

Article

Displacement Profile Equations for Performance-Based Seismic Design of Concentrically Braced Steel Frames

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Abstract

This research focuses on characterizing typical displacement patterns in concentrically braced frame (CBF) systems for use in the direct displacement-based seismic design (DDBD) methodology. Using the finite-element program SeismoStruct, two-dimensional models were developed for nonlinear time-history analysis (NLTHA), employing scaled real accelerograms, conventional gravity loads, and detailed numerical models. Thirty varied CBF configurations with different numbers of storeys, spans, and bracing types were evaluated. It was found that the conventional displacement profiles, commonly used for moment-resisting frames, do not accurately represent the actual behavior of CBFs in the inelastic range. Therefore, fitted equations were developed and validated to accurately represent the actual displacements of CBF systems, accounting for factors such as the fundamental vibration period and equivalent system damping. These improvements enable the seismic design optimization, advanced displacement and drift control, and strengthen structural safety and performance in high-seismicity zones in the region. This contribution is relevant to performance-based engineering, facilitating a plausible update to regulations and best practices for seismic-resistant design.

Keywords: concentrically braced frames; direct displacement-based design; equivalent damping; nonlinear time history analysis; displacement patterns

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1. Introduction

The social and economic impact of the catastrophic consequences of seismic events, and the extreme vulnerability of buildings, has pushed the implementation of modern rules on the design and evaluation of structures in seismic areas [1]. In the early 20th century, the primary method of analysis involved simulating the seismic response to lateral forces proportional to the structure's mass, which were resisted by elastic forces governed by the structural stiffness of the system elements [2]. This method, known as the force-based design procedure, does not guarantee achieving the desired performance objectives by a mere reduction in seismic forces [3].

By 1995, performance-based earthquake engineering (PBEE) was first described in the SEAOC Vision 2000 document, which defined this methodology as a structural design approach aimed at ensuring compliance with predefined performance objectives [4]. In

PBEE, human safety, costs, and serviceability levels appropriate for users, owners, and society are considered [5]. In 1996, ATC-40 introduced the capacity spectrum method as a performance-based analysis derived from the pushover procedure [6]. Subsequently, FEMA-273 established several analysis and design methods for multiple performance levels (structural and non-structural), ranging from linear static to nonlinear dynamic procedures, and proposed targets or limits for allowable drift at each performance level [7]. At present, ASCE/SEI 41-23 specifies six structural performance levels: S-1 Immediate Occupancy, S-2 Damage Control, S-3 Life Safety, S-4 Limited Safety, S-5 Collapse Prevention, and S-6 Structural Performance Not Considered [8]. The more clearly and precisely these desired performance objectives are formulated, and the more appropriately the provisions associated with their satisfaction are defined, the more confidently it can be stated that the designed buildings will meet the intended performance targets. These provisions must be derived from the actual seismic behavior of structures [9].

The Direct Displacement-Based Design (DDBD) method, introduced by Priestley in 1993 within the Performance-Based Earthquake Engineering framework, is a design methodology that directly relates target displacement demands to seismic performance objectives. The approach defines global parameters such as interstorey drift and ductility capacity, while also verifying deformation limit states at the member level to ensure local performance criteria are satisfied [10]. The DDBD approach is based on creating a substitute structure that represents a multi-degree-of-freedom structure with an equivalent single-degree-of-freedom system (SDOF) (Figure 1a) characterized by a secant or effective stiffness K_e at the target displacement level (Figure 1b). The target or design displacement Δd is set by the performance criteria for the building, such as plastic deformation limit states in structural elements or interstorey drift levels for non-structural elements [11]. Considering that the response will be nonlinear under intense seismic excitation, the SDOF structure will be characterized by an equivalent viscous damping value ξ_{eq} that depends on the ductility demand (Figure 1c). Then, the design displacement spectrum is scaled to the equivalent viscous damping value, and a required effective period T_e is identified from the design displacement (Figure 1d). Based on the effective mass of the structure m_e and the required effective period, the required effective stiffness K_e and the design base shear V_b are computed [12].

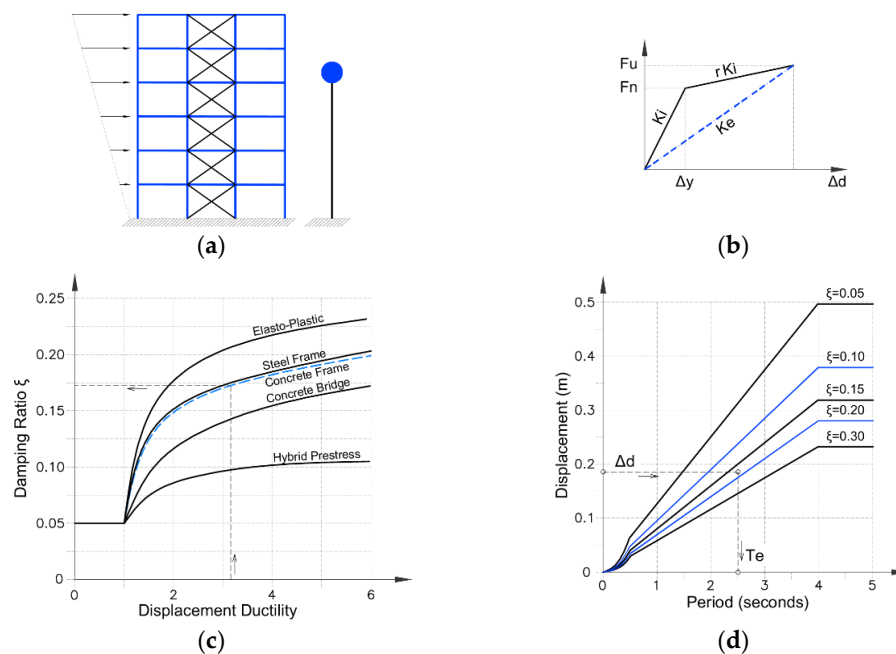


Figure 1. Conceptualization of DDBD procedure (adapted from Priestley [10]). (a) SDOF simulation. (b) Effective stiffness K_e . (c) Equivalent damping versus ductility. (d) Design displacement spectra.

Among the various structural systems where Direct Displacement-Based Design can be applied, steel concentrically braced frames (CBFs) are of particular interest due to their widespread use and seismic efficiency for low- and mid-rise buildings. They are characterized as efficient and cost-effective structural systems, since they are relatively simple to design, fabricate, and construct [13]. Compared with moment-resisting frame systems, they exhibit high lateral stiffness, significant global strength, and a wide range of possible bracing configurations. In CBFs, during high-intensity earthquake events, the alternate cyclic tension-yielding and post-buckling deformation of the braces dissipate the energy imparted to the structural system [14]. Other elements, such as beams and columns, will behave elastically and are not expected to yield to dissipate energy [15]. Despite the advantages of CBFs, the DDBD method is not commonly applied to these systems because studies on this methodology remain limited.

The first application of DDBD to CBF systems was developed by Medhekar and Kennedy, which included design examples subjected to real accelerograms. However, their study was restricted to only two case studies and an effective damping ratio of 5% [16]. By 2006, the study of [17] examined local member deformations and global response in frames ranging from 3 to 20 storeys, with 3 and 6 bays. Although 13 different accelerograms were used and the inelastic deformation of the elements was successfully captured, the scatter between the recorded displacements and those computed using the proposed equations is significant for frames taller than six storeys.

In 2008, Della Corte and Mazzolani proposed mathematical expressions for displacement patterns based on analytical relationships between interstorey drift and the local deformation of structural members, accounting for the yielding regions of the CBF. The use of only five ground-motion records for a single case study (inverted-V configuration) was the main limitation of their work [18]. Subsequently, ref. [19] focused on 4-storey and 8-storey CBFs with X and inverted-V bracing, performing NLTHA under 30 different ground-motion records, which enabled an initial proposal to refine Priestley's equations. However, these equations cannot be directly extended to mid- or high-rise frames because they were calibrated only on those particular case studies [19]. Coupled equations for several CBF configurations were developed for the first time by [20], based on average peak storey displacements obtained from nonlinear time-history analyses that considered local brace buckling and low-cycle fatigue. The resulting displacement patterns were compared with the results computed using Priestley's equations [10], with particular emphasis on increases and decreases in interstorey drifts for 16 cases of CBFs with X and inverted-V bracing [20].

More recently, the authors of [21] proposed a direct displacement-based design methodology for dissipative bracings used in the seismic retrofit of gravity-load-designed reinforced concrete frames. Their approach is formulated in terms of interstorey drift control, using an equivalent linearization to distribute stiffness and damping of the braces. This work reinforces the importance of accurately capturing displacement and drift profiles in the PBEE of braced systems, where these profiles are explicitly characterized and parametrized for steel concentrically braced frames. In 2024, ref. [22] proposed a displacement-based design procedure for the seismic retrofit of existing buildings using self-centering dissipative braces. The method defines target performance in terms of maximum roof displacement and interstorey drift limits, and then proportionally distributes the strength and stiffness of these systems along the height of the structure to achieve the desired displacement profile while exploiting the hysteretic shape response. By explicitly linking retrofit design to target displacement demands, this work further emphasizes the

central role of height-wise displacement patterns in displacement-based seismic design, where such patterns are directly characterized and parametrized for concentrically braced steel frames.

Considering the above statements, the present study aims to complement and further strengthen the DDBD methodology for CBFs by providing a larger set of case studies, including a greater range of storey heights, bracing typologies, and bay numbers. The pre-design of each frame illustrates the applicability of the DDBD procedure. The adaptation, modification, and comparison of the equations proposed in previous studies, such as [10,11,20], which purport to offer a better fit for the actual displacements of the thirty different CBFs case studies considered here, will constitute a significant contribution to the state of the art on displacement profiles.

The remainder of this paper is organized as follows. Section 2 presents the configuration of the case studies and their pre-design sections using the DDBD method. The selection, shortening, and scaling of seismic records based on seismic hazard are described in Section 3. Section 4 presents the conditions for the nonlinear analytical modeling of the CBFs. The run criteria for the NLTHA and the displacement profiles obtained are presented in Section 5. Section 6 details the proposed numerical expressions and compares them with those in similar studies. Finally, Section 7 summarizes the conclusions of this research.

2. CBFs Considered in This Study

Thirty CBFs were selected for this study, including three different bracing typologies, various numbers of storeys, and different numbers of bays (the geometry of the case studies, number of bays, and location and number of braced bays is shown in Figure 2 and Table 1). The studied case frames have 3, 4, 5, and 6 bays. The number of storeys varies between 4, 8, 12, 16, and 20. The number of bays is reduced for the taller frame typologies. Three bracing configurations are considered: X bracing, single-diagonal bracing, and inverted-V bracing. The storey height is 3.4 m, and the width of each braced and unbraced bay is 7.0 m. All these layouts are assumed to represent the most common geometries and dimensions observed in buildings across Ecuador.

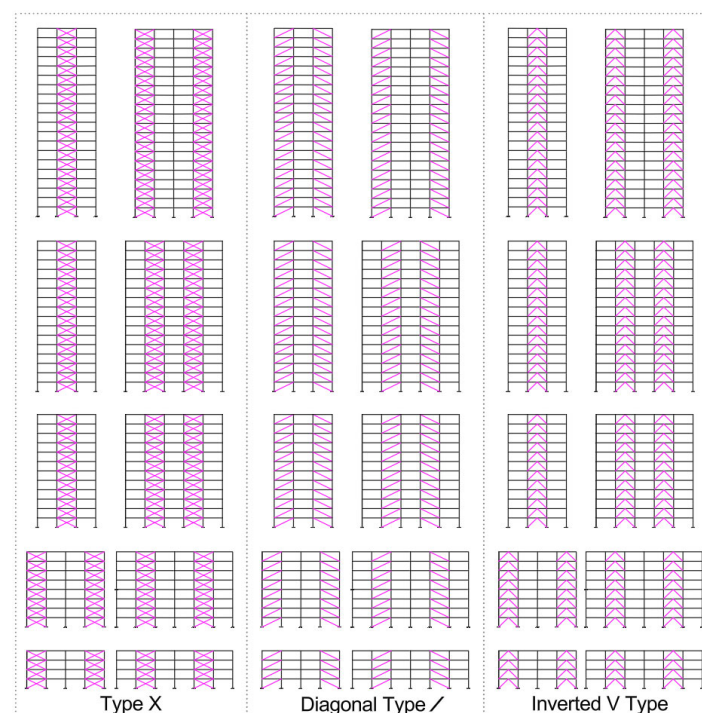


Figure 2. Geometry and bracing layout of case study frames.**Table 1.** Details of the analytical CBFs typologies.

Storey Number	Bay Typology		Bracing Type		Number of Frames
	Type ¹	Number	Type	Number	
4	4, 6	2	X, Λ , /	3	6
8	4, 6	2	X, Λ , /	3	6
12	3, 5	2	X, Λ , /	3	6
16	3, 5	2	X, Λ , /	3	6
20	3, 4	2	X, Λ , /	3	6
Total CBFs number:					30

¹ Taller CBFs systems have a lower number of bays.

The geometry of the case studies, number of bays, and location and number of braced bays is shown in Figure 2.

All thirty CBF configuration cases are designed according to the DDBD procedure described in [23], by modeling an equivalent single-degree-of-freedom (SDOF) system and computing an equivalent damping ratio based on ductility. The design displacement of the SDOF system is computed from the displacement profile as follows: Equation (1) for 4-storey frames; Equation (2) for 8, 12, and 16-storey frames; and Equation (3) for 20-storey frames. Each equation is selected to obtain the smallest displacement demand for each frame type, as a function of the number of storeys n , the target drift θ_t , the storey height H_i , and the total frame height H_n .

$$\text{For } n \leq 4, \text{ according to [10]:} \quad \delta i = \theta t * H_i \quad (1)$$

$$\text{For } 4 < n < 20, \text{ according to [11]:} \quad \delta i = \theta t * H_i \left[1 - \frac{0.5}{16 * H_n} (n - 4) * H_i \right] \quad (2)$$

$$\text{For } n > 20, \text{ according to [11]:} \quad \delta i = \theta t * H_i * \left(1 - \frac{0.5 * H_i}{H_n} \right) \quad (3)$$

Equations (1)–(3) were selected among existing Priestley-type profiles to avoid excessive overestimation of global displacement while enforcing a drift-based Life Safety performance limit at every storey, consistent with PBEE and DDBD principles.

Seismic demand is determined from the acceleration response spectrum defined by the NEC-SE-DS code [24] for a seismic zone V (high seismic hazard), with a PGA value of 0.40 for a return period of 475 years, an importance factor of 1.0, and a soil profile type D [24]. The 5%-damped displacement spectrum is used to design each frame, with a corner period T_c equal to 2.86 s. To adjust the displacement spectrum to other damping levels, the scale factor $\eta = (7/(2 + \xi))^{0.25}$ prescribed in Eurocode 8 is adopted [25].

The design assumptions adopted for the braced systems are as follows: in accordance with ASCE/SEI 7-22, the target interstorey drift is taken as 2% for a Life Safety performance level [26], each braced bay is assumed to resist the same portion of the building's lateral load; the axial tension and compression forces in the braces are assumed to carry the entire seismic shear at each storey; beam–column and brace–beam–column joints are modeled as force–transfer connections (no moment transmission); and axial forces in the braces due to gravity loads are neglected [23]. A typical dead load of 670 kg/m² is assumed for steel buildings, including the weight of the slab, beams, columns, braces, masonry, façades, and finishes. The live load is taken as 250 kg/m² for office occupancy, in accordance with NEC-SE-CG [27]. In addition, the effective seismic mass is defined as 100% of the dead load plus 15% of the live load, following the recommendations of [28] for residential and office buildings in the region.

For each bracing typology (X, single-diagonal /, and inverted-V), a dedicated pre-design algorithm was developed. This is because the transfer of seismic lateral forces to the braces, and subsequently to columns and beams, differs depending on the bracing configuration; that is, the magnitude, direction, and alignment of the internal forces vary with the CBF typology. Braces are designed using HSS tubular members, whereas columns and beams are designed with wide flange W sections in all case studies.

Tables 2–4 show the brace, column, and beam sections used in all CBFs types, for 4 and 8 storeys, 12 and 16 storeys, and 20 storeys, respectively.

Table 2. HSS brace sections and wide flange column beam sections in CBFs of 4 and 8 storeys.

Frame Label	Elements	Type X	Type Diagonal /	Type V Inverted
4 storeys 4 bays	Braces	Lv. 1–2 HSS 177.8 × 177.8 × 7.9	HSS 228.6 × 228.6 × 9.5	HSS 177.8 × 177.8 × 9.5
		Lv. 3–4 HSS 139.7 × 139.7 × 7.9	HSS 177.8 × 177.8 × 9.5	HSS 152.4 × 152.4 × 9.5
	Columns	Lv. 1–2 W 760 × 265 × 173	W 690 × 250 × 125	W 690 × 250 × 170
		Lv. 3–4 W 460 × 190 × 89	W 460 × 190 × 97	W 410 × 180 × 75
	Beams	Lv. 1–2 W 410 × 180 × 75	W 530 × 210 × 123	W 530 × 210 × 82
		Lv. 3–4 W 310 × 200 × 60	W 460 × 190 × 97	W 410 × 180 × 75
4 storeys 6 bays	Braces	Lv. 1–2 HSS 254 × 254 × 7.9	HSS 254 × 254 × 19	HSS 228.6 × 228.6 × 12.7
		Lv. 3–4 HSS 203.2 × 203.2 × 7.9	HSS 254 × 254 × 12.7	HSS 228.6 × 228.6 × 9.5
	Columns	Lv. 1–2 W 920 × 310 × 271	W 920 × 310 × 271	W 920 × 310 × 289
		Lv. 3–4 W 530 × 210 × 138	W 460 × 280 × 144	W 610 × 230 × 125
	Beams	Lv. 1–2 W 530 × 210 × 101	W 760 × 265 × 220	W 610 × 230 × 125
		Lv. 3–4 W 410 × 180 × 85	W 460 × 280 × 144	W 610 × 230 × 113
8 storeys 4 bays	Braces	Lv. 1–4 HSS 152.4 × 152.4 × 12.7	HSS 254 × 254 × 9.5	HSS 203.2 × 203.2 × 7.9
		Lv. 5–8 HSS 152.4 × 152.4 × 7.9	HSS 203.2 × 203.2 × 9.5	HSS 177.8 × 177.8 × 7.9
	Columns	Lv. 1–4 W 920 × 420 × 344	W 920 × 310 × 289	W 610 × 325 × 341
		Lv. 5–8 W 610 × 325 × 155	W 610 × 230 × 125	W 610 × 230 × 153
	Beams	Lv. 1–4 W 610 × 230 × 101	W 690 × 250 × 140	W 530 × 210 × 82
		Lv. 5–8 W 530 × 210 × 72	W 530 × 210 × 109	W 410 × 180 × 75
8 storeys 6 bays	Braces	Lv. 1–4 HSS 254 × 254 × 9.5	HSS 304.8 × 304.8 × 15.9	HSS 228.6 × 228.6 × 15.9
		Lv. 5–8 HSS 228.6 × 228.6 × 7.9	HSS 254 × 254 × 15.9	HSS 203.2 × 203.2 × 12.7
	Columns	Lv. 1–4 W 920 × 420 × 588	W 920 × 420 × 656	W 920 × 420 × 725
		Lv. 5–8 W 920 × 310 × 271	W 610 × 325 × 285	W 920 × 310 × 289
	Beams	Lv. 1–4 W 610 × 230 × 113	W 920 × 310 × 238	W 610 × 230 × 153
		Lv. 5–8 W 530 × 210 × 92	W 610 × 325 × 174	W 530 × 210 × 123

Table 3. HSS brace sections and wide flange column beam sections in CBFs of 12 and 16 storeys.

Frame Label	Elements	Type X	Type Diagonal /	Type V Inverted
12 storeys 3 bays	Braces	Lv. 1–4 HSS 177.8 × 177.8 × 12.7	HSS 203.2 × 203.2 × 9.5	HSS 228.6 × 228.6 × 9.5
		Lv. 5–8 HSS 177.8 × 177.8 × 9.5	HSS 177.8 × 177.8 × 9.5	HSS 203.2 × 203.2 × 9.5
		Lv. 9–12 HSS 152.4 × 152.4 × 7.9	HSS 139.7 × 139.7 × 7.9	HSS 177.8 × 177.8 × 6.3
	Columns	Lv. 1–4 W 920 × 420 × 588	W 920 × 310 × 289	W 920 × 420 × 588
		Lv. 5–8 W 920 × 420 × 344	W 610 × 325 × 174	W 920 × 420 × 344
		Lv. 9–12 W 760 × 265 × 173	W 610 × 180 × 82	W 690 × 250 × 125
	Beams	Lv. 1–4 W 610 × 230 × 113	W 530 × 210 × 109	W 530 × 210 × 109
		Lv. 5–8 W 530 × 210 × 92	W 460 × 190 × 97	W 530 × 210 × 101
		Lv. 9–12 W 410 × 180 × 67	W 410 × 180 × 67	W 530 × 165 × 74
12 storeys 5 bays	Braces	Lv. 1–4 HSS 228.6 × 228.6 × 9.5	HSS 304.8 × 304.8 × 12.7	HSS 203.2 × 203.2 × 15.9
		Lv. 5–8 HSS 228.6 × 228.6 × 7.9	HSS 304.8 × 304.8 × 9.5	HSS 203.2 × 203.2 × 12.7

16 storeys 3 bays	Columns	Lv. 9–12	HSS 177.8 × 177.8 × 7.9	HSS 254 × 254 × 7.9	HSS 177.8 × 177.8 × 9.5
		Lv. 1–4	W 1000 × 400 × 748	W 1000 × 400 × 642	W 920 × 420 × 787
		Lv. 5–8	W 920 × 420 × 449	W 920 × 420 × 368	W 920 × 420 × 491
		Lv. 9–12	W 920 × 310 × 201	W 690 × 250 × 170	W 760 × 265 × 185
	Beams	Lv. 1–4	W 530 × 210 × 101	W 760 × 265 × 196	W 530 × 210 × 138
		Lv. 5–8	W 530 × 210 × 92	W 760 × 265 × 161	W 530 × 210 × 123
		Lv. 9–12	W 410 × 180 × 75	W 690 × 250 × 125	W 410 × 180 × 85
	Braces	Lv. 1–4	HSS 228.6 × 228.6 × 9.5	HSS 177.8 × 177.8 × 15.9	HSS 203.2 × 203.2 × 15.9
		Lv. 5–8	HSS 228.6 × 228.6 × 7.9	HSS 177.8 × 177.8 × 12.7	HSS 203.2 × 203.2 × 12.7
		Lv. 9–12	HSS 203.2 × 203.2 × 7.9	HSS 177.8 × 177.8 × 9.5	HSS 177.8 × 177.8 × 12.7
		Lv. 13–16	HSS 139.7 × 139.7 × 7.9	HSS 139.7 × 139.7 × 7.9	HSS 177.8 × 177.8 × 6.3
	Columns	Lv. 1–4	W 920 × 420 × 970	W 920 × 420 × 491	W 1000 × 400 × 976
		Lv. 5–8	W 920 × 420 × 588	W 920 × 310 × 313	W 920 × 420 × 656
		Lv. 9–12	W 920 × 420 × 344	W 760 × 265 × 173	W 920 × 420 × 368
		Lv. 13–16	W 690 × 250 × 152	W 610 × 180 × 82	W 610 × 230 × 125
	Beams	Lv. 1–4	W 530 × 210 × 101	W 690 × 250 × 140	W 530 × 210 × 138
		Lv. 5–8	W 530 × 210 × 92	W 610 × 230 × 125	W 530 × 210 × 123
		Lv. 9–12	W 460 × 190 × 82	W 610 × 230 × 101	W 530 × 210 × 101
		Lv. 13–16	W 410 × 180 × 67	W 610 × 180 × 82	W 530 × 210 × 72
16 storeys 5 bays	Braces	Lv. 1–4	HSS 304.8 × 304.8 × 9.5	HSS 304.8 × 304.8 × 19	HSS 254 × 254 × 15.9
		Lv. 5–8	HSS 254 × 254 × 9.5	HSS 304.8 × 304.8 × 15.9	HSS 254 × 254 × 12.7
		Lv. 9–12	HSS 254 × 254 × 7.9	HSS 254 × 254 × 15.9	HSS 228.6 × 228.6 × 12.7
		Lv. 13–16	HSS 177.8 × 177.8 × 7.9	HSS 177.8 × 177.8 × 15.9	HSS 177.8 × 177.8 × 12.7
	Columns	Lv. 1–4	W 920 × 420 × 1269	W 920 × 420 × 1194	W 920 × 420 × 1377
		Lv. 5–8	W 920 × 420 × 787	W 920 × 420 × 787	W 920 × 420 × 970
		Lv. 9–12	W 920 × 420 × 491	W 920 × 420 × 449	W 920 × 420 × 588
		Lv. 13–16	W 920 × 310 × 201	W 610 × 325 × 174	W 920 × 310 × 238
	Beams	Lv. 1–4	W 610 × 230 × 125	W 920 × 310 × 271	W 760 × 265 × 173
		Lv. 5–8	W 530 × 210 × 109	W 920 × 310 × 238	W 610 × 230 × 140
		Lv. 9–12	W 460 × 190 × 97	W 610 × 325 × 174	W 530 × 210 × 138
		Lv. 13–16	W 410 × 180 × 75	W 530 × 210 × 138	W 360 × 250 × 91

Table 4. HSS brace sections and wide flange column beam sections in CBFs of 20 storeys.

Frame Label	Elements	Type X	Type Diagonal /	Type V Inverted
20 storeys 3 bays	Braces	Lv. 1–4	HSS 203.2 × 203.2 × 15.9	HSS 203.2 × 203.2 × 15.9
		Lv. 5–8	HSS 203.2 × 203.2 × 15.9	HSS 203.2 × 203.2 × 15.9
		Lv. 9–12	HSS 177.8 × 177.8 × 15.9	HSS 177.8 × 177.8 × 15.9
		Lv. 13–16	HSS 152.4 × 152.4 × 15.9	HSS 152.4 × 152.4 × 15.9
		Lv. 17–20	HSS 114.3 × 114.3 × 12.7	HSS 114.3 × 114.3 × 12.7
	Columns	Lv. 1–4	W 920 × 420 × 1377	W 920 × 420 × 1377
		Lv. 5–8	W 920 × 420 × 1077	W 920 × 420 × 1077
		Lv. 9–12	W 920 × 420 × 656	W 920 × 420 × 656
		Lv. 13–16	W 920 × 420 × 368	W 760 × 265 × 196
		Lv. 17–20	W 690 × 250 × 170	W 530 × 210 × 82
	Beams	Lv. 1–4	W 610 × 230 × 140	W 760 × 265 × 161
		Lv. 5–8	W 610 × 230 × 140	W 760 × 265 × 161
		Lv. 9–12	W 460 × 280 × 113	W 530 × 210 × 138
		Lv. 13–16	W 360 × 250 × 101	W 530 × 210 × 123
		Lv. 17–20	W 410 × 180 × 75	W 460 × 190 × 82

20 storeys 4 bays	Braces	Lv. 1–4	HSS 203.2 × 203.2 × 15.9	HSS 304.8 × 304.8 × 15.9	HSS 228.6 × 228.6 × 15.9
		Lv. 5–8	HSS 203.2 × 203.2 × 12.7	HSS 304.8 × 304.8 × 12.7	HSS 228.6 × 228.6 × 12.7
		Lv. 9–12	HSS 203.2 × 203.2 × 12.7	HSS 304.8 × 304.8 × 12.7	HSS 203.2 × 203.2 × 12.7
		Lv. 13–16	HSS 177.8 × 177.8 × 9.5	HSS 228.6 × 228.6 × 12.7	HSS 203.2 × 203.2 × 9.5
		Lv. 17–20	HSS 139.7 × 139.7 × 7.9	HSS 177.8 × 177.8 × 9.5	HSS 152.4 × 152.4 × 7.9
	Columns	Lv. 1–4	W 920 × 420 × 1377	W 920 × 420 × 1194	W 920 × 420 × 1269
		Lv. 5–8	W 920 × 420 × 970	W 920 × 420 × 970	W 920 × 420 × 970
		Lv. 9–12	W 920 × 420 × 656	W 920 × 310 × 576	W 920 × 420 × 588
		Lv. 13–16	W 920 × 420 × 344	W 920 × 310 × 289	W 920 × 310 × 313
		Lv. 17–20	W 690 × 250 × 152	W 610 × 230 × 125	W 530 × 210 × 92
	Beams	Lv. 1–4	W 610 × 230 × 140	W 920 × 310 × 238	W 690 × 250 × 152
		Lv. 5–8	W 530 × 210 × 123	W 690 × 250 × 192	W 610 × 230 × 140
		Lv. 9–12	W 530 × 210 × 123	W 690 × 250 × 192	W 610 × 230 × 113
		Lv. 13–16	W 460 × 190 × 89	W 610 × 230 × 153	W 460 × 190 × 97
		Lv. 17–20	W 410 × 180 × 67	W 460 × 190 × 97	W 410 × 180 × 67

3. Seismic Hazard and Earthquake Ground Motions

From the NGA-West2 (Next Generation Attenuation—West 2) ground-motion database of the Pacific Earthquake Engineering Research Center [29], 11 seismic records were selected in accordance with the recommendations of ASCE/SEI 7-22 [26]. Based on [30], the expected hazard for Quito, Ecuador, is characterized by a 475-year return period spectrum for soil type *D*, with *PGA* > 0.25 *g*, magnitudes *M_w* > 5.9, and rupture distances *R_{rup}* > 5 km.

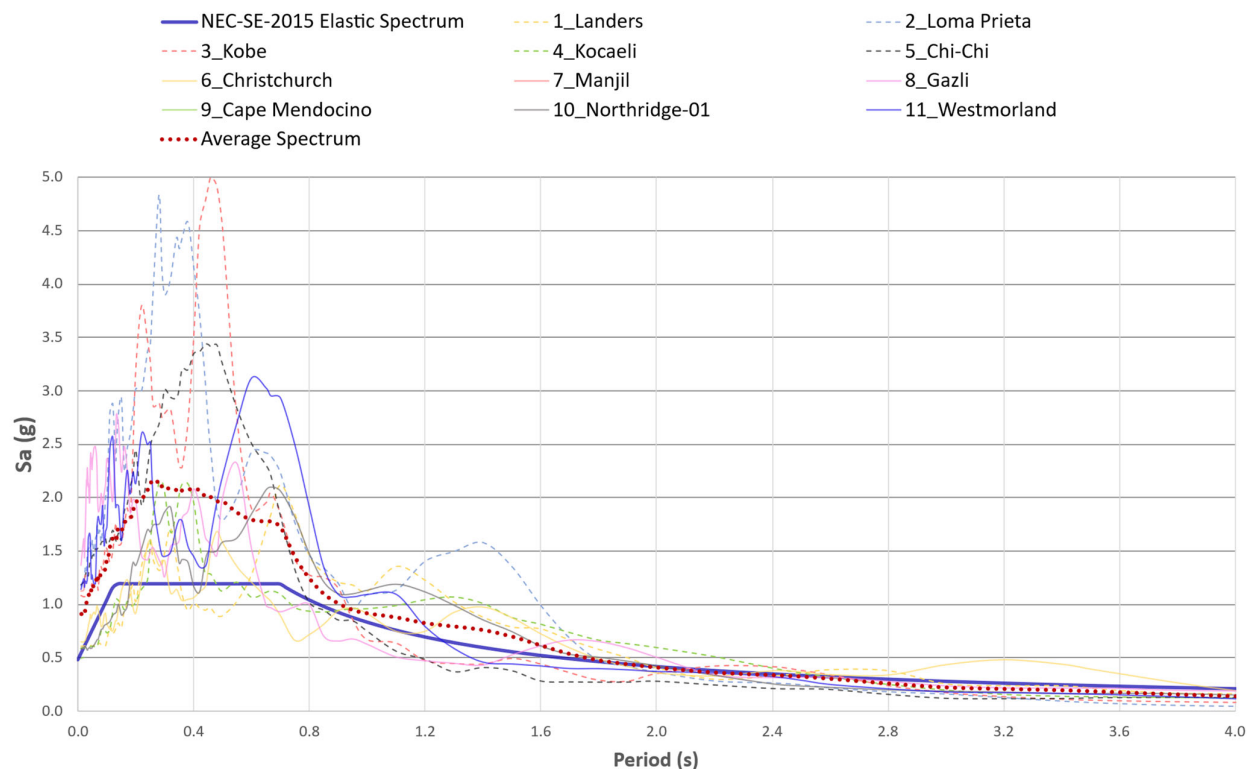
Only horizontal components were used, combined through the geometric mean. This conventional spectral-scaling procedure, which yields scale factors of up to about 2.5, accounts for the adjustment from recorded motions to the relatively high design intensity and ensures that the mean spectrum of the 11 scaled records is consistent with the target code spectrum, in line with current recommendations for NLTHA in PBEE applications.

Each selected ground-motion record is truncated using the Arias Intensity procedure, with an initial time corresponding to 5% of the cumulative seismic energy and a final time at 95% of the total energy [31]. Using the PEER NGA-West2 tool, a scale factor is computed for each record over a period range that covers the fundamental mode of vibration of the thirty case study frames. The lower bound is taken as 0.5 times the minimum fundamental period of the 4-storey frames (*T*₁ = 0.36 s). The upper bound is 1.5 times the maximum fundamental period identified for the 20-storey frames (*T*₂ = 5.61 s).

Table 5 lists 11 records used in the analysis, detailing the duration and number of data points for the trimmed seismic records, as well as the scale factor expressed in units of gravitational acceleration *g*. The matching between the NEC-SE-DS target acceleration spectrum and the spectra of each accelerogram modified by the scale factors in Table 5, and the mean spectrum of the 11 selected seismic records, is illustrated in Figure 3.

Table 5. Details of selected seismic records.

Item	Accelerogram	Magnitude Mw	Year	Rrup (km)	Time Step (seg)	Trimmed Record Duration (seg)	Data Number	Scale Factor (g)
1	Landers	7.28	1992	11.03	0.02	26.02	1301	2.12
2	Loma Prieta	6.93	1989	15.23	0.005	11.905	2381	2.28
3	Kobe	6.9	1995	7.08	0.01	11.22	1122	2.21
4	Kocaeli	7.51	1999	15.37	0.005	10.855	2171	1.61
5	Chi-Chi	7.62	1999	26	0.005	10.805	2161	2.49
6	Christchurch	6.2	2011	5.55	0.005	9.62	1924	1.16
7	Manjil	7.37	1990	12.55	0.02	29.06	1453	2.13
8	Gazli	6.8	1976	5.46	0.0066	6.376	966	1.40
9	Cape Mendocino	7.01	1992	19.32	0.005	11.84	2368	1.34
10	Northridge-01	6.69	1994	5.92	0.02	5.52	276	0.97
11	Westmorland	5.9	1981	6.5	0.005	6.88	1376	2.29

**Figure 3.** Elastic design spectrum NEC-SE-DS [24] and pseudo acceleration spectra scaled to T1 and T2 periods.

4. Analytical Modeling

To analyze the performance of steel CBF systems, nonlinear dynamic analyses are conducted using SeismoStruct 2023 finite-element software [32]. The frames are modeled in 2-D, and the braces can exhibit out-of-plane buckling under high compression loads. According to [33], concentrically configured frames fail by buckling under severe shaking. The columns and beams' response is limited to an in-plane displacement by constraining the degrees of freedom in the out-of-plane directions. All connections are modeled as pinned, and columns are modeled as continuous members with continuous moments and force transmission. Base nodes are assigned fixed supports with displacement restraints

along the translational in-plane axis. Furthermore, brace “out-of-plane buckling” is not represented as a physical displacement of the frame out of the plane, but as the loss of stiffness and strength associated with axial–flexural instability of the HSS brace elements under compression, captured through the geometrically nonlinear formulation and the cyclic steel constitutive model at the section level.

The type of material used is structural steel Gr. 50 (yield strength of 50 ksi). Columns, beams, and braces are modeled as inelastic frame members with force-based formulation (infrmFB “inelastic frame-force based”) as suggested by [19]. The number of fibers for the columns and beams is 80, and three integration sections are assigned according to the recommendations for steel frames in the SeismoStruct user manual [32]. For brace elements, discretization is done with 150 fibers and five integration sections, to optimize the modeling procedure of hollow square sections (HSS) braces [34]. Figure 4 shows a typical section discretization in HSS and wide flange W sections.

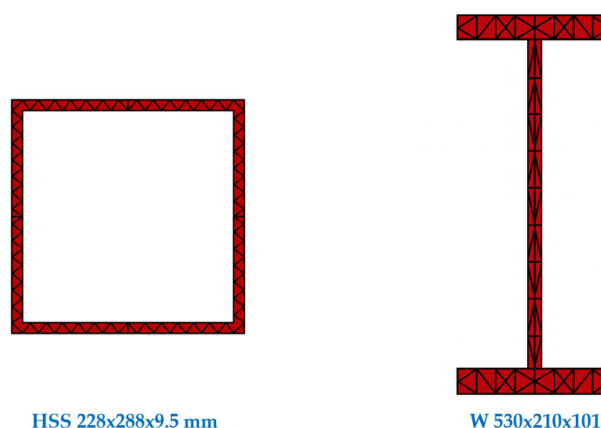


Figure 4. Typical fiber section discretization in HSS and wide flange W sections. Example for HSS 228 × 228 × 9.5 and W 530 × 210 × 101.

A “master node” is assigned to each floor, and the other nodes on each level are set as “slave nodes” to reduce the number of degrees of freedom per storey and accelerate nonlinear analysis, thereby simplifying the CBFs’ structural analysis. The gravitational load w corresponds to the reactive seismic mass assigned in all CBF beams. The values of these loads are presented in kN/m in Table 6.

Table 6. Gravitational loads, effective damping, and effective period in CBF systems.

Frame Label	w (kN/m)	Type X		Type Diagonal /		Type V Inverted	
		T_{eff} (seg)	ξ_{CBF} (%)	T_{eff} (seg)	ξ_{CBF} (%)	T_{eff} (seg)	ξ_{CBF} (%)
4 storeys 4 bays	97.14	1.22	14.53%	1.26	17.10%	1.22	15.00%
4 storeys 6 bays	145.71	1.28	18.24%	1.28	18.86%	1.28	18.20%
8 storeys 4 bays	97.14	2.04	14.07%	2.16	18.13%	2.12	16.67%
8 storeys 6 bays	145.71	2.16	18.71%	2.18	19.47%	2.14	17.53%
12 storeys 3 bays	72.85	2.7	15.27%	2.7	15.47%	2.78	17.23%
12 storeys 5 bays	121.42	2.78	17.53%	2.86	19.87%	2.74	16.60%
16 storeys 3 bays	72.85	2.86	16.15%	2.86	16.15%	2.86	16.73%
16 storeys 5 bays	121.42	2.86	16.15%	2.86	16.15%	2.86	16.73%
20 storeys 3 bays	72.85	2.84	14.58%	2.84	14.58%	2.86	15.29%
20 storeys 4 bays	97.14	2.84	14.58%	2.84	14.58%	2.86	15.29%

5. Nonlinear Time–History Analysis (NLTHA) and Displacement Patterns

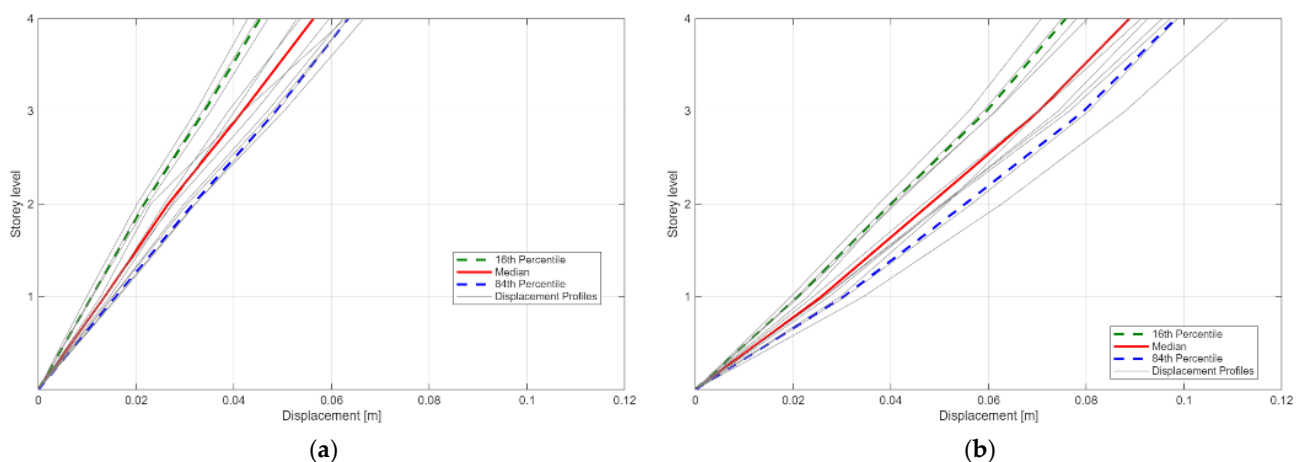
To run the NLTHA simulations, each seismic record is applied as a “Time–History Dynamic Load” in the X direction for all base nodes. The scale factor for accelerograms is a multiplier applied to the acceleration time–history expressed in m/s^2 .

Each nonlinear model in SeismoStruct is configured with the effective damping ξ_{CBF} and effective period T_{eff} of the equivalent single-degree-of-freedom system (SDOF) for automatic calculation of the tangent stiffness of the system [20]. Both parameters are calculated during the pre-design phase by the DDBD procedure. Table 6 shows the parameter values for each case study.

The SeismoStruct calculation engine is configured to account for geometric nonlinearities in frame systems, using Lagrange multipliers to improve coupling between inelastic elements. A maximum of 40 iterations is used as the global iterative strategy, with 35 updates to the system stiffness matrix. The convergence tolerance for each loop is 0.0001, and the maximum number of iterations per loop is 30 for the force-based fiber element constitutive model. Following the recommendations in [12], the Newmark Integration Scheme is used with $\beta = 0.25$ and $\gamma = 0.5$.

The DDBD pre-design and member checks were intended to preclude soft-storey formation, and all analyses were stable and were fully included in the averaging process used to derive the proposed displacement patterns. To accelerate the time–history analysis, the SeismoStruct output is limited to the global displacements of each “master node” in the X direction.

In the 30 case studies, lateral displacement values are extracted at the “master node” of each storey at every stage of the time–history analysis. Using the maximum value at each level over the trimmed duration of the seismic event (Table 5), the displacement profile is plotted. The 11 earthquakes analyzed in each frame describe the shape of the displacement profile by averaging at each storey. Drifts are calculated using a typical inter-storey height of 3.40 m. In Figures 5–14, the gray lines correspond to the storey displacement and drift profiles obtained from each individual ground-motion record; the average displacement and drift pattern are shown, along with the graphs of the 16th and 84th percentile envelopes of the dataset across all seismic records.



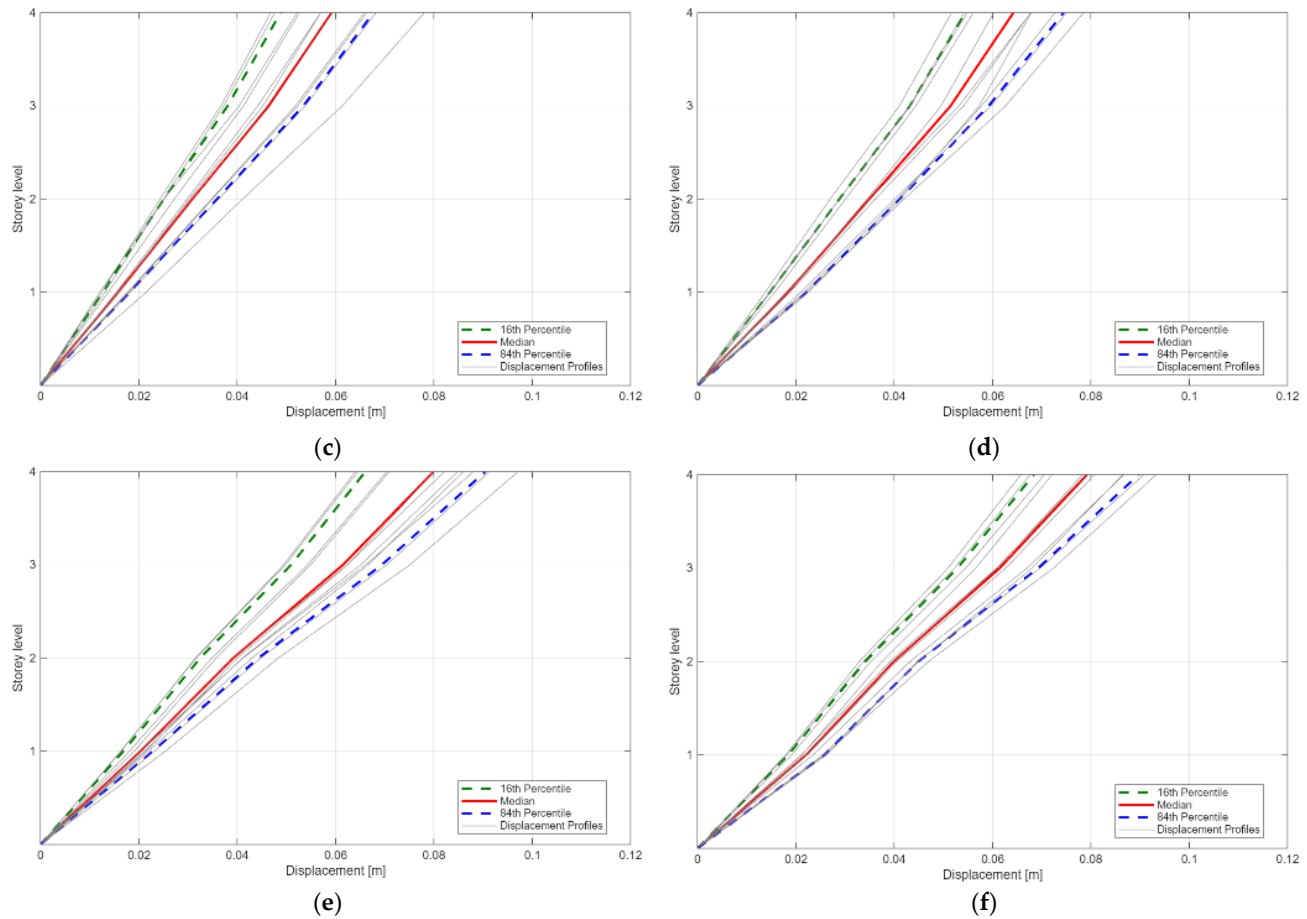
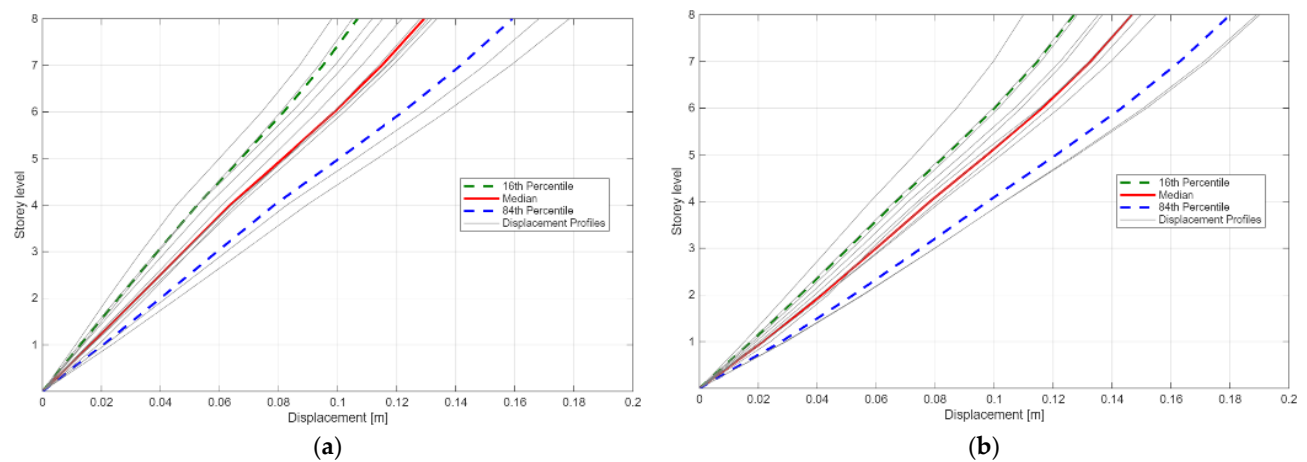


Figure 5. Displacement patterns in 4-storey CBFs. (a) 4 bays type X, (b) 6 bays type X, (c) 4 bays type Vi, (d) 6 bays type Vi, (e) 4 bays type /, (f) 6 bays type /.



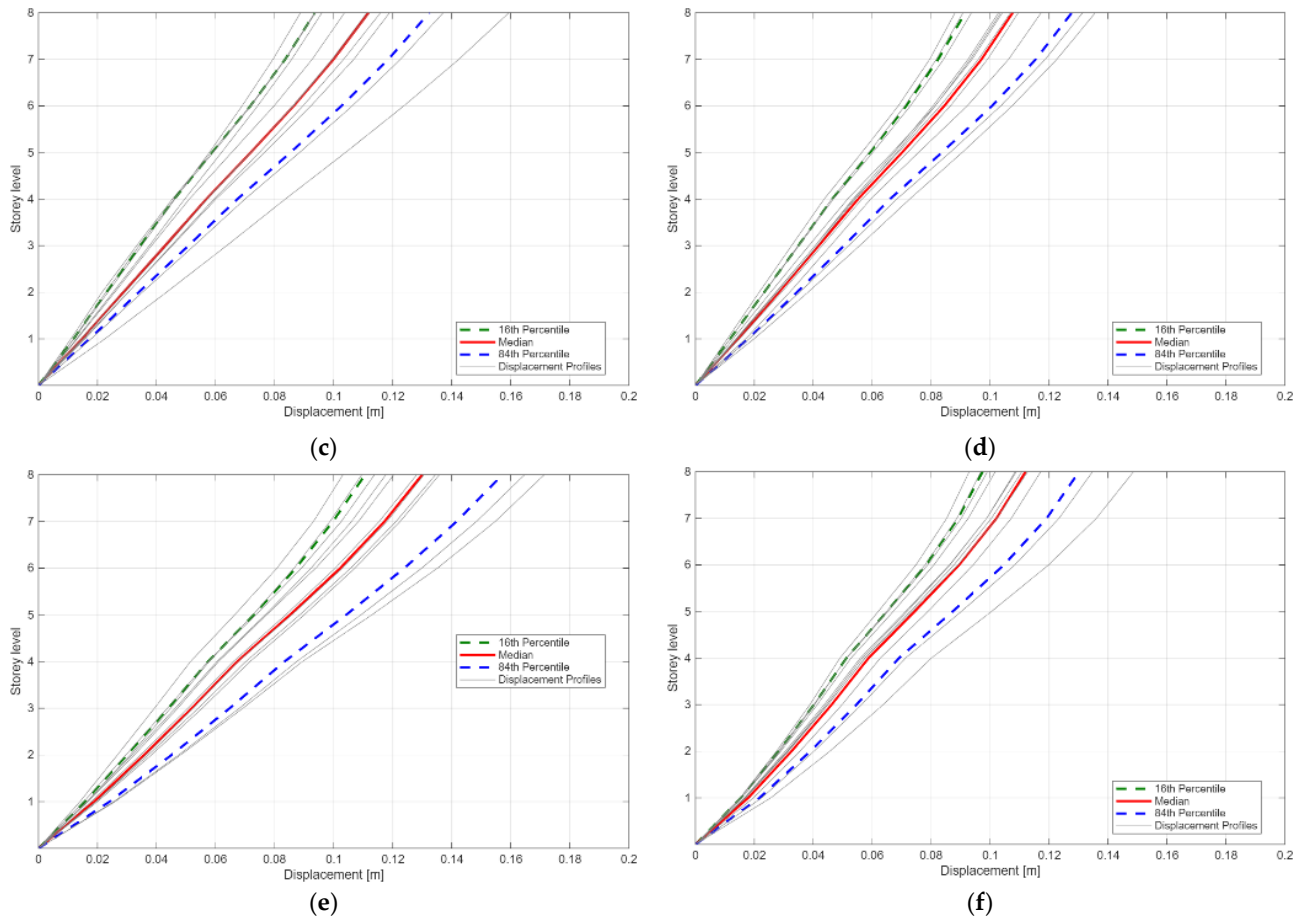
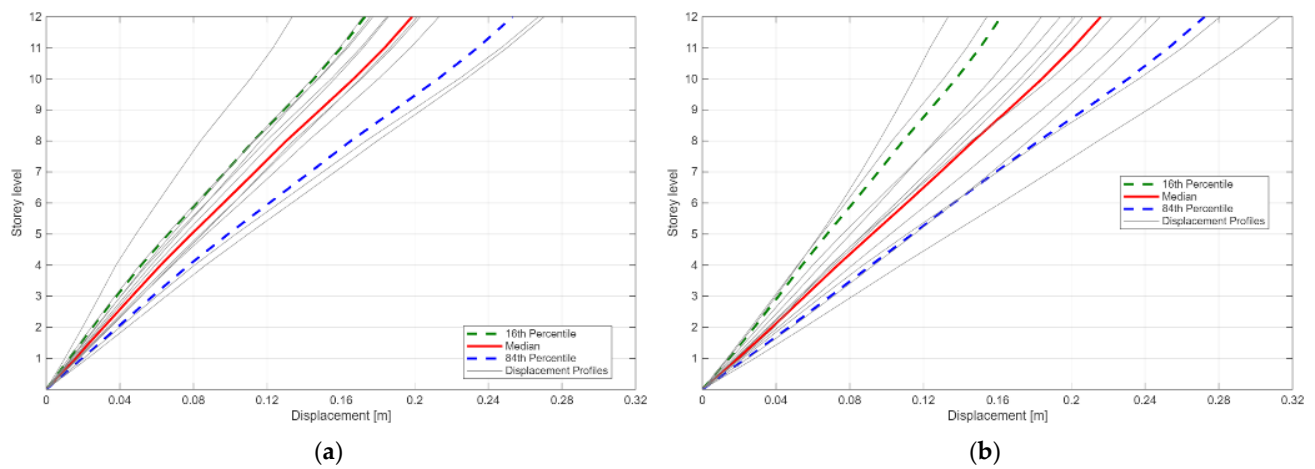


Figure 6. Displacement patterns in 8-storey CBFs. (a) 4 bays type X, (b) 6 bays type X, (c) 4 bays type Vi, (d) 6 bays type Vi, (e) 4 bays type /, (f) 6 bays type /.



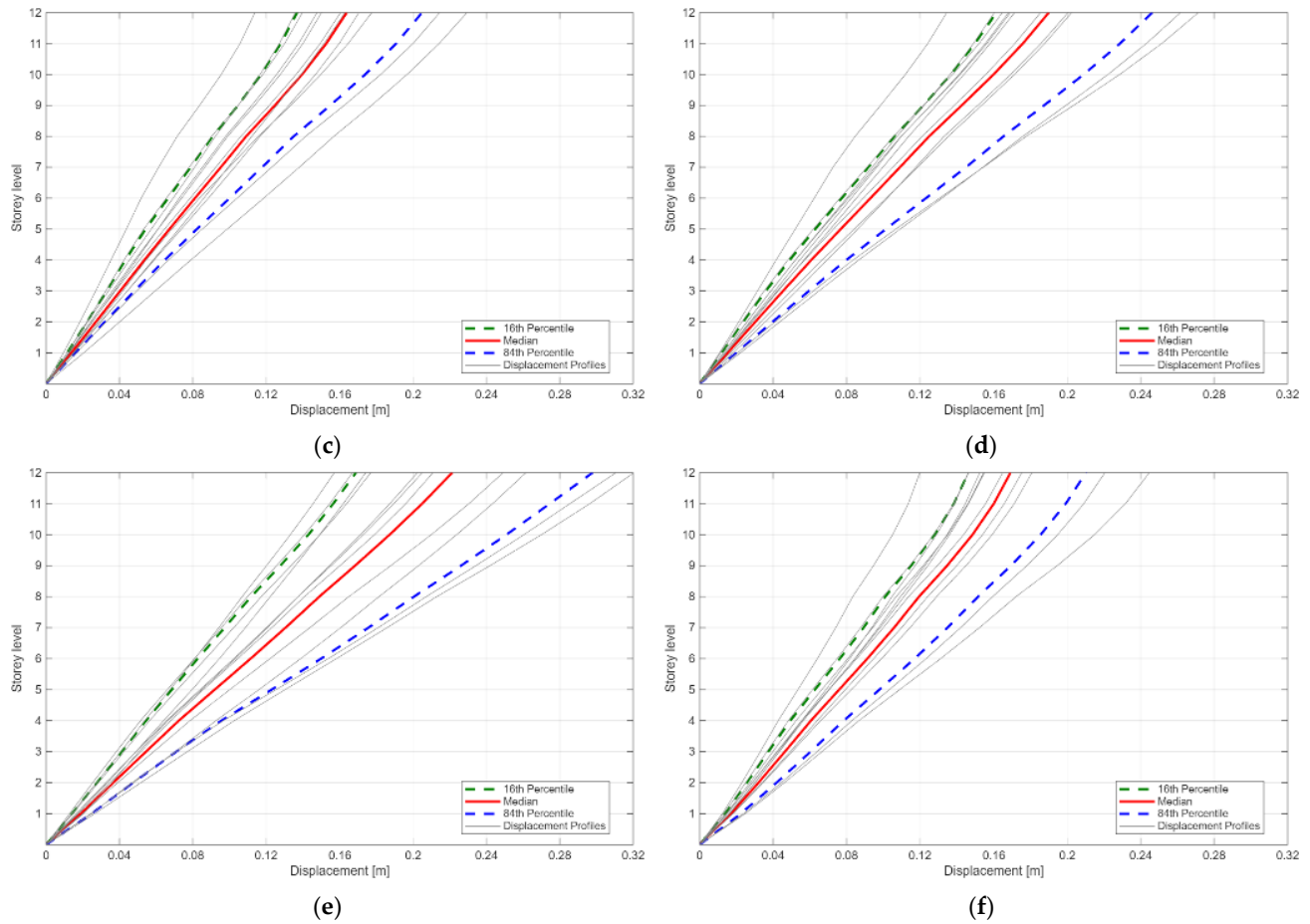
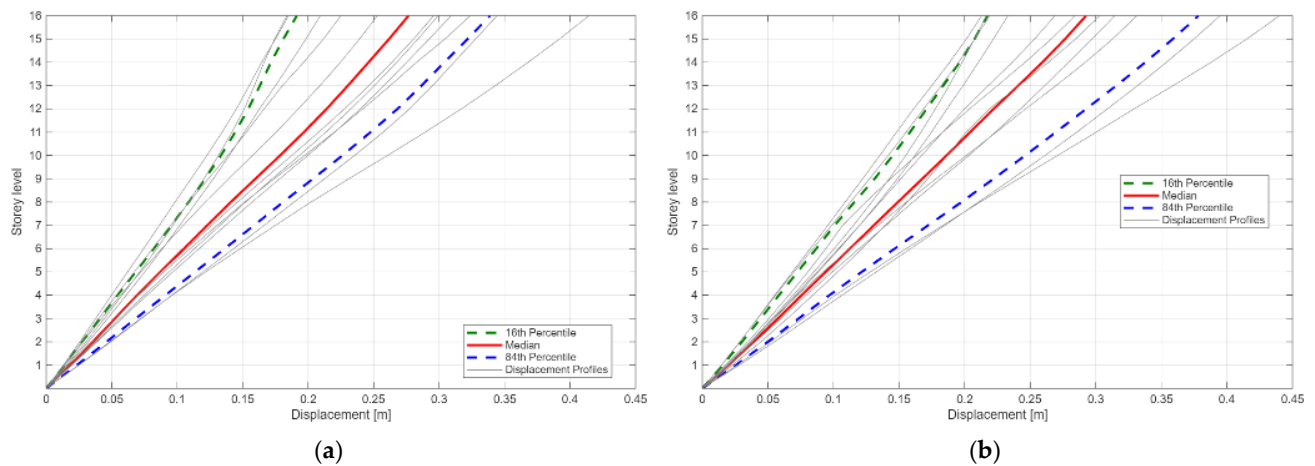
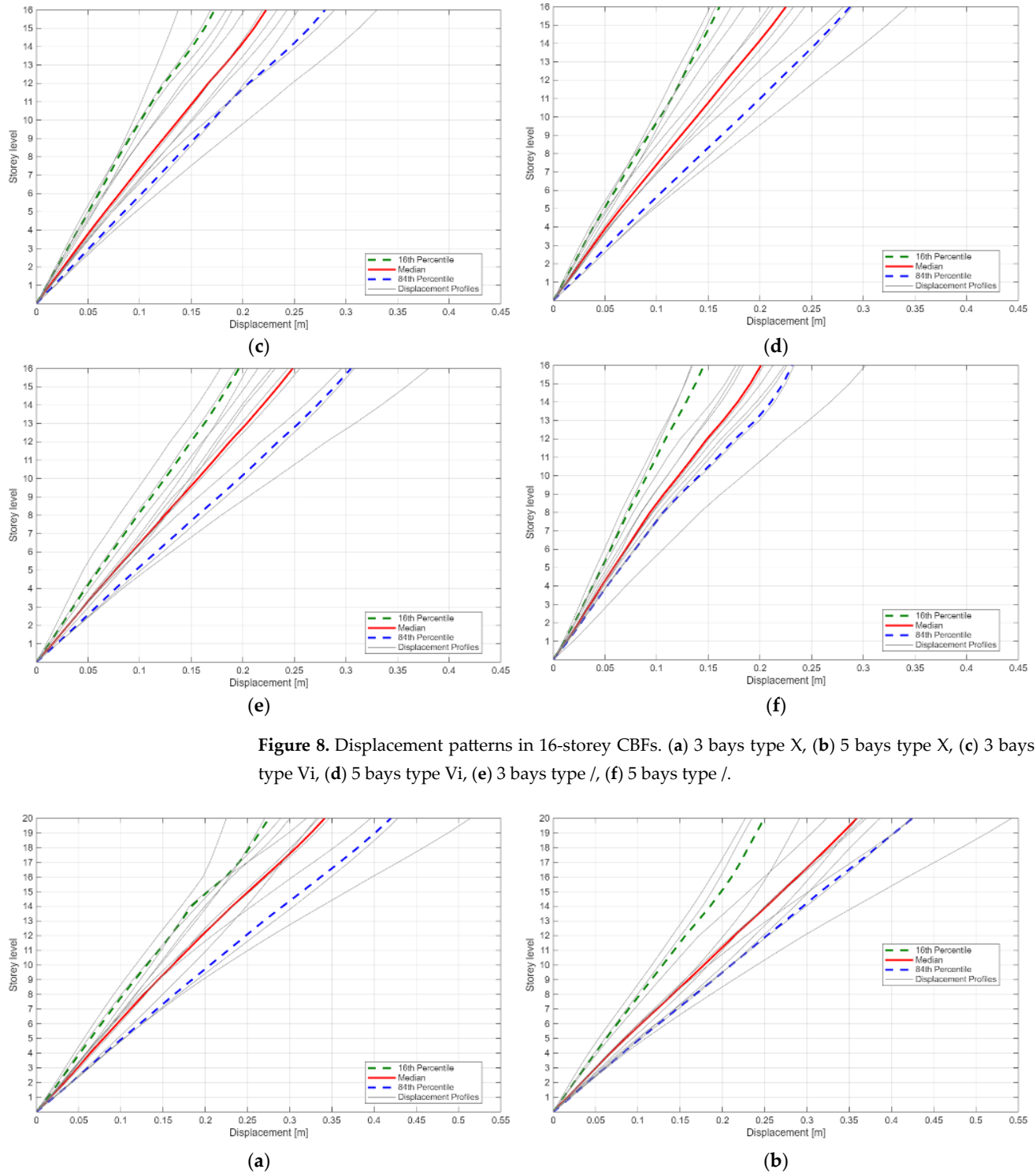


Figure 7. Displacement patterns in 12-storey CBFs. (a) 3 bays type X, (b) 5 bays type X, (c) 3 bays type Vi, (d) 5 bays type Vi, (e) 3 bays type /, (f) 5 bays type /.





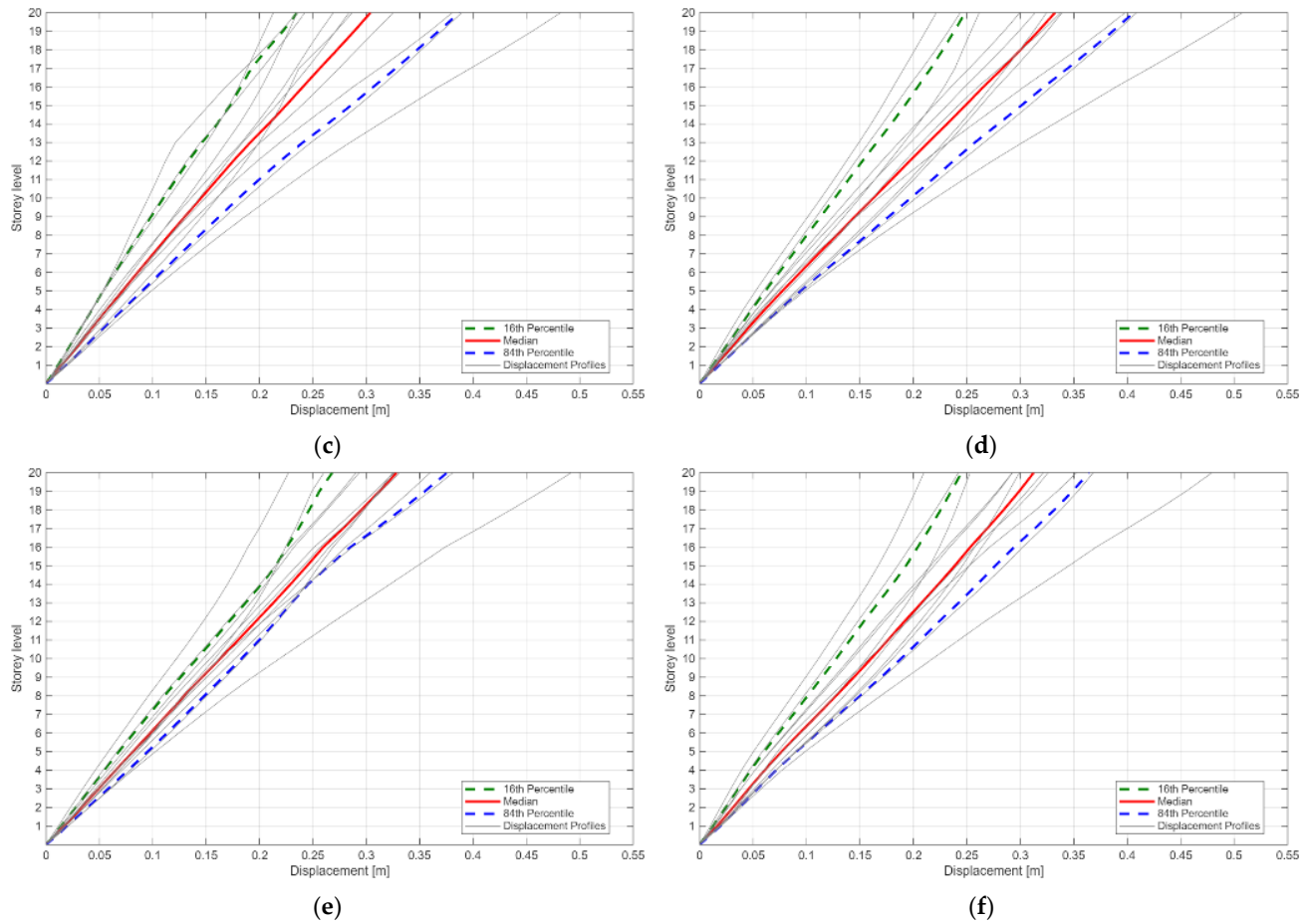
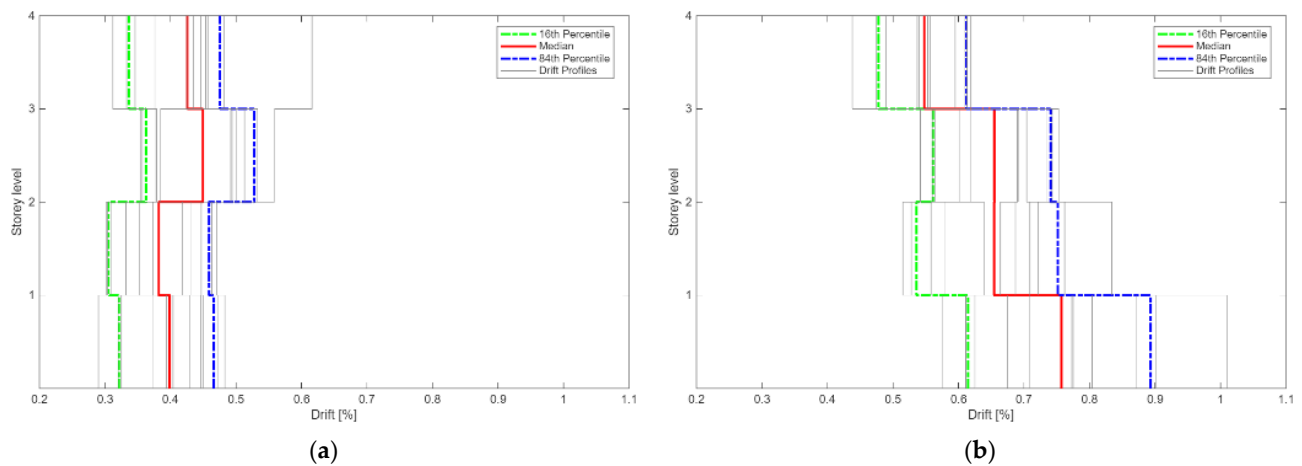


Figure 9. Displacement patterns in 20-storey CBFs. (a) 3 bays type X, (b) 4 bays type X, (c) 3 bays type Vi, (d) 4 bays type Vi, (e) 3 bays type /, (f) 4 bays type /.



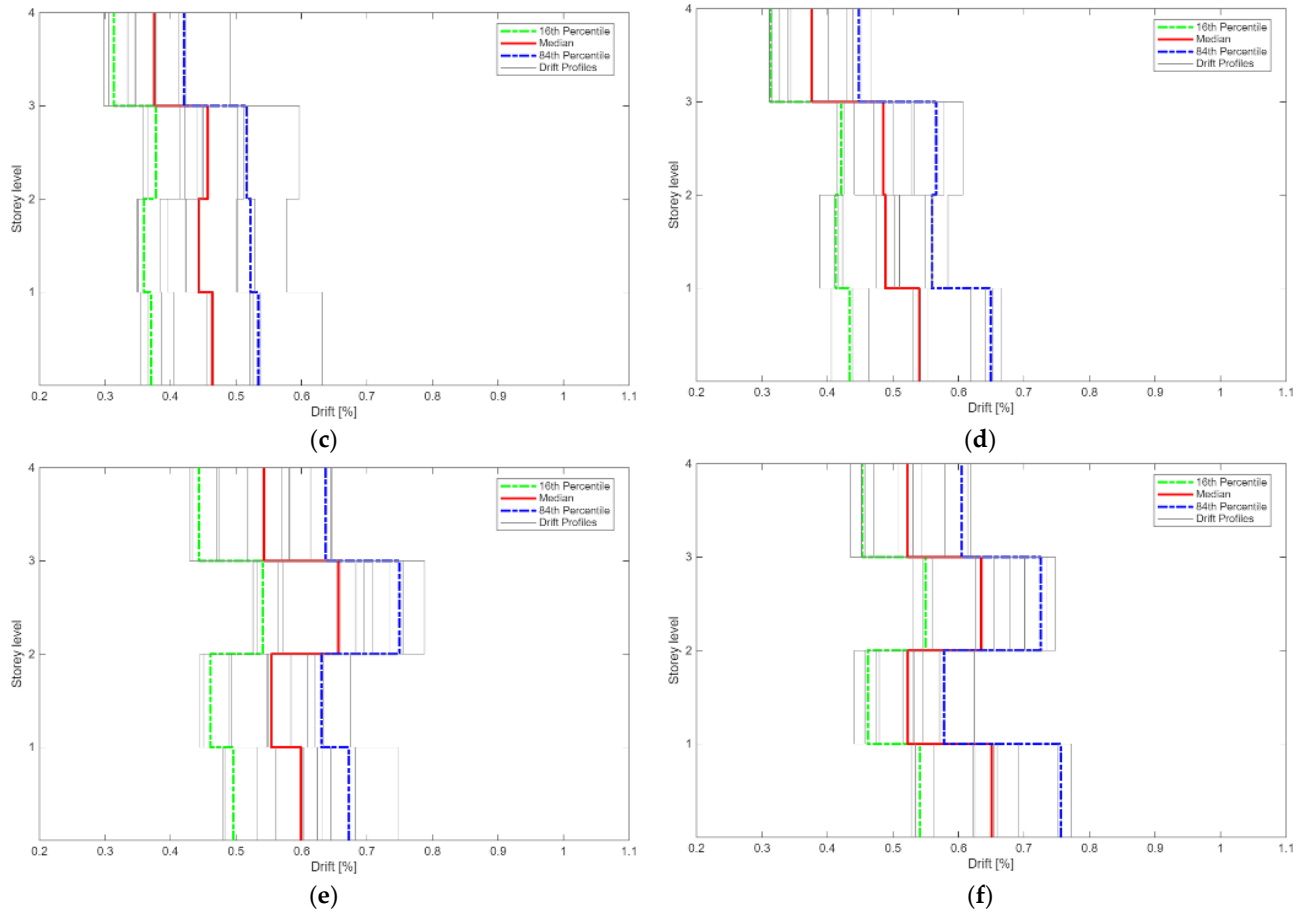
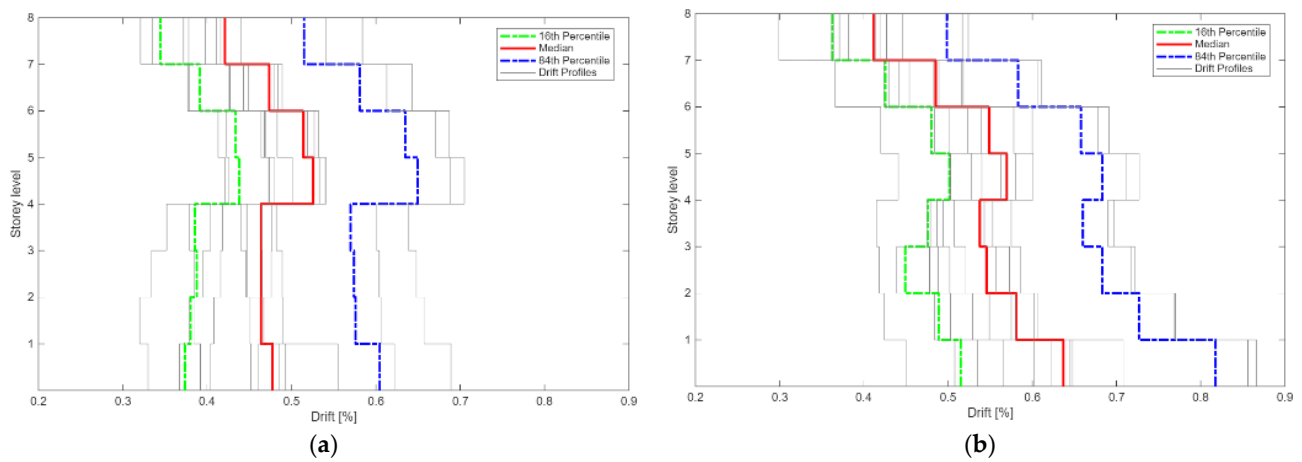


Figure 10. Drift patterns in 4-storey CBFs. (a) 4 bays type X, (b) 6 bays type X, (c) 4 bays type Vi, (d) 6 bays type Vi, (e) 4 bays type /, (f) 6 bays type /.



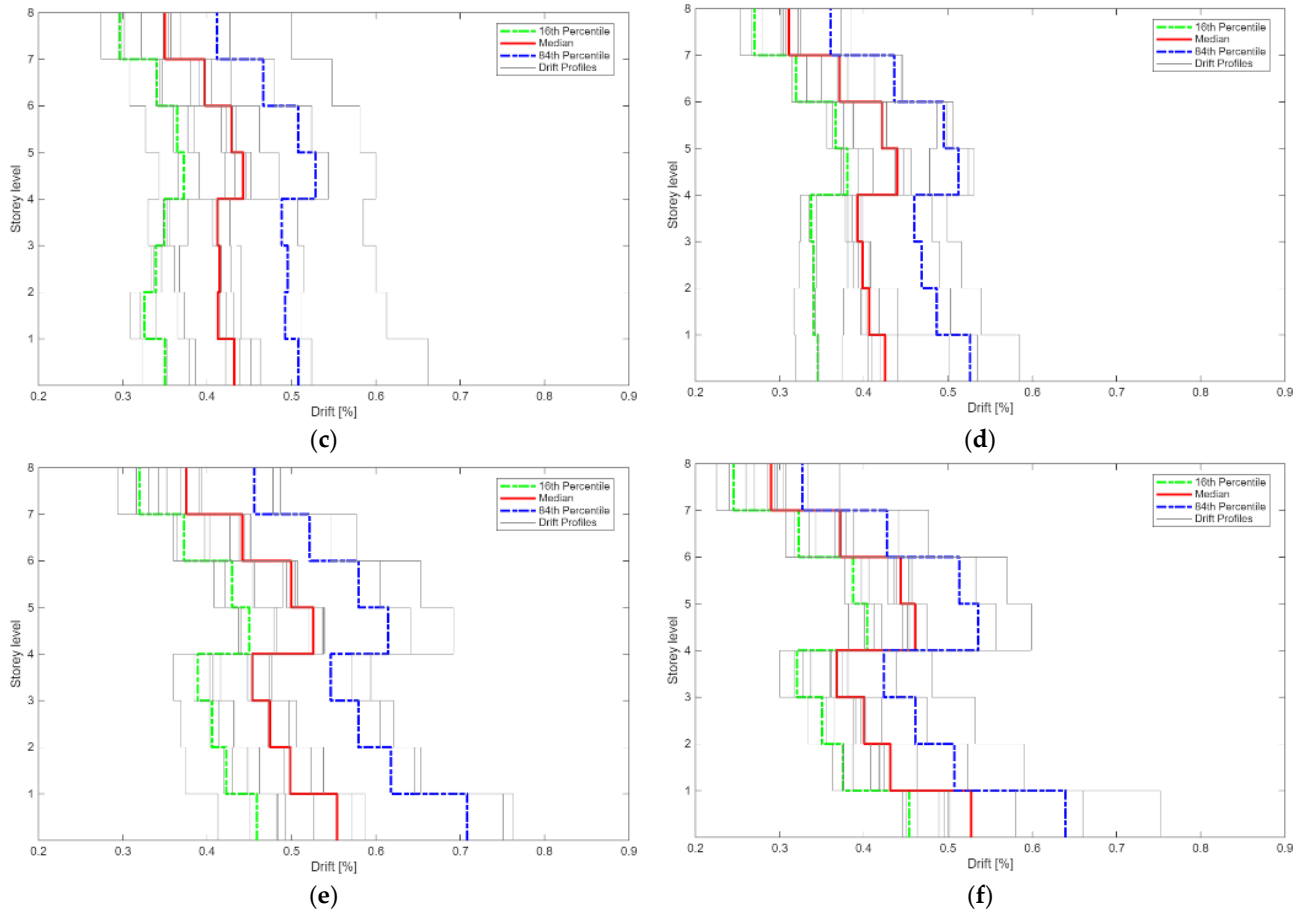
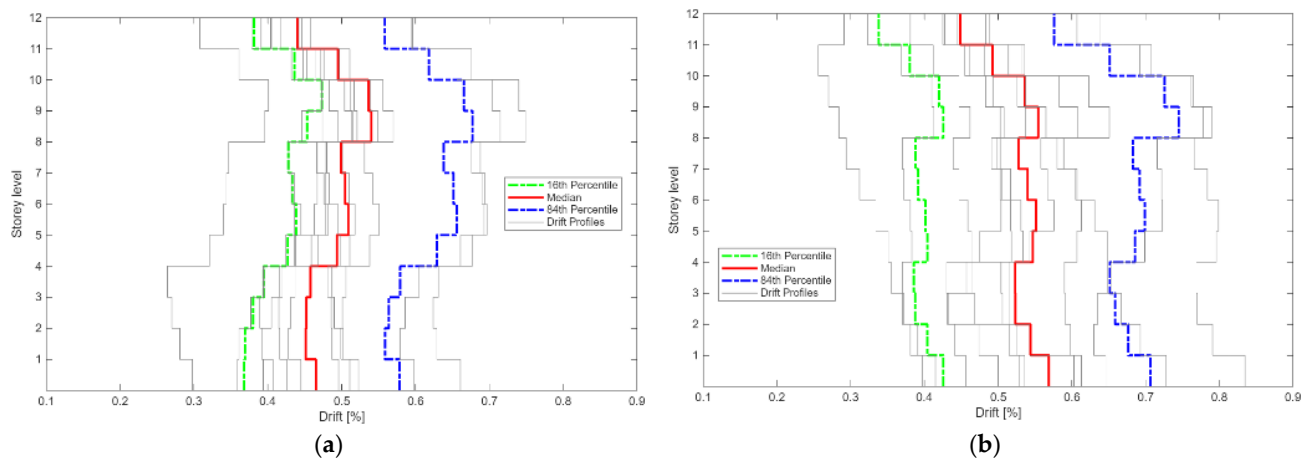


Figure 11. Drift patterns in 8-storey CBFs. (a) 4 bays type X, (b) 6 bays type X, (c) 4 bays type Vi, (d) 6 bays type Vi, (e) 4 bays type /, (f) 6 bays type /.



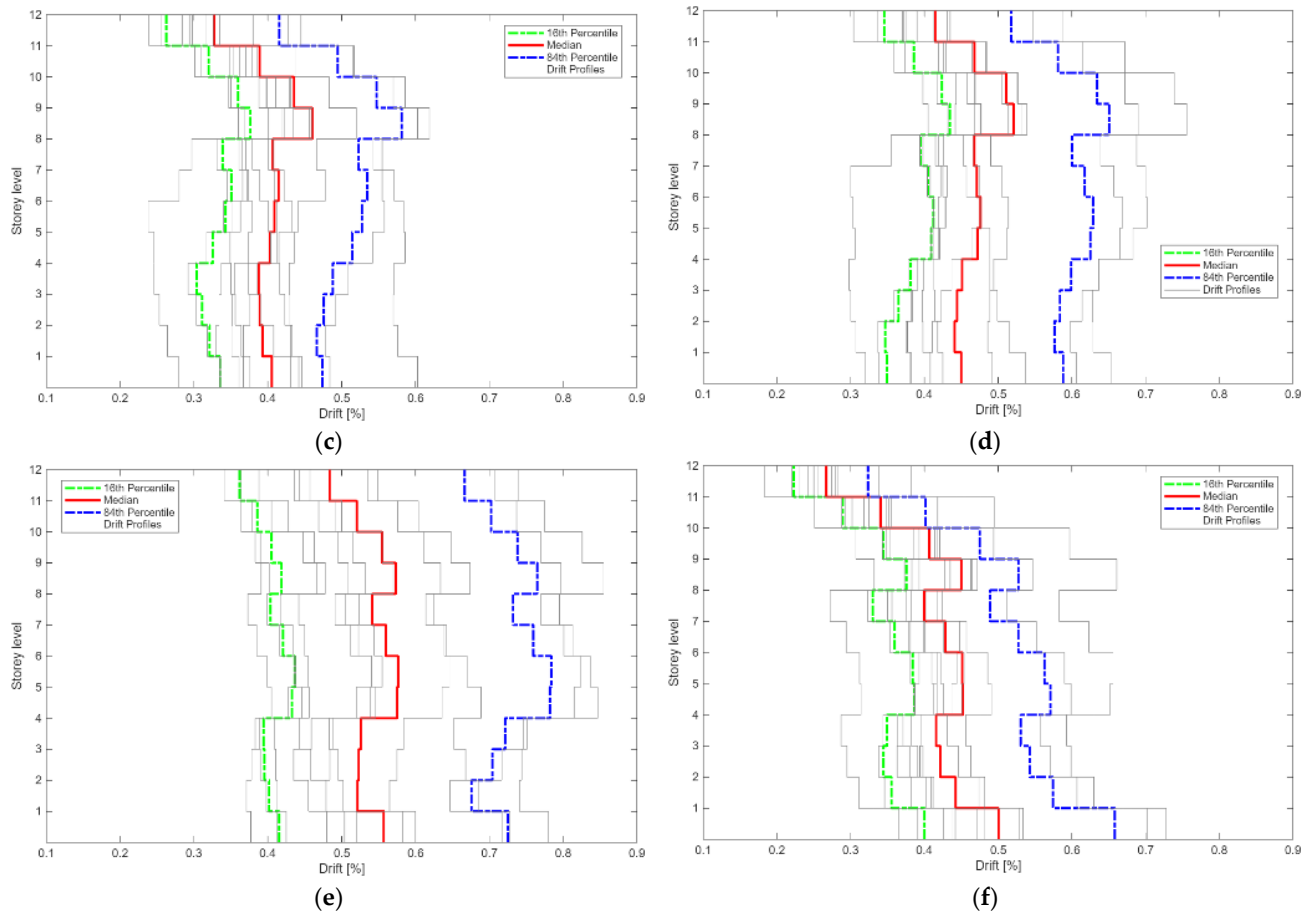
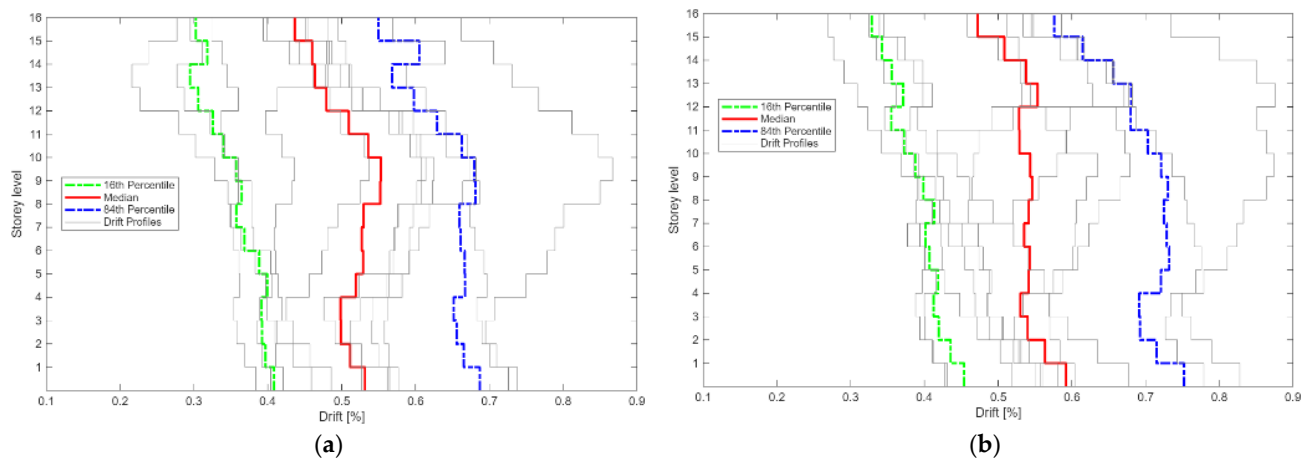


Figure 12. Drift patterns in 12-storey CBFs. (a) 3 bays type X, (b) 5 bays type X, (c) 3 bays type Vi, (d) 5 bays type Vi, (e) 3 bays type /, (f) 5 bays type /.



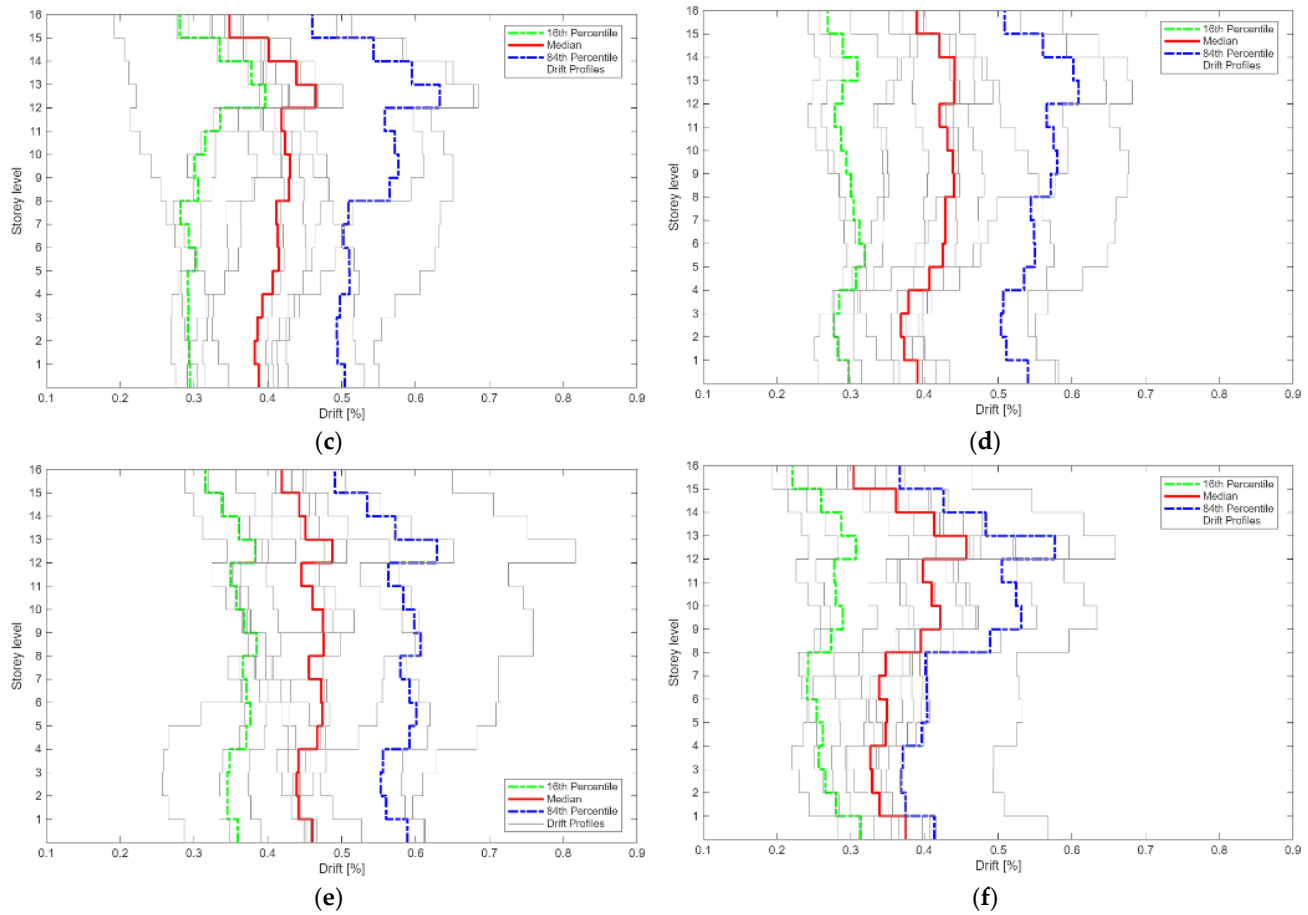
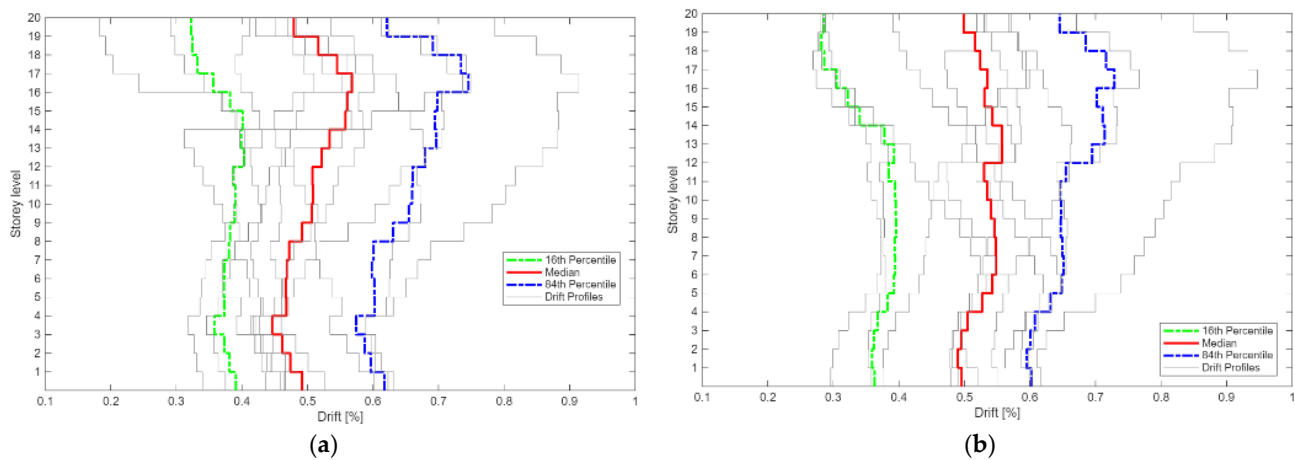


Figure 13. Drift patterns in 16-storey CBFs. (a) 3 bays type X, (b) 5 bays type X, (c) 3 bays type Vi, (d) 5 bays type Vi, (e) 3 bays type /, (f) 5 bays type /.



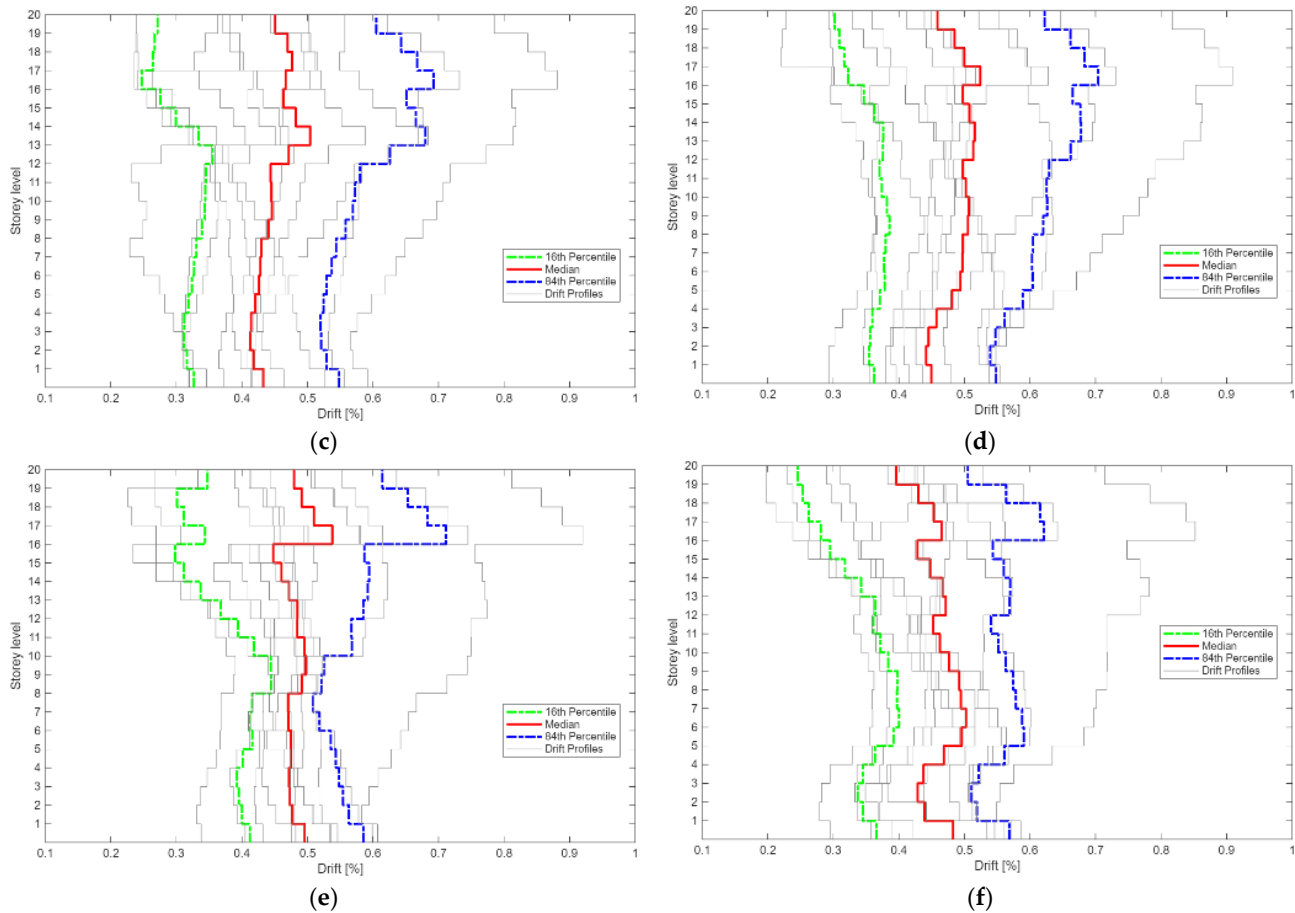


Figure 14. Drift patterns in 20-storey CBFs. (a) 3 bays type X, (b) 4 bays type X, (c) 3 bays type Vi, (d) 4 bays type Vi, (e) 3 bays type /, (f) 4 bays type /.

6. Results and Discussion

6.1. Proposed Equations

Based on the average displacement profile of each frame, three equation typologies were analyzed to assess their fit with the numerical results and to identify the formulation that most accurately reproduces the computed displacement pattern. Starting from the mathematical forms proposed in recent related studies, such as [11,19,20], the following expressions are adopted for estimating storey displacements in CBFs:

$$\text{Option 1:} \quad \delta i = \left[\frac{\theta t * H_i}{b * N b} \right]^a \quad (4)$$

$$\text{Option 2:} \quad \delta i = \left[\frac{\theta t * H_i}{b * N b} \right]^a * \left(1 - \frac{H_i}{4 * H n} \right) \quad (5)$$

$$\text{Option 3:} \quad \delta i = \left[\frac{\theta t * H_i}{b * N b} \right]^a * \left[7 - 8 \left(\frac{H_i}{H n} \right) + 5 \left(\frac{H_i}{H n} \right)^2 \right] \quad (6)$$

In Equations (4)–(6), the floor displacement δi is calculated based on the number of storeys n , the target system drift θt , the height of each storey H_i , the total height of the CBFs $H n$, the number of bays in the frame $N b$, the non-dimensional exponential constant a , and the non-dimensional factor-type constant b . Although the proposed equations have been calibrated and adjusted using models designed for Life Safety performance, the inclusion of the target drift parameter improves their applicability across different performance levels.

Table 7 presents the values of a and b used for the curve fitting of the three alternative equations. The visual comparison of the patterns computed with these equations is shown in Figure 15 for two representative case studies: an 8-storey, 4-bay X braced frame (Figure 15a) and a 20-storey, 4-bay inverted-V frame (Figure 15b).

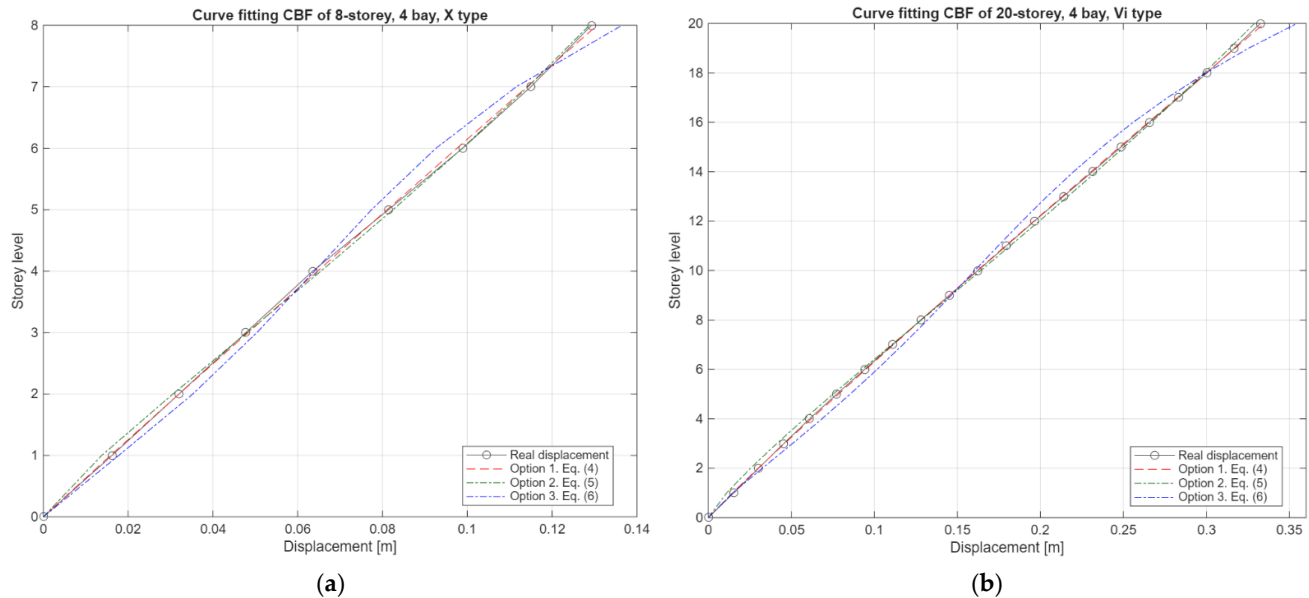


Figure 15. Curve fitting for alternative equations. (a) CBF of 8-storey, 4-bay X-type, and (b) CBF of 20-storey, 4-bay inverted-V.

Table 7. Alternative equations parameters for curve fitting in the CBFs of an 8-storey, 4-bay X-type, and the CBF of 20-storey, 4-bay inverted-V.

Parameter	Symbol	8-Storey, 4-Bay X-Type	20-Storey, 4-Bay Inverted-V
Common Variables	n	8	20
	Nb	4	4
	θt	2.00%	1.71%
	Hn (m)	27.2	68.0
Option 1. Equation (4)	a	1.014	1.047
	b	1.012	0.826
Option 2. Equation (5)	a	1.198	1.226
	b	0.592	0.570
Option 3. Equation (6)	a	1.181	1.231
	b	2.375	2.082

The visual differences shown in Figure 15, and the error minimization statistics of the curve fitting for the three options of equations (Table 8), indicate that the best agreement with the NLTHA results is achieved with option 1, represented by Equation (4). This thesis holds for all bracing typologies analyzed, namely X, inverted-V, and single-diagonal / configurations.

Table 8. Comparison of curve fitting for the three options of equations.

Option	CBF of 8-Storey, 4-Bay X-Type					CBF of 20-Storey, 4-Bay Inverted-V				
	a	b	R^2 *	SSE **	RMSE ***	a	b	R^2	SSE	RMSE
Option 1. Equation (4)	1.014	1.012	0.9996	0.00002	0.0009	1.047	0.826	0.9999	0.00003	0.0008
Option 2. Equation (5)	1.198	0.592	0.9991	0.00005	0.0014	1.226	0.570	0.9994	0.00011	0.0027

Option 3. Equation (6)	1.181	2.375	0.9914	0.00010	0.0045	1.231	2.082	0.9936	0.00142	0.0086
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* R²: coefficient of determination; a value close to 1 indicates that the model provides a good fit with the data. ** SSE: sum of squared errors; a lower value indicates a better fit. *** RMSE: root mean square error; a smaller value indicates a better fit.

The curve fitting results for the proposed Equation (4) are reported in Table 9 for all 30 frames considered in this study.

Table 9. Values of “*a*” and “*b*” for the study frames in the proposed Equation (4).

Type X CBF		Type Vi CBF		Type / CBF	
4 storeys, 4 bays	4 storeys, 6 bays	4 storeys, 4 bays	4 storeys, 6 bays	4 storeys, 4 bays	4 storeys, 6 bays
<i>a</i>	1.052	0.890	0.943	0.890	1.000
<i>b</i>	1.048	0.684	1.353	0.980	0.846
8 storeys, 4 bays	8 storeys, 6 bays	8 storeys, 4 bays	8 storeys, 6 bays	8 storeys, 4 bays	8 storeys, 6 bays
<i>a</i>	1.014	0.917	0.980	0.967	0.939
<i>b</i>	1.012	0.724	1.256	0.894	1.179
12 storeys, 3 bays	12 storeys, 5 bays	12 storeys, 3 bays	12 storeys, 5 bays	12 storeys, 3 bays	12 storeys, 5 bays
<i>a</i>	1.053	0.980	1.021	1.039	1.008
<i>b</i>	1.251	0.771	1.580	0.802	1.202
16 storeys, 3 bays	16 storeys, 5 bays	12 storeys, 3 bays	16 storeys, 5 bays	16 storeys, 3 bays	16 storeys, 5 bays
<i>a</i>	0.990	0.976	1.043	1.058	1.010
<i>b</i>	1.177	0.686	1.365	0.796	1.286
20 storeys, 3 bays	20 storeys, 4 bays	20 storeys, 3 bays	20 storeys, 4 bays	20 storeys, 3 bays	20 storeys, 4 bays
<i>a</i>	1.067	1.029	1.055	1.047	1.007
<i>b</i>	1.067	0.785	1.206	0.826	1.180

The 30 case studies were condensed into 12 final equations by grouping CBF systems for which the coefficients *a* and *b* did not differ significantly. For X-type and single-diagonal / frames, the same *a* and *b* values are adopted, whereas distinct *a* and *b* values are assigned to inverted-V frames. These 12 averaged equations, differentiated by ranges of the number of storeys *n* and the number of bays *N_b*, are summarized in Table 10.

Table 10. Values of “*a*” and “*b*” for the proposed averaged equations.

Parameter	<i>n</i> = 4 Type X, Type /		<i>n</i> = 4 Type Vi	
	For <i>N_b</i> ≤ 4	For <i>N_b</i> > 4	For <i>N_b</i> ≤ 4	For <i>N_b</i> > 4
<i>a</i>	1.026	0.918	0.943	0.890
<i>b</i>	0.947	0.671	1.353	0.980
Parameter	<i>n</i> = 8, <i>n</i> = 12 Type X, Type /		<i>n</i> = 8, <i>n</i> = 12 Type Vi	
	For <i>N_b</i> ≤ 4	For <i>N_b</i> > 4	For <i>N_b</i> ≤ 4	For <i>N_b</i> > 4
<i>a</i>	1.004	0.934	1.001	1.003
<i>b</i>	1.161	0.891	1.418	0.848
Parameter	<i>n</i> = 16, <i>n</i> = 20 Type X, Type /		<i>n</i> = 16, <i>n</i> = 20 Type Vi	
	For <i>N_b</i> < 4	For <i>N_b</i> ≥ 4	For <i>N_b</i> < 4	For <i>N_b</i> ≥ 4
<i>a</i>	1.019	1.019	1.049	1.053
<i>b</i>	1.177	0.816	1.286	0.811

Based on the averaged values of *a* and *b* reported in Table 10, the proposed equation can be derived for each frame typology. For example, for a 12-storey, 4-bay CBF with single-diagonal / bracing, Equation (7) is proposed.

$$\delta i = \left[\frac{\theta t * H_i}{0.891 * N b} \right]^{0.934} \quad (7)$$

From Figures 16–20, a comparison is presented between the mean storey displacements obtained from the NLTHA and those computed using the proposed averaged equations (Table 10). The displacement values given by Equations (1)–(3), which were used in the pre-design of CBFs, are also included. To ease visual interpretation of the variation among these values, a 1:1 trend line (identity line) has been plotted using the median displacement measured at each level in NLTHA.

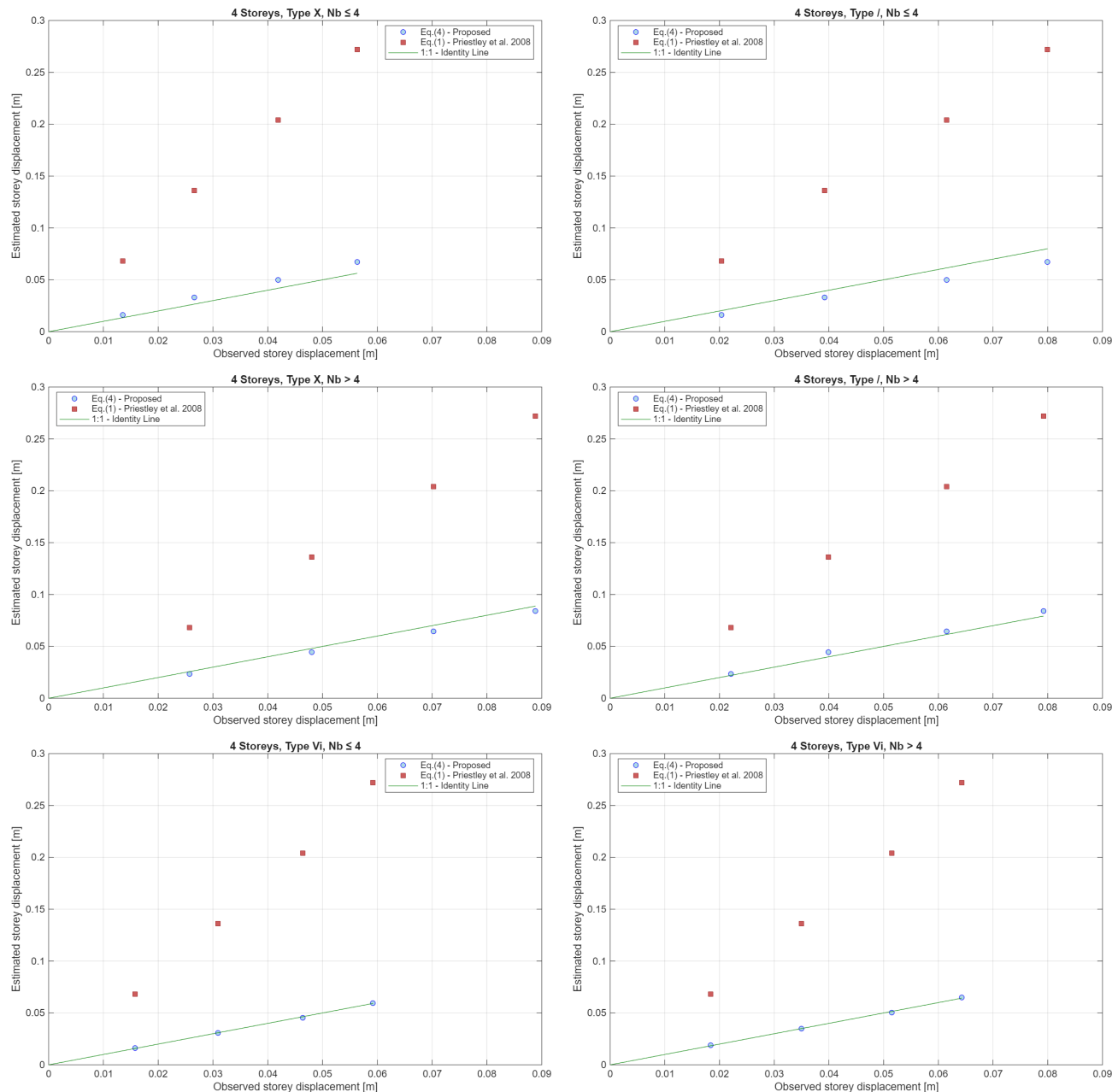


Figure 16. Comparison of displacement pattern calculations for 4-storey frames.

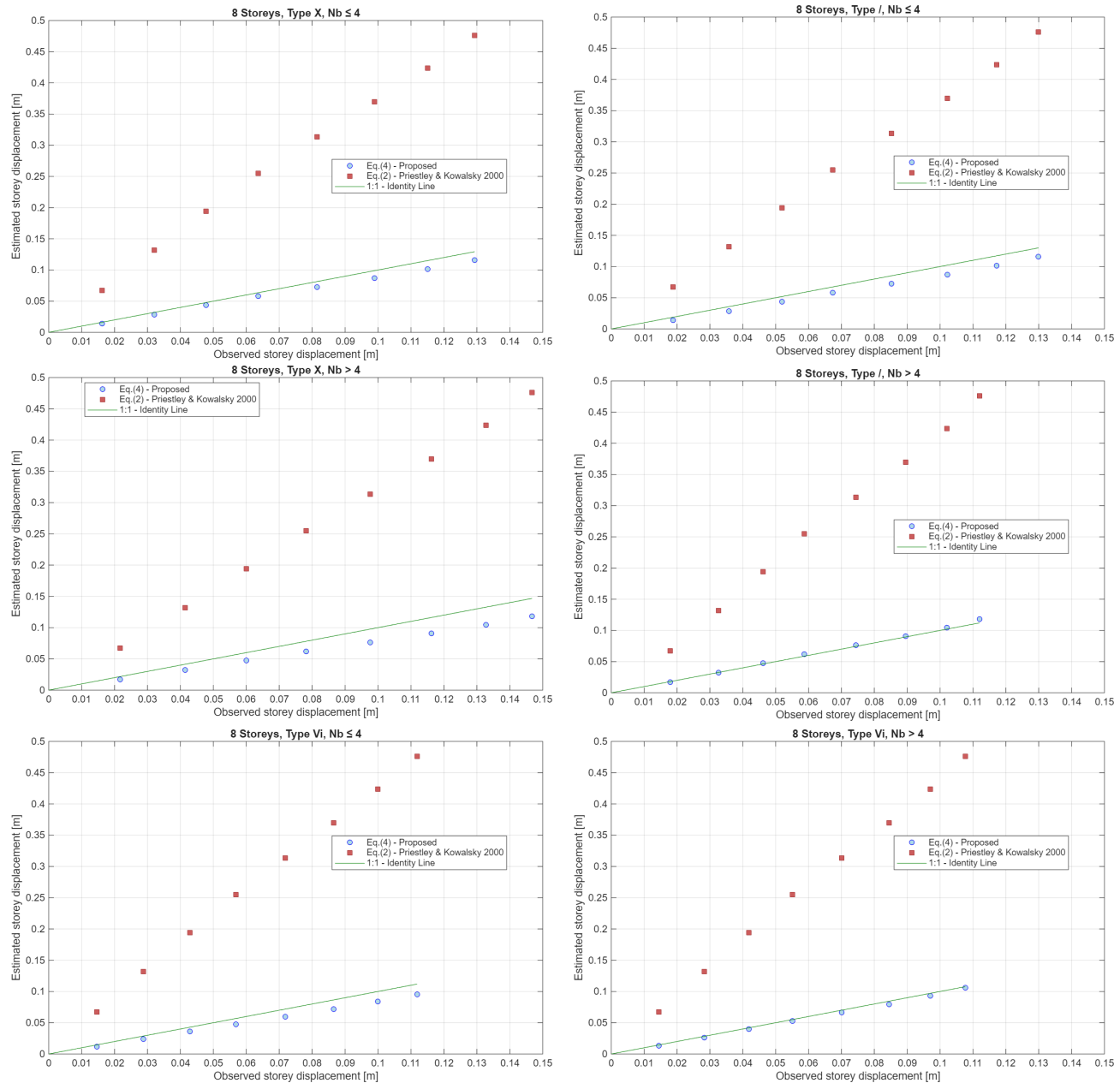
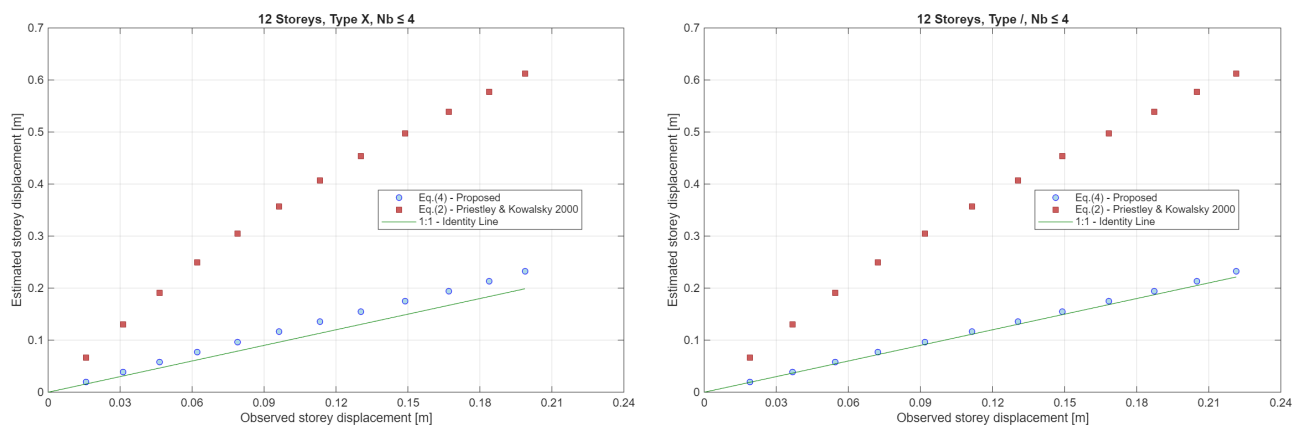


Figure 17. Comparison of displacement pattern calculations for 8-storey frames.



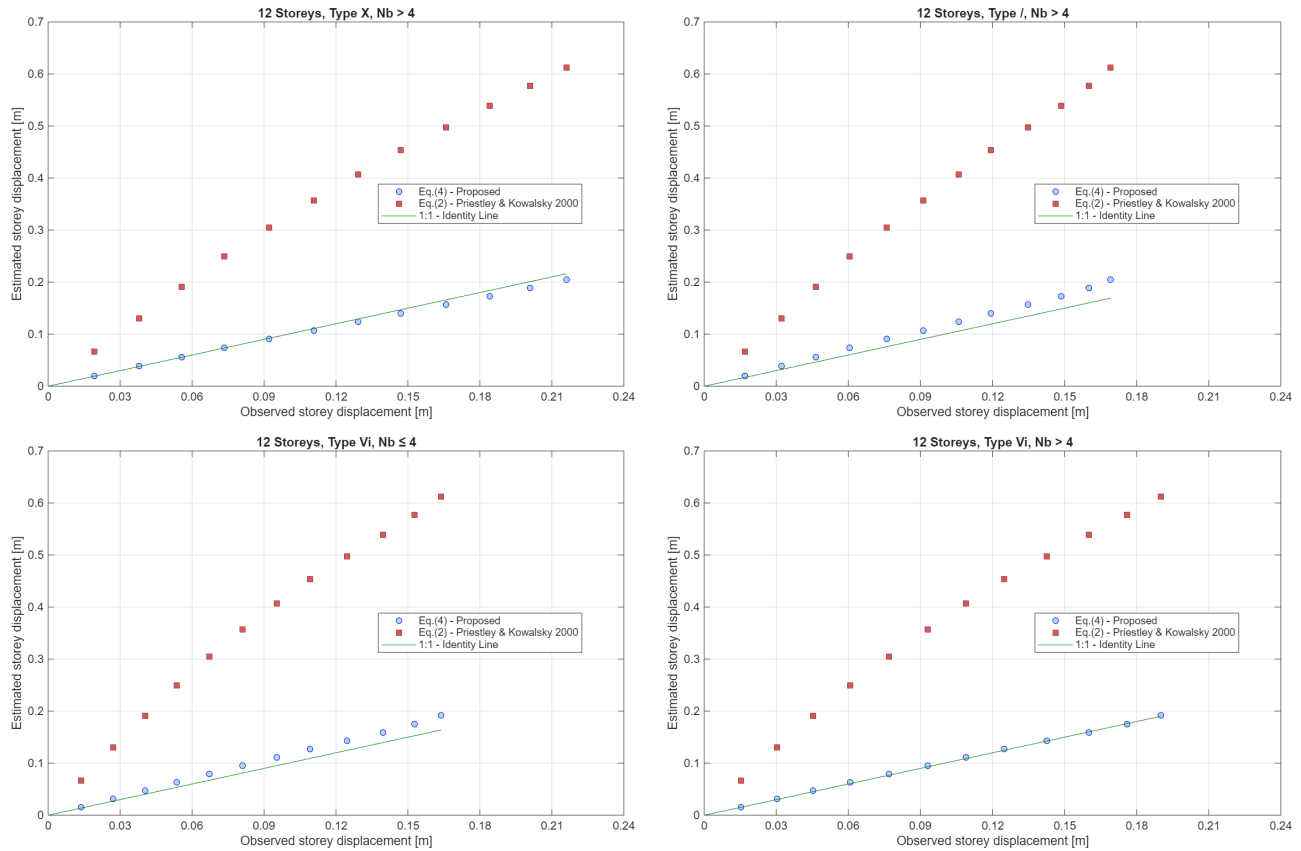
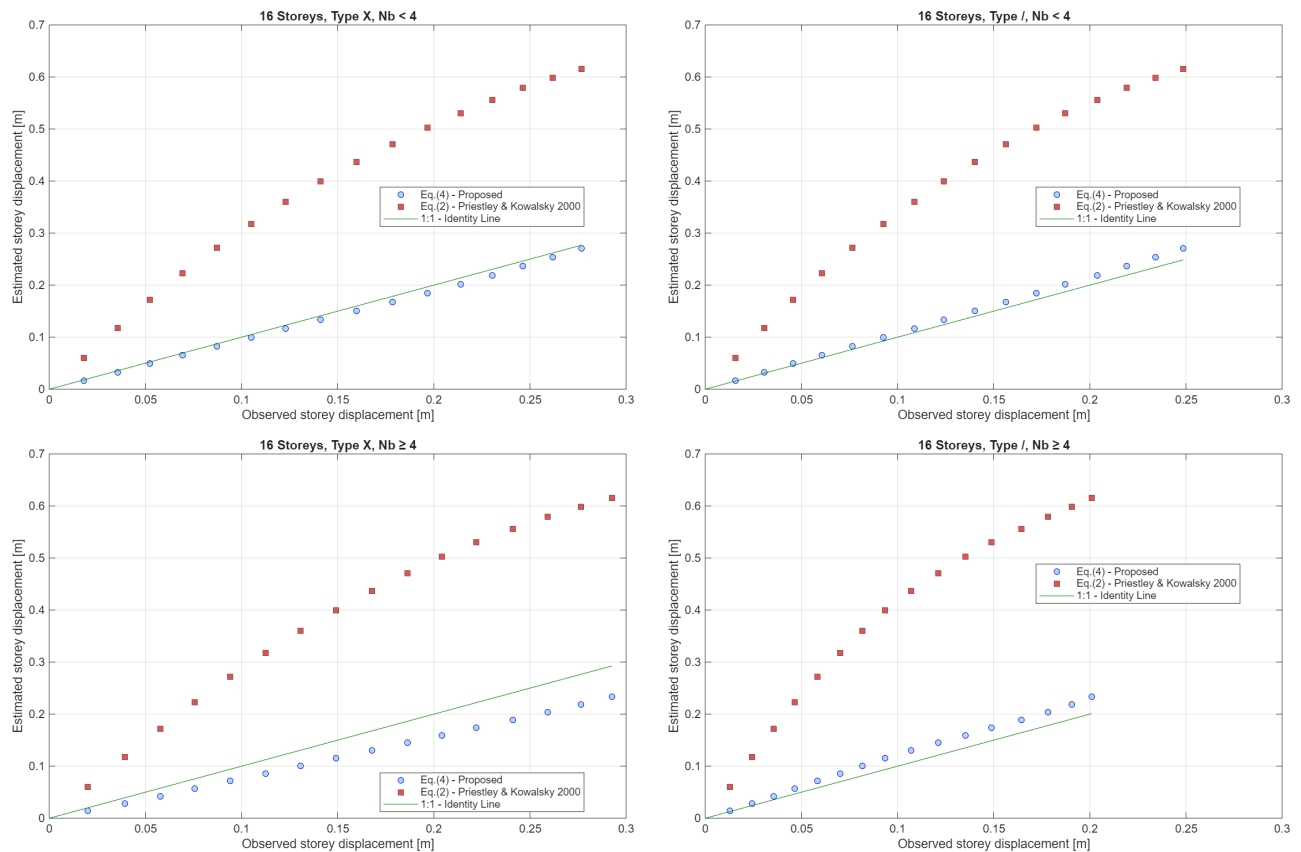


Figure 18. Comparison of displacement pattern calculations for 12-storey frames.



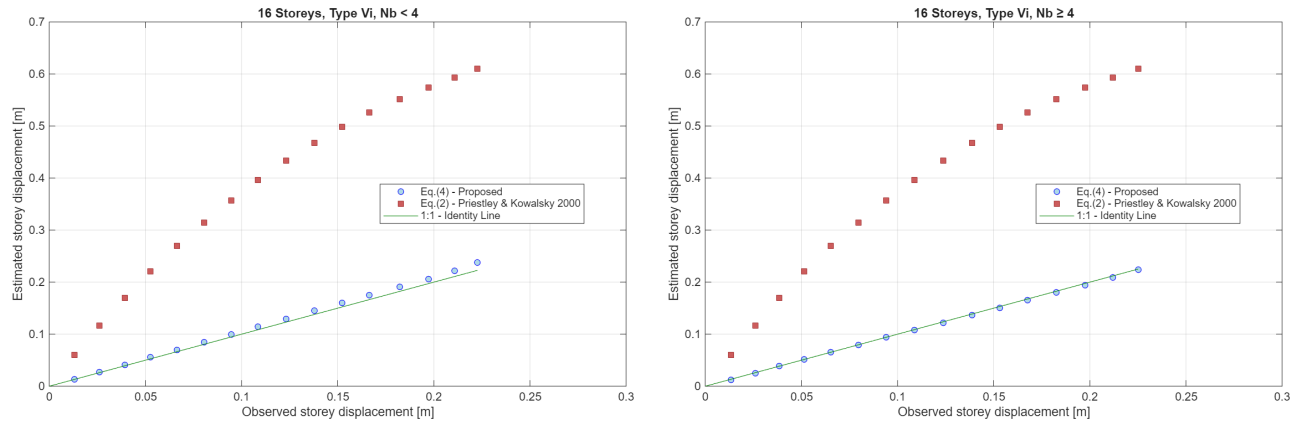


Figure 19. Comparison of displacement pattern calculations for 16-storey frames.

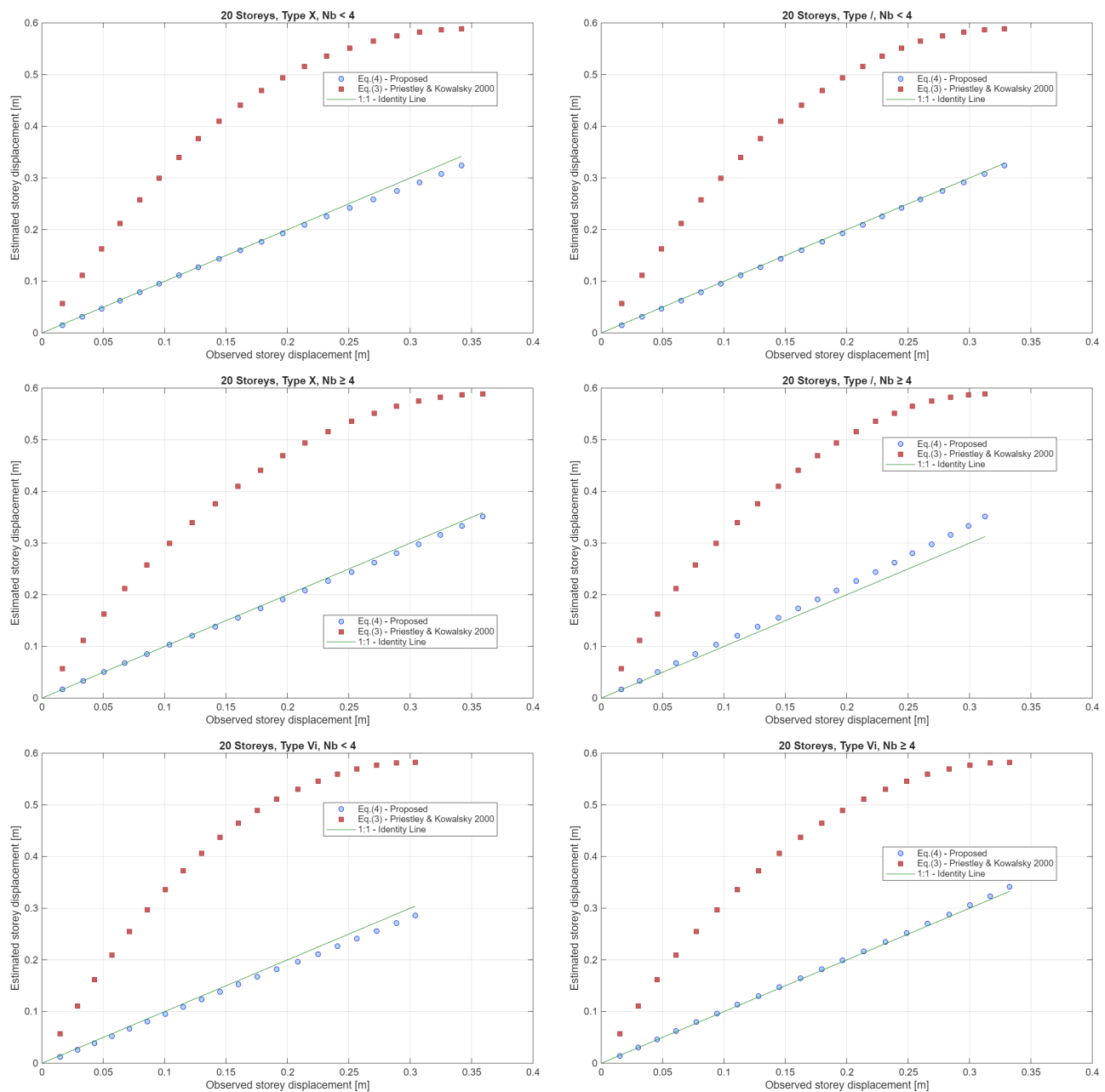


Figure 20. Comparison of displacement pattern calculations for 20-storey frames.

6.2. Discussion of Results

Based on the results shown in Figures 5–9, the displacement profile obtained from the NLTHA for 4-storey frames exhibits an approximately linear shape with a slight curvature that an exponential function with an exponent close to unity can represent. This curvature decreases, and may even vanish, in the displacement patterns of the 8- and 12-storey frames. The displacement profile becomes essentially linear for the taller frames (16 and 20 storeys).

Across the three bracing typologies, the drift response varies (see Figures 10–14) with the number of storeys: for 4-storey frames, the interstorey drift decreases steadily along the height; for 8-storey frames, the drift generally decreases with storey level but exhibits a slight jump at the fourth storey. For 12-, 16-, and 20-storey frames, the response becomes more irregular, because in X and single-diagonal / frames the drift pattern is similar to the 4- and 8-storey frames, whereas in inverted-V frames the drift tends to increase consistently with height, with pronounced drift jumps, particularly at the eighth or tenth storey.

The displacements computed using the modified equations derived from previous studies exhibit varying levels of agreement with the NLTHA results, with option 1 providing the best fit, as indicated by both graphical interpretation and error minimization statistics. The inclusion of the denominator factor “ b ” and the exponential factor “ a ” in Equation (4) completes the general form of the proposed equation for all CBF typologies investigated. The factor “ b ” differentiates the displacement response according to the number of bays in the structure and, consequently, the magnitude of the reactive seismic mass supported by the lateral load-resisting system. In turn, the factor “ a ” captures the curvature shapes observed in the average displacement patterns of the simulated CBFs.

The a and b values were obtained from direct calibration to NLTHA results. The grouping into ranges of n and Nb is a deliberate choice to maintain accuracy and avoid misleading generalization, and the development of continuous functional relationships for a and b is identified as a promising direction for future research once a broader parametric study is completed.

The case studies for which the values of “ a ” and “ b ” did not differ significantly were grouped, yielding a final set of 12 distinct equations. These equations can capture displacement patterns: a nearly linear shape with slight curvature in 4-storey buildings, minimal or imperceptible curvature in 8- and 12-storey frames, and an essentially linear shape in the taller frames (16 and 20 storeys).

The displacement patterns estimated with the twelve proposed equations are compared with the NLTHA results in Figures 16–20. Maximum variations of 19.0%, 19.4%, 21.1%, 20.2%, and 12.4% between the predicted and calculated roof displacements for the 4-, 8-, 12-, 16-, and 20-storey frames are shown, respectively. In these figures, the displacements obtained in the pre-design stage are also plotted using equations from similar studies, and a considerable deviation from the 1:1 trend line is observed with respect to the actual displacement values.

A variation of 19% to 21% is considered acceptable, given that when equations from previous studies are used (particularly mathematical expressions derived for unbraced steel frames), the divergence is even more pronounced compared with the proposal of this research. A short-term challenge is to explore alternative groupings and combinations of the coefficients “ a ” and “ b ” in order to further reduce these variation percentages.

7. Conclusions

To illustrate the DDBD methodology and the applicability of the proposed displacement patterns, 30 case study frames were pre-designed using this procedure. In this way,

the magnitudes and directions of the forces to be resisted by the structural members were established. The steel brace, beam, and column sections were selected to satisfy slenderness, ductility, axial, shear, and flexural capacity requirements.

Displacement patterns in CBF structures exhibit specific behaviors that depend on the number of storeys, the number of bays, and the bracing typology, with apparent variation in the shape and curvature of the displacement profile along the building height. This exponential-type curvature is more evident in 4- and 8-storey frames. At the same time, a predominantly linear pattern is observed for the taller buildings, based on the average response to the 11 ground motions applied to each case study.

The stability and convergence of the nonlinear time-history models run in SeismoStruct depend initially on the constitutive model adopted for the inelastic material and on the fiber-discretized steel sections, but fundamentally on the global stiffness of the system, which is conceptualized from the effective fundamental period of the frame T_{eff} and the effective damping ratio ξ_{CBF} computed in the pre-design stage.

Twelve averaged equations were developed and validated. All of these are calibrated for different ranges of the number of storeys, the number of bays, and bracing configurations, and they reproduce the displacement profiles from NLTHA with high accuracy (minor errors and coefficients of determination close to 1). The divergence in displacement values between the inverted-V braced models and the X and diagonal / braced models led to the coupling of the X and diagonal / braced types into a single equation group.

The inclusion of parameters such as target drift and the number of bays, along with the discretization of equations for different bracing configurations and numbers of storeys, is essential to capture the actual response of the structural system under seismic loads in the study area of Quito, Ecuador.

A maximum variation of 21% was found between the mean value obtained from the NLTHA and the value computed with the proposed equations. This variation is larger for the 16- and 20-storey frames. The level of variation is deemed acceptable relative to results obtained using equations from previous studies, for which the discrepancy is greater. A future challenge is to explore alternative parameter groupings for “a” and “b” to further reduce this variation.

The set of equations proposed in this study significantly reduces the variation between calculated and predicted displacements, improving on the accuracy achieved with existing mathematical expressions developed primarily for unbraced steel frames. This advancement enhances the efficiency of the DDBD methodology and strengthens structural safety in the seismic design of CBF systems.

The present study develops and calibrates displacement pattern equations using NLTHA results from 30 steel CBF configurations (different heights, numbers of bays, and bracing typologies), explicitly linking the proposed profiles to the fundamental period and equivalent damping within a DDBD framework. This broader parametric basis and direct integration into DDBD make the proposed equations a novel contribution that complements and extends previous, more-case-specific displacement pattern formulations.

The development and application of specific equations for different bracing configurations and several building height ranges have been a significant contribution to the DDBD methodology. For this reason, the present study significantly improves the DDBD procedure for CBFs by providing, through the proposed equations, a more reliable and predictive model for this type of lateral load-resisting system.

It is recommended that the proposed displacement equations be adopted to enhance the seismic design of structures with concentrically braced frames, particularly for buildings employing the three bracing typologies examined in this study.

The proposed expressions are best regarded as design-oriented tools to support the direct displacement-based design of CBFs, to be used together with conventional drift

checks and engineering judgment, rather than as definitive code provisions. It will also be essential to promote the dissemination and practical adoption of the DDBD seismic performance methodology among structural engineers and designers to ensure its proper implementation in the construction industry.

Disseminating the study's conclusions and recommendations will help establish and position the DDBD method and the design of CBF systems within the structural engineering community, thereby strengthening seismic resilience in Ecuador and other high-seismic-hazard regions of Latin America.

8. Limitations and Future Work

Further experimental and numerical validation of these CBF models is required across a broader range of concentric bracing configurations, vertical load cases, and seismic scenarios to strengthen their applicability. Future research should also assess the influence of additional variables, such as soil–structure interaction, the proportion of braced bays, connection and base-restraint flexibility, and the contribution of nonstructural systems to the overall seismic response.

The displacement pattern equations proposed in this study are calibrated for steel CBF systems pre-designed under Ecuadorian seismicity conditions ($PGA \approx 0.40\text{ g}$), soil type D, and conventional Grade 50 steel, for target interstorey drift ratios around 2% associated with a Life Safety performance level. Within this calibration domain, the averaged coefficients “ a ” and “ b ” reproduce the non-dimensional displacement profiles obtained from NLTHA of the 30 case study frames. However, for structural systems that fall outside this range, such as those designed for substantially different performance objectives (Immediate Occupancy or Collapse Prevention), alternative steel grades, different brace slenderness ratios, or soil classes that lead to significantly different spectral shapes, the proposed equations should be applied with caution and complemented by verification through nonlinear analysis.

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