

Article

Calculation of Stability Capacity for Elastically Restrained Sway Reinforced Concrete Slender Columns Based on Elastoplastic Stiffness

Shuwei Lan ¹, Peng Zhou ^{2,*}, Difei Zhao ^{3,*} , Wei Zhang ⁴, Jiansheng Zhang ⁵ and Hongyu Chen ¹ 

¹ Faculty of Civil Engineering and Mechanics, Kunming University, Kunming 650214, China; lanshuwei@kmu.edu.cn (S.L.); 385603495@kmu.edu.cn (H.C.)

² Yunnan Longke Construction Engineering Co., Ltd., Kunming 651701, China

³ School of Computer Science and Technology/School of Artificial Intelligence, China University of Mining & Technology, Xuzhou 221116, China

⁴ Department of Spatial Planning, TU Dortmund University, 44227 Dortmund, Germany; wei8.zhang@tu-dortmund.de

⁵ School of Construction Engineering, Yunnan Vocational University of Land and Resources, Kunming 650504, China; 201310164@yngtxy.edu.cn

* Correspondence: 201310405@yngtxy.edu.cn (P.Z.); difei.zhao@cumt.edu.cn (D.Z.)

Abstract

With the continuous advancement of urban renewal and the renovation and utilization of existing buildings, a large number of existing reinforced concrete slender columns face challenges in capacity evaluation. Quick and accurate calculation of their stability capacity, which represents the upper limit of member capacity, holds significant importance. These columns often exhibit plastic characteristics such as concrete cracking and steel yielding. Moreover, the bracing restraint provided by adjacent columns typically falls between that of a sway frame and a non-sway frame, classifying them as elastically restrained sway frame columns. Current design codes lack appropriate effective length factor tables for such columns, while the stiffness degradation induced by material nonlinearity is difficult to quantify accurately. To address these issues, the frame column is isolated from the overall structure and modeled as a rigid compression member system with three springs. The influence of bracing stiffness on the column's critical load is revealed, leading to a formula for the elastic critical load of elastically restrained sway frame columns. Based on tests of reinforced concrete columns under compression, the influence mechanisms of eccentricity ratio and longitudinal reinforcement ratio on flexural stiffness degradation are elucidated. The obtained elastoplastic stiffness is then integrated into the stability calculation framework for elastically restrained sway frame columns, resulting in a method for determining the elastoplastic stability capacity of reinforced concrete columns that accounts for both geometric and material nonlinearities. This method avoids solving complex transcendental equations and offers a straightforward calculation process, providing a simple and practical hand-calculation tool for evaluating the stability capacity of reinforced concrete columns in existing buildings.

Keywords: reinforced concrete slender columns; elastically restrained sway; geometric nonlinearity; material nonlinearity; stability capacity; elastoplastic stiffness



Academic Editor: Salvatore Verre

Received: 10 June 2026

Revised: 3 July 2026

Accepted: 10 July 2026

Published: 14 July 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article

distributed under the terms and

conditions of the [Creative Commons](#)

[Attribution \(CC BY\)](#) license.

1. Introduction

The current construction industry is facing a transition from a phase of extensive new construction to a phase of renovation and utilization of existing old buildings and built structures. A large number of reinforced concrete slender columns exist in these existing buildings. As the service time increases, cracks often appear in these concrete columns, leading to a reduction in their capacity and stiffness. The stability capacity of reinforced concrete slender columns, representing the upper limit of member capacity, can be used to assess the magnitude of their capacity. The calculation of elastoplastic stability capacity for slender reinforced concrete columns requires comprehensive consideration of two types of nonlinear effects: firstly, the geometric nonlinearity induced by longitudinal flexural deflection; and secondly, the material nonlinearity arising from concrete cracking, crushing, and steel reinforcement yielding. Although existing studies have developed distinct analytical approaches for each of these two nonlinear problems individually, simplified computational methods that account for their coupled effects remain a further investigation.

For the calculation of stability capacity considering geometric nonlinearity, the effective length factor method is primarily employed. This method utilizes the calculation tables for effective length factors of frame columns provided in the *Code for Design of Steel Structures* [1–4]. However, this method is applicable under conditions of uniformly distributed loads between columns and in the absence of mutual bracing effects. In practical engineering, most frame columns are often subjected to bracing effects from adjacent columns. Such bracing often cannot meet the stiffness requirements for non-sway frame columns, and the columns commonly operate in a state between non-sway and free-sway frame columns. Columns of this type are referred to as elastically restrained sway frame columns. Currently, the steel structure code lacks corresponding calculation tables for determining the stability capacity of such frame columns. Tong [5] derived the expressions for the critical load of frame columns under various bracing types based on the equilibrium method. Slimani [6,7] introduced dimensionless parameters to characterize the coupling effect between the axial force and the flexural stiffness of columns, and established a solution framework for the elastic buckling critical load. The above studies provide a theoretical basis for understanding the influence of bracing stiffness on buckling behavior; however, they are predominantly based on elastic analysis and do not account for the modification of the critical load due to stiffness degradation caused by material nonlinearity.

For the calculation of stability capacity considering material nonlinearity, various national codes generally adopt a uniform reduction factor applied to the elastic flexural stiffness. The Chinese code GB-50010 [8] adopts a reduction factor of 0.60; the American code ACI-318 [9] adopts 0.70; and the New Zealand standard NZS-3101 [10] specifies the factor in a graded manner according to the axial load ratio. However, the elastoplastic flexural stiffness of reinforced concrete columns is influenced by numerous factors, including not only the axial load ratio but also the reinforcement ratio, eccentricity, and other parameters. The use of a unified equivalent reduction factor may introduce considerable errors in the stability capacity calculation of reinforced concrete slender columns. To address this issue, Elwood [11] proposed a three-component effective stiffness model based on experimental responses, accounting for flexural, shear, and anchorage-slip deformations. Kumar [12] provided upper and lower bound estimates of effective stiffness within common parameter ranges through parametric analysis. Wei [13] and Hou [14], respectively, derived regression formulae for effective stiffness for solid square piers, thin-walled hollow piers, and circular hollow piers, using axial load ratio, shear span ratio, and reinforcement ratio as controlling parameters. However, the above effective stiffness models are primarily intended for stiffness evaluation in seismic design, and have not yet been directly linked to the calculation of elastoplastic stability capacity of slender columns. Liu [15] employed non-

linear finite element analysis to investigate the influence of material stiffness degradation on the critical load and presented a numerical method for determining the elastoplastic stability capacity; however, the model is relatively complex and inconvenient for manual engineering calculations. Sapkota [16] constructed a predictive model for effective stiffness based on machine learning, but paid insufficient attention to the stiffness degradation during the full plastic development process under static loading. Huang [17–19] employed ABAQUS nonlinear finite element analysis to investigate the effects of axial compression ratio, torsion–bending ratio, and eccentricity on the combined loading performance of RC columns, and proposed a unified bearing capacity calculation method under combined torsion. In addition, through edge-column-removal tests and numerical simulations on a two-story RC frame, they studied the progressive collapse resistance of multi-story frames and revealed the respective load resistance contributions of slabs and beams. Furthermore, for HPFL-BSP strengthened columns, they considered the influences of loading conditions, pre-damage levels, and strengthening methods, and introduced strengthening influence coefficients and torsion influence coefficients to modify the restoring force model. However, its direct engineering application still requires further simplification.

In summary, the current state of research on the stability capacity calculation of elastically restrained sway reinforced concrete slender columns still suffers from the following deficiencies. First, at the level of geometric nonlinearity, the effective length factor method lacks a rational basis for elastically restrained sway frame columns that fall between the free-sway and non-sway conditions; moreover, existing modification approaches are mostly grounded in elastic buckling analysis and have not effectively integrated the influence of material nonlinearity. Second, at the level of material nonlinearity, the uniform reduction factor or the axial load ratio based on graded reduction adopted by current codes cannot fully account for the coupled effects of multiple parameters such as reinforcement ratio and eccentricity; in addition, the existing effective stiffness models are primarily developed for seismic design applications, and their direct relevance to the elastoplastic stability capacity of slender columns remains inadequate. Third, with regard to the coupling between the two nonlinearities, there is a notable lack of simplified calculation methods that simultaneously consider both geometric and material nonlinearities while remaining amenable to routine engineering practice.

In response to the above issues, this paper presents the following investigations. First, a mechanical model of a rigid-bar system with three springs under compression is established to reveal the variation in the critical load of elastically restrained sway frame columns as a function of bracing stiffness, to provide a nomogram for the critical bracing stiffness, and to propose simplified formulae for the critical load and the effective length factor of elastically restrained sway frame columns. Second, through elastoplastic mechanical performance tests on nine reinforced concrete slender columns with varying eccentricity ratios and reinforcement ratios, the influence of eccentricity ratio and reinforcement ratio on flexural stiffness degradation is examined, and a regression formula for the elastoplastic flexural stiffness applicable to slender columns is derived. Third, by integrating the elastoplastic flexural stiffness with the effective length factor for elastically restrained sway, a practical calculation method for the elastoplastic stability capacity of elastically restrained sway reinforced concrete slender columns is established, which can serve as a reference for the stability assessment of reinforced concrete columns in existing buildings.

2. Relationship Between Bracing Stiffness and Critical Load of Frame Columns

In practical engineering, although frame columns are restrained by adjacent beams, the bracing stiffness is often insufficient to meet the requirement for strong bracing; hence,

such columns are classified as weakly braced elastically restrained sway frame columns. However, the effective length factor method specified in current codes only provides tabulated values for the two limiting cases, namely unbraced free-sway and strongly braced non-sway, without offering any basis for determining the effective length factor for weakly braced elastically restrained sway frame columns that fall between these two extremes. To address this deficiency, it is essential to first establish a quantitative relationship between the bracing stiffness and the critical load of frame columns, which will subsequently provide a foundation for the calculation of elasto-plastic stability capacity under the coupled effects of geometric and material nonlinearities. The accuracy of the effective length factor method in determining the structural stability capacity depends on the selection of boundary conditions during the isolation process of the frame column. When isolating the column, consideration should be given to the lateral deformation of the structure (i.e., sway and non-sway cases), as well as the restraint conditions at the column ends. The restraints at both ends of the isolated column can be simulated by three or two springs. As shown in Figure 1 for a compression column, three springs are present at each column end, representing rotational stiffness and lateral stiffness, denoted by c_1 , c_2 , and c_w . In practice, the spring stiffness can be determined through the following approaches: the rotational spring stiffness is obtained from the linear stiffness ratio of the connected beams and columns via joint rotational analysis, while the translational spring stiffness is determined by the equivalent lateral stiffness provided by the floor lateral bracing system and the adjacent columns. For cases where the restraint stiffness in existing buildings cannot be precisely quantified, it is recommended to calibrate the stiffness parameters using field inspection data. For ease of analysis, relative stiffness is introduced, defined as $R_1 = c_1/6i_c$, $R_2 = c_2/6i_c$, $\bar{c}_w = c_w l^2/i_c$; i_c is the linear stiffness of the column, l is the height of the isolated column, and $E_c I_c$ is the flexural stiffness of the column.

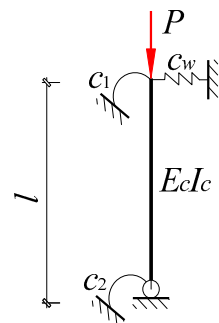


Figure 1. Separation frame column restrained by three springs.

The three-spring isolated frame column of the concrete structure shown in Figure 1 is considered. The story height l is 300 cm, and the frame column has a rectangular cross-section of 30×40 cm, with a section width of 30 cm and a section depth of 40 cm. The elastic modulus is 2000 kN/cm^2 . The rotational spring stiffness c_1 and c_2 at both column ends is taken as $6,400,000 \text{ kN}\cdot\text{cm}$. An axial load of 200 kN is applied at the column top, while the horizontal spring stiffness c_w is varied. Elastic buckling analysis is performed using the finite element software ANSYS 17.0 to investigate the variation in the critical load of the isolated column. In the ANSYS solution, the column is modeled using BEAM188 elements, and the springs are modeled using COMBIN39 elements, with axial deformation ignored. The column is meshed into 10 elements. The Block Lanczos eigenvalue solver is employed to extract the first ten buckling modes and their corresponding critical load factors. The critical buckling capacity of the frame column is obtained as the product of the critical load factor and the applied load.

As shown in Table 1, when the horizontal spring stiffness is zero (Type ①), i.e., the lateral bracing stiffness of the frame column is zero, the column behaves as a free-sway frame column, and its critical load is the smallest. As the horizontal spring stiffness gradually increases, the lateral bracing stiffness of the frame column also increases; from Type ② to Type ⑧, the critical load of the braced frame column gradually increases, and these columns are classified as elastically restrained sway frame columns. From Type ⑨ to Type ⑩, the horizontal spring stiffness increases by 50%, meaning the lateral bracing stiffness also increases significantly, but the critical load remains almost unchanged, indicating that the column behaves as a non-sway frame column. To more intuitively illustrate the relationship between the relative stiffness of the horizontal spring and the critical load of the structure, Figure 2 is plotted.

Table 1. Comparative analysis of the relationship between bracing stiffness and critical load variation in frame columns.

Type	Relative Stiffness of the Horizontal Spring $\bar{c}_w$	ANSYS Critical Load/kN	Calculation Formula for Critical Load Increment	Increment Ratio
①	0	20,223		
②	2	26,916	$ \textcircled{2} - \textcircled{1} / \textcircled{1}$	0.331
③	4	33,581	$ \textcircled{3} - \textcircled{1} / \textcircled{1}$	0.661
④	6	40,215	$ \textcircled{4} - \textcircled{1} / \textcircled{1}$	0.989
⑤	8	46,814	$ \textcircled{5} - \textcircled{1} / \textcircled{1}$	1.315
⑥	10	53,375	$ \textcircled{6} - \textcircled{1} / \textcircled{1}$	1.639
⑦	12	59,893	$ \textcircled{7} - \textcircled{1} / \textcircled{1}$	1.962
⑧	16	72,779	$ \textcircled{8} - \textcircled{1} / \textcircled{1}$	2.599
⑨	20	85,419	$ \textcircled{9} - \textcircled{1} / \textcircled{1}$	3.224
⑩	30	85,767	$ \textcircled{10} - \textcircled{1} / \textcircled{1}$	3.241

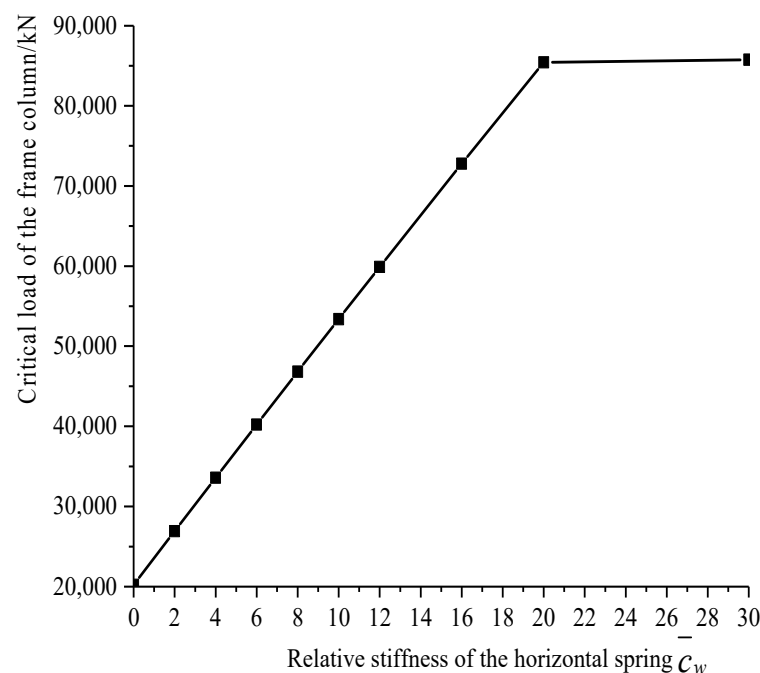


Figure 2. Relationship between relative stiffness of the horizontal spring and critical load of the frame column.

As shown in Figure 2, as the relative stiffness of the horizontal spring increases, the critical load of the frame column also increases, exhibiting a clear linear upward trend.

When the relative stiffness of the horizontal spring increases to a certain extent, i.e., when the bracing stiffness reaches a certain value, further increasing the relative stiffness of the horizontal spring or the bracing stiffness no longer enhances the critical load capacity of the frame column. This stiffness value is termed the critical bracing stiffness. When the bracing stiffness of the frame column lies between zero and the critical bracing stiffness, the column is referred to as an elastically restrained sway frame column, and its critical load falls between those of a free-sway frame column and a non-sway frame column. From the above analysis, it can be seen that, given the two rotational spring stiffnesses c_1 and c_2 at the top and bottom of the isolated column, as the horizontal constraint spring stiffness c_w provided by the bracing increases from zero, the critical load of the column gradually increases. When c_w increases to a certain value, the buckling mode of the isolated frame column becomes non-sway instability, and the critical load for non-sway instability equals that for sway instability. Beyond this point, further increasing the bracing stiffness does not increase the critical load of the frame column. This specific value is defined as the critical bracing stiffness of the isolated column. The dimensionless relative stiffness can be obtained by the following equation [20]:

$$\bar{c}_{wT} = \varepsilon_1^3 [(\varepsilon_1^2 - 36R_1R_2) \tan \varepsilon_1 - 6(R_1 + R_2)\varepsilon_1] / [\varepsilon_1^3 \tan \varepsilon_1 + 36R_1R_2(2 \tan \frac{\varepsilon_1}{2} - \varepsilon_1) \tan \varepsilon_1 + 6(R_1 + R_2)\varepsilon_1(\tan \varepsilon_1 - \varepsilon_1)] \quad (1)$$

where R_1 and R_2 represent the ratios of the sum of the beam linear stiffness at the top and bottom ends of the column to the sum of the column linear stiffness, respectively, and ε_1 is the dimensionless characteristic coefficient of the non-sway frame column, which can be calculated by the following formula:

$$\varepsilon_1 = \pi \frac{3 + 2(R_1 + R_2) + 1.28R_1R_2}{3 + 1.4(R_1 + R_2) + 0.64R_1R_2} \quad (2)$$

Equation (1) involves numerous parameters for solving the critical bracing stiffness of frame columns, making engineering calculations inconvenient. To facilitate engineering application, a series of calculations for different parameter values has been performed and compiled into the nomograph shown in Figure 3.

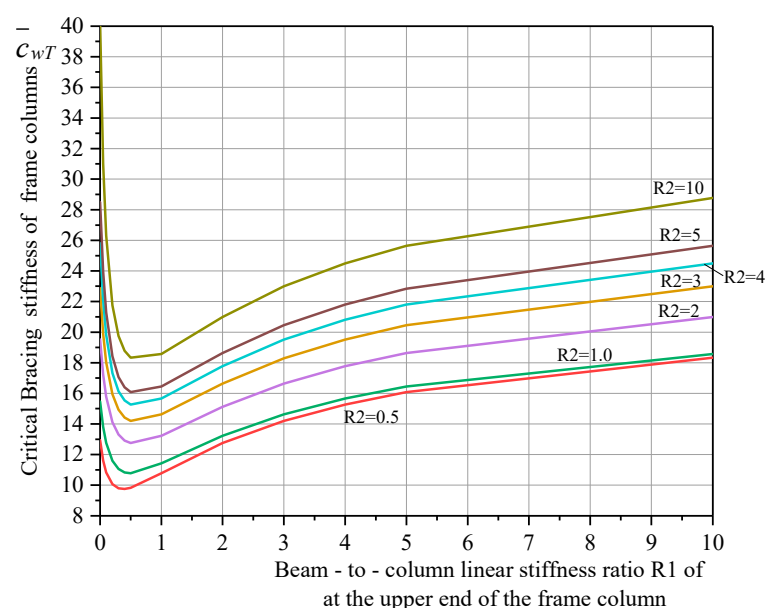


Figure 3. Nomograph of critical bracing stiffness for frame columns.

3. Calculation of Elastic Stability Capacity of Elastically Restrained Sway Frame Columns

On the basis of the quantitative relationship established between bracing stiffness and critical load, it is imperative to develop a practical calculation formula for the critical load of frame columns with elastic sway, so as to facilitate engineering applications and to provide a basis for subsequent nonlinear coupling analysis. For elastically restrained sway frame columns, the critical load lies between that of free-sway frame columns and non-sway frame columns. As can be seen from Figure 2, before the bracing stiffness reaches the critical bracing stiffness, the critical load of the frame column increases linearly with the increase in bracing stiffness. Furthermore, through extensive computational analysis and verification, the solution results are fitted by a function to obtain the approximate expression for the critical load of elastically restrained sway frame columns:

$$P_{cr} = P_{0cr} + (P_{1cr} - P_{0cr}) \frac{\bar{c}_w}{\bar{c}_{wT}} \quad (3)$$

where P_{0cr} and P_{1cr} are the critical loads of the free-sway frame column and the non-sway frame column, respectively, and P_{cr} is the critical load of the elastically restrained sway frame column.

The following relationship exists between the critical load of an elastically restrained sway frame column and its effective length factor μ :

$$P_{cr} = \frac{\pi^2 EI}{(\mu l)^2} \quad (4)$$

The expression for the effective length factor of the elastically restrained sway frame column can be obtained from Equations (3) and (4):

$$\frac{1}{\mu^2} = \frac{1}{\mu_0^2} + \left(\frac{1}{\mu_1^2} - \frac{1}{\mu_0^2} \right) \frac{\bar{c}_w}{\bar{c}_{wT}} \quad (5)$$

where μ_0 and μ_1 are the effective length factors of the free-sway frame column and the non-sway frame column, respectively, which can be calculated by Equations (6a) and (6b) [21,22].

$$\mu_0 = \sqrt{\frac{1.52 + 4(R_1 + R_2) + 7.5R_1R_2}{R_1 + R_2 + 7.5R_1R_2}} \quad (6a)$$

$$\mu_1 = \frac{3 + 1.4(R_1 + R_2) + 0.64R_1R_2}{3 + 2(R_1 + R_2) + 1.28R_1R_2} \quad (6b)$$

Once the parameters R_1 and R_2 of the frame column are determined, they can be substituted into Equation (6) to calculate the corresponding effective length factor, and then the effective length factor and critical load capacity of the elastically restrained sway frame column can be obtained.

4. Calculation of Elastoplastic Stability Capacity of Elastically Restrained Sway Frame Columns

To calculate the elastoplastic stability capacity of slender reinforced concrete columns, it is necessary to comprehensively consider the coupling effect of geometric nonlinearity and material nonlinearity. Since longitudinal reinforcement is arranged in reinforced concrete columns, the reinforcement ratio $\rho = A_s/bh$ directly affects the flexural stiffness of the cross-section, thereby influencing the elastoplastic capacity of the structure. In addition, concrete column sections usually have an initial eccentricity, and the change in the load

application point during loading also causes stiffness variation; therefore, the influence of the eccentricity ratio e_0/h must be introduced. The strength grades of steel and concrete also affect nonlinear behaviors such as concrete cracking and steel yielding during loading, thus significantly influencing the elastoplastic buckling load of the structure. To analyze the effects of factors such as axial compression ratio, reinforcement ratio, eccentricity ratio, and slenderness ratio on stiffness degradation, and to modify the previously proposed formula for elastic stability capacity, elastoplastic mechanical tests on slender reinforced concrete columns are carried out.

4.1. Test Scheme for Elastoplastic Mechanical Properties of Slender Reinforced Concrete Columns

Given that higher strength of steel reinforcement and concrete generally improve the ultimate capacity of columns, delay cracking, and restrain crack width development, this test primarily focuses on two basic variables: eccentricity ratio and reinforcement ratio. A total of nine reinforced concrete column specimens are designed and fabricated. All specimens have a length of 2200 mm and adopt a rectangular cross-section with symmetric reinforcement. The concrete strength grade is C25, the reinforcement is HRB400 with a yield strength of 360 N/mm², and the stirrups are $\phi 8@75/150$. The concrete is supplied by a ready-mixed concrete plant, with a slump of 80–100 mm. During casting, it is placed in layers and compacted by internal vibration, then covered with plastic sheeting and cured by water spraying. The hardened properties, among others, satisfy the testing requirements. A schematic diagram of the specimens is shown in Figure 4a, and detailed design parameters are listed in Table 2.

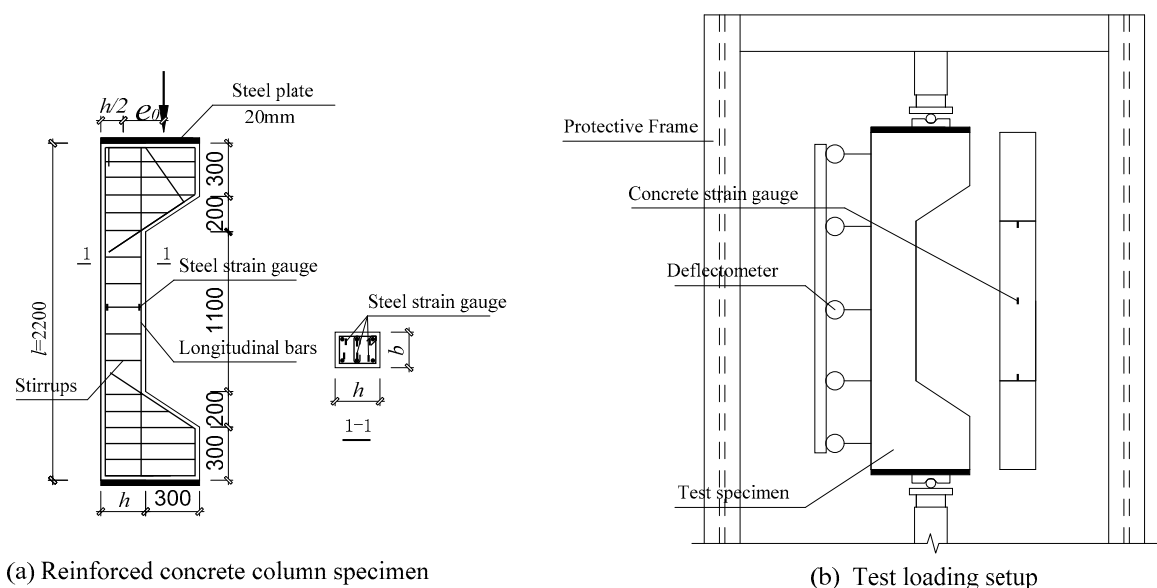


Figure 4. Schematic diagram of the reinforced concrete specimen and test loading setup.

This test was conducted on a 1000-ton hydraulic press, and the test loading setup is shown in Figure 4b. Monotonic loading and constant displacement loading methods were adopted to measure the specimen deflection, longitudinal reinforcement strain, and concrete strain.

Prior to the formal testing, a preloading procedure was carried out up to 20% of the estimated ultimate load to eliminate inelastic deformation and to verify the proper functioning of the instrumentation. The formal loading employed a hybrid load–displacement control strategy. During the load-controlled phase, the load was applied incrementally at steps of 10% of the estimated ultimate load, with each load level sustained for 10 min until deformation stabilization was achieved before proceeding to the next increment. Upon

reaching 80% of the ultimate load, the control mode was switched to displacement control, with a displacement rate of 0.02 mm/min, and loading continued until the load dropped to 50% of the ultimate capacity, at which point the test was terminated. The failure modes of the specimens exhibited typical characteristics of large-eccentricity and small-eccentricity compression. For the large-eccentricity specimens (A-1 to A-6), the failure process was characterized as follows: at load levels between 15% and 25% of the ultimate load, transverse cracks first appeared in the mid-span region on the tension side. As the load increased, these cracks propagated and increased in number. When the load reached 75% to 90% of the ultimate load, the tensile reinforcement yielded, the concrete in the compression zone was crushed, and the member deflection increased significantly, exhibiting a ductile failure mode. In contrast, the small-eccentricity specimens (A-7 to A-9) showed less pronounced cracking; near the ultimate load, longitudinal cracks developed in the compression zone, and failure occurred suddenly with limited deflection growth, displaying a brittle failure pattern. In this case, the reinforcement on the compression side yielded, whereas that on the opposite side did not.

Table 2. Test specimen parameters for elastoplastic mechanical properties of slender reinforced concrete columns.

Parameters	Unit	SJ-1	SJ-2	SJ-3	SJ-4	SJ-5	SJ-6	SJ-7	SJ-8	SJ-9
Section size $b \times h$	mm	250×200	250×200	250×200	250×200	250×200	250×200	250×200	250×200	250×200
Eccentricity e_0	mm	35	35	35	80	80	80	140	140	140
Longitudinal reinforcement	-	6 ϕ 12	6 ϕ 14	6 ϕ 16	6 ϕ 12	6 ϕ 14	6 ϕ 16	6 ϕ 12	6 ϕ 14	6 ϕ 16
Longitudinal reinforcement ratio ρ	%	1.33	2.41	3.77	1.33	2.41	3.77	1.33	2.41	3.77
Eccentricity ratio e_0/h	-	0.175	0.175	0.175	0.40	0.40	0.40	0.70	0.70	0.70

4.2. Analysis of Elastoplastic Mechanical Properties of Slender Reinforced Concrete Columns

During the test loading process, as the axial force N gradually increases, the reinforced concrete column exhibits significant nonlinear characteristics. Initially, the column remains in the elastic stage. As the load increases, cracks begin to appear in the concrete, the material enters the nonlinear stage, and the column gradually transitions from elastic to elastoplastic state. When the axial force further increases, the strain in the reinforcement grows until reaching the yield point, ultimately leading to failure as the entire cross-section of the column enters the plastic stage. The reinforced concrete specimens during the loading process are shown in Figure 5.

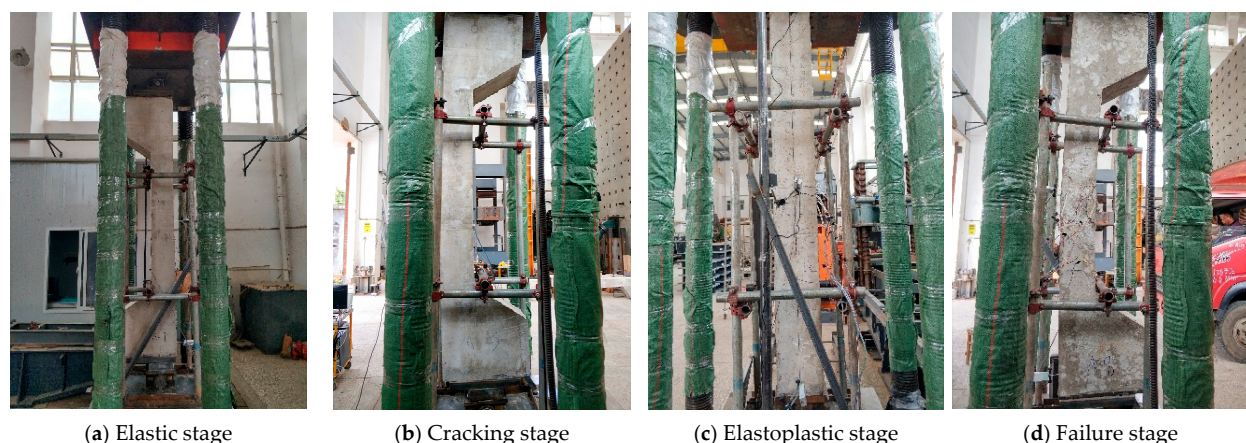


Figure 5. Diagram of the loading process of reinforced concrete specimens.

Based on the geometric relationship of strains and the constitutive relationship of the column section, the following can be obtained [23,24]:

$$EIy'' = -M = EI\phi = EI \frac{\varepsilon_{st} - \varepsilon_{sc}}{h_0 - a'_s} \quad (7)$$

where ϕ is the curvature; ε_{st} and ε_{sc} are the tensile strain and compressive strain of the reinforcement, respectively; EI is the elastoplastic flexural stiffness of the reinforced concrete column; h_0 is the effective depth of the section; and a'_s is the distance from the resultant force point of the reinforcement in the compression zone to the compression edge of the section.

As can be seen from Equation (7), the relationship between the bending moment M and the curvature ϕ of a reinforced concrete column effectively reflects the evolution process of the sectional flexural stiffness. The slope of the moment–curvature curve represents the instantaneous flexural stiffness of the section. As the load increases, the bending moment gradually increases. When the strain of the concrete edge fibers exceeds the ultimate tensile strain, the member cracks and enters the plastic state. As the load continues to increase, the strain in the reinforcement increases and may eventually yield. Therefore, by analyzing the variation in the slope of the moment–curvature curves of reinforced concrete specimens (SJ-1 to SJ-9) under different eccentricity ratios and reinforcement ratios, the degradation law of flexural stiffness of reinforced concrete columns during loading can be revealed.

4.2.1. Effect of Eccentricity Ratio on Sectional Flexural Stiffness

The variation in load application position significantly affects the mechanical behavior and failure mode of reinforced concrete columns, and its influence on flexural stiffness can be quantitatively analyzed through the eccentricity ratio. To investigate the effect of eccentricity ratio on the flexural stiffness of reinforced concrete eccentrically loaded columns, two groups of specimens with the same longitudinal reinforcement ratio but different eccentricity ratios were selected, and their moment–curvature curves during monotonic vertical loading up to the ultimate load were plotted. For the first group, specimens SJ-1, SJ-4, and SJ-7 all had a longitudinal reinforcement ratio of 1.33%, with corresponding eccentricity ratios of 0.175, 0.40, and 0.70, respectively; their moment–curvature curves are shown in Figure 6a. For the second group, specimens SJ-2, SJ-5, and SJ-8 all had a longitudinal reinforcement ratio of 2.41%, with eccentricity ratios also of 0.175, 0.40, and 0.70, respectively; their corresponding curves are shown in Figure 6b. By comparing the moment–curvature curves of specimens with the same reinforcement ratio but different eccentricity ratios, the influence of eccentricity ratio on the evolution process of sectional flexural stiffness can be revealed.

As shown in Figure 6, each moment–curvature curve exhibits a steep upward trend before cracking, reflecting a high initial tangent modulus of the section at this stage. As the load increases, the specimen cracks and enters the elastoplastic stage, where the slope of the curve decreases significantly. This change directly reflects the gradual degradation of sectional flexural stiffness due to the development of material nonlinearity. Further comparison of the moment–curvature curves for specimens with the same reinforcement ratio but different eccentricity ratios shows that the overall slope of each curve decreases as the eccentricity ratio increases. This indicates that the eccentricity ratio is an important factor affecting the flexural stiffness of reinforced concrete columns: throughout the entire loading process, specimens with larger eccentricity ratios exhibit more pronounced stiffness degradation and a correspondingly smaller flexural stiffness level.

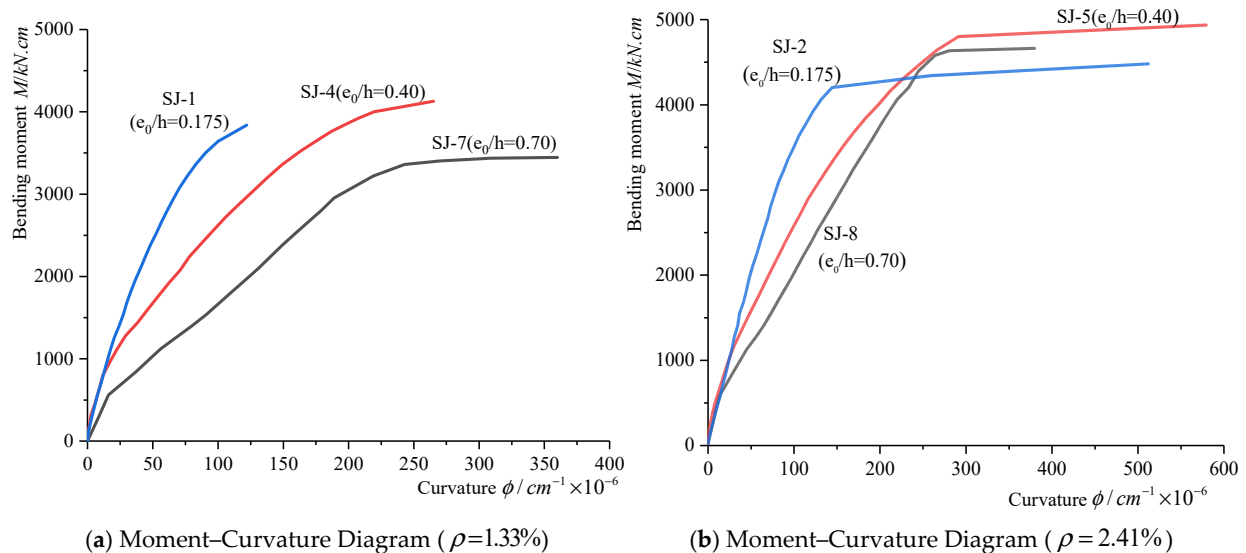


Figure 6. Moment–curvature diagram of reinforced concrete specimens under different eccentricities.

4.2.2. Effect of Reinforcement Ratio on Sectional Flexural Stiffness

To investigate the influence of the longitudinal reinforcement ratio on the flexural stiffness of reinforced concrete columns, based on the aforementioned test data, specimens with different reinforcement ratios but the same eccentricity ratio were selected to plot their moment–curvature curves. Two groups of specimens with the same eccentricity ratio but different longitudinal reinforcement ratios were chosen, and their moment–curvature curves during monotonic vertical loading up to the ultimate load were plotted. For the first group, specimens SJ-4, SJ-5, and SJ-6 all have an eccentricity ratio of 0.40, with corresponding longitudinal reinforcement ratios of 1.33%, 2.41%, and 3.77%, respectively; their moment–curvature curves are shown in Figure 7a. For the second group, specimens SJ-7, SJ-8, and SJ-9 all have an eccentricity ratio of 0.70, with corresponding longitudinal reinforcement ratios also of 1.33%, 2.41%, and 3.77%, respectively; their moment–curvature curves are shown in Figure 7b. By comparing the curve characteristics of specimens with the same eccentricity ratio but different reinforcement ratios, the influence of the reinforcement ratio on the evolution law of sectional flexural stiffness can be further revealed.

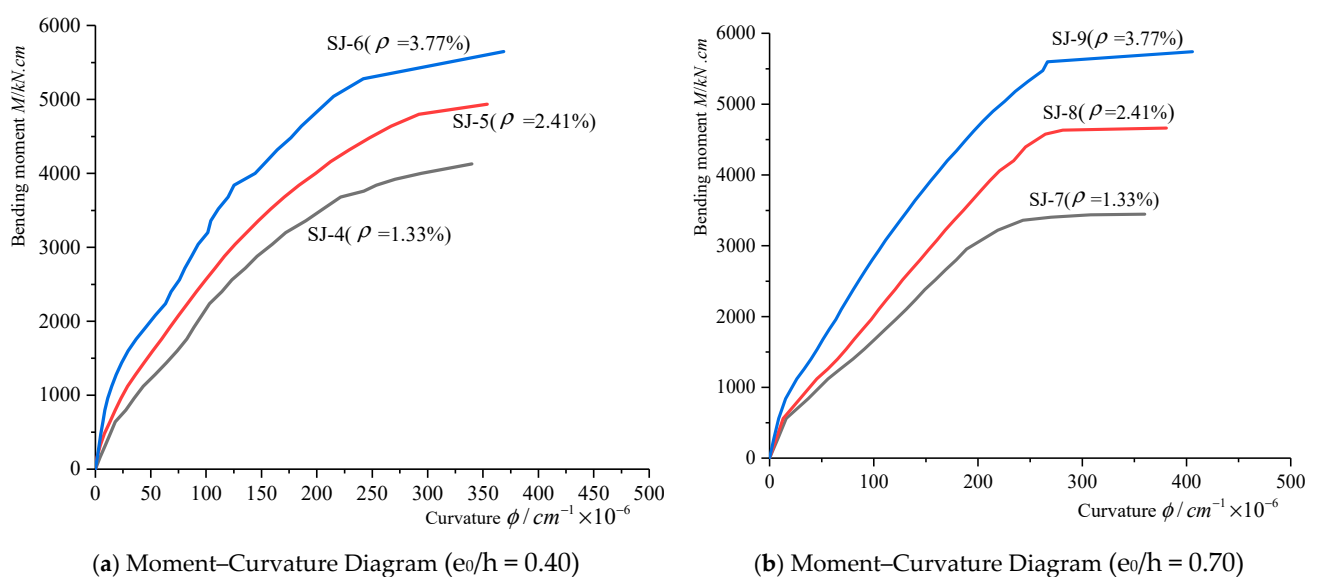


Figure 7. Moment–curvature diagram of reinforced concrete specimens under different longitudinal reinforcement ratios.

As shown in Figure 7, each group of moment–curvature curves exhibits a high initial slope before cracking, indicating a relatively large sectional flexural stiffness at this stage. As the load increases, the specimen cracks and enters the elastoplastic phase, where the slope of the curves gradually decreases, reflecting the degradation of sectional stiffness due to the development of material nonlinearity. Furthermore, comparing the curves of specimens with the same eccentricity ratio but different reinforcement ratios reveals that a higher reinforcement ratio leads to a corresponding increase in the overall slope of the curves, indicating that increasing the longitudinal reinforcement ratio helps enhance the sectional flexural stiffness and delay the stiffness degradation process.

In Figure 6, specimens SJ-1 and SJ-2 with low eccentricity ratios ($e_0/h = 0.175$) did not exhibit an obvious inflection point on the moment–curvature curves after concrete cracking, whereas specimens SJ-7 and SJ-8 ($e_0/h = 0.70$) showed significant stiffness degradation at relatively low bending moment levels. This difference is closely related to the controlling effect of eccentricity ratio on the cross-sectional stress distribution. In low-eccentricity specimens, axial compression predominates, and the section is either fully compressed or has only a very small tension zone; consequently, concrete cracking is substantially suppressed, with limited crack width and propagation range, so that stiffness degradation is not clearly manifested on the macroscopic moment–curvature response. As the eccentricity ratio increases, the tension zone expands, allowing more extensive concrete cracking with notably larger crack widths and numbers, thus rendering the sectional stiffness degradation more pronounced. Furthermore, the stiffness changes in Figure 7 appear more significant, mainly because the transverse comparisons of different reinforcement ratios at eccentricity ratios of 0.40 and 0.70 make the slope variations easier to identify and compare. In summary, post-cracking stiffness degradation objectively exists in all specimens, but the degree of its macroscopic manifestation depends on the extent to which the eccentricity ratio governs the internal force distribution and crack development across the section.

4.3. Calculation Formula for Elastoplastic Stability Capacity of Reinforced Concrete Frame Columns

The flexural stiffness of reinforced concrete columns exhibits significant stage evolution characteristics during loading. Before cracking, both concrete and steel reinforcement are in a linear elastic state, and the column stiffness is the elastic flexural stiffness, with the moment–curvature relationship presenting a linear segment with the maximum slope. As the load continues to increase, the concrete in the tension zone cracks and ceases to work, the stress in the steel reinforcement gradually increases, and the member enters the cracked elastoplastic working stage. At this stage, the flexural stiffness transforms into elastoplastic stiffness, and shows a continuous degradation trend during the development of plasticity, with the slope of the moment–curvature relationship curve gradually decreasing. When calculating the elastoplastic stability capacity of reinforced concrete frame columns, the elastoplastic flexural stiffness of the frame column should be adopted. The test results show that the elastoplastic stiffness of the specimens is mainly influenced by factors such as eccentricity ratio, reinforcement ratio, and vertical load level. By analyzing the slopes of the moment–curvature relationship curves of specimens SJ-1 to SJ-9, and performing regression fitting on the test data, a calculation formula for the elastoplastic flexural stiffness of reinforced concrete columns is established:

$$EI = (0.85 + 25 \frac{A_s}{A}) (1 - \frac{e_0}{h} - 0.5n) E_c I_c \quad (8)$$

where EI is the elastic–plastic flexural stiffness of the reinforced concrete column, with its value ranging between $0.35E_c I_c$ and $0.85E_c I_c$; $E_c I_c$ is the elastic flexural stiffness of the reinforced concrete column, defined as E_c is the elastic modulus of concrete and I_c is the

moment of inertia of the reinforced concrete column; $\frac{A_s}{A}$ is the ratio of the longitudinal reinforcement area to the cross-sectional area of the column, i.e., the reinforcement ratio; $\frac{e_0}{h}$ is the eccentricity ratio of the column, defined as e_0 is the eccentricity of the column and h is the height of the column; n is the axial compression ratio, calculated as the ratio of the design value of the axial compressive force on the column to the product of the gross cross-sectional area of the column and the design value of the concrete compressive strength.

To verify the rationality of the proposed regression formula, it is compared with the provisions of GB-50010 (reduction factor 0.60), ACI-318 (reduction factor 0.70), and NZS-3101 (with reduction factors specified according to axial compression ratio levels). These codes primarily adopt the axial compression ratio as the governing parameter, without considering the coupled effects of reinforcement ratio and eccentricity ratio. The proposed formula introduces the reinforcement ratio and eccentricity ratio in addition to the axial compression ratio: an increase in reinforcement ratio retards stiffness degradation, whereas a higher eccentricity ratio aggravates stiffness decay, a trend consistent with experimental observations. Taking an axial compression ratio of 0.5 as an example, the proposed reduction factor is 0.68 for a reinforcement ratio of 1.33% and an eccentricity ratio of 0.175, which is close to the NZS-3101 value of 0.62; however, for a reinforcement ratio of 2.41% and an eccentricity ratio of 0.40, the reduction factor is 0.51, which deviates considerably from the code values. This indicates that the uniform code-specified values may overestimate the actual stiffness in some cases and underestimate it in others. The range of the proposed reduction factor, from 0.35 to 0.85, is determined based on the present experimental data and with reference to the reduction values corresponding to different axial compression ratios in NZS-3101, so as to avoid excessive underestimation or overestimation of the actual stiffness due to improper selection of the factor. It should be noted that Equation (8) is an empirical formula derived from regression analysis of the test data of nine rectangular reinforced concrete slender-to-medium columns (section dimensions 250 mm × 200 mm, concrete grade C25, reinforcement HRB400, reinforcement ratio ranging from 1.33% to 3.77%, and eccentricity ratio from 0.175 to 0.70). The applicability of this formula is, in principle, limited to the aforementioned parameter ranges. Nevertheless, the mathematical form of the formula has a clear mechanical basis: the reinforcement ratio term reflects the enhancement of sectional flexural stiffness by longitudinal reinforcement; the eccentricity ratio term characterizes the aggravating effect of load eccentricity on sectional stiffness degradation; and the axial compression ratio term captures the influence of axial load level on plastic development. All three variables are key factors affecting the elastoplastic flexural stiffness, and the functional form is extendable to a broader range of parameters. For columns beyond the experimental parameter ranges (e.g., different cross-sectional dimensions, concrete strength grades, higher reinforcement ratios, etc.), the applicability of the formula requires further verification. However, the proposed method for calculating the elastoplastic stability capacity of frame columns, that is, the coupling of the elastic sway effective length factor and the elastoplastic stiffness, is general in its framework. Users can conduct supplementary tests or numerical analyses for specific parameters. For instance, for reinforced concrete columns strengthened with materials such as carbon fiber-reinforced polymer sheets [25–27], the stability capacity calculation procedure presented herein can still be adopted after regressing the corresponding stiffness formulae.

It can be seen from Equation (8) that the elastoplastic stiffness of a reinforced concrete column is related to the reinforcement ratio, eccentricity ratio, and axial compression ratio. Using this equation to calculate the elastoplastic stiffness of a reinforced concrete frame column can account for the gradual variation in flexural stiffness during the elastoplastic process. Once the elastoplastic flexural stiffness of the reinforced concrete column is

determined, it can be substituted into Equation (4) to obtain the calculation formula for the elastoplastic stability capacity of the elastically restrained sway reinforced concrete frame column:

$$P_{cr} = \frac{\pi^2 EI}{(\mu l)^2} = \frac{\pi^2 E_c I_c}{(\mu l)^2} (0.85 + 25 \frac{A_s}{A}) (1 - \frac{e_0}{h} - 0.5n) \quad (9)$$

where the effective length factor μ of the elastically restrained sway reinforced concrete frame column can be calculated by Equation (5). Using this equation to determine the elastoplastic stability capacity of the frame column can comprehensively account for the effects of both geometric nonlinearity and material nonlinearity.

To verify the correctness and reliability of the proposed method, the elastoplastic stability capacities of specimens SJ-1 to SJ-9 calculated by Equation (9) are compared with the experimentally measured capacities. The results indicate that the relative errors between the calculated and experimental values range from 6% to 18%. These errors primarily stem from the simplifying assumptions introduced in the regression fitting of the elastoplastic flexural stiffness formula (Equation (8)), such as neglecting shear deformation and assuming an ideal elastoplastic constitutive model for the material, as well as the inherent discrepancies between the measured material properties in the tests and the design values. Overall, this level of error is within the acceptable range for engineering practice, demonstrating that the proposed calculation method can predict the elastoplastic stability capacity of slender-to-medium reinforced concrete columns in elastically restrained sway frames with reasonable accuracy. This comparative validation further substantiates the rationality and engineering applicability of the coupled treatment of geometric and material nonlinearities.

5. Example

As shown in Figure 8, an existing reinforced concrete isolated compression column in a building has a column height of 7500 mm and a cross-sectional size of 300 mm × 400 mm. The relative stiffness of the equivalent horizontal spring at the column top is $\bar{c}_w = 2$. The ratios of the sum of beam linear stiffness to the sum of column linear stiffness at both the top and bottom ends of the isolated column are both 1.5. The concrete strength grade is C30, with elastic modulus $E_c = 3 \times 10^4$ N/mm² and design compressive strength $f_c = 14.3$ N/mm². The steel reinforcement grade is HRB400 with a yield strength of $f_y = 360$ N/mm². The longitudinal reinforcement consists of 8φ12. The design axial load is $N = 600$ kN, and the design bending moment is $M = 60$ kN·m. The elastoplastic stability capacity of this existing reinforced concrete column is calculated using the proposed method.

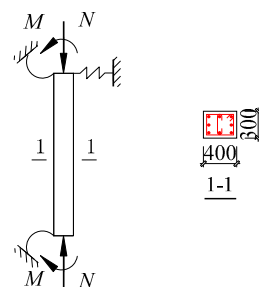


Figure 8. Calculation diagram of reinforced concrete separation column.

- ① Determination of the effective length factor of the frame column

Based on the given information, the relative stiffness of the equivalent horizontal spring at the column top is $\bar{c}_w = 2$, and the ratio of the sum of beam linear stiffness to the sum of column linear stiffness at both the top and bottom ends of the isolated column is $R_1 = R_2 = 1.5$. According to Equation (5), the effective length factor of this elastic sway frame column is obtained as $\mu = 1.12$.

② Calculation of main parameters

$$e_0 = M/N = 60/400 = 0.15 \text{ m} = 150 \text{ mm}$$

$$n = N/f_c A = 600,000/(14.3 \times 300 \times 400) = 0.35$$

Substituting the previously calculated values into Equation (8) yields the elastoplastic stiffness of the column:

$$\begin{aligned} EI &= (0.85 + 25 \frac{A_s}{A}) (1 - \frac{e_0}{h} - 0.5n) E_c I_c \\ &= (0.85 + 25 \times \frac{888}{120,000}) (1 - \frac{150}{400} - 0.5 \times 0.35) E_c I_c = 0.466 E_c I_c \end{aligned}$$

Satisfies: $0.35 E_c I_c < EI < 0.85 E_c I_c$.

③ Calculation of the elastoplastic stability capacity of existing reinforced concrete frame columns

Substituting the obtained elastoplastic stiffness into Equation (9) yields:

$$P_{cr} = \frac{\pi^2 E_c I_c}{(\mu l)^2} (0.85 + 25 \frac{A_s}{A}) (1 - \frac{e_0}{h} - 0.5n) = \frac{3.14^2 \times 3 \times 10^4 \times 16 \times 10^8}{(1.12 \times 7500)^2} \times 0.466 = 3125 \text{ kN}$$

Based on the above calculation and analysis, the following results are obtained: the vertical load carried by the existing concrete column is 600 kN, and the elastoplastic stability capacity of the reinforced concrete frame column is 3125 kN. Therefore, the stability verification of this existing reinforced concrete frame column is satisfied, and this value can be taken as the upper-bound capacity of the slender frame column for preliminary assessment in the safety evaluation of existing buildings. For comparative purposes, calculations are also performed using the reduction factors of 0.60 specified in Chinese code GB-50010 and 0.70 specified in American code ACI 318, corresponding to elastoplastic flexural stiffnesses of $0.60 E_c I_c$ and $0.70 E_c I_c$, respectively. The resulting capacities are 4024 kN and 4695 kN, respectively, while the value obtained by the proposed method is 3125 kN. Compared with the proposed method, the GB-50010 result is 28.8% higher, and the ACI-318 result is 50.2% higher. The reason for these discrepancies lies in the fact that both GB-50010 and ACI 318 adopt uniform reduction factors without considering the reinforcement ratio and eccentricity ratio of the specific column, whereas the proposed method, based on the experimental regression formula, quantitatively accounts for the coupled influence of reinforcement ratio and eccentricity ratio.

6. Conclusions

- (a) The three-spring rigid bar model is used to simulate the end restraints of any isolated column in a reinforced concrete structure. By varying the horizontal spring stiffness, the influence of bracing stiffness on the critical load of the frame column is revealed, and the physical meaning of the critical bracing stiffness is clarified. When the horizontal spring stiffness is less than the critical bracing stiffness, the critical load varies linearly. Once the bracing stiffness reaches the critical value, the column buckling mode transitions from sway to non-sway, and the critical load no longer increases. A nomograph of the critical bracing stiffness is provided, which serves

as a practical tool for the preliminary classification of free-sway versus non-sway frame columns.

- (b) Based on the linear variation in the critical load with bracing stiffness, approximate expressions for the critical load of elastically restrained sway frame columns are derived by a linear interpolation between the free-sway and non-sway critical loads, avoiding the need to solve complex transcendental equations. This provides a simple and practical hand calculation tool for the elastic stability design of elastically restrained sway frame columns in engineering practice.
- (c) Experimental investigations on the elastoplastic behavior of reinforced concrete columns reveal that both eccentricity ratio and reinforcement ratio significantly influence the degradation of flexural stiffness. A regression formula for the elastoplastic flexural stiffness is established based on the test data. By combining this formula with the effective length factor of elastically restrained sway frame columns, a calculation method for the elastoplastic stability capacity of slender reinforced concrete columns in elastically restrained sway frames is proposed. This method accounts for the coupling effect of geometric nonlinearity and material nonlinearity, and the calculation process is straightforward, providing a reference for the stability assessment of reinforced concrete columns in existing buildings.
- (d) The proposed method is applicable to moderately slender reinforced concrete columns. For short columns, stability effects are not significant, and design can be directly governed by strength control. For columns with large slenderness ratios, second-order effects become substantially more pronounced, and the applicability of the present method requires further verification. In addition, all test specimens in this study adopted rectangular cross-sections with symmetrical reinforcement. The applicability of the regression formula to columns with unsymmetrical reinforcement or with circular, T-shaped, L-shaped, or other cross-sectional profiles needs to be further extended and validated. Furthermore, the influences of long-term load effects, geometric initial imperfections, and complex boundary conditions on stability capacity should be investigated further through combined nonlinear finite element analyses and additional experiments.

Author Contributions: Conceptualization, S.L. and P.Z.; Methodology, S.L.; Software, D.Z., W.Z., J.Z. and H.C.; Validation, D.Z. and H.C.; Formal Analysis, S.L. and D.Z.; Investigation, S.L., D.Z., J.Z., P.Z. and H.C.; Resources, D.Z. and H.C.; Data Curation, S.L., J.Z. and H.C.; Writing—Original Draft, S.L.; Writing—Review and Editing, D.Z.; Visualization, S.L. and W.Z.; Supervision, D.Z. and W.Z.; Project Administration, S.L.; Funding Acquisition, S.L. and H.C. All authors have read and agreed to the published version of the manuscript.

Funding: This study was financially supported by Basic Research Project in Yunnan province of China (202401AT070031), the Special Basic Cooperative Research Programs of Yunnan Provincial Undergraduate Universities' Association (202501BA070001-075, 202501BA070001-083) and the National Natural Science Foundation of China (51868034).

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: Author Peng Zhou was employed by the company Yunnan Longke Construction Engineering Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as potential conflicts of interest.

References

1. *ANSI/AISC 360-16; Specification for Structural Steel Buildings*. The AISC Committee on Specifications: Chicago, IL, USA, 2016.

2. EN 1993-1-1; Eurocode 3: Design of Steel Buildings. The Authority of the Standards Policy and Strategy Committee: London, UK, 2005.
3. GB 50017-2017; Standard for Design of Steel Structure. China Architecture & Building Press: Beijing, China, 2018.
4. AS 4100-1998; Standards Australia Steel Structures. Sail Global International Limited: Sydney, Australia, 1998.
5. Tong, G. *In-Plane Stability of Steel Structures*; China Architecture & Building Press: Beijing, China, 2004; pp. 146–188.
6. Slimani, A.; Ammari, F.; Adman, R. The effective length of columns in in unsymmetrical frames asymmetrically loaded. *Asian J. Civ. Eng.* **2018**, *19*, 487–499. [[CrossRef](#)]
7. Slimani, A.; Belaid, T.; Saidani, M.; Ammari, F.; Adman, R. A new method for determining the effective length factor of columns in partially braced frames on elastic supports. *Struct. Eng. Mech.* **2023**, *85*, 825–835. [[CrossRef](#)]
8. GB 50010-2010; Code for Design of Concrete Structures. China Architecture & Building Press: Beijing, China, 2018.
9. ACI 318-14; Building Code Requirements for Structural Concrete and Commentary. American Concrete Institute: Michigan, MI, USA, 2014.
10. NZS3101; New Zealand Standard concrete structures Standard. Government of New Zealand: Wellington, New Zealand, 2006.
11. Elwood, K.J.; Eberhard, M.O. Effective stiffness of Reinforced concrete columns. *ACI Struct. J.* **2009**, *106*, 476–484. [[CrossRef](#)] [[PubMed](#)]
12. Kumar, R.; Singh, Y. Stiffness of reinforced concrete frame members for seismic analysis. *ACI Struct. J.* **2010**, *107*, 607–615. [[CrossRef](#)]
13. Wei, W.; Shao, C.; Xiao, Z.; Qi, Q. Experimental study on effective stiffness of reinforced concrete hollow piers. *China Civ. Eng. J.* **2019**, *52*, 101–110. [[CrossRef](#)]
14. Hou, Z.; Li, G.; Wu, Z.; Kong, K. Study on effective stiffness of circular hollow reinforced concrete piers under seismic load. *World Earthq. Eng.* **2024**, *40*, 174–185. [[CrossRef](#)]
15. Liu, X.; Wei, W.; Bai, S. Study on inelastic stability checking method of reinforced concrete frame column members. *Spec. Struct.* **2019**, *36*, 1–9. [[CrossRef](#)]
16. Sapkota, S.C.; Das, S.; Saha, P. Optimized machine learning models for prediction of effective stiffness of rectangular reinforced concrete column sections. *Structures* **2024**, *62*, 106155. [[CrossRef](#)]
17. Huang, H.; Guo, M.; Zhang, W.; Zeng, J.; Yang, K.; Bai, H. Numerical investigation on the bearing capacity of RC columns strengthened by HPFL-BSP under combined loadings. *J. Build. Eng.* **2021**, *39*, 102266. [[CrossRef](#)]
18. Huang, H.; Huang, M.; Zhang, W.; Guo, M.; Liu, B. Progressive collapse of multistory 3D reinforced concrete frame structures after the loss of an edge column. *Struct. Infrastruct. Eng.* **2022**, *18*, 249–265. [[CrossRef](#)]
19. Huang, H.; Guo, M.; Zhang, W.; Huang, M. Seismic behavior of strengthened RC columns under combined loadings. *J. Bridge Eng.* **2022**, *27*, 05022005. [[CrossRef](#)]
20. Geng, X.; Zhou, D. Universal charts for determining the effective length of compression columns. *Eng. Mech.* **2014**, *31*, 154–160. [[CrossRef](#)]
21. Zhang, H.; Wang, K.; Jia, J.; Cai, L.; Wu, Z.; Zhang, L.; Liu, J. Effective length factor of flexural stiffness released columns in sidesway uninhibited and inhibited frames. *Int. J. Steel Struct.* **2026**, *26*, 31–45. [[CrossRef](#)]
22. Zhang, W. *In-Plane Stability of Steel Structures*; Wuhan University of Technology Press: Wuhan, China, 2018; pp. 45–107.
23. Xiao, J.; Shuang, C.; Pan, W.; Su, H.; Sun, G.; Han, C. Solving the overall critical load capacity of sway frame based on the spring-pendulum model. *Structures* **2024**, *61*, 106013. [[CrossRef](#)]
24. Zhang, S.; Chen, X.; Lan, S. Analytical solution of inverse algorithm for compressive and flexural bearing capacity of annular concrete section. *J. China Three Gorges Univ. (Nat. Sci. Ed.)* **2022**, *44*, 82–87. [[CrossRef](#)]
25. Qian, K.; Gao, G.; Yu, X.; Li, Z.; He, Y. Interpretable probabilistic resistance model for RC structures against progressive collapse. *Eng. Struct.* **2026**, *360*, 122825. [[CrossRef](#)]
26. Xie, Q.; Sinaei, H.; Shariati, M.; Khorami, M.; Mohamad, E.T.; Bui, D.T. An experimental study on the effect of CFRP on behavior of reinforce concrete beam column connections. *Steel Compos. Struct.* **2019**, *30*, 433–441. [[CrossRef](#)]
27. Luo, Z.Y.; Sinaei, H.; Ibrahim, Z.; Shariati, M.; Jumaat, Z.; Wakil, K.; Pham, B.T.; Mohamad, E.T.; Khorami, M. Computational and experimental analysis of beam to column joints reinforced with CFRP plates. *Steel Compos. Struct.* **2019**, *30*, 271–280. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.