

Article

Shear Correction Factor for Porous Eco-Materials: Mechanical Characterization of a Heterogeneous Medium

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Abstract

Classical formulas for the shear correction factor k_s , typically derived for homogeneous continua, are unsuitable for porous media exhibiting local density gradients, irregular pore morphologies, and spatially varying stiffness. This paper presents a generalized analytical–numerical methodology for evaluating the shear correction factor in a wide class of porous eco-materials. The approach is based on the strain energy equivalence principle and uses a continuous stiffness model that reflects density-dependent elastic properties. A voxel-based microstructural representation is employed to validate the analytical predictions and to quantify the influence of heterogeneity on the shear stress distribution. Perlite is used as a representative case study, demonstrating how classical homogeneous formulas may produce errors exceeding 40%, while the proposed method provides significantly improved agreement with numerical benchmarks. The framework is applicable to a broad range of porous materials and offers a consistent basis for predicting transverse shear stiffness in lightweight fillers, thermal barriers, and fire-protective building components where shear deformation is critical.

Keywords: porous materials; perlite; shear correction factor; Timoshenko theory; heterogeneous media; lightweight eco-materials; microstructure modeling; strain energy equivalence

1. Introduction

The growing demand for environmentally responsible construction materials has reshaped contemporary building technologies, motivating the development and adoption of lightweight, low-carbon, and resource-efficient alternatives to conventional mineral composites [1,2]. Among these innovations, porous eco-materials (such as expanded perlite, pumice–cement composites, foamed concretes, mycelium-based biocomposites, and other cellular solids of natural origin) have become an essential component of the sustainable construction landscape [3–7]. Their unique combination of low density, excellent thermal insulation [8], and reduced environmental footprint aligns directly with global decarbonization strategies and the principles of the circular economy. For this reason, the present study highlights cutting-edge research aimed at transforming material engineering toward greener and more efficient solutions [9,10].

Despite their increasing use in structural and semistructural applications, the mechanical characterization of porous eco-materials remains significantly underdeveloped [2,11]. In particular, their response to transverse shear (central to predicting deflections,

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ensuring serviceability, and assessing load-bearing capacity in beams, slabs, sandwich panels, and prefabricated insulation elements) continues to pose a methodological challenge [12,13]. Classical formulations of the shear correction factor k_s , derived for homogeneous and prismatic continua [14,15], assume uniform stiffness distributions and do not capture the intrinsic heterogeneity of cellular and porous microstructures. As a result, applying such simplified formulas to highly inhomogeneous materials may lead to substantial errors in predicted shear stiffness, inaccurate modeling of deformation mechanisms, and unreliable structural performance assessments [16,17].

Porous media exhibit a rich variety of microstructural features [18] (nonuniform pore sizes, density gradients, irregular void geometries, anisotropic skeleton arrangements, and local necking phenomena) that strongly influence the distribution of transverse shear stresses [19,20]. These characteristics inherently violate the assumptions underlying classical Timoshenko beam and Mindlin plate theories, where the shear correction factor has traditionally been expressed in closed form for isotropic solids with rectangular or circular cross-sections [13–15,21]. The deviation from homogeneity is not merely a minor perturbation; instead, it represents a fundamental departure from the continuum hypothesis, causing the actual shear strain energy to differ substantially from that predicted by simplified models.

In practical structural engineering, transverse shear effects are commonly addressed within the framework of classical beam and plate theories, as reflected in design standards such as Eurocode-based formulations and national codes, where shear stiffness is incorporated through correction factors. Experimental determination of shear-related parameters is typically performed indirectly via bending tests, short-span beam tests, or plate deflection measurements rather than through direct shear stress measurement.

In the context of green construction technologies, accurate mechanical modeling is of particular importance. Many porous eco-materials are used in applications where shear deformation governs the overall behavior: thermal insulation panels subjected to in-plane racking [22,23], lightweight fillers in composite floor systems, fire-resistant claddings [24–26], or hollow and partially foamed prefabricated elements operating under combined bending and shear [27]. Recent studies emphasize the role of perlite in maintaining structural integrity under high-temperature conditions, suggesting its suitability for fire-resistant bituminous and cementitious composites [3]. Underestimating or overestimating the transverse shear stiffness may result in either overly conservative designs (which undermine the ecological and economic benefits of these materials) or unsafe structural assessments, especially in systems involving bending-dominated response [28].

Existing research has addressed parts of this challenge but remains fragmented [29]. Some studies adopt homogenization strategies, deriving effective stiffnesses based on averaged porosity or empirical density–modulus correlations [30–33]. Others employ full microstructural finite element models [34] to capture local stress concentrations and strain energy distributions [28,35,36]. Parallel to these mechanical approaches, data-driven techniques using machine learning and deep learning have recently emerged to estimate flow-related parameters and optimize geoenvironmental material properties, though they rarely address transverse shear mechanics directly. However, current approaches often lack a direct link to the shear correction factor formulation required by Timoshenko-type theories [12]. Moreover, there is still no widely accepted framework that connects microstructural heterogeneity with an equivalent macroscale shear correction factor that can be systematically applied in engineering analyses [37,38].

To address these limitations, the present paper proposes a generalized analytical–numerical methodology for evaluating the shear correction factor k_s for a wide class of porous eco-materials. The approach extends the classical strain energy equivalence principle [39] by introducing a spatially varying, density-dependent stiffness field that reflects

the true mechanical heterogeneity of the medium. A voxel-based microstructural representation is employed to resolve local shear energy contributions, enabling a direct comparison between analytical predictions and numerical benchmarks. This dual-scale formulation not only enhances accuracy but also ensures that the method can be readily adapted to materials with arbitrary pore morphologies, including those produced through additive manufacturing or modern foaming technologies [40].

Perlite—a lightweight, naturally occurring material widely used in eco-insulation systems, lightweight mortars, and fire-resistant composites [1,41–43]—is selected as the representative case study [1,44,45]. Research indicates that while perlite significantly reduces density, its interaction with binders—such as polylactic acid or polypropylene—can be tuned to enhance specific mechanical performance. Moisture permeability, however, remains a critical factor in hygrothermal assessment. Its highly variable porosity and nonlinear relationship between density and elastic parameters make it an ideal material for evaluating the limitations of classical homogeneous formulas [46,47]. As will be demonstrated, standard expressions for k_s may produce errors exceeding 70% or more [48] when applied to such materials, while the proposed methodology achieves substantially improved agreement with numerical simulations. These findings have practical implications for the design of sustainable structural components, where reliable shear stiffness predictions are essential for performance optimization [35,49].

Overall, this work contributes to the ongoing transformation of the construction industry toward low-impact, high-efficiency material solutions. What distinguishes the present work from existing approaches is the consistent integration of an analytical strain energy formulation with a microstructure-resolved voxel model, enabling a direct quantification of how local density gradients and pore morphology influence the effective transverse shear stiffness. Unlike classical derivations, which rely on idealized assumptions of homogeneity [13,15,50], the proposed method explicitly accounts for spatial variations in stiffness and provides a shear correction factor that emerges naturally from the heterogeneous energy distribution. This dual-scale formulation allows for transparent identification of the mechanisms responsible for deviations from classical k_s values and offers a unified procedure that can be applied to materials with arbitrary pore architectures. Although expanded perlite is employed here as a representative benchmark material, the proposed framework is not material-specific and can be directly applied to other porous eco-materials, including foamed concretes, pumice-based composites, bio-derived cellular solids, and additively manufactured porous structures, provided that a voxelized microstructural representation and a local stiffness field are available.

In the concluding part of this study, we demonstrate how the method bridges analytical theory and numerical simulation, showing excellent consistency between predicted and computed shear strain energies, and we present a clear interpretation of how microstructural features drive the observed discrepancies. The resulting framework not only yields accurate k_s values but also provides practical guidelines for designers working with lightweight eco-materials, illustrating which microstructural characteristics most strongly govern shear behavior and how they can be optimized in advanced sustainable construction applications. The novelty of the present work lies in coupling strain energy equivalence with voxel-resolved stiffness fields, providing a direct and physically transparent link between microstructural heterogeneity and effective transverse shear stiffness.

2. Materials and Methods

This section develops a generalized analytical–numerical methodology for evaluating the shear correction factor k_s in heterogeneous porous eco-materials. The formulation begins with the classical strain–energy equivalence for homogeneous materials and then systematically extends it to cross-sections with spatially varying stiffness. A voxel-based

discretization permits direct use of microstructural data and accommodates arbitrary pore morphologies.

2.1. Classical Definition of the Shear Correction Factor

In Timoshenko beam theory, the effective transverse shear stiffness is expressed as

$$GA_{\text{eff}} = k_s GA, \quad (1)$$

where A is the cross-sectional area, G is the shear modulus, and k_s is the shear correction factor (SCF) [51–54]. The SCF is defined via the equivalence between the real shear strain energy of the cross-section and the simplified energy assumed in Timoshenko theory [55].

The real shear energy for a shear force V is

$$U_{\text{real}} = \int_A \frac{\tau(x, y)^2}{2G} dA, \quad (2)$$

while the Timoshenko model assumes

$$U_{\text{Timo}} = \frac{V^2}{2k_s GA}. \quad (3)$$

Introducing the normalized shear stress function

$$\tau(x, y) = V \varphi(x, y), \quad \int_A \varphi(x, y) dA = 1, \quad (4)$$

the shear correction factor for a homogeneous material becomes

$$k_s = \frac{1}{A \int_A \varphi(x, y)^2 dA}. \quad (5)$$

This expression applies only when the material is homogeneous and the modulus G is constant. For a homogeneous rectangular cross-section of width b and height h , the exact shear stress distribution under a shear force V is

$$\tau(y) = \frac{3V}{2bh} \left(1 - \frac{4y^2}{h^2} \right). \quad (6)$$

Substituting Equation (6) into Equation (2) and integrating over the height yields the exact real shear energy

$$U_{\text{real}} = \frac{3V^2}{5GA}. \quad (7)$$

Equating Equations (3) and (7) gives the classical shear correction factor for a homogeneous rectangular section:

$$k_s = \frac{5}{6}. \quad (8)$$

This value serves as the reference baseline for all heterogeneous cases.

2.2. Geometric Shear Stress Distribution

For an arbitrary prismatic cross-section subjected to a shear force V , the exact shear stress distribution obtained from sectional equilibrium is

$$\tau(y) = \frac{V Q(y)}{I b(y)}, \quad (9)$$

where: $b(y)$ is the material width at height y , $Q(y) = \int_y^{y_{\text{max}}} \bar{y} b(\bar{y}) d\bar{y}$ is the first moment of area, and $I = \int_A y^2 dA$ is the second moment of area.

The corresponding normalized form is

$$\varphi(y) = \frac{Q(y)}{I b(y)}. \quad (10)$$

Equation (10) is purely geometric and remains valid for both homogeneous and heterogeneous materials. Material heterogeneity enters the formulation only through the energy integral.

2.3. Generalized Shear Correction Factor for Heterogeneous Materials

Porous eco-materials exhibit spatial variation in stiffness. The shear modulus becomes a function of position:

$$G(x, y) = G(\rho(x, y)). \quad (11)$$

Substituting Equation (4) into Equation (2) yields the heterogeneous real shear energy:

$$U_{\text{real}} = \frac{V^2}{2} \int_A \frac{\varphi(x, y)^2}{G(x, y)} dA. \quad (12)$$

Using the Timoshenko energy (3), the heterogeneous shear correction factor becomes

$$k_s = \frac{1}{G_{\text{ref}} A \int_A \frac{\varphi(x, y)^2}{G(x, y)} dA}, \quad (13)$$

where G_{ref} is a reference modulus (e.g., matrix modulus, maximum local modulus, or the average modulus of the solid phase). Equation (13) reduces to the classical homogeneous expression (5) when $G(x, y) = G_{\text{ref}}$ everywhere.

Scale transition is handled through strain energy equivalence: the microstructure-resolved shear strain energy is condensed into a single effective shear correction factor k_s , which can be directly employed within Timoshenko beam or Mindlin plate theories at the structural scale, without explicitly resolving the microstructure in the global analysis.

2.4. Voxel-Based Representation of the Microstructure

Real porous materials cannot be described analytically; therefore, the cross-section is discretized into voxels with:

- material indicator $\chi_{ij} \in \{0, 1\}$;
- local modulus G_{ij} ;
- pixel area $\Delta A = \Delta x \Delta y$.

The voxel-based microstructural data employed in this study were generated from representative grayscale images reflecting typical porous perlite morphologies reported in the literature, which were subsequently processed to obtain spatial distributions of density and shear modulus for methodological evaluation.

The geometric properties become

$$A = \sum_{ij} \chi_{ij} \Delta A, \quad (14)$$

$$\bar{y} = \frac{\sum_{ij} \chi_{ij} y_j \Delta A}{A}, \quad (15)$$

$$I = \sum_{ij} \chi_{ij} (y_j - \bar{y})^2 \Delta A, \quad (16)$$

$$b_j = \left(\sum_i \chi_{ij} \right) \Delta x, \quad (17)$$

$$Q_j = \sum_{k \geq j} \left(\sum_i \chi_{ik} \right) (y_k - \bar{y}) \Delta A. \quad (18)$$

The geometric shear form becomes

$$\varphi_j = \frac{Q_j}{I b_j}. \quad (19)$$

The heterogeneous energy integral (13) is approximated by

$$S = \sum_{ij} \frac{\varphi_j^2}{G_{ij}} \Delta A. \quad (20)$$

Hence the final discrete form of the shear correction factor is

$$k_s = \frac{1}{G_{\text{ref}} A S}. \quad (21)$$

Equation (21) is the core formula used in all subsequent numerical analyses. Implementation details are given in Appendix A.

Voxel convergence was verified by repeating the calculations for increasing resolutions (from 27×27 to 270×270 pixels) while preserving the same porosity and pore morphology. The resulting shear correction factors differed by less than 2–3%, even for highly irregular pore distributions, confirming that the adopted voxel resolution is sufficient for stable evaluation of the shear strain energy.

2.5. Computational Procedure

The voxel-based algorithm proceeds as follows (see Figure 1).

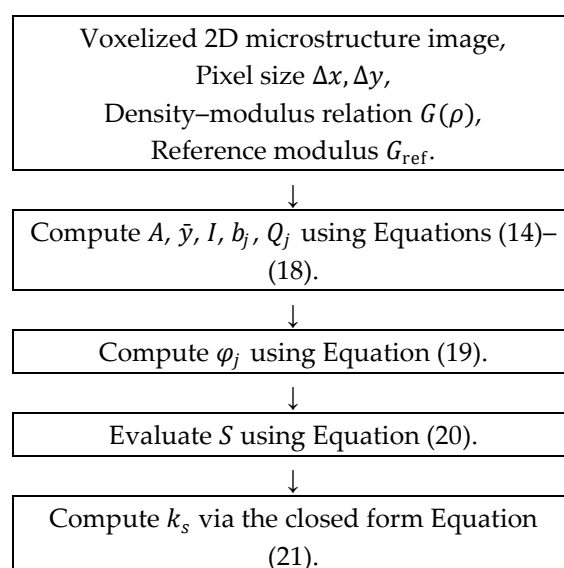


Figure 1. Flowchart of pixel-based method.

The computational procedure was implemented using in-house scripts written in MATLAB (MathWorks, version R2024a), enabling full control over voxel-based data processing and strain energy evaluation. The classical homogeneous value is expressed in

Equation (8). Comparing Equations (21) and (8) quantifies the influence of heterogeneity, porosity, and pore morphology.

3. Results

This section presents a quantitative assessment of the proposed analytical–numerical methodology for determining the shear correction factor k_s in porous eco-materials. The discussion focuses on: (i) the density-dependent stiffness distribution in a perlite-based composite; (ii) the resulting shear stress and shear energy fields obtained from the voxel-based analytical model; (iii) the deviation of the heterogeneous correction factor k_s from the classical homogeneous value $k_s = 5/6$; and (iv) the sensitivity of k_s to relative density and pore morphology. Perlite is used as the representative case study; however, the analysis is general and applicable to a broad class of porous materials.

3.1. Density-Dependent Stiffness Distribution

The first step of the evaluation involves generating a spatially varying stiffness field $G(x, y)$, derived from the experimentally calibrated density–modulus relationship. Figure 2 presents the resulting distribution for the representative perlite sample.

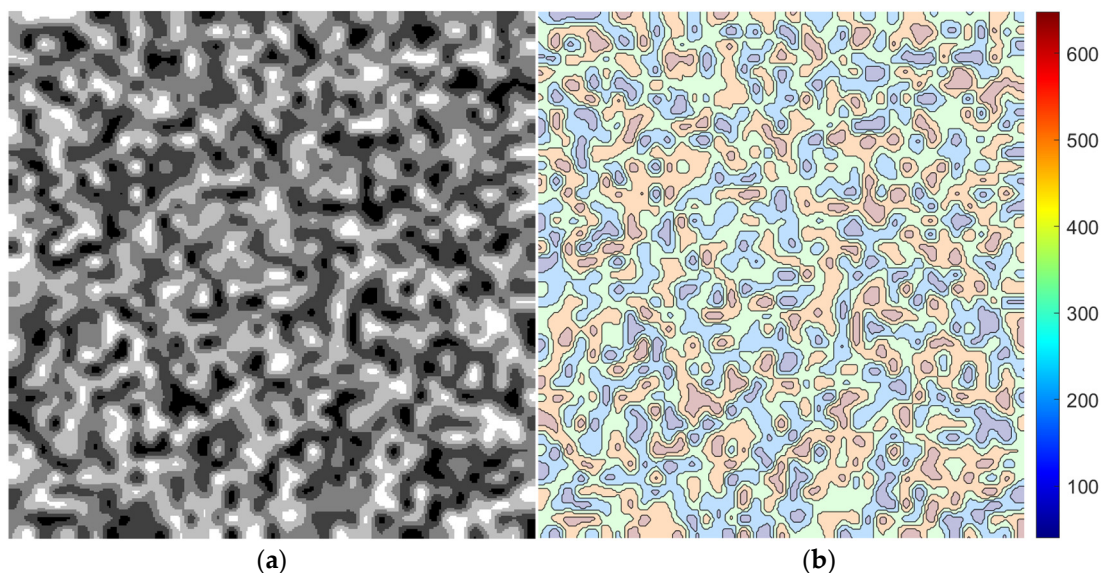


Figure 2. Spatial distribution of density and shear modulus $G(x, y)$ [MPa] in the perlite microstructure: (a) porosity distributions in perlite-based materials; (b) stiffness gradients.

The images shown in Figure 2 represent generated voxel-based microstructural patterns reflecting typical porosity distributions in perlite-based materials, derived from representative grayscale datasets reported in the literature and processed for numerical evaluation rather than acquired directly using experimental imaging devices. As this study focuses on methodological development, image-based microstructures are used as representative patterns rather than direct experimental reconstructions, which limits the method to effective stiffness evaluation rather than detailed failure or damage analysis. The plot highlights strong stiffness gradients, particularly between solid grains and highly porous regions. These variations directly influence the local shear energy and show that assuming constant stiffness for such materials is physically unjustified.

3.2. Shear Stress and Shear Energy Distribution from the Voxel-Based Model

Using the geometric procedure described in Section 2, the normalized shear stress form φ_j is computed row by row from Equation (5), and the heterogeneous energy density is obtained from Equation (6). Figure 3 illustrates the resulting shear stress profile in the cross-section for a unit shear force $V = 1$.

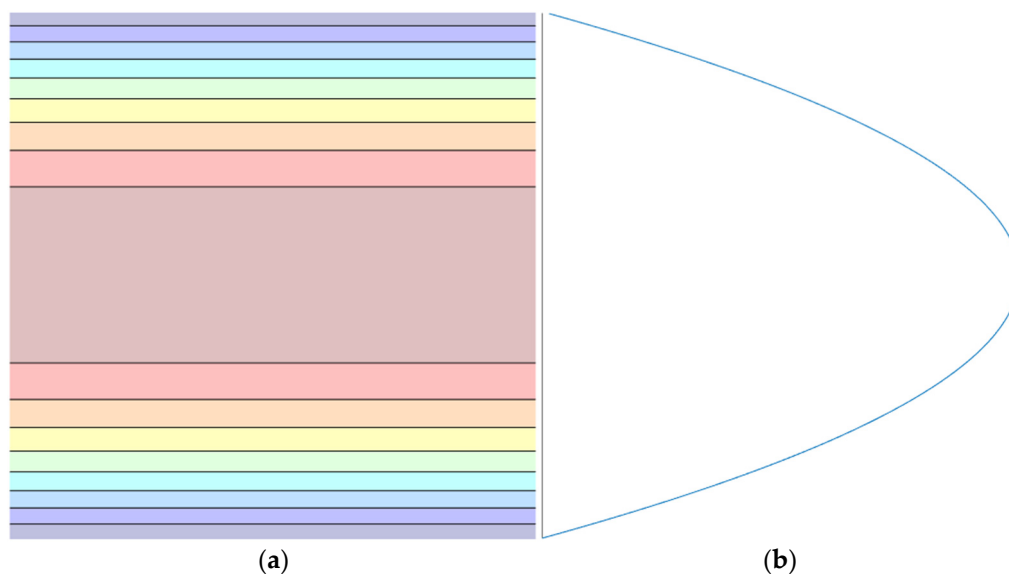


Figure 3. Geometric shear stress distribution $\tau(x, y) = \varphi(y)$ obtained from the voxel-based analytical model for a unit shear force: (a) contour of normalized shear stress; (b) plot of normalized shear stress.

To evaluate how heterogeneity affects energy storage, the local contribution to the shear energy is proportional to φ_j^2/G_{ij} . Figure 4 shows the corresponding shear energy density

$$u_{xy}(x, y) \propto \frac{\varphi(y)^2}{G(x, y)}. \quad (22)$$

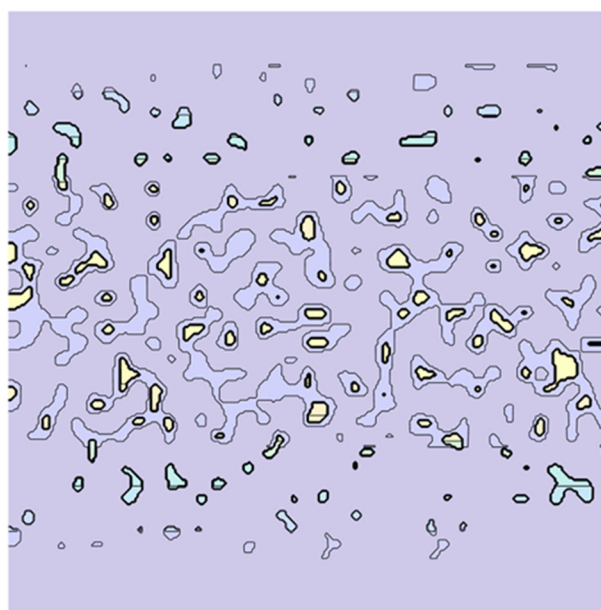


Figure 4. Normalized transverse shear energy density in the voxelized microstructure.

These results demonstrate that the global shear response is dominated by a subset of the microstructure exhibiting either low stiffness or geometric concentration of shear, confirming the need for an energy-based correction factor.

3.3. Generalized Shear Correction Factor vs. Classical Homogeneous Value

For each microstructure considered, the heterogeneous shear correction factor k_s is computed from Equation (6). The classical homogeneous value $k_s^{\text{ref}} = 5/6$ obtained in Equation (3) serves as a baseline corresponding to a beam with the same geometry but uniform modulus equal to G_{ref} . Tables 1–4 summarize the results for several density levels and corresponding microstructures (e.g., obtained by varying overall porosity while keeping geometry fixed, see Figure 5).



Figure 5. Patterns: (a) pattern1 (27×27 [pixels])—3 levels; (b) pattern1 (27×27 [pixels])—5 levels; (c) pattern2 (27×27 [pixels])—3 levels; (d) pattern2 (27×27 [pixels])—5 levels; (e) pattern3 (270×270 [pixels])—3 levels; and (f) pattern3 (270×270 [pixels])—5 levels.

Table 1. Shear correction factor of heterogeneous perlite sections compared with the classical homogeneous value. Pattern 1–3 levels.

ρ/ρ_0 [-]	k_s [-]	k_s/k_s^{ref} [-]
0.5152	0.2481	0.2978
0.5407	0.4044	0.4853
0.5663	0.5104	0.6125
0.5918	0.5860	0.7032
0.6173	0.6419	0.7703
0.6428	0.6842	0.8211
0.6683	0.7169	0.8603
0.6938	0.7425	0.8911
0.7193	0.7628	0.9153
0.7449	0.7788	0.9346
0.7959	0.8018	0.9622
0.8469	0.8162	0.9795
0.8979	0.8248	0.9898
0.9490	0.8294	0.9953
1.0000	0.8311	0.9973

Table 2. Shear correction factor of heterogeneous perlite sections compared with the classical homogeneous value. Pattern 1–5 levels.

ρ/ρ_0 [-]	k_s [-]	k_s/k_s^{ref} [-]
0.5172	0.3810	0.4572
0.5426	0.5311	0.6373
0.5680	0.6155	0.7386
0.5934	0.6707	0.8048
0.6188	0.7097	0.8516
0.6442	0.7386	0.8863
0.6697	0.7606	0.9127
0.6951	0.7777	0.9332
0.7205	0.7910	0.9492
0.7459	0.8014	0.9617
0.7967	0.8160	0.9792
0.8475	0.8247	0.9897
0.8984	0.8294	0.9952
0.9492	0.8312	0.9975
1.0000	0.8311	0.9973

Table 3. Shear correction factor of heterogeneous perlite sections compared with the classical homogeneous value. Pattern 3–3 levels.

ρ/ρ_0 [-]	k_s [-]	k_s/k_s^{ref} [-]
0.5247	0.3418	0.4101
0.5497	0.5069	0.6082
0.5747	0.6029	0.7235
0.5998	0.6649	0.7979
0.6248	0.7077	0.8492
0.6498	0.7385	0.8862
0.6748	0.7613	0.9136
0.6998	0.7787	0.9344
0.7248	0.7920	0.9504
0.7498	0.8024	0.9629
0.7999	0.8168	0.9802
0.8499	0.8255	0.9906

0.8999	0.8304	0.9965
0.9500	0.8328	0.9994
1.0000	0.8333	1.0000

Table 4. Shear correction factor of heterogeneous perlite sections compared with the classical homogeneous value. Pattern 3–5 levels.

ρ/ρ_0 [-]	k_s [-]	k_s/k_s^{ref} [-]
0.5258	0.5165	0.6198
0.5508	0.6311	0.7574
0.5757	0.6883	0.8259
0.6007	0.7244	0.8692
0.6256	0.7498	0.8997
0.6506	0.7686	0.9224
0.6755	0.7831	0.9398
0.7005	0.7945	0.9534
0.7255	0.8035	0.9642
0.7504	0.8106	0.9727
0.8003	0.8209	0.9850
0.8503	0.8272	0.9927
0.9002	0.8310	0.9972
0.9501	0.8328	0.9994
1.0000	0.8333	1.0000

3.4. Effect of Material Heterogeneity and Relative Density on k_s

To evaluate the robustness of the method, additional microstructures are generated for multiple density levels and pore architectures. Figure 6 summarizes the relationship between relative density ρ/ρ_s and the resulting shear correction factor.

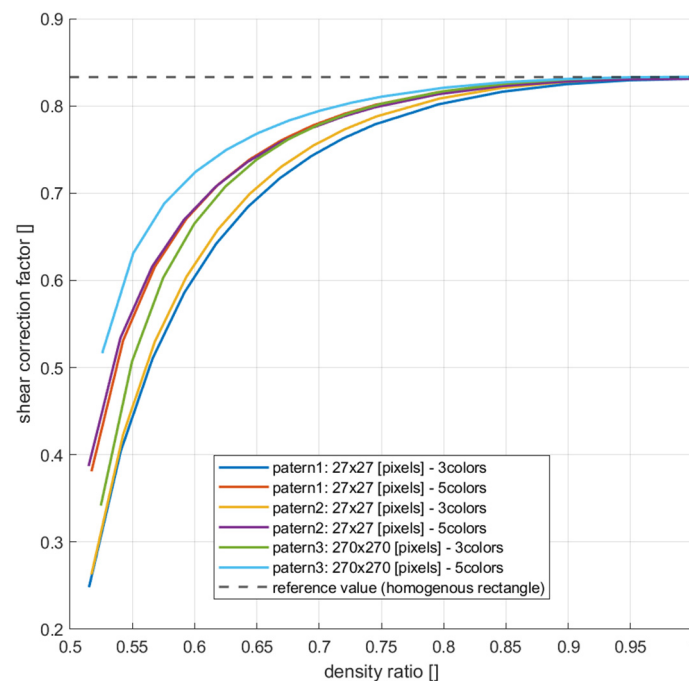


Figure 6. Influence of relative density ρ/ρ_s on the heterogeneous shear correction factor k_s ; the horizontal line marks the classical homogeneous reference $k_s = 5/6$.

The results typically reveal a monotonic approach of k_s toward $5/6$ as the material becomes denser. For highly porous microstructures, k_s significantly exceeds $5/6$,

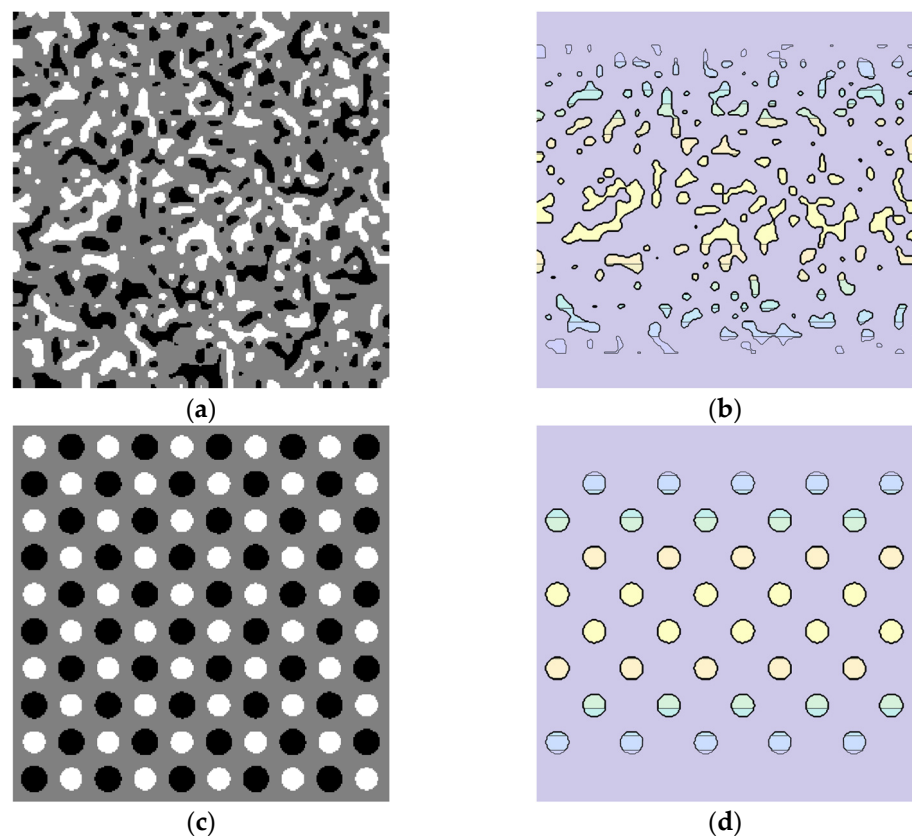
reflecting strong shear stress nonuniformity and energy localization. As porosity decreases, the microstructure becomes more homogeneous, and the correction factor gradually converges to the classical value.

It should be emphasized that the proposed formulation separates geometric effects from material heterogeneity. The geometric shear stress distribution $\varphi(y)$ depends exclusively on the cross-sectional shape, while material heterogeneity enters the formulation solely through the local shear modulus $G(x, y) = G(\rho(x, y))$. Consequently, the influence of the assumed density–modulus relationship is quantitative rather than qualitative. Different admissible functional forms of $G(\rho)$, consistent with experimental observations for porous materials, lead to moderate variations in the numerical value of the shear correction factor k_s but do not alter the monotonic convergence toward the classical value $5/6$ with increasing relative density. This indicates that porosity distribution and stiffness contrast dominate the transverse shear response, whereas the exact analytical form of $G(\rho)$ plays a secondary role.

The results are obtained through systematic parametric variation in porosity and microstructural patterns rather than statistical analysis of multiple physical samples; therefore, conventional statistical error measures are not applicable.

3.5. Sensitivity to Pore Morphology

To isolate the role of geometry, three idealized pore patterns are evaluated at comparable relative densities: (a) random irregular pores (representative of real perlite), (b) periodic circular voids, and (c) and (d) periodic/shifted circular voids representing foaming. Figure 7 compares the shear energy distributions among these morphologies, and Table 5 collects the corresponding values of k_s .



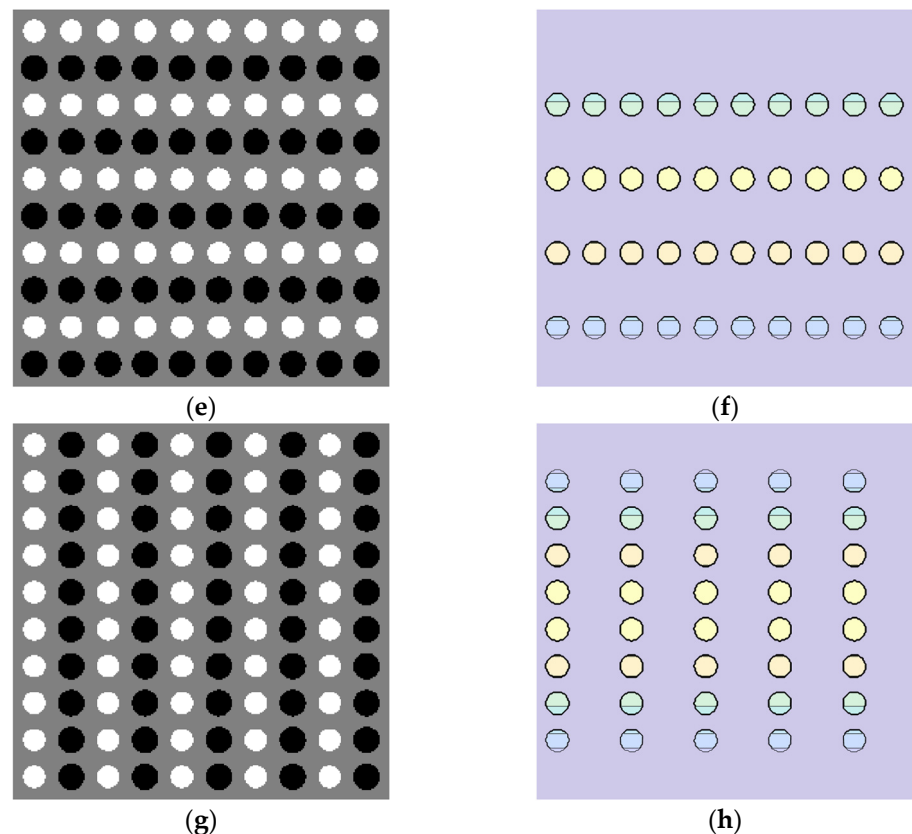


Figure 7. Effect of pore morphology (random, circular-shifted, circular-rows, and circular-columns) on shear energy localization in perlite-like microstructures. (a,b) random irregular pore distribution; (c,d) shifted circular pores; (e,f) circular pores arranged in rows; and (g,h) circular pores arranged in columns.

The reference numbers of pixels shown in Figure 7a,b are 10,501 for white and 14,509 for black. In all three regular patterns, this corresponds to 50 white circles with a radius of 8.18 px and 50 black circles with a radius of 9.61 px.

Table 5. Comparison of shear correction factors from different patterns.

Morphology Type	Relative Density ρ/ρ_s	Standard Deviation of G_{ij}	k_s (Heterogeneous)	Deviation from 5/6 (%)
Perlite	0.5505	2.1918×10^8	0.35326	57.61
Shifted	0.5505	2.1934×10^8	0.34971	58.03
Rows	0.5505	2.1934×10^8	0.34971	58.03
Columns	0.5505	2.1934×10^8	0.34971	58.03

All patterns lead to the strongest energy localization and therefore the lowest k_s , while periodic circular morphologies exhibit the most uniform shear transfer, yielding higher correction factors. This clearly demonstrates that mainly porosity, not pore shape, is a key driver of transverse shear response in porous eco-materials.

4. Discussion

Unlike classical shear correction formulations derived for homogeneous continua, the present approach integrates spatially varying stiffness directly into an energy-based framework, enabling explicit quantification of microstructural heterogeneity effects that are not accessible through standard homogenization techniques.

The results presented in Section 3 offer a comprehensive perspective on the transverse shear behavior of porous eco-materials and highlight the limitations of classical models when applied to heterogeneous microstructures. A central observation is that the heterogeneous shear correction factor k_s , obtained from Equation (6) systematically deviates from the classical homogeneous value $k_s^{\text{ref}} = 5/6$, with the discrepancy increasing as porosity and stiffness contrast grow. This is a direct manifestation of the fact that shear forces are transmitted preferentially through a sparse solid skeleton, while surrounding voids and weak zones contribute little to shear resistance.

The voxel-based energy maps reveal a pronounced localization of shear strain energy in narrow load paths and in regions of reduced stiffness. Only a subset of the microstructure contributes significantly to the global shear response, which explains why simplified homogeneous models misrepresent the effective stiffness. In particular, for low-density perlite microstructures, the present method predicts a shear correction factor substantially lower than $5/6$, indicating that a naive application of the classical formula would overestimate the effective shear stiffness and underestimate deflections.

The sensitivity study with respect to pore morphology further clarifies the underlying mechanisms. Elongated voids and anisotropic pore clusters (typical of gravity-driven foaming or directional expansion) induce strong stiffness gradients and break the continuity of the solid skeleton, leading to elevated k_s . In contrast, more isotropic, circular pores provide smoother stress paths and lower correction factors at the same relative density. This implies that density alone is insufficient to characterize shear behavior; microstructural shape descriptors are equally important. The proposed methodology naturally captures these effects because both density and morphology enter the formulation through the spatial field G_{ij} and the geometry of the voxelized cross-section.

For microstructures exhibiting pronounced anisotropy or preferred pore alignment, the transverse shear response may become direction-dependent. In the present work, a single scalar correction factor is evaluated for a prescribed loading direction; however, the same energy-based procedure may be applied independently for different shear directions, enabling the definition of directional shear correction factors if required.

An important outcome of this study is the emergence of a smooth and physically interpretable relationship between relative density and k_s . As the material transitions from highly porous to nearly dense, the predicted correction factor gradually approaches $5/6$, recovering the classical Timoshenko result in the dense limit. This provides a continuous bridge between the microstructure-resolved description and the homogeneous beam idealization and suggests that the proposed framework can serve as a basis for simplified design formulas linking k_s to measurable morphological indicators.

Finally, although the current work relies exclusively on analytical energy arguments and voxel-based quadrature, the methodology remains fully compatible with more detailed numerical models, such as finite element simulations, should they be required in future studies. In its present form, however, the approach already delivers quantitatively meaningful correction factors and offers diagnostic insight into the role of microstructural heterogeneity. This combination of physical transparency and computational simplicity makes it attractive for engineering applications involving perlite-based composites and other porous eco-materials used in sustainable construction.

The present study does not include direct experimental measurements of transverse shear stiffness. The primary objective is to establish a physically consistent analytical-numerical framework for evaluating the shear correction factor in heterogeneous porous media. Experimental uncertainty associated with density measurements or modulus identification may affect the quantitative value of k_s ; however, such uncertainty can be naturally propagated through the proposed formulation via the local stiffness field $G(x, y)$. Importantly, the framework is fully compatible with experimental input data and can be

readily calibrated when suitable measurements become available. Experimental verification of the proposed framework is beyond the scope of the present study and is identified as a natural direction for future work.

5. Conclusions

This study proposes a generalized analytical–numerical framework for evaluating the transverse shear correction factor k_s in porous eco-materials characterized by pronounced microstructural heterogeneity. The formulation is based on the strain energy equivalence principle extended to a spatially varying shear modulus field and implemented using a voxel-based representation of the material microstructure.

The results demonstrate that classical homogeneous shear correction factors, such as the commonly adopted value $k_s = 5/6$ for rectangular sections, may significantly misrepresent the effective transverse shear stiffness of highly porous materials. For low-density microstructures, the proposed method predicts substantially reduced shear stiffness, highlighting the risk of unconservative deformation estimates when homogeneous assumptions are applied in structural analysis.

A key outcome of this study is the identification of porosity and stiffness contrast as the dominant factors governing the transverse shear response, whereas pore morphology plays a secondary role at comparable relative densities. The computed shear correction factors exhibit a smooth and physically consistent convergence toward the classical value as the material becomes denser, providing a continuous link between microstructure-resolved descriptions and classical beam and plate theories.

From a practical design perspective, the proposed framework offers a straightforward pathway for incorporating microstructural effects into structural-scale analyses. Once evaluated, the shear correction factor can be directly introduced into standard beam or plate finite elements as a modified shear stiffness parameter, without the need for repeated microstructural simulations. This enables efficient use of the method in routine engineering design involving porous eco-materials.

Overall, the presented methodology enables more accurate and physically transparent modeling of transverse shear behavior in heterogeneous porous materials and provides a robust basis for extending classical structural theories toward sustainable and lightweight material systems.

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Appendix A

This appendix provides a clear computational recipe for reproducing the heterogeneous shear correction factor k_s defined in Equation (5), (13) and (21), without resorting to finite element simulations.

Appendix A.1. Input

- 2D image IMG (grayscale, CT, or optical micrograph).
- Pixel size: dx , dy .
- Mapping from intensity to modulus: $G_{ij} = G(\rho_{ij})$.
- Reference modulus: G_{ref} (G_{ref}).
- Material indicator χ_{ij} (χ_{ij} , 1 = material, 0 = void).

Appendix A.2. Compute Geometry

```

A = 0
num = 0
for each row j:
    y_j = y coordinate of row j
    for each column i:
        if chi_ij == 1:
            A += dx*dy
            num += y_j*dx*dy
y_bar = num/A
I = 0
for each row j:
    y_j = y coordinate of row j
    for each column i:
        if chi_ij == 1:
            I += (y_j - y_bar)^2*dx*dy

```

Appendix A.3. Compute Row Width and First Moment

```

for each row j:
    b_j = 0
    for each column i:
        if chi_ij == 1:
            b_j += dx
for each row j:
    Q_j = 0
    for each row k >= j:
        y_k = y coordinate of row k
        row_width = 0
        for each column i:
            if chi_ik == 1:
                row_width += dx
        Q_j += row_width*(y_k - y_bar)*dx*dy

```

Appendix A.4. Compute Shear Form Function φ_j

```

for each row j:
    if b_j > 0:
        phi_j = Q_j/(I*b_j)
    else:
        phi_j = 0

```

Appendix A.5. Compute Heterogeneous Shear Energy Term

```

sum_phi2_over_G = 0
for each row j:
    for each column i:
        if chi_ij == 1 and G_ij > 0:
            sum_phi2_over_G += phi_j^2*(1.0/G_ij)*dx*dy

```

Appendix A.6. Compute Shear Correction Factor

Using Equation (6),

```

k_s = 1.0/(G_ref*A*sum_phi2_over_G)

```

Appendix A.7. Comparison with Homogeneous Reference

The following is used to evaluate the influence of heterogeneity:

```

k_ref = 5.0/6.0 # classical homogeneous value for a rectangular section
error = (k_s - k_ref)/k_ref*100.0 # relative deviation in percent

```

Values of k_s and error for different density levels or pore morphologies populate Tables 1–4 in the Results Section.

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