

Article

Probabilistic Model Optimization and Safety Assessment Methods for Existing Masonry Structures

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Abstract

The practice of the assessment of the safety of existing masonry structures is related to the safety of people's lives and property. However, the current assessment method, described in “GB50292-2015 Standard for appraisal of reliability of civil buildings”, fails to fully consider the uncertainty-related characteristics of the structures, which easily leads to unreasonable assessment results. This paper proposes a method of safety assessment for existing masonry structures that considers the updating of resistance and load probability models and different member weights. First, based on the resistance probability model measured in the field, the resistance model in the current code (GB50292-2015) is updated through Bayesian theory. Then, the variable load model is updated for different subsequent working years through the equal-exceeding-probability method. Finally, the safety grade of the existing masonry structure is obtained by the analytic hierarchy process, using the affiliation set as the assessment index. This method of analysis solves the problem relating to the “jump” in the middle of the break-point of the traditional safety grading standard. It also fully considers the uncertainty-related characteristics of the existing structure, and its evaluation results align with the existing structure's actual situation, which is critical to the assessment of the safety of the existing masonry structure.



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1. Introduction

Existing masonry structures are a considerable presence in the field of construction engineering. Due to the influences of time and environment, the material performance is degraded, resulting in lower levels of structural safety and reliability, which affects the safety of people's lives and property. The structural safety assessment of existing masonry structures is important in urban renewal. At present, the assessment of the safety of existing masonry structures in China is mainly based on the “GB 50292-2015 Standard for appraisal of reliability of civil buildings” [1], which characterizes the safety level of the members through the coefficient $R/(\gamma_0 S)$, then describes the safety level of the subunit through the percentage of the safety level of the subunit's different members, and finally qualifies the safety level of the identified unit through the subunit's lowest assessment level. However, the current safety assessment method does not consider the weights of different types of members with respect to the structural safety, nor does it take into account the changes in

the probabilistic models of the resistance and loading of the existing structure, which leads to assessment results that are on the conservative side [2]. For example, the load probability model in current codes does not account for the structure's subsequent service life, nor does it specify investigations into the actual load distribution on the structure. The resistance model refers only to measured samples of in situ material strength, disregarding the effects of geometric characteristics and computational uncertainty, as well as the limitations of the sample size. Therefore, the current assessment method merely serves as a compliance check which determines whether existing building structures meet design specifications, and fails to adequately account for the actual conditions of existing buildings. There is an urgent need for the study of the safety assessment methods applicable to existing masonry structures.

The measurement of structural safety has gradually changed from member reliability to structural-system reliability. Currently, the primary research on structural-system safety includes three aspects: (i) finding and constructing the main failure modes of structural systems; (ii) calculating the mode failure probability based on the limit state equations of the failure modes; and (iii) calculating the failure probability of the system from the mode failure probability of the main failure modes [3]. Among the previous works on the topic, Freudenthal [4] has pioneered the analogy of structural systems to series systems. Based on this, Hong [5] proposed the probabilistic network assessment technique in 1975. With Ditlevesen's establishment of a narrow-bound formula for the reliability of structural systems in 1979 [6], the paradigm of overall structural reliability analysis based on the series-parallel system model took shape. Subsequently, failure mode screening techniques and equivalent system modeling methods were developed to provide a theoretical basis for the overall structural reliability analysis [7–12].

At present, scholars within China and abroad have attempted to introduce fuzzy mathematical theory [13,14] and the analytic hierarchy process (AHP) into the reliability assessment of existing building structures [15–25]. On this basis, Zheng [26] combined the affiliation rating method and the AHP to propose a system reliability-assessment method for existing frame structures. However, the AHP depends on the weights of substructures and does not apply to high-rise and super-high-rise buildings due to the large numbers of members, which decreases the weight coefficients of the underlying members. At the same time, the above analysis methods cannot reflect the real physical development of the structure from linear to nonlinear; for this reason, Li Jie et al. [27–35], proposed the probability density evolution theory based on the principle of probability conservation, and at the same time, utilizing the principle of equivalent extreme value, solved the problem of the combined explosion of the traditional structural overall reliability analysis. The chief difficulty in the overall reliability analysis of complex structural systems is solved by synthesizing the theory of probability density evolution and the principle of equivalent extreme-value events. The reliability assessment described above provides a crucial basis for structural rehabilitation [36].

Although a wealth of research achievements have been made relating to the assessment of the safety of structural systems, in practical application, practical assessment methods, fuzzy assessment methods, and physical-synthesis assessment methods have their advantages and disadvantages. There is still a lack of simple, reasonable, and feasible reliability-assessment methods for existing structural systems. At the same time, the reliability-assessment method associated with existing masonry building systems under the current code is based on the reliability assessments of elements; ignores the influences of the weights of element types, locations, importance, and other factors; and does not take into account the characteristics of the existing structural loading and resistance uncertainty.

Therefore, the assessment methodology of the current code needs to be updated to be congruent with the actual characteristics of existing building structures.

In this paper, firstly, with respect to the resistance probability model derived from small samples of field measurements of existing structures, the resistance model in the current code is updated through the Bayesian method, to make it conform to the actual characteristics of the existing structures and to overcome the disadvantage of the significant errors associated with the small-sample probability model. Then, the variable load model, which results in different determinations of subsequent working years, is updated through the equal-exceeding-probability method. Based on the updated model, the reliability and affiliations of the members are calculated, and the reliability of the members is characterized by the affiliation set. Finally, the overall reliability of the existing structure is evaluated using the AHP, taking into account the differences in the weights of the members of the existing structure. Therefore, the main innovations of this paper are as follows: (1) a method for modifying the resistance and load probability models of existing structures is proposed; (2) safety assessment criteria for existing structures based on reliability indices and affiliation sets are advanced; and (3) a methodological framework for the overall reliability assessments of existing structures is established. The findings provide significant guidance for the assessment of the safety of existing structures and offer a theoretical basis for revising subsequent relevant standards.

2. Update of the Member-Resistance Model for Existing Masonry Structures

2.1. On-Site Measured Material Strength Models

Currently, the inspection of existing masonry structures generally utilizes non-destructive testing. The rebound method is one of the most commonly used non-destructive testing methods, and is widely used in the engineering inspections of existing masonry structures. According to “GB/T 50315-2011 Technical Standard for On-site Inspection of Masonry Engineering” [37], when using the rebound method to determine the strength of masonry materials, it is necessary to use the rebound method to determine the brick strength (f_1) and the mortar strength (f_2), and then calculate the average value of the masonry material strength (f_m) according to Equation (1):

$$f_m = k_1 f_1^\alpha (1 + 0.07 f_2) k_2 \quad (1)$$

where f_1 is the brick compressive strength, f_2 is the bond mortar compressive strength, $\alpha = 0.5$, $k_1 = 0.78$, and when $f_2 < 1$, $k_2 = 0.6 + 0.4 f_2$.

If the mean and variance of the brick strength (f_1) and the mean and variance of the mortar strength (f_2) are obtained from the actual measurements, the mean and variance of the masonry material strength (f) can be obtained by the Taylor expansion method. If the function $Z = g(x_1, x_2, \dots, x_n)$ is assumed, the mean and variance can be expressed by Equations (2) and (3), respectively:

$$\mu_Z \approx g(\mu) + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 g}{\partial X_i \partial X_j} \bigg|_{\mu} \text{Cov}(X_i, X_j) \quad (2)$$

where μ is a vector of variable means.

$$\sigma_Z^2 \approx \sum_{i=1}^n \sum_{j=1}^n \frac{\partial g}{\partial X_i} \bigg|_{\mu} \frac{\partial g}{\partial X_j} \bigg|_{\mu} \text{Cov}(X_i, X_j) \quad (3)$$

According to “GB 50003-2011 Code for the Design of Masonry Structures” [38], the masonry strength standard value (f_k) should have a 95% guarantee rate, i.e., $f_k = f_m - 1.645\sigma$, so the mean coefficient of the measured material strength $\kappa_f = f_m / f_k$. From Equations (1)–(3), the mean coefficient (κ_f) and variation coefficient (δ_f) of the measured masonry strength can be obtained. Thus, the uncertainty-related characteristics κ_f^* and δ_f^* of the measured material properties can be obtained.

2.2. Resistance Model Update

Due to the limited number of measured samples, measured probability distribution parameters generally do not accurately reflect the actual situation of the structure. However, the prior probability model and distribution parameters are known during the structural design process, so the prior probability model can be updated based on the measured probability model, and a posterior probability distribution that is congruent with the characteristics of the existing building can be obtained through the Bayesian equation, Equation (4).

$$\pi(\theta|x) = \frac{\pi(\theta)p(x|\theta)}{\int_{-\infty}^{+\infty} \pi(\theta)p(x|\theta)d\theta} \quad (4)$$

where $\pi(\theta)$ is the prior probability density function; $p(x|\theta)$ is the conditional density function (i.e., the likelihood function).

If the prior probability distribution of a random variable is normal $N(\mu', \sigma'^2)$, the likelihood function $L(\theta)$ is normal $N(\mu^*, \sigma^{*2})$ and the posterior distribution is also normal $N(\mu'', \sigma''^2)$; the mean μ'' and variance σ''^2 are determined as follows:

$$\mu'' = \frac{\mu^* \sigma'^2 + \mu' \sigma^{*2}}{\sigma'^2 + \sigma^{*2}} \quad (5)$$

$$\sigma''^2 = \frac{(\sigma' \sigma^*)^2}{\sigma'^2 + \sigma^{*2}} \quad (6)$$

If the mean coefficient and coefficient of variation of the prior probability distribution of material performance uncertainty are κ'_f and δ'_f , respectively, then the mean coefficient and coefficient of variation of the posterior probability distribution of the masonry material performance uncertainty-related characteristics, as updated by the Bayesian method, are κ''_f and δ''_f , respectively. According to “GBJ 68-84 Unified Standard for Reliability Design of Building Structures” [39], the uncertainty-related characteristics of the resistance consist of computational model uncertainty, the uncertainty related to geometric characteristics, and material performance uncertainty. Assuming that the uncertainty of the computational model and the uncertainty associated with the geometric characteristics of the resistance of the existing masonry structure follow the a priori distribution characteristics, the updated mean coefficient of resistance and the coefficient of variation are as follows.

$$\kappa_R = \kappa_P \cdot \kappa_a \cdot \kappa''_f \quad (7)$$

where κ_R is the updated mean coefficient of resistance, κ_P is the mean coefficient of geometric characteristics uncertainty, κ_a is the mean coefficient of computational model uncertainty, and κ''_f is the mean coefficient of material performance uncertainty updated by the Bayesian method.

$$\delta_R = \sqrt{\delta_P^2 + \delta_a^2 + \delta''^2_f} \quad (8)$$

where δ_R is the updated variation coefficient of resistance, δ_P is the variation coefficient of geometric characteristics uncertainty, δ_a is the variation coefficient of computational model

uncertainty, and δ_f'' is the variation coefficient of material performance uncertainty updated by the Bayesian method.

The a priori distribution parameters of resistance uncertainty for brick masonry structures are shown in Table 1, and the a posteriori distribution parameters of resistance can be obtained through Equations (1)–(8).

Table 1. Statistical parameters of resistance uncertainty for the design of brick masonry members.

Types		κ	δ	Types		κ	δ
Ω_f	Axial compression	1.00	0.174	Ω_p	Axial compression	1.0922	0.2059
	Eccentric compression	1.00	0.174		Eccentric compression	1.1814	0.2159
	Shear	1.00	0.240		Shear	1.017	0.1260
Ω_a	Axial compression	1.00	0.023	R	Axial compression	1.0922	0.2705
	Eccentric compression	1.00	0.023		Eccentric compression	1.1814	0.2811
	Shear	1.00	0.036		Shear	1.0170	0.2734

Notes: $\kappa = \mu_X / X_k$, μ_X is the mean of the variable X ; X_k is the standard value of variable X ; and δ_X is the coefficient of variation of the variable X .

2.3. Load Model Update

A variable load obeys the Gumbel distribution, and the distribution characteristics of the variable load under different subsequent working-life conditions can be obtained by the equal-exceeding-probability method [40], as shown in Equation (9).

$$F_{Q_{T'}}(x) = [F_{Q_i}(x)]^{\frac{T'}{T}m} \quad (9)$$

where a is the probability distribution of loads for the subsequent working life T' ; b is the probability distribution model of loads at any point in time, as shown in Equation (10); T is the design working life; and m is the number of times the maximum value of the load occurs during the design reference period.

$$F_{Q_i}(x) = \exp \left[-\exp \left(-\frac{x - \theta}{\alpha} \right) \right] \quad (10)$$

where $\alpha = \sqrt{6}\sigma / \pi$, $\theta = \mu - \zeta\alpha$, μ is the mean, σ is the standard deviation, and ζ is Euler's constant $\zeta = 0.5772$.

From Equations (9) and (10), the probability distribution parameters for variable loads with different subsequent working years can be obtained, as shown in Table 2. The permanent loads follow a normal distribution and do not consider the effects of subsequent working years.

Table 2. Load statistical parameters for different numbers of subsequent working years.

Subsequent Working Years	S_G		Residential Live Loads		S_Q		Wind	
	κ	δ	κ	δ	κ	δ	κ	δ
10a	1.06	0.07	0.486	0.308	0.371	0.407	0.455	0.256
20a			0.554	0.270	0.437	0.346	0.673	0.225
30a			0.594	0.252	0.475	0.318	0.822	0.210
40a			0.622	0.241	0.502	0.300	0.928	0.201
50a			0.644	0.233	0.524	0.288	1.000	0.194

Notes: $\kappa = \mu_X / X_k$, μ_X is the mean of the variable X ; X_k is the standard value of variable X ; and δ_X is the coefficient of variation of the variable X .

3. Reliability Assessments of Existing Masonry Structural Systems

3.1. Affiliation Set

The standard “GB 50068-2018 Unified Standard for Reliability Design of Building Structures” [41] specifies the safety grading standards for existing masonry components with a safety grade of Grade II, as well as the corresponding target reliability indices, as shown in Table 3. Although using the member’s reliability index in order to rate existing masonry members is more accurate, there is the problem of sudden changes in the rating near the boundary region. For example, although $\beta = 3.1999$ and $\beta = 3.2001$ are very close, they span two safety levels. The problem of sudden rating changes in the boundary region can be solved by the affiliation rating method. The affiliation functions of the a_u level, b_u level, c_u level, and d_u level are shown in Equations (11)–(14), respectively, and their function images are as shown in Figure 1. The affiliation set $\mu_x = \{\mu_a(x), \mu_b(x), \mu_c(x), \mu_d(x)\}$ of the members can be obtained from the affiliation function.

Table 3. Safety classification of existing masonry members of structural safety class II.

a_u	b_u	c_u	d_u
$R/(\gamma_0 S) \geq 1$ $\beta \geq 3.7$	$1 > R/(\gamma_0 S) \geq 0.95$ $3.7 > \beta \geq 3.45$	$0.95 > R/(\gamma_0 S) \geq 0.9$ $3.45 > \beta \geq 3.2$	$R/(\gamma_0 S) < 0.9$ $\beta < 3.2$

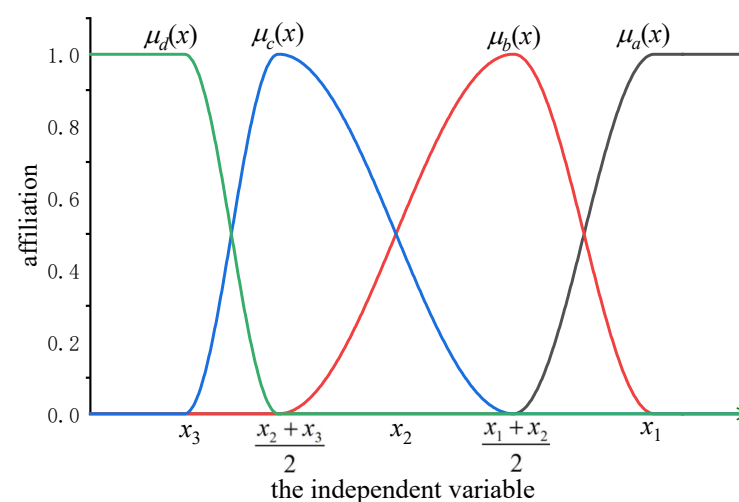


Figure 1. Affiliation function.

The affiliation function for the a_u level is

$$\mu_a(x) = \begin{cases} 1 & x_1 < x \\ 0.5 + 0.5 \sin \frac{2\pi}{x_1 - x_2} \left(x - \frac{3x_1 + x_2}{4} \right) & \frac{x_1 + x_2}{2} < x \leq x_1 \\ 0 & x \leq \frac{x_1 + x_2}{2} \end{cases} \quad (11)$$

The affiliation function for the b_u level is

$$\mu_b(x) = \begin{cases} 0 & x_1 < x \\ 0.5 - 0.5 \sin \frac{2\pi}{x_1 - x_2} \left(x - \frac{3x_1 + x_2}{4} \right) & \frac{x_1 + x_2}{2} < x \leq x_1 \\ 0.5 + 0.5 \sin \frac{2\pi}{x_1 - x_3} \left(x - \frac{x_1 + x_3 + 2x_2}{4} \right) & \frac{x_2 + x_3}{2} < x \leq \frac{x_1 + x_2}{2} \\ 0 & x \leq \frac{x_2 + x_3}{2} \end{cases} \quad (12)$$

The affiliation function for the c_u level is

$$\mu_c(x) = \begin{cases} 0 & \frac{x_1+x_2}{2} < x \\ 0.5 - 0.5 \sin \frac{2\pi}{x_1-x_2} \left(x - \frac{x_1+x_3+2x_2}{4} \right) & \frac{x_2+x_3}{2} < x \leq \frac{x_1+x_2}{2} \\ 0.5 + 0.5 \sin \frac{2\pi}{x_2-x_3} \left(x - \frac{3x_3+x_2}{4} \right) & x_3 < x \leq \frac{x_2+x_3}{2} \\ 0 & x \leq x_3 \end{cases} \quad (13)$$

The affiliation function for the d_u level is

$$\mu_d(x) = \begin{cases} 1 & x \leq x_3 \\ 0.5 - 0.5 \sin \frac{2\pi}{x_2-x_3} \left(x - \frac{3x_3+x_2}{4} \right) & x_3 < x \leq \frac{x_2+x_3}{2} \\ 0 & \frac{x_2+x_3}{2} < x \end{cases} \quad (14)$$

When the member reliability index is used as the independent variable, x_1 , x_2 , and x_3 , when corresponding to masonry members with structural safety class II, can be considered to be 3.7, 3.45, and 3.2, respectively. In other words, if the member reliability indices are $\beta_1 = 3.3$ and $\beta_2 = 3.4$, the corresponding affiliation sets are $[0, 0, 0.9045, 0.0955]$ and $[0, 0.1744, 0.8256, 0]$, respectively. The first-order reliability method (FORM) can calculate the reliability indices for existing masonry elements. The member's limit state function is shown in Equation (15).

$$Z = R - S_G - S_Q \quad (15)$$

where R is the resistance, S_G is the permanent load, and S_Q is the variable load.

3.2. Analytic Hierarchy Process (AHP)

3.2.1. Hierarchical Recursive

The recursive hierarchical model for a masonry structure is shown in Figure 2. The top layer (target layer) is the overall reliability index A; the middle layer (criterion layer) contains member damage hazard B1 and inter-component correlation coefficient B2, which affect the top layer (target layer); and the bottom layer (scheme layer) consists of basic independent elements such as element type (C1), location (C2), and number of floors (C3), which are independent and cross-influence the criterion layer (e.g., a single program-level factor can act on single/multiple criterion-layer factors). At the same time, criterion and scheme layers are sometimes interchangeable. For example, in masonry structures, where different floors have different effects on the overall structural safety, floors can also be used as the second layer, and element types and locations as the third layer.

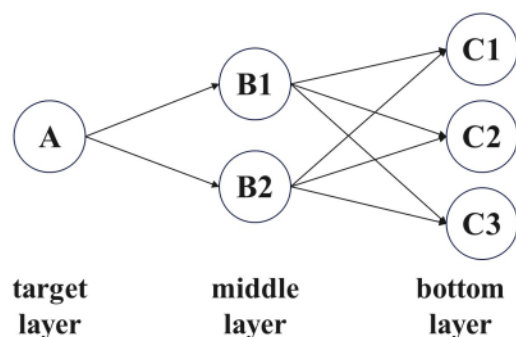


Figure 2. The hierarchical recursive model.

3.2.2. Judgment Matrix

The judgment matrix is constructed based on the nine-level scale method, and the scale values are set based on psychological experiments and practical-application experience. The specific meanings and usage scenarios are shown in Table 4.

Table 4. The ratio scale and the meanings associated with the judgment matrix.

Scale e_{ij}	Meaning	Example
1	Elements i and j are of equal importance	Beams and columns have a comparable impact on the overall reliability of the structure.
3	Element i is slightly more important than element j	Main beams are slightly more important than secondary beams.
5	Element i is significantly more important than element j	Framing columns have a significantly higher risk of failure than infill walls.
7	To a strong degree, element i is more important than element j	Foundation elements are much more important than decorative elements.
9	Element i is extremely more important than element j	Bearing wall failures can lead to a direct collapse of the structure.
2,4,6,8	Indicates the intermediate values for the adjacent judgments above	It is difficult to clearly distinguish between “slightly important” and “obviously important”.
$1/(1\sim9)$	The degree of unimportance of element i over element j	The importance of secondary beams is 1/3 of the importance of the main beams.

According to Figure 2, the recursive hierarchical structure, the judgment matrix E of a certain level consists of the square matrix of each element scale of the next level. For example, the safety of each layer of a five-story masonry structure has a different degree of importance relative to the safety of the entire structure; therefore, the safety of the entire structure is the target layer, each floor is a middle layer, and the judgment matrix for the target layer is a 5×5 square matrix. If it is assumed that the safety of the first floor has the most significant influence on the structure and the safety of the fifth floor has the least influence on the structure, the judgment matrix is as shown in Equation (16).

$$E_{55} = \begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} & e_{15} \\ e_{21} & e_{22} & e_{23} & e_{24} & e_{25} \\ e_{31} & e_{32} & e_{33} & e_{34} & e_{35} \\ e_{41} & e_{42} & e_{43} & e_{44} & e_{45} \\ e_{51} & e_{52} & e_{53} & e_{54} & e_{55} \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 & 7 & 9 \\ 1/3 & 1 & 3 & 5 & 7 \\ 1/5 & 1/3 & 1 & 3 & 5 \\ 1/7 & 1/5 & 1/3 & 1 & 3 \\ 1/9 & 1/7 & 1/5 & 1/3 & 1 \end{bmatrix} \quad (16)$$

3.2.3. Hierarchical Ordering

The weights of each element of this layer relative to the previous layer can be solved for by the judgment matrix. Generally, the maximum eigenvalue of the judgment matrix ($\lambda_{\max}$) and its corresponding eigenvector ($w_{\max}$) can be solved first. The eigenvector corresponding to the maximum eigenvalue can be normalized to obtain the weight of each element of this layer relative to the previous layer, as shown in Equation (17).

$$EW = \lambda W \quad (17)$$

where $\mathbf{E}$ is the judgment matrix, $\mathbf{W}$ is the eigenvector matrix, $\mathbf{W} = \{w_1, w_2, w_3 \dots w_n\}$, λ is the eigenvector, $\lambda = \{\lambda_1, \lambda_2, \lambda_3 \dots \lambda_n\}$, and $\lambda_{\max} = \max\{\lambda_1, \lambda_2, \lambda_3 \dots \lambda_n\}$.

When calculating the weight of each element of the scheme layer relative to the target layer, each layer weight vector $w_{\max i}$ should be calculated individually at first, and then divided equally according to the number of elements accounted for in the different types to determine the weight of each element accounted for in the overall reliability, as shown in Equation (18).

$$\bar{\mathbf{W}} = w_{\max 1}^T w_{\max 2} \quad (18)$$

where $\bar{\mathbf{W}}$ is the weight matrix of the bottom elements relative to the target layer, $w_{\max 1}$ is the middle-layer weight vector, and $w_{\max 1} = \{v_1, v_2, v_3 \dots v_n\}$, $w_{\max 2}$ is the bottom weight vector, $w_{\max 2} = \{u_1, u_2, u_3 \dots u_n\}$.

3.2.4. Judgment Matrix Consistency Test

A consistency test should be performed on the judgment matrix when calculating weight. If the consistency test is not passed, the judgment matrix must be readjusted. A consistency test is generally characterized by the consistency ratio indicator CR , as shown in Equation (19). When $CR < 0.1$, the consistency test returns a passing value.

$$CR = \frac{CI}{RI} \quad (19)$$

where CI is the index of deviation consistency of judgment matrix, as shown in Equation (20); RI is the average stochastic consistency index of judgment matrix, as shown in Table 5.

$$CI = \frac{\lambda_{\max} - n}{n - 1} \quad (20)$$

where $\lambda_{\max}$ is the maximum eigenvalue of the judgment matrix; n is the maximum order of the judgment matrix.

Table 5. The index of average random consistency.

n	1	2	3	4	5	6	7	8	9
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45

The total hierarchical ranking is calculated layer by layer from top to bottom. Taking the middle layer B as an example, if some influencing factors have a single sort consistency index of (CI_j) for the target layer (A_j) and the corresponding average random consistency index (RI_j), then the total sort random consistency ratio of the middle layer B can be expressed by Equation (21). When $CR_B < 0.1$, the total ranking consistency of the hierarchy is considered to be adequate; otherwise, the judgment matrix needs to be further adjusted until the consistency is satisfied.

$$CR_B = \frac{\sum_{i=1}^n W_j CI_j}{\sum_{i=1}^n W_j RI_j} \quad (21)$$

3.2.5. Assessment of Structural Systems

The set associated with the affiliation evaluation of the overall structure can be expressed by Equation (22).

$$\bar{u} = u_{4 \times n} \cdot \bar{\mathbf{W}}_{n \times 1} \quad (22)$$

where $u_{4 \times n}$ is the affiliation matrix of a single component of the scheme layer; $\overline{W}1_{n \times 1}$ is the weight matrix of a single element of the scheme layer with respect to the target layer, which is obtained by transforming $\overline{W}$.

The structural reliability grading criteria are shown in Table 6. A grade of A_u means the overall load-bearing capacity of the building meets the requirements for safe use, and measures should be taken with respect to a very small number of general components. A grade of B_u means the overall load-bearing capacity of the building basically meets the requirements for safe use, while some general members or a few major members should be subject to measures, and the overall load-bearing capacity of the building is not yet affected. A grade of C_u means the safety of the building does not meet the requirements for safe use, and a few major components should be subject to immediate measures to significantly change the overall bearing capacity of the building. A grade of D_u means the building has serious safety hazards, the overall load-bearing is seriously affected, and measures should be taken immediately.

Table 6. Safety classification of existing masonry structures.

Overall Reliability Rating	Residential Building	Industrial Building
A_u	$\mu_c = \mu_d = 0$ and $\mu_b \leq 0.25$	$\mu_c = \mu_d = 0$ and $\mu_b \leq 0.3$
B_u	$\mu_c = \mu_d = 0$ and $\mu_b > 0.25$	$\mu_c = \mu_d = 0$ and $\mu_b > 0.3$
C_u	$\mu_d \leq 0.05$ and $0 < \mu_c \leq 0.5$	$\mu_d \leq 0.05$ and $0 < \mu_c \leq 0.5$
D_u	$\mu_d > 0.05$ or $\mu_c > 0.5$	$\mu_d > 0.05$ and $\mu_c > 0.5$

The steps for rating the overall safety of an existing masonry structure are set out in the following, and a related flow chart is shown in Figure 3.

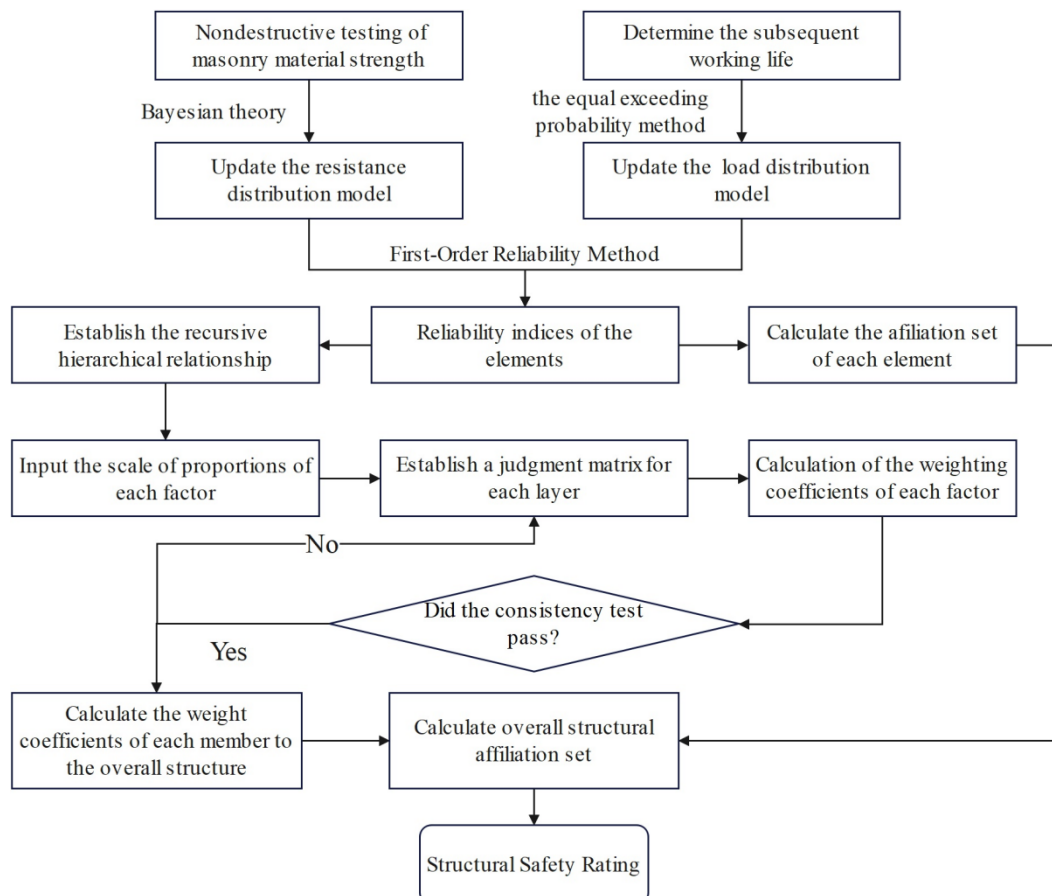


Figure 3. The overall safety evaluation process for existing masonry structures.

- (1) Conduct nondestructive testing of the strength of masonry materials and determine the subsequent working life.
- (2) Update the resistance distribution model based on Bayesian theory and the load distribution model based on the equal-exceeding-probability method.
- (3) Calculate the reliability index for each component using the first-order reliability method and calculate the affiliation set for each element.
- (4) Establish the recursive hierarchical relationship, build the judgment matrix based on ratio scaling, and calculate the weight coefficients of each factor for each single level.
- (5) Conduct a consistency test of the judgment matrix.
- (6) Calculate the weight coefficients of each element relative to the overall structure.
- (7) Synthesize (3) and (6) to calculate the overall structural affiliation set; this determines the overall safety rating for the existing masonry structure.

4. Assessment Case

The structure is a five-story existing brick masonry house with a subsequent working life of 30 years; the height of the first floor is 3.500 m, and the height of the other floors is 2.700 m. The strength grade of the brick is MU10 (the average value of the on-site measurements is 11.0 MPa, and the coefficient of variation is 0.18), and the strength grade of the mortar is M1.0 (the average value of the on-site measurements is 1.1 MPa, and the coefficient of variation is 0.04). The thickness of both the internal and external brick walls is 240 mm. The masonry house is load-bearing by means of its transverse walls. The masonry weight is 18 kN/m^3 , the permanent floor load is 4.0 kN/m^2 , the floor live load is 2.0 kN/m^2 , the stairwell permanent load is 7.0 kN/m^2 , the stairwell live load is 3.5 kN/m^2 , the roof permanent load is 5.0 kN/m^2 , and the roof live load is 0.5 kN/m^2 , while the first floor and the standard floor plan are structurally planned as shown in Figure 4. Walls with numerical axis designations are transverse walls, while walls with alphabetic axis designations are longitudinal walls. Components are numbered in the format “number1–number2”, where the first digit represents the wall number and the second digit denotes the wall segment number. For example, component 15-1 in Figure 4a indicates the first wall segment of transverse wall 15.

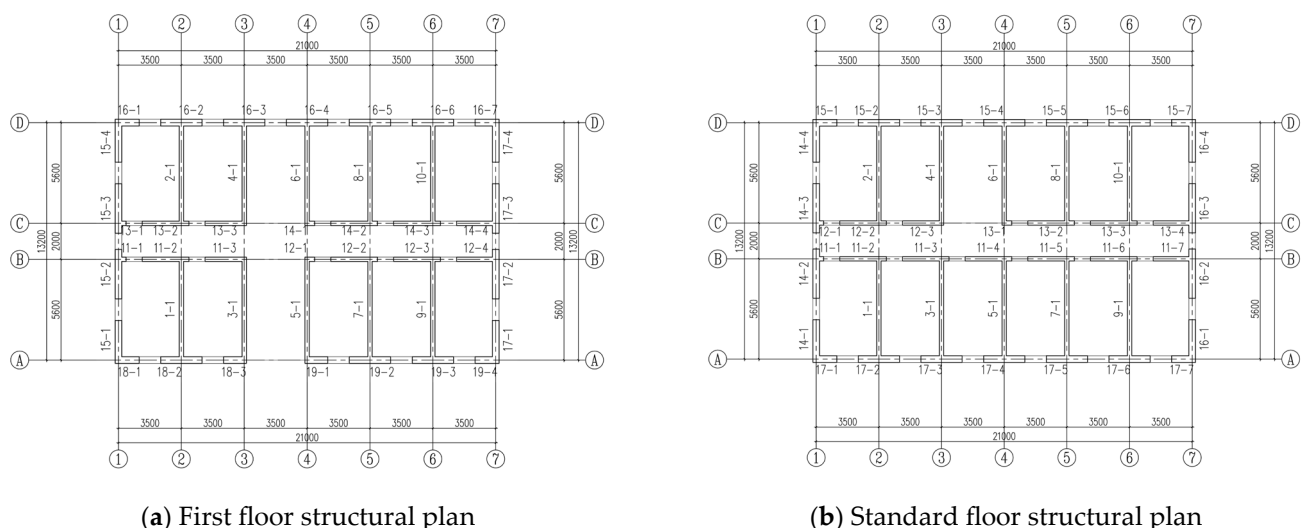


Table 7. The safety grades of each member according to the “GB 50292-2015 Standard for appraisal of reliability of civil buildings” are shown in Table 7. Since the safety grades of the members from the third to the fifth floor are the same as those of the second floor, this paper only shows the information for the first and second floors. According to the “GB 50292-2015 Standard for appraisal of reliability of civil buildings”, the proportion of d_u -grade members is 10.9%, which exceeds the 5% limit, and the superstructure is rated as D_u -grade. This means the building has serious safety hazards, the overall load-bearing is seriously affected, and measures should be taken immediately.

Table 7. Safety grades for the first- and second-floor compression members per GB 50292-2015.

First Floor						Second Floor					
Number	R_d/kN	S_{Gk}/kN	S_{Qk}/kN	$R_d/(\gamma_0 S)$	Grade	Number	R_d/kN	S_{Gk}/kN	S_{Qk}/kN	$R_d/(\gamma_0 S)$	Grade
1-1	1161.21	795.98	166.6	0.9	c_u	1-1	1220.54	623.5	127.40	1.22	a_u
2-1	1161.21	795.98	166.6	0.9	c_u	2-1	1220.54	623.5	127.40	1.22	a_u
3-1	1161.21	946.34	176.52	0.78	d_u	3-1	1220.54	623.5	127.40	1.22	a_u
4-1	1161.21	912.4	235.32	0.75	d_u	4-1	1220.54	709.71	179.06	1.02	a_u
5-1	1161.21	976.46	181.91	0.75	d_u	5-1	1220.54	623.5	127.40	1.22	a_u
6-1	1161.21	928.96	240.71	0.74	d_u	6-1	1220.54	724.6	183.17	1.00	a_u
7-1	1161.21	795.98	166.6	0.9	c_u	7-1	1220.54	623.5	127.40	1.22	a_u
8-1	1161.21	795.98	166.6	0.9	c_u	8-1	1220.54	623.5	127.40	1.22	a_u
9-1	1161.21	795.98	166.6	0.9	c_u	9-1	1220.54	623.5	127.40	1.22	a_u
10-1	1161.21	795.98	166.6	0.9	c_u	10-1	1220.54	623.5	127.40	1.22	a_u
11-1	118.59	66.71	9.13	1.18	a_u	11-1	121.92	50.87	6.96	1.59	a_u
11-2	602.96	268.38	36.75	1.49	a_u	11-2	619.94	204.65	28.00	2.01	a_u
11-3	533.39	247.10	36.07	1.43	a_u	11-3	619.94	204.65	28.00	2.01	a_u
12-1	103.98	82.47	12.2	0.84	d_u	11-4	619.94	204.65	28.00	2.01	a_u
12-2	602.96	268.38	36.75	1.49	a_u	11-5	619.94	204.65	28.00	2.01	a_u
12-3	602.96	268.38	36.75	1.49	a_u	11-6	619.94	204.65	28.00	2.01	a_u
12-4	548.47	210.67	27.61	1.81	a_u	11-7	563.9	153.78	21.00	2.44	a_u
13-1	118.59	66.71	9.13	1.18	a_u	12-1	121.92	50.87	6.96	1.59	a_u
13-2	602.96	268.38	36.75	1.49	a_u	12-2	619.94	204.65	28.00	2.01	a_u
13-3	533.39	226.65	36.75	1.54	a_u	12-3	548.41	177.13	27.48	2.02	a_u
14-1	103.98	75.13	12.2	0.9	c_u	13-1	106.82	59.33	9.29	1.17	a_u
14-2	602.96	268.38	36.75	1.49	a_u	13-2	619.94	204.65	28.00	2.01	a_u
14-3	602.96	268.38	36.75	1.49	a_u	13-3	619.94	204.65	28.00	2.01	a_u
14-4	548.47	210.67	27.61	1.81	a_u	13-4	563.9	153.78	21.00	2.44	a_u
15-1	881.57	372.42	41.64	1.61	a_u	14-1	904.12	285.66	31.84	2.16	a_u
15-2	1057.09	443.54	41.66	1.65	a_u	14-2	1073.42	337.81	31.86	2.20	a_u
15-3	1057.09	443.54	41.66	1.65	a_u	14-3	1073.42	337.81	31.86	2.21	a_u
15-4	881.57	372.42	41.64	1.61	a_u	14-4	904.42	285.66	31.84	2.16	a_u
16-1	535.21	158.54	0.00	2.6	a_u	15-1	542.21	118.55	0.00	3.52	a_u
16-2	922.08	317.09	0.00	2.24	a_u	15-2	934.14	237.1	0.00	3.03	a_u
16-3	922.08	317.09	0.00	2.24	a_u	15-3	934.14	237.1	0.00	3.03	a_u
16-4	922.08	317.09	0.00	2.24	a_u	15-4	934.14	237.1	0.00	3.03	a_u
16-5	922.08	317.09	0.00	2.24	a_u	15-5	934.14	237.1	0.00	3.03	a_u
16-6	922.08	317.09	0.00	2.24	a_u	15-6	934.14	237.1	0.00	3.03	a_u
16-7	535.21	158.54	0.00	2.6	a_u	15-7	542.21	118.55	0.00	3.52	a_u
17-1	881.57	372.42	41.64	1.61	a_u	16-1	904.12	285.66	31.84	2.16	a_u
17-2	1057.09	443.54	41.66	1.65	a_u	16-2	1073.42	337.81	31.86	2.21	a_u
17-3	1057.09	443.54	41.66	1.66	a_u	16-3	1073.42	337.81	31.86	2.21	a_u
17-4	881.57	372.42	41.64	1.61	a_u	16-4	904.42	285.66	31.84	2.16	a_u
18-1	535.21	158.54	0.00	2.6	a_u	17-1	542.21	118.55	0.00	3.52	a_u
18-2	922.08	317.09	0.00	2.24	a_u	17-2	934.14	237.1	0.00	3.03	a_u
18-3	509.15	203.98	0.00	1.92	a_u	17-3	934.14	237.1	0.00	3.03	a_u
19-1	509.15	203.98	0.00	1.92	a_u	17-4	934.14	237.1	0.00	3.03	a_u
19-2	922.08	317.09	0.00	2.24	a_u	17-5	934.14	237.1	0.00	3.03	a_u
19-3	922.08	317.09	0.00	2.24	a_u	17-6	934.14	237.1	0.00	3.03	a_u
19-4	535.21	158.54	0.00	2.6	a_u	17-7	542.21	118.55	0.00	3.52	a_u

4.2. Safety Assessment Based on Model Updating and Affiliations

The process for evaluating the safety of existing masonry structures according to Figure 1 is as follows:

(1) The resistance model is updated based on the measured values. The probability distribution model for the material performance uncertainty before and after the update is shown in Figure 5, and it can be observed that after the update, the probability density function curve for the material strength uncertainty is changed from “short and fat” to “thin and high”; i.e., the mean coefficient of material strength uncertainty is increased, and the coefficient of variation is decreased. The resistance distribution parameters are updated according to Equations (1)–(7), with $\kappa_R = 1.3385$, $\delta_R = 0.2325$, and obeying a log-normal distribution. According to Table 2, the parameters of variable load distribution are selected for a subsequent working life of 30 years, with $\kappa_{SQ} = 0.594$, $\delta_{SQ} = 0.252$, and obeying the Gumbel distribution. The permanent-load distribution parameters are $\kappa_{SG} = 1.06$, $\delta_{SG} = 0.07$, obeying a normal distribution.

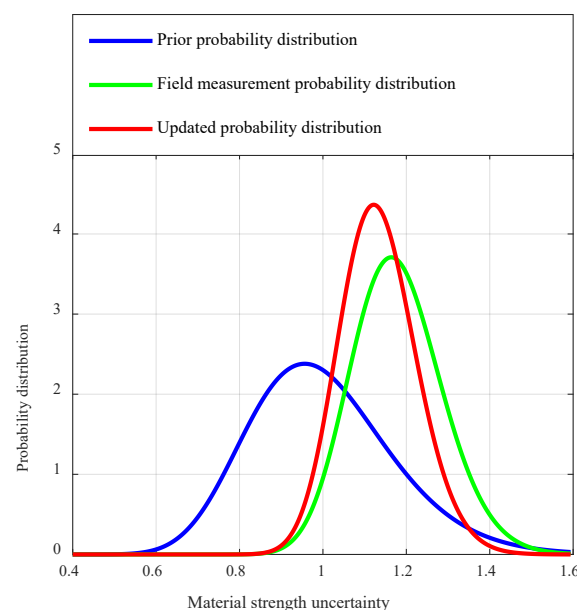


Figure 5. Probability density function, before and after material strength update.

(2) The safety class of this masonry structure is class II, and its standard value of resistance $R_k = 1.6R_d$. The reliability index of each member can be obtained according to the first-order reliability method, and then the affiliation set for each member can be calculated according to Equations (11) and (14), as shown in Table 8.

Table 8. Affiliation information for elements associated with the first or second level.

First Floor					Second Floor				
Number	B	Element Type	Affiliation Set	Weights	Number	B	Element Type	Affiliation Set	Weights
1-1	3.59	Bearing wall	[0.0402,0.9598,0,0]	0.0136	1-1	4.82	Bearing wall	[1,0,0,0]	0.0069
2-1	3.59	Bearing wall	[0.0402,0.9598,0,0]	0.0136	2-1	4.82	Bearing wall	[1,0,0,0]	0.0069
3-1	2.96	Bearing wall	[0,0,0,1]	0.0136	3-1	4.82	Bearing wall	[1,0,0,0]	0.0069
4-1	2.86	Bearing wall	[0,0,0,1]	0.0136	4-1	4.11	Bearing wall	[1,0,0,0]	0.0069
5-1	2.83	Bearing wall	[0,0,0,1]	0.0136	5-1	4.82	Bearing wall	[1,0,0,0]	0.0069
6-1	2.78	Bearing wall	[0,0,0,1]	0.0136	6-1	4.02	Bearing wall	[1,0,0,0]	0.0069
7-1	3.59	Bearing wall	[0.0402,0.9598,0,0]	0.0136	7-1	4.82	Bearing wall	[1,0,0,0]	0.0069
8-1	3.59	Bearing wall	[0.0402,0.9598,0,0]	0.0136	8-1	4.82	Bearing wall	[1,0,0,0]	0.0069
9-1	3.59	Bearing wall	[0.0402,0.9598,0,0]	0.0136	9-1	4.82	Bearing wall	[1,0,0,0]	0.0069

Table 8. Cont.

First Floor					Second Floor				
Number	B	Element Type	Affiliation Set	Weights	Number	B	Element Type	Affiliation Set	Weights
10-1	3.59	Bearing wall	[0.0402,0.9598,0,0]	0.0136	10-1	4.82	Bearing wall	[1,0,0,0]	0.0069
11-1	4.68	Bearing wall	[1,0,0,0]	0.0136	11-1	5.92	Bearing wall	[1,0,0,0]	0.0069
11-2	5.65	Bearing wall	[1,0,0,0]	0.0136	11-2	6.89	Bearing wall	[1,0,0,0]	0.0069
11-3	5.45	Bearing wall	[1,0,0,0]	0.0136	11-3	6.89	Bearing wall	[1,0,0,0]	0.0069
12-1	3.21	Bearing wall	[0,0,0.0186,0.9814]	0.0136	11-4	6.89	Bearing wall	[1,0,0,0]	0.0069
12-2	5.65	Bearing wall	[1,0,0,0]	0.0136	11-5	6.89	Bearing wall	[1,0,0,0]	0.0069
12-3	5.65	Bearing wall	[1,0,0,0]	0.0136	11-6	6.89	Bearing wall	[1,0,0,0]	0.0069
12-4	6.28	Bearing wall	[1,0,0,0]	0.0136	11-7	7.68	Bearing wall	[1,0,0,0]	0.0069
13-1	4.68	Bearing wall	[1,0,0,0]	0.0136	12-1	5.92	Bearing wall	[1,0,0,0]	0.0069
13-2	5.65	Bearing wall	[1,0,0,0]	0.0136	12-2	6.89	Bearing wall	[1,0,0,0]	0.0069
13-3	5.74	Bearing wall	[1,0,0,0]	0.0136	12-3	6.90	Bearing wall	[1,0,0,0]	0.0069
14-1	3.54	Bearing wall	[0,0.8644,0.1356,0]	0.0136	13-1	4.65	Bearing wall	[1,0,0,0]	0.0069
14-2	5.65	Bearing wall	[1,0,0,0]	0.0136	13-2	6.89	Bearing wall	[1,0,0,0]	0.0069
14-3	5.65	Bearing wall	[1,0,0,0]	0.0136	13-3	6.89	Bearing wall	[1,0,0,0]	0.0069
14-4	6.28	Bearing wall	[1,0,0,0]	0.0136	13-4	7.68	Bearing wall	[1,0,0,0]	0.0069
15-1	5.96	Mountain wall	[1,0,0,0]	0.0166	14-1	7.17	Mountain wall	[1,0,0,0]	0.0084
15-2	6.06	Mountain wall	[1,0,0,0]	0.0166	14-2	7.25	Mountain wall	[1,0,0,0]	0.0084
15-3	6.06	Mountain wall	[1,0,0,0]	0.0166	14-3	7.25	Mountain wall	[1,0,0,0]	0.0084
15-4	5.96	Mountain wall	[1,0,0,0]	0.0166	14-4	7.17	Mountain wall	[1,0,0,0]	0.0084
16-1	≥ 10.00	Window wall	[1,0,0,0]	0.0038	15-1	≥ 10.00	Window wall	[1,0,0,0]	0.0020
16-2	≥ 10.00	Window wall	[1,0,0,0]	0.0038	15-2	≥ 10.00	Window wall	[1,0,0,0]	0.0020
16-3	≥ 10.00	Window wall	[1,0,0,0]	0.0038	15-3	≥ 10.00	Window wall	[1,0,0,0]	0.0020
16-4	≥ 10.00	Window wall	[1,0,0,0]	0.0038	15-4	≥ 10.00	Window wall	[1,0,0,0]	0.0020
16-5	≥ 10.00	Window wall	[1,0,0,0]	0.0038	15-5	≥ 10.00	Window wall	[1,0,0,0]	0.0020
16-6	≥ 10.00	Window wall	[1,0,0,0]	0.0038	15-6	≥ 10.00	Window wall	[1,0,0,0]	0.0020
16-7	≥ 10.00	Window wall	[1,0,0,0]	0.0038	15-7	≥ 10.00	Window wall	[1,0,0,0]	0.0020
17-1	5.96	Mountain wall	[1,0,0,0]	0.0166	16-1	7.17	Mountain wall	[1,0,0,0]	0.0084
17-2	6.06	Mountain wall	[1,0,0,0]	0.0166	16-2	7.25	Mountain wall	[1,0,0,0]	0.0084
17-3	6.06	Mountain wall	[1,0,0,0]	0.0166	16-3	7.25	Mountain wall	[1,0,0,0]	0.0084
17-4	5.96	Mountain wall	[1,0,0,0]	0.0166	16-4	7.17	Mountain wall	[1,0,0,0]	0.0084
18-1	≥ 10.00	Window wall	[1,0,0,0]	0.0038	17-1	≥ 10.00	Window wall	[1,0,0,0]	0.0020
18-2	≥ 10.00	Window wall	[1,0,0,0]	0.0038	17-2	≥ 10.00	Window wall	[1,0,0,0]	0.0020
18-3	≥ 10.00	Window wall	[1,0,0,0]	0.0038	17-3	≥ 10.00	Window wall	[1,0,0,0]	0.0020
19-1	≥ 10.00	Window wall	[1,0,0,0]	0.0038	17-4	≥ 10.00	Window wall	[1,0,0,0]	0.0020
19-2	≥ 10.00	Window wall	[1,0,0,0]	0.0038	17-5	≥ 10.00	Window wall	[1,0,0,0]	0.0020
19-3	≥ 10.00	Window wall	[1,0,0,0]	0.0038	17-6	≥ 10.00	Window wall	[1,0,0,0]	0.0020
19-4	≥ 10.00	Window wall	[1,0,0,0]	0.0038	17-7	≥ 10.00	Window wall	[1,0,0,0]	0.0020

(3) Definition of judgment matrices. The middle-layer judgment matrix B is defined according to the number of floors (five floors in total, defined as a 5×5 square matrix). The judgment matrix C of the scheme layer is defined according to the types of elements (differentiated as bearing wall, mountain wall, or window wall, and defined as a 3×3 square matrix). Combined with engineering experience, the judgment matrices B and C are constructed according to the scale in Table 4

$$B = \begin{bmatrix} 1 & 3 & 5 & 7 & 9 \\ 1/3 & 1 & 3 & 5 & 7 \\ 1/5 & 1/3 & 1 & 3 & 5 \\ 1/7 & 1/5 & 1/3 & 1 & 3 \\ 1/9 & 1/7 & 1/5 & 1/3 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 3 & 5 \\ 1/3 & 1 & 3 \\ 1/5 & 1/3 & 1 \end{bmatrix}$$

And when the consistency test is conducted on the judgment matrix according to Equations (17)–(21), it can be determined that $CR_B = 0.053 < 0.1$, $CR_C = 0.033 < 0.1$, and the consistency test is passed.

(4) By solving for the maximum eigenvalue and the maximum vector, the weight coefficients $Weight_B$ and $Weight_C$ for a single level can be obtained. The weight coefficient matrix $Weight_A$ for each component with respect to the overall structure can be obtained by multiplying the judgment matrix weight coefficient vectors ($Weight_B^T \cdot Weight_C$).

$$Weight_B = \{0.5218, 0.2615, 0.1290, 0.0634, 0.0333\},$$

$$Weight_C = \{0.6370, 0.2583, 0.1047\},$$

$$Weight_A = Weight_B^T \cdot Weight_C = \begin{matrix} & & \begin{matrix} bearing \\ wall \end{matrix} & \begin{matrix} mountain \\ wall \end{matrix} & \begin{matrix} window \\ wall \end{matrix} \\ \begin{matrix} 1st floor \\ 2nd floor \\ 3rd floor \\ 4th floor \\ 5th floor \end{matrix} & & \begin{bmatrix} 0.3267 & 0.1325 & 0.0537 \\ 0.1666 & 0.0675 & 0.0274 \\ 0.0822 & 0.0333 & 0.0135 \\ 0.0404 & 0.0164 & 0.0066 \\ 0.0212 & 0.0086 & 0.0035 \end{bmatrix} \end{matrix}$$

The weight coefficients for each member for the structure are then obtained by dividing by the number of members of the same type in the weight coefficient matrix; the weight coefficients for each member associated with the first floor or the second floor are shown in Table 8.

(5) Based on the affiliation set and the weight coefficients of each member, the affiliation set of the structure can be obtained according to Equation (22). The $\bar{\mu} = \{0.840, 0.090, 0.002, 0.068\}$. By checking Table 6, it can be determined that the existing masonry structure is rated D_u .

Table 9 presents a comparison of the assessment results for components and structures under the current method and the proposed method. The proportions of c_u - and d_u -grade components are 15% and 11%, based on the current method, while the proportions of c_u - and d_u -grade components are 17% and 19%. The rating results for the structure-safety rating according to the standard in the “GB 50292-2015 Standard for appraisal of reliability of civil buildings” and the method outlined in this paper reflect consistent assessment results (both D_u -grade); that is, the masonry structure has serious safety hazards, and immediate measures should be taken.

Table 9. Comparison of the two assessment methods.

First Floor							Second Floor					
Number	Grade1	Grade2	Grade3	Number	Grade1	Grade2	Number	Grade1	Grade2	Number	Grade1	Grade2
1-1	c_u	c_u	b_u	14-4	a_u	a_u	1-1	a_u	a_u	13-4	a_u	a_u
2-1	c_u	c_u	b_u	15-1	a_u	a_u	2-1	a_u	a_u	14-1	a_u	a_u
3-1	d_u	d_u	c_u	15-2	a_u	a_u	3-1	a_u	a_u	14-2	a_u	a_u
4-1	d_u	d_u	c_u	15-3	a_u	a_u	4-1	a_u	a_u	14-3	a_u	a_u
5-1	d_u	d_u	c_u	15-4	a_u	a_u	5-1	a_u	a_u	14-4	a_u	a_u

Table 9. Cont.

First Floor							Second Floor					
Number	Grade1	Grade2	Grade3	Number	Grade1	Grade2	Number	Grade1	Grade2	Number	Grade1	Grade2
6-1	d_u	d_u	c_u	16-1	a_u	a_u	6-1	a_u	a_u	15-1	a_u	a_u
7-1	c_u	c_u	b_u	16-2	a_u	a_u	7-1	a_u	a_u	15-2	a_u	a_u
8-1	c_u	c_u	b_u	16-3	a_u	a_u	8-1	a_u	a_u	15-3	a_u	a_u
9-1	c_u	c_u	b_u	16-4	a_u	a_u	9-1	a_u	a_u	15-4	a_u	a_u
10-1	c_u	c_u	b_u	16-5	a_u	a_u	10-1	a_u	a_u	15-5	a_u	a_u
11-1	a_u	a_u	a_u	16-6	a_u	a_u	11-1	a_u	a_u	15-6	a_u	a_u
11-2	a_u	a_u	a_u	16-7	a_u	a_u	11-2	a_u	a_u	15-7	a_u	a_u
11-3	a_u	a_u	a_u	17-1	a_u	a_u	11-3	a_u	a_u	16-1	a_u	a_u
12-1	d_u	c_u	b_u	17-2	a_u	a_u	11-4	a_u	a_u	16-2	a_u	a_u
12-2	a_u	a_u	a_u	17-3	a_u	a_u	11-5	a_u	a_u	16-3	a_u	a_u
12-3	a_u	a_u	a_u	17-4	a_u	a_u	11-6	a_u	a_u	16-4	a_u	a_u
12-4	a_u	a_u	a_u	18-1	a_u	a_u	11-7	a_u	a_u	17-1	a_u	a_u
13-1	a_u	a_u	a_u	18-2	a_u	a_u	12-1	a_u	a_u	17-2	a_u	a_u
13-2	a_u	a_u	a_u	18-3	a_u	a_u	12-2	a_u	a_u	17-3	a_u	a_u
13-3	a_u	a_u	a_u	19-1	a_u	a_u	12-3	a_u	a_u	17-4	a_u	a_u
14-1	c_u	c_u	b_u	19-2	a_u	a_u	13-1	a_u	a_u	17-5	a_u	a_u
14-2	a_u	a_u	a_u	19-3	a_u	a_u	13-2	a_u	a_u	17-6	a_u	a_u
14-3	a_u	a_u	a_u	19-4	a_u	a_u	13-3	a_u	a_u	17-7	a_u	a_u
The proportions of the different grades of components							Structural safety rating					
Live load is 1.5 kN/m ²	Current method			c_u -grade: 15%, d_u -grade: 11%			The proportion of d_u -grade exceeds 5%, the structure is D_u -grade.					
	New method			c_u -grade: 17%, d_u -grade: 9%			$\bar{\mu} = \{0.840, 0.090, 0.002, 0.068\}$, based on Table 6, the structure is D_u -grade.					
	New method			b_u -grade: 17%, c_u -grade: 9%			$\bar{\mu} = \{0.738, 0.262, 0.000, 0.000\}$, based on Table 6, the structure is B_u -grade.					

Notes: Grade1 denotes component rating according to current standards; Grade2 denotes component rating using the method described herein; and Grade3 denotes component rating using the method described herein after adjustment for live loads.

However, when the floor live load was adjusted to 1.5 kN/m², the assessment results for the current method and the new method showed significant differences, as shown in Table 9. The assessment results under the current method remained unchanged. However, when evaluated using the new method, the proportion of b_u -grade components in the first floor is 17% and that of c_u -grade components is 9%, and there are no d_u -grade components. The assessment results for the second-floor components remain unchanged. The affiliation set of the structure changes to $\bar{\mu} = \{0.738, 0.262, 0.000, 0.000\}$. According to Table 6, the structure is classified as B_u -grade, meaning that it meets requirements and only requires repairs to some components.

The above case demonstrates that the two assessment methods may present different results; since the method described in this paper takes into account the change characteristics of the resistance and load model of the existing structure, as well as the weights of different types of components relative to the overall reliability of the structure, the assessment results are more in line with the actual situation of the existing structure, and also have broad applicability.

5. Results and Discussion

The number of existing masonry buildings is plentiful, but the assessment methods for existing structures do not fully consider the changes of resistance and load probability models, and do not solve the problem of sudden changes of endpoints between safety assessment intervals; these factors lead to unreasonable safety assessments of existing masonry structures. Therefore, this paper proposes a safety assessment method that

considers both the update of resistance and load probability models and the different weights of members, and its main conclusions are as follows:

(1) The resistance probability model for existing masonry structures is one-size-fits-all, and based on limited testing samples, a resistance probability model that conforms to the actual resistance probability model of the existing masonry structure can be obtained by updating the a priori probability model through Bayesian theory.

(2) The load probability model for existing masonry structures is related to the subsequent work years, and a load probability model matching the subsequent work years can be obtained through the equal-exceeding-probability method.

(3) Based on the modified resistance and load probability model, the contributions of different members to the overall reliability of the structure can be effectively considered by assigning weight coefficients and affiliation sets to different members through the affiliation analysis method and the hierarchical analysis method; based on the affiliation assessment indices, assessment results can be obtained in line with the actual assessment results appropriate to the existing masonry structure.

However, the method proposed in this paper still has certain limitations in its application.

(1) While it performs well on multi-level structures with established rules, its applicability to complex structures, high-level structures, and irregular structures may be insufficient and require further investigation.

(2) The construction of the judgment matrix is subjective and empirical, and it needs to be implemented reasonably in combination with the structure's type, layout, and importance. Therefore, subsequent research may focus on developing various types of judgment matrices for use by engineers and researchers.

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