

Article

Restrained Torsional Response of Composite Box Girder Bridge with Corrugated Steel Webs During Balanced Cantilever Construction

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Abstract

During the balanced cantilever construction of long-span composite box-girder bridges with corrugated steel webs (CSWs), eccentric construction loads induce restrained torsion in addition to bending and shear. If this torsional contribution is not adequately accounted for, the resulting warping normal stresses in the concrete slabs and shear stresses at the CSW–slab interfaces may be significantly underestimated—particularly during stages when the cantilever section remains partially open and torsional stiffness is reduced. Although restrained torsion has been extensively studied for prismatic or fully closed box sections, the response of variable-depth composite girders with CSWs under asynchronous pouring construction (APC)—where the cantilever tip may be temporarily unclosed—has not been systematically clarified. To address this gap, this study proposes an equivalent modeling approach that incorporates the orthotropic characteristics of CSWs. Based on the restrained-warping theories of Umanskii and Vlasov, the governing differential equations for an eccentrically loaded cantilever are derived and solved via a finite-difference scheme with appropriate end-boundary conditions, yielding closed-form expressions for warping normal and shear stresses. The proposed analytical method is validated against three-dimensional finite-element (FE) simulations of an actual bridge. The validated FE model is then used to simulate the cantilever erection process, systematically evaluating the effects of construction scheme, diaphragm casting sequence, and critical APC stages on the torsional response. Results indicate that the equivalent CSW model accurately captures the torsional behavior of variable-depth composite girders. Restrained torsion from eccentric loading produces substantial secondary stresses: warping normal stresses in the bottom and top slabs reach up to 26% and 18% of their bending counterparts, respectively, whereas warping shear stresses in the CSWs account for approximately 22% of the shear stress. Torsional resistance is highly sensitive to the construction method; APC sequences, in particular, induce highly variable warping stresses near the cantilever tip, where eccentric loading should be avoided. Critical-stage analysis further reveals that the long-cantilever stage with an unclosed section represents the most vulnerable condition, owing to abrupt changes in torsional stiffness and local stress concentrations at the end-section transition. Internal diaphragms promote a more uniform distribution of warping deformation from the fixed support to the cantilever end; accordingly, it is recommended that each diaphragm be cast promptly upon completion of its corresponding segment. Overall, the proposed analytical–numerical framework offers a practical tool for rapid restrained-torsion assessment, identification of critical construction stages and



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regions, control of eccentric construction loads, and rational selection of diaphragm casting sequences—thereby supporting construction-stage risk mitigation for long-span bridges with CSWs.

Keywords: corrugated steel web; composite box girder; balanced cantilever construction; restrained torsion; warping stress; diaphragm; construction stage

1. Introduction

Conventional long-span prestressed concrete box-girder (PCBG) bridges are plagued by several inherent deficiencies, including the substantial self-weight, a high susceptibility to cracking, and relatively low prestressing efficiency. These limitations collectively impede their further development toward larger spans, lighter structural systems, and more efficient construction practices [1–5]. In response, composite box-girder bridges with corrugated steel webs (BGCSWs) have attracted increasing attention from both the engineering community and the research field. In this structural system, the conventional concrete webs are replaced by corrugated steel webs (CSWs), which exhibit high shear buckling resistance and low axial stiffness. This configuration enables the concrete top slab, concrete bottom slab, and CSWs to respectively resist bending moments and shear forces, thereby achieving highly efficient load-carrying behavior [6,7]. Furthermore, the low axial stiffness of CSWs enhances prestressing efficiency, while the reduced web weight significantly decreases the overall structural self-weight and eliminates the risk of cracking in concrete webs. To date, BGCSWs have been widely adopted in long-span and ultra-long-span continuous girder bridges, continuous rigid-frame bridges, and various other structural forms [8,9].

The conventional cantilever method with form travelers is currently one of the most commonly adopted construction techniques for long-span BGCSWs. Previous studies and engineering applications [10–14] have demonstrated that this method eliminates the need for ground-supported falsework and exhibits strong adaptability to complex site conditions, making it particularly suitable for crossings over mountainous valleys, rivers, and existing transportation corridors. However, these studies have primarily focused on construction feasibility, equipment arrangement, and general process adaptability, with relatively limited attention paid to the refined stress response induced by eccentric actions during cantilever erection.

In addition to the bending and shear effects conventionally considered in construction-stage analysis, eccentric construction loads can induce restrained torsion [15–17]. Existing studies have shown that such torsional restraint generates additional normal stresses in the concrete top and bottom slabs, as well as supplementary shear stresses at the web-to-slab connections, thereby modifying the overall stress state of the structure. Nevertheless, the available discussions are largely based on conventional reinforced or prestressed concrete box girders under simplified loading and boundary conditions; the applicability of these findings to variable-section BGCSWs during staged cantilever construction has not yet been systematically examined.

Furthermore, eccentric loading may also trigger distortion of the box-girder cross-section [18–21], leading to relative deformation of the sectional shape and consequent redistribution of local stresses. Prior studies have established the importance of distortion effects in box-girder analysis; however, they have generally treated restrained torsion and sectional distortion as separate or idealized problems, without fully addressing their coupled influence during the construction process of BGCSWs. In addition, this issue is further complicated by the reduced axial stiffness of the corrugated webs, the distinct

shear-transfer mechanism at the web-to-slab interfaces, and the progressive evolution of the cantilever section during segmental erection. As a result, neglecting restrained torsion while considering only bending and shear may lead to a underestimation of actual stress levels at critical structural locations. Therefore, a more targeted investigation into the restrained torsion effects of long-span BGCSWs under eccentric loading during cantilever construction is warranted to close this knowledge gap and provide a rational basis for construction-stage safety assessment.

Significant progress has been made in theoretical studies on the restrained torsion effects of long-span box girders. Chen & Zeng [22] derived analytical solutions for members with mono-symmetric I-sections and channel sections. Hu et al. [23] and Zhang et al. [24] proposed analytical methods applicable to box-girder sections symmetric about the vertical axis. Zhou et al. [25] developed an efficient simplified method for torsion–distortion analysis of box girders with arbitrary asymmetric cross-sections. Arici et al. [26] proposed a practical analytical method for bridge structures that accounts for bending, torsion, nonuniform torsion, distortion, and shear-lag effects; this method is theoretically consistent with higher-order beam theories and overcomes the difficulties encountered when conventional numerical solution methods are implemented within a finite-element framework. Additionally, Francisco et al. [18] and Li et al. [20] systematically investigated the torsional and distortional behavior of box-girder bridges and corresponding practical analysis methods. Collectively, these studies have provided an important theoretical foundation for analyzing the torsional behavior of box girders.

The load-bearing mechanism of BGCSWs differs significantly from that of conventional PCBGs [27–29]. Owing to the low axial stiffness of CSWs, they barely participate in flexural resistance, resulting in a negligible contribution to the warping normal stress induced by torsion. Moreover, the shear stiffness of CSWs differs from that of concrete webs, leading to a distinct distribution of torsional shear stress under torsional loading [30–32]. To facilitate torsional analysis of BGCSWs, metrics such as the equivalent elastic modulus, equivalent shear modulus, and equivalent web thickness are commonly adopted to achieve a reasonable equivalence of the CSWs [33–35]. Existing research on the load-bearing mechanism of BGCSWs has primarily focused on flexural behavior, shear behavior, shear-lag effects, stability and dynamic characteristics [33,34,36]. Studies have further demonstrated that the geometric parameters of CSWs, the detailing of shear connectors [37–39], and the arrangement of diaphragms all influence the overall spatial mechanical performance [40,41].

Furthermore, several scholars have investigated the spatial mechanical behavior of such composite box girders under eccentric loads, employing thin-walled box-girder theory, energy methods, or finite-element methods to analyze the distributions of warping normal stress, torsional shear stress, and distortional stress [42–45]. Shen et al. [43] and Zhu et al. [35] conducted experimental and theoretical studies on the pure torsional behavior of BGCSWs, analyzing the torque–twist relationship and the distribution of free torsional shear stress. In actual engineering structures, the torsion problem of box girders almost invariably manifests as a restrained torsional effect [44]. The significant additional stresses induced by eccentric loads in the concrete top and bottom slabs and at the web-to-flange connections are critical for the crack-resistance verification of the slabs and the shear check of the CSWs, and they also represent one of the primary factors limiting the span length of BGCSWs. Li et al. [45] performed load tests and finite-element analyses on a cable-stayed bridge with BGCSWs. The results indicated that under both symmetric and eccentric loads, the shear force distribution among the webs was uneven; the concrete top and bottom slabs could share a certain portion of the shear force; and the diaphragms provided a restraining effect on distortional deformation.

Most existing studies have focused on prismatic BGCSWs, whereas research on the restrained torsion of variable-section BGCSWs during cantilever construction remains limited. In particular, under asynchronous pouring construction (APC), the cantilever end has not yet formed a complete closed box section, resulting in local torsional stiffness, warping restraint conditions, and distortional deformation characteristics that differ from those of conventional cantilever box girders with fully closed sections, as shown in Figure 1. Furthermore, the casting sequence of internal diaphragms further influences the torsional stress distribution during construction. Therefore, there is a clear need to establish a corresponding analytical method for the restrained torsion problem of variable-section composite box-girder bridges with CSWs under eccentric loads during cantilever construction, and to integrate this method with finite-element models to elucidate the influence mechanisms of factors such as construction scheme and diaphragm casting sequence on the structural torsional response.

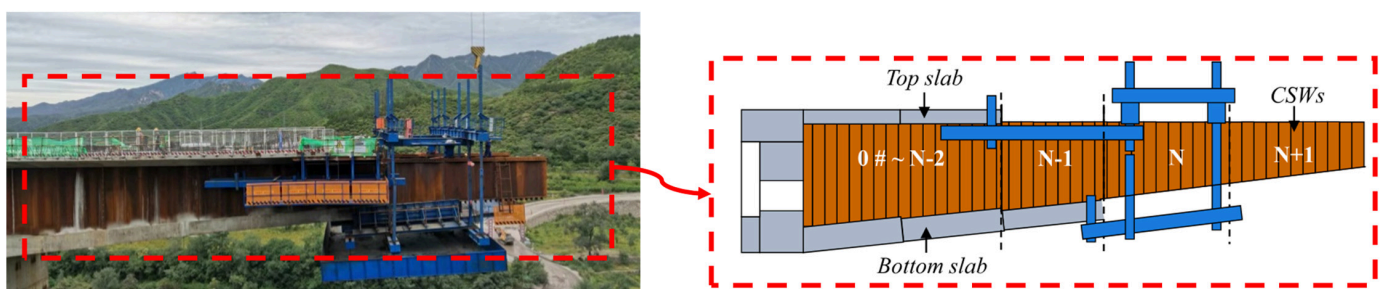


Figure 1. PC box-girder bridges with CSWs using APC.

Based on the foregoing, this paper takes a long-span BGCSW as a case study and investigates the restrained torsional effect of a variable-section BGCSW under eccentric loads during cantilever construction. First, an equivalent decomposition method for eccentric vertical loads is established. Subsequently, based on Umanskii's restrained torsion theory, the governing differential equations for restrained torsion of a variable-section cantilever box girder are derived, and a finite-difference scheme is employed to solve for warping normal stresses and torsional shear stresses. A three-dimensional finite-element model is then developed to verify the reliability of the proposed theoretical method. Furthermore, the restrained torsional response is examined by considering the influences of the asynchronous pouring construction (APC) method, diaphragm casting sequence, and critical construction stages. The findings are expected to provide practical guidance for eccentric load control, torsional safety assessment, and construction scheme optimization during the construction phase of such bridges.

2. Methodology

2.1. Equivalence of Eccentric Loading Effects

When an eccentric vertical load is applied to the top flange of a box girder with CSWs, the load can be decomposed into a bending component and an antisymmetric component, as shown in Figure 2. The antisymmetric component is further partitioned into torsional and distortional load components. All geometric and material parameters used in the subsequent analytical calculations were determined based on the design specifications of the prototype bridge considered in this study. These parameters were consistently applied throughout the theoretical derivations and the corresponding numerical simulations.

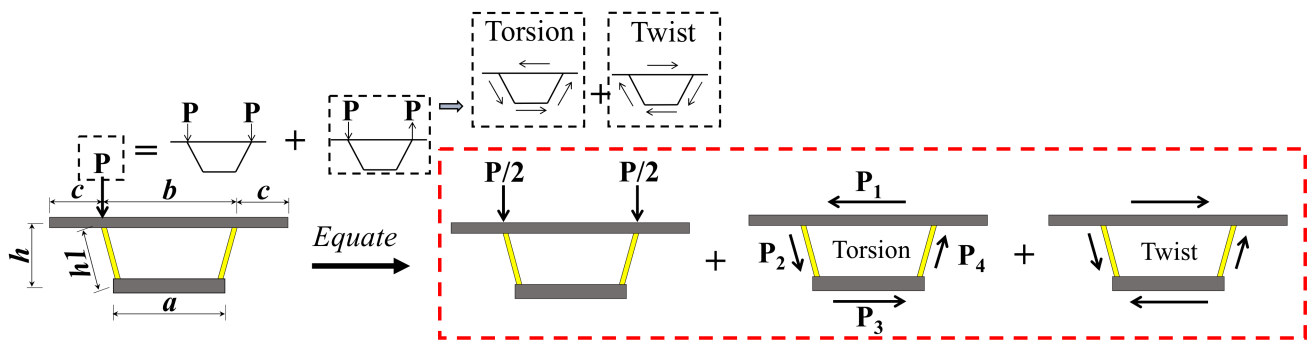


Figure 2. Decomposition of a vertical eccentric load into equivalent force systems.

For the torsional load case, the equivalent forces are given by

$$P_1 = \frac{Pb^2}{2(a+b)h} \quad (1)$$

$$P_2 = P_4 = \frac{Pbh_1}{2(a+b)h} \quad (2)$$

$$P_3 = \frac{Pab}{2(a+b)h} \quad (3)$$

The resultant external torque M_k acting on the cross-section is obtained as the moment of these decomposed forces about the section's shear center.

2.2. Equivalent Modeling of a Box Girder with CSW

The geometric parameters of the CSW are defined in Figure 3. Based on the condition of equal axial deformation between a flat web and a corrugated web, the equivalent longitudinal elastic modulus of the CSW can be expressed as

$$E_x = \frac{a_s + b_s}{3a_s + c_s} E_0 \left(\frac{t_s}{h_s} \right)^2 \quad (4)$$

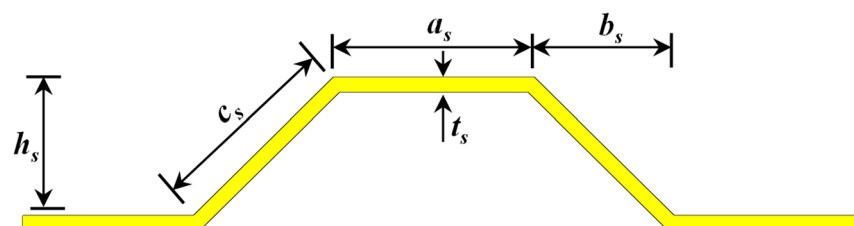


Figure 3. Geometrical parameters of the CSW.

The equivalent shear modulus G_{eq} is calculated as

$$G_{eq} = G_0 \frac{a_s + b_s}{a_s + c_s} \quad (5)$$

Applying the principle of equal total shear force in the web, the CSW can be replaced by a concrete web with equivalent thickness:

$$t_{eq} = t_s \frac{G_{eq}}{G_c} \quad (6)$$

For torsional analysis, the longitudinal stiffness E_x of the CSW can be neglected, as the web primarily resists the shear stress induced by eccentric loading. Through the

equivalences established above, a box girder with CSWs can be represented as an equivalent concrete box girder for torsion-related calculations.

3. Restrained-Torsion Analysis

3.1. Restrained-Torsion Differential Equation

Following Umanskii's second theory, the differential equation governing restrained torsion is given by

$$E\beta'''(z)I_{\bar{\omega}} - G\theta'(z)I_d = -M_k \quad (7)$$

where $\beta(z)$ is the warping intensity; $\theta(z)$ is the angle of twist; M_k is the total torque applied to the cross-section; I_d is the Saint-Venant torsional constant; and $I_{\bar{\omega}}$ is the generalized warping moment of inertia, defined as

$$I_{\bar{\omega}} = \oint \bar{\omega}^2 dA \quad (8)$$

$$\bar{\omega} = \int_0^s \rho ds - \frac{I_d}{\Omega} \int_0^s \frac{ds}{t} \quad (9)$$

where Ω is twice the area enclosed by the midline of the cross-section.

For the equivalent section of a composite girder with CSWs, the relevant dimensions are shown in Figures 2 and 3, where t_t and t_b denote the thicknesses of the concrete top and bottom slabs, respectively. The torsional constant is given by

$$I_d = \frac{\Omega^2}{\frac{b}{t_t(1-\beta)} + \frac{a}{t_b(1-\beta)} + \frac{2h_1}{t_{eq}(1+\beta)}} + \frac{2}{3}ct_t^3 \quad (10)$$

Differentiating Equation (7) with respect to the longitudinal coordinate z and letting $m_t = dM_k/dz$ denote the distributed torque, one obtains

$$E\beta''''(z)I_{\bar{\omega}} - G\theta''(z)I_d = -m_t \quad (11)$$

Additionally,

$$\frac{M_k}{GI_\rho} = \theta'(z) - \mu\beta'(z) \quad (12)$$

where $I_\rho = \oint t\rho^2 ds$ is the polar moment of inertia, and $\mu = 1 - \frac{I_d}{I_\rho}$ is the section-constraint coefficient. Hence,

$$\theta''(z) = \mu\beta''(z) + \frac{m_t}{GI_\rho} \quad (13)$$

Substituting Equation (13) into Equation (11) yields

$$\beta''''(z) - k^2\beta''(z) = -\frac{\mu}{EI_{\bar{\omega}}}m_t; \quad k^2 = \mu \frac{GI_d}{EI_{\bar{\omega}}} \quad (14)$$

3.2. Restrained Torsion Induced Stresses

The warping normal stress is expressed as

$$\sigma_w = \frac{B_{\bar{\omega}} \cdot \bar{\omega}}{I_{\bar{\omega}}} \quad (15)$$

where the bi-moment $B_{\bar{\omega}}$ is defined as

$$B_{\bar{\omega}} = -E\beta''(z) \oint \bar{\omega}^2 dA = -E\beta''(z)I_{\bar{\omega}} \quad (16)$$

The torsional shear stress can be written as

$$\tau_w = \frac{M_k}{t\Omega} - \frac{M_{\bar{w}}\bar{S}_{\bar{w}}}{I_{\bar{w}}t} \quad (17)$$

where M_k represents the total applied torque; $M_{\bar{w}}$ is the warping torque; and $S_{\bar{w}}$ is the generalized warping static moment, given by

$$M_{\bar{w}} = -E\beta'''(z)I_{\bar{w}} \quad (18)$$

$$\bar{S}_{\bar{w}} = S_{\bar{w}} - \frac{\oint S_{\bar{w}}\rho ds}{\Omega} \quad (19)$$

where $S_{\bar{w}}$ denotes the sectorial static moment, defined as $S_{\bar{w}} = \int_0^s \bar{w}t ds$. The aforementioned sectional properties $\bar{w}$, I_{ρ} , $I_{\bar{w}}$, I_d , $S_{\bar{w}}$, $\bar{S}_{\bar{w}}$, μ , k^2 , and others are determined from the known geometry; convenient calculation formulas are available in Refs. [23,24], and are therefore not repeated here.

Thus, the essential stresses induced by restrained torsion in a variable-depth cantilever girder are obtained by solving the differential Equation (14) together with the physical relations for the bi-moment and warping torque.

3.3. Finite Difference Solution

The problem is solved using the finite difference method [40]. Taking the free end of the cantilever as the coordinate origin, a longitudinal coordinate z is established. The girder is divided into n equal segments of length d , with nodes are numbered $i = 0, 1, 2, \dots, n$ (Figure 4).

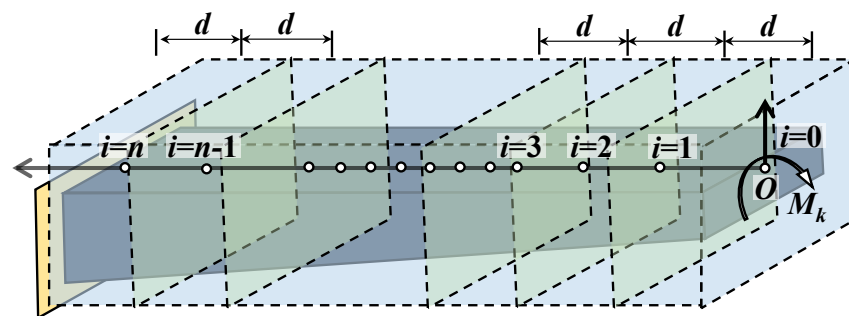


Figure 4. Finite segment model of a variable-depth cantilever beam.

At the free end, where warping is unrestrained and no restraining shear exists,

$$B_0 = 0; \quad \beta_0''' = 0 \quad (20)$$

At the fixed end, where both twist and warping are prevented,

$$\theta_n = 0; \quad \beta_n' = 0 \quad (21)$$

Substituting $B_{\bar{w}}'' = -E\beta''''(z)I_{\bar{w}}$ into Equation (11) gives

$$-B_{\bar{w}}'' - GI_d \left(\mu\beta''(z) + \frac{m_t}{GI_{\rho}} \right) = -m_t \quad (22)$$

Substitution of Equation (16) leads to

$$B_{\bar{w}}'' - \mu \frac{GI_d}{EI_{\bar{w}}} B_{\bar{w}} = \left(1 - \frac{I_d}{I_{\rho}} \right) m_t \quad (23)$$

i.e.,

$$B''_{\bar{\omega}} - k^2 B_{\bar{\omega}} = \mu m_t \quad (24)$$

The second-order finite difference approximation for $B''_{\bar{\omega}}$ is

$$B''_{\bar{\omega}} = \frac{B_{i+1} - 2B_i + B_{i-1}}{d_i^2} \quad (25)$$

Inserting Equation (25) into Equation (24) yields

$$B_{i+1} - (2 + k_i^2 d^2) B_i + B_{i-1} = \mu_i m_i d^2 \quad (i = 1, 2, \dots, n-1) \quad (26)$$

For the node at the fixed end ($i = n$), Equations (7), (12), (18) and (21) give

$$\frac{M_k^n}{G I_{\rho}^n} = \theta'_n - \mu \beta'_n = \theta'_n \quad (27)$$

$$E \beta_n''' I_{\bar{\omega}}^n - G \theta_n' I_d^n = -M_k^n \quad (28)$$

$$M_{\bar{\omega}}^n = -E \beta_n''' I_{\bar{\omega}}^n \quad (29)$$

$$\mu_n = 1 - \frac{I_d^n}{I_{\rho}^n} \quad (30)$$

Solving these simultaneously,

$$M_{\bar{\omega}}^n = M_k^n - \frac{M_k^n I_d^n}{I_{\rho}^n} = \mu_n M_k^n \quad (31)$$

The first-order finite difference formulation for $M_{\bar{\omega}}^n$ is

$$M_{\bar{\omega}}^n = \frac{B_{n+1} - B_{n-1}}{2d} \quad (32)$$

Substituting Equation (32) into Equation (31) gives

$$B_{n+1} = B_{n-1} + 2\mu_n d M_k^n \quad (33)$$

Finally, inserting Equation (33) into Equation (26) for $i = n$ yields

$$B_{n-1} - \left(1 + \frac{1}{2} k_n^2 d^2\right) B_n = \frac{\mu_n m_n d^2}{2} - \mu_n d M_k^n \quad (34)$$

Equation (26) (for $i = 1$) and Equation (34) constitute a linear system of n equations for the unknown bi-moments $B_1, B_2, \dots, B_n$, which can be solved efficiently by matrix methods.

Once the bi-moments are known, the warping torques follow from

$$M_{\bar{\omega}}^i = \frac{B_{i+1} - B_{i-1}}{2d} \quad (35)$$

and for $i = n$, they follow directly from Equation (31). The warping normal stress σ_w and torsional shear stress τ_w in each segment are then obtained from Equations (15) and (17). The complete solution procedure is summarized in Figure 5.

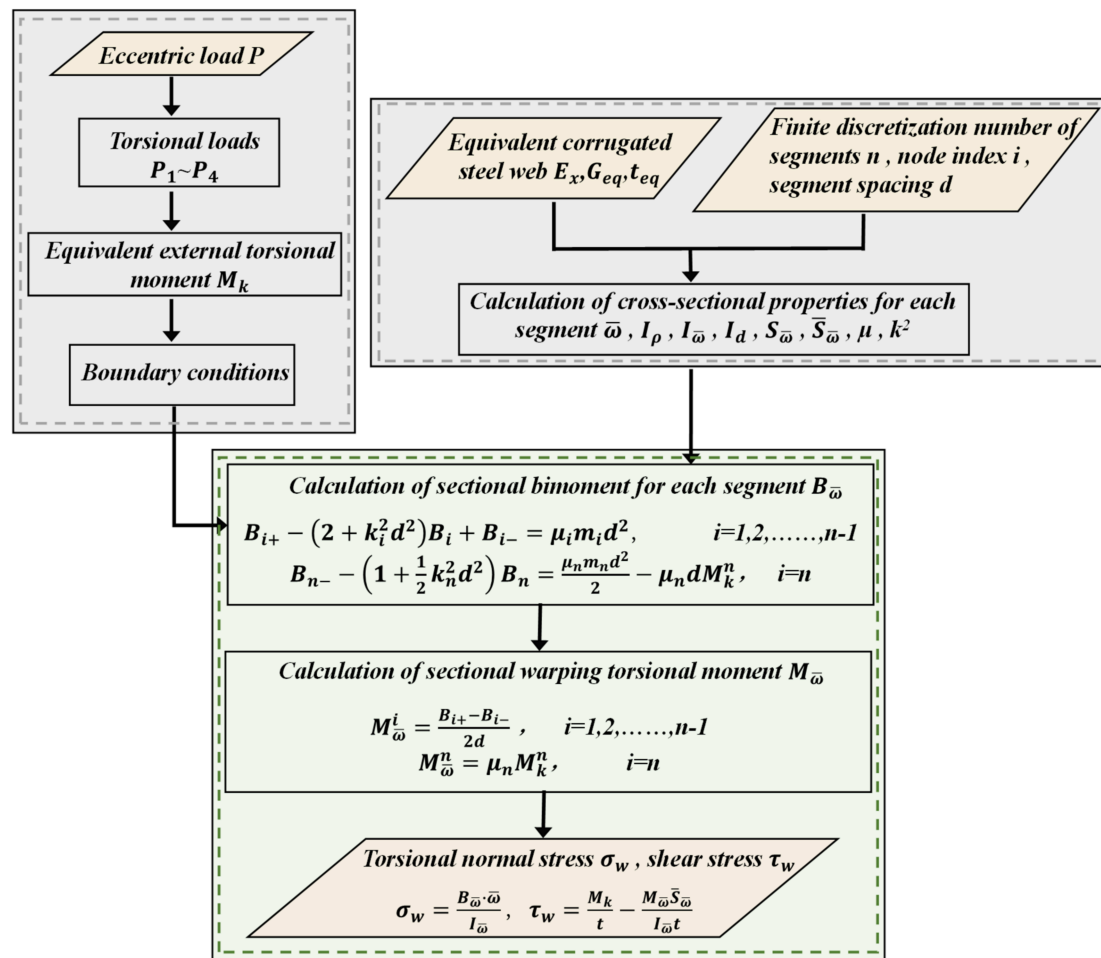


Figure 5. Flowchart of the finite difference solution for composite girders with CSWs.

4. Case Study

4.1. Parameters of the BGCSWs

The Beijing Damogou Bridge is adopted as a case study for verification. The superstructure consists of variable-depth rigid-frame continuous composite girders with CSWs, constructed using the APC method. The bridge has a total length of 330 m, with a span configuration of (60 + 105 + 105 + 60) m. The cross-section is a single-cell box girder with vertical webs. The girder depth varies parabolically from 7.0 m at the pier support to 3.5 m at the midspan and side piers. For comparative purposes, the analysis focuses on the segment from the support section to the location corresponding to the maximum cantilever length of 48 m, with appropriate sectional simplifications. The key geometric parameters are illustrated in Figure 6.

4.2. Comparison of Calculation Results

In the ABAQUS (v2020) model, as illustrated in Figure 7, the concrete top and bottom slabs are discretized using C3D8R solid elements, while the CSWs are modeled with S4R shell elements. The material properties adopted in the FE model were taken to be consistent with those of the prototype bridge. Specifically, C55 concrete was used for the top and bottom slabs, and Q345qD structural steel was adopted for the CSWs. The main mechanical properties employed in the numerical analysis are summarized in Table 1.

crement sizes were considered: 0.8 m, 1.6 m, and 2.4 m, corresponding to half, one, one and a half wavelengths of the corrugated steel web, respectively. The most critical eccentric loading case was selected, and the maximum warping normal stresses in the top and bottom slabs were used as the evaluation criteria.

As shown in Table 2, with the results from the 1.6 m mesh taken as the reference, the average percentage differences in the warping normal stresses for the top and bottom slabs were 1.43% and 1.7% for the 0.8 m and 2.4 m discretizations, respectively. These minor deviations indicate that, within the considered range, the discretization length has only a limited effect on the computed normal stresses. Therefore, a discretization increment of 1.6 m was deemed appropriate and consistently adopted throughout this study.

Table 2. Sensitivity of warping normal stresses to the finite-difference discretization length.

Discretization Length Increment	Normal Stress in the Top Slab	Deviation Relative to the 1.6 m	Normal Stress in the Bottom Slab	Deviation Relative to the 1.6 m
0.8	0.0584	1.04%	−0.336	1.82%
1.6	0.0578	0	−0.33	0
2.4	0.0567	−1.9%	−0.325	−1.5%

Taking the free end as the origin ($i = 0$), the cross-section of the composite girder with CSWs at each nodal point is transformed into an equivalent concrete box-girder section. The torsional section properties—namely $\bar{\omega}$, I_p , $\bar{I}_\omega$, I_d , $S_{\bar{\omega}}$, $\bar{S}_{\bar{\omega}}$, μ , and k^2 —are computed for each cross-section. These values are then incorporated into the fixed-end and free-end boundary conditions of the cantilever beam. Solving the resulting system of matrix equations yields the warping normal and shear stresses at each discretized cross-section.

As shown in Figure 8, the warping normal stresses at three characteristic locations—Point A (cantilever tip of the concrete top slab), and corner Points B and C—are compared with FE results. Under a concentrated torque applied at the free end, the theoretical predictions along the top and bottom slabs exhibit satisfactory overall agreement with the FE simulations.

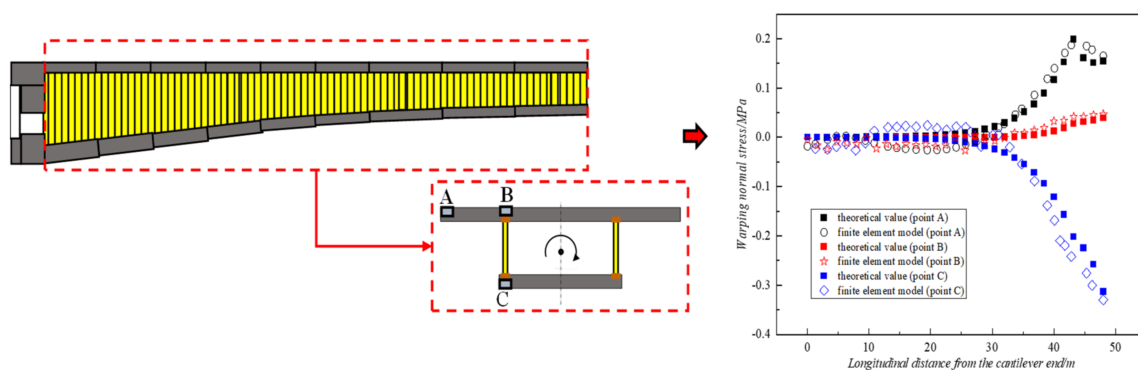


Figure 8. Comparison between theoretical solutions and FE results of torsional warping normal stress.

For a more rigorous quantitative evaluation, we statistically analyzed the differences between the theoretical and FE results at all longitudinal positions for Points A, B, and C. The error metrics—maximum absolute error (MaxAE), mean absolute error (MAE), and root mean square error (RMSE)—are summarized in Table 3.

Table 3. Error statistics between the theoretical and FE results at Points A, B, and C.

Point	Maximum Absolute Error (MaxAE)	Mean Absolute Error (MAE)	Root Mean Square Error (RMSE)
A	13%	4.2%	5.9%
B	18.1%	6.4%	7.7%
C	22.4%	9.1%	11.5%

Among the three points, Point A shows the closest agreement, with a maximum deviation of 13%. The relatively larger discrepancies at corner Points B and C are mainly due to the modeling differences between the FE method and the theoretical formulation. In the FE model, concrete slabs are meshed with solid elements, corrugated steel webs with shell elements, and the steel–concrete interface is simulated using embedded constraints. This approach inevitably introduces local stress concentrations and three-dimensional stress redistribution near the connection regions between the web flanges and concrete slabs. In contrast, the proposed theoretical model is based on an equivalent-section approach and Vlasov’s restrained torsion theory, which are intended to capture the global warping response and do not explicitly account for localized phenomena such as connection stress concentrations, shear lag, or three-dimensional load-transfer mechanisms at the steel–concrete interface.

Additionally, the higher MaxAE values at Points B and C are partly amplified by their lower absolute stress levels, which make the relative errors more sensitive to small absolute differences. Nevertheless, the theoretical results consistently reproduce both the magnitude and the longitudinal variation in the stresses predicted by the FE model, thereby confirming the overall reliability and accuracy of the proposed method.

In general, the FE results are slightly higher than the theoretical predictions. The absolute value of the warping normal stress is largest at the lower corner Point C, followed by Point A at the top slab tip, and smallest at the upper corner Point B. This trend is primarily governed by the variation in generalized sectorial coordinates across the cross-section. Near the free end, within the region influenced by the applied torque, warping normal stresses remain relatively modest, and the discrepancies between the FE and theoretical results in this zone are mainly attributable to Saint-Venant boundary effects. From the segment end section (Figure 8) toward the fixed support, the warping normal stresses at all monitored points exhibit a pronounced increasing trend.

Figure 9 presents the warping normal stress distributions at critical locations where pronounced warping effects are observed—namely, at Segment #1 and at the end of Segment #3 (as identified in Figure 6). Additionally, the total torsional shear stress distributions are shown at the ends of Segments #1 and #8, which are situated near the fixed support and the free end, respectively.

The results indicate that significant warping normal stresses develop at the cantilever tip of the top slab and at the web–bottom slab junctions. Under an eccentric load applied at the free end, the total torsional shear stress in Segment #8—located closer to the point of load application—exceeds that in Segment #1 near the fixed support. The shear stress distribution in Segment #8 exhibits characteristics of free torsion, with a relatively minor contribution from secondary shear stresses. In contrast, for Segment #1 near the fixed end, the constrained warping gives rise to a substantial secondary shear stress component. Across all segment cross-sections, elevated shear stresses are consistently observed at the midpoints of the top and bottom slabs, as well as in the lower central regions of the webs.

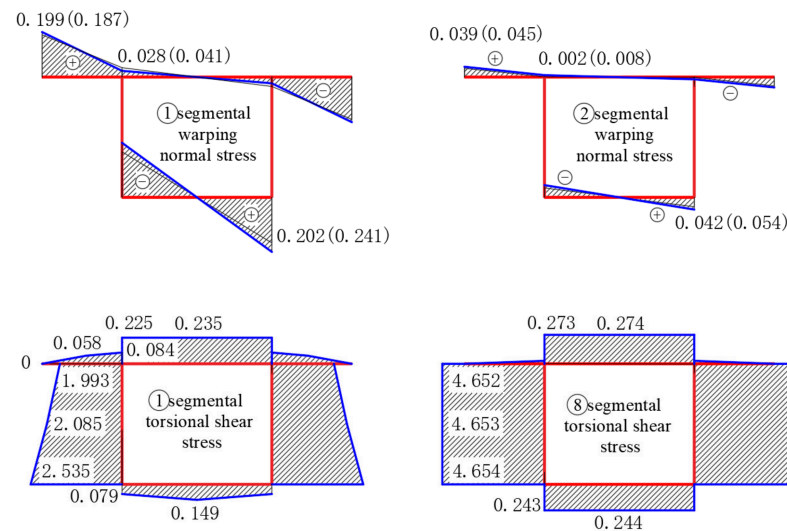


Figure 9. Distribution of torsional normal stress and shear stress at key cross-sections (unit: MPa).

The bending normal stresses, shear stresses, and the relative contributions of torsional stresses at key locations along each segment were determined using classical beam theory [45], and are summarized in Table 4.

Table 4. Stresses and proportions at selected points of each construction segment in the maximum cantilever state under eccentric loading.

Section Points of Each Construction Segment in the Maximum Cantilever State Under Eccentric Loading	Fixed End	Segment #1	Segment #3	Segment #5	Segment #7	Segment #8	Segment #9
Warping normal stress at A/MPa	0.155	0.199	0.039	0.006	0.001	0	0
Top slab bending normal stress/MPa	0.868	1.900	1.776	1.512	0.955	0.510	0
Proportion	17.8%	10.5%	2.2%	0.4%	0.1%	0.0%	0.0%
Warping normal stress at C/MPa	−0.313	−0.202	−0.042	−0.007	−0.001	0	0
Bottom slab bending normal stress/MPa	−1.197	−1.438	−1.697	−1.735	−1.378	−0.835	0
Proportion	26.2%	14.0%	2.5%	0.4%	0.1%	0.0%	0.0%
Torsional shear stress at the mid-point of the top slab/MPa	0.062	0.235	0.195	0.218	0.257	0.274	0.281
Torsional shear stress at the mid-point of the bottom slab/MPa	0.175	0.149	0.117	0.148	0.211	0.243	0.258
Torsional shear stress at the mid-point of the web/MPa	2.788	2.085	2.549	3.340	4.361	4.653	4.781
Web bending shear stress/MPa	9.828	9.463	11.431	14.936	19.593	20.958	21.56
Proportion	28.4%	22.0%	22.3%	22.4%	22.3%	22.2%	22.2%

The results indicate that the total torsional shear stress at each key point increases progressively from the fixed support toward the free end. The warping normal stress attains its maximum value in the segment nearest to the fixed support (Segment #1) and subsequently decreases markedly—a response characteristic of constrained torsion in a cantilever subjected to a concentrated end torque. Under the considered eccentric loading, the warping normal stress induced in the bottom slab reaches up to 26% of the corresponding bending normal stress, while that in the top slab attains 18%. Furthermore, the torsional shear stress in the CSW accounts for approximately 22% of the bending shear stress. These findings demonstrate that the stresses arising from constrained torsion in variable-depth composite girders with CSWs under eccentric loading are non-negligible. Accordingly, during construction, careful attention should be paid to eccentric load conditions, including temporary construction loads and asymmetric prestressing.

5. FE Parametrical Analyses

Unlike conventional cantilever construction, the bridge is installed using a multi-working-face APC method. Consequently, the structural states during construction differ between the conventional and APC cantilever schemes [11,49]. Moreover, the influence of the construction sequence and the design parameters of key components on the restrained torsion response throughout the construction process warrants comprehensive investigation.

5.1. Influence of Cantilever Construction Method

To evaluate the impact of different construction techniques, detailed FE models were developed based on the actual bridge geometry, representing both the APC method and the conventional balanced cantilever method. In the APC method, the terminal segment at the cantilever tip comprises a U-shaped composite section consisting of a concrete bottom slab and a CSW, whereas the remaining segments are consistent with those in the traditional method. Figure 10 compares the longitudinal distribution of the torsional warping normal stress at the maximum cantilever configuration, for an applied torque of $M_k = 4000 \text{ kN} \cdot \text{m}$ acting at the shear center of the free-end section.

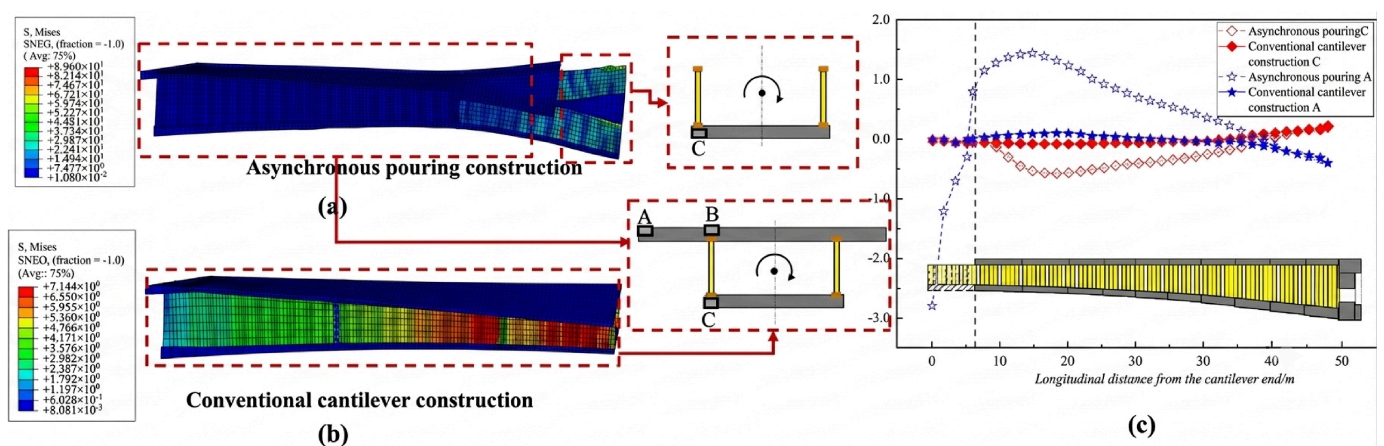


Figure 10. Stress contribution and warping normal stresses: (a) APC method; (b) conventional cantilever; (c) warping normal stresses.

As shown in Figure 10a,b, compared with the conventional cantilever construction method, the cantilever end region of the APC method remains a critical zone sensitive to stress concentration and deformation. As shown in Figure 10c, the results indicate that the longitudinal distributions of restrained torsional warping normal stress under the two cantilever construction methods are broadly similar. Within the 14 m region adjacent to the fixed support (Segments #1–3 in Figure 6), the stress magnitudes are nearly identical. However, for the remaining Segments #4–9, notable discrepancies emerge, with the most pronounced difference observed at corner Point C.

Owing to the composite cross-section of the free-end segment (Segment #9) in the APC method—comprising only the bottom slab and CSW—no warping normal stress develops at the top slab tip (Point A) within the 0–6.4 m range. The maximum compressive stress at this location reaches 0.6 MPa at approximately 17 m from the free end (Segment #7), after which the stress diminishes toward the fixed support, transitioning into tension near the root. At corner Point C, the stress within the 0–6.4 m range of Segment #9 shifts abruptly from compression to tension, attaining a peak value of 1.5 MPa at about 14 m from the free end (Segment #7). Proceeding further toward the fixed support, the stress gradually reverts to compression, with a magnitude of 0.4 MPa at the fixed end.

Because the stiffness variation is most pronounced in the section-transition region, a mesh-convergence analysis was conducted specifically for this region. Mesh sizes of 100, 200, and 300 mm were considered, and the normal stresses in the BGCSW were compared. The results show that the maximum difference in normal stress between the finer and coarser meshes was less than approximately 3%. Moreover, the locations of local stress concentrations near the transition region and the overall stress-distribution patterns remained essentially unchanged. Therefore, the adopted mesh is sufficiently accurate for analyzing the global restrained torsional response and the distribution of warping stresses.

The foregoing analysis indicates that the torsional resistance of composite girders with CSWs differs significantly between the conventional balanced cantilever construction and the APC method. Under the APC method, the warping stresses in the free-end segments exhibit pronounced variation and are of considerable magnitude. This behavior stems from the differing structural configurations of the cantilever tip associated with each construction method. In the conventional method, the cantilever consistently maintains a closed composite cross-section comprising the top slab, web, and bottom slab—consistent with the theoretical formulation adopted herein.

It should be noted, however, that the present analytical formulation, based on Uman-skii's theory, is strictly applicable to closed-box sections. In the APC case, the open-to-closed section transition exhibits complex stress distributions and pronounced boundary effects. Since the torsional theories for open and closed thin-walled sections cannot be readily unified to describe this transition analytically, the present theoretical solution is applied only to the region where the closed-box section has been formed, while the open U-section and the transition region near the cantilever tip are evaluated primarily using the three-dimensional FE model.

To further clarify the applicability of the proposed theoretical model under APC conditions, an additional comparison between the theoretical predictions and FE results is presented in Figure 11. Within approximately two to three segments adjacent to the fixed end, where closed box cells have been formed, the two sets of results show good overall agreement. As the cross-section gradually transitions toward an open U-shaped section toward the free end, the discrepancy progressively increases. These observations indicate that the analytical model based on Umanskii's theory can reasonably characterize the global restrained torsional response in regions with closed box cells, but cannot rigorously predict the local warping stresses in open-section and section-transition regions.

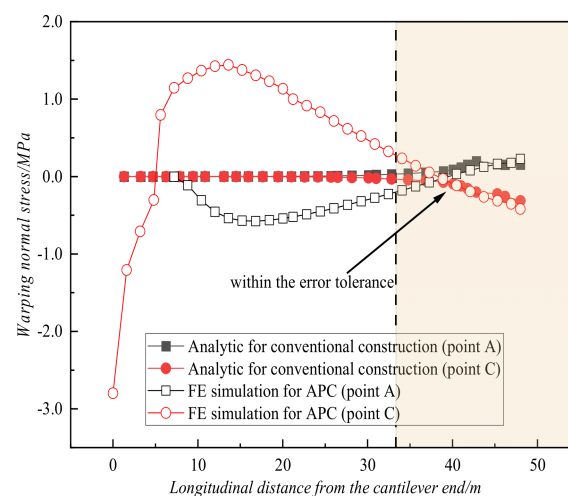


Figure 11. Comparison between analytic results (conventional construction) and FE simulation (APC).

Compared with the conventional cantilever construction method, the APC method results in a free end that invariably assumes an open composite cross-section consisting solely of the bottom slab and CSW at various construction stages. Consequently, particular attention should be directed to the stress state over an extended region near the free end when eccentric loads are applied during construction.

Therefore, in discussing the APC condition, the global response of the closed-section region is interpreted primarily on the basis of the theoretical analysis, whereas the stress response in the open-section region and the section-transition zone is evaluated using the three-dimensional finite-element results. Future work may extend the present formulation by integrating Vlasov's thin-walled theory for open sections with Umanski's theory for closed sections, thereby establishing a unified governing differential equation for restrained torsion amenable to solution via finite difference methods.

5.2. Effect of Diaphragm Casting Sequence

Diaphragms serve as deviators for external prestressing tendons and may be cast either during cantilever construction or after closure, prior to tendon stressing. To evaluate the influence of diaphragm casting timing on the torsional response, the longitudinal distribution of warping normal stress under the maximum cantilever configuration is compared for cases with and without diaphragms (Figure 12). In the figure, the white-filled regions along the cantilever and the corresponding dashed lines on the x-axis indicate the locations of the diaphragms.

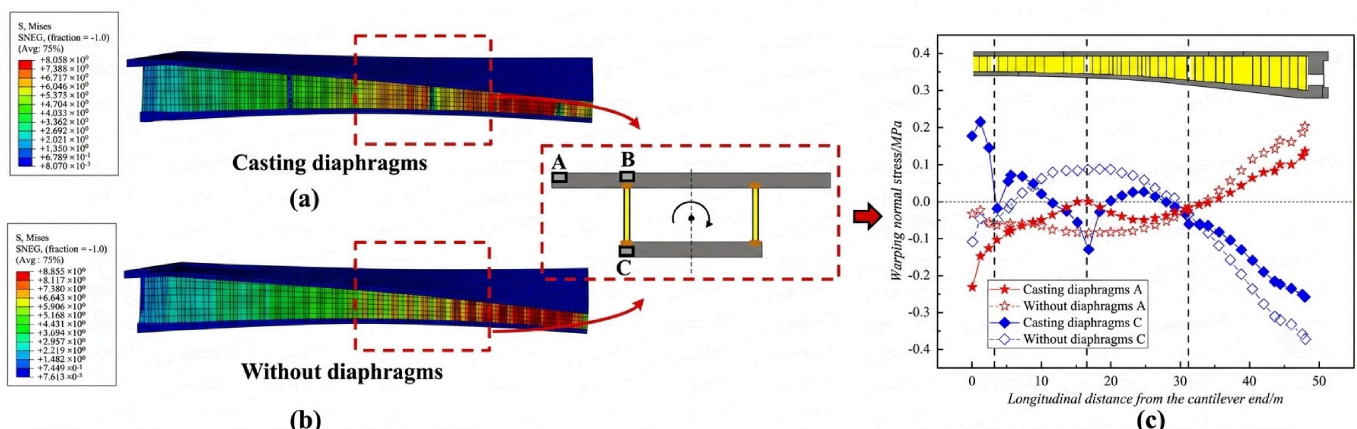


Figure 12. Stress contribution and warping normal stress: stress contribution (a) with diaphragm and (b) without diaphragm; (c) warping normal stress.

As shown in Figure 12a,b, the BGCSW with diaphragms exhibits a lower overall stress level than that without diaphragms. As shown in Figure 12c, under an external torque applied at the cantilever tip, the warping normal stresses in the free-end segment are higher when the diaphragm is cast than without. In contrast, the tensile and compressive stresses in the remaining segments are reduced after diaphragm casting, with the most pronounced decrease observed near the intermediate diaphragm located 16 m from the free end. In this region, the warping stress magnitude is substantially diminished, and the stress levels at the fixed end are also moderated to varying degrees.

Overall, the stress envelopes at the top slab tip (Point A) and the bottom slab corner (Point C) are reduced in the presence of diaphragms, indicating that diaphragms promote a more uniform distribution of warping deformation along the span under concentrated eccentric loads at the cantilever tip. The diaphragms restrain cross-sectional distortion and warping deformation, thereby reducing the overall warping stress in the girder segments behind the diaphragm. However, the segment between the free end and the first diaphragm

behaves as a short locally restrained torsional region. Large warping deformation tends to develop near the free end, whereas deformation is significantly restricted at the diaphragm. This compatibility condition redistributes the bimoment and warping stress within the short end segment, which may lead to a local stress increase near the free end.

The magnitude of this local increase is expected to depend on diaphragm stiffness, thickness, location, and spacing. A stiffer diaphragm generally provides stronger restraint against distortion and warping, while potentially intensifying the local stress redistribution in the adjacent end segment. During cantilever construction, it is advisable to cast each diaphragm promptly after completion of its corresponding segment—although the diaphragm near the free end may be cast later depending on site conditions—to enhance overall torsional resistance.

This observation is consistent with the established restraining role of diaphragms in box-girder structures reported in prior studies [23,24], which have demonstrated that diaphragms effectively reduce distortion and warping effects under service loads. However, those studies have primarily focused on completed structures, whereas the present work extends this understanding to variable-depth composite girders with CSWs during staged cantilever construction. Our results further indicate that diaphragm-casting timing is particularly critical in this context; delayed diaphragm installation may increase the risk of excessive warping deformation and local stress concentration in the unclosed or weakly restrained cantilever region. This finding constitutes a distinctive contribution of the present study, as it highlights a construction-stage effect that has not been systematically addressed in previous investigations.

Nevertheless, the present observation corresponds specifically to the diaphragm configuration and casting sequences considered in this study. The effects of diaphragm stiffness, spacing, and arrangement on the torsional response have not been exhaustively examined, and these factors may significantly influence the magnitude and distribution of warping stresses under different construction scenarios. Future work will therefore extend the current framework to systematically investigate these parametric effects.

5.3. Critical Stages Analysis of APC Method

During the APC process, the cross-sectional configuration of the BGCSW varies continuously due to the sequential installation of the CSWs and the staged casting of the concrete bottom and top slabs. The cross-sections encountered during the APC process can be categorized into three types, as shown in Figure 13a:

- Type 1—Composite section consisting of both top and bottom concrete slabs and CSWs, applicable to Segments #0 through #N-2 within Area A;
- Type 2—Composite section comprising the bottom slab and CSWs only, applicable to Segment #N-1 within Area B;
- Type 3—Steel section consisting solely of CSWs, applicable to Segment #N within Area C.

The three sectional configurations encountered during APC were not modeled by assigning equivalent sectional properties to different regions; instead, they were generated directly from the model geometry. The staged casting sequence was reproduced by progressively activating the corresponding slab components at each construction stage. Accordingly, the transition from the closed composite section to the bottom slab-CSW section, and subsequently to the CSW-only section, was represented solely by the presence or absence of the relevant slab components. At the interfaces between adjacent section types, the upper and lower flanges of the CSWs remained embedded in the adjoining slab components. Displacement continuity, and thus the transfer of shear flow and deformation across the interfaces, was ensured through the embedded-region constraints. Based on

the evolution of cross-sectional types during the APC process, four representative critical construction stages are selected for further analysis, as shown in Figure 13b.

- Critical Stage 1 corresponds to the initial cantilever stage, at which the top slab of Segment #1 (Area A) and the bottom slab of Segment #2 (Area B) have been cast, while the CSWs of Segment #3 (Area C) have been installed.
- Critical Stage 2 corresponds to the maximum cantilever stage, at which the top slab of Segment #7 (Area A) and the bottom slab of Segment #8 (Area B) have been cast, while the CSWs of Segment #9 (Area C) have been installed.
- Critical Stage 3 represents the stage at which the top slab of Segment #8 (Area A) and the bottom slab of Segment #9 (Area B) have been cast.
- Critical Stage 4 corresponds to the completion of the full closure of the BGCSW box-girder section, at which the top slab of Segment #9 (Area A) has been cast, and the girder essentially forms a continuous closed cross-section along its entire longitudinal direction.

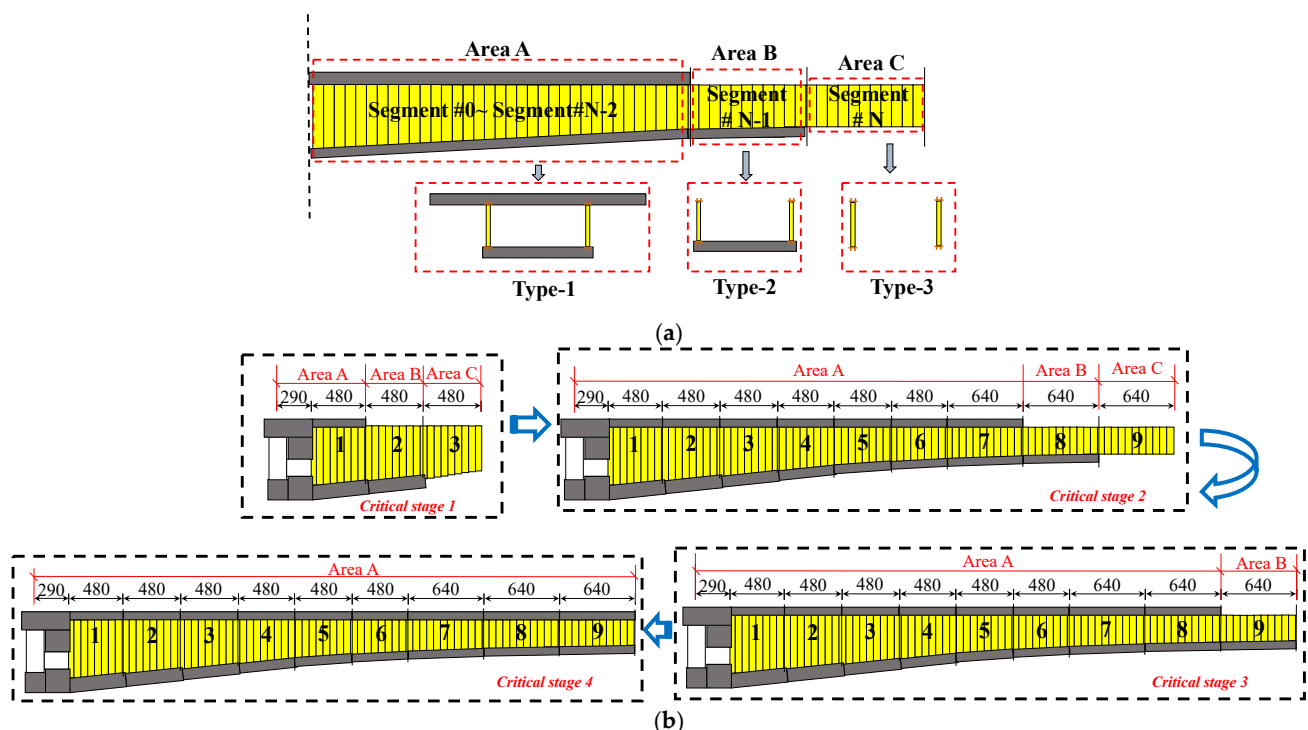


Figure 13. (a) Cross-section types for different segments and (b) critical stages during APC process.

The stress distribution and deformation corresponding to the four critical stages are presented in Figure 14. The BGCSW exhibits distinctly different torsional responses throughout the APC process.

In Critical Stage 1, the overall stress level and deformation remain relatively small, owing to the short cantilever length and the strong restraint provided by the fixed end. Nevertheless, local stress concentrations still occur near the transition zone between Areas B and C, attributable to the abrupt change in sectional configuration and torsional stiffness.

In Critical Stage 2, the overall deformation increases substantially, with pronounced torsional deformation and stress concentrations observed in Areas B and C. This indicates that Critical Stage 2 represents the most critical torsional state during the APC process. The pronounced response is primarily attributed to the long cantilever length, which substantially reduces the restraining effect of the fixed support. In addition, Areas B and C are composed of Type-2 and Type-3 sections, respectively, both of which possess

considerably lower torsional stiffness than the Type-1 section. Consequently, these regions are more susceptible to significant twist and local distortion.

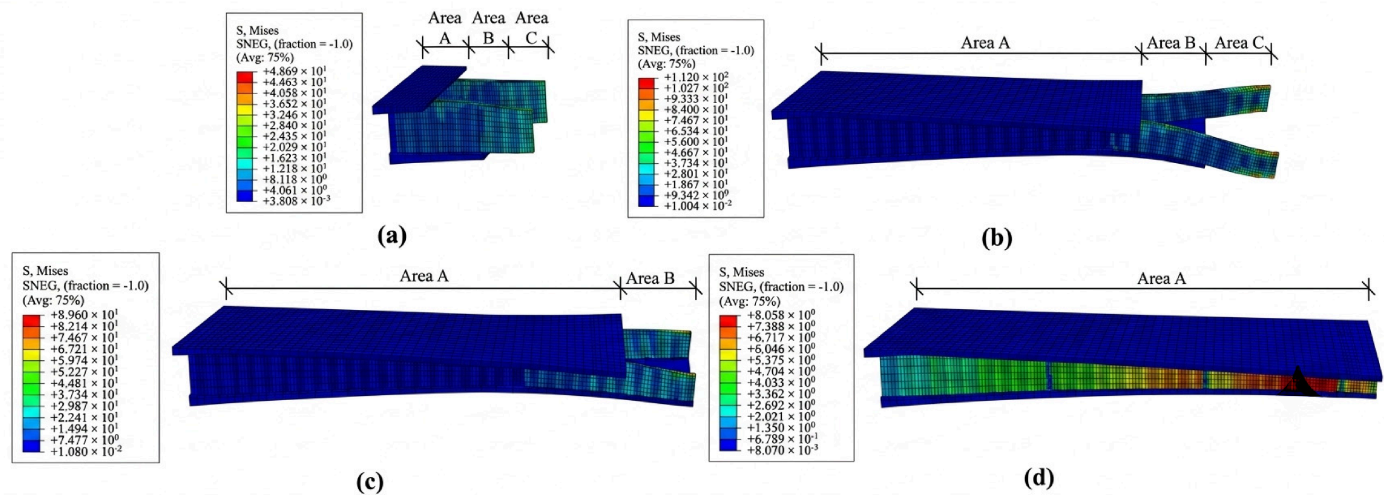


Figure 14. Stress distribution diagram under critical stages: (a) Critical Stage 1; (b) Critical Stage 2; (c) Critical Stage 3; (d) Critical Stage 4.

In Critical Stage 3, the overall deformation and stress level decrease compared with those in Critical Stage 2. This reduction can be attributed to the improved sectional closure resulting from the continued casting of the top and bottom slabs, which enhances both the torsional stiffness and the warping-restraint capacity of the BGCSW.

In Critical Stage 4, the BGCSW exhibits smaller deformation and a more uniform stress distribution under torsional loading. This is because the structure consists exclusively of Type-1 cross-sections at this stage, which provide enhanced torsional stiffness and warping-restraint capacity.

Overall, these results indicate that the BGCSW is more susceptible to torsional deformation and stress concentration when the cantilever is long and the cross-section remains unclosed. Therefore, special attention should be paid to open cross-sections and transition zones between different section types during the long-cantilever stages of the APC method.

Figure 15 presents the longitudinal distribution of warping normal stress along the BGCSW under different critical stages. When torsion is applied at the free end of the cantilever, the top slab (Points A and B) is primarily in compression, whereas the bottom slab (Point C) is primarily in tension. Overall, the warping normal stress is most pronounced under Critical Stages 2 and 3, with the peak normal stress in both stages occurring at 16 m from the free end—coinciding with the location of the middle diaphragm.

This behavior can be attributed to the longitudinal subdivision of the BGCSW by the middle diaphragms. These diaphragms divide the structure into several regions, including the fixed-end-to-diaphragm, diaphragm-to-diaphragm, and diaphragm-to-free-end regions. The boundary condition at the diaphragm lies between that of the free end and the fixed end; it is neither a free-warping boundary nor a fully restrained warping boundary. Consequently, the diaphragm partially interrupts the transmission of warping deformation along the BGCSW. When concentrated torsion is applied at the free end, each box region develops a relatively independent restrained torsional warping response. As a result, warping normal stress peaks occur in the vicinity of the diaphragms, which explains the observed distribution characteristics of the warping stress.

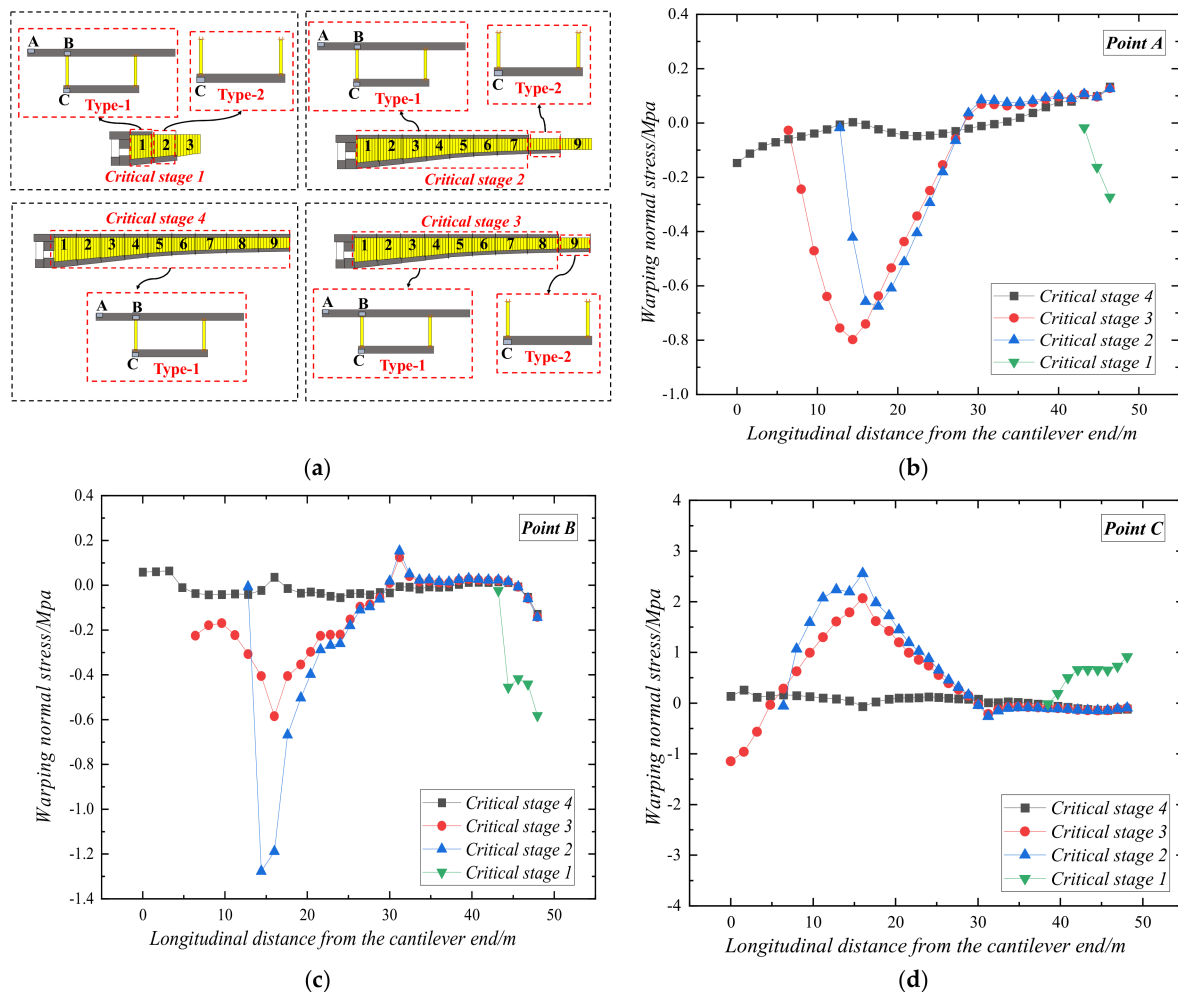


Figure 15. Warping normal stress distribution under critical stages: (a) critical normal stress point; (b) Point A; (c) Point B; (d) Point C.

Under Critical Stage 2, the stress level reaches its maximum. The largest peak tensile stress occurs at Point C (approximately 2.56 MPa), whereas the largest peak compressive stress occurs at Point B (approximately -1.28 MPa). Notably, the peak compressive stress at Point A is only about -0.67 MPa (substantially lower than that at Point B) suggesting that the stress response near the CSW-to-top slab connection is more sensitive to torsional loading. This sensitivity arises because Point B is influenced not only by the global torsional response of the BGCSW, but also by local force transfer from the CSW.

Under Critical Stage 3, both the peak compressive and tensile stresses occur at Point C, with values of approximately -1.15 MPa and 2.07 MPa, respectively. The peak tensile stress decreases by about 19.2% relative to Critical Stage 2. Nevertheless, Critical Stage 2 still exhibits a relatively large peak tensile stress, warranting particular attention during the APC process.

Under Critical Stage 4, the warping normal stresses at all points are low. The stress level under Critical Stage 1 is slightly higher than that under Critical Stage 4, but remains much lower than under Critical Stages 2 and 3. This is attributable to the short cantilever length and the strong restraint provided by the fixed support, which together limit the overall torsional deformation and prevent significant increases in warping normal stress.

Overall, the warping normal stress is most unfavorable when the BGCSW has a long cantilever length and its cross-section has not yet been fully closed. In particular, the section at the first diaphragm closest to the free end of the cantilever constitutes the critical location.

Therefore, the stress development in this region should be given particular attention during the APC process.

Figure 16 presents the torsional shear stress distribution of the BGCSW under different critical stages. Although some variation exists among the stages, the overall shear stress level remains relatively low. Point E generally exhibits higher shear stress than Points D and F, indicating that the CSW-to-bottom slab connection is a location of relative shear stress concentration. This is because the CSW acts as the primary shear-resisting component of the box girder; as most of the shear force is transferred through the CSW, relatively high shear stresses develop at the CSW–bottom slab connection.

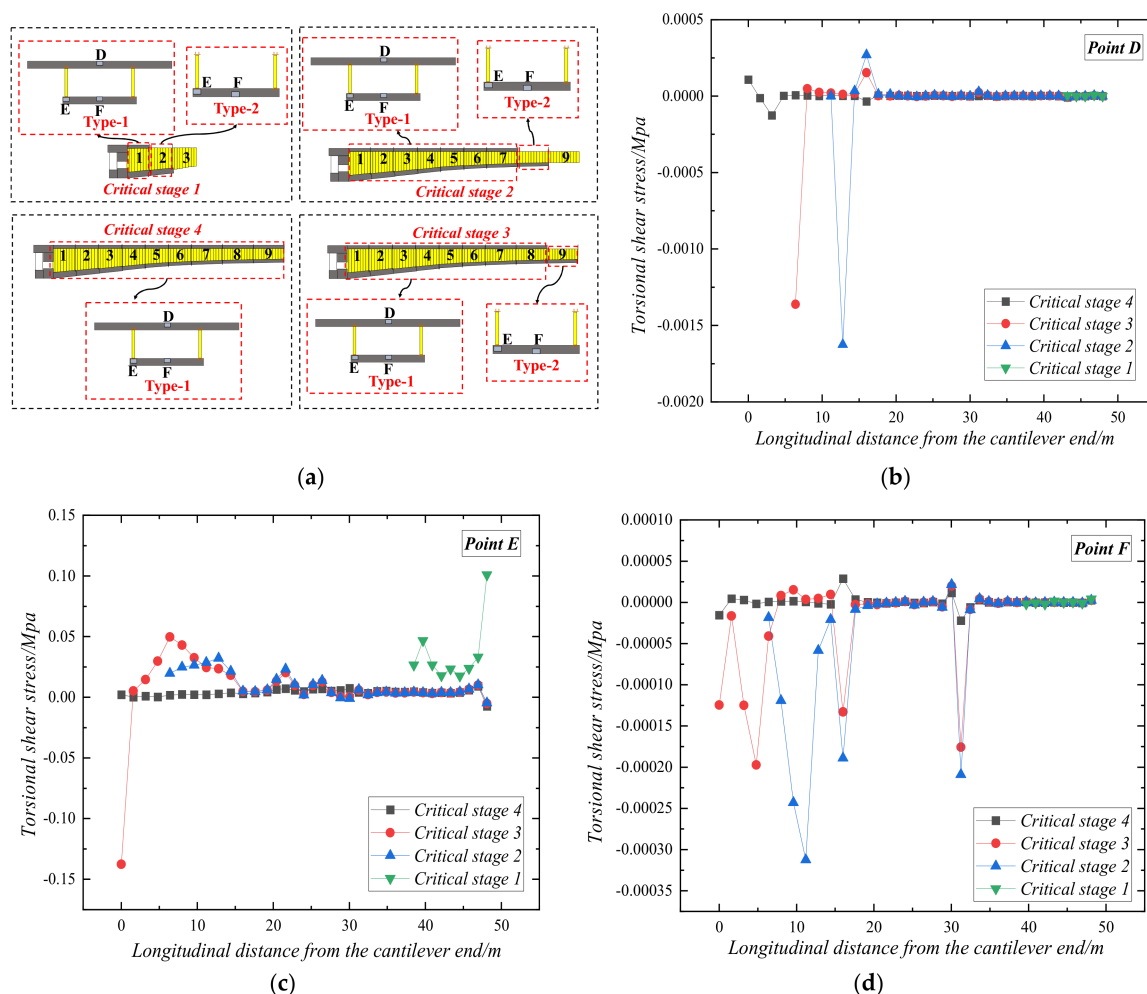


Figure 16. Torsional shear stress distribution under critical stages: (a) critical shear stress point; (b) Point D; (c) Point E; (d) Point F.

Under Critical Stages 1 and 4, the shear stress varies gently along the BGCSW and remains at a low overall level. In contrast, under Critical Stages 2 and 3, distinct shear stress peaks occur at the transition zones between different cross-section types, indicating that changes in cross-sectional form induce local stress redistribution and shear stress concentration at these zones.

6. Conclusions

- (1) A finite-difference solution to the governing differential equation for constrained torsion in variable-depth composite girders with CSWs is established and validated through three-dimensional finite-element simulations of an actual bridge, confirming

the correctness of the derived formulations. The solution method is straightforward and well-suited for rapid engineering calculations.

- (2) Under eccentric loading, stresses induced by constrained torsion are non-negligible: warping normal stress reaches 26% of bending stress in the bottom slab and 18% in the top slab, while torsional shear stress in the web accounts for approximately 22% of the total. When a concentrated torque is applied at the cantilever tip, the total torsional shear stress near the free end exceeds that at the fixed support, exhibiting free-torsion characteristics.
- (3) In the APC method, warping normal stresses in the free-end segments exhibit pronounced variation and considerable magnitude due to the differing structural configuration of the cantilever tip. While the conventional balanced cantilever method aligns with the present formulation, the APC method requires an extended approach combining the present derivations with thin-walled theory for both open and closed sections to establish a unified governing differential equation.
- (4) Diaphragms promote a more uniform distribution of warping deformation along the cantilever. It is advisable to cast each diaphragm promptly after completing its corresponding segment to enhance torsional resistance.
- (5) Critical stage analysis reveals that the maximum cantilever stage with unclosed cross-sections is the most critical state, where low torsional stiffness causes severe warping stresses and local stress concentrations at CSW-to-slab connections. Eccentric construction loads should therefore be strictly controlled during this stage to mitigate the risk of concrete cracking.
- (6) This study establishes a construction-stage torsional risk assessment and control framework for long-span BGCSWs constructed using the APC method. The framework integrates the proposed equivalent analytical model, the identification of critical construction stages and vulnerable regions—such as long cantilevers with unclosed sections, free-end zones, diaphragm locations, and CSW-to-slab connections—and practical recommendations for diaphragm casting sequencing. It provides a systematic basis for eccentric-load control, timely diaphragm installation, and targeted stress verification, thereby helping to reduce local stress concentrations and mitigate the risk of concrete cracking during construction.

Future work will extend the proposed method to a wider range of structural configurations, including bridges with different span lengths, web geometries, and cross-sectional shapes. More complex loading scenarios—such as combined eccentric dead and live loads, wind effects, and temperature gradients—will also be incorporated. Additionally, the influence of varying boundary restraints, segment casting sequences, diaphragm installation timing, and closure procedures on the torsional response will be systematically examined. To further enhance the practical reliability of the proposed approach, field-monitoring data from actual bridge construction sites will be collected and used to validate and calibrate the analytical and numerical models.

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