Abstract: The post-construction improvement technology of bored pile tips has been widely used in the construction of gravel pebble layers, but the diffusion law of the grouting slurry remains to be revealed. To study the diffusion law of the grouting slurry at the tip of bored piles in gravel pebble layers, an equivalent relationship between the seepage continuity equation and the mass diffusion continuity equation was established based on an in-depth comparative analysis; further relying on the Diluted Species Transport Module in Porous Media of Comsol Multiphysics, a three-dimensional numerical model of slurry diffusion was established to systematically study the impact of key factors such as different porosities, grouting times, grouting pressures, slurry diffusion coefficients, and the ratio of vertical to horizontal slurry diffusion coefficients on the diffusion radius of the slurry. The calculation results show that with the continuation of grouting, the increase in porosity, grouting pressure, and slurry diffusion coefficient, or the decrease in the ratio of vertical to horizontal slurry diffusion coefficients, the diffusion range of the slurry increases accordingly. Furthermore, comparatively speaking, the impact of porosity is the smallest, while the impact of the slurry diffusion coefficient is the greatest. The research findings reflect the diffusion trend of the slurry in the gravel pebble layer, which has guiding significance for the quality control of the actual design and construction of grouting.

Keywords: drilled pile; slurry diffusion; numerical simulation; seepage; post-grouting

1. Introduction

China has many mountains and rivers, and the geological conditions are complex. In areas with more mountains and rivers, sandstone and gravel soils are widely developed, and the geological conditions are relatively good. To reach or cross the gravel layer, bored piles are commonly used as the foundation for urban buildings. However, the soil cover layer of the foundation is relatively shallow and mostly consists of moderately to slightly weathered rock layers, with the sediment from bored piles mainly being gravel. To eliminate the adverse effects of the sediment at the bottom of the hole, the engineering community often uses the post-grouting improvement technology at the bottom of the bored piles. This technology involves grouting to allow the slurry to seep into the smallest gaps of the pile end sediment, combining with the sediment to form a cement coagulation block. The cement slurry further seeps into the bearing layer at the pile end, forming an enlarged head that continuously increases with the progress of grouting. Obviously, it has a compaction effect on the bearing layer at the pile end, increasing the bearing area at the pile end, thereby enhancing the bearing capacity of the bored piles.

The post-grouting improvement technology at the pile end has been proven effective in enhancing the bearing capacity of pile foundations since its first use in the construction of...
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the Maracaibo Bridge in 1958 [1]. It is understandable that the enhancement of the bearing capacity of bored piles is closely related to the diffusion law of the post-grouting slurry, and many domestic and foreign scholars have conducted numerous studies on this; for example, Sun Bintang et al. [2] established the basic differential equation of seepage for post-grouting slurry at the pile end; a series of existing experimental tests by Bezuijen [3], Talmon [4], and Ding [5] have shown that the slurry has rheological characteristics as a Bingham fluid and grouting diffusion during the grouting process; Yang Xiuju et al. [6] derived a formula for calculating the effective diffusion radius of Bingham fluid slurry during permeation grouting in sandy soil; Liu Jian et al. [7] systematically studied the diffusion law of cement slurry in planar fractures under static and dynamic water conditions through model testing and numerical simulation; He Jianlong et al. [8] used the FLAC3D program to simulate the range of cement slurry diffusion at the pile end; Lin Yuanjun et al. [9] conducted numerical simulation research on the seepage field and displacement field of soft rock roadways, analyzing the impact of grouting pressure, rock permeability, and slurry viscosity on reinforcement effects; Huang Yaoguang et al. [10,11] established models of slurry diffusion under different grouting pressures, grouting times, and anchor spacing to study the seepage and diffusion laws of full-section anchor grouting slurry in tunnel surrounding rocks. Zhang Jian et al. [12] developed a formula for calculating the diameter and the climb height of the cement core of jet grouting (CCJG). It must be acknowledged that the aforementioned research has to some extent revealed the diffusion law of post-grouting slurry at the pile end, but due to the great diversity of engineering geological conditions, theoretical analysis and experimental simulation are also difficult to universalize, and there is currently a lack of systematic research on the factors affecting the diffusion law of post-grouting slurry in gravel layers and their sensitivity [13–20].

Therefore, this paper establishes a three-dimensional numerical model of post-grouting slurry diffusion at the pile end of bored piles in gravel layers based on the finite element software Comsol Multiphysics (Version 6.1.0.357). Based on the seepage continuity equation and the material diffusion equation, sensitivity analysis is conducted on the influence of grouting time, porosity, grouting pressure, slurry diffusion coefficient, and the ratio of vertical to horizontal slurry diffusion coefficients on the diffusion radius and rebound height of the slurry, revealing the diffusion law of post-grouting slurry at the pile end of bored piles in gravel layers. The conclusions have guiding significance for the design and construction of post-grouting at the pile end of bored piles in gravel layers.

2. Model Establishment

2.1. The Slurry Diffusion Model Based on the Continuity Equation of Seepage

To study the impact of various factors on the diffusion effect of the slurry, the following assumptions are made here: (1) the soil mass is continuous, homogeneous, and isotropic; (2) the porosity and permeability coefficient of the soil mass are constant values and do not change during the grouting process; (3) the slurry is a homogeneous, incompressible Newtonian fluid. At this point, the diffusion process of the slurry in the soil can be regarded as the seepage process of groundwater in the soil, which satisfies the continuity equation of unsteady seepage motion [21]:

\[- \left( \frac{\partial (\rho v_x)}{\partial x} + \frac{\partial (\rho v_y)}{\partial y} + \frac{\partial (\rho v_z)}{\partial z} \right) = S_s \frac{\partial p}{\partial t} \]  \hspace{1cm} (1)

\[S_s = \rho g (a + n\beta)\]  \hspace{1cm} (2)

In the equation: \(\rho\) represents the slurry density (kg/m\(^3\)); \(v_x, v_y, v_z\) represents the tensor of the seepage velocity of the slurry in each direction (m/s); \(S_s\) represents the storage coefficient (dimensionless); \(p\) represents the grouting pressure (kPa); \(a\) represents the soil compression coefficient (dimensionless); \(n\) represents the porosity (dimensionless); \(\beta\) represents the slurry compressibility coefficient (dimensionless).
According to Darcy’s Law, since the soil mass is isotropic, the seepage velocity is related to the slurry pressure by the following relationship:

\[
\begin{align*}
  v_x &= -\frac{K}{\gamma} \frac{\partial p}{\partial x} \\
  v_y &= -\frac{K}{\gamma} \frac{\partial p}{\partial y} \\
  v_z &= -\frac{K}{\gamma} \frac{\partial p}{\partial z}
\end{align*}
\] (3)

In the equation: \(K\) is the slurry diffusion coefficient, and \(\gamma\) is the slurry unit weight (kN/m\(^3\)).

By combining Equations (1) to (3), we can obtain:

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{S_s}{K} \frac{\partial p}{\partial t}
\] (4)

We use the Comsol Multiphysics module for dilute species transport in porous media to simulate the diffusion process of the slurry in the soil. Assuming that no chemical reactions occur between the slurry and the soil during the transport process, and that the velocity in the soil’s flow field is zero, the governing equation for the material diffusion module of this module is:

\[
-D \left[ \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2} \right] = n \frac{\partial c}{\partial t}
\] (5)

In the equation: \(D\) is the diffusion coefficient of the substance (dimensionless), \(n\) is the porosity of the porous medium (dimensionless), and \(c\) is the concentration of the substance (mol/m\(^3\)).

Comparing Equations (4) and (5), and setting:

\[
D = \frac{nK}{S_s}
\] (6)

\[
c = \frac{p}{\gamma}
\] (7)

Obviously, an equivalent relationship can be established between Equations (1) and (5), therefore, this module can be used to simulate the diffusion pattern of the slurry.

2.2. Model Establishment Process

The study utilizes the Diluted Species Transport Module for porous media to conduct the research. The model is configured as a 20 × 20 × 40 (m) cube, with the bored pile located at the center of the model. The pile has a diameter of 1 m and a length of 20 m, and grouting pipes with an inner diameter of 5.0 cm are placed on both sides at the bottom of the pile. Figure 1 illustrates the computational model and its mesh division.

Simultaneously, based on the actual conditions of post-grouting at the pile tip, it is assumed that the slurry is injected from the bottom of the grouting pipe and diffuses towards the surrounding area and bottom of the soil. The lower surface of the grouting pipe is set as the initial grouting pressure surface boundary, with the surrounding and bottom surfaces of the cube defined as outflow surface boundaries, and the remaining surfaces as no-flow surfaces. The four sides and bottom of the cube are defined as “outflow” boundaries, simulating the diffusion path of the slurry in the soil. The diffusion path of the slurry in the soil should be from the grouting pipe mouth into the soil, and under the action of gravity and grouting pressure, it flows towards the bottom and surrounding areas. It is worth noting that although there is a certain upward reflow of the slurry during
the grouting process, this is due to the infiltration speed of the slurry lagging behind the grouting speed and does not mean that the upper surface is also an outflow boundary.

Figure 1. Finite element mesh.

Furthermore, based on the case analysis of bored pile projects in the gravel pebble layer and a review of relevant specifications, the empirical values of porosity, grouting time, slurry diffusion coefficient, and grouting pressure as shown in Table 1 [22–27] are selected to study the impact of these parameters on the diffusion pattern of the slurry.

Table 1. Numerical simulation calculation parameters.

<table>
<thead>
<tr>
<th>Slurry Diffusion Coefficient k/(m s⁻¹)</th>
<th>Slurry Storage Rate Ss</th>
<th>Slurry Bulk Density γ(kN/m³)</th>
<th>Porosity n</th>
<th>Grouting Pressure/MPa</th>
<th>Grouting Time t/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 × 10⁻⁴–6 × 10⁻⁴</td>
<td>0.3</td>
<td>16.0</td>
<td>0.2–0.4</td>
<td>2.0–4.0</td>
<td>0–300</td>
</tr>
</tbody>
</table>

Figure 2 illustrates the schematic diagram of slurry diffusion for a specific working condition obtained using Comsol, along with its corresponding front view and top view.

Figure 2. (a) Schematic diagram of slurry diffusion. (b) Front view of the 3D model. (c) Top view of the 3D model.

3. Numerical Simulation

3.1. Slurry Diffusion Pattern Analysis

The study of the slurry diffusion pattern takes time as the independent variable and the slurry diffusion radius as the dependent variable to explore the extent of the impact of various factors. According to the previous context, the calculation time is selected from
0 to 300 s with a time step of 10 s. After calculation and processing, the change in the slurry diffusion range over time and the contour cloud map of the slurry diffusion range can be obtained.

Assuming initial grouting conditions with a porosity of 0.3, a slurry diffusion coefficient of $3 \times 10^{-4}$ m/s, and a grouting pressure of 2 MPa, the effect of different factors on slurry diffusion is studied by changing the value of a certain factor. Figure 3 shows the contour cloud map of the slurry diffusion radius changing with grouting time under given influencing factors. Since the soil mass is assumed to be isotropic, the diffusion pattern of the slurry is symmetrically distributed in all directions. By selecting a vertical section passing through the center of the pile as the object of observation, it can be seen that the slurry diffusion on both sides first forms a smaller enlarged head near the grouting mouth on both sides of the pile end. As time goes on, the slurry diffusion radius increases, and the slurry on both sides begins to converge and wrap up the entire bottom of the pile.

Figure 3. Contour cloud diagram of slurry hydraulic head with different grouting time for the case with $n = 0.3$, $K = 0.3 \times 10^{-4}$ m/s. (a) $n = 0.3$, $p = 2$ MPa, $K = 3 \times 10^{-4}$ m/s, $t = 30$ s; (b) $n = 0.3$, $p = 2$ MPa, $K = 3 \times 10^{-4}$ m/s, $t = 300$ s.

3.2. The Influence of Grouting Pressure

To study the impact of grouting pressure on the diffusion range of the slurry, the relationship between the diffusion radius of the slurry, the rebound height, and time was investigated under the condition that the initial grouting conditions remain unchanged, with a variation in grouting pressure from 2 MPa to 4 MPa. Figures 4 and 5, respectively, show the change curves of the slurry diffusion radius and rebound height under different pressures as a function of grouting time.

Figure 4. Comparison of diffusion radii at different grouting pressures for different grouting times.
To verify the rationality of the calculation model, the model’s computational results are compared with the research findings of Liu Jun et al. [28]. Figures 4 and 6 demonstrate the relationship between the diffusion radius and grouting pressure in the gravel layer. Although the grouting pressures differ, the underlying pattern is similar, allowing the two to corroborate each other, and it is clear that there is a positive correlation between the diffusion radius and the grouting pressure.

3.3. The Influence of the Porosity of the Gravel Layer at the Pile Tip

Furthermore, to study the impact of porosity on the diffusion range of the slurry, the porosity of the gravel layer at the pile tip was varied, with calculations performed for porosities of 0.2, 0.25, 0.3, 0.35, and 0.4. Figures 7 and 8 present the variation curves...
of the slurry diffusion radius and the rebound height at different porosities over the grouting time.

**Figure 7.** Comparison of diffusion radii at different porosities for different grouting times.

**Figure 8.** Comparison of slurry returns at different porosities for different grouting times.

From Figures 7 and 8, it can be seen that the increase in the porosity of the gravel layer at the pile tip has no significant effect on the grout diffusion radius and the grout return height. In the early stage of grouting, the grout diffusion is insufficient, and there is no significant difference in the grout diffusion radius and grout return height between different porosities. When the grouting time reaches 300 s and the porosity increases from 0.2 to 0.4, the grout diffusion radius and grout return height only increase by 10.4% and 7.5%, respectively. It can be seen that the influence of the porosity of the gravel layer at the pile tip on grout diffusion is not simply a matter of promotion or inhibition, but the result of the interplay of various factors, which should be considered comprehensively. On the one hand, it is assumed that the soil at the pile tip is homogeneous and isotropic, and according to Darcy’s law, the lower the porosity, the faster the grout passes through the soil pores under the same grouting pressure, thus the faster the grout diffusion. Based on this, an increase in porosity is not conducive to grout diffusion. On the other hand, as can be seen from Equation (6), the grout diffusion coefficient is positively correlated with the porosity, and an increase in porosity is beneficial to grout diffusion. It can be seen that only by considering the above situations can we correctly analyze the impact of porosity on the grout diffusion pattern.
Although the increase in the pile tip porosity does not have a significant effect on grout diffusion, and porosity is difficult to change through actual engineering measures, this result also indicates that the post-grouting reinforcement technology at the pile tip is universally applicable to gravel layers with different porosities and is a feasible method.

To verify the rationality of the calculation model, the model’s computational results are compared with the research findings of Liu Jun et al. [28]. Figures 7 and 9 reveal the relationship between the diffusion radius and the porosity of the pebble layer. Moreover, when the grouting time is 70 s and the porosity is 0.4, the diffusion radii corresponding to both figures are around 0.6 m, which verifies the rationality of the calculation model presented in this paper.

3.4. The Influence of the Slurry Diffusion Coefficient and Its Anisotropy

Based on the initial grouting conditions, the study investigated the relationship between the slurry diffusion radius, rebound height, and grouting time with the variation of the slurry diffusion coefficient ranging from $3 \times 10^{-4}$ m/s to $6 \times 10^{-4}$ m/s. The computational results are shown in Figures 8 and 9.

Figures 10 and 11 indicate that the slurry diffusion coefficient has a significant impact on the diffusion radius of the slurry. As can be seen from the figures, when the grouting time is 300 s and the slurry diffusion coefficient is $6 \times 10^{-4}$ m/s, the slurry diffusion radius reaches 1.36 m, which is a 33.3% increase compared to the case where the slurry diffusion coefficient is $3 \times 10^{-4}$ m/s. This is because the slurry diffusion coefficient directly affects the mobility of the slurry in the soil; the larger the slurry diffusion coefficient, the stronger the diffusion capability, and the greater the diffusion range of the slurry in the soil. Since the permeability coefficient of the slurry is related to the viscosity of the slurry, the increase in the slurry diffusion coefficient can be achieved by altering the water–cement ratio of the slurry. Therefore, to facilitate construction and ensure quality, the water–cement ratio should be between 0.5 and 0.7.

Furthermore, to study the impact of the ratio of vertical to horizontal slurry diffusion coefficients ($K_V:K_H$) on the diffusion of the slurry, calculations were performed with different $K_V:K_H$ ratios of 1:1, 1:5, 1:10, 1:20, and 1:50.

Figure 12 indicates that as the $K_V:K_H$ ratio decreases, the slurry diffusion radius increases. When the grouting time is 300 s, compared to the scenario with a $K_V:K_H$ of 1:1, the diffusion radius of the slurry increases by 4.9%, 8.8%, 10.8%, and 12.7% for $K_V:K_H$ ratios of 1:5, 1:10, 1:20, and 1:50, respectively. It can be observed that the increase in the slurry diffusion radius is not significant as the $K_V$ to $K_H$ ratio decreases.
Figure 10. Comparison of diffusion radii at different slurry diffusion coefficients for different grouting times.

Figure 11. Comparison of slurry returns at different slurry diffusion coefficients for different grouting times.

Figure 12. Comparison of diffusion radii at different $K_V:K_H$ ratios for different grouting times.

Figure 13 shows the rebound height changing with grouting time under different $K_V:K_H$ conditions. The figure demonstrates that when the grouting time is 300 s, the
rebound height significantly decreases when the $K_V:K_H$ ratio changes from 1:1 to 1:5, reducing from 0.84 m to 0.4 m. However, when the $K_V:K_H$ ratio is 1:10, 1:20, and 1:50, the change in rebound height between these levels is minimal, indicating that altering the $K_V:K_H$ ratio has a limited effect on reducing the rebound height at this point. Considering the impact of various factors on the rebound height, the computational results suggest that the sensitivity to the $K_V:K_H$ ratio is the highest. This is because the permeability coefficient of the slurry in the soil is related to the properties of the soil, and in engineering practice, the horizontal permeability coefficient of the slurry in the soil is generally greater than the vertical permeability coefficient.

![Graph showing comparison of slurry returns at different $K_V:K_H$ ratios for different grouting times.](image)

**Figure 13.** Comparison of slurry returns at different $K_V:K_H$ ratios for different grouting times.

4. Conclusions

This paper, on the basis of comparing and analyzing the continuity equation of seepage flow and the continuity equation of material diffusion, has identified the equivalence between the two; relying on the porous medium dilute substance transfer module of Comsol Multiphysics, a three-dimensional numerical model of grout diffusion in the post-boring grouting of bored piles at the pile tip in a cobble layer has been established, and a systematic study has been conducted on the impact of key factors such as different porosities, grouting time, grouting pressure, grout permeability coefficient, and the ratio of vertical to horizontal grout permeability coefficients on the diffusion radius of the grout. The conclusions are as follows:

1. The diffusion process of the grouting slurry at the pile tip in the soil is regarded as the seepage process of water in the soil. By comparing the continuity equation of seepage and the continuity equation of substance diffusion, an equivalent relationship between the two is established. A 3D numerical model for slurry diffusion is established using the Diluted Species Transport Module for porous media in Comsol Multiphysics, and the contour cloud map of the slurry diffusion trend is obtained;

2. In the early stage of grouting, since the diffusion of the slurry is not sufficient, the impact of various influencing factors on the slurry diffusion radius is relatively small. As the grouting time extends, the impact of these factors on the slurry diffusion radius becomes increasingly significant;

3. With the continuation of grouting, the increase in pile-end porosity, grouting pressure, and slurry diffusion coefficient, or the decrease in the ratio of vertical to horizontal slurry diffusion coefficients, leads to an expansion of the slurry diffusion range. Among these, the pile-end porosity has the least impact on the slurry diffusion range, while the slurry diffusion coefficient has the greatest impact. To facilitate construction and ensure quality, the water–cement ratio should be between 0.5 and 0.7;
(4) In practical engineering, the rebound height can be controlled, and the quality of grouting and reinforcement effect can be improved by adjusting the grouting pressure and changing the water-cement ratio of the slurry.

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