{3,0}^{(\pm)} &= \varepsilon_{33}^T (1 - k_{31}^2) E_{3,0}^{(\pm)}, \psi_{,2}^{(\pm 2)} = \pm \frac{k_{31}^2}{2(1 - k_{31}^2)} \frac{1}{d_{31}} \kappa \\ E_{3,0}^{(\pm)} &= -\frac{2V}{h_p} + (h + h_e) \frac{k_{31}^2}{2(1 - k_{31}^2)} \frac{1}{d_{31}} \kappa, E_{3,1}^{(\pm 2)} = \mp \frac{k_{31}^2}{1 - k_{31}^2} \frac{1}{d_{31}} \kappa \\ \sigma_{1,0}^{(\pm 2)} &= \pm \left(\frac{2V}{h_p} \frac{d_{31}}{s_{11}^E} - (h + h_e) \frac{k_{31}^2}{2(1 - k_{31}^2)} \frac{1}{s_{11}^E} \kappa \right), \sigma_{1,1}^{(\pm 2)} = \frac{1}{s_{11}^E} \frac{1}{1 - k_{31}^2} \kappa \end{aligned} \quad (15)$$

To obtain a formula for calculating bending deformation from the measured electric potential difference, we perform integration in Equation (6).

As a result, we obtain the following two formulas for determining bending deformation with an arbitrary dependence of the body's stress-strain state on time:

$$\frac{d\kappa}{dt} = \frac{2d_{31}}{k_{31}^2 (h + h_e)} \left(\frac{VY}{\varepsilon_{33}^T \Omega} + \frac{2}{h_p} \frac{dV}{dt} (1 - k_{31}^2) \right) \quad (16)$$

and for the case of harmonic vibrations of a deformable body according to the law $e^{-i\omega t}$ (ω is the circular frequency of vibrations):

$$\kappa = \frac{2V}{h_p} \frac{d_{31}}{k_{31}^2 (h + h_e)} \left(i \frac{Yh_p}{\varepsilon_{33}^T \omega \Omega k_{31}^2} + \frac{2(1 - k_{31}^2)}{k_{31}^2} \right) \quad (17)$$

On the right-hand sides of Formulas (16) and (17), all quantities are known.

Using Formula (16) or (17), we can determine the bending deformation at any point of the deformable body.

4. The Electromechanical Coupling Coefficient

Piezoelectric materials are widely used as converters of electrical energy into mechanical energy and vice versa, so it is very important to numerically evaluate the efficiency of energy conversion.

This characteristics of efficiency are commonly referred to as the electromechanical coupling coefficient (EMCC), but in the modern theory of electroelasticity, there is still no general agreement on how to calculate it. This paper is an extension of our previous research [17] on the EMCC where three different ways of calculating the EMCC, using the examples of beams, plates, and cylindrical shells, were analyzed.

The first formula is the most popular for calculating the EMCC [18], which we denote k_s , and it is given by:

$$k_s^2 = \frac{U_m^2}{U_e U_d} \quad (18)$$

where U_e , U_d , and U_m are the elastic, electric and interaction energy, respectively.

Formula (18) is widely used to determine the piezoelectric characteristics of a piezoelectric material, traditionally denoted k_{33} , k_{31} , etc. [19]. They are found by solving the static problems for electroelastic bodies of the simplest geometry, and these solutions are independent of the coordinates of points of the body and of time. Henceforth, we shall call these electroelastic states uniform states, and the EMCC k_s calculated from Equation (18) will be called the static EMCC.

As a rule, the electroelastic state of an actual piezoelectric element is not uniform. For non-uniform electroelastic states, the EMCC depends on many parameters, such as vibration frequency, the geometry of the piezoelectric element and its electrodes, and the mechanical and electrical boundary conditions. The values of the EMCC for actual piezoelectric elements are often smaller than the tabulated values of k_s for uniform states.

The second formula is Mason's formula [18]. It is often used to determine the EMCC in dynamic problems for oscillations at frequencies near resonance. We will call the EMCC calculated from this formula the dynamic EMCC and denote it k_d :

$$k_d^2 = \frac{\omega_a^2 - \omega_r^2}{\omega_a^2} \quad (19)$$

where ω_r^2 is the resonance frequency of the vibrations and ω_a^2 is the corresponding anti-resonance frequency of the vibrations.

The third formula for the EMCC is called the energy formula [17]. We shall call the EMCC calculated from this formula the energy EMCC and denote it k_e :

$$k_e^2 = \frac{U^{(d)} - U^{(sh)}}{U^{(d)}} \quad (20)$$

where $U^{(d)}$ is the internal energy of the piezoelectric element with disconnected electrodes and $U^{(sh)}$ is the internal energy for the element with short-circuited electrodes. To calculate k_e , we first solve the initial problem and then calculate the internal energy of the piezoelectric element with disconnected electrodes $U^{(d)}$ and the internal energy for the element with short-circuited electrodes, assuming that the strains are known. When calculating $U^{(d)}$, the potential difference on the disconnected electrodes is found from the integral condition at the disconnected electrodes:

$$\int_s D_3^{(d)} ds = 0 \quad (21)$$

where s is the surface of the electrode, $D_3^{(d)}$ is the component of the elastic induction vector normal to the electrode surface, and the dot above $D_3^{(d)}$ denotes a derivative with respect to time. For the conditions of short-circuited electrodes, the electric potential $\varphi^{(sh)}$ must be zero:

$$\varphi^{(sh)} = 0 \quad (22)$$

The energy method of determining the EMCC was discussed previously in [17]. As shown in [17], the energy EMCC (20) is valid for any problems. In the case of a uniform state, the values of k_e coincide with the values of k_s and near resonance k_e coincides with k_d .

To calculate EMCC k_e we will use Formula (20).
Electric current is determined by the formula:

$$I = -i\omega \int_{\Omega} D_{3,0} d\Omega = -i\omega \Omega \left[-\frac{2V}{h_2} \varepsilon_{33}^T (1 - k_{31}^2) + (z_2 + z_1) \frac{\varepsilon_{33}^T k_{31}^2}{2d_{31}l} \frac{dw_0}{d\xi} \Big|_{\xi=1} + \frac{d_{31}}{s_{11}^E} u_0 \Big|_{\xi=1} \right] \quad (23)$$

Since the quantities $U^{(d)}$ and $U^{(sh)}$ are even functions of the variable x_3 , we will calculate $U^{(d)}$ and $U^{(sh)}$ only for values of $x_3 \geq 0$. The superscripts $(\pm)$ should be omitted from the formulas.

Let us consider the case of open electrodes:

$$\begin{aligned} D_{3,0}^{(d)} &= 0, \quad E_{3,0}^{(d)} = -\frac{2V^{(d)}}{h_p} + (h + h_e) \frac{k_{31}^2}{2(1-k_{31}^2)} \frac{1}{d_{31}} e_{1,1} = 0, \\ V^{(d)} &= (h + h_e) h_p \frac{k_{31}^2}{4(1-k_{31}^2)} \frac{1}{d_{31}} e_{1,1}, \quad E_{3,1}^{(d)} = -\frac{k_{31}^2}{1-k_{31}^2} \frac{1}{d_{31}} e_{1,1}, \\ \sigma_{1,1}^{(d)} &= \frac{1}{s_{11}^E} \frac{1}{1-k_{31}^2} e_{1,1}, \quad \sigma_{1,0}^{(d)} = 0. \end{aligned} \quad (24)$$

For the case of short-circuited electrodes, the following formulas apply:

$$\begin{aligned} V^{(sh)} &= 0, \quad E_{3,0}^{(sh)} = (h + h_e) \frac{k_{31}^2}{2(1-k_{31}^2)} \frac{1}{d_{31}} e_{1,1}, \quad D_{3,0}^{(sh)} = (h + h_e) \frac{k_{31}^2}{2} \frac{\varepsilon_{33}^T}{d_{31}} e_{1,1} \\ E_{3,1}^{(sh)} &= -\frac{k_{31}^2}{1-k_{31}^2} \frac{1}{d_{31}} e_{1,1}, \quad \sigma_{1,1}^{(sh)} = \frac{1}{s_{11}^E} \frac{1}{1-k_{31}^2} e_{1,1}, \quad \sigma_{1,0}^{(sh)} = -(h + h_e) \frac{k_{31}^2}{2(1-k_{31}^2)} \frac{1}{s_{11}^E} e_{1,1} \end{aligned} \quad (25)$$

Using Formulas (24) and (25) above, we derive the formulas for the internal energy of the gauge with open electrodes $U^{(d)}$ and its internal energy in the case of short-circuited electrodes $U^{(sh)}$:

$$\begin{aligned} U^{(d)} &= \int_V \gamma \sigma_1^{(1)(d)} \gamma e_1^{(1)} dv + U^{(e)} = \frac{(h^3 - h_e^3) e_1^{(1)2}}{3(1-k^2) s_{11}^E} + U^{(e)} \\ U^{(e)} &= \int_V \gamma \sigma_1^{(1)(e)} \gamma e_1^{(1)} dv = \frac{h_e^3}{3} E e_1^{(1)2} \end{aligned} \quad (26)$$

$$\begin{aligned} U^{(sh)} &= \int_V [(\sigma_1^{(0)(sh)} + \gamma \sigma_1^{(1)(sh)}) \gamma e_1^{(1)} + (E_3^{(0)(sh)} + \gamma E_3^{(1)(sh)}) D_3^{(0)(sh)}] dv + U^{(e)} = \\ &= \frac{1}{1-k^2} \frac{e_1^{(1)2}}{s_{11}^E} \left(-\frac{(h^2 - h_e^2)(h + h_e)k^2}{2} + \frac{h^3 - h_e^3}{3} + h_p \frac{(h + h_e)^2 k^2}{4} \right) + U^{(e)} = \\ &= \frac{1}{1-k^2} \frac{e_1^{(1)2}}{s_{11}^E} \left(-h_p \frac{(h + h_e)^2 k^2}{4} + \frac{h^3 - h_e^3}{3} \right) + U^{(e)} \end{aligned} \quad (27)$$

Let us calculate EMCC k_e using the Formula (20):

$$k_e^2 = \frac{U^{(d)} - U^{(sh)}}{U^{(d)}} = \frac{3}{4} k_{31}^2 \frac{h_p (h + h_e)^2}{h^3 + h_e^3 (Es_{11}^E - 1)} \quad (28)$$

The last equation can be rewritten as follows:

$$\frac{k_e}{k_{31}} = \frac{1}{2} \sqrt{\frac{3(1-x)(1+x)^2}{1+x^3(Es_{11}^E - 1)}}, \quad x = \frac{h_e}{h}, \quad h = h_e + h_p \quad (29)$$

where h_p and h_e are the thicknesses of the elastic and piezoelectric layers, respectively (Figure 1).

By changing the thickness of the elastic layer, we can increase EMCC and reduce the fragility of the gauge. The dependence of the ratio k_e/k_{31} on variable x is presented in

Figure 3. Here, the value $x = h_e/h$ is equal to the ratio of the thickness of the elastic layer h_e to the total thickness of the gauge h .

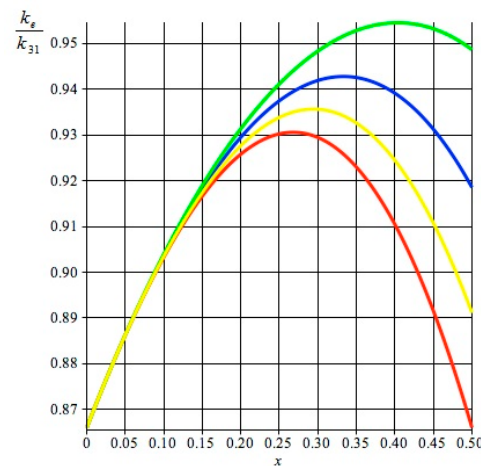


Figure 3. Dependence of the ratio k_e/k_{31} on the variable $x = h_e/h$. The red, yellow, blue, and green curves correspond to values of Es_{11}^E of 2, 1.5, 1.0, and 0.5, respectively.

In Figure 3 the red, yellow, blue, and green curves correspond to values of Es_{11}^E equal to 2, 1.5, 1.0, and 0.5, respectively. From Figure 3, it can be seen that EMCC increases in the presence of an elastic layer. The largest increase in k_e , about 10 percent, was obtained at $Es_{11}^E = 0.5$ and $x = 0.404$ ($h_e = 0.68h_p$). In this case, for piezoceramics Z29 [20], k_e is 0.35.

5. Determination of Principal Bending Deformations Using Gauge Sockets

The transformation of any components of the strain tensor when moving from one coordinate system to another is performed using the same formulas. For example, let us place three bending strain gauges on the surface of the body along straight lines A , B , C (Figure 4). The gauges are shown as rectangles.

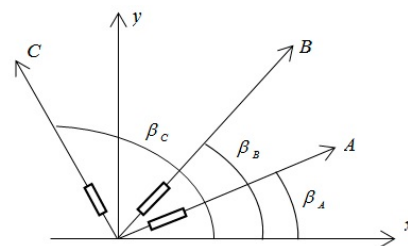


Figure 4. Three gauges oriented at arbitrary angles relative to the x-axis.

For given angles β_A , β_B and β_C , and known results of strain measurements κ_A , κ_B and κ_C components of the strain tensor in the Cartesian coordinate system (κ_{xx} , κ_{yy} and τ_{xy}) are found by solving the system of Equation (30).

$$\begin{aligned}\kappa_A &= \kappa_{xx} \cos^2 \beta_A + \kappa_{yy} \sin^2 \beta_A + \tau_{xy} \sin \beta_A \cos \beta_A, \\ \kappa_B &= \kappa_{xx} \cos^2 \beta_B + \kappa_{yy} \sin^2 \beta_B + \tau_{xy} \sin \beta_B \cos \beta_B, \\ \kappa_C &= \kappa_{xx} \cos^2 \beta_C + \kappa_{yy} \sin^2 \beta_C + \tau_{xy} \sin \beta_C \cos \beta_C.\end{aligned}\quad (30)$$

The principal deformations κ_1 and κ_2 , as well as their directions, are calculated using the formulas:

$$\begin{aligned}\kappa_1 &= \frac{1}{2}(\kappa_{xx} + \kappa_{yy}) + \frac{1}{2}\sqrt{(\kappa_{xx} - \kappa_{yy})^2 + \tau_{xy}^2}, \\ \kappa_2 &= \frac{1}{2}(\kappa_{xx} + \kappa_{yy}) - \frac{1}{2}\sqrt{(\kappa_{xx} - \kappa_{yy})^2 + \tau_{xy}^2}, \\ 2\beta &= \arctg\left(\frac{\tau_{xy}}{(\kappa_{xx} - \kappa_{yy})}\right).\end{aligned}\quad (31)$$

where β is the angle of inclination of the strain κ_1 to the axis x .

6. Experimental Results for Piezoelectric Gauges

Piezoelectric gauges of longitudinal [1–3] and bending deformations are based on the use of the direct piezoelectric effect and belong to the same group of measuring devices. As noted in the Introduction, one of the important advantages of these sensors is their ease of use: a voltmeter measures the electrical signal coming from the electrodes, then the corresponding strain component is calculated from the measured electrical signal. Apart from a voltmeter, no other additional equipment or external energy sources are required. For comparison, recall that the Thomson strain gauge requires an external power source and additional equipment.

The experimental technique for piezoelectric gauges of longitudinal and bending strains is identical. In [2,3], the theory of the piezoelectric gauge was constructed and experiments were performed. For the experiments, we took a round steel plate with a radius of 0.08 m and a thickness of 0.0018 m clamped along the edge. Piezoelectric ceramic elements produced in Russia, similar in properties to the PZT-4 material, were used as a gauge. The plate vibrated harmonically near the first and second resonances.

In the vicinity of the first resonance in the center of the plate, three measurements of the electric potential difference were taken as it approached the resonance. The components of the longitudinal deformation of the plate $\varepsilon_r = \varepsilon_\varphi$ were calculated (r and φ are polar coordinates). The measurement and experimental results are presented in Table 1.

Table 1. Electric potential differences in V and strain ε_r around the first resonance at the center of the plate.

V [mv]	$\varepsilon_r \times 10^{-6}$
260	0.54
475	1.00
542	1.14

In the vicinity of the second resonance at the point $r = 0.04$ m, three measurements of the electric potential difference were performed, as the second resonance was approached by two gauges oriented in the circular φ and radial r directions, from which the longitudinal strain components ε_r and ε_φ were calculated. The measurement and experimental results are presented in Table 2:

Table 2. Differences in electric potential and deformation near the second resonance in the region of the highest deformation amplitudes.

V_r [mv]	V_φ [mv]	$\varepsilon_r \times 10^{-6}$	$\varepsilon_\varphi \times 10^{-6}$
90	200	1.25	0.40
155	340	2.10	0.68
200	420	2.60	0.90

To confirm the accuracy of the experimental results obtained using piezoelectric gauges, we performed strain measurements using another method—holographic interferometry. Figure 5 shows a hologram of plate vibration near the first resonance.

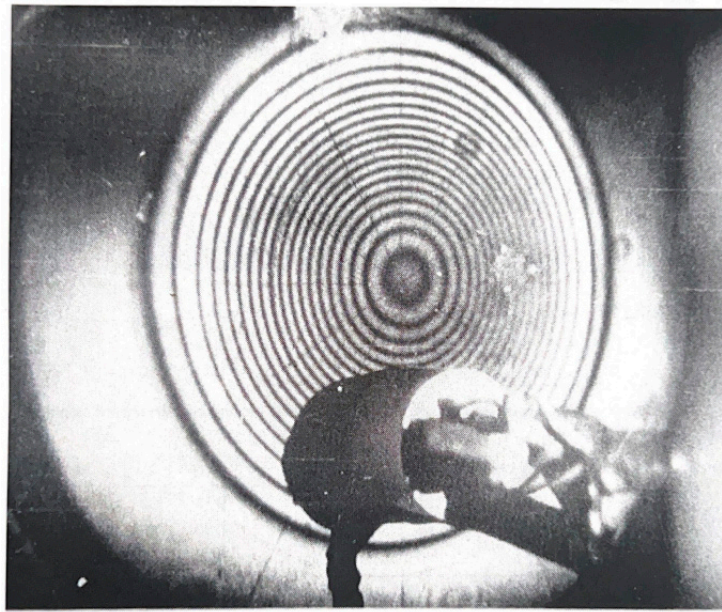


Figure 5. Hologram of a vibrating plate near the first resonance.

The measurements and experimental results obtained using the holographic interferometry method near the first resonance are presented in Table 3.

Table 3. Strains in the center of the plate near the first resonance, obtained by holographic interferometry.

V [mv]	$\varepsilon_r \times 10^{-6}$
260	0.53
475	0.98
542	1.14

In the vicinity of the second resonance, the hologram of plate vibration has the form shown in Figure 6.

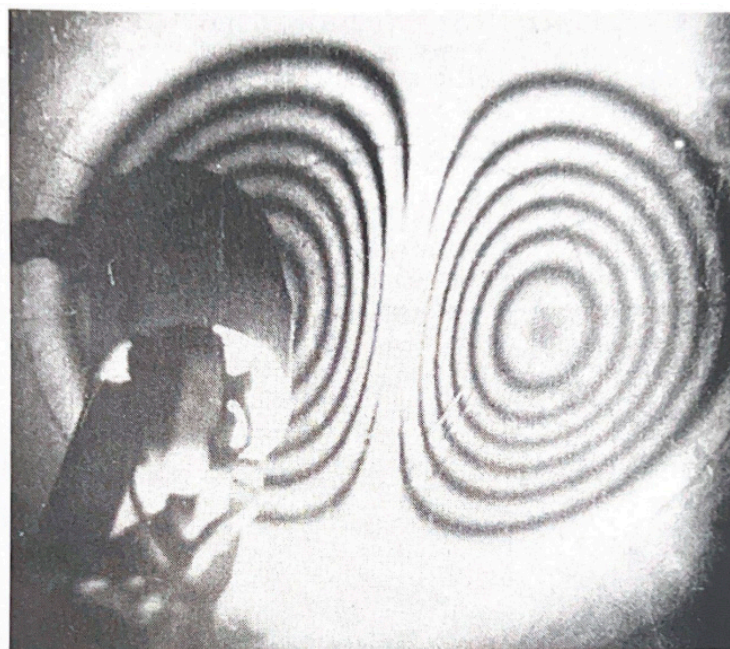


Figure 6. Hologram of a vibrating plate near the second resonance.

The measurements and experimental results obtained using the holographic interferometry method near the second resonance are presented in Table 4.

Table 4. Deformations near the second resonance in the region of the highest deformation amplitudes, obtained by holographic interferometry.

V_r [mv]	V_φ [mv]	$\varepsilon_r \times 10^{-6}$	$\varepsilon_\varphi \times 10^{-6}$
90	200	1.25	0.54
155	340	1.87	0.80
200	420	2.50	1.10

The results of measuring longitudinal deformation obtained by two experimental methods coincide with good accuracy.

7. Conclusions

In our paper, a mathematical model of a gauge is based on the linear theory of electroelasticity. When constructing the mathematical model, it was taken into account that the thickness of the gauge is small compared to its length, so it should be considered a thin-walled structure. As shown in numerous papers on the linear theory of thin-walled structures (see, for example, [21,22]), the use of linear theory imposes restrictions on the value of the required quantities: the value of the components of the displacement vector of the gauge points should be less than its thickness, and the rotation angles of the gauge elements should be less than its relative thickness (the relative thickness of the gauge is equal to the ratio of its thickness to its length). For example, let us estimate the maximum bending angle for our gauge with a relative thickness of 1/20. This means that the bending angle described by the linear theory should be less than 0.05 (in radians) or 2.860° (in degrees). For a thinner gauge with a relative thickness of 1/50, the bending angle should be less than 0.02 (in radians) or 1.150° (in degrees).

Our piezoelectric bending strain gauge, together with a tangential strain gauge, allows us to measure all components of the strain tensor on the surface of a deformable body, which makes it possible to improve the quality of operation of the structure.

We are continuing to develop this topic and are ready to offer a new modified small bending strain gauge in which composite piezoelectric elements (stacks) will be used instead of piezoelectric layers. The EMCC of such a gauge doubles. The fragility of the ceramics will be significantly reduced, since thin layers of piezoelectric ceramics are enclosed in a metal matrix.

In another new piezoelectric bending strain gauge, which we are currently working on, we took a piezoelectric film as a piezoelectric material. Such a gauge can be used for structures made of soft materials.

Since we work at a civil engineering university, we began to study the possibilities of applying piezoelectric gauges to various civil engineering structures. Our purpose is to obtain mathematical models of problems and find their solutions. For example, when studying the failure of a structure, it is necessary to determine the presence of cracks and their evolution. Obviously, the use of gauges alone is not sufficient, since it is impossible to obtain the internal stress–strain state of the object from measured deformations on the surface of an object. In addition to gauges, it is necessary to use actuators to excite different types of vibrations and study the dynamic behavior of the structure.

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