

Article

A Simplified Method for Calculating the Lateral Bearing Capacity of Rectangular Piles in Sloping Ground

Tao Chen ¹, Nan Ge ¹, Xuanbin Yang ², Yongchao Du ³, Zhen Guo ¹, Shangle Xie ², Mingxing Zhu ^{2,*} and Zhengzhao Liang ⁴

¹ Ningbo High-Grade Highway Construction Management Center, Ningbo 315000, China

² School of Civil Engineering and Architecture, Jiangsu University of Science and Technology, Zhenjiang 212000, China

³ Liaoning Provincial Transportation Planning and Design Institute Co., Ltd., Shenyang 110166, China

⁴ State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian 116024, China

* Correspondence: mxingzhu@just.edu.cn

Abstract

Evaluating the lateral bearing behavior of rectangular piles in sloping ground is computationally demanding due to the complex three-dimensional spatial pile-soil interaction. To circumvent the inefficiency of full-scale numerical modeling and the limitations of conventional p-y methods, this study proposes a simplified analytical framework for rapid preliminary design. By systematically isolating the topographical slope effect and the cross-sectional shape effect, a series of mathematical modification factors, namely the ultimate lateral capacity factor K_H , the maximum bending moment factor K_M , and the maximum reverse shear force factor K_Q , were established utilizing a standard level-ground square pile as the computational baseline. The results indicate that increasing the slope angle from 0° to 30° reduces the lateral capacity by approximately 14–20%, whereas increasing the aspect ratio from 1 to 4 effectively compensates for this reduction, improving the capacity by approximately 75%. However, the correspondingly enhanced flexural stiffness simultaneously triggers a substantial non-linear amplification in both the maximum bending moment and the deep reverse shear force. Benchmark comparisons show a maximum deviation of 4.58% for lateral capacity, while the maximum deviations in bending moment and reverse shear force are 10.43% and 9.78%, respectively. By integrating these modification factors with conventional equivalent calculation methods, this study provides an efficient analytical tool for rectangular pile foundation design in sloping ground.

Keywords: rectangular pile; sloping ground; lateral bearing capacity; simplified calculation method; modification factors

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1. Introduction

Pile foundations are critical deep foundation systems for resisting lateral loads induced by wind, vehicular braking, earthquakes, and similar actions in high-rise buildings, bridges, transmission towers, and related structures [1,2]. As transportation, energy, and urban development continue to expand into mountainous, hilly, and coastal areas, large numbers of piles are unavoidably installed near slope crests or on slope faces [3,4]. Compared with level ground, a slope destroys the geometric symmetry of the surrounding soil:

the passive soil wedge on the free-face side is truncated, while the lateral confinement and initial stiffness of the shallow soil are significantly reduced, causing larger horizontal displacements and higher internal pile forces under the same load level [5]. Consequently, the effect of sloping ground on the lateral bearing behavior of piles has become a key issue constraining pile foundation design in mountainous and other complex terrains.

For piles in sloping ground, extensive studies have been carried out through model tests, centrifuge tests, and three-dimensional finite element analyses. Georgiadis and Georgiadis [1] established depth-dependent formulations for the ultimate resistance factor and initial stiffness reduction by simultaneously considering slope angle and pile-soil adhesion on the basis of three-dimensional elastoplastic finite element analysis, and showed that the reduction in initial stiffness is mainly concentrated within a depth of approximately six pile diameters below the slope crest. Sand model tests conducted by Muthukkumaran [2] indicated that, under downslope loading at the slope crest, the lateral capacity of a single pile may decrease by as much as 66% to 81% relative to that in level ground. Finite element results reported by Sawant and Shukla [4] showed that pile-head displacement at the crest increased by about 37% and the maximum bending moment by 16% to 26% relative to level ground, and they proposed a critical edge distance of 6 to 9 pile diameters beyond which the slope effect becomes negligible. Through model tests in soft clay, Rathod et al. [6] extracted measured p - y curves for piles in sloping ground and found that, when the slope steepened from level ground to 1V:1H, the maximum pile bending moment increased by 15% to 41%. They also showed that the slope simultaneously reduces the ultimate soil resistance and the initial stiffness mainly within a depth range of about 10 to 11.5 pile diameters, whereas the influence largely disappears in deeper soil. Liu and Jiang [7] developed a modified p - y curve model for offshore piles adjacent to sandy slopes by incorporating the coordinated deformation mechanism of the pile-soil-slope system, while simultaneously accounting for the slope-induced reductions in ultimate soil resistance and initial stiffness. Their study showed that the slope effect is manifested not only in the degradation of ultimate resistance, but also in the alteration of the actual soil compression through lateral movement of the slope surface soil, thereby modifying the p - y curve framework itself. Jiang et al. [3], Yin et al. [8], and Liu et al. [5] further demonstrated that increasing slope angle enlarges the maximum bending moment and its embedment depth and shifts the zero point of soil resistance downward, while also developing corresponding simplified nonlinear solution methods. These studies consistently confirm that the slope effect is objective, significant, and cannot be neglected in the design of laterally loaded piles in sloping ground; however, almost all of them were conducted for piles with circular cross-sections.

Meanwhile, with the development of trenching and diaphragm wall construction technologies, piles with rectangular cross-sections have been used increasingly in bridges, deep excavations, and tall structures. Compared with circular piles, rectangular piles can significantly increase the cross-sectional moment of inertia by orienting the long side along the loading direction, thereby improving flexural stiffness and effectively controlling horizontal displacement [9–11]. Using a variational energy method, Basu et al. [9,12] established a semi-analytical model for rectangular piles in layered soil and confirmed that, in the elastic stage, the second moment of area of the cross-section is the key geometric quantity governing pile displacement and bending-moment response [11,13]. In terms of ultimate resistance, the two-dimensional plane-strain finite element limit analysis of Keawsawasvong and Ukritchon [14] showed that, for the same characteristic width, the ultimate lateral resistance of rectangular piles at depth is 25% to 40% higher than that of circular piles. Centrifuge tests by Truong and Lehane [15] further showed that, in soft clay, the ultimate lateral resistance of square and H-piles is approximately 25% and 38% higher, respectively, than that of circular piles, and they accordingly proposed shape modification

factors. Nevertheless, these studies on rectangular piles were also carried out almost exclusively under level-ground conditions [16].

It follows that most studies on piles in sloping ground are concerned with circular piles, whereas most studies on rectangular piles are concerned with level ground, and research directly combining sloping ground and rectangular piles remains extremely limited. Among the available studies, only Jiang et al. [17] compared circular and rectangular piles under clay slope conditions and found that, owing to their greater flexural stiffness, rectangular piles can suppress the deformation aggravation induced by the slope more effectively; however, because of the moment-attraction effect, they develop a larger peak bending moment at a greater depth. That study, however, kept the pile-slope distance fixed and did not propose a rapid calculation method for bearing capacity that could be used directly in engineering design. More importantly, the widely used p-y curve method encounters multiple difficulties when applied to rectangular piles in sloping ground. First, the p-y method discretizes the continuous soil mass into mutually independent springs and neglects continuous shear transfer within the soil [18]. Second, the slope effect is highly complex and difficult to represent by a unified p-y modification; reported critical influence distances range from 6 to 15 pile diameters, and the proposed correction schemes vary substantially [2–4,19], indicating limited generality. Third, back-calculating p-y curves from finite element results is cumbersome, usually requiring high-order polynomial fitting of pile bending moments or shears followed by differentiation in order to suppress numerical noise [3,6,17]. Fourth, classical p-y models exhibit serious bias for rigid piles with large diameters and low slenderness ratios, tending to overestimate deep-soil stiffness while ignoring additional resistance components such as pile-base shear, pile-base moment, and side friction, although these components may contribute 30% to 50% of the total resistance of a rigid pile [20–22]. Of particular importance is that, after the cross-section changes to a rectangular shape, side-wall friction becomes much more significant [23], and this mechanism is difficult to capture by relying solely on lateral p-y springs.

In view of the above, this study abandons the cumbersome and applicability-limited p-y curve route and proposes a rapid calculation method for the lateral bearing capacity of rectangular piles in sloping ground based on parametric fitting of finite element results. Using the published 3D-FEA dataset reported in [24], this study extracts the ultimate lateral capacity, maximum bending moment, and maximum reverse shear force as macroscopic characteristic quantities. By abandoning complex micro-scale modeling, the present work shifts the focus entirely to establishing a rapid mathematical evaluation framework. Next, taking a square pile with an aspect ratio of 1:1 in level ground as the reference case, these characteristic quantities are fitted parametrically, and quantitative relationships are established for the ultimate capacity modification factor K_H , the maximum bending moment modification factor K_M , and the maximum shear modification factor K_Q as functions of aspect ratio and slope angle. The pile response of the reference square pile in level ground is first calculated using the transfer matrix method. The resulting reference solution is then combined with K_H , K_M , and K_Q to reconstruct the ultimate lateral capacity, maximum bending moment, and maximum reverse shear force reported for different combinations of slope angle and aspect ratio in [24]. The comparison is intended to examine the numerical consistency and benchmark-reproduction capability of the proposed framework within the investigated parameter range. With a computational cost substantially lower than that of three-dimensional finite-element analysis, the proposed method provides a rapid analytical reference for preliminary assessment within the investigated parameter range.

2. Geometric Definition and Baseline Database

To establish a rapid evaluation framework for piles in sloping ground, the geometric boundary conditions must first be clearly defined. Figure 1 presents the schematic diagram of a laterally loaded rectangular pile embedded in a slope. The key geometric variables controlling the asymmetric soil resistance are the slope angle α and the cross-sectional aspect ratio η . The aspect ratio is defined as $\eta = L/B$, where L is the side length parallel to the horizontal loading direction and B is the side length perpendicular to the loading direction. To eliminate the influence of material volume and isolate the shape effect, the cross-sectional area of the rectangular pile is maintained constant across all variations of the aspect ratio.

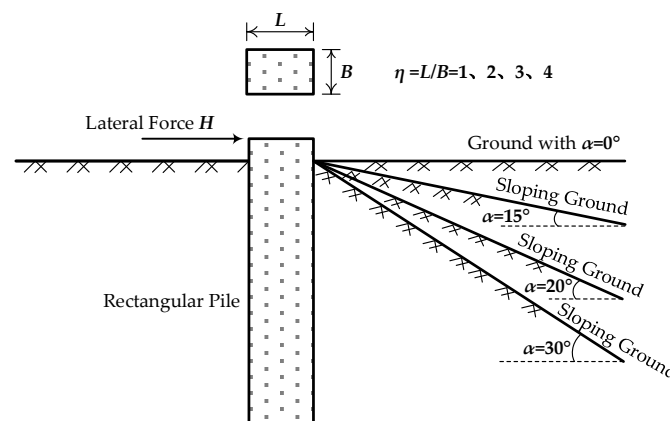


Figure 1. Schematic model of rectangular pile for slope.

The present study uses the published numerical dataset reported in [24] and does not claim the development of a new three-dimensional finite-element database. The finite-element assumptions, material parameters, pile–soil contact settings, boundary conditions, loading procedure, displacement criterion, and the 16 combinations of slope angle and cross-sectional aspect ratio are inherited from [24]. The present study uses the ultimate lateral capacity, maximum bending moment, and maximum reverse shear force reported for these cases as source data. The new contribution begins with the normalization and analytical treatment of these characteristic quantities, followed by the derivation of K_H , K_M , and K_Q under physically constrained zero-intercept conditions and their integration with the transfer matrix method to establish a rapid design-oriented calculation framework. In the source numerical dataset, only the slope angle α and the cross-sectional aspect ratio η were varied. The soil profile and constitutive parameters, pile material and overall dimensions, cross-sectional area, pile position relative to the slope, slope geometry other than α , groundwater condition, loading mode, and displacement criterion were retained at the values adopted in [24]. Accordingly, the modification factors developed in this study are calibrated only for a single rectangular pile under monotonic lateral loading in homogeneous soil, within the ranges $0^\circ \leq \alpha \leq 30^\circ$ and $1 \leq \eta \leq 4$. Their application to conditions outside this domain requires additional assessment and, where necessary, recalibration.

Based on the published dataset in [24], three engineering characteristic quantities were extracted for sixteen combined topographical and geometric conditions. These conditions comprise slope angles of 0° , 15° , 20° , and 30° , along with aspect ratios of 1, 2, 3, and 4. All characteristic quantities were determined under a strict serviceability limit state, corresponding to a lateral pile-head displacement of 0.1 m [25]. The extracted datasets for the ultimate lateral capacity H_u , the maximum bending moment $M_{\max}$, and the maximum reverse shear force $|Q_{\max}^-|$ are summarized in Table 1, Table 2, and Table 3, respectively.

These macroscopic limit states serve as the fundamental input dataset for deriving the modification factors in the subsequent sections.

Table 1. Lateral ultimate bearing capacity of rectangular piles, H_u (kN).

$\alpha \backslash \eta$	1	2	3	4
0°	1603.861	2020.867	2429.721	2640.277
15°	1450.408	1857.854	2261.212	2459.443
20°	1399.341	1802.641	2202.036	2399.033
30°	1286.695	1681.149	2067.518	2260.149

Table 2. Maximum bending moment of rectangular piles, $M_{\max}$ (MN·m).

$\alpha \backslash \eta$	1	2	3	4
0°	12.90	18.42	23.99	27.37
15°	12.23	17.70	23.44	26.44
20°	12.04	17.44	23.12	26.12
30°	11.56	16.82	22.43	25.30

Table 3. Maximum reverse shear force of rectangular piles, $|Q_{\max}^-|$ (MN).

$\alpha \backslash \eta$	1	2	3	4
0°	1.255	1.452	1.701	1.873
15°	1.171	1.379	1.646	1.808
20°	1.142	1.353	1.621	1.785
30°	1.095	1.288	1.566	1.73

3. Establishment of Modification Coefficients for Rectangular Piles in Sloping Ground

3.1. Modification Factor for Ultimate Lateral Capacity (K_H)

Based on Table 1, to achieve rapid evaluation of the lateral ultimate bearing capacity, this section takes the bearing capacity H_{u0} of the standard square pile in level ground ($\alpha = 0^\circ$, $\eta = 1$) as the reference, and establishes a coupled correction coefficient K_H considering both the slope and the aspect ratio. For a case within the investigated parameter domain, the ultimate lateral capacity $H_u(\alpha, \eta)$ of a rectangular pile in sloping ground can be expressed as:

$$K_H = \frac{H_u(\alpha, \eta)}{H_{u0}} = [1 - \mu \cdot \sin(\alpha)] \cdot \eta^\beta \quad (1)$$

where K_H is modification factor for ultimate lateral capacity, α is the slope angle, η is the cross-sectional aspect ratio of the rectangular pile, and μ and β are the coefficients to be fitted.

Based on Equation (1) and Table 1, the calculated values of K_H are listed in Table 4.

Table 4. Modification factor for ultimate horizontal bearing capacity, K_H .

$\alpha \backslash \eta$	1	2	3	4
0°	1.000	1.260	1.515	1.646
15°	0.904	1.158	1.410	1.533
20°	0.872	1.124	1.373	1.496
30°	0.802	1.048	1.289	1.409

To eliminate the influence of sloping terrain, the three-dimensional finite element data under the level-ground reference condition were extracted. In this case, the slope-effect weakening term $1 - \mu \sin(\alpha)$ is equal to 1, and the objective function is reduced to a pure section-effect model:

$$K_H(0^\circ, \eta) = \eta^\beta \quad (2)$$

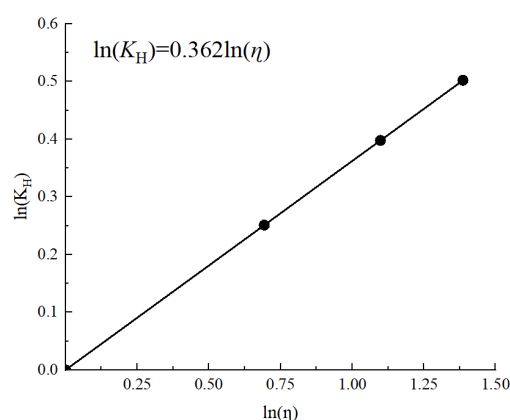
To avoid the possibility that direct nonlinear regression may fall into a local optimum, the natural logarithm was taken on both sides of the equation. The original problem was then transformed into a zero-intercept linear regression problem, with $\ln(\eta)$ as the independent variable and $\ln(K_H)$ as the dependent variable:

$$\ln[K_H(0^\circ, \eta)] = \beta \ln(\eta) \quad (3)$$

It is important to emphasize that the zero-intercept constraint applied in this log-linear regression is not merely a mathematical adjustment, but a strict representation of the fundamental physical boundary conditions. Specifically, when the cross-section of the rectangular pile degrades into a standard square with $\eta = 1$ and the terrain degrades into level ground with $\alpha = 0^\circ$, the correction coefficient K_H must logically equal unity. This constraint ensures that the formulation returns to the reference response when $\eta = 1$ and $\alpha = 0^\circ$, prevents the regression from yielding non-physical results, and anchors the analytical framework to the defined baseline condition.

According to Figure 2, the following result can be obtained:

$$\ln[K_H(0^\circ, \eta)] = 0.362 \ln(\eta) \quad (4)$$

**Figure 2.** Fitting comparison of coefficient β .

The zero-intercept log-linear regression yields $R^2 = 0.999$ indicating a close fit to the calibration data. The resulting coefficient β is:

$$\beta = 0.362 \quad (5)$$

The regression coefficients are obtained from a bounded FEM dataset covering slope angles from 0° to 30° and aspect ratios from 1 to 4, ensuring consistency within the calibration domain. After obtaining the preliminary solution of coefficient β , it was substituted into the full sample space. The normalized bearing-capacity coefficient after eliminating the section effect was defined as $K_H^* = K_H/\eta^\beta$. At this point, the objective function was transformed into a linear equation related to the slope angle:

$$1 - K_H^* = \mu \cdot \sin(\alpha) \quad (6)$$

Taking $\sin(\alpha)$ as the horizontal coordinate and $1 - K_H^*$ as the vertical coordinate, the terrain weakening coefficient μ can be directly separated. The data fitting curve is shown in Figure 2, and the fitting accuracy is relatively high ($R^2 = 0.963$). Therefore, the coefficient μ is obtained as:

$$\mu = 0.322 \quad (7)$$

Figure 3 shows that, under the same α , the data points corresponding to different η values exhibit slight dispersion. This is because changes in η alter the boundary of the shallow sliding wedge, leading to three-dimensional mechanical coupling between the slope and the section. Although the decoupling assumption of the model $K_H = [1 - \mu \sin(\alpha)]\eta^\beta$ converts this weak coupling into residual error, the fitting degree still reaches $R^2 = 0.963$. The remaining dispersion reflects the weak interaction between slope angle and cross-sectional aspect ratio that is not explicitly represented by the multiplicative formulation. Accordingly, the proposed expression should be interpreted as a simplified representation of the calibration dataset within the investigated parameter domain.

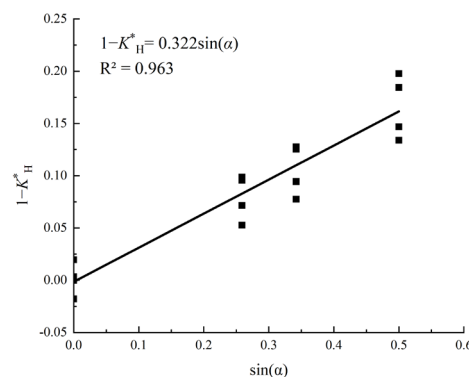


Figure 3. Fitting comparison of coefficient μ .

Therefore, the final form of the macroscopic coupled correction model for bearing capacity in Equation (1) is:

$$K_H(\alpha, \eta) = [1 - 0.322 \cdot \sin(\alpha)] \cdot \eta^{0.362} \quad (8)$$

To visually reveal the evolution of bearing capacity under the combined effects of slope weakening and section enhancement, this study constructed a three-dimensional response surface of K_H based on the fitted formula and mapped the discrete finite element data points for spatial comparison, as shown in Figure 4. Figure 4 shows that the spatial surface of K_H is jointly governed by section compensation and terrain weakening. Along the η -axis, the surface rises in an upward-convex form ($\beta = 0.362$), indicating that increasing the loading-face width can effectively mobilize deep soil resistance, although the increase in bearing capacity shows a diminishing marginal effect. Along the α -axis, the surface decreases monotonically ($\mu = 0.322$), reflecting the mechanical nature that an increase

in slope angle leads to the loss of shallow passive-zone soil. The agreement between the fitted response surface and the 16 finite-element data points indicates that the proposed decoupled formulation provides a consistent representation of the dataset within the calibration domain.

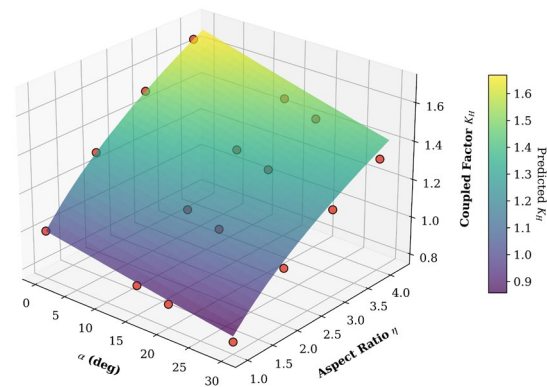


Figure 4. Combined response surface of slope and cross-sectional effects for K_H .

The fitted-versus-benchmark comparison in Figure 5 shows that the calculated values are distributed close to the 1:1 line within the calibration domain. This agreement indicates that the two-variable multiplicative formulation provides a compact representation of the published dataset, while the residual dispersion reflects the simplification of the coupled three-dimensional response.

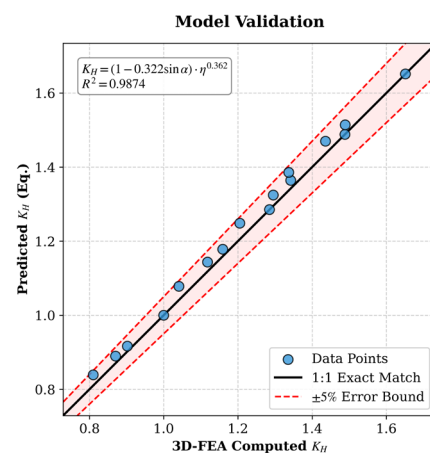


Figure 5. Comparison of the fitted K_H values with the published FEA benchmark values.

3.2. Modification Factor for Maximum Bending Moment (K_M)

Following the modeling approach used for the bearing-capacity model, this section introduces a coupled correction coefficient K_M considering both the slope and the aspect ratio. Taking the maximum bending moment $M_{\max 0}$ of the standard square pile in level ground ($\alpha = 0^\circ$, $\eta = 1$) as the reference, the maximum bending moment $M_{\max}(\alpha, \eta)$ of a rectangular pile in a slope within the investigated parameter domain can be expressed as:

$$K_M = \frac{M_{\max}(\alpha, \eta)}{M_{\max 0}} = [1 - \mu_M \cdot \sin(\alpha)] \cdot \eta^{\beta_M} \quad (9)$$

where K_M is modification factor for maximum bending moment, and μ_M and β_M are the coefficients to be fitted.

According to Equation (9) and Table 2, the macroscopic coupled correction coefficient K_M for maximum bending moment can be further obtained, as shown in Table 5.

Table 5. Modification factor for maximum bending moment, K_M .

$\alpha \backslash \eta$	1	2	3	4
0°	1.000	1.428	1.860	2.122
15°	0.948	1.372	1.817	2.050
20°	0.933	1.352	1.792	2.025
30°	0.896	1.304	1.739	1.961

This section continues to adopt the same procedure as Section 3.1. Based on the data in Table 5, distribution fitting was performed, and the following fitting equation was obtained:

$$K_M(\alpha, \eta) = [1 - 0.194 \cdot \sin(\alpha)] \cdot \eta^{0.483} \quad (10)$$

Figure 6 presents the three-dimensional response surface of the correction coefficient K_M and the finite element data points. Compared with the bearing-capacity model K_H , K_M rises more steeply along the aspect-ratio axis η , and its section amplification exponent $\beta = 0.483$ is significantly higher than 0.362. The mechanical reason is that, as η increases, the section moment of inertia I increases cubically. The high-stiffness wide pile shows relatively rigid behavior and forces the resistance to transfer to deeper layers. This “stiffness attraction effect” makes it difficult for the pile shaft to release bending through flexural deformation, thereby inducing an exponential amplification of the peak bending moment. Therefore, this reveals that while increasing the aspect ratio effectively compensates for the slope-induced loss of lateral capacity, it simultaneously demands a significantly higher flexural capacity from the pile shaft. In practical engineering, this implies that designers cannot blindly increase η . The amplification of the bending moment requires a proportional increase in longitudinal reinforcement, stricter crack control, and enhanced ductility detailing to ensure structural safety.

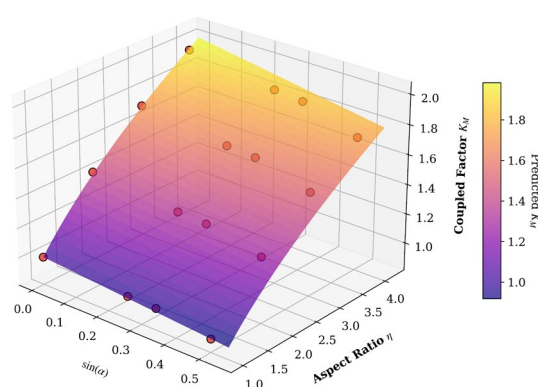


Figure 6. Combined response surface of slope and cross-sectional effects for K_M .

As shown in Figure 7, the response surface attenuates gently along the slope-angle direction. The terrain weakening coefficient $\mu_M = 0.194$ is significantly lower than that of the bearing-capacity model, which is 0.322, indicating that the maximum bending moment is less sensitive to sloping terrain. The mechanism is that the narrowing of the loading face, namely the reduction in geometric resistance, and the loss of shallow soil, namely the terrain weakening effect, form a superimposed “secondary weakening” effect, causing

the effective shallow restraint to be almost completely lost. At this stage, the deep load-transfer mechanism becomes dominant, and the formation of the maximum bending moment depends strongly on deep anchorage and high flexural stiffness. Therefore, variations in the shallow slope angle α have a much weaker influence on the extreme value of deep internal force, which is macroscopically reflected by the decrease in μ_M .

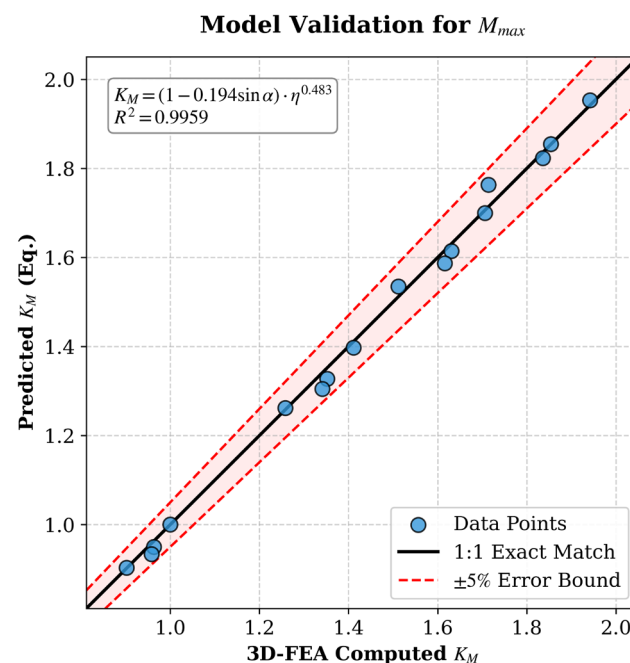


Figure 7. Comparison of the fitted K_M values with the published FEA benchmark values.

3.3. Modification Factor for Maximum Reverse Shear Force (K_Q)

The maximum reverse shear force is another core control indicator in the cross-sectional design of pile foundations. Particularly in sloping ground, it is imperative to prevent shear failure, as it is a brittle failure mode with catastrophic consequences. While the maximum shear force in the shallow layer equals the lateral load applied at the pile head, the exact magnitude of the maximum reverse shear force developed in the deep strata remains unquantified. Furthermore, under the influence of the slope effect, how the maximum reverse shear force evolves compared to that in level ground is still unclear. Therefore, this section develops a simplified calculation expression for the deep maximum reverse shear force. Similarly, taking the square pile (aspect ratio $\eta = 1$) in level ground (slope angle $\alpha = 0^\circ$) as the reference baseline, denoted as $|Q_{\max 0}^-|$, the maximum reverse shear force of a rectangular pile within the investigated ranges of slope angle α and aspect ratio η , denoted as $|Q_{\max}^-(\alpha, \eta)|$, can be obtained by multiplying the reference value by a coupled modification factor, K_Q . This approach facilitates a rapid evaluation of the maximum reverse shear force for rectangular piles in sloping ground based on existing engineering computational capabilities. Following this rationale, the coupled modification factor K_Q , which jointly incorporates the slope effect and the cross-sectional aspect ratio, is established as follows:

$$K_Q = \frac{|Q_{\max}^-(\alpha, \eta)|}{|Q_{\max 0}^-|} = [1 - \mu_Q \cdot \sin(\alpha)] \cdot \eta^{\beta_Q} \quad (11)$$

where K_Q is modification factor for maximum reverse shear force, and μ_Q and β_Q are the coefficients to be fitted.

According to Equation (11) and Table 3, the macroscopic coupled correction coefficient K_Q for maximum reverse shear force can be further obtained, as shown in Table 6.

Table 6. Modification factor for maximum reverse shear force, K_Q .

$\alpha \backslash \eta$	1	2	3	4
0°	1.000	1.157	1.355	1.492
15°	0.933	1.099	1.312	1.441
20°	0.910	1.078	1.292	1.422
30°	0.873	1.026	1.248	1.378

Similarly, based on the data in Table 6, distribution fitting was performed, and the following fitting equation was obtained:

$$K_Q = [1 - 0.230 \cdot \sin(\alpha)] \cdot \eta^{0.296} \quad (12)$$

As shown in Figure 8, the cross-sectional amplification index for K_Q ($\beta_Q = 0.296$) is substantially lower than those for the maximum bending moment (0.483) and the ultimate lateral capacity (0.362). This sharp contrast elucidates the depth-dependent spatial evolution of the pile-soil load-sharing mechanism. The maximum bending moment occurs at shallow depths, where the slope-induced loss of lateral soil confinement is most severe. Consequently, as the aspect ratio η increases, the enhanced flexural stiffness forces the pile to resist the external load predominantly by itself, triggering an exponential surge in the bending moment. Conversely, the maximum reverse shear develops in the deeper strata, where the slope effect diminishes. The intact deep soil provides sufficient passive resistance to share a considerable portion of the load, thereby alleviating the internal shear concentration induced by the increased pile stiffness and minimizing the sensitivity of the deep shear peak to cross-sectional shape variations. Furthermore, the K_Q surface exhibits a steady monotonic decrease along the slope angle α axis. The topographic weakening coefficient ($\mu_Q = 0.230$) indicates that the loss of shallow terrain still moderately reduces the absolute magnitude of the deep reverse shear.

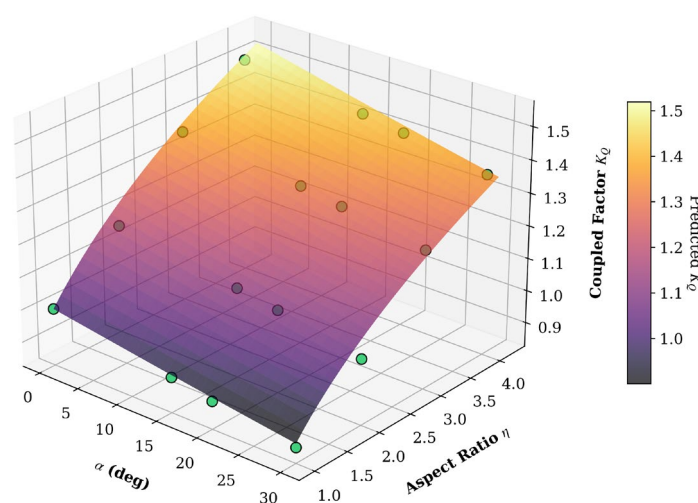


Figure 8. Combined response surface of slope and cross-sectional effects for K_Q .

As illustrated in Figure 9, the fitted K_Q values are distributed close to the 1:1 line for the 16 calibration cases, with $R^2 = 0.9674$. This result indicates that the proposed expression captures the principal variation in the published K_Q dataset within the calibration domain.

The comparison represents goodness of fit to the calibration data and should not be interpreted as independent predictive validation.

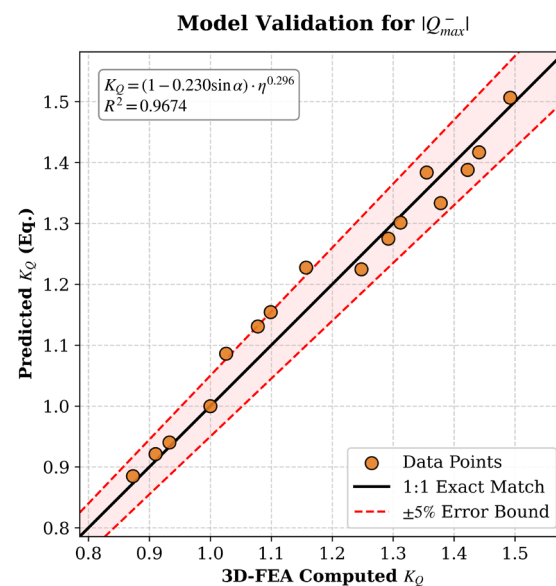


Figure 9. Comparison of the fitted K_Q values with the published FEA benchmark values.

4. Model Integration and Reproduction of Published Benchmark Responses

4.1. Calibration of the Transfer-Matrix Reference Solution for the Level-Ground Square Pile

The preceding sections established the modification factors for the macroscopic characteristic responses of rectangular piles in sloping ground, specifically the ultimate lateral capacity, maximum bending moment, and maximum reverse shear force. These modification factors are exclusively benchmarked against the load-bearing response of a reference square pile (aspect ratio $\eta = 1$) in level ground (slope angle $\alpha = 0^\circ$). Consequently, to derive the ultimate bearing solutions for rectangular piles under combined topographical and geometrical conditions, the bearing response of the conventional square pile in level ground must first be accurately determined. Figure 10 shows the loading model of a rectangular pile in level ground with an aspect ratio $\eta = 1$.

Research on calculation methods for the lateral bearing capacity of pile foundations is already extensive. Representative methods include the power series method proposed by Zhang et al. [26,27], the transfer matrix method represented by Zhu Mingxing et al. [28,29], the variational method [30,31], and the finite difference method in pure mathematical form [32,33]. Since these methods are already mature, their specific procedures are not discussed here.

In this section, the standard square pile in level ground ($\alpha = 0^\circ$, $\eta = 1$) is used as the reference. The pile response is first calculated using the transfer matrix method [28,29]. Then, combined with the correction coefficients for bearing capacity, maximum bending moment, and reverse shear force, the bearing characteristics of rectangular piles in slopes are estimated. The parameters of the reference square pile are as follows: side length $D = 2$ m, flexural stiffness $EI = 4.2 \times 10^7$ kN·m², and the m -method is adopted for the pile-soil interaction curve $p = mbzy$, with $m = 4500$ kN/m⁴. To reflect the plastic characteristics of the p - y curve, the ultimate soil resistance p_u is determined by displacement control [29], with $y_u = 3 \times D/80 = 0.075$ m.

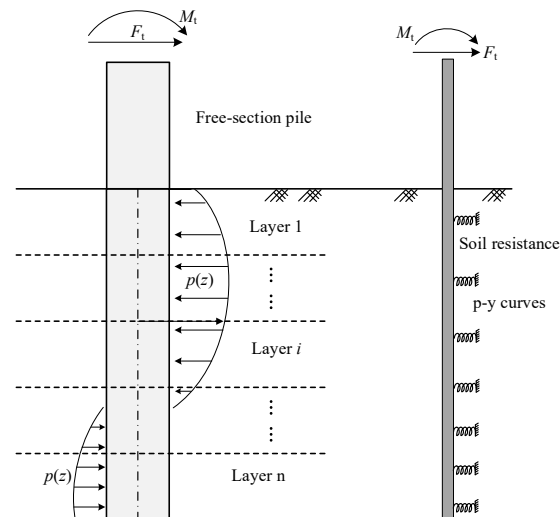


Figure 10. Schematic of a horizontally loaded pile on level ground.

The comparison between the theoretical calculation and finite element results is shown in Figure 11. Under the condition of $\alpha = 0^\circ$ and $\eta = 1$, the bending moment and shear force distributions of the rectangular pile agree well with the theoretical results.

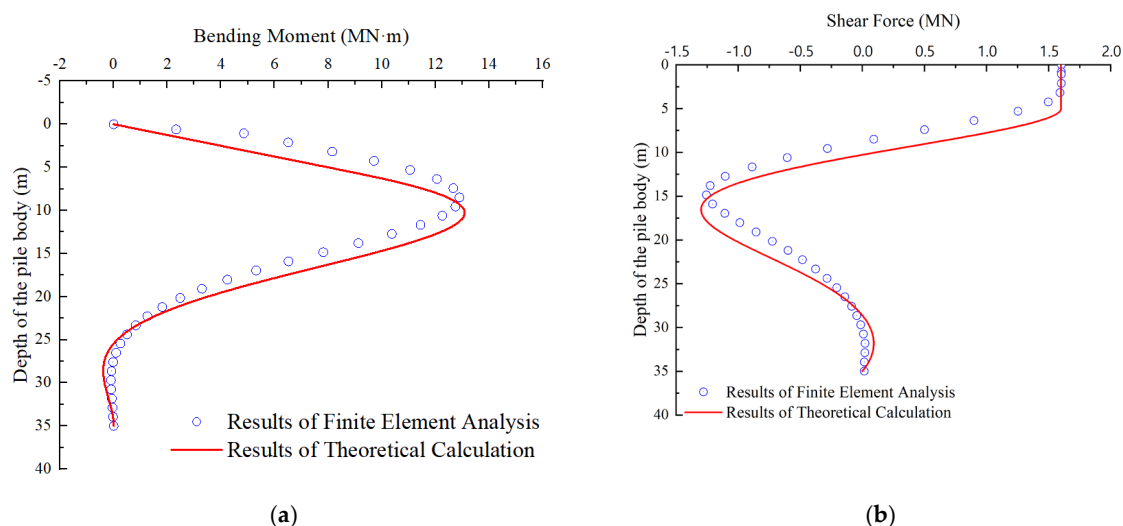


Figure 11. Calibration comparison between the transfer-matrix solution and the published FEA benchmark for $\alpha = 0^\circ$, $\eta = 1$. (a) Bending moment comparison; (b) Shear force comparison.

The theoretical maximum bending moment is 13.09 MN·m, and the maximum reverse shear force is 1.298 MN. Compared with the finite element results of 12.90 MN·m and 1.255 MN, the errors are 1.45% and 3.31%, respectively. The relatively small deviations indicate that the selected transfer-matrix parameters provide a numerically consistent reference response for the level-ground square-pile case.

As shown in Figure 12, after calibrating the transfer-matrix parameters for the reference square pile ($\alpha = 0^\circ$, $\eta = 1$), the same calculation procedure was applied to the level-ground cases with $\eta = 2, 3$, and 4. The calculated characteristic values were compared with the corresponding finite-element benchmark values reported in [24], as summarized in Table 7. This comparison does not constitute an independent validation because the modification factors were derived from the same published dataset. Instead, it examines whether the transfer-matrix reference solution and the proposed modification factors can be consistently integrated to reproduce the published benchmark responses. The resulting

deviations provide a quantitative measure of the internal numerical consistency of the integrated calculation framework.

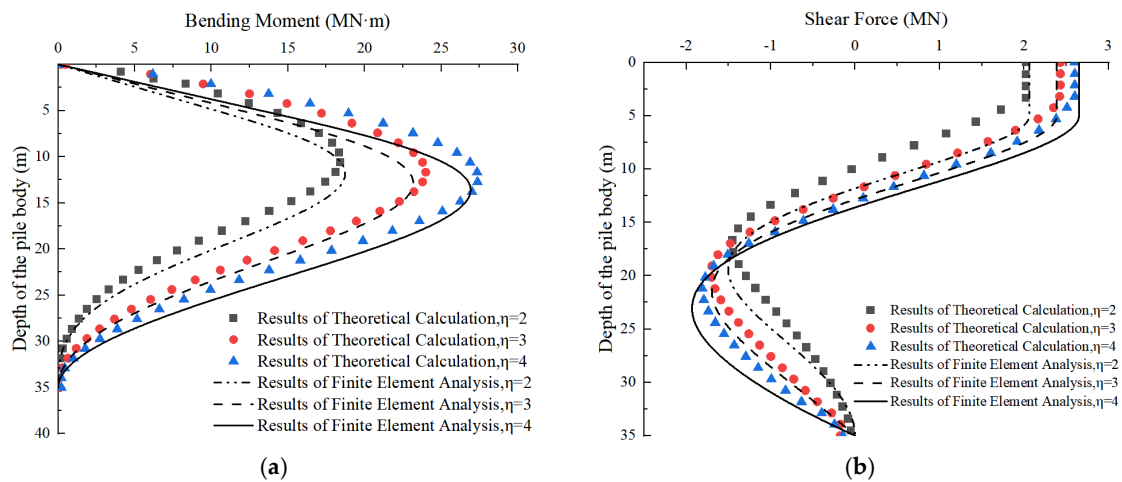


Figure 12. Benchmark comparison between the integrated calculation results and the published FEA values for $\alpha = 0^\circ$, $\eta = 2, 3$, and 4 . (a) Bending moment comparison; (b) Shear force comparison.

Table 7. Comparison of response characteristic values of rectangular piles under $\alpha = 0^\circ$, $\eta = 2, 3$, and 4 .

Response Characteristic		$\eta = 2$		$\eta = 3$		$\eta = 4$		Average (%)
		Result	Error (%)	Result	Error (%)	Result	Error (%)	
Ultimate lateral load (kN)	FEA	2020.87	/	2429.72	/	2640.28	/	/
	Theoretical	2061.29	+2	2387.18	−1.75	2649.19	+0.34	+0.20
	Predicted	2061.29	+2	2387.18	−1.75	2649.19	+0.34	+0.20
Maximum bending moment (MN·m)	FEA	18.42	/	23.99	/	27.37	/	/
	Theoretical	18.73	+1.68	23.17	−3.42	26.92	−1.64	−1.13
	Predicted	18.30	−0.65	22.25	−7.25	25.57	−6.58	−4.83
Maximum reverse shear force (MN)	FEA	1.452	/	1.701	/	1.873	/	/
	Theoretical	1.504	+3.58	1.697	−0.24	1.930	+3.04	+2.13
	Predicted	1.594	+9.78	1.795	+5.53	1.956	+4.43	+6.58

4.2. Reproduction of Published Benchmark Responses for Rectangular Piles in Sloping Ground

The comparisons presented in this section use the published finite-element results in [24] as benchmark values. Because these data also form the basis for deriving the modification factors, the comparisons are not intended as an independent or out-of-sample validation. Their purpose is to assess whether the integrated analytical framework can consistently reconstruct the published responses over the investigated combinations of slope angle and aspect ratio.

(1) Benchmark Comparison for Ultimate Lateral Capacity

Since a generally accepted modified p – y formulation for rectangular piles in sloping ground is not currently available, the integrated correction-coefficient framework described above is applied to reconstruct the characteristic responses reported in [24].

According to Equation (8), the reconstructed values of the lateral ultimate bearing capacity $H_u(\alpha, \eta)$ of rectangular piles in slopes can be obtained, as summarized in Table 8. The comparison with the results in Table 1 is shown in Figures 13 and 14.

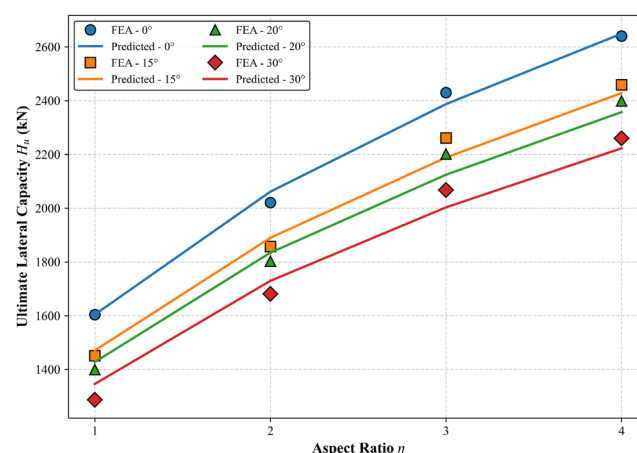


Figure 13. Comparison of reconstructed lateral capacities with the published FEA benchmark values.

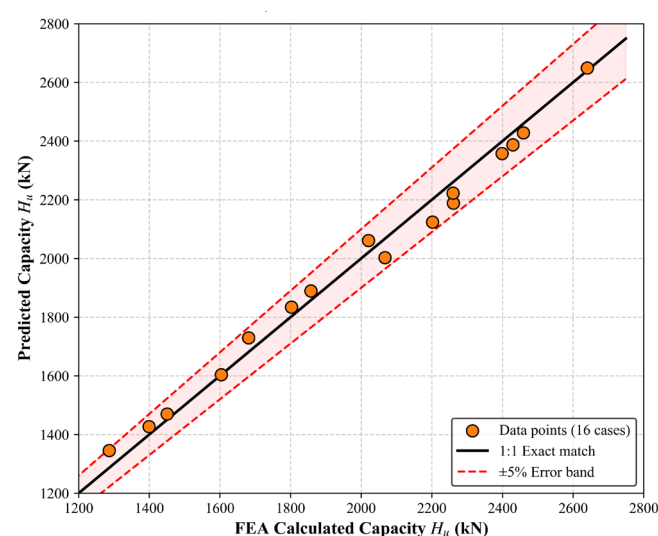


Figure 14. Deviation of reconstructed lateral capacities from the published FEA benchmark values.

Table 8. Reconstructed lateral ultimate bearing capacity $H_u(\alpha, \eta)$ of rectangular piles in slopes (kN).

$\alpha \backslash \eta$	1	2	3	4
0°	1603.86	2061.29	2387.18	2649.19
15°	1470.21	1889.51	2188.24	2428.40
20°	1427.23	1834.28	2124.28	2357.43
30°	1345.64	1729.42	2002.84	2222.67

As shown in Figure 13, the prediction formula agrees well with the three-dimensional finite element results. For the same slope angle α , H_u increases significantly with η . Although the effective pile-soil width becomes smaller, the increase in the section moment of inertia greatly improves the flexural stiffness EI , allowing the load to be transferred more effectively to deeper soil layers. For the same η , H_u decreases clearly as α increases, with the highest value at 0° and the lowest at 30°. This is because the loss of soil in front of the slope reduces the effective overburden stress in the shallow passive zone, intensifies plastic yielding, and greatly weakens the ultimate resistance of the foundation.

As shown in Figure 14, the proposed prediction formula accurately reflects the non-linear coupling between slope weakening and stiffness enhancement. For the 16 working conditions, the theoretical predictions agree well with the finite element results. Across

the 16 investigated cases, the signed deviations between the reconstructed lateral capacities and the published benchmark values range from -3.53% to $+4.58\%$, with a mean absolute percentage deviation of 2.06% and a maximum absolute deviation of 4.58% . The largest deviation occurs at $\alpha = 30^\circ$ and $\eta = 1$. These results indicate that the proposed framework consistently reproduces the lateral-capacity variation represented by the published benchmark dataset within the investigated parameter domain.

(2) Benchmark Comparison for Maximum Bending Moment

Based on the predicted values of $M_{\max}(\alpha, \eta)$ from Equation (10), the results are summarized in Table 9, and the comparison with Table 2 is shown in Figures 15 and 16. Across the 16 investigated cases, the signed deviations between the reconstructed maximum bending moments and the published benchmark values range from -10.43% to $+2.25\%$, with a mean absolute percentage deviation of 5.18% and a maximum absolute deviation of 10.43% . Fourteen of the 16 cases fall within $\pm 10\%$, whereas the cases with $\alpha = 20^\circ, \eta = 3$ and $\alpha = 30^\circ, \eta = 3$ exhibit deviations of approximately -10.16% and -10.43% , respectively. The predominantly negative deviations at larger aspect ratios indicate that the simplified formulation tends to underestimate the benchmark maximum bending moments. This tendency should be considered when the method is used for preliminary flexural assessment.

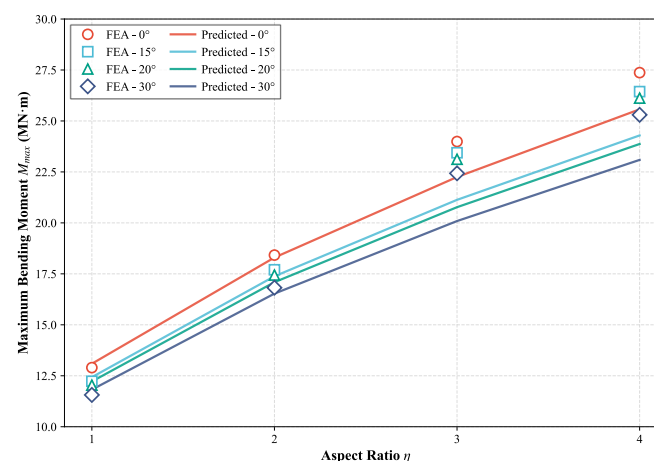


Figure 15. Comparison of reconstructed maximum bending moments with the published FEA benchmark values.

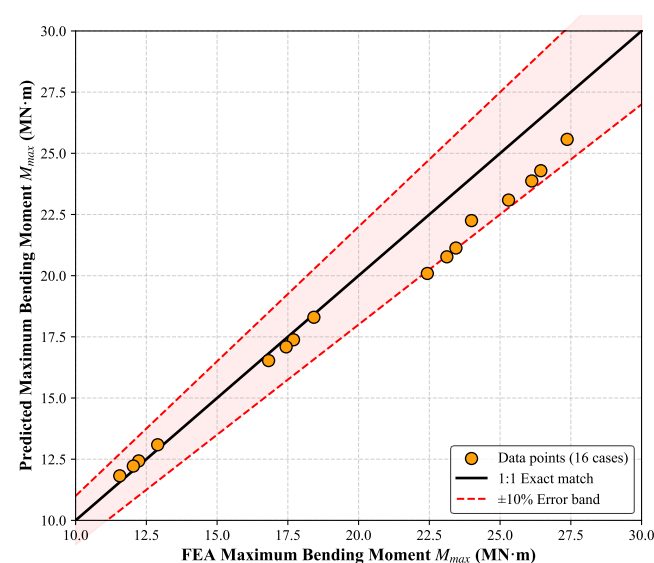


Figure 16. Deviation of reconstructed $M_{\max}$ from the published FEA benchmark values.

Table 9. Predicted maximum bending moment $M_{\max}(\alpha, \eta)$ of rectangular piles in slopes (MN·m).

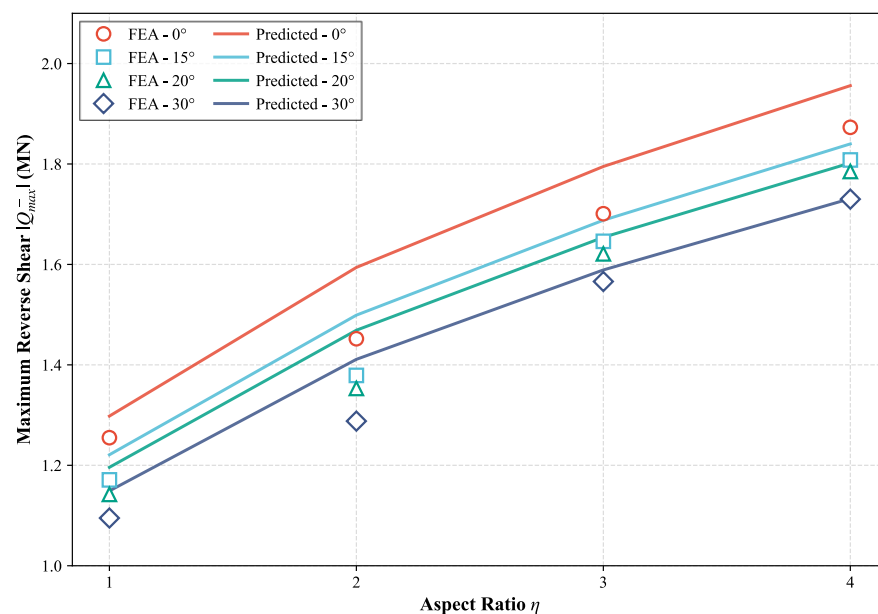
$\alpha \backslash \eta$	1	2	3	4
0°	13.09	18.30	22.25	25.57
15°	12.43	17.38	21.13	24.29
20°	12.22	17.09	20.77	23.87
30°	11.82	16.53	20.09	23.09

(3) Benchmark Comparison for Maximum Reverse Shear Force

The complete predicted values for the maximum reverse shear force $|Q_{\max}^-(\alpha, \eta)|$ of rectangular piles in sloping ground are obtained using Equation (11) and summarized in Table 10. Figures 17 and 18 illustrate the comparison between the results in Table 10 and those in Table 3. Across the 16 investigated cases, the signed deviations between the reconstructed maximum reverse shear forces and the published benchmark values range from +0.06% to +9.78%, with a mean absolute percentage deviation of 4.55% and a maximum absolute deviation of 9.78%. All reconstructed values are slightly higher than the corresponding benchmark values, indicating a modest conservative tendency for maximum reverse shear within the investigated parameter range. The simplified formulation also reproduces the general increase in reverse shear with aspect ratio and its gradual decrease with slope angle.

Table 10. Predicted maximum reverse shear force $|Q_{\max}^-(\alpha, \eta)|$ of rectangular piles in slopes (MN).

$\alpha \backslash \eta$	1	2	3	4
0°	1.298	1.594	1.795	1.956
15°	1.221	1.499	1.688	1.840
20°	1.196	1.469	1.654	1.802
30°	1.149	1.411	1.589	1.731

**Figure 17.** Comparison of reconstructed maximum reverse shear forces with the published FEA benchmark values.

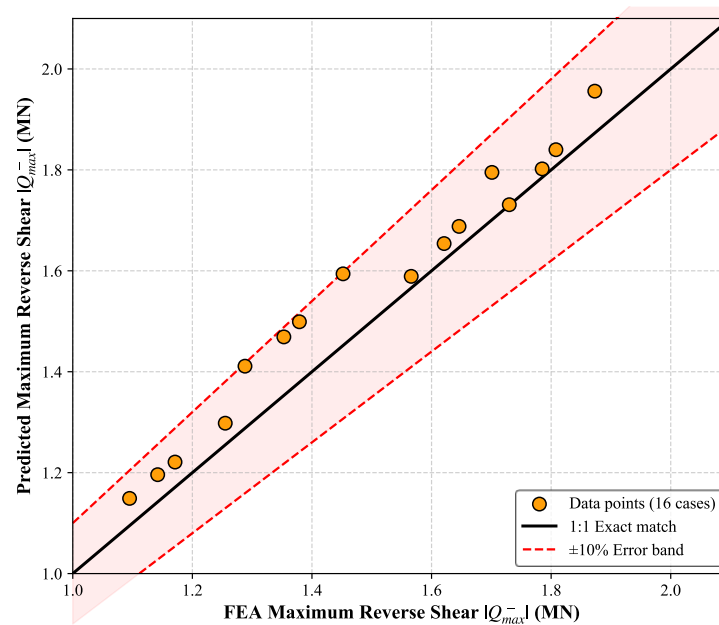


Figure 18. Deviation of reconstructed $|Q_{max}|$ from the published FEA benchmark values.

4.3. Discussion

The primary objective of this study is to establish a rapid and practical evaluation framework for laterally loaded rectangular piles in sloping ground, addressing the limitations of conventional p - y curve methods and the high computational cost of full 3D-FEA. By decoupling the effects of slope geometry and cross-sectional shape, the proposed modification factors K_H , K_M , and K_Q enable efficient prediction of pile response based on conventional level-ground solutions, significantly improving computational efficiency for preliminary design.

Comparison with the published benchmark dataset yields mean absolute percentage deviations of 2.06%, 5.18%, and 4.55% for the ultimate lateral capacity, maximum bending moment, and maximum reverse shear force, respectively. The corresponding maximum absolute deviations are 4.58%, 10.43%, and 9.78%. The relatively larger deviation in maximum bending moment occurs mainly at larger aspect ratios, where the simplified formulation tends to underestimate the benchmark response. This behavior reflects the residual error introduced when complex three-dimensional stress redistribution is represented by decoupled multiplicative correction factors. Consequently, the reconstructed bending moments should be interpreted with greater caution than the reconstructed lateral capacities and reverse shear forces.

It should be emphasized that the proposed method is a first-order analytical tool developed from the published numerical dataset in [24]. The comparisons in Section 4 assess the reproduction of that benchmark dataset and the numerical consistency of the integrated framework; they do not constitute an independent out-of-sample validation. The present formulation is calibrated only for a single rectangular pile under monotonic lateral loading and under the homogeneous-soil and fixed model conditions adopted in [24]. Within this model configuration, only the slope angle and cross-sectional aspect ratio were varied, over the ranges $0^\circ \leq \alpha \leq 30^\circ$ and $1 \leq \eta \leq 4$. In addition, the present framework does not account for cyclic loading, combined vertical–lateral loading, pile-group interaction, layered or spatially heterogeneous soil, strain-softening behavior, groundwater or seepage effects, or progressive slope failure. These conditions may alter the pile–soil load-transfer mechanism and internal-force distribution; therefore, application of the proposed factors to such cases requires additional assessment and, where necessary, recalibration.

The soil strength and stiffness parameters, pile dimensions and cross-sectional area, pile position relative to the slope, slope height and other geometric conditions, ground-water condition, and loading procedure were not varied. Therefore, direct extrapolation to different soil strengths or layered profiles, different slope heights or pile–slope positions, different groundwater or seepage conditions, different pile dimensions, cyclic or combined loading, or pile-group conditions should be avoided. Such applications require additional numerical or experimental assessment and, where necessary, recalibration of the modification factors. Independent assessment using experimental measurements, field monitoring data, or numerical datasets not involved in model development is required before the method can be generalized beyond the present domain and will be pursued in future work.

5. Conclusions

This study investigates the lateral bearing behavior of rectangular piles in sloping ground and establishes a rapid evaluation framework for preliminary assessment within the investigated numerical domain. By innovatively decoupling the topographical slope effect and the cross-sectional shape effect, a macroscopic analytical method is proposed. The main conclusions are drawn as follows:

- (1) Based on the available three-dimensional finite element simulations of rectangular piles under various slope angles ranging from 0° to 30° and cross-sectional aspect ratios ranging from 1 to 4, modification factors were mathematically established utilizing a level-ground square pile as the baseline, which include the ultimate lateral capacity factor K_H , the maximum bending moment factor K_M , and the maximum reverse shear force factor K_Q .
- (2) For the investigated aspect ratios, increasing the slope angle from 0° to 30° reduces the ultimate lateral capacity by 14.40–19.78%, corresponding to a maximum reduction of approximately 20%. Under a 30° slope, increasing the aspect ratio from 1 to 4 increases the lateral capacity by approximately 75.7%.
- (3) Both the maximum bending moment and the deep maximum reverse shear force increase nonlinearly as the aspect ratio η increases, and they exhibit a gradual decay as the slope angle α increases. Additionally, the positions of the peak bending moment and the extreme shear force shift downward along the pile shaft under the influence of the sloping ground.
- (4) A rapid evaluation method was developed by combining the proposed modification factors with the conventional level-ground square-pile solution. Within the investigated parameter range, the maximum absolute deviations in H_u , $M_{\max}$, and $|Q_{\max}|$ are 4.58%, 10.43%, and 9.78%, respectively.

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