

Article

Small-Scale and Large-Scale Modeling of Fiber-Reinforced Concrete Girders

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Abstract: Experimental and computational research on the behavior of small-scale and large-scale fiber-reinforced concrete (FRC) beams is presented in this paper. The experimental part included the small-scale bending tests, which were conducted on three 1.3 m long by 0.1 m wide by 0.15 m high rectangular simply supported beams, and the large-scale test that was conducted on 12.8 m long by 0.2 m wide by 1.3 m two-chords girder. The concrete mixture in the large-scale test was designed with environmentally more justifiable supplementary materials (binder and fibers), striving for sustainable excellence. To accurately predict the mechanical behavior of tested models, a numerical model incorporating the real nonlinear materials laws is used. A numerical model based on finite element analysis (FEA) is developed. The FEA model is created using a smeared crack approach with a constitutive law for the tensile behavior of FRC derived from an inverse analysis based on prism bending tests. The numerical model is validated against experimental results and the accuracy of numerical predictions based on finite element modeling showed a good correlation with the test data. The FEA-based model makes it easier to predict how FRC structures fail under transversal loading and can serve as a foundation for creating new design processes. Additionally, the presented research is aimed at the feasibility of recycled steel FRC field application in building structures. The usage of recycled steel fibers could achieve environmental benefits through the adoption of sustainable materials. The present study showcased the possibility of modeling reinforced concrete structural building parts made with recycled steel fibers using available software.



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1. Introduction

Fiber-reinforced concrete (FRC) is increasingly finding applications in structures, which raises the need for appropriate numerical models to describe this material. The behavior of the FRC is more challenging to describe when compared to regular concrete (RC). The FRC is a cementitious composite that combines randomly oriented fibers made from steel, synthetic polymers, glass, etc. Fibers are applied in different percentages in relation to the volume of concrete [1,2].

FRC with steel fibers is used due to its numerous advantages, such as durability, enhanced toughness, high tensile strength, shear strength, and energy absorption capacity which makes it a significant material to be applied as structural elements in engineering structures [3,4].

Finding the right input material properties to effectively represent FRC is challenging. Specifically, it is necessary to accurately estimate the tensile characteristics that are significant for FRC. The parameters can be directly entered into the material model using the measured response of the direct tensile test. Unfortunately, the test is not conducted frequently since sample preparation and testing are difficult. Usually, three-point or four-point bending tests are used [5]. Although the results cannot be immediately incorporated into the model, the results can be used for the material model. To properly determine the computational model parameters, inverse analysis of the data must be done [6,7].

The contribution of the reinforcing fibers on the structural behavior of beams depends on several factors, which are the fiber volume fraction [8,9]; the geometrical and mechanical properties of the fibers, such as geometric profile aspect ratio and tensile strength [10]; the specimen sizes [11]; and the fiber distribution within the volume of the composite [12]. The concrete elements made with traditional steel bars with the addition of fibers are sometimes referred to as hybrid-reinforced concrete (HRC). When considering HRC beams, among other factors, the steel bar reinforcement ratio (ρ) and the fiber volume fraction (V_f) determine the flexural behavior of an HRC beam subjected to bending until failure [13–15].

The flexural response in beams with lower ρ is characterized by a plastic plateau because of the yielding of the longitudinal steel bars. In these situations, the specimen's ability to support loads is increased by the fiber addition [16]. In a reinforced concrete beam with high ρ the plastic plateau is followed by crushing of concrete, ultimately causing the beam to break. In this instance, adding fiber has very little effect on the load-bearing capacity because regular reinforcement makes up the majority of the contribution. In this instance, adding fiber has very little effect on the load-bearing capacity because regular reinforcement makes up the majority of the contribution. However, the crushing branch of the response becomes much more gradual because of the fibers [17].

For HRC, reinforcing fibers play a key role related to the bridging action, which affects both the macrocracks and the microcracks to prevent their opening and growth. Debonding, sliding, and frictional pulling-out between the concrete and fibers are the causes of bridge-toughening mechanisms. The bridging crack model is an LEFM-based method that can describe the mechanical response of fiber-reinforced beams in bending considering the brittle concrete behavior and the fiber-reinforcing action, which may be expressed as either yielding or slippage. Over the last few decades, various versions of the bridging crack model have been used to describe the crack propagation process in the FRC [18,19]. The main advantage of this model is that it describes the concrete composites' constitutive flexural response, such as snap-back and snap-through instabilities, in the postcracking phase. A global structural response of an element, which can range from ductile to catastrophic, can be predicted by the model. The brittleness number can be used to characterize this ductile-to-brittle transition. In HRC, post-cracking structural behavior can be described by the bar-reinforcement brittleness number, the fiber-reinforcement brittleness number, and the pull-out brittleness number [20]. An effective relationship between the two reinforcement brittleness numbers is used to define the minimum reinforcement conditions for HRC beams to ensure a ductile post-peak response without reducing their structural bearing capability.

The present tendency to make concrete using environmentally more acceptable alternative materials leads many times to challenges regarding mechanical and durability demands. The literature has shown that fiber reinforcement proved to be the recommendable solution for green concretes [21,22]. In reinforced concrete structures, commercial steel fibers are usually used, while for other applications, recycled steel fibers recovered from used tires can provide an environmentally more acceptable solution [23–25]. The impurities left with the steel fibers after the extraction process many times are considered a limiting factor for wider utilization as fiber reinforcement in concrete. On the other hand, these fibers can provide some benefits also. The utilization of recycled steel fibers from used tires addresses a significantly reduced carbon footprint [26], less cost, and additionally can provide improved mechanical properties to the resulting FRC [27].

A deeper analysis of the environmental benefits of recycled steel FRC should be considered from two aspects simultaneously: reduction of steel rebars by steel fibers in concrete and reduction of the environmental impact by applying recycled steel fibers as raw material instead of industrial made. As shown in [28] the Life-cycles assessment analysis of FRC vs. RC is very complex even if it is limited to a concrete girder. The various effects of bending moment capacities, different spans, and concrete compressive strength had significantly affected the global warmth potential (GWP) but the average reduction was about 37% for FRC. The environmental impact of the use of recycled steel fibers is a

topic of much research [25,26,29], indicating a possibility of even more remarkable carbon emission reduction.

The effect of size is important to consider when assessing the bearing capacity of a concrete member subjected to various loading conditions. A size effect in concrete elements denotes that the nominal structural strength and material brittleness decrease with increasing concrete element size under tension [30,31]. Thus, a concrete element is more ductile on a small scale, while on a sufficiently large scale, a concrete element exhibits perfectly brittle behavior. The studies of size effect are motivated by the issues of concrete structures for which there exists a large gap between laboratory tests and the scales of real-life structures, such as in large-span girders, bridges, reactor containments, dams, etc. [32]. The mechanical behavior of a small-scale beam may be different from a full-scale beam due to the size effect. The size effect is comprehended as the influence of the characteristic structure dimension on the nominal strength of the structure when geometrically similar structures are compared. In order to understand and reduce the size effect, it is often necessary to conduct a full-size numerical and experimental study.

Currently, most of the structural design codes for concrete structures do not consider the size of elements. Since quasi-brittle materials, such as plain and fiber-reinforced concrete, fail by the formation of cracks, a size effect needs to be implemented. On a small scale, those materials follow the results from the theory of plasticity (or strength theory), which are characterized by yield stress or material strength. On a large scale, quasi-brittle materials obey linear elastic fracture mechanics (LEFM), which are characterized by material strength and fracture energy [33]. A number of size effect laws have been proposed to model the reduction in nominal tensile strength with element size, such as in Bažant et al. [30,34], Carpinteri et al. [35], and Kim et al. [36]. The size effect law provides a smooth transition from failures that are closer to strength failures at small sizes to failures controlled by LEFM at large sizes. A size effect could be practically eliminated with proper geometric scaling of all beam components including the reinforcement layout, cover thickness, and maximum aggregate size [37]. Considering those recommendations, the presented research on small-scale models applies “strict geometric similarity” in the ratio 1:3.3 where, all dimensions were scaled 3.3 times.

Despite its efficiency, experimental testing to investigate the behavior of HRC elements under various loads is limited by high cost and time. Therefore, finite element analysis (FEA) was introduced as one of the most effective methods to mitigate these problems in analyzing HRC structural elements. Numerical investigations are usually conducted on small- to medium-sized HRC models [38–40] but testing on large-scale models is fairly rare, particularly on two-chord roof girders. The presented research is filling up the last gap towards the feasibility of recycled steel FRC field application in building structures. While the research studies regarding recycled steel fibers so far, were dominantly focused on material properties, and environmental effects and the applications were mostly limited to reinforced concrete pavements; the present study showcases the possibility of modeling reinforced concrete structural building parts made with recycled steel fibers using available software.

This paper provides research results regarding the computational modeling of fiber-reinforced concrete materials using the computer program DIANA FEA 10.7 [41], which is a finite element-based software used for a nonlinear analysis of reinforced concrete structures. The primary benefit of the program is that it offers special material models for FRC, which include enhanced ductility from the shape of the tensile softening branch. The main issue in computational analysis is to find the appropriate tensile parameters to successfully model HRC. The objective of this research is to present the basic principles of HRC modeling and understanding of the material model behavior applied on a large-scale two-chord roof girder.

Computational studies can benefit the investigation of full-size specimens because it is difficult to conduct experiments on a large number of specimens. A further extensive investigation could be used to analyze the effect of major design-oriented parameters such as concrete compressive and tensile strength, amount of steel reinforcement, volume fraction of steel fibers; and applied constitutive models. To fully explore the enhancement of the HRC girders' flexural capacity to support loads and material efficiency, research on a ductile-to-brittle transition in the ultimate structural response needs to be carried out. This analysis would allow the definition of the minimum reinforcement condition to ensure a safe flexural bearing capacity of the large-scale HRC girder.

2. Experimental Research

In this research experimental testing was conducted on two types of girders. In the first case, testing was performed on small-scale models. These models were used as an introduction to further research on the second case model, which is a full-scale real-life girder to be used as a roof structure on an industry hall.

2.1. Small-Scale FRC Beam Testing

The experimental part of the small-scale study was conducted on the model of the beam made from FRC. Three beam specimens were tested under a four-point bending load until failure. The dimensions of the beams and reinforcement are shown in Figure 1, while the testing of the beam is shown in Figure 2. The values of the load-bearing capacities recorded in tests ($N_{u,i}$), the mean value of the capacities ($N_{u,exp}$), and the standard deviation (S_n) and variation of the results (COV) are given in Table 1. The fresh concrete mixture was: cement (CEM II/B-M(V-L) 32.5R (Lafarge, Beočin, Serbia)) 421 kg/m³, water 240 kg/m³, coarse aggregate (4–8 mm) (Leget agregati, Sremska Mitrovica, Serbia) 455 kg/m³, fine aggregate (0–4 mm) (Leget agregati, Sremska Mitrovica, Serbia) 1099 kg/m³, hooked steel fibers (Spajic, Negotin, Serbia) 39 kg/m³, and super-plasticizer 1.85 kg/m³ (Sika Srbija, Šimanovci, Serbia). The analyzed beam included the addition of steel fibers in the amount of 0.5% in relation to the total mass of concrete. The steel fibers were 30 mm long, 0.3 mm in diameter, and had hooks at both ends. Standard cube and cylinder test specimens were used to determine the properties of hardened concrete. The mean values for the cylinder (f_c) and cube compressive strength (f_{cu}), the flexural tensile strength (f_{ct}), elastic modulus (E_c), Poisson's ratio (μ_c), and volumetric weight of concrete are listed in Table 2. The reinforcement was made from steel with yield tensile strength of $f_y = 400$ MPa, modulus of elasticity $E_s = 200$ GPa, and Poisson's ratio: $\mu_s = 0.30$.

The test setup included measuring vertical deformations in three places using displacement transducers, which were positioned in the middle and in the quarter of the span. Additionally, two dial gauges were used to monitor the movement of the main beam supports. The force was applied using a hydraulic press and steel beam with two roller supports. The intensity of the load was measured using the displacement transducer.

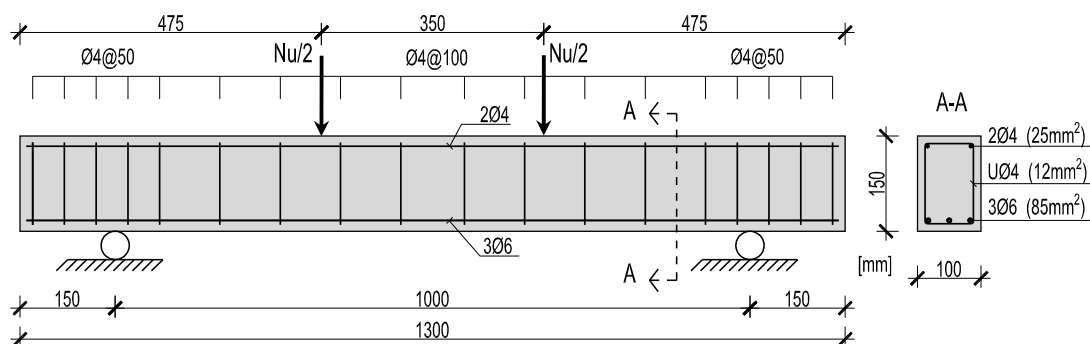


Figure 1. Dimensions of the beam and reinforcement.

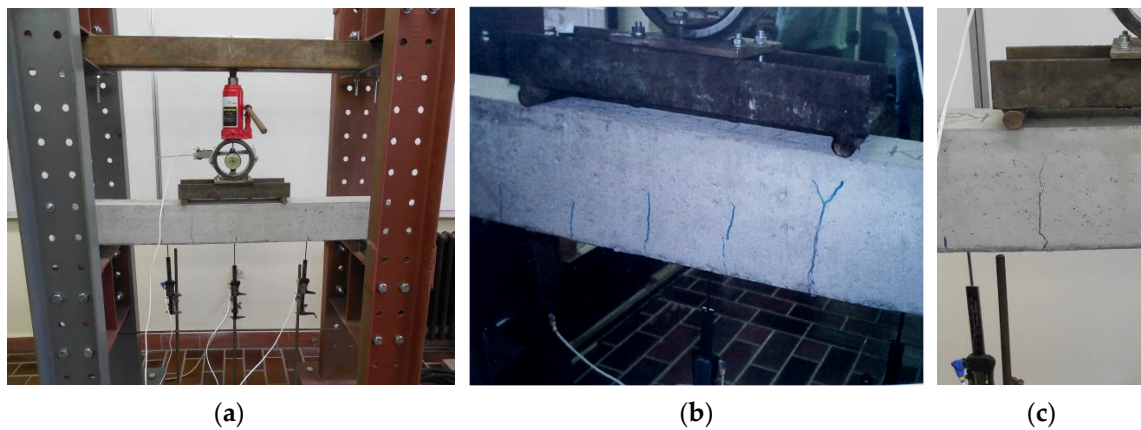


Figure 2. (a) Test setup of the beam; (b) cracks on the side of the beam; (c) crack at failure.

Table 1. Beam models—ultimate load forces.

N_{u1} (kN)	N_{u2} (kN)	N_{u3} (kN)	$N_{u,exp}$ (kN)	S_n (kN)	COV (%)
32.50	33.75	34.67	33.64	1.1	3.2

COV—coefficient of variation.

Table 2. Properties of hardened concrete.

f_c (N/mm ²)	f_{cu} (N/mm ²)	f_{ct} (N/mm ²)	E_c (N/mm ²)	μ_c (-)	γ_c (kg/m ³)
22.4	29.9	4.34	24,257	0.13	2252

During the loading process, the presence of cracks was checked on all sides of the beams. The first crack was observed at a force of around 10 kN. Afterward, with increasing load, vertical cracks appeared under the loading apparatus spaced at a 100 mm distance, indicating pure bending conditions (Figure 2b). Each registered crack was marked and noted in the log. Before reaching the ultimate load, one large crack opened just under the left force at a load level of around 30 kN (Figure 2c). The bearing capacity was reached at forces noted in Table 1. The failure of beams was accompanied by large deformations while retaining the load capacity. The displayed behavior indicates that the yield stress in the main reinforcement has been reached.

2.2. Large-Scale FRC Girder Testing

In the second case, one large-scale girder was tested under loading conditions that simulate real-life cases. The girder was a two-chord HRC beam with a span of 12.8 m and a constant width of 200 mm. Figure 3 shows the geometry of the girder, and Figure 4 shows reinforcement in the girder. The fresh concrete mixture was: cement (CEM II/B-M (V-L) 32.5R (Lafarge, Beočin, Serbia)) 890 kg/m³, silica fume (Sika Srbija, Šimanovci, Serbia) 90 kg/m³, water 240 kg/m³, coarse aggregate (2–8 mm) (Leget agregati, Sremska Mitrovica, Serbia) 305 kg/m³, fine aggregate (0–4 mm) (Leget agregati, Sremska Mitrovica, Serbia) 749 kg/m³, steel fibers 235 kg/m³, and super-plasticizer (Sika Srbija, Šimanovci, Serbia) 19 kg/m³. While higher-class cement is commonly used for high-strength concrete, considering the need to ensure good workability of concrete with a large amount of fiber, lower-class types of cement can provide sufficient results and reduce significantly the environmental impact of the concrete. The mixture was made with the addition of recycled steel fibers from used tires in the amount of 3% of the total mass of the concrete (Figure 5a,b). The fiber length varied between 5–50 mm, while the diameters were 0.05–1 mm. The basic physical and mechanical characteristics of the concrete from which the girder is made are

presented in Table 3. Although this research is focused on modeling and a detailed LCA analysis was not included it is important to note that the mechanical characteristics of FRC were achieved using CEM II/B-M cement and recycled steel fibers. The reduction of GWP for the use of CEM II/B-M instead of CEM I is 27%, according to [42]. The reduction of GWP for the use of recycled steel fibers instead of industrially made fibers is near 50% [28,43]. The longitudinal reinforcement was B500-B with a yield strength of $f_y = 500$ MPa, a modulus of elasticity of $E_s = 200$ GPa, and a Poisson's ratio of $\mu_s = 0.30$, while the stirrups were made from steel with a yield strength of $f_y = 240$ MPa. The longitudinal steel bar reinforcement ratio was $\rho = 0.56\%$.

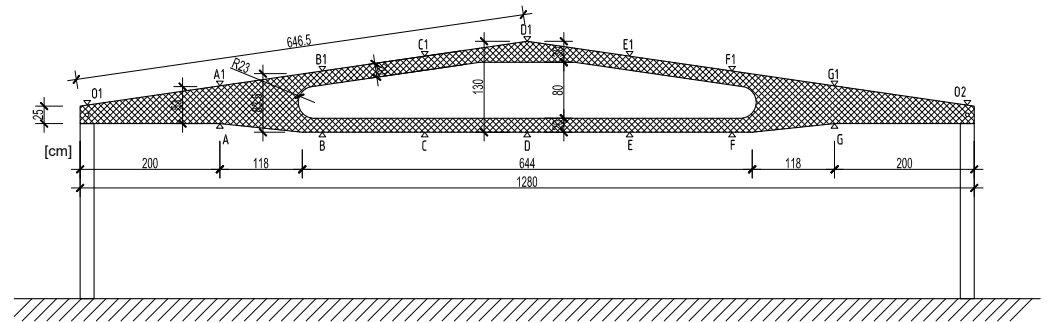


Figure 3. The geometry of the girder.

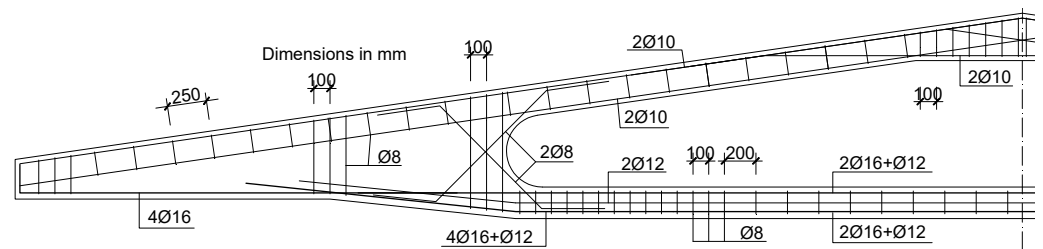


Figure 4. Reinforcement of the girder.



Figure 5. (a) Applied recycled steel fibers; (b) distribution of recycled steel fibers in concrete.

Table 3. Properties of hardened concrete.

f_c (N/mm ²)	f_{cu} (N/mm ²)	f_{ct} (N/mm ²)	E_c (N/mm ²)	μ_c (-)	γ_c (kg/m ³)
80	95	10.1	43,000	0.22	2510

The global deformations of the girder were registered by geodetic measurement with a 0.5 mm accuracy at the specified points shown in Figure 4. Measuring points A1 ÷ G1 are positioned on the top of the girder, while A ÷ G are points positioned on the bottom part of the girder. Additional points were positioned at the supports O1 and O2. Registration of specific longitudinal extensions was performed with digital displacement sensors. The extensions were measured on both chords in the section located halfway between axes through points B and C. Additionally—in axis B, at the point of rounding of the girder—extensions were measured with an additional displacement sensor with a base of 100 mm. That location was chosen based on the expected distribution of normal stresses in the girder.

For safety reasons involving the large forces and size of the girder, testing included only the load levels that are under the load-bearing capacity. The load to the girder was applied indirectly via a series of tensioners (green belts in Figure 6). Concrete beams were used as a load (ballast). The beams were placed under the main girder, and then they were attached to the main girder with tensioners. After applying the initial force to each tensioner, the girder was ready for testing. Two hydraulic presses were installed at the place of the left and right supports (Figure 7a) and used to lift the entire assembly of the girder and ballast beams. In this way, the girder was raised evenly until the ballast beams were lifted from the ground, which achieved the full loading of the girder. After that, the instruments were read and the change in geometry of the girder was measured. While the girder was loaded, the presence of cracks was checked on all sides of the girder (Figure 7b,c). Each observed crack was marked and noted in the log, and then the width of the crack opening was measured (Figure 8). The cracks were symmetrically positioned around the vertical axis through the midpoint of the girder.



Figure 6. Girder with second-phase loading.

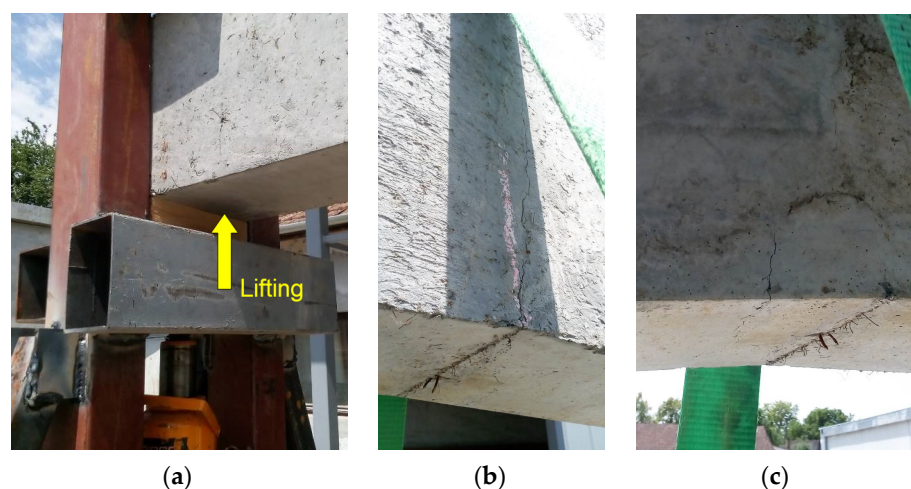


Figure 7. (a) Lifting of the support; (b) cracks in the final load phase at the measuring Point G; (c) cracks in the final load phase at the measuring Point A.

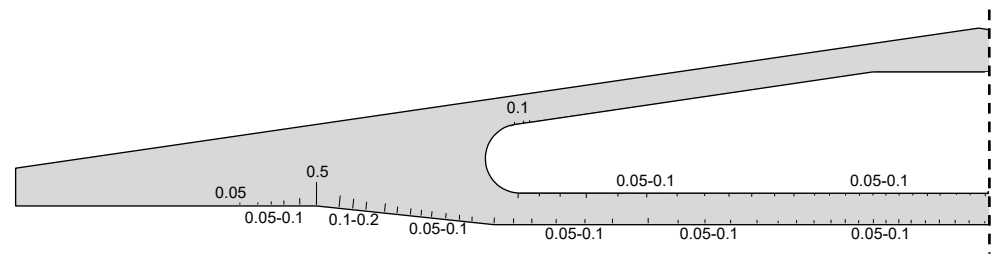


Figure 8. Crack distribution and width in the final load phase.

One of the challenges in large-scale testing was the uniform application of forces. The loading procedure was verified over several iterations to ensure a precise lifting process. In the first phase of lifting the hydraulic presses applied force in steps of 50 kN. Then, measurement of instruments was conducted and the results were compared to the values from the FE analysis.

3. Computational Modeling of FRC Girders

3.1. Model Description

The presented study is focused on the computational modeling of HRC beams and girders that were experimentally tested. The computational model has to effectively predict the structural response and provide accurate information regarding their structural behavior. The model based on the finite element method (FEM) considers the geometry, test setup simulation, applied loading, and applied boundary conditions. The results from the numerical modeling of the large girder were used as a reference for the experimental testing.

Computational simulation was performed by FEM using computer software DIANA FEA [41]. The simulation included modeling of the material nonlinearities for the concrete with the steel fibers and steel reinforcement. Both finite element models were two-dimensional with plane stress elements for concrete, and embedded truss elements for longitudinal and transversal reinforcement. The concrete was simulated using a smeared cracking approach with a total strain model and rotating crack behavior. The reinforcement steel was modeled using the hardening plasticity model. The steel plates for support and load application were modeled with linear elastic material properties. Interface elements with no-tension behavior have been used between the steel loading/support plates and the concrete beam to simulate contact lines.

For models with a smeared crack approach, if the critical fracture energy G_F is exceeded, an equivalent stiffness reduces to zero. Consequently, no visual discontinuity is present in the model. A smeared crack model has been shown to have improved accuracy in modeling reinforced concrete [44]. The rotating crack approach is often implemented in modeling reinforced concrete beams with stirrups since a model with a fixed crack might have considerable stress concentrations at the tip of a crack, which can lead to convergence issues.

The crack bandwidth, as a measure for the crack length, depends on the type and distortion of finite elements. Since finite elements mesh had sides which are close to an aspect ratio of 1:1 Rots crack bandwidth specifications were used in both models. When cracks start to develop, Poisson's ratio reduces from the initial value down to zero as the residual tensile stress becomes zero. A damage-based reduction model was used to describe Poisson's ratio reduction. The effects of the biaxial stress state on the compressive strength of concrete are taken into account. The compressive strength reduction due to lateral cracking follows the Vecchio and Colins reduction model with a lower bound reduction factor of $\beta = 0.4$.

The small-scale model is controlled by forced displacement loading and therefore, energy balance is chosen as convergence criteria with a solution norm of 0.01. On a large-scale model, which is force-loaded, a displacement and energy balance norm is used with

0.01 tolerance. The load on both was applied in many small static load steps using an implicit form.

3.1.1. Small Scale Model—Beam

The computational model was created for two-dimensional conditions (Figure 9). The geometry of the beam included the reinforcements (longitudinal and transversal), and the loading and support plates. The beam has a total length of 1300 mm, a depth of 150 mm, and a width of 100 mm. The geometry, reinforcement, boundary, and loading conditions are presented in Figures 1 and 2.

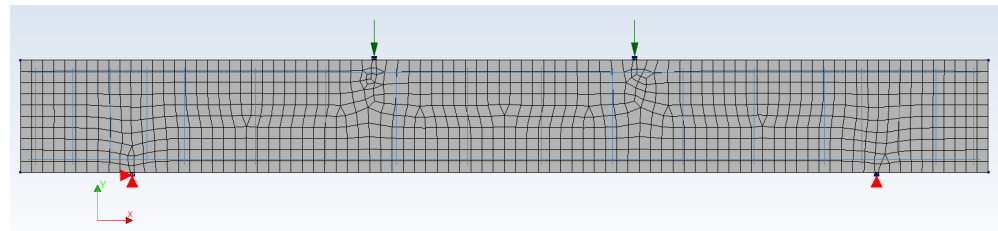


Figure 9. Finite element mesh, supports, and loading.

The FRC beam is modeled using a quadrilateral isoparametric CQ16M plane stress element with eight nodes and sixteen degrees of freedom [45]. This element type was chosen as the best to describe plane stress states considering large displacements and crack development. The thickness of elements is equal to the width of the beam (100 mm). Axes of the local element are specially defined to ensure that axes have the same directions in all elements and that the number of integration points is set to high. This procedure increases the accuracy of the results, which is especially relevant when comparing the results with experimental data [46]. The steel plates for the supports and loading positions are defined with the elastic properties modulus of elasticity $E_{sp} = 200$ GPa and the Poisson's ratio $\mu_{sp} = 0$. Shell elements with a thickness equal to 200 mm (width of the beam) are also applied. A high number of integration points is also set. A line interface between the steel plates and the concrete beam is considered in the analysis. The definition of the structural interface properties was based on the reference Belletti et al. [46] with the normal stiffness modulus $k_y = 1000$ N/mm³ and the shear stiffness modulus $k_x = 10$ N/mm³. The CL12I interface element with a quadratic interpolation scheme is used [45]. The sources of the interface are the edges of the beam, while the targets are the plate's edges. The thickness of the structural interface is set to 100 mm, which is equal to the width of the beam. Following the procedure from the reference [46] the reduced Lobatto three-point integration scheme is utilized. The longitudinal and transversal reinforcements are modeled using embedded reinforcement elements. The geometries of reinforcement bars are defined with an equivalent cross-section area of the bar: bottom main reinforcement of 3Ø6 bars as three times the area of Ø6 rebar ($3 \times 28.3 = 84.9$ mm²), top longitudinal reinforcement of 2Ø4 bars as two times the area of Ø4 rebar ($2 \times 12.6 = 25.2$ mm²), and transversal reinforcement with stirrups of 2Ø4 bars as two times the area of Ø4 rebar ($2 \times 12.6 = 25.2$ mm²).

Boundary conditions are set in the mid-point of the lower edge of the support plates. The left support plate is fixed for translations in the horizontal and vertical direction, while the right support plate is restricted for the translation only in the vertical direction. A static nonlinear analysis is performed with increasing forced displacement until failure. Imposed deformation in the vertical direction is considered as the point load with a value of -1 mm. To replicate the experimental test, which is governed by displacement control, the prescribed deformation is used in the model instead of force. The self-weight of the beam is not considered in the analysis.

A mesh sensitivity study was performed to optimize computational time with the convergence of results. The quadratic elements with an approximate element size of 15 mm are used to simulate concrete. Just under loading plates and in support zones, the density

of the mesh was increased to reduce the influence of the support and loading points. The FEM analysis was conducted on a model with around 1000 finite elements.

3.1.2. Large-Scale Model—Girder

The computational model of the large-scale girder was created for two-dimensional plane stress conditions. The geometry of the girder included longitudinal and transversal reinforcements, loading, and support plates. The main principles that were determined from small-scale modeling were used in modeling the large-scale girder. The same CQ16M plane stress elements were used for modeling concrete and for modeling loading and support plates.

Additionally, the same material properties as in the previous model were used for the loading and support plate, and line interface. The thickness of all shell elements is 200 mm, which is equal to the width of the girder. The geometries of reinforcement bars are defined with an equivalent cross-sectional area of the bar in a similar way as for small-scale models.

Computational analysis was performed for the loads that were used during the experimental testing of the supports. The total load during the test was 16,320 kg, and that load was used in FEM modeling. Figure 10 shows a schematic representation of the load on the girder.

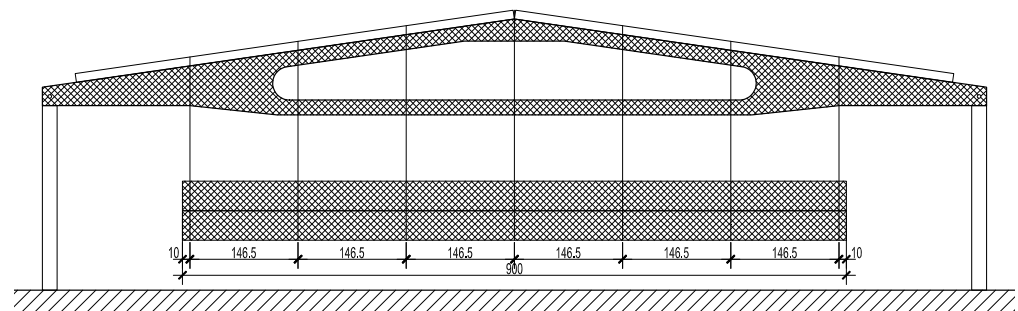


Figure 10. Schematic representation of the maximum load.

The load force from the ballast beams is transferred to the girder through seven-point suspension using the tensioner system. The forces in the tensioners are determined as reactions of the continuous beam (Figure 11) so that the supports correspond to a certain individual tensioner. The forces in the tensioners are included in the computational analysis as active forces acting on the upper chord of the main girder (Figure 12). The position of the forces corresponds to the position of the tensioner in the experiment. A static nonlinear analysis is performed with increasing forces up until failure. Forces in the vertical direction are considered as the point load with a value of 16.32 times the reaction forces from Figure 11. The experimental test, which is governed by force control, is replicated in that way. The self-weight of the girder is considered in the analysis as a distributed load implemented in the software.

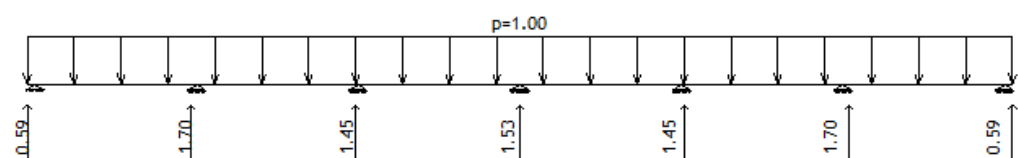


Figure 11. Reactions of a continuous beam due to unit distributed load.

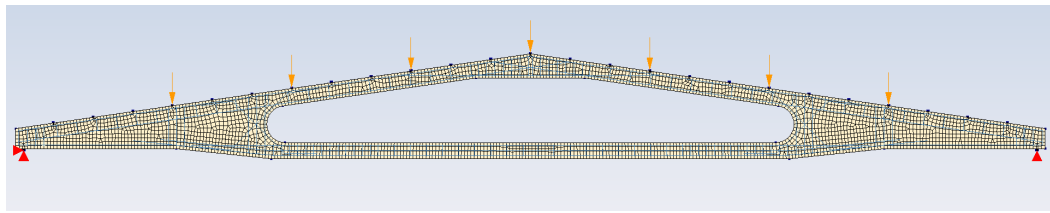


Figure 12. Finite element mesh, supports, and loading of the large girder.

Figure 12 shows the generated mesh of finite elements in concrete. The reinforcement is modeled with embedded finite elements. The analyzed beam has supports that correspond to the supports of a static simple beam system. The mesh density is determined so that the complex geometry of the girder can be accurately described. Elements with an approximate element size of 40 mm are used to simulate concrete. The FEM analysis was conducted on a model with around 10,000 finite elements.

3.2. Specimen Properties

3.2.1. Reinforcement

The reinforcing steel was modeled using the von Mises plasticity material model with a strain-hardening effect. The material was also observed to be linear up to yield stress and bilinear up to ultimate stress as shown in Figure 13. The material properties listed in Table 4 are Young's modulus E_s , Poisson's ratio μ_s , yielding stress f_y , ultimate stress f_u , ultimate strain ϵ_u , and hardening modulus E_{sT} .

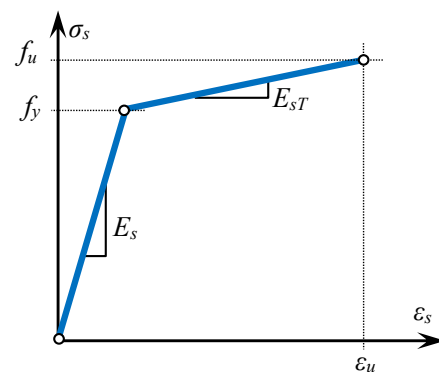


Figure 13. Stress–strain relationship of reinforcing steel.

Table 4. Mechanical properties of reinforcing steel.

	E_s (MPa)	μ_s (-)	f_y (MPa)	f_u (MPa)	ϵ_u (-)	E_{sT} (MPa)
Main reinforcement—Beam	200,000	0.3	400	450	0.05	1042
Main reinforcement—Girder	200,000	0.3	500	550	0.05	1053
Stirrups	210,000	0.3	240	360	0.25	482

3.2.2. Fiber-Reinforced Concrete

The software covers the complex behavior of concrete reinforced with steel micro-fibers. For the computational analysis, the total strain-based model was adopted to describe the behavior of concrete following the fib fiber reinforced concrete parameters.

Most of the material properties of the concrete are determined from tests on the standard specimens. For a parameter that was not tested, the value was estimated according to the Model Code 2010 [47]. The properties of the fiber-reinforced concrete used for the modeling of small-scale beams and large-scale girders are listed in Table 5. The stress–strain

relationship of the fiber concrete uses a softening law in tension presented in Figure 14. A parabolic law, used to describe behavior in compression, is based on the definition of the fracture energy in compression, determined as $G_c = 250 G_F$ according to [48].

Table 5. Mechanical properties of concrete.

Parameter			Beam	Girder
Young's modulus	E _c	(GPa)	24.26	40.37
Poisson's ratio	μ _c	(-)	0.12	0.22
Compressive strength	f _{cm}	(MPa)	22.41	85.10
Fracture energy in compression	G _c	(N/mm)	31.941	40.612
Fracture energy in tension	G _F	(N/mm)	0.1278	0.1624
Mass density	ρ	(T/mm ³)	2.25 × 10 ⁻⁹	2.46 × 10 ⁻⁹

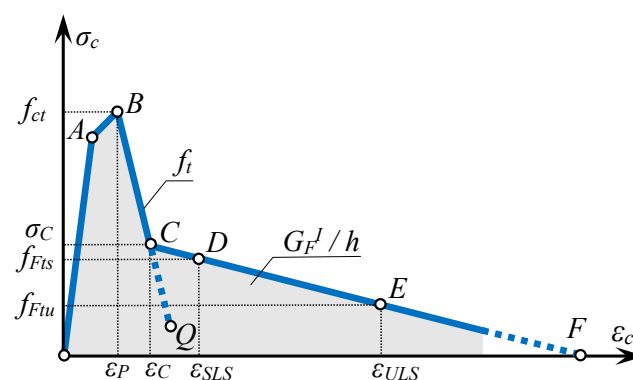


Figure 14. Stress–strain response of FRC in tension.

The tensile behavior of concrete is the most important aspect when modeling structures made from FRC. The mechanical parameters of the FRC were determined through laboratory tests on the three-point notched bending specimens. The diagram of the force versus the deformation can be produced from the test, where the deformation is expressed in terms of crack mouth opening displacement (CMOD). The FRC strength parameters are described by the residual flexural tensile strength ($f_{R,i}$) which is determined from the Force–CMOD diagram [10]. The uniaxial tensile behavior of the FRC was determined according to the Model Code 2010 [4,47].

Point B (Figure 14) corresponds to the value of the tensile stress of the FRC, which is determined from the characteristic value of the tensile stress (f_{ct}). Point A is defined as a tensile stress value equal to $0.9 f_{ct}$. Point C defines the drop of the tensile stress after the peak value, which is determined as the intersection of the softening branch of the plain concrete curve with the extrapolated branch of the FRC model defined by Points D and E. Points D and E are determined from the values of the residual flexural tensile strength for serviceability and ultimate limit states. It is considered that when a crack is opened to the width of an 8 mm concrete tensile strength reduces to zero which is represented by Point F. The values for the specific points that define the tensile constitutive relations for the FRC of the small-scale beam model and large-scale girder model are presented in Table 6.

Table 6. Mechanical properties of concrete.

Point	Beam		Girder	
	f_t (MPa)	CMOD (mm)	f_t (MPa)	CMOD (mm)
A	2.73	0.0011	6.37	0.0016
B	3.04	0.0020	7.08	0.0025
C	2.28	0.025	5.31	0.025
D	2.01	0.5	4.67	0.5
E	1.12	2.5	2.62	2.5
F	0	8	0	8

4. Analysis of the Results

4.1. Small Scale FRC Beam

In order to verify the accuracy of the FEM model, the results of the FEM analysis were compared to the test results. Factors explored in the computational analysis included normal and shear stresses as well as elastic and plastic strains, cracks, deflections, and reaction forces. The load-bearing capacity of the beam obtained from the numerical analysis ($N_{u,FEM}$) was 34.3 kN. It can be observed that the FEM result matched the peak load from the experimental study with the ratio $N_{u,FEM}/N_{u,exp} = 1.02$. Mean absolute error (MAE) and root mean squared error (RMSE) are calculated from absolute differences between the FEM estimate and experimental values. Values of MAE = 0.90 kN and RMSE = 1.10 kN indicate that the forecasted value has low errors, and the small difference between RMSE and MAE implies consistent error size. The diagrams of force (P) in comparison to the mid-span deflection (δ) from the experimental and the numerical analysis are presented in Figure 15a. The results from the numerical analysis showed very good agreement with the test results. The FEM model was found to have great predictions in terms of overall strength and structural behavior of the beam. The FRC beam failed through a combination of concrete cracking and reinforcement yielding, which are accompanied by large deformations of the test specimens.

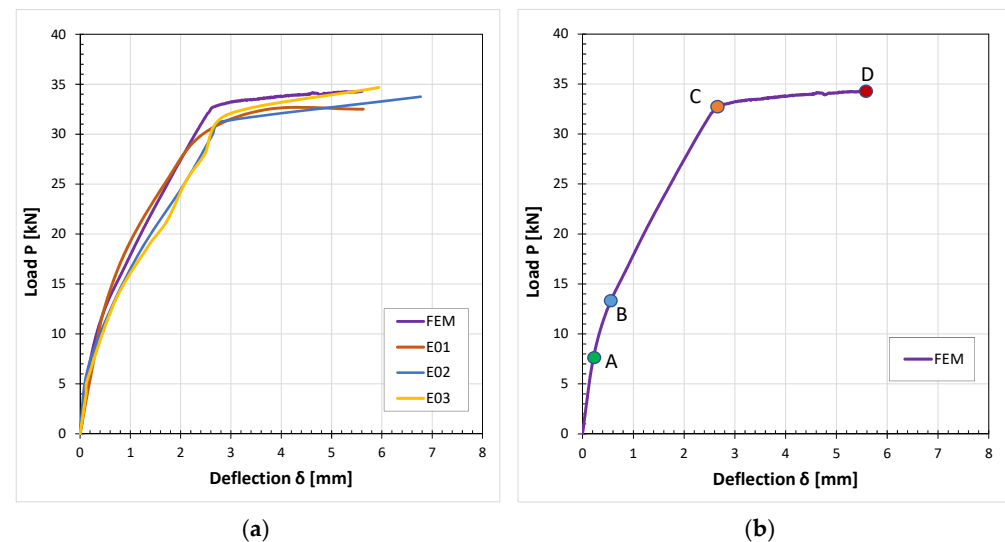


Figure 15. Relationship between load and deflection at the mid-span: (a) experimental and FEM results; (b) characteristic points from FEM analysis.

The behavior of lightly reinforced HRC beams is primarily governed by the tensile cracking of the concrete [18]. Many experimental studies described the behavior of beams made from the HRC in three distinct stages in order to define the flexural response in terms of the applied load vs. deflection diagram. The structural response starts as a linearly increasing branch until the development of the first cracks. The post-cracking stage begins

after this, and from that point, the behavior is dependent on the reinforcement ratio and fiber volume fraction among other factors. The presence of fibers induces a significant bridging stress between open crack faces which leads to a relatively higher fracture toughness and ductility. In the final stage, the flexural response is characterized by a plastic plateau due to the yielding of the longitudinal reinforcement bars. Similar behavior was registered during experimental research, as presented in Figure 15a. Considering the deformation curves, the post-peak response is characterized by a plastic plateau shape, and the snap-back or snap-through instabilities were not observed. Because of the limited number of tests, a brittleness number analysis and a ductile-to-brittle transition were not performed.

The behavior of the beam can be divided into four regions, as presented in the force-deflection diagram in Figure 15b. The presented diagram is obtained from FEM analysis and it shows different stages of the beam deformation. Point A on the $P-\delta$ curve is the instant of appearance of the first micro-crack at load of a 7.7 kN. Up until that point, the load capacity of the beam depends predominantly on the tensile strength of the concrete. Point B represents the value when the FRC reaches its maximum tensile stress at a force value of 13.4 kN. After that, cracks with a pronounced opening appear and the main reinforcement takes over most of the load-bearing process. Point C depends on the tensile strength of the steel reinforcement and represents the state when the yield stress in the main rebar develops. This point is reached at a force of 32.8 kN, while the corresponding deformation was 2.6 mm. At that point, large displacement of the beam develops and plastic deformation of the reinforcement begins. After this point, the beam will quickly reach failure, which is represented by Point D. The failure happened due to a force of 34.3 kN and a deformation of 5.7 mm.

While stress development predominantly depends on the tensile capacity of the concrete (Point A, Figure 15b), all four curves from Figure 15a almost completely coincide. After that, the load-carrying process starts to depend on the presence of the steel microfibers in the concrete. The small differences in the results could be explained by the different distribution of the steel microfibers in concrete and the variation of material properties.

To compare the failure mode between the FEM and experimental results distribution of normal stresses was developed. Up until the appearance of the first cracks, FRC behaves in the same way as ordinary concrete, but from FRC's higher tensile strength, an increase in ductility is expected since the micro-reinforcement has properties that prevent the opening of the formed crack. As long as the concrete remains without cracks, the distribution of normal stresses in the concrete is homogeneous, as shown in Figure 16a. When the maximum tensile strength is reached, the load on the beam is carried by the main reinforcement, which means that the stress concentration around the crack becomes inhomogeneous, as in Figure 16b.

Based on this process, the moment when the FRC will exceed its tensile strength can be determined, which was registered due to a force of 13.4 kN (Point B, in Figure 15b). The value of the crack opening was 1.7×10^{-3} mm, and the stress state in the tension zone in the concrete was 3.88 MPa, which is greater than the maximum tensile strength of the concrete itself. The load step that has significance from the experimental testing is the force of 26.77 kN, for which the crack value of 0.2 mm was registered. The mentioned crack width was adopted as the maximal crack opening that would be accepted in exploitation conditions. In the numerical analysis, the force of 26.8 kN caused a crack opening of 0.14 mm, which is a slightly smaller value than the experimental result. Cracks in the beam at peak load are shown in Figure 17. At peak load, flexural cracks are dominant. It can be noticed that crack distribution from the FEM analysis corresponds to cracks from the experimental analysis (Figure 2b).

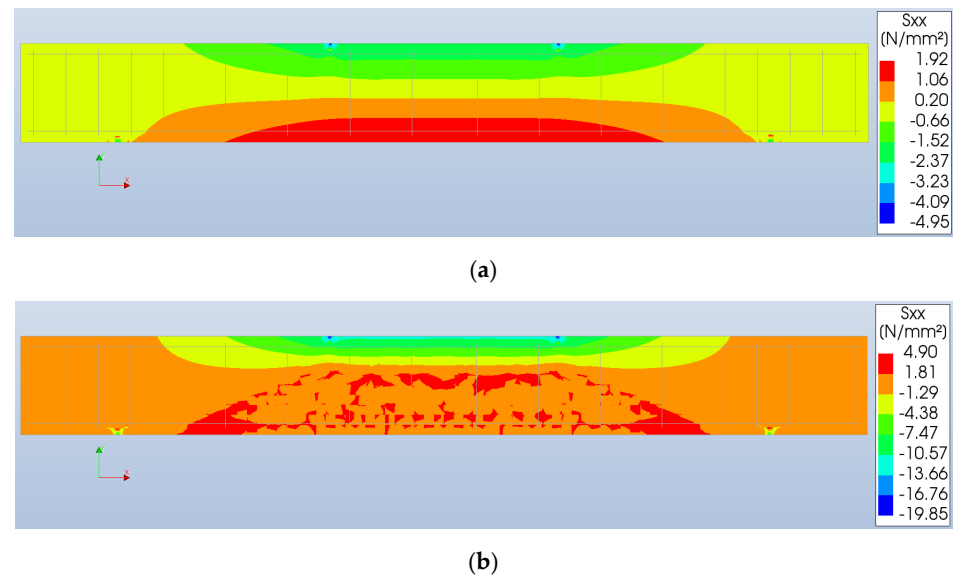


Figure 16. Normal stresses in beam: (a) before crack development; (b) after crack development.

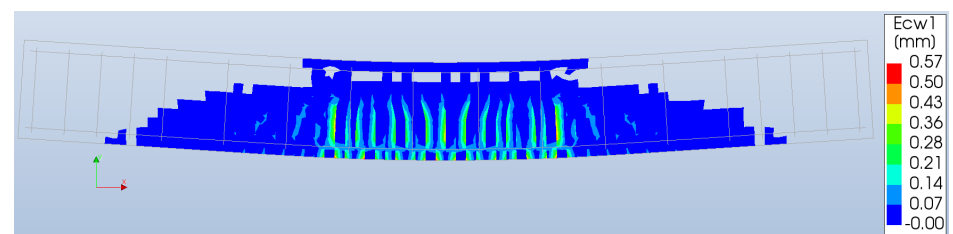


Figure 17. Cracks in beam at peak load.

The distribution of normal stresses in reinforcement at peak load is presented in Figure 18. The maximal value of stress in the bottom longitudinal reinforcement exceeded the yield stress value of 400 MPa. From those results, it can be concluded that the yielding of steel has occurred, which is followed by the large deformations of the beam and opening of the cracks.

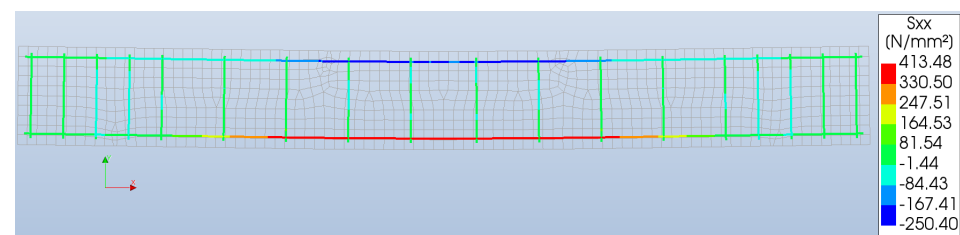


Figure 18. Normal stresses in reinforcement at peak load.

4.2. Large-Scale FRC Girder

The computational analysis included normal and shear stresses, crack development and distribution, global deformations and deflections, and reaction forces. The relationship between the load and the deflection of the point on the lower chord in the middle of the span is shown in Figure 19. The diagram shows the results from the experimental and FEM analysis. The maximum load level under expected service conditions is indicated by a horizontal dashed line. Three different slopes of the deformation curve obtained by FEM analysis can be observed. The initial slope corresponds to the state of the beam without cracks with fully elastic behavior (from 0 up to Point A). After that, a significant development of deformations is noticeable due to the change in stiffness after the formation

and propagation of cracks (the region between points A and B). At a load of about 230 kN, yield stress occurs in the longitudinal reinforcement. A further increase in the load leads to a further decrease in the stiffness of the girder, rotation of the cross-sections, and eventual failure (Point C). The coefficient of safety against breakage in relation to the maximum operating load is 1.54. The experimental load phases (LF) are represented by the blue dots. Point LF1 represents the first loading phase, LF2 the second, and LF3 the third, while point U represents the unloaded state. In LF1, the girder is without any cracks and in elastic behavior, which corresponds to Point A from the FEM analysis. The initial stiffness of the FEM model was to some extent higher, which could be explained by possible differences in material characteristics in the FEM and test model. The LF2 point is the point at which pronounced deformations are registered and cracks in concrete develop. In this phase, the girder is still almost in an elastic state. Upon reaching point LF2, a large number of cracks were registered (Figure 7b,c) with the development of deflections. All points registered during testing are close to the results from FEM analysis as shown in Figure 19.

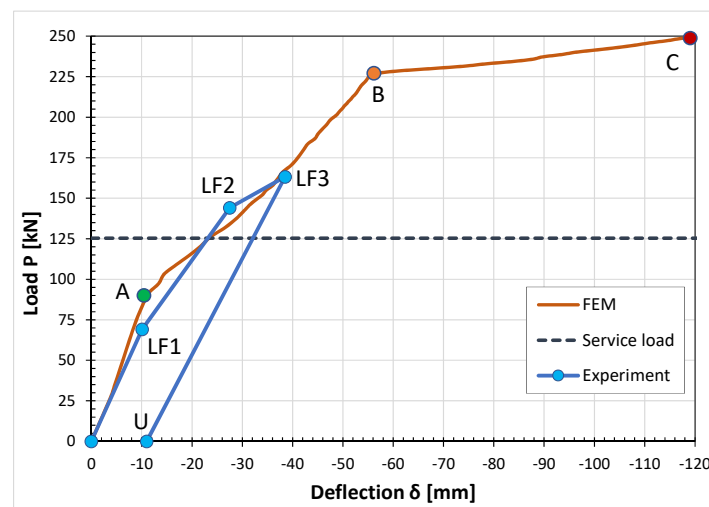


Figure 19. Relationship between load and deflection at the mid-span.

The results from the experimental analysis showed very good agreement with the FEM results in registered deflection, stresses, and load response. The FEM model was also used for predictions in terms of overall strength and structural behavior of the test girder. The large FRC model failed through a combination of concrete cracking and reinforcement yielding, which are accompanied by large deformations of the specimen. Deformations of the girder at maximal service load level are presented in Figure 20a. The vertical deflection in the middle of the lower chord was around 30 mm. The maximal deformations at peak load just before failure are presented in Figure 20b. A critical zone where the breaking of the girder will occur is noticeable at both ends of the girder.

The state of normal stresses in concrete for the maximum load during the service conditions is presented in Figure 21. Places where the highest normal stresses occur are visible. The highest compressive stresses are registered in a small zone on the upper chord, while the tensile stresses are present across the bottom part of the girder. From the presented stress values, it can be noticed that the compressive stresses in the concrete are not over the compressive strength of the FRC. At the same time, the tensile stresses in the concrete are at the values that correspond to the tensile strength of the concrete. Excess tensile stresses will result in the increased development of cracks, which will lead to an increase in tensile stresses in lower longitudinal reinforcement. Cracks due to the self-weight of the girder and maximal experimental load are presented in Figure 22. The majority of the cracks with a width from 0.05 mm to 0.15 mm are observed on the lower chord of the girder. Another critical zone is noticeable near supports at both ends of the girder. In those zones, a high

shear force led to high principal tensile stresses that exceeded the tensile capacity of the FRC. As a consequence, cracks with openings larger than 0.2 mm developed.

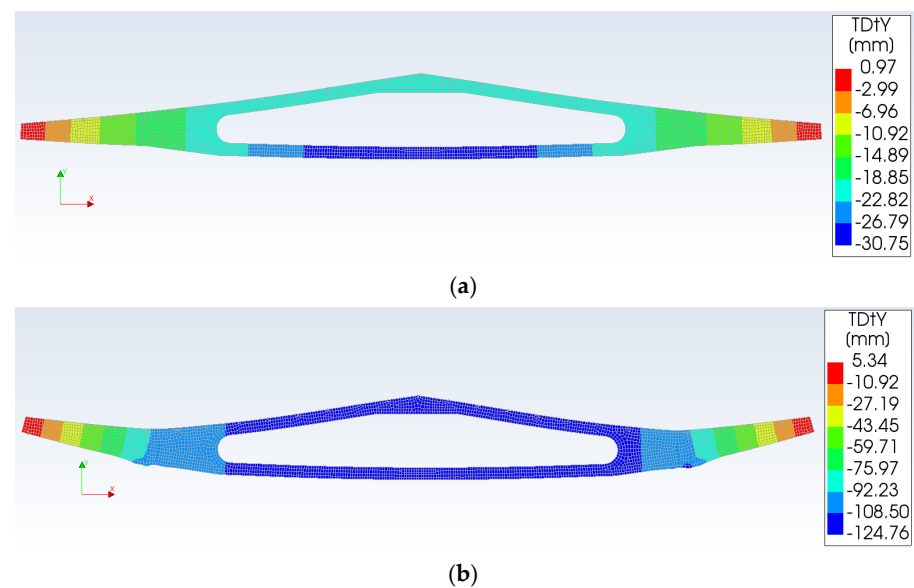


Figure 20. Deformations of the girder: (a) at service load; (b) at peak load.

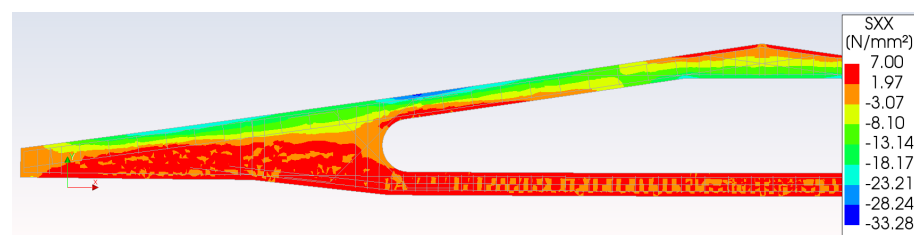


Figure 21. Normal stresses in girder.

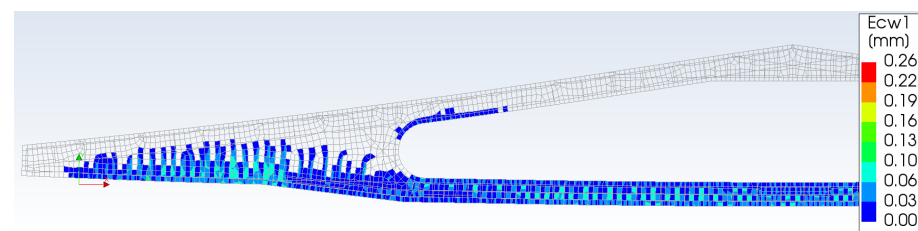


Figure 22. Cracks due to self-weight and maximal experimental load.

Cracks in the beam at the final load step that converged are shown in Figure 23. Presented deformations are given in normalized form to better show the global response of the girder. Critical zones with large cracks where fracture of the girder is expected are clearly noticeable. The same critical points were observed during the experiment.

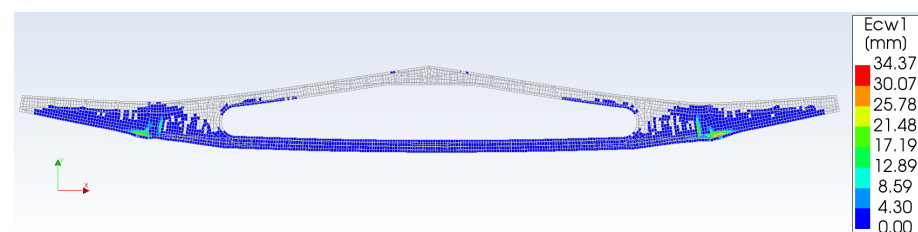


Figure 23. Cracks in beam at peak load.

After the FEM analysis, it can be observed that the computational model largely corresponds to the conditions observed during the experimental test. Through this model, the critical places on the girder are indicated, which were also shown during the test. Additionally, all conditions can be followed up to failure and the safety coefficient can be determined, which would be very difficult to achieve through an experiment in safe conditions considering the large magnitude of the breaking load. Various parameters can be analyzed through the calculation model and their influence on the bearing capacity can be determined. The results of the presented analysis indicate a high degree of accuracy of the model. The given calculation model could be used for further optimization of the geometry of the supports, the characteristics of the concrete, and the amount of reinforcement.

From the FEM analysis, a brittle type of failure of the large-scale girder could be observed. To fully explore the flexural capacity of the HRC girders, research on a ductile-to-brittle transition in the ultimate structural response needs to be carried out. This analysis would allow the definition of the minimum reinforcement and fiber volume to ensure a safe flexural bearing capacity of the large-scale HRC girder. As recommended in [49] the structural problem of size-scale effects can be quantitatively resolved by the tools provided by fracture mechanics—particularly by dimensional analysis and fractal geometry—since damage, strain localization, and fracture are phenomena are not always interpretable within the framework of continuum mechanics. Further research will investigate the flexural and shear behavior of HRC beams with extended experimental testing focused on size-scale effects.

5. Conclusions

Based on the experimental and computational analysis, the following conclusions were established:

- (1) The behaviors of the small-scale beam and large-scale girder are as expected for this static system and load case.
- (2) The load-bearing capacity of the large girder in the observed case depends on the load-bearing capacity of the reinforcement in the critical section, which indicates that the high mechanical characteristics of concrete cannot be fully utilized.
- (3) The given girder for the maximum operational load has a coefficient of safety in relation to breakage of 1.54. The safety coefficient is practically even higher because the service load, in addition to constant loads, includes the simultaneous effect of snow and wind at maximum intensity, which is an extremely rare situation in reality.
- (4) The calculation model indicates the possibility of further analysis in order to optimize both the geometry of the girder, the strength class of concrete, and the amount and form of used reinforcement.
- (5) Further analysis could provide concrete recommendations for the constructive application of concrete made with the addition of steel fibers.
- (6) Future research and development activities would focus more on sustainability enhancement by adopting other supplementary cementitious materials towards cement-free concrete and implementing recycled concrete aggregate in the FRC mix compositions.

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