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A Simplified Theoretical Model for Progressive Collapse Resistance of Steel Girders: Focusing on Load–Displacement Behavior Under Three Concentrated Loads

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Abstract

Progressive collapse is characterized by disproportionate structural failure triggered by localized damage, such as column loss under extreme loading conditions. The objective of this study is to develop a simplified analytical model that is applicable in engineering practice without the need for high-fidelity nonlinear finite element analysis. Although current design guidelines (GSA and DoD) provide analytical procedures and acceptance criteria, they do not explicitly address the tensile resistance of girders after the acceptance criteria are satisfied, particularly under large deformation and connection failure. To address this limitation, this study proposes a simplified theoretical load–displacement model for a fixed-end girder subjected to three concentrated loads, considering the effects of secondary beams and focusing on the local girder response under a column-removal scenario. The proposed model incorporates moment–axial force interactions at plastic sections in the large-deformation range. Based on one-dimensional finite element analysis results, an early-developed axial force of $0.15F_p$ at the onset of the transition stage and a residual bending moment of $0.3M_p$ during the catenary action stage are explicitly introduced to better represent actual structural behavior. The girder response is idealized using five characteristic points: yielding (Y), full plasticity (P), transition initiation (T), pure catenary action initiation (C), and collapse governed by connection failure (F_{conn}). Stress distributions at plastic sections are analyzed using three-dimensional finite element models to establish stress-based formulations and a rational procedure for estimating axial force at collapse. The validity of the proposed model is verified through comparisons with finite element analysis results for girders with different span-to-depth ratios. The results demonstrate reasonable agreement in terms of collapse load and displacement, particularly for slender girders, confirming the applicability of the proposed model for progressive collapse assessment.

Keywords: progressive collapse; catenary action; moment–axial force interaction; simplified theoretical girder model; column–tree connection failure

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1. Introduction

Progressive collapse refers to a phenomenon in which local structural damage, initiated by abnormal loads such as impact events, gas explosions, or terrorist attacks, propagates to adjacent structural members, leading to the partial or total collapse of a structure. Because the extent of the final collapse is significantly greater than the initial local damage, progressive collapse is generally characterized by disproportionality. To address this issue, the General Services Administration (GSA) and the Department of Defense (DoD) of the United States have published the GSA 2016 [1] and DoD 2024 [2] guidelines, respectively, which are aimed at the progressive collapse-resistant design and analysis of federal civilian facilities and military or defense-related buildings. These guidelines emphasize graded risk management, quantitative analysis, and multiple protective strategies, and both propose column removal scenarios at locations considered to have a high risk of progressive collapse. Furthermore, acceptance criteria corresponding to prescribed loading conditions are provided based on linear and nonlinear static and dynamic analysis methods, as well as the detailing of girder–column connections. However, the post-acceptance load tensile resistance associated with large deformations of flexural members under extreme loading conditions is not explicitly considered in these guidelines.

Based on GSA 2016 [1] and DoD 2024 [2], numerous researchers (Sadek et al. [3], Main and Sadek [4], Lee et al. [5], Na et al. [6], and Wang et al. [7]) have developed and evaluated computational progressive collapse analysis models using nonlinear static and dynamic analyses. While these numerical approaches can provide detailed insight into structural behavior, they require a high level of expertise and computational experience, which limits their practical applicability in routine structural design. Therefore, there is a clear need for simplified analytical models capable of evaluating the progressive collapse resistance of structures.

Previous studies on theoretical girder models indicate that Izzuddin [8], Izzuddin et al. [9], Stylianidis et al. [10], and Harry et al. [11] developed simplified analytical models for simply supported girders subjected to uniformly distributed loads by considering the interaction between bending moment and axial force at plastic sections during the development of catenary action. In these studies, a single plastic hinge formed at the midspan of the girder was assumed. Lee and Kim [12] and Lee et al. [13], based on ABAQUS [14] numerical analysis results, derived relationships between beam behavior adjacent to the removed column and the span-to-depth ratio, and proposed an analytical model for a simply supported beam subjected to a concentrated load at midspan, including the stiffness associated with the pure catenary action stage. Li et al. [15] and Chen et al. [16] developed a theoretical model for fixed-end girders under uniformly distributed loads by defining girder behavior in five distinct stages based on strength variations at plastic sections. In addition, Deputy et al. [17] and Guo et al. [18] proposed analytical models for fixed-end beams subjected to a single concentrated load, incorporating the detailed interaction behavior between bending moment and axial force at plastic sections after the onset of catenary action. In these models, three plastic hinges forming at the beam ends and midspan were considered. However, none of the aforementioned studies explicitly defined the structural collapse point within their analytical frameworks.

While research on numerical modeling for progressive collapse assessment has been active, its practical application to large-scale buildings remains limited due to the requirement for high-level expertise, time-consuming modeling, and the generation of massive datasets. In particular, dynamic finite element analysis considering both material and geometric nonlinearities entails prohibitive computational costs and execution times, making it challenging to implement in routine structural design practice. This study aims to develop a simplified theoretical girder model for evaluating the progressive collapse resistance of structures through relatively straightforward computational procedures. A

fixed-end girder subjected to three concentrated loads, including the effects of secondary beams, is considered, and the stress distribution at plastic sections is analyzed at each loading stage to establish a corresponding stress-based analytical model. Furthermore, the loads and displacements acting on the girder following the failure of Column–Tree connections are quantified, and a complete load–displacement relationship from the girder yielding point to the connection failure point is proposed. By incorporating the early development of axial force and the residual moment capacity of plastic sections—obtained from numerical analysis results—into the analytical model, this study seeks to improve the reliability of the proposed model compared to existing simplified moment–axial force interaction approaches.

2. Overview of the Theoretical Girder Model

Figure 1 illustrates the load–displacement relationship of a girder subjected to typical vertical loading conditions [9,11,13,15]. The response is divided into five distinct stages: the elastic stage (Stage 1), plastic stage (Stage 2), transition stage (Stage 3), catenary transition stage (Stage 4), and pure catenary action stage (Stage 5). In the elastic and plastic stages of a fixed-end girder, the applied load is primarily resisted by bending moments. After the first yielding occurs at the yield point (Point Y), plastic hinges form at both ends and at the midspan at the plastic point (Point P), at which the girder reaches its maximum flexural resistance.

As the load continues to increase and reaches the onset of the transition stage (Point T), the bending moments at the girder ends and midspan attain their full plastic moment capacity and can no longer increase. Consequently, axial force begins to increase rapidly to resist the additional applied load. During the catenary transition stage, both the midspan and end sections remain in a plastic state while the axial force continues to develop. At the onset of pure catenary action (Point C), plastic axial force develops in the girder, resulting in a pronounced catenary mechanism. In the pure catenary action stage, the structural response is governed predominantly by axial force rather than bending.

At the girder tensile failure point (Point F), the maximum stress corresponding to the tensile strength of steel is reached at the plastic section, leading to tensile rupture of the girder. However, considering that connection failure may occur at a lower load level prior to girder tensile rupture, a connection failure point (Point F_{conn}) is defined. This point is adopted as the collapse point of the proposed theoretical girder model.

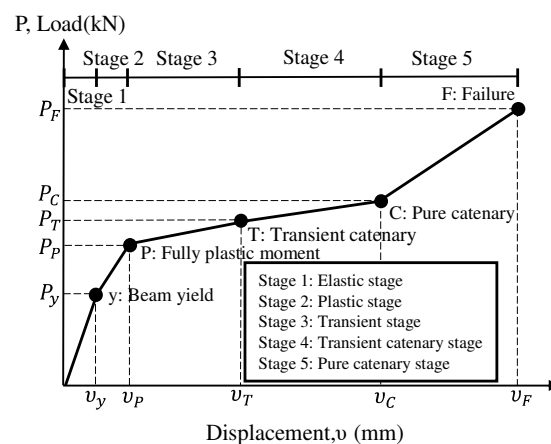


Figure 1. Load–displacement relationship of the theoretical model of the girder.

The target structure considered in this study is a steel intermediate moment-resisting frame with Column–Tree connections, as investigated in the progressive collapse study by Li [19] (Figure 2a,b). The scope of the theoretical model is defined as the lowest-story model

(2bayx1bayx1 story) (Figure 2d) extracted from a simplified sub-structural model (Figure 2c), which includes all stories (2bayx1bay) adjacent to the column removal location in the short-span direction of a multi-story moment-resisting frame (Figure 2a). In addition, the gravity loads from the composite slab steel deck, transferred through secondary beams with the pinned connection in the short-span direction, are represented by a fixed-end girder subjected to three concentrated loads.

It should be noted that this fixed-end girder model has inherent limitations in fully replicating the complex mechanical behavior of real-world structures. As it assumes idealized fixed boundary conditions and uniform span lengths, it may not explicitly account for the flexibility of beam–column joints, including joint translation and inward rotation, potential torsional effects arising from unequal spans, or the stiffness contribution of surrounding members. However, this study intentionally adopts this simplified approach to establish a fundamental load–displacement relationship of a girder under a column-removal scenario, represented by three equivalent concentrated loads transferred from secondary beams. (Figure 2b). Previous studies (Na et al. [6], Li [19], and Wu [20]) have shown that, under similar girder-level sub-assembly conditions, the differences in peak load and global load–displacement response remain within a limited range.

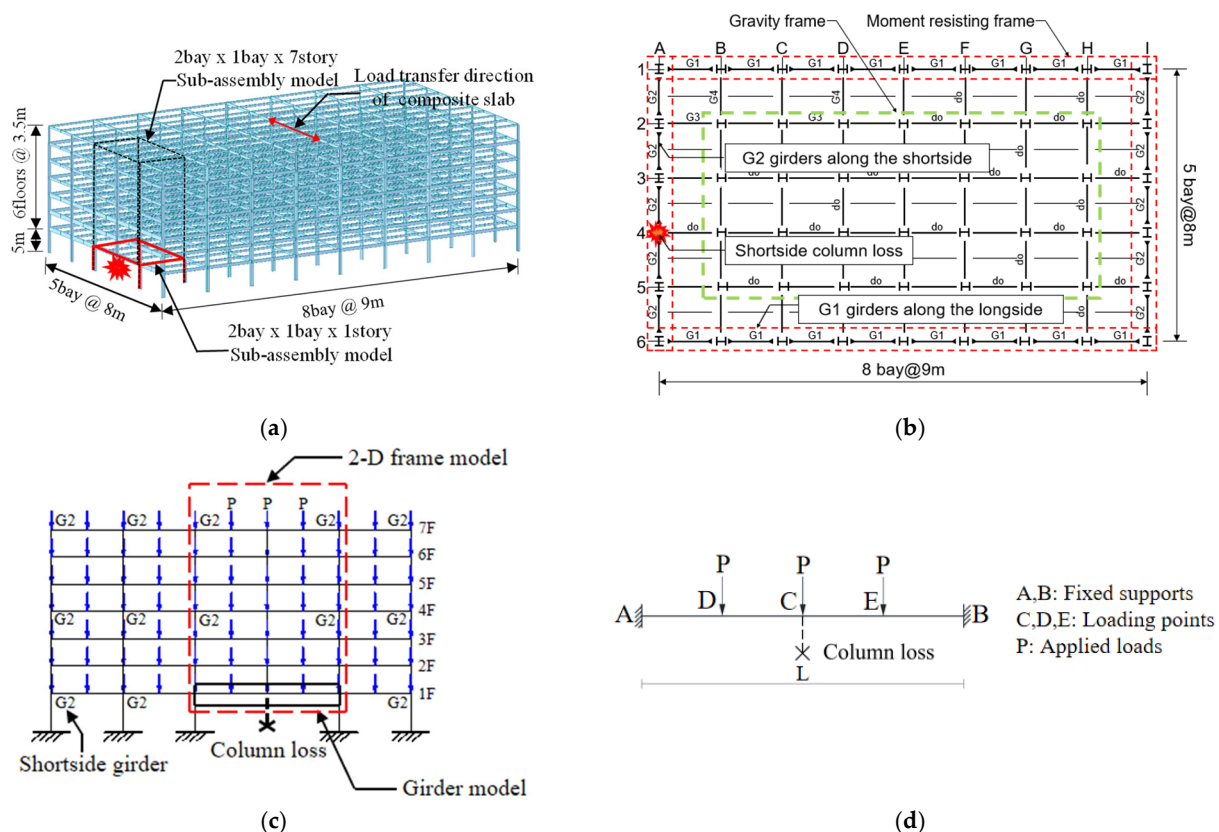


Figure 2. Scope of the simplified model (Li [19]). (a) Entire frame; (b) Structural plan; (c) Sub-assembly model of the shortside column removal scenario; (d) Analysis system for a fixed beam under three concentrated loads.

In this study, the relationship between bending moment and axial force at plastic hinges of a fixed-end girder subjected to three concentrated loads was investigated as a function of the span-to-depth ratio (L/D). Finite element analyses were performed using the commercial program ABAQUS [14], in which the girder was modeled as a fixed-end member using two-node one-dimensional beam elements (B31). The material properties of the girder were defined consistently with the numerical model presented by Li [19], adopting SS400 structural steel and a bilinear constitutive material model. Based on the

analysis results, the relationships among the normalized bending moment at fixed supports (M/M_{p_End}) and mid position (M/M_{p_Mid}), normalized axial force (F/F_{p_End} , F/F_{p_Mid}), and vertical displacement were derived, as shown in Figure 3.

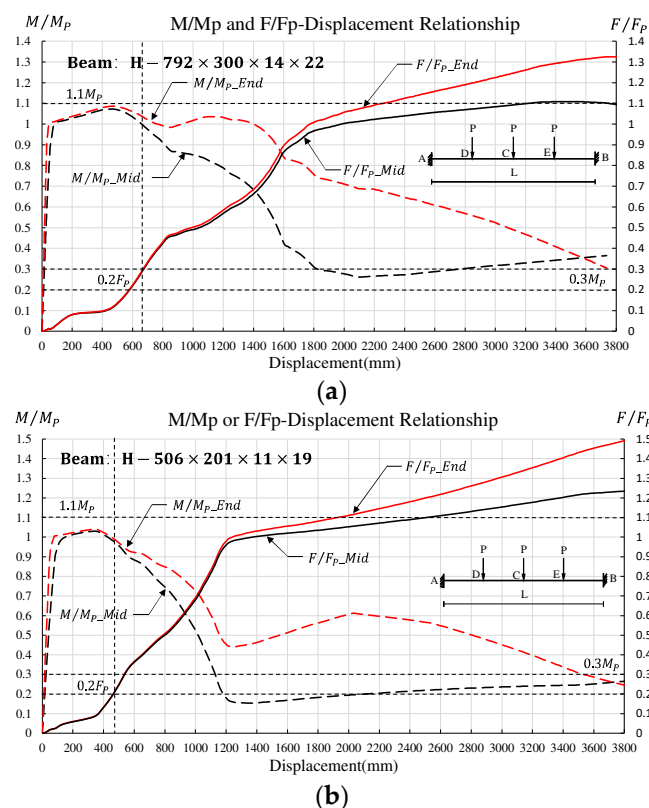
For girders with an H-shaped cross-section, the plastic moment capacity M_p and the plastic axial force capacity F_p can be calculated using Equations (1) and (2), respectively.

$$M_p = \sigma_y Z \quad (1)$$

$$F_p = \sigma_y A \quad (2)$$

Here, σ_y denotes the yield strength of the material, and Z and A represent the plastic section modulus and the cross-sectional area of the H-shaped girder, respectively.

Figure 3 presents the analysis results of the L/D of 20, 30 and 40 with respect to the different span lengths of 16 m to 18 m. When compared with the simplified model represented by the black line in Figure 4—which has been adopted in previous studies by Iz-zuddin [8], Lee and Kim [12], Stylianidis [10], and Deputy et al. [17]—it can be observed that axial force develops from an early stage of loading, and that the bending moment does not vanish even during the pure catenary action stage. The early development of axial force is directly related to the slope of the load–displacement curve in the transition stage (Stage 3) shown in Figure 1, while the residual moment in the pure catenary action stage significantly influences the determination of the girder collapse point.



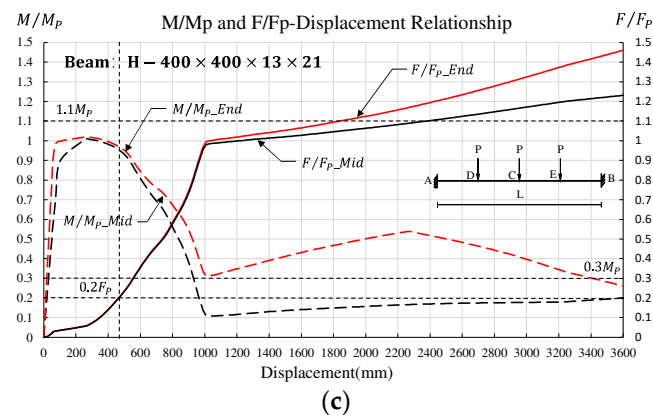


Figure 3. Relationship between M/M_p and F/F_p depending on span-depth ratio (L/D) of the girders. (a) $L/D = 20$; (b) $L/D = 30$; (c) $L/D = 40$.

Based on the numerical analysis results, the maximum normalized moment M/M_p of the three models with respect to the L/D ratio ranged from 1.0 to 1.1. At the end of the analysis, the residual moment ratios M/M_p at both girder ends and at midspan were found to be in the range of 0.2–0.38. Lee et al. [13] considered material strain hardening and adopted the plastic hinge moment as $1.1M_p$. In addition, AISC 360 [21] presents an approximate average interaction curve for the moment–axial force relationship of H-shaped sections, indicating that the flexural moment begins to decrease when the axial force exceeds 15% of the plastic axial capacity ($F \geq 0.15F_p$).

According to the above numerical results and the findings of previous studies, the axial force at the end of the transitional stage was assumed to be 15% of the plastic axial force ($F \geq 0.15F_p$), and the residual moment in the pure catenary action stage was taken as 30% of the plastic moment ($0.3M_p$), as illustrated by the red line in Figure 4. In this study, an improved load–displacement relationship for a fixed–fixed girder subjected to three concentrated loads was formally established in Figure 4 by incorporating the effects of early-developing axial force and residual moment.

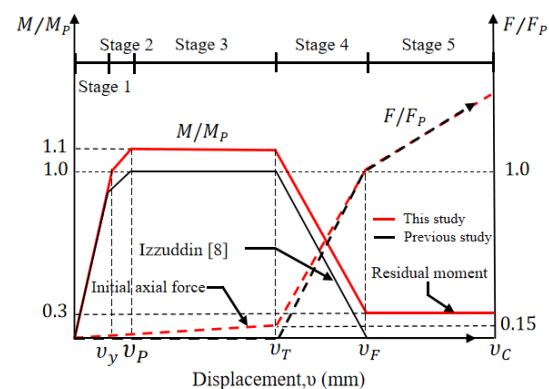


Figure 4. Interaction relationship between moment and axial force [8].

3. Development of Stress Models for Plastic Sections

To identify the stress distribution in the plastic section of the girder and the stress state at final failure corresponding to each characteristic point on the load–displacement curve shown in Figure 1, a numerical analysis was conducted using a fixed–fixed girder model discretized with three-dimensional solid elements. Based on the numerical results, stress models of the plastic section corresponding to each characteristic point were derived. Ultimately, the theoretical failure point of the girder is determined by analyzing the stress distribution at the onset of tensile failure.

3.1. Three-Dimensional Solid Element Modeling of the Girder

The simplified model representing the short-span column removal scenario of the entire structural frame (Figure 2d) consists of an 8 m span girder (H-792 × 300 × 14 × 22) connected to columns with a height of 1 m (H-498 × 432 × 45 × 70). The girder–column connections were modeled as rigid connections, and fixed-end boundary conditions were applied at both ends of the girder. Displacement control was imposed at the top section of the column, and the analysis was performed until collapse.

As shown in Figure 5, the nominal stress–strain relationships specified in KBC 2016 [22] were adopted for the girder and column materials: SS400 steel ($F_y = 235$ MPa, $F_u = 400$ MPa) and SM490 steel ($F_y = 325$ MPa, $F_u = 490$ MPa). The elastic modulus was taken as $E = 205$ GPa. A bilinear material model was employed by assuming an ultimate strain of 0.2.

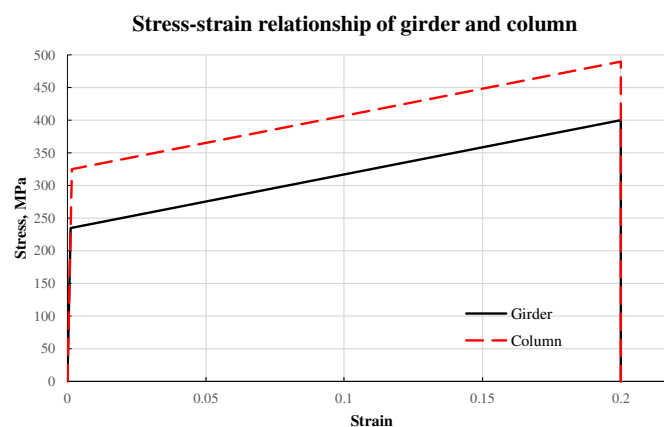


Figure 5. Stress–strain relationship of girder and column.

Both the girder and columns were discretized using 8-node solid elements (C3D8). The interaction between the girder and column was modeled using the *Constraint* function provided in ABAQUS [14], and a *Tie* constraint was applied to represent rigid connections in both horizontal and vertical directions at the contact surfaces. Considering the web thickness of 14 mm and the flange thickness of 22 mm, the mesh sizes of the web and flange were set to 14 mm and 11 mm, respectively, resulting in a total of 5284 nodes. To improve numerical convergence, a quasi-static analysis was conducted.

To simulate element failure modes, the *Ductile Damage* and *Element Deletion* options available in ABAQUS [14] were employed. When the stress in a girder element reached the material strength, damage initiation occurred, and the fracture energy after damage initiation was assumed to be zero, allowing the element to be deleted. This approach enabled realistic simulation of sectional fracture in the girder.

3.2. Numerical Analysis Results

The relationship between the applied load on the girder and the vertical displacement at the column removal location is shown in Figure 6a. Plastic behavior of the girder initiated at a vertical displacement of approximately 65.86 mm under a load of 449.4 kN. After reaching a vertical displacement of 1323.45 mm and a load of 2343.32 kN, elements adjacent to the fixed ends began to fail, resulting in a reduction in load-carrying capacity. At a displacement of 1538.25 mm, multiple elements near the fixed ends fractured, and the analysis was terminated.

The objective of this study is to develop a theoretical model for the girder and to verify its validity by comparison with numerical results obtained from analyses using one-dimensional beam elements. To avoid the influence of local buckling in the web, the

sectional response was evaluated at a location 14 mm away from the fixed end. The relationship between bending moment and axial force at this section is shown in Figure 6b. The fully plastic moment M_p and plastic axial force F_p , calculated based on the sectional dimensions, were 1654.48 kN·m and 5720 kN, respectively.

As shown in Figure 6b, at the plastic point (P), the bending moment in the plastic section reached approximately $1.05M_p$, while the axial force was negligible. At the transitional catenary point (T), the maximum bending resistance of approximately $1.10M_p$ was observed, accompanied by an early development of axial force equal to $0.17F_p$. As the catenary action developed further, the axial force increased while the bending moment gradually decreased. At the onset of pure catenary action (C), the axial force in the plastic section reached $1.0F_p$, indicating that the load resistance was governed primarily by axial force. At the failure point (F), corresponding to the ultimate displacement, the bending moment and axial force were $0.37M_p$ and $1.36F_p$, respectively.

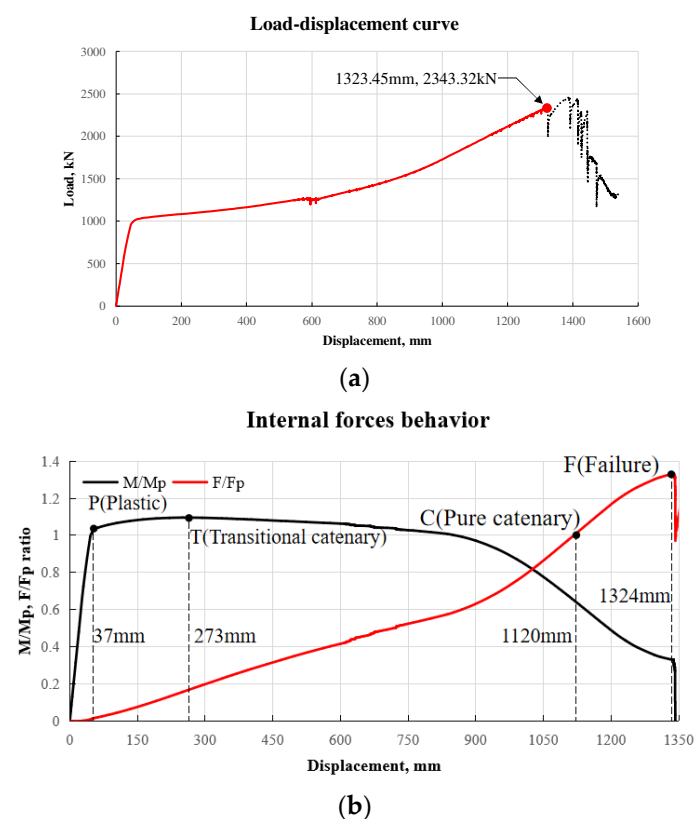


Figure 6. Analysis results of FEA model using 3-D element. (a) Load–displacement relationship; (b) M–F relationship of plastic cross-section.

3.3. Stress Models Based on Moment–Axial Force–Displacement Relationships of the Plastic Section

To analyze the stress distribution characteristics of the plastic section at each characteristic point, von Mises stresses and principal stresses were extracted from 58 nodes along the web height. The stress distributions at points P, T, C, and F are presented in Figure 7a. In Figure 7a–c, the horizontal axis represents the signed von Mises stress following the sign convention of the principal stress, and the vertical axis denotes the sectional height measured from the centroid of the cross-section.

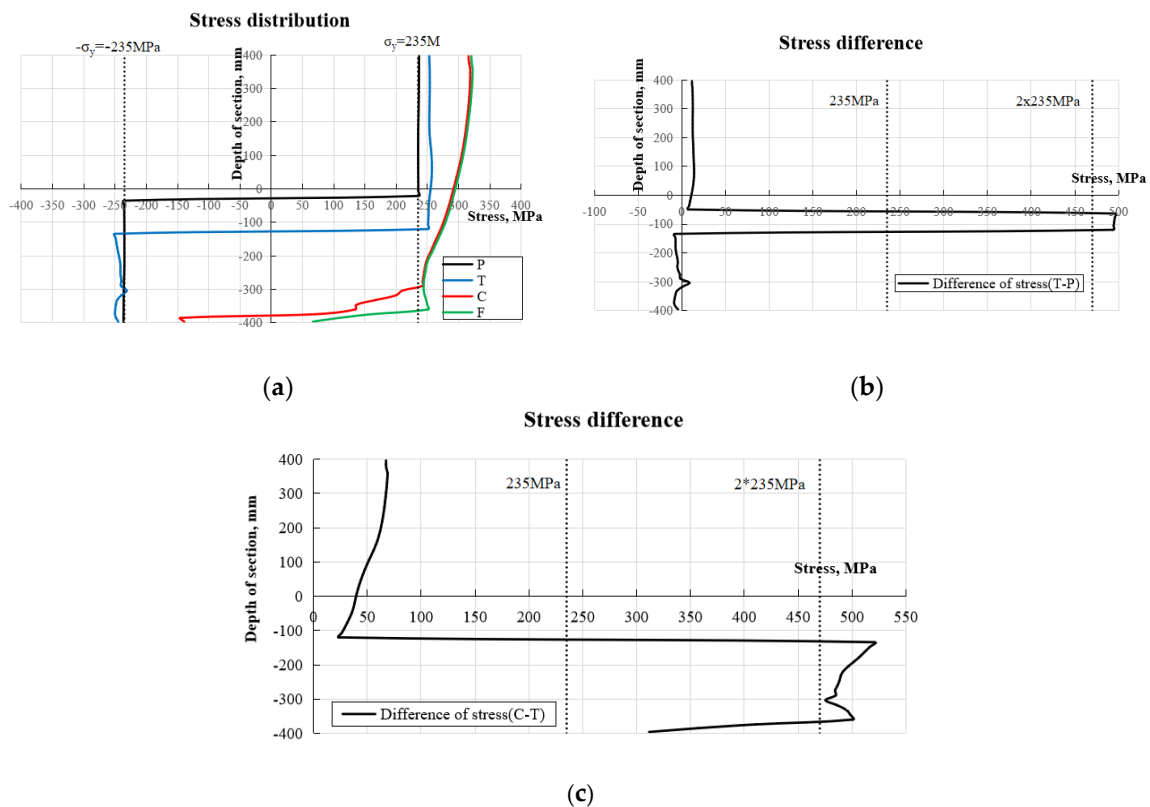


Figure 7. Analysis results of stress in the plastic section. (a) Stress distribution in the plastic section; (b) Stress difference between point T and point P; (c) Stress difference between point C and point T.

3.3.1. Fully Plastic State: Point P

At the fully plastic state, the stress distribution across the section is shown in Figure 7a. The stresses at the upper and lower portions of the section reached the yield stress of $\pm\sigma_y = \pm 235\text{ MPa}$. Due to the relatively small axial force, the neutral axis shifted downward by approximately 14 mm.

Since the influence of axial force on the bending-dominated stress distribution is negligible at point P, the stress model for this state assumes that the stresses at both the upper and lower portions of the section are equal to the yield stress $\pm\sigma_y$, with the neutral axis fixed at the sectional centroid, as illustrated in Figure 8.

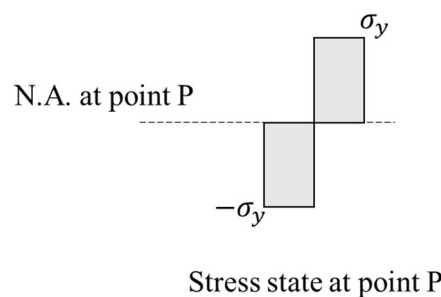


Figure 8. Stress model at point P.

3.3.2. Onset of Transitional Catenary Action: Point T

As shown in Figure 6b, the bending moment at point T increased slightly by approximately $0.05M_p$ compared to point P, while the axial force increased to $0.17F_p$. From the stress distribution of the section shown in Figure 7a, both tensile and compressive stresses

slightly exceeded the yield stress, and the neutral axis shifted downward by approximately 120 mm.

From the stress variation between points P and T illustrated in Figure 7b, it can be observed that within an approximately 105 mm region of the web located between the neutral axes at points P and T, the sectional stress increased from $-\sigma_y$ to approximately +250 MPa, corresponding to an increase of about twice the yield stress. This stress variation is attributed to the development of the axial force of $0.17F_p$.

At this stage, the axial force tends to achieve equilibrium through the minimum work principle. Consequently, the axial force is primarily mobilized in the smallest effective portion of the section adjacent to the neutral axis at point P, converting compressive yield stress ($-\sigma_y$) into tensile yield stress ($+\sigma_y$). Due to the eccentricity between the axial force and the neutral axis at point P, a positive bending moment is induced. To satisfy overall sectional equilibrium, a counteracting negative bending moment develops, resulting in an increase in both tensile and compressive stresses at the upper and lower portions of the section ($\sigma_{point T} > \sigma_y$).

With respect to the magnitude of the early axial force observed in the numerical analysis ($0.17F_p$), the approximate average interaction curve defined in AISC 360 [21] was referenced. AISC 360 [21] provides an approximate average curve for the moment–axial force interaction of H-shaped sections, describing the gradual reduction in bending moment capacity as axial force increases under fully plastic bending. According to this interaction relationship, when the axial force exceeds approximately 0.15 times the plastic axial force ($F \geq 0.15F_p$), the bending moment begins to decrease. Therefore, to enhance the general applicability of the proposed stress model, the axial force at point T was conservatively taken as $0.15F_p$.

The sectional stress model at point T is constructed by superimposing the developed axial force of $0.15F_p$, as shown in Figure 9, onto the fully plastic bending stress distribution in equilibrium at point P while additionally considering the counteracting bending moment required to maintain sectional equilibrium.

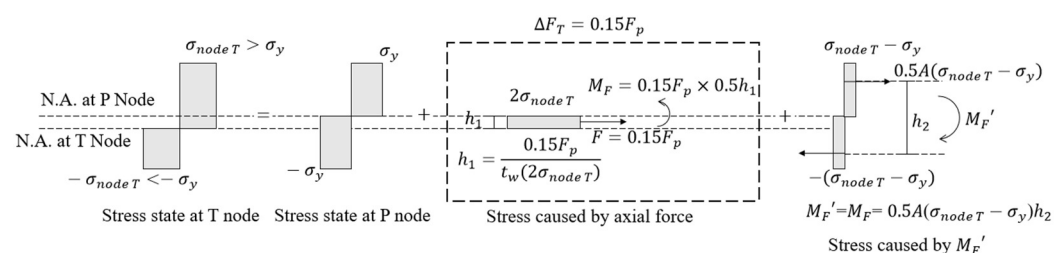


Figure 9. Stress model at point T.

In this formulation, the axial force of $0.15F_p$ transforms the stress in the web over a height h_1 below the neutral axis at point P from $-\sigma_y$ to $+\sigma_{point T}$. The height h_1 can be determined using Equation (3), and the bending moment induced by the axial force, M_{FM} , can be calculated using Equation (4). By substituting Equation (3) into Equation (4) and solving simultaneously with Equation (5), both h_1 and $\sigma_{point T}$ can be obtained.

$$h_1 = \frac{0.15F_p}{t_w(2\sigma_{point T})} \quad (3)$$

$$M_F = 0.15F_p \times 0.5h_1 \quad (4)$$

$$M_F' = 0.5A(\sigma_{point T} - \sigma_y) \times h_2 \quad (5)$$

Here, t_w denotes the web thickness; $\sigma_{point\ T}$ and σ_y represent the sectional stress at point T and the yield stress, respectively. A is the total cross-sectional area, and h_2 is the distance between the centroids of the tensile and compressive zones associated with M'_F .

3.3.3. Onset of Pure Catenary Action: Point C

At point C, the bending moment decreased to $0.65M_p$, corresponding to a reduction of 35% in the plastic moment, while the axial force significantly increased to $1.0F_p$, indicating the full development of catenary action in the girder. The neutral axis shifted substantially and was located within the lower flange. The stress variation between points T and C is shown in Figure 7c, where the stresses in the section above the neutral axis at point T increased by approximately 50 MPa on average. In the lower portion of the section excluding the flange, the increase in stress exceeded twice the yield stress. Due to the reduction in bending moment, some regions of the lower flange experienced a transition from compressive stress to tensile stress, while compressive stresses were maintained in the remaining regions.

To construct a simplified yet physically representative stress model for point C, the residual bending moment at this stage was assumed to be $0.3M_p$, as stated in Section 2, and the stress below the neutral axis at point P shown in Figure 7a was limited to $+\sigma_y$. The increase in axial force from point T to point C, amounting to $0.85F_p$, reduces the bending moment by $0.7M_p$, leaving a residual moment of $0.3M_p$.

The increased axial force was decomposed into two components, $F_{C,1}$ and $F_{C,2}$. The force $F_{C,1}$ increases the stress below the neutral axis at point T from $-\sigma_{point\ T}$ to $+\sigma_y$, and the eccentricity between the centroid of $F_{C,1}$ and the neutral axis at point P, denoted by h_4 , results in a reduction in the bending moment by $0.7M_p$. The remaining axial force increment $F_{C,2}$, was uniformly applied to the entire cross-section after the application of $F_{C,1}$, such that the total axial force at point C equals $1.0F_p$, as illustrated in Figure 10.

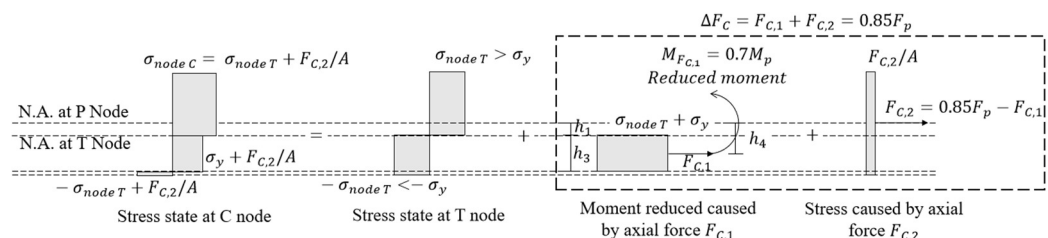


Figure 10. Stress model at point C.

Since part of the lower flange may experience tensile stress during the reduction in the bending moment by $0.7M_p$ (Figure 11), the values of $F_{C,1}$ and h_4 were determined using Equations (6)–(9). After determining h_3 , the sectional stress at the upper flange at point C, $\sigma_{point\ C}$, can be calculated using Equation (9) based on the forces $F_{C,1}$ and $F_{C,2}$.

$$M_{F_{C,1}} = 0.7M_p = F_{C,1} \times h_4 = F_{w,F_{C,1}} \times y_w + F_{f,F_{C,1}} \times y_f \quad (6)$$

$$F_{C,1} = F_{w,F_{C,1}} + F_{f,F_{C,1}} \quad (7)$$

$$F_{C,2} = F_p - F_{C,1} \quad (8)$$

$$\sigma_{point\ C} = \sigma_{point\ T} + \frac{F_{C,2}}{A} \quad (9)$$

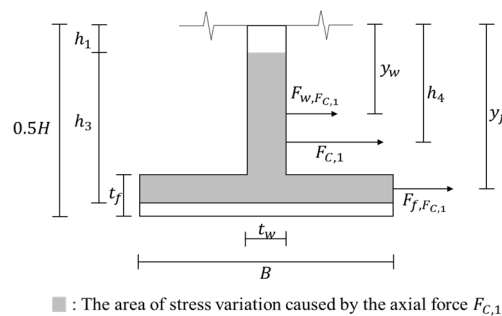


Figure 11. Stress caused by axial force $F_{C,1}$.

3.3.4. Failure Point: Point F

A comparison between points C and F in Figure 7a indicates a significant increase in stress in the lower portion of the section, accompanied by a slight stress increase over the remaining cross-section. The tensile stress at failure reached approximately 322 MPa, which corresponds to about 81% of the tensile strength ($\sigma_u = 400$ MPa).

This discrepancy in the failure stress arises from the fact that, as described in Section 2, the stress behavior shown in Figure 7a was evaluated at a section located 14 mm away from the girder end, where local buckling of the web occurs concurrently. Consequently, the analyzed section does not fully capture the nominal tensile fracture capacity of the girder. In the experimental observations, the final failure mode of the overall system was governed by connection failure prior to tensile rupture of the girder. Therefore, the determination of the girder tensile failure point (point F) primarily serves to provide a reliable slope between points C and F on the global load–displacement curve, rather than to represent the actual tensile fracture of the girder.

Accordingly, the tensile strength at point F was taken as the nominal tensile strength σ_u , instead of the reduced stress level observed in the numerical analysis shown in Figure 7a.

As illustrated in Figure 12, the stress model at point F assumes that, after reaching point C, the bending moment remains constant while only the axial force continues to increase. Upon reaching point F, the stresses in the section above the neutral axis at point T reach the tensile strength, resulting in fracture of the plastic section. The increment in axial force, $F_F - F_P$, can be obtained from the difference between the tensile strength σ_u and the sectional stress at point C, $\sigma_{point C}$, multiplied by the effective sectional area. Consequently, the axial force at collapse F_F , can be evaluated using Equations (10) and (11).

The axial force increment ΔF defined in Equations (10) and (11) plays a key role in the formalization of the failure point presented in Section 4.

$$F_F = F_P + \Delta F \quad (10)$$

$$\Delta F = (\sigma_u - \sigma_{point C})A \quad (11)$$

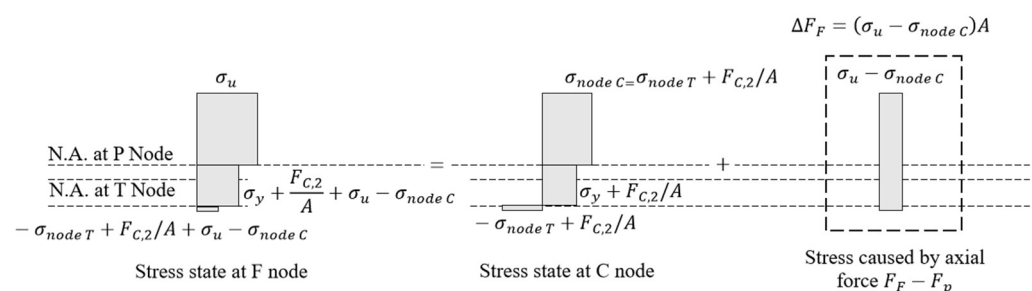


Figure 12. Stress model at point F.

4. Formulation of the Load–Displacement Relationship

In this section, a theoretical load–displacement model is formulated for a fixed–fixed girder subjected to three concentrated loads. The elastic limit point (Y) and the fully plastic point (P) are derived based on elastic theory and plastic theory, respectively. The onset of the transitional catenary action (point T) is subsequently determined based on the moment–axial force interaction.

Throughout the progressive collapse process, a residual bending moment of $0.3M_p$ is assumed to be maintained, while only the axial force continues to increase. Based on the stress model at the failure point (F), the axial force corresponding to tensile fracture of the girder is evaluated. By considering the deformation of a rigid–plastic mechanism, the onset of pure catenary action (point C) and the tensile failure point (point F) are derived in terms of the relationships between axial force and applied load, as well as between axial deformation and vertical displacement. The load–displacement response between these characteristic points is then idealized using linear interpolation.

The derivation of the load–displacement relationships at the onset of transitional catenary action (point T) and pure catenary action (point C) is based on the moment–axial force interaction model of plastic hinges proposed by Izzuddin [8], together with the differential and integral formulations of axial force increments. These formulations are reorganized to be consistent with the assumptions and the applicable range of the present study.

4.1. Formulation of the Onset of Plastic Behavior: Point Y

For the fixed–fixed girder subjected to three concentrated loads, as shown in Figure 2d, the yield moment M_y first develops at the girder sections adjacent to the fixed ends. The relationship between the applied load P_y corresponding to the yield moment M_y and the mid-span vertical displacement v_y can be obtained using elastic beam theory, as expressed in Equations (12) and (13) and illustrated in Figures 13 and 14.

$$P_y = \frac{16M_y}{6L} \quad (12)$$

$$v_y = \frac{P_y L^3}{96EI} = \frac{M_y L^2}{30EI} \quad (13)$$

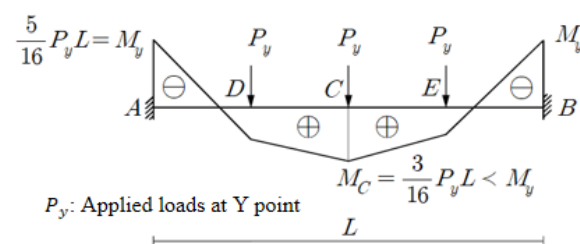


Figure 13. Moment diagram at the point Y.

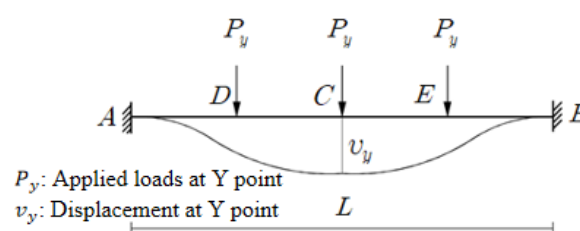


Figure 14. Deformation diagram at the point Y.

4.2. Formulation of the Onset of the Transitional Stage: Point P

After the elastic stage, as the applied load P increases, the bending moments at the end sections A and B reach the fully plastic moment M_p , leading to the formation of plastic hinges at the fixed ends. Following the development of end plastic hinges, moment redistribution occurs, and the bending moment at the mid-span section C also reaches the fully plastic moment M_p , resulting in the formation of a plastic hinge at the center of the girder, as illustrated in Figure 15.

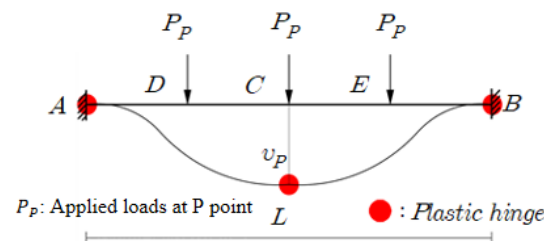


Figure 15. Moment diagram at the point P of the girder end.

At this stage, the deformation shape of the girder corresponds to that shown in Figure 15. By applying plastic beam theory, the load P_p and the corresponding vertical displacement v_p at the onset of the transitional stage (point P) can be evaluated using Equations (14) and (15).

Lee and Kim [12] reported that the transitional-stage onset load P_p is not significantly influenced by the span-to-depth ratio (L/D) and suggested that P_p be amplified by a factor of 1.1 to account for strain hardening effects.

$$P_p = 1.1 \frac{4M_p}{L} \quad (14)$$

$$v_p = \frac{7M_p L^2}{96EI} \quad (15)$$

4.3. Formulation of the Onset of Transitional Catenary Action: Point T

During the transitional stage, the bending moments at both the girder ends and the mid-span reach the fully plastic moment and can no longer increase. As shown in Figure 16, the girder sections at the ends and at the mid-span behave as rotational hinges, leading to a rapid increase in rotation. To resist further increases in the applied load, axial force begins to develop rapidly in the girder. In this stage, the load–displacement relationship is derived by explicitly considering the interaction between bending moment and axial force.

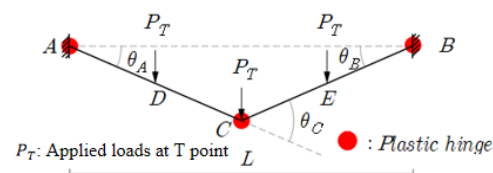


Figure 16. Rotation angle at the point T.

The derivation of the axial force increment equation for determining point T is based on the methodology proposed by Izzuddin [8]. Izzuddin [8], Yin and Wang [23], Li et al. [15], Harry et al. [11], and Deputy et al. [17] idealized the interaction between bending moment and axial force at a plastic hinge during the transitional stage, which can be expressed as Equations (16) and (17).

$$\frac{M}{M_p} + \frac{F}{F_p} = 1 \quad (16)$$

$$r_p = \frac{M_p}{F_p} \quad (17)$$

Here, r_p and F_p denote the plastic interaction radius and the plastic axial force, respectively. The total axial elongation of the girder, $\Delta(v)$, can be expressed as a function of the mid-span displacement v by applying a Taylor series expansion, as given in Equation (18).

$$\Delta(v) = \frac{2v^2}{L} \quad (18)$$

As illustrated in Figure 17, plastic hinges are formed at the mid-span section C and at the end sections A and B. The elastic axial elongation of the girder, Δ_e , can be calculated from the total elongation Δ and the plastic elongations at sections A, B, and C using Equation (19). The axial force in the girder, F , can then be expressed according to Hooke's law as shown in Equation (20), where k_e is the axial stiffness of the girder, A is the cross-sectional area, and E is the elastic modulus.

$$\Delta_e = \Delta - \Delta_A - \Delta_B - \Delta_C = \Delta - 2\Delta_A - \Delta_C \quad (19)$$

$$F = k_e \Delta_e = k_e \left(\frac{2v^2}{L} - 2\Delta_A - \Delta_C \right) \quad (20)$$

$$k_e = \frac{EA}{L} \quad (21)$$

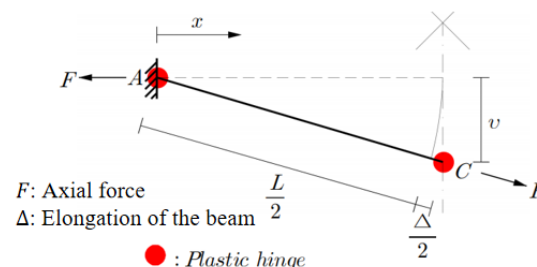


Figure 17. Elongation of the beam at the point T.

Following Izzuddin [8], differentiation of Equation (20) leads to Equation (22).

$$\delta F = k_e \left(4 \frac{v \delta v}{L} - 8 \frac{r_p \delta v}{L} \right) \quad (22)$$

From Equation (22), the axial force can increase ($\delta F > 0$) only when the condition $4 \frac{v \delta v}{L} > 8 \frac{r_p \delta v}{L}$ is satisfied, which implies that the mid-span displacement v_p must exceed $2r_p$. Therefore, the displacement at the onset of transitional catenary action, v_T , is defined as the larger of v_p and $2r_p$, as expressed in Equation (23). For girders with a very large span-to-depth ratio, the condition $v_p > 2r_p$ may govern the determination of v_T .

$$v_T = \max(v_p, 2r_p) \quad (23)$$

During the transitional stage, since the bending moments at the end and mid-span plastic sections have reached the fully plastic moment and no further increase in bending moment occurs, the load–displacement relationship exhibits a horizontal response. However, considering the actual girder behavior shown in Figure 4, the axial force in the plastic section at the onset of transitional catenary action is taken as 15% of the plastic axial force. The load increment δP_T required to generate an axial force of $0.15F_p$ can be obtained

from the triangular geometric configuration of the girder corresponding to the displacement v_T , as illustrated in Figure 17. Accordingly, the load at the onset of transitional catenary action, P_T , can be expressed as follows.

$$P_T = 1.1 \frac{4M_P}{L} + \delta P_T = 1.1 \frac{4M_P}{L} + \left(\frac{v_T}{\sqrt{v_T^2 + (0.5L)^2}} \right) 0.15F_P \quad (24)$$

4.4. Formulation of the Onset of Pure Catenary Action: Point C

At this stage, the bending moments at the end and mid-span sections decrease to a residual moment of $0.3M_P$, while the axial force continues to increase. Consequently, the plastic hinges develop significant plastic axial force, and the girder model transitions from a beam-dominated mechanism to a truss-like mechanism, as illustrated in Figure 18.

Following the approach proposed by Izzuddin [8], Equation (22) is integrated over the displacement range between v_T and v_C , which introduces additional elastic elongation in the girder excluding the plastic hinges. By applying $v_T = 2r_P$, the relationship between the axial force F and the vertical displacement at the column removal location v_C can be expressed as Equation (25).

$$F = \int_{v_T}^{v_C} \delta F = \int_{v_T}^{v_C} k_e \left(4 \frac{v \delta v}{L} - 8 \frac{r_P \delta v}{L} \right) = k_e \frac{2(v_C - v_T)^2}{L} \quad (25)$$

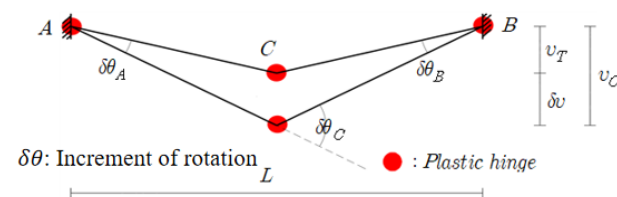


Figure 18. Rotation angle and displacement increment at the point C.

Pure catenary action is initiated when the axial force reaches the plastic axial force F_P . By rearranging Equation (23), the corresponding displacement v_C can be obtained as given in Equation (26).

$$v_C = 2r_P + \sqrt{\frac{F_P L}{2k_e}} \quad (26)$$

As shown in Figure 19, once pure catenary action is established, the relationship between the applied load P_C and the displacement v_C can be derived from the moment equilibrium at point C by considering the residual bending moments of $0.3M_P$ at both the supports and the mid-span section, resulting in Equation (27).

$$P_C = 2 \frac{H v_C}{L} + \frac{1.2M_P}{L} \quad (27)$$

Here, H is $F_P \cos \theta_C$.

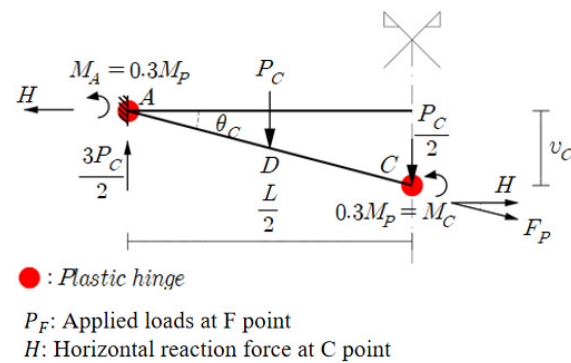


Figure 19. Free body diagram at the point C.

4.5. Formulation of the Tensile Failure Point: Point F

After the onset of pure catenary action, the residual bending moment remains constant at $0.3M_P$, and the increasing load is resisted solely by the axial force. In addition, shear deformation of the girder and its influence on the load–displacement response under large deflection are not included in the present formulation. Based on the developed stress model at the collapse point, the increment in axial force from the onset of pure catenary action to failure can be evaluated. Using the triangular proportional relationship between axial deformation and the vertical displacement at the column removal location, the collapse load P_F can be determined.

The increments of axial force and axial deformation in the plastic stage are assumed to exhibit linear behavior. Referring to Equation (21), the axial force increment δF and the corresponding axial deformation increment Δ_p after point C can be expressed as Equation (28).

$$\Delta_p = \frac{\delta F}{k_p} \quad (28)$$

$$k_p = \frac{E_p A}{L} \quad (29)$$

Here, k_p denotes the axial stiffness of the girder in the plastic region, and E_p represents the slope of the stress–strain relationship in the plastic region.

As illustrated in Figure 20, the deformed girder length at point C, L_C , can be evaluated from the vertical displacement at point C and the original girder length L using Equation (30). The increase in axial force from point C to point F is obtained as ΔF_F from the stress model. Accordingly, the axial force at collapse, F , can be calculated using Equation (31), and the axial deformation induced by the axial force increment can be expressed as Equation (32). The increment in vertical displacement at failure, δv_F , and the vertical displacement at point F, v_F , can be determined using Equation (33).

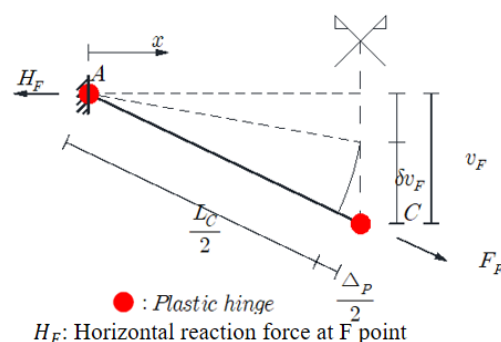


Figure 20. Free body diagram at the point F.

$$L_C = \sqrt{L^2 + v_C^2} \quad (30)$$

$$F_F = \Delta F_F + F_P \quad (31)$$

$$\Delta_p = \frac{\Delta F_F}{E_p A} \quad (32)$$

$$v_F = v_C + \delta v_F = v_C + \sqrt{(0.5L_C + 0.5\Delta_p)^2 - (0.5L_C)^2} \quad (33)$$

By considering the residual bending moments of $0.3M_p$ at both the supports and the mid-span section, the relationship between the applied load P_F and the vertical displacement v_F at tensile failure can be derived from the moment equilibrium at point C. This relationship is analogous to that at point C and is expressed as Equation (34).

$$P_F = 2 \frac{H_F v_F}{L} + \frac{1.2M_p}{L} \quad (34)$$

Here, H_F is equal to $F_F \cos \theta_F$.

5. Failure Point of the Column–Tree Connection

The theoretical model described thus far is a simplified model that does not explicitly account for connection behavior. In addition, global tying forces transmitted to adjacent beams, columns, and floor systems, as well as joint translation and rotation after column loss, are not explicitly considered in the present formulation. Experimental results of three-dimensional frame–slab systems conducted by Fu et al. [24] and Wang et al. [7] showed that, after the applied load reached the maximum resistance capacity, fracture of the bolts and flanges in the girder–column connections (End-plate bolted connections and WUF-B connections) occurred, leading to a rapid degradation of the load-resisting capacity of the girders. This stage was defined as the collapse point of the structural system.

In this study, to reflect the influence of girder–column connections on the failure of the frame–slab system, the failure point (denoted as point F_{conn}) of the Column–Tree connection located 900 mm from the column flange during the catenary action stage was evaluated based on the bearing-type connection performance and determined through five sequential steps.

As the applied load increases in the plastic region, the shear force acting on the girder increases monotonically. Since the slip-critical strength of the high-strength friction bolts in the Column–Tree connection can be assumed to have already been exceeded, the bearing strength of the bolts and splice plates must be examined. During the catenary action stage, the bending moment is significantly reduced to $0.3M_p$; therefore, the shear force acting on the flange bolts is relatively small compared to that acting on the web shear bolts. Consequently, the bearing strength of the Column–Tree connection is governed by the high-strength shear bolts in the web splice, which resist the applied shear force.

As shown in Figure 21, during the girder catenary action stage, it is assumed that the applied load P and the axial force F in the girder increase linearly with the vertical displacement v . Based on this assumption, the relationships between the concentrated load acting on the girder at connection failure P_{conn} and the corresponding girder axial force F_{conn} , as well as the vertical displacement at connection failure v_{conn} , are formulated as Equations (35) and (36), respectively. These expressions are obtained by applying equilibrium conditions at the connection failure state.

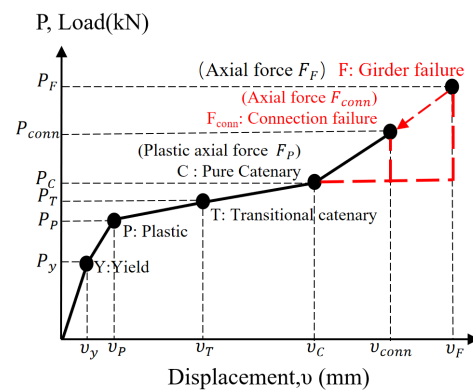


Figure 21. Load & axial force–displacement relationship at the catenary stage.

$$F_{conn} = \frac{(P_{conn} - P_C)(F_F - F_P)}{P_F - P_C} + F_P \quad (35)$$

$$v_{conn} = \frac{(v_F - v_C)(F_F - F_P)}{F_{conn} - F_F} + v_C \quad (36)$$

5.1. Analysis of the Connection Failure Process

The failure process of the connection was analyzed through the following five calculation steps. In Steps 1 and 2, the failure mode of the web splice is determined as the smaller value between the sum of the shear strengths of the high-strength bolts and the sum of the bearing strengths of the bolt holes. In Step 3, the load and displacement acting on the girder at the instant of web splice failure are evaluated. In Step 4, the possibility of flange splice failure at the moment of web splice failure is examined. In Step 5, it is assumed that, after the web splice failure, both the applied load and axial force acting on the girder are fully transferred to the flange splice, and the failure of the flange splice is subsequently investigated.

5.1.1. Step 1: Evaluation of the Shear Strength of High-Strength Web Bolts

The shear strength of the high-strength web bolts, F_{wb} , is calculated using Equation (37) based on the number of web bolts n , the bolt cross-sectional area A_b , the nominal shear strength F_{nv} , and the number of shear planes N_s . In this study, it is assumed that the threaded portion of the bolt is not included in the shear plane; therefore, the nominal shear strength F_{nv} is taken as 0.5 times the nominal tensile strength of the bolt F_t , as given in Equation (38).

$$F_{wb} = nA_b \times F_{nv} \times N_s \quad (37)$$

$$F_{nv} = 0.5F_t \quad (38)$$

5.1.2. Step 2: Evaluation of the Bearing Strength of Web Bolt Holes and Identification of the Failure Mode

According to AISC 360 [21], the bearing strength of the bolt holes in the web splice, F_{bh} , is calculated using Equation (39), and the failure mode of the web splice is identified by comparing F_{bh} with the shear strength of the high-strength web bolts F_{wb} . Since the magnitude of deflection at failure has not yet been determined and the directions of the shear force and axial force acting on the connection are not clearly defined, the bearing strength calculation is simplified and conservatively evaluated. As shown in Figure 22, the clear distance between bolt holes L_c is taken as the smaller value between the clear

distances evaluated in the girder axial direction (x) and the shear direction (y), denoted as $L_{c,x}$ and $L_{c,y}$, respectively, as expressed in Equation (40).

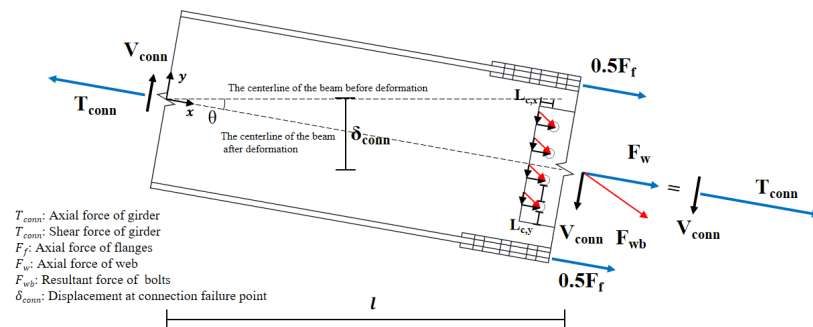


Figure 22. Freebody diagram of Column-Tree connection.

$$F_{bh} = 1.5nL_c t F_u \leq 3.0ndt F_u \quad (39)$$

$$L_c = \min(L_{c,x}, L_{c,y}) \quad (40)$$

Here, t is the thickness of the connected plate, F_u is the nominal tensile strength of the connected plate, and d is the bolt diameter.

Using Equations (37) and (39), the shear strength of the high-strength web bolts F_{wb} and the bearing strength of the bolt holes F_{bh} are calculated and summarized in Table 1. As shown in Table 1, the shear strength of the web shear bolts in the Column-Tree connections of the girders at each story is smaller than the corresponding bearing strength of the bolt holes, indicating that the failure mode of the web splice is governed by bolt shear failure.

Table 1. Load & displacement when web splices fracture.

Connection Location (Figure 2b)	Shear Strength of Bolts, F_{wb} (kN)	Bearing Strength of Bolt Holes, F_{bh} (kN)	Expected Fracture Type
G2_1-3F	3768	111,484	Bolt shear
G2_4-6F	1884	5742	Bolt shear
G2_7F	1256	2784	Bolt shear

5.1.3. Step 3: Load P_{conn} and Axial Force F_{conn} at Web Splice Failure

Figure 22 illustrates the shear force V_{conn} and axial force T_{conn} generated in the Column-Tree connection by three concentrated loads P_{conn} acting on the girder at the moment of connection failure. The shear force V_{conn} acting on the cross-section is transmitted through the web splice shear bolts, whereas the axial force T_{conn} , which is the sum of the catenary-induced axial force and the girder load, is transmitted through both the flange and web splice bolts, as expressed in Equations (41) and (42).

The horizontal forces transmitted to the flange splice and web splice, F_f and F_w , respectively, are determined by distributing the axial force T_{conn} in proportion to the flange area A_f and web area A_w , as given in Equations (43) and (44).

$$V_{conn} = 1.5P_{conn} \cos \theta \quad (41)$$

$$T_{conn} = F_{conn} + 1.5P_{conn} \sin \theta \quad (42)$$

$$F_f = \frac{A_f}{A} T_{conn} \quad (43)$$

$$F_w = \frac{A_w}{A} T_{conn} \quad (44)$$

$$\theta = \frac{v_{conn}}{0.5L} \quad (45)$$

The resultant force combining the shear force V_{conn} and the axial force F_w acting on the web splice, denoted as $F_{w,conn}$, is given by Equation (46).

$$F_{w,conn} = \sqrt{V_{conn}^2 + F_w^2} \quad (46)$$

Since the resultant force at web splice failure is equal to the maximum resistance (F_{wb}) capacity of the bolt group $F_{w,conn}$, substituting Equations (35), (37) and (39) into Equation (46) and rearranging using Equation (47) yields a nonlinear equation for the concentrated load P_{conn} at Column–Tree connection failure. This equation can be solved using an appropriate numerical method.

$$F_{w,conn} = F_{wb} \quad (47)$$

Through the above procedure, the concentrated load P_{conn} and the corresponding displacement v_{conn} acting on the fixed–fixed girder at the instant of shear failure of the web bolts in the Column–Tree connections at the 1st, 4th, and 7th stories are summarized in Table 2. These results are incorporated into the theoretical girder model as the connection failure point.

Table 2. Load, axial force & displacement at connection failure.

Connection Location (Figure 2b)	F_{wb} (kN)	F_{conn} (kN)	v_{conn} (mm)	P_{conn} (kN)
G2_1-3F	3768	6578	1814	1628
G2_4-6F	1884	5294	1286	1021
G2_7F	1256	2310	1861	632

5.1.4. Step 4: Evaluation of Flange Splice Failure at the Instant of Web Splice Failure

At the instant of web splice failure, the stress in the high-strength web bolts reaches the nominal shear strength, and the potential failure of the girder flange, flange splice plates, and flange high-strength bolts must be examined. When a concentrated load P_{conn} and an axial force F_{conn} act on the girder, an axial force of $0.5F_f$ is applied to each flange, as shown in Figure 23, and is transmitted to the flange splice plates through the high-strength bolts.

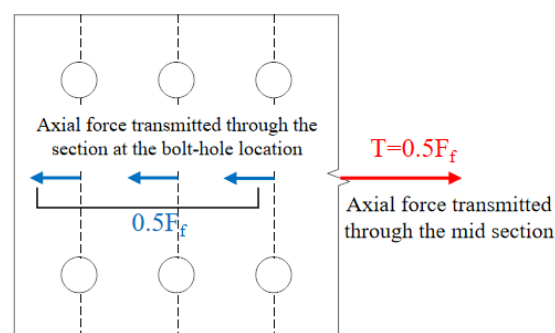


Figure 23. Axial force transmitted through the mid-section and the section at the bolt-hole location.

The stresses on the flange shear plane (σ_f), the shear stress in the flange bolts (σ_{fb}), and the stress on the shear plane of the flange splice plates at the bolt line ($\sigma_{fs,bh}$) are

calculated using Equations (48), (49), and (50), respectively. Since the midsection of the flange splice plate is the region through which the girder flange axial force is transferred and is therefore vulnerable to potential failure, the stress at this section, $\sigma_{fs,mid}$, is additionally evaluated using Equation (51), as illustrated in Figure 23.

$$\sigma_f = \frac{0.5F_f}{n_b A_{f,n}} \quad (48)$$

$$\sigma_{fb} = \frac{0.5F_f}{n A_b N_s} \quad (49)$$

$$\sigma_{fs,bh} = \frac{0.5F_f}{n_b A_{fs,n}} \quad (50)$$

$$\sigma_{fs,mid} = \frac{0.5F_f}{n_b A_{fs}} \quad (51)$$

Here, n_b is the number of bolt rows, $A_{f,n}$ is the net sectional area of the flange, n is the number of shear planes, $A_{fs,n}$ is the net sectional area of the flange splice plate excluding bolt holes, and A_{fs} is the gross sectional area of the flange splice plate.

Using Equations (48)–(51), the stresses in the girder flange, the flange splice plate at the bolt line, and the flange bolts at the instant of web splice failure are calculated and compared with the tensile strength of SS400 steel ($\sigma_u = 400$ MPa) and the nominal shear strength of the bolts ($F_{nv} = 500$ MPa), as summarized in Table 3. The results indicate that the stresses in the flanges at each story are 49%, 28%, and 52% of the tensile strength, respectively, while the stresses in the flange splice plates at the bolt line remain below 47% of the tensile strength. The stresses in the flange bolts are within 33% of the nominal shear strength, and the stress at the midsection of the flange splice plate is within 86% of the tensile strength, confirming that flange splice failure does not occur at this stage.

Table 3. Stresses at flange, flange splices and flange bolts when web splices fracture.

Connection Elements	Stress (MPa)	Connection Location (Figure 2b)		
	Stress Ratio (%)	G2_1-3F	G2_4-6F	G2_7F
Flange	σ_f	194	111	209
	σ_f/σ_u	49	28	52
Flange bolts	σ_{fb}	164	132	153
	σ_{fb}/F_{nv}	33	26	31
Flange splices at the mid-section	$\sigma_{fs,mid}$	343	266	321
	$\sigma_{fs,mid}/\sigma_u$	86	66	80
Section at the bolt-hole location of flange splices	$\sigma_{fs,bh}$	183	102	194
	$\sigma_{fs,bh}/\sigma_u$	46	26	47

5.1.5. Step 5: Evaluation of Flange Splice Failure Immediately After Web Splice Failure

Immediately after web splice failure, when the concentrated load P_{conn} and axial force F_{conn} act on the girder, a horizontal force T_{conn} and a vertical force V_{conn} are applied to the flange splice as shown in Figure 22. Based on the von Mises yield criterion, the equivalent plane stress considering two-dimensional stress components is calculated for the flange ($\sigma_{VM,f}$), the flange splice plate at the bolt line ($\sigma_{VM,fs,bh}$), and the midsection of the flange splice plate ($\sigma_{VM,fs,mid}$) using Equations (52)–(60).

$$\sigma_{VM,f} = \sqrt{\sigma_{11,f}^2 + 3\sigma_{12,f}^2} \quad (52)$$

$$\sigma_{VM,fs,bh} = \sqrt{\sigma_{11,fs,bh}^2 + 3\sigma_{12,fs,bh}^2} \quad (53)$$

$$\sigma_{VM,fs,mid} = \sqrt{\sigma_{11,fs,mid}^2 + 3\sigma_{12,fs,mid}^2} \quad (54)$$

$$\sigma_{11,f} = \frac{0.5T_{conn}}{n_b A_{f,n}} \quad (55)$$

$$\sigma_{12,f} = \frac{V_{conn}}{n_b A_{f,n}} \quad (56)$$

$$\sigma_{11,fs,bh} = \frac{0.5T_{conn}}{n_b A_{fs,n}} \quad (57)$$

$$\sigma_{12,fs,bh} = \frac{0.5V_{conn}}{n_b A_{fs,n}} \quad (58)$$

$$\sigma_{11,fs,mid} = \frac{0.5T_{conn}}{A_{fs}} \quad (59)$$

$$\sigma_{12,fs,mid} = \frac{0.5V_{conn}}{A_{fs}} \quad (60)$$

Here, σ_{11} is the stress component due to axial force, and σ_{12} is the stress component due to shear force. Considering that the flange high-strength bolts transfer the horizontal force T_{conn} , the stress in the flange bolts σ_{fb} is calculated using Equation (61).

$$\sigma_{fb} = \frac{0.5T_{conn}}{n A_b N_s} \quad (61)$$

Using Equations (52)–(54) and (61), the equivalent stresses in the flange, the flange splice plate at the bolt line, the midsection of the flange splice plate, and the stresses in the flange bolts immediately after web splice failure are calculated for each story and compared with the tensile strength of the steel material and the nominal shear strength of the bolts. As summarized in Table 4, the stresses in the flange and the flange splice plate at the bolt line range from 28% to 55% of the tensile strength of SS400 steel (400 MPa), indicating no failure in these components. The stresses in the flange high-strength bolts are within 66% of the nominal shear strength. However, stresses of 618 MPa, 412 MPa, and 521 MPa develop at the midsection of the flange splice plate for the 1st–3rd stories, 4th–6th stories, and the 7th story, respectively, corresponding to 155%, 103%, and 130% of the tensile strength of the steel.

Table 4. Stresses at flange, flange splices and flange bolts at the instant after web splices fracture.

Connection Elements	Stress (MPa)	Connection Location (Figure 2b)		
	Stress Ratio (%)	G2_1-3F	G2_4-6F	G2_7F
Flange	$\sigma_{VM,f}$	296	164	267
	$\sigma_{VM,f}/\sigma_u$	49	28	52
Flange bolts	σ_{fb}	327	263	307
	σ_{fb}/F_{nv}	66	56	61
Mid-section of flange splices	$\sigma_{VM,fs,mid}$	618	412	521
	$\sigma_{VM,fs,mid}/\sigma_u$	155	103	130
Section at the bolt-hole location of flange splices	$\sigma_{VM,fs,bh}$	222	141	197
	$\sigma_{VM,fs,bh}/\sigma_u$	55	35	49

Based on the results of Steps 1–5, shear failure of the high-strength web bolts occurs due to the combined action of axial force and shear force. Immediately after the web splice failure, both the concentrated load and axial force acting on the girder are fully transferred to the midsection of the flange splice plate, leading to a sudden fracture of this section and complete failure of the Column–Tree connection. Accordingly, the Column–Tree connection failure point defined in this study, P_{conn} and v_{conn} , is determined as the applied load and displacement at the instant of shear failure of the high-strength web bolts.

6. Validation of the Theoretical Model

The load–displacement relationship of a fixed–fixed girder subjected to three concentrated loads, as formulated in the preceding sections, was applied to the example listed in Table 5 and compared with the results obtained from ABAQUS [14] numerical analyses to verify the validity of the theoretical model developed in this study. Table 6 summarizes the calculated loads and displacements at the characteristic points of the theoretical model, namely Points Y, P, T, C, and F. The load and displacement corresponding to the connection failure point (Point F_{conn}) are presented in Table 2.

Table 5. Specifications of the applicable model.

Section Size: H-792 × 300 × 14 × 22			
Length (L)	16 m	Yield moment (M_y)	1451 kN × m
Young's modulus (E)	205 GPa	Plastic moment (M_p)	1654 kN × m
Moment of inertia (I)	$2.54 \times 10^9 \text{ mm}^4$	Axial stiffness (k_e)	311,856 kN/m
Yield strength (σ_y)	235 MPa	Plastic axial force (F_p)	5720 kN
Ultimate strength (σ_u)	400 MPa	Plastic interaction radius (r_p)	297 mm

Table 6. Calculation results of load and displacement at each characteristic points.

Characteristic Points	Equation Number	Load/Displacement	Results
Point Y	(12)	P_y	301.5 kN
	(13)	v_y	24.7 mm
Point P	(14)	P_p	455 kN
	(15)	v_p	59.3 mm
Point T	(24)	P_T	539.7 kN
	(23)	v_T	594 mm
Point C	(27)	P_C	822.7 kN
	(26)	v_C	977 mm
Point F	(34)	P_F	3996 kN
	(33)	v_F	4285 mm

6.1. Overview of the Numerical Analysis Model

To validate the theoretical model derived in this study, one-dimensional (1D) finite element analyses were performed using ABAQUS [14], considering cases with and without the application of Column–Tree connections, as shown in Figure 24. The girder specifications were identical to those listed in Table 5, and a bilinear material model was adopted for SS400 structural steel.

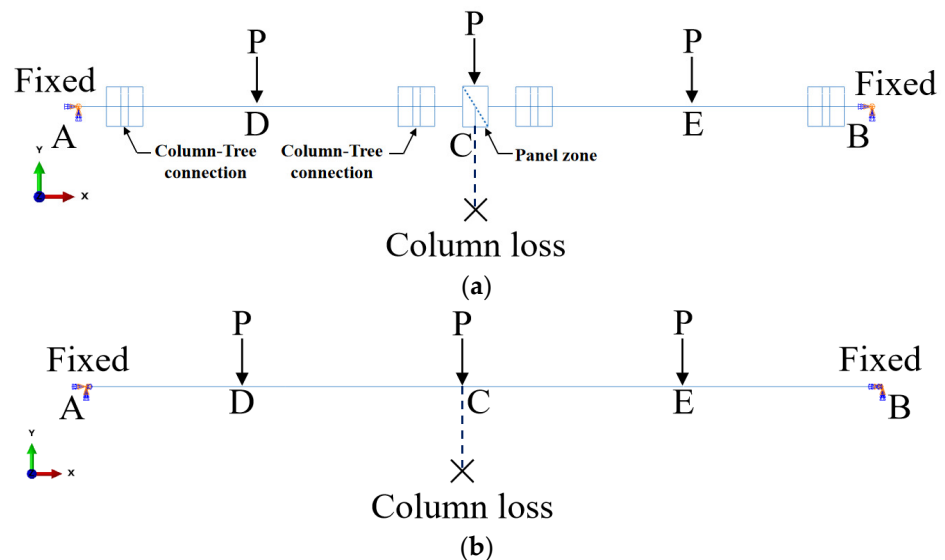
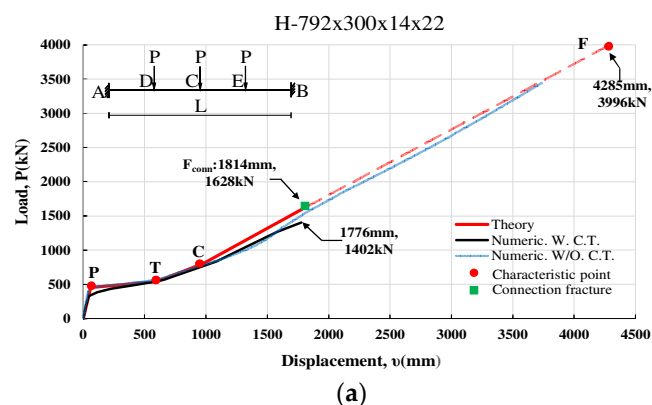


Figure 24. FEM model using 1-D El. (a) with Column-Tree connection & panel zone; (b) without Column-Tree connection.

Three modeling cases with identical span lengths but different cross-sectional dimensions were considered. The girder cross-sections were selected to match those of the G2 girders in the target structure and were defined as H-792 × 300 × 14 × 22, H-400 × 400 × 13 × 21, and H-400 × 200 × 8 × 13. For models incorporating connection behavior, the one-dimensional box-shaped Column-Tree modeling approach proposed by Li [19] was adopted (Figure 24a). As shown in Figure 24, the three concentrated load(P) applied to points C, D, E of girder.

6.2. Comparison Between the Theoretical Model and Numerical Analysis Results

For the 1st-, 4th-, and 7th-story girders of the seven-story building considered in this study, the load–displacement relationships including each characteristic points Y (elastic limit point), P (fully plastic point), T (transitional catenary action), C (pure catenary action), and F_{conn} (connection failure) of the theoretical model subjected to three concentrated loads were evaluated and compared with the corresponding numerical analysis results, as shown in Table 7 and Figure 25. The overall trends of the load–displacement curves obtained from the theoretical model and the finite element model are in good agreement.



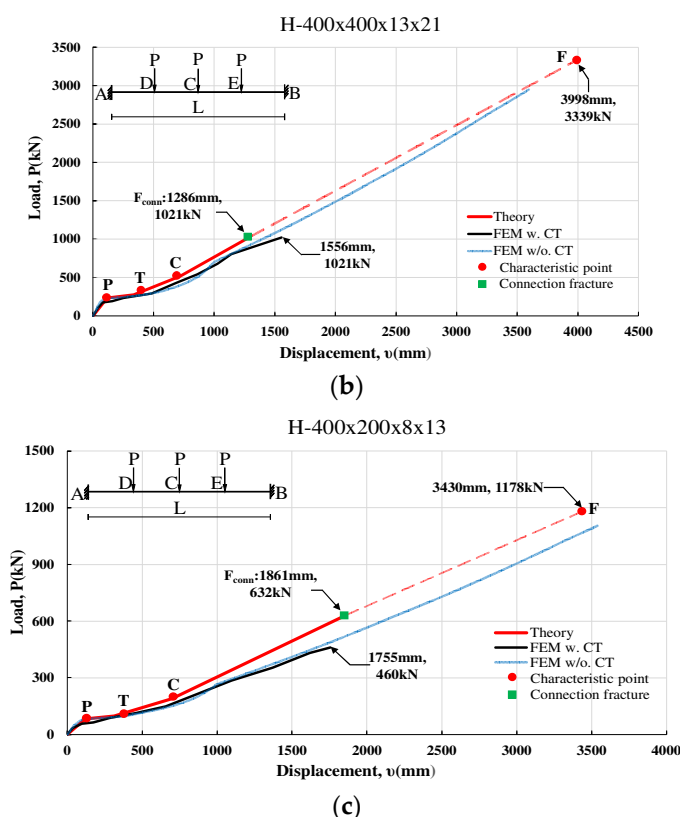


Figure 25. Comparison of $P - \delta$ curves from theoretical and FEM model for girder. (a) 1st story; (b) 4th story; (c) 7th-story.

Table 7. Comparison of the connection failure point between FEM and theory.

Floor	Beam Size	P or v	FEM	Theory	Error
1st	H-792 × 300 × 14 × 22	P_{conn}	1402 kN	1628 kN	16%
		v_{conn}	1776 mm	1814 mm	2%
4th	H-400 × 400 × 13 × 21	P_{conn}	1021 kN	1021 kN	0%
		v_{conn}	1556 mm	1286 mm	17%
7th	H-400 × 200 × 8 × 13	P_{conn}	460 kN	632 kN	37%
		v_{conn}	1755 mm	1861 mm	6%

For the girder with the H-792 × 300 × 14 × 22 section ($L/D = 20$), the load and displacement at the connection failure point (Point F_{conn}) predicted by the theoretical model were 1628 kN and 1814 mm, respectively, whereas the collapse load and displacement obtained from the numerical model with Column–Tree connections (FEM w. CT) were 1402 kN and 1776 mm. The corresponding discrepancies were 16% in load and 2% in displacement.

For the H-400 × 400 × 13 × 21 girder ($L/D = 40$), the collapse loads and displacements predicted by the theoretical model and the FEM w. CT model were 1021 kN and 1286 mm, and 1021 kN and 1556 mm, respectively. For the H-400 × 200 × 8 × 13 girder ($L/D = 40$), the collapse points were obtained as 632 kN and 1861 mm from the theoretical model and 460 kN and 1755 mm from the FEM w. CT model.

As shown in Figure 25, the discrepancies in collapse load for the 1st- and 4th-story girders are 16% and 0%, respectively, while the corresponding displacement differences are 2% and 17%. In contrast, the 7th-story girder exhibits the largest discrepancy in collapse load, reaching 37%. It is observed that the theoretical model based on stress models at characteristic points shows better agreement with numerical results for girders with relatively larger cross-sections. This tendency can be attributed to the fact that the girder

stress model used in this study was calibrated based on the numerical results of the H-792 $\times$ 300 $\times$ 14 $\times$ 22 section, and the tensile failure point of the girder (Point F) was determined accordingly.

A comparison between the theoretical model, including the girder tensile failure point (Point F), and the numerical model without Column–Tree connections (FEM w/o. CT) indicates that the theoretical model consistently exhibits higher stiffness than the numerical results. This difference is attributed to the rigid–plastic mechanism assumption adopted in the theoretical model, in which plastic hinges are idealized as discrete points, thereby neglecting the spread of plasticity along the member length.

In addition, as shown in Figure 3, when the girder reaches the tensile failure point (Point F), the bending moment at the midspan does not reduce to the simplified residual moment level of $0.3M_p$. Consequently, the stress developed in the plastic section is higher than that predicted by the simplified theoretical stress model, leading to failure at lower loads and smaller displacements in the numerical analysis.

7. Conclusions

In this study, an improved theoretical load–displacement model for a fixed–fixed girder subjected to three concentrated loads was developed by incorporating the moment–axial force interaction of plastic sections in the large-deformation range. Based on one-dimensional finite element analysis results, an early-developed axial force at the onset of the transition stage (Point T) and a residual bending moment of $0.3M_p$ remaining in the plastic section during the catenary action stage (Point C) were explicitly considered to better represent the actual structural behavior. By formulating the load and displacement at each characteristic point using equilibrium conditions at the supports, the girder response was idealized as a theoretical model consisting of five characteristic points: the yield point (Y), the plastic hinge formation point (P), the transition stage initiation point (T), the pure catenary action initiation point (C), and the collapse point governed by connection failure (F_{conn}).

The main conclusions of this study can be summarized as follows:

- (1) Systematic Definition of Structural Response: The girder load–displacement relationship was systematically defined using five characteristic points—Y, P, T, C, and F—corresponding to yielding, full plasticity, initiation of the transition stage, initiation of pure catenary action, and collapse. This formulation enables a clear physical interpretation of each deformation stage and provides a rational framework for representing progressive collapse behavior.
- (2) Improved Axial-Flexural Interaction Model: To better capture the actual structural response, an early-developed axial force and a residual bending moment at the plastic section were incorporated. This approach improves upon conventional simplified interaction models by more accurately reflecting the gradual transition from flexural-dominated behavior to axial-force-dominated catenary action.
- (3) Rational Estimation of Collapse Resistance: Stress distributions in plastic sections were analyzed using three-dimensional solid-element models to propose a procedure for estimating axial force at collapse. The derived formulations play a critical role in formalizing the collapse point (F_{conn}) and enhance the mechanical consistency of the model across various structural stages.
- (4) Model Validation and Practical Implementation: The validity of the model was verified through comparisons with finite element analysis for girders with different span-to-depth ratios (L/D), showing reasonable agreement in collapse load (16–37% error) and displacement (2–17% error). The model exhibited improved accuracy for girders with larger L/D ratios, although it consistently predicted higher stiffness due to the rigid–plastic assumption. While the model adopts idealized boundary conditions that

may not fully account for joint flexibility or torsional effects, it serves as a validated baseline for continuous beams. For practical use, the procedures and equations for calculating these characteristic points are schematically summarized in Figure 26.

- (5) Future Research and Extensions: The proposed framework can be extended to cases involving uniformly distributed loads through the same conceptual approach. While this study provides a necessary theoretical foundation, further research is required to generalize the model to more complex structural systems. This includes investigating a broader range of girder dimensions, beam spans, and steel grades, as well as explicitly incorporating high-order parameters—such as the spread of plasticity, joint flexibility, and torsional effects—to better represent realistic behavior in multi-story structures.

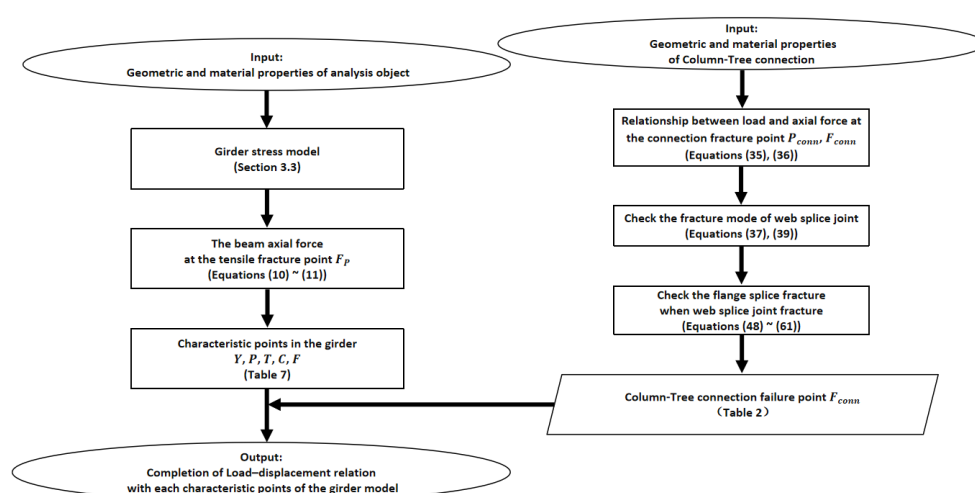


Figure 26. Design scheme of theoretical girder model.

The proposed model is intended for girder-level assessment under a column-removal scenario and does not represent the global progressive collapse behavior of an entire structural system, including joint deformability, three-dimensional load redistribution, and tying action through adjacent members and slabs.

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