

Article

Study on Parameter Optimization of a Multi-Mass Pendulum for a Wind-Induced Vibration Control System

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Abstract

The tuned mass damper (TMD) exhibits good performance in suppressing wind-induced vibrations of high-rise structures. However, a single TMD has a limited control bandwidth and poor robustness. The multiple-pendulum tuned mass damper (MPTMD) offers advantages, such as a wider control bandwidth, stronger robustness, and a simple structural configuration, while its working frequency can be easily adjusted by varying the pendulum lengths. With two optimization objectives, namely displacement and acceleration, this study derives the displacement and acceleration dynamic amplification factors of the primary structure equipped with the MPTMD under external excitation and examines the interrelationships among the optimal parameters and their underlying mechanisms. The accuracy of the proposed optimization method and the effectiveness of the MPTMD are validated by fitting the theoretically derived optimal parameter curves with results from numerical simulations. Finally, the control performance of MPTMD and TMD is compared through a numerical example subjected to realistic wind load excitations, verifying the control effectiveness of MPTMD. Nevertheless, several limitations should be acknowledged. The present optimization is based on a single-degree-of-freedom (SDOF) primary structure and targets only the first translational mode; the effects of higher modes and multi-degree-of-freedom (MDOF) coupling are not considered. Additionally, the wind load is represented by a synthetic time history with a fixed return period, and uncertainties in real wind fields are not fully addressed. Future work should extend the proposed method to multi-modal control, nonlinear behavior, and experimental validation.



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Keywords: multiple-pendulum tuned mass damper; wind-induced vibration; optimal parameters

1. Introduction

The Tuned Mass Damper (TMD) is a dynamic vibration absorber system composed of an inertial mass, a stiffness element, and a damping element. The concept of the TMD originated from the work of Frahm [1], who used a spring absorber to control the rolling motion of ships. Subsequently, based on Frahm's research, scholars proposed the classical TMD and established design parameter expressions and optimization methods for structures equipped with TMDs [2,3]. Furthermore, to improve the control performance of

TMDs, various TMD variants have been proposed in the literature, such as the negative-stiffness TMD, extended KDamper, and adaptive eddy current TMD, among others [4–8].

To achieve optimal control performance, the natural frequency of the TMD must be tuned to the fundamental frequency of the primary structure. Once the fundamental frequency of the primary structure shifts, the control effectiveness of the TMD decreases significantly [9]. Some researchers have proposed using multiple TMDs with linearly distributed operating frequencies, collectively forming a Multiple Tuned Mass Damper (MTMD) system. The effective frequency band of an MTMD system is much wider than that of a single TMD, thereby enhancing the frequency robustness of the control system [10]. Yamaguchi, Abe, Harnpornchai, Kabelova et al. [11–14] investigated the effectiveness of MTMDs in structural vibration control and provided insights into their control mechanisms, design methods, and parameter expressions. Igusa and Xu [15] investigated the vibration control effectiveness of MTMDs with operating frequencies distributed around the fundamental frequency of the primary structure and optimized the MTMD. The results indicated that MTMDs are more effective and stable than a TMD of equivalent mass. Abe and Igusa [16] analyzed the structural vibration control performance of MTMDs under harmonic loading. They pointed out that structures with closely spaced natural frequencies can experience modal coupling due to second-order effects, which influence structural amplitudes. Lin et al. [17] addressed the stroke issue of MTMDs. They developed a suitable MTMD vibration mitigation scheme that reduces the stroke of the MTMD while ensuring control effectiveness. The rationality of the scheme was validated through shaking table tests on a three-story building with and without the MTMD.

With numerous parameters involved in MTMDs, obtaining optimal parameters through reasonable optimization methods has become crucial for realizing the vibration mitigation performance of such control systems. Bakre and Jangid [18] derived optimization formulas for the damping ratio, tuning-frequency ratio, and frequency bandwidth of MTMD systems, using the minimization of primary structure displacement under base harmonic excitation. Zheng et al. [19] investigated the control performance of MTMDs on the wind-induced vibration response of high-rise structures. They established a numerical analysis model of the MTMD-primary structure system and employed a genetic algorithm to search for the optimal MTMD parameters. Patil, Jangid, and Bandivadekar [20–22] employed numerical methods to obtain the optimal parameters of MTMDs and investigated the influence of design parameters such as the mass ratio, damping ratio, number of dampers, tuning frequency, and frequency bandwidth on the robustness and control effectiveness of MTMDs. Bozer and Ozsariyildiz [23] introduced an artificial bee colony optimization algorithm to explore the true optimal values of the stiffness and damping parameters of individual TMDs within an MTMD system, as well as the optimal values of the MTMD parameters, under constraints such as uniform distributions of stiffness, mass, or frequency spacing.

This paper investigates the parameter optimization of an SDOF primary structure equipped with an MPTMD under wind loads. Compared with existing studies on MTMD/MPTMD, the main novel features and contributions of this work are summarized as follows:

- Dual-objective optimization framework. Unlike previous studies that typically optimize MTMD parameters based solely on displacement or acceleration response, this work derives the dynamic amplification factors for both displacement and acceleration objectives. The optimal MPTMD parameters are obtained by minimizing the peaks of these two factors simultaneously, providing a more comprehensive design basis.
- Theoretical derivation of optimal parameters under wind excitation. While prior research has focused on MTMD optimization under seismic or harmonic

excitations [9–14], this paper establishes closed-form analytical expressions for the optimal frequency ratio, damping ratio, and frequency bandwidth of MPTMD systems specifically under wind loads, filling a gap in the current literature.

- Cross-validation between theoretical and numerical solutions. Instead of relying solely on either analytical or numerical methods, this work employs both approaches: theoretical optimization via dynamic amplification factor minimization and numerical optimization using the acceleration response of a 20-story benchmark model under realistic wind loads. The consistency between the two sets of results validates the correctness and applicability of the theoretical method.
- Practical engineering relevance. The use of a full-scale 20-story benchmark structure with realistic wind field data bridges the gap between theoretical parameter studies and practical engineering design, demonstrating the real-world feasibility and effectiveness of the proposed MPTMD configuration.

These findings collectively advance the understanding of MPTMD parameter optimization and provide a reliable design methodology for wind-induced vibration control of tall buildings.

2. Derivation of the Dynamic Magnification Factor of the MPTMD System

In this section, we derive the equation of motion and the expression for the dynamic magnification factor of an SDOF system equipped with MPTMD. To facilitate subsequent analyses, a set of normalized parameters is defined.

The natural period of a pendulum mass can be calculated as follows:

$$T = 2\pi\sqrt{\frac{l}{g}} \quad (1)$$

where l is the pendulum length and g is the gravitational acceleration.

The MPTMD system consists of n sub-PTMDs, with frequencies denoted as $\omega_1, \omega_2, \omega_3, \dots, \omega_n$. These PTMDs are tuned around the controlled structural frequency and linearly distributed over a certain frequency bandwidth with equal frequency steps. As shown in Figure 1, the mass, stiffness, and damping of the primary structure are denoted by M , K , and C , respectively, while its controlled frequency and damping ratio are ω_s and ζ_M . The mass and the restoring force under wind load for each pendulum mass are m_j and F_{cj} , respectively. Moreover, let x_M and x_j denote the displacements of the primary structure and each pendulum mass under wind load, respectively.

Based on d'Alembert's principle and the concept of the transfer function, the basic analytical model of MPTMD can be established [24]. The equation of motion for a structure equipped with MPTMD under load $p(t)$ is expressed in Equation (2):

$$\begin{cases} M\ddot{x}_M + C\dot{x}_M + Kx_M = \sum_{j=1}^n F_{cj} + p(t) \\ m_j\ddot{x}_j + F_{cj} = 0 \end{cases} \quad (2)$$

The restoring force expression of each pendulum mass under load is as follows:

$$F_{cj} = \frac{m_j g_0}{l_j} (x_j - x_M) \quad (3)$$

Let the relative displacement be $x_{dj} = x_j - x_M$. Substituting it into the equation of motion yields

$$\begin{cases} s^2 M X_M(s) + s C X_M(s) + K X_M(s) = \sum_{j=1}^n \frac{m_j g_0}{l_j} X_{dj}(s) + P(s) \\ s^2 m_j (X_{dj}(s) + X_M(s)) + \frac{m_j g_0}{l_j} X_{dj}(s) = 0 \end{cases} \quad (4)$$

Therefore, the transfer function of the primary structure equipped with MPTMD is

$$(s^2 M + s C + K) X_M(s) + \sum_{j=1}^n \frac{m_j g_0 s^2}{s^2 l_j + g_0} X_M(s) = P(s) \quad (5)$$

Furthermore, the following normalized parameters are defined:

$$\omega_M^2 = \frac{K}{M}, \tilde{\zeta}_M = \frac{C}{2M\omega_M}, \omega_j^2 = \frac{k_j}{m_j}, f_j = \frac{\omega_j}{\omega_M}, \mu_j = \frac{m_j}{M}, g = \frac{\omega}{\omega_M} \quad (6)$$

Let $s = i\omega$. Then, the frequency response function of the primary structure equipped with MPTMD is obtained as follows:

$$G(i\omega) = \frac{1}{K} \cdot \frac{1}{1 - g^2 - \sum_{j=1}^n \frac{\mu_j g^2}{1 - g^2/f_j^2} - M\omega^2 + i2\tilde{\zeta}_M g} \quad (7)$$

Separating the real and imaginary parts of the frequency characteristic expression yields $G(i\omega) = (\omega) + iV_0(\omega)$. Represented in polar coordinates, it can be written as $G(i\omega) = R(\omega)e^{i\theta(\omega)}$, which, upon rearrangement, gives $R(\omega) = [U_0(\omega)^2 + V_0(\omega)^2]^{1/2}$. Here, $R(\omega)$ denotes the amplitude of $G(i\omega)$, expressed as follows:

$$R(\omega) = \left(\frac{A_1^2 + B_1^2}{C_1^2 + D_1^2} \right)^{1/2} \quad (8)$$

$$A_1 = 1, B_1 = 0, C_1 = \frac{1}{K} \left[1 - g^2 - \sum_{j=1}^n \frac{\mu_j g^2}{1 - g^2/f_j^2} \right], D_1 = \frac{1}{K} \cdot 2\tilde{\zeta}_M g$$

Define the lateral displacement of the primary structure under the static load P_0 as $x_0 = \frac{P_0}{K}$. Finally, the dynamic amplification factor of the primary structure equipped with MPTMD is obtained as follows:

$$A_D(g) = \frac{R_M(\omega) \cdot P_0}{x_0} = R_M(\omega) \cdot K = \left[\frac{1}{\left[1 - g^2 - \sum_{j=1}^n \frac{\mu_j g^2}{1 - g^2/f_j^2} \right]^2 + (2\tilde{\zeta}_M g)^2} \right]^{1/2} \quad (9)$$

Similarly, the acceleration dynamic amplification factor is obtained as follows:

$$A_A(g) = \frac{R_A(\omega) P_0}{\ddot{x}_0} = g^2 \cdot R_A(\omega) \cdot K = g^2 \cdot A_D(g) \quad (10)$$

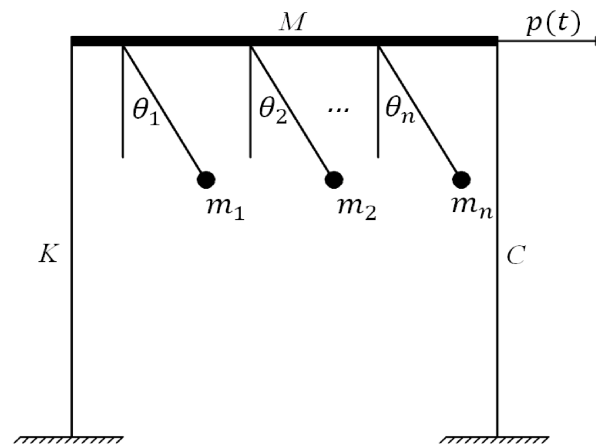


Figure 1. SDOF primary structure equipped with MPTMD.

3. Parameter Optimization Analysis with Displacement as the Objective

In this section, we conduct parameter optimization with displacement control performance as the optimization objective and analyze the effects of the parameters on the control performance as well as the interaction patterns among these parameters.

In the parameter optimization of the MPTMD control system, displacement is first taken as the optimization objective to meet the requirements of structural stiffness design. Based on the formula of the displacement dynamic amplification factor, the corresponding dynamic amplification factors under different parameter combinations can be calculated. By minimizing the peak value of the dynamic amplification factor under different mass ratios, the optimal damping ratio, optimal frequency ratio, optimal bandwidth, and other important parameters can be obtained.

In the following analysis, the mass ratio is fixed at $\mu = 0.01$ unless otherwise stated, and the number of pendulum masses is varied from $n = 3$ to $n = 60$. The upper bound $n = 60$ was selected because the optimal parameters were found to converge beyond this value, while $\mu = 0.01$ represents a commonly used baseline in both theoretical and practical contexts.

By varying the parameters of the MPTMD system, the influence of each parameter on the system's dynamic amplification factor can be analyzed. The damping of the primary structure has a non-negligible effect; therefore, a damping ratio of $\zeta_M = 0.01$ is adopted for the primary structure in the analysis. The following conclusions can be drawn from Figure 2:

1. When the mass ratio $\mu = 0.01$, the bandwidth parameter $\beta = 0.2$, and the average tuning-frequency ratio $f_T = 1$, the curves of the displacement dynamic amplification factor for different numbers of pendulum masses n fluctuate significantly within the dimensionless frequency range $g = 0.9\text{--}1.1$, exhibiting multiple peaks. A larger n leads to more concentrated peaks and more regular fluctuations in the curve.
2. When $\mu = 0.01$ and $n = 21$, the dynamic amplification factor curve exhibits two relatively large peaks as the bandwidth parameter β varies from 0 to 0.3. This indicates that the peak magnitude can be adjusted by varying the bandwidth within a certain range. Consequently, with other parameters fixed, the optimal value of the bandwidth parameter β can be determined by minimizing the peak of the dynamic amplification factor.
3. When $\mu = 0.01$ and $n = 21$, the displacement dynamic amplification factor reaches its minimum at the average tuning-frequency ratio $f_T = 1$. When f_T deviates from 1, the curve increases sharply and displays multiple pronounced peaks.

In summary, the displacement dynamic amplification factor of the MPTMD system is highly sensitive to variations in its parameters. To achieve optimal control effectiveness, when some parameters are given, the remaining target parameters must be optimized accordingly, thereby enabling the MPTMD system to attain its best performance.

3.1. Influence of Mass Ratio on Optimal System Parameters

When studying the influence pattern of the mass ratio on the optimal parameters of the MPTMD control system, damping ratios of 0, 0.02, 0.05, and 0.1 for the primary structure are considered to investigate the influence of the structural damping ratio on the optimal parameters.

3.1.1. Influence of Mass Ratio on the Optimal Bandwidth

The variation in the optimal bandwidth β^{opt} with respect to the mass ratio μ is illustrated in Figure 2. This section investigates the values and trends of the optimal bandwidth parameter as the mass ratio changes. For the MPTMD control system, different mass ratios correspond to different requirements for the optimal bandwidth β^{opt} . With the total mass ratio of the system held constant, this study analyzes the influence of the number of pendulum masses n (considering three cases: $n = 5, 15$, and 25) on the variation curve for the optimal bandwidth. The results indicate that for a given mass ratio, the optimal bandwidth value varies with the number of pendulum masses. In general, the system's demand for optimal bandwidth increases with an increase in the mass ratio μ .

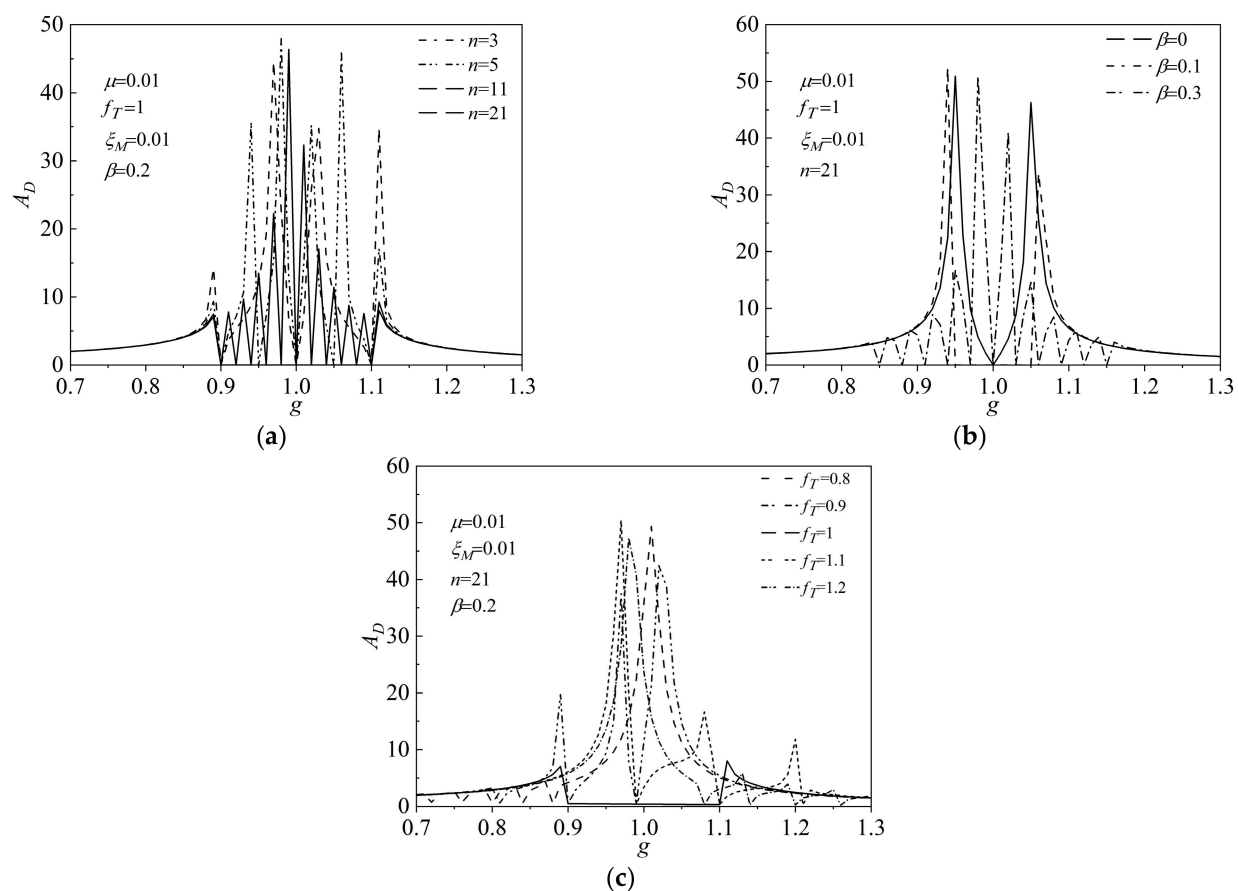


Figure 2. Influence of the MPTMD system parameters on the dynamic amplification factor of the primary structure response. (a) Influence of quantity n . (b) Influence of bandwidth β . (c) Influence of the average tuning-frequency ratio f_T .

Furthermore, to investigate the influence of the primary structure's damping ratio ξ_M , Figure 3 shows that under the same mass ratio condition, a larger damping ratio ξ_M of the primary structure leads to a greater required optimal bandwidth for the system. Therefore, the damping ratio of the primary structure has a non-negligible effect on the selection of the optimal bandwidth for the MPTMD system.

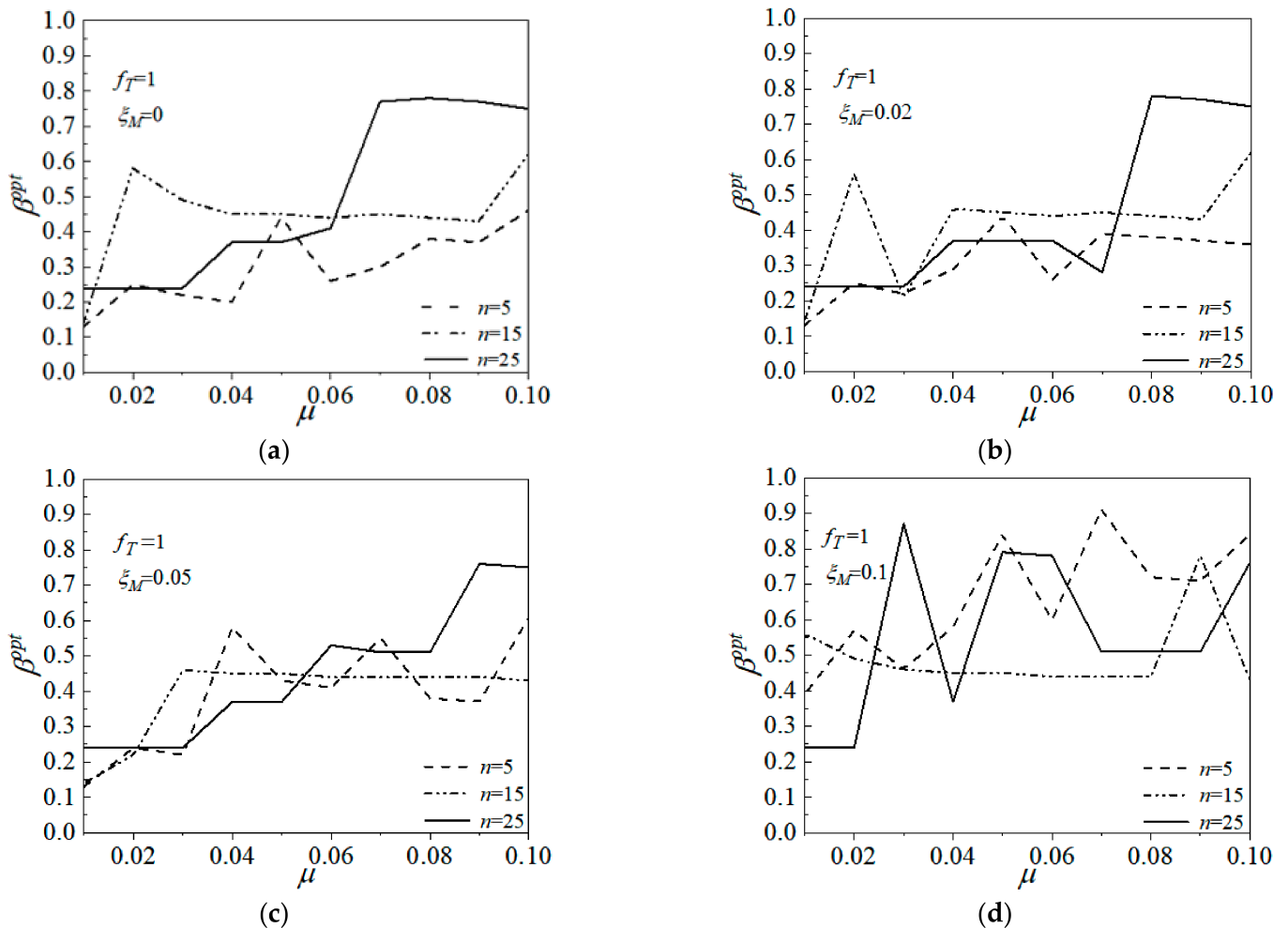


Figure 3. Variation trend of optimal bandwidth with mass ratio. (a) $\xi_M = 0$; (b) $\xi_M = 0.02$; (c) $\xi_M = 0.05$; (d) $\xi_M = 0.1$.

3.1.2. Influence of Mass Ratio on the Optimal Average Tuning-Frequency Ratio

The variation in the optimal average tuning-frequency ratio with the mass ratio is shown in Figure 4. Since the MPTMD system itself has no damping, determining the optimal average tuning-frequency ratio is crucial for tuning the system frequency during the design stage. As shown in Figure 4, for different damping ratios of the primary structure, the optimal average tuning-frequency ratio curves for different numbers of pendulum masses follow consistent trends as the mass ratio increases. For a given mass ratio, the required optimal average tuning-frequency ratio differs depending on the number of pendulum masses. Consequently, in the design of an MPTMD control system, approximate values should not be adopted simply because mass ratios are similar. Instead, the precise optimal value should be selected based on the optimal average tuning-frequency ratio curve to ensure the effectiveness of the control system.

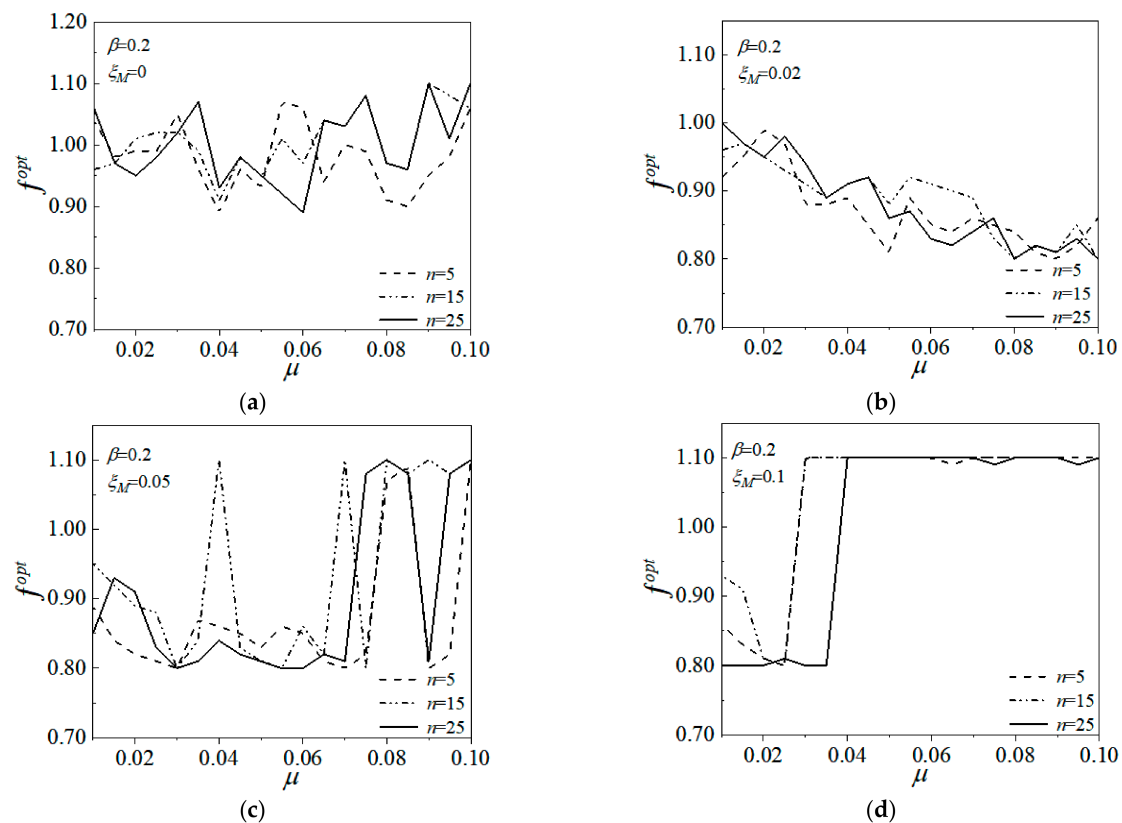


Figure 4. Variation trend of optimal average tuning-frequency ratio with mass ratio. (a) $\zeta_M = 0$; (b) $\zeta_M = 0.02$; (c) $\zeta_M = 0.05$; (d) $\zeta_M = 0.1$.

3.2. Influence of the Number of Pendulum Masses on Optimal System Parameters

3.2.1. Influence of the Number of Pendulum Masses on the Optimal Bandwidth

Figure 5 shows the variation in the optimal bandwidth β^{opt} with the number of pendulum masses n . In studying the variation in the optimal bandwidth value with the number, it can be observed that when the structural damping ratio $\zeta_M < 0.1$, the optimal bandwidth value exhibits regular fluctuations as the number of pendulum masses increases. Due to the absence of damping in the pendulum masses themselves, the optimal bandwidth values corresponding to adjacent numbers differ significantly. Figure 5 indicates that the required optimal bandwidth values for different numbers of pendulum masses vary substantially. Therefore, designing the MPTMD system based on the optimal bandwidth variation curve is of critical importance.

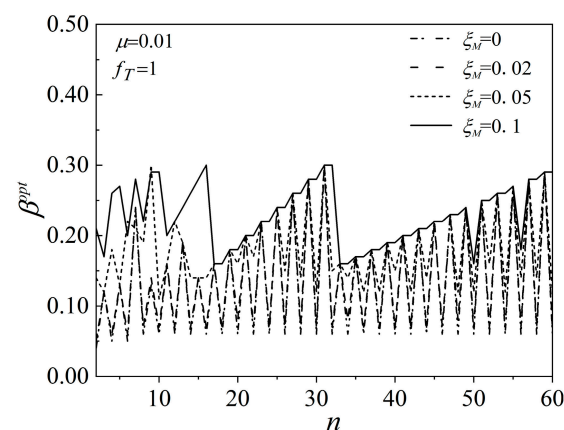


Figure 5. Variation trend of the optimal bandwidth with the number of pendulum masses.

3.2.2. Influence of the Number of Pendulum Masses on the Optimal Average Tuning-Frequency Ratio

The variation in the optimal average tuning-frequency ratio f^{opt} with the number n is illustrated in Figure 6. It is evident that as the number of pendulum masses increases, the optimal average tuning-frequency ratio of the MPTMD system fluctuates within the range of 0.9 to 1.1 for different damping ratios of the primary structure. This indicates that $f_T = 1$ is not the optimal value under all conditions. Furthermore, although the variation trends of the curves corresponding to different structural damping ratios are similar, the specific values of the optimal average tuning-frequency ratio differ. Therefore, when investigating the relationship between the number and the optimal average tuning-frequency ratio, the influence of the damping ratio of the primary structure must be taken into account.

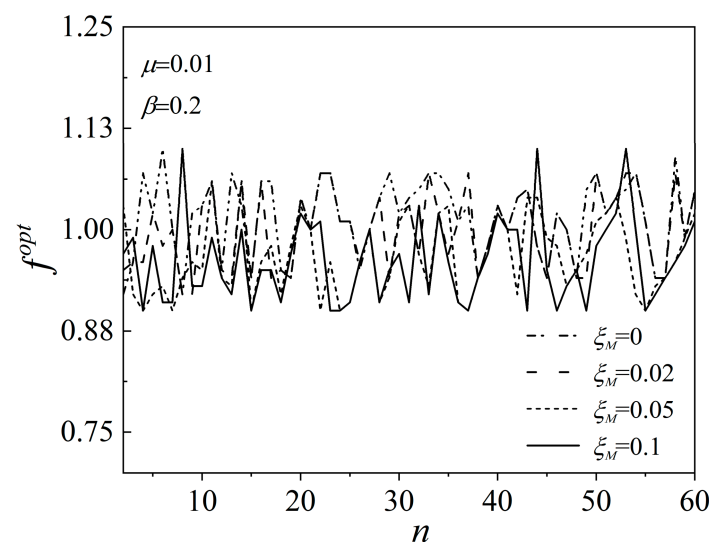


Figure 6. Variation trend of the optimal average tuning-frequency ratio with the number of pendulum masses.

4. Parameter Optimization Analysis with Acceleration as the Objective

Based on the formula for the acceleration dynamic amplification factor of the primary structure equipped with the MPTMD system, the corresponding factors under different parameter combinations can be calculated. The optimal values of the target parameters for specific working conditions are obtained by minimizing the peak of this factor.

By varying the parameters of the MPTMD system, the influence of each parameter on the acceleration dynamic amplification factor can be analyzed. Here, the damping ratio of the primary structure is taken as $\xi_M = 0.1$, and the mass ratio of the MPTMD system is $\mu = 0.01$. The following conclusions can be drawn from Figure 7:

1. When the mass ratio $\mu = 0.01$, the bandwidth $\beta = 0.2$, and the average tuning-frequency ratio $f_T = 1$, the acceleration dynamic amplification factor curves corresponding to different numbers of pendulum masses n fluctuate significantly within the dimensionless frequency range $g = 0.9 \sim 1.1$, exhibiting multiple peaks. A larger number of pendulum masses, n , leads to more concentrated peaks and more regular fluctuations in the curves.
2. When the mass ratio $\mu = 0.01$ and the number of pendulum masses $n = 21$, the acceleration dynamic amplification factor curves consistently show two relatively large peaks as the bandwidth β varies from 0 to 0.3. This indicates that the magnitude of the peaks can be adjusted by changing the value of the bandwidth β . Therefore, with other parameters fixed, the optimal value of the bandwidth β can be determined by minimizing the peak of the acceleration dynamic amplification factor.

3. When the mass ratio $\mu = 0.01$ and the number of pendulum masses $n = 21$, the acceleration dynamic amplification factor reaches its minimum at the average tuning-frequency ratio $f_T = 1$. When f_T deviates from 1, the curve increases sharply and displays multiple pronounced peaks. This pattern is similar to the influence curve of the optimal average tuning-frequency ratio when displacement is used as the optimization objective.

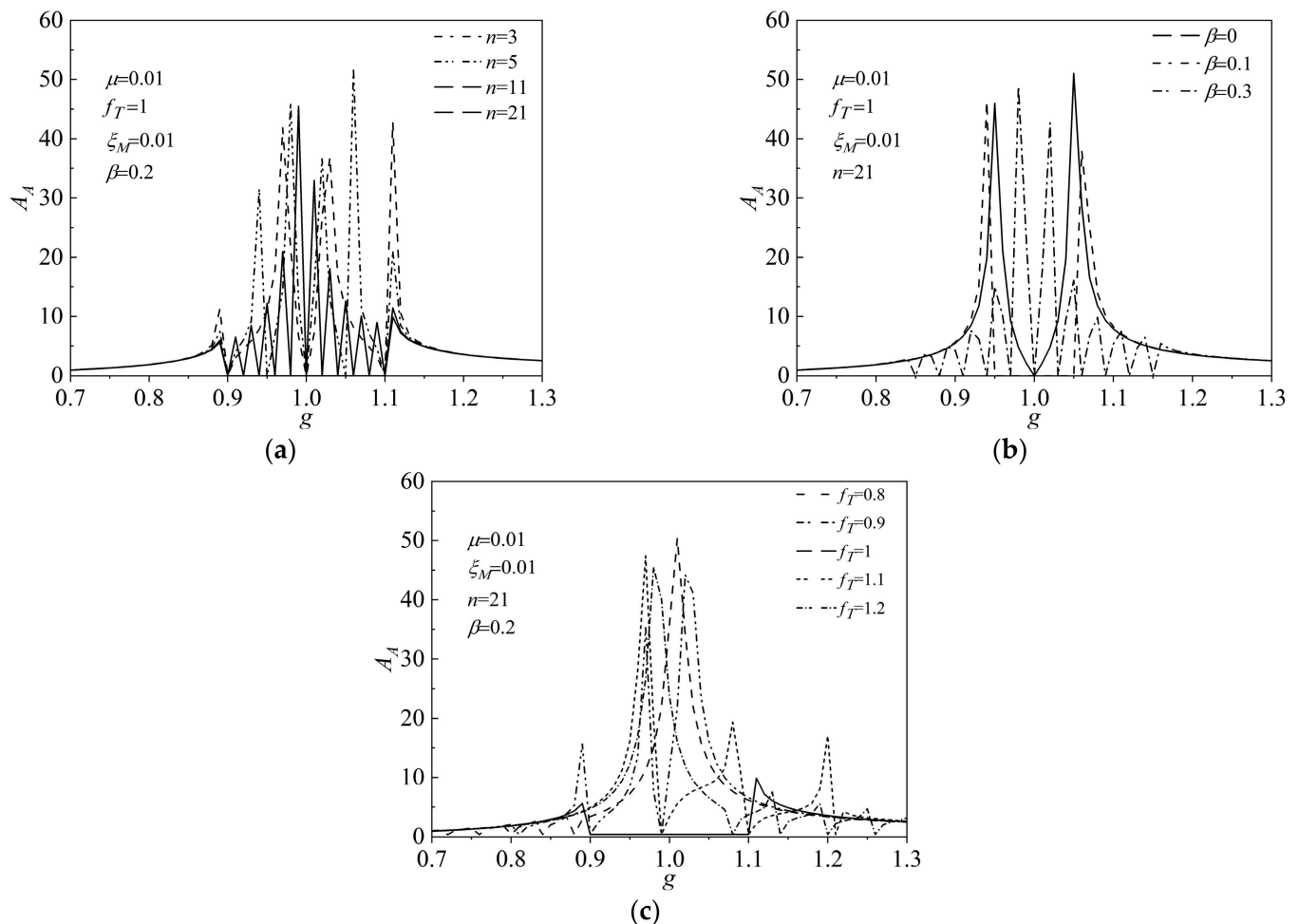


Figure 7. Influence of the MPTMD system parameters on the dynamic amplification factor of the primary structure response. (a) Influence of quantity n . (b) Influence of bandwidth β . (c) Influence of the average tuning-frequency ratio f_T .

The acceleration dynamic amplification factor curves of the MPTMD system indicate that variations in its parameters significantly affect dynamic amplification. To achieve optimal control performance, the target parameters must be optimized. This section takes acceleration as the optimization objective and considers the mass ratio and the number of pendulums as variables to investigate the parameter optimization of the MPTMD system under load for different mass ratios and numbers of pendulums.

4.1. Influence of Mass Ratio on Optimal System Parameters

4.1.1. Influence of Mass Ratio on the Optimal Bandwidth

The variation in the optimal bandwidth β^{opt} with the mass ratio μ is shown in Figure 8. Compared to the case of the displacement dynamic amplification factor, the variation trend of the mass ratio versus the optimal bandwidth remains the same when acceleration is taken as the optimization objective. Specifically, the value of the optimal bandwidth β^{opt} increases

with an increase in the mass ratio μ . As shown in Figure 8, as the structural damping ratio increases, the optimal bandwidth curves corresponding to different mass ratios all exhibit a continuous upward trend. Furthermore, the optimal value curves corresponding to different numbers of pendulum masses, n , essentially coincide, with no clear pattern observed in the fluctuations among the individual curves.

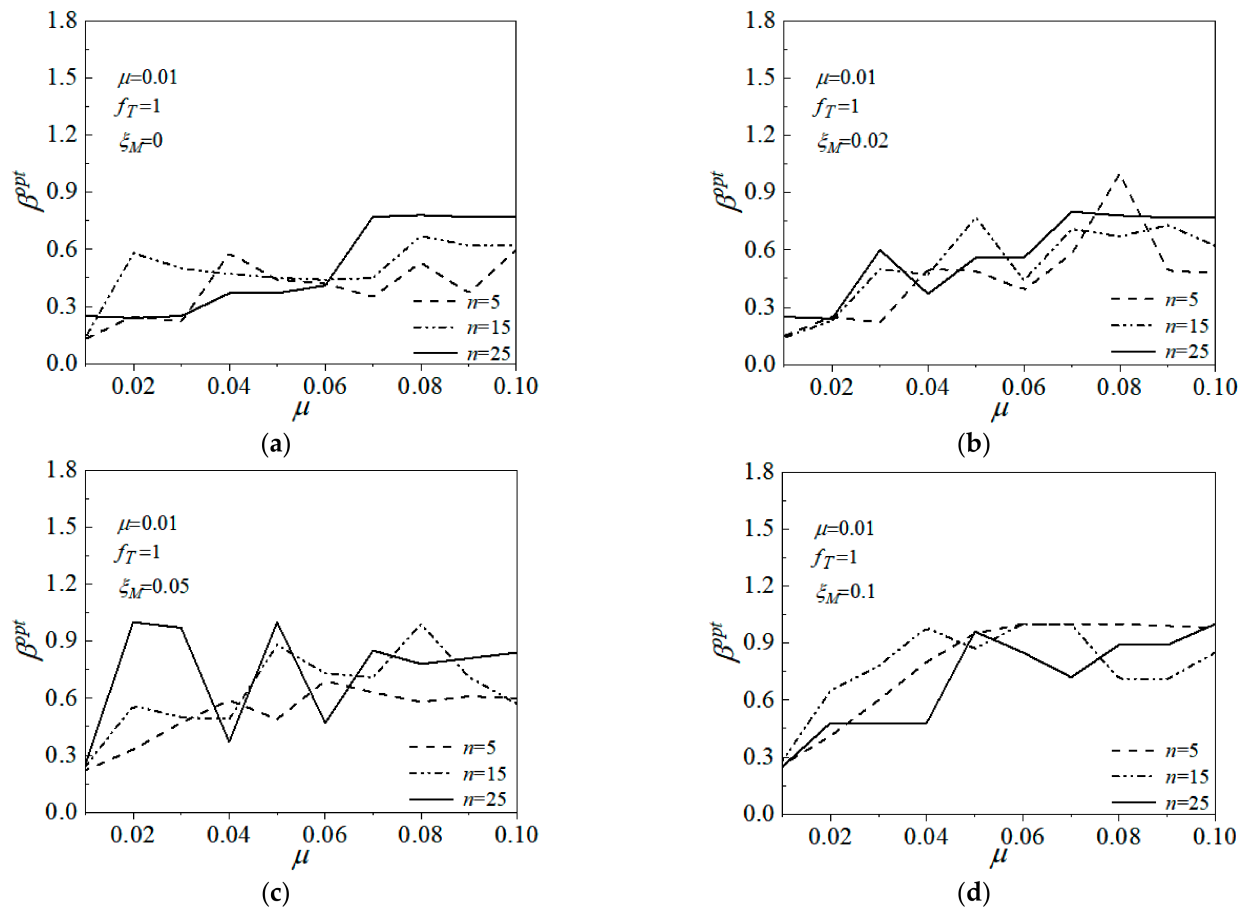


Figure 8. Variation trend of optimal bandwidth with mass ratio. (a) $\xi_M = 0$; (b) $\xi_M = 0.02$; (c) $\xi_M = 0.05$; (d) $\xi_M = 0.1$.

4.1.2. Influence of Mass Ratio on the Optimal Average Tuning-Frequency Ratio

The variation trend of the optimal average tuning-frequency ratio f^{opt} with the mass ratio μ is illustrated in Figure 9. For the same damping ratio of the primary structure, the variation trends of the optimal average tuning-frequency ratio curves corresponding to different numbers of pendulum masses n are consistent. However, their specific optimal values differ under the same mass ratio. As can be seen from Figure 9:

1. When the structural damping ratio $\xi_M < 0.05$, the optimal average tuning-frequency ratio increases with an increase in the mass ratio.
2. When the structural damping ratio $\xi_M > 0.1$, the optimal average tuning-frequency ratio exhibits a decreasing trend as the mass ratio increases, with its optimal value stabilizing around 0.8.
3. When the damping ratio of the primary structure $\xi_M = 0.1$, for mass ratios $\mu > 0.3$, the optimal average tuning-frequency ratio curves corresponding to different numbers of pendulums essentially coincide and converge to a stable value.

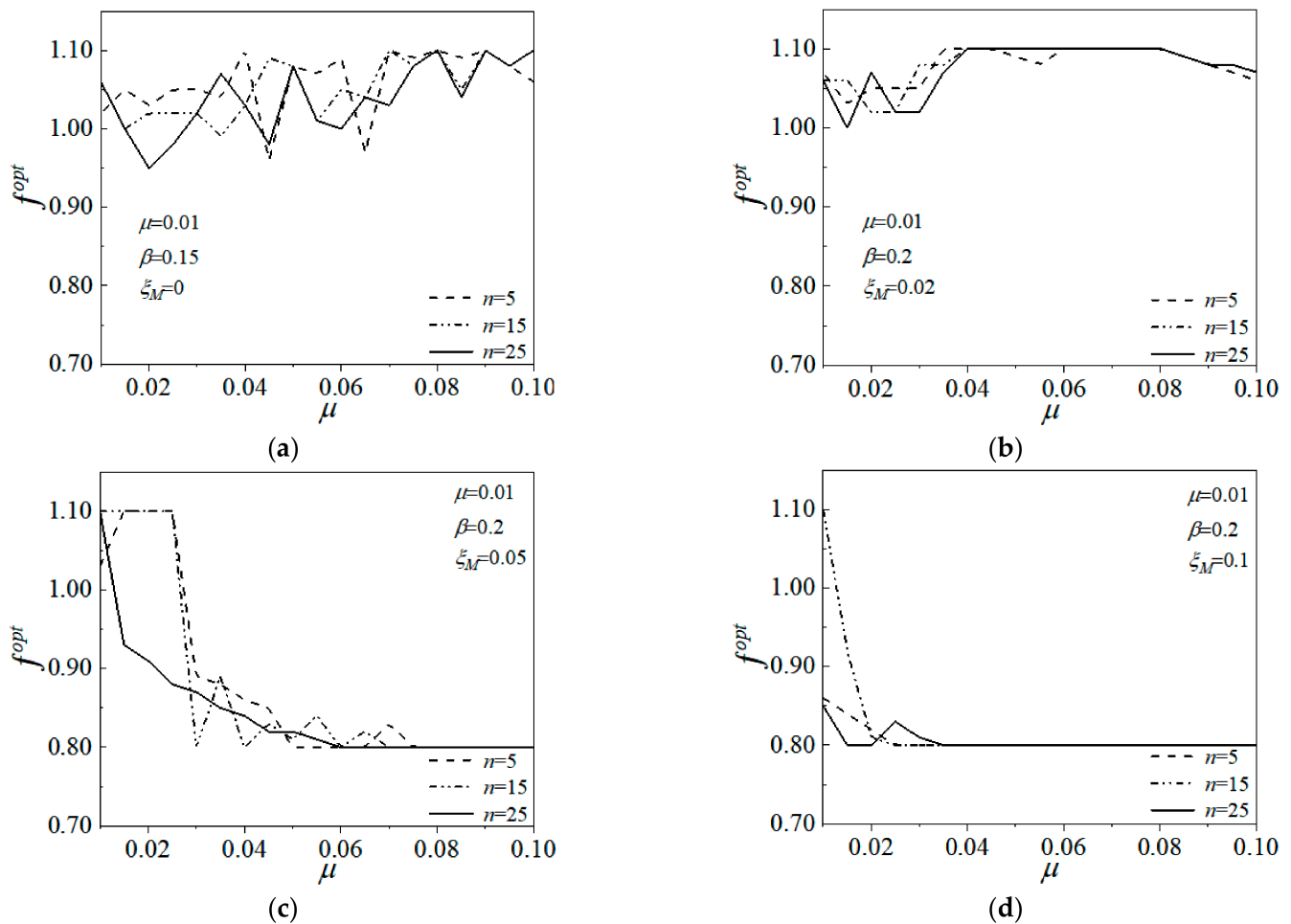


Figure 9. Variation trend of optimal average tuning-frequency ratio with mass ratio. (a) $\xi_M = 0$; (b) $\xi_M = 0.02$; (c) $\xi_M = 0.05$; (d) $\xi_M = 0.1$.

In conclusion, for the MPTMD system, the influence of the primary structure's damping ratio is non-negligible. Furthermore, approximately taking $\xi_M = 0.05$ as a boundary, the variation trend of the optimal average tuning-frequency ratio with the mass ratio is opposite under lower and higher structural damping ratios.

4.2. Influence of the Number of Pendulum Masses on Optimal System Parameters

4.2.1. Influence of the Number of Pendulum Masses on the Optimal Bandwidth

The variation in the optimal bandwidth β^{opt} with the number n is shown in Figure 10. Compared with the case where acceleration is the optimization objective, it can be observed from Figure 10 that when the structural damping ratio $\xi_M < 0.02$, the number-optimal bandwidth curve is essentially consistent with the result obtained using displacement as the optimization objective. Therefore, when the damping ratio of the primary structure is relatively small, the optimal bandwidth for the MPTMD system can be selected according to the same optimal value curve based on the number of pendulum masses n . When the structural damping ratio is relatively large, the optimization curves obtained from the two objectives differ significantly. Therefore, the optimal parameter values should be determined separately according to their respective optimal parameter curves.

4.2.2. Influence of the Number of Pendulum Masses on the Optimal Average Tuning-Frequency Ratio

The variation in the optimal average tuning-frequency ratio f^{opt} with the number n is shown in Figure 11. As the number of pendulum masses n increases, the optimal average tuning-frequency ratio of the MPTMD system fluctuates within the range of 0.9 to 1.1 for different values of the damping ratio ξ_M of the primary structure. Although the fluctuation trends of the number-optimal average tuning-frequency ratio curves are similar for different structural damping ratios, the corresponding optimal values of the average tuning-frequency ratio are not identical. When $\xi_M = 0.1$, the optimal value curve of the average tuning-frequency ratio exhibits larger fluctuation amplitudes as the number increases. Therefore, the influence of the damping ratio of the primary structure cannot be neglected when investigating the relationship between the number and the optimal average tuning-frequency ratio.

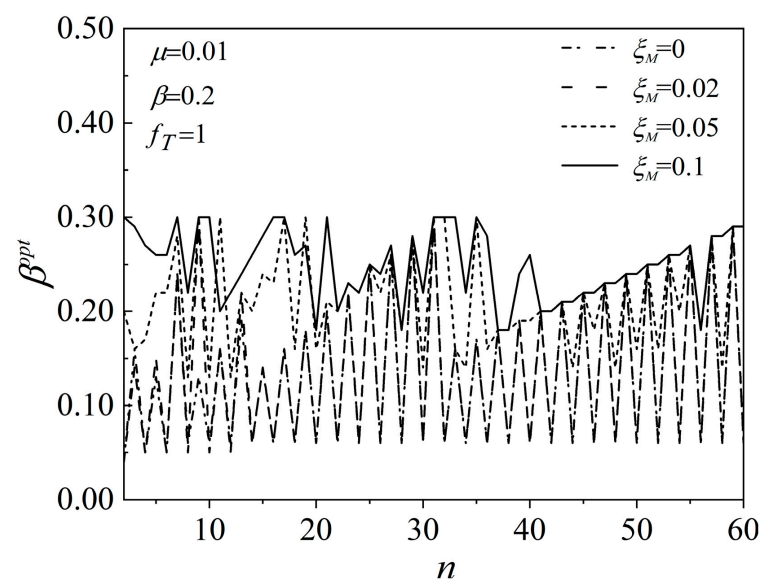


Figure 10. Variation trend of the optimal bandwidth with the number of pendulum masses.

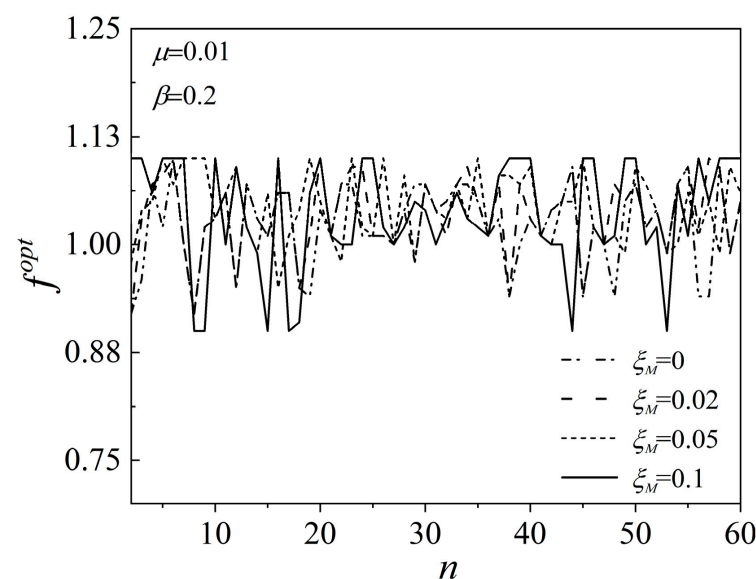


Figure 11. Variation trend of the optimal average tuning-frequency ratio with the number of pendulum masses.

5. Numerical Analysis Verification

To further verify the validity of the theoretical solutions for the optimal MPTMD parameters, we conduct parameter optimization on the benchmark numerical model in this section to obtain the corresponding numerical solutions. The numerical solutions are compared with their theoretical counterparts, which also demonstrates the engineering applicability of the optimal parameters.

Based on the information on the 20-story benchmark steel structure, a finite element model is developed. The control system mass was simulated using lumped mass elements, the stiffness components were modeled with LINK elements, and the dampers were simulated using Maxwell-type viscous damper elements. The finite element model of the benchmark steel structure is shown in Figure 12. The MPTMD control system is attached to the top floor of the model, targeting the first-order Y-direction translational mode (modal period $T_1 = 3.85$ s). A synthetically generated wind load time history with a 10-year return period is input to calculate the structural acceleration response. The control target parameters are varied within a defined range, and the optimal parameters are determined by analyzing the control effectiveness on the acceleration response of the primary structure equipped with the MPTMD.

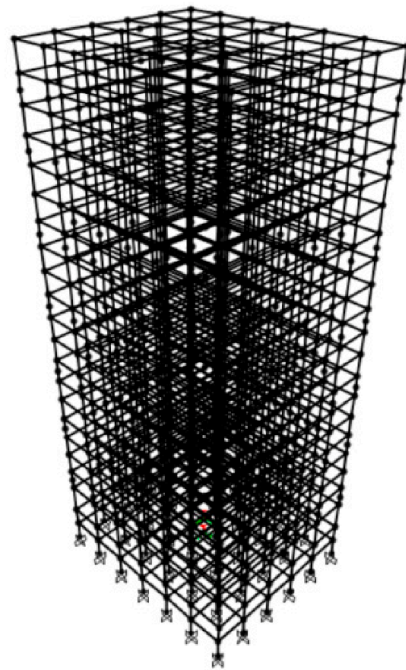


Figure 12. 20-story benchmark steel structure finite element model.

5.1. Variation Relationship Between Optimal Parameters and the Mass Ratio

5.1.1. Mass Ratio—Optimal Bandwidth

To validate the correctness of the theoretically derived optimal bandwidth curves corresponding to different numbers of pendulums, the structural damping ratio is set as $\zeta_M = 0.02$ and the average tuning-frequency ratio as $f_T = 1$. Numerical analyses are conducted for two cases with the number of pendulum masses $n = 3$ and $n = 9$ to obtain the numerical solutions of the optimal bandwidth that yield the best wind-induced vibration control performance. As shown in Figure 13, the numerical results lie on both sides of the theoretical optimal bandwidth curve and follow the same variation trend, indicating good agreement between the two.

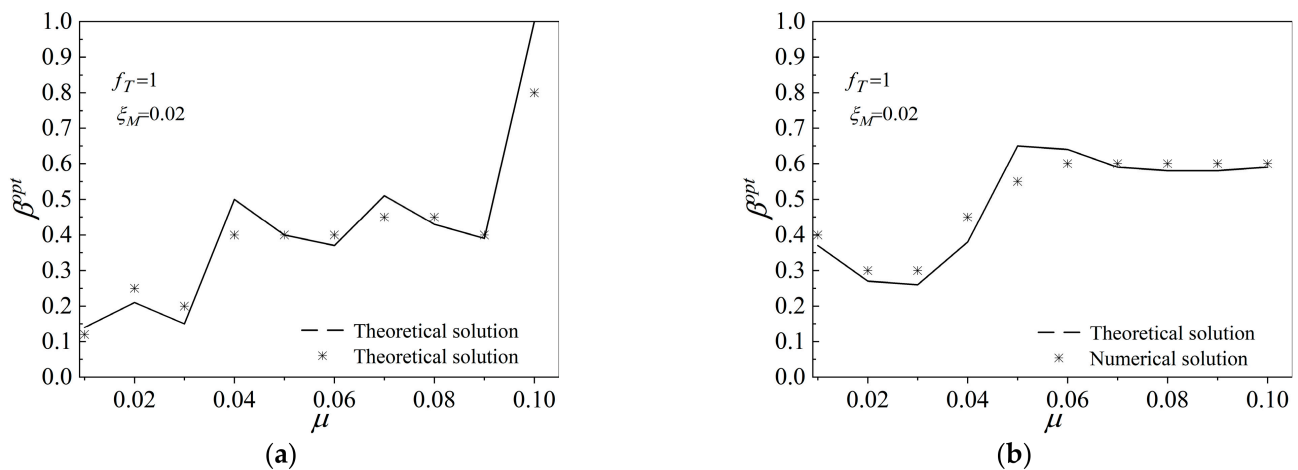


Figure 13. Comparison between the theoretical solution and the numerical solution. (a) $n = 3$; (b) $n = 9$.

5.1.2. Mass Ratio—Optimal Average Tuning-Frequency Ratio

Taking the number of pendulum masses as $n = 3$ and $n = 9$, with the structural damping ratio controlled at $\zeta_M = 0.02$ and the bandwidth set to a constant value of $\beta = 0.2$, numerical analysis of the structural response under wind load is performed by varying the average tuning-frequency ratio f_T within the range of 0.9 to 1.1. This process determines the optimal value of the average tuning-frequency ratio corresponding to different mass ratios. The results from the numerical analysis are then compared with the mass ratio—optimal average tuning-frequency ratio curve, as shown in Figure 14.

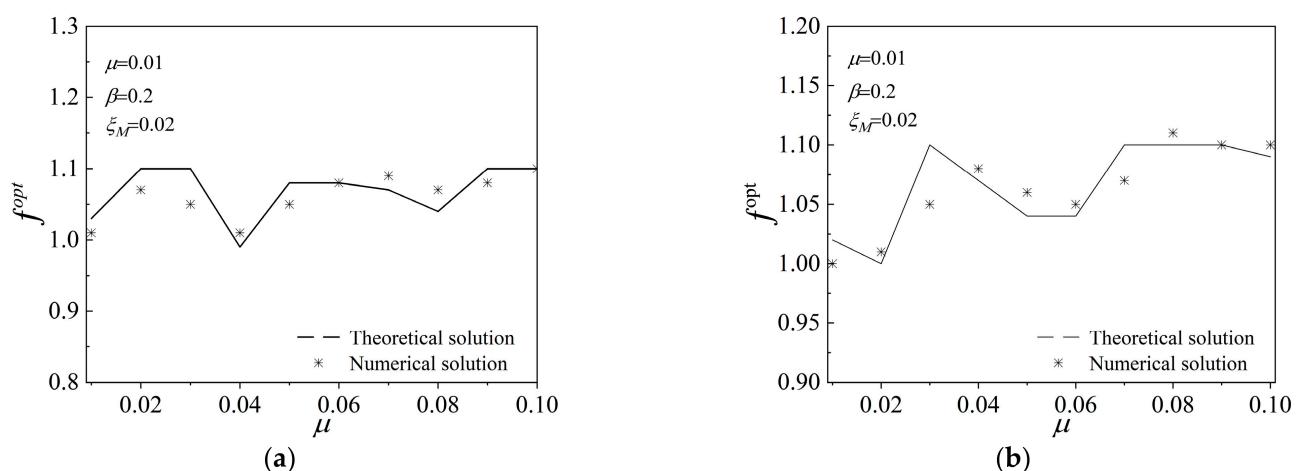


Figure 14. Comparison between the theoretical solution and the numerical solution. (a) $n = 3$; (b) $n = 9$.

5.2. Variation Relationship Between Optimal Parameters and the Number of Pendulum Masses

5.2.1. The Number of Pendulum Masses—Optimal Bandwidth

The relationship between the number of pendulum masses and the optimal bandwidth is shown in Figure 15, where the bandwidth β varies between 0 and 0.3. As shown in the figure, the theoretical solution and the numerical solution correspond well for different numbers of pendulum masses. Both can be used to guide the parametric design of the MPTMD.

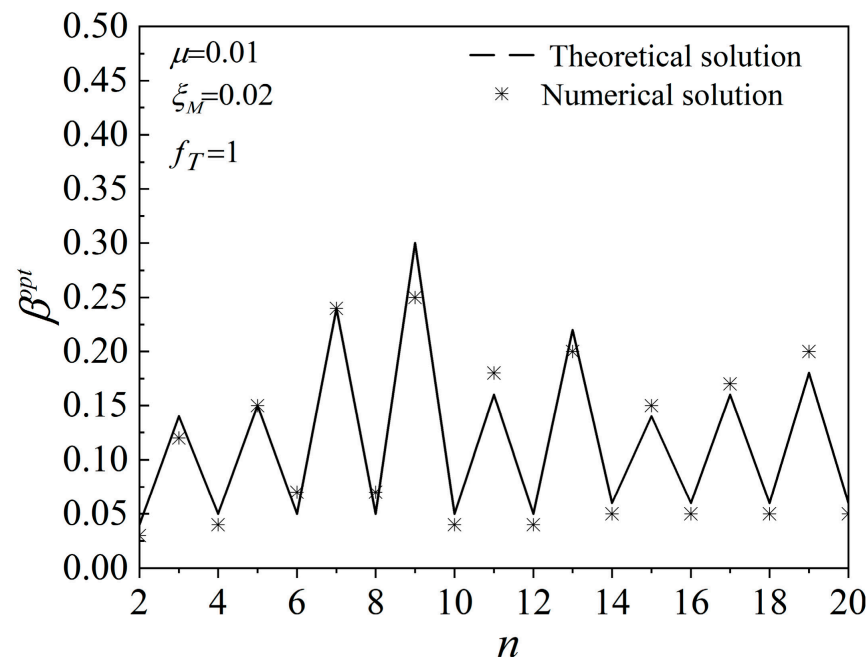


Figure 15. Variation in the optimal bandwidth.

5.2.2. The Number of Pendulum Masses—Optimal Average Tuning-Frequency Ratio

When verifying the curve of the quantity-optimal average tuning-frequency ratio, with the bandwidth β controlled at 0.2, the optimal average tuning-frequency ratio generally falls between 0.9 and 1.1. The optimal average tuning-frequency ratio corresponding to each mass ratio is determined based on numerical analysis results, as shown in Figure 16. Although the MPTMD system was not specifically optimized for the actual wind load characteristics, the theoretical solution still agrees well with the numerical results. This demonstrates the reliability of the theoretical solution in guiding the parameter design of the MPTMD.

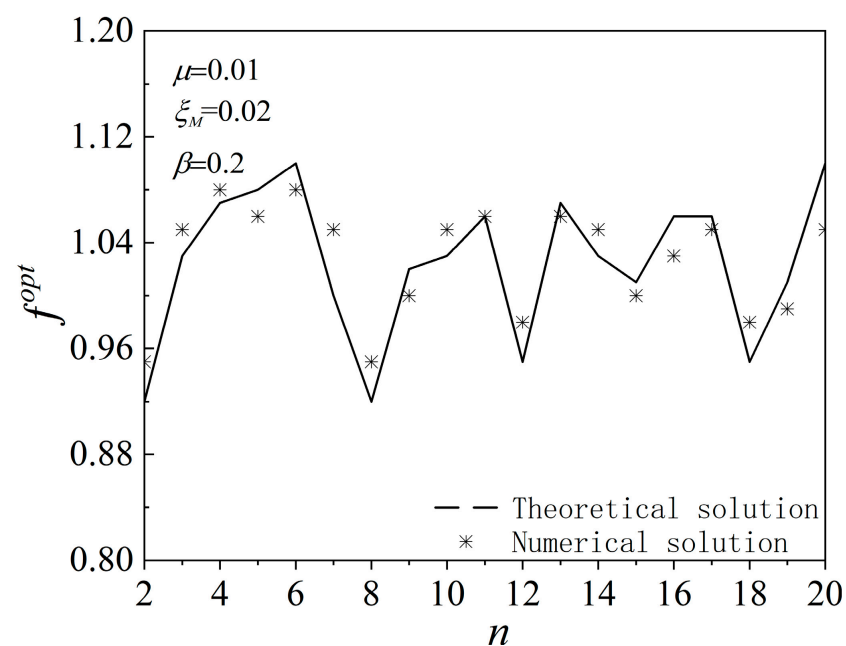


Figure 16. Variation in the optimal average tuning-frequency ratio.

6. Research on the Wind-Induced Vibration Control Effect of the MPTMD

In this section, the control performance of the MPTMD is verified through a wind-induced vibration response example of the benchmark model. Modal analysis is conducted on the primary structure, as illustrated in Table 1. Due to the significant difference in translational lateral stiffness between the two principal directions of the structure, comfort is governed by the weaker stiffness direction (y-direction). Therefore, the target mode shape for the MPTMD and TMD control systems is the first-order translation in the y-direction. Wind loads are simulated using the harmonic synthesis method with the following parameters: ground roughness Category C, a basic wind pressure of 0.45 kN/m^2 (10-year return period), and a simulation duration of 600 s. The time-history responses of the primary structure equipped with MPTMD and TMD systems (total mass ratio $\mu_t = 0.01$) are calculated, as shown in Figure 17. The number of pendulums of the MPTMD is set to $n = 10$.

Table 1. Modal analysis results.

Dynamic Characteristics	Modal (Translation in the x-Direction)			Modal (Translation in the y-Direction)		
	Mode2	Mode5	Mode8	Mode1	Mode4	Mode7
Period/s	1.41	0.50	0.28	3.85	0.59	0.31
Modal Mass	74	11	3.34	74	11	3.49
Participation Factor/%						

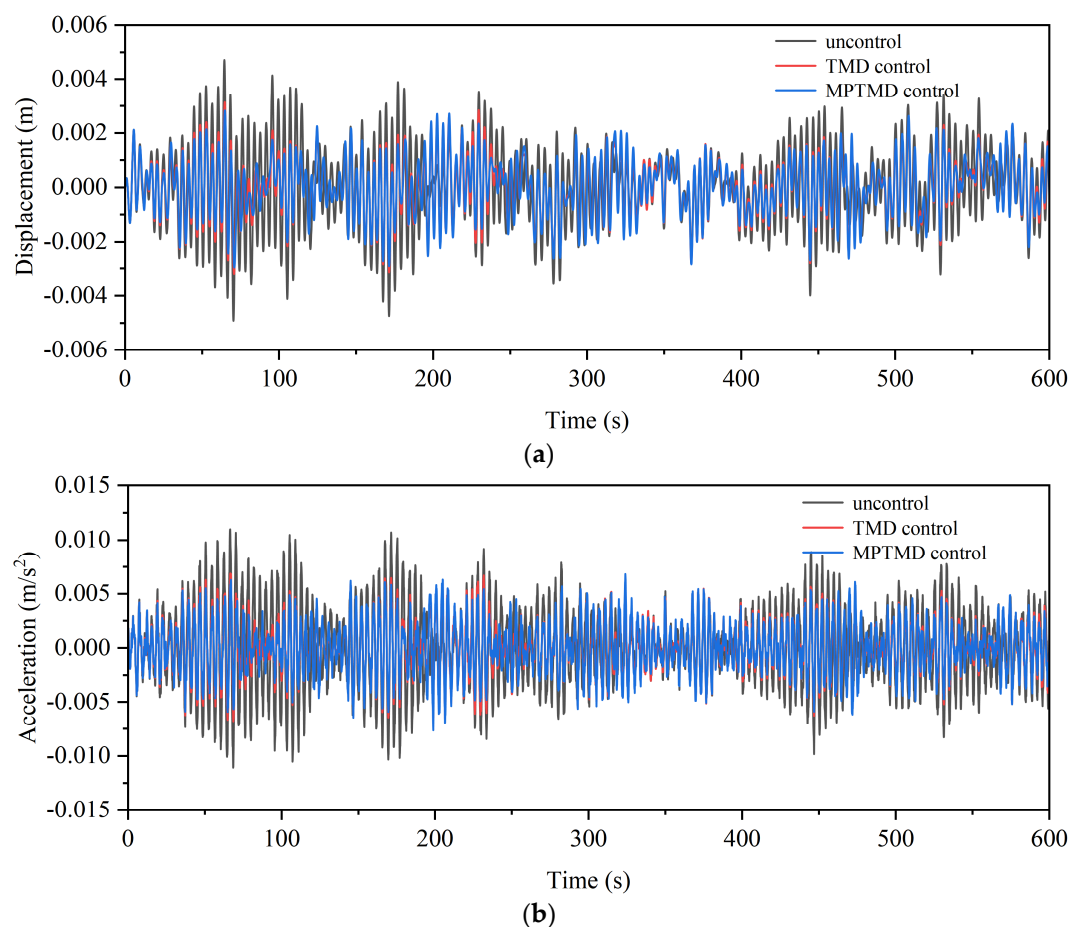


Figure 17. Time histories of wind-induced vibration responses of the primary structure under TMD and MPTMD control. (a) Displacement response. (b) Acceleration response.

The results demonstrate that both systems can effectively suppress structural responses under fluctuating wind loads, and MPTMD outperforms TMD in terms of acceleration and displacement suppression. Further analysis evaluates the vibration control performance of the two control systems under three different mass ratios, as shown in Table 2. Across all mass ratios, MPTMD consistently outperforms TMD in vibration control, with a particularly significant advantage in peak displacement control. This can be attributed to the fact that displacement is more sensitive to low-frequency components near resonance, where the MPTMD's frequency spread is most beneficial. Acceleration, being proportional to the square of frequency, is influenced by higher-frequency content that lies outside the MPTMD's optimized band.

Table 2. Wind-induced vibration control performance of MPTMD and TMD.

μ_t	Control System	Displacement Root Mean Square Reduction Rate (%)	Peak Displacement Reduction Rate (%)	Acceleration Root Mean Square Reduction Rate (%)	Peak Acceleration Reduction Rate (%)
0.01	MPTMD	30.61	40.06	36.10	37.86
	TMD	30.39	33.47	35.08	37.36
0.015	MPTMD	34.23	45.25	40.29	42.53
	TMD	33.84	38.55	39.34	42.13
0.02	MPTMD	36.40	48.96	43.32	44.70
	TMD	36.17	41.84	42.30	44.22

7. Conclusions

This study focuses on parameter optimization of an MPTMD system for wind-induced vibration control. With displacement and acceleration as the optimization objectives, respectively, the expressions for the dynamic amplification factors of the primary structure equipped with the MPTMD are derived. The optimal parameters of the control system are determined by minimizing the maximum value of the dynamic amplification factor. Furthermore, the influencing relationships among the optimal parameters of the MPTMD system are investigated. Finally, the reliability of the theoretical solution for the MPTMD system is validated through numerical simulations. The main conclusions are as follows:

1. The equation of motion for a single-degree-of-freedom primary structure equipped with the MPTMD system is established, and the system's transfer function and dynamic amplification factors are solved.
2. Using displacement and acceleration as optimization objectives, parameter optimization design is performed for the MPTMD system. Taking the mass ratio and the number of pendulum masses as variables, the influencing relationships among the optimal parameters of the MPTMD system are studied.
3. A 20-story benchmark model is established and subjected to wind loads to simulate the control performance of the MPTMD. With acceleration as the control target, the numerical solutions for the optimal parameters are determined. A comparison between the numerical results and theoretical solutions verifies the correctness of the latter.
4. The control performance of MPTMD and TMD is compared by calculating the dynamic responses of the benchmark model under wind load excitations.

Despite the contributions listed above, this study has several limitations that merit attention. First, the theoretical derivations assume an SDOF primary structure, whereas real tall buildings exhibit MDOF behavior and modal coupling. Second, the numerical verification is conducted using a synthetic wind load with a 10-year return period; the sensitivity of the optimal parameters to different wind spectra and turbulence intensities has not been examined.

Accordingly, future research could focus on the following aspects: (1) extending the optimization framework to MDOF structures with multiple controlled modes; (2) incorporating realistic stochastic wind fields and uncertainty quantification into the parameter design; (3) conducting experimental tests (e.g., shake table or wind tunnel) to validate the theoretical and numerical findings; (4) exploring hybrid control strategies that combine MPTMD with active or semi-active devices; and (5) investigating the fatigue performance and life-cycle cost of MPTMD systems in long-term service.

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Abbreviations

The following abbreviations are used in this manuscript:

TMD	Tuned Mass Damper
MPTMD	Multiple-Pendulum Tuned Mass Damper

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