

Article

A Hybrid Integration Approach for Milling Stability Prediction of Regenerative Chatter Using Simpson and Hermite Methods

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Abstract

This paper proposes an enhanced numerical integration technique for predicting milling stability. The underlying milling dynamics model incorporates regenerative chatter effects, formulated as a system of linear time-delay differential equations. The computational methodology begins by dividing the tool engagement period into the free vibration and forced vibration intervals, followed by uniform discretization of the forced vibration interval. A numerical integration method is primarily carried out using Simpson's and Hermite's rules. Thus, a discrete dynamic mapping that correlates the system's current state with its previous state is constructed. Based on this, the milling stability is ultimately determined by applying Floquet theory. Furthermore, the mean squared error metric is introduced to quantify the prediction accuracy of stability lobe diagrams. Through comprehensive comparative analyses, the predicted efficiency and accuracy of the proposed method are systematically benchmarked against the conventional approaches. The simulated and experimental results demonstrate that the proposed method achieves high computational efficiency alongside good accuracy, and its engineering practicality is rigorously validated through milling experiments.

Keywords: milling stability; numerical integration; Simpson formula; Hermite formula; mean squared error

1. Introduction

The limited stiffness of thin-walled workpieces makes them prone to regenerative chatter in milling processes. This vibration phenomenon not only severely degrades machining accuracy and surface integrity, but also accelerates tool wear, often resulting in cutting edge chipping and reduced tool life [1,2]. Furthermore, it jeopardizes critical machine tool components, necessitating vibration suppression through reduced cutting parameters, thereby significantly constraining production efficiency [3,4]. Consequently, a thorough investigation of chatter is essential to enhance both machining precision and productivity.

Extensive research has focused on three primary approaches for addressing milling chatter: chatter prediction, identification [5,6], and suppression [7,8]. Among these, this paper focuses on chatter prediction. The dynamics of regenerative milling are

Academic Editor: Konstantinos-Dionysios Bouzakis

Received: 16 January 2026

Revised: 31 January 2026

Accepted: 6 February 2026

Published: 8 February 2026

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characterized by delay differential equations (DDEs). Solving these DDEs generates stability lobe diagrams (SLDs), which are essential for predicting chatter-free machining parameters [9,10].

Researchers have developed various methods to predict SLDs. Among these, discretization methods and numerical integration techniques have received particular attention. The semi-discretization method (0thSDM) was first introduced by Insperger et al. [11]. Subsequent refinements led to the first-order SDM (1stSDM) [12] for improved accuracy, followed by the development of various higher-order SDMs. Ding et al. [13] later developed the full-discretization method (1stFDM), which was followed by the second-order FDM (2ndFDM) [14] for greater precision. Later, higher-order FDMs were developed.

Ding et al. [15] pioneered a numerical integration method (NIM) based on Newton–Cotes formulas and a Gaussian quadrature. Subsequently, Zhang et al. [16] incorporated structural mode coupling effects alongside regenerative effects into an improved Newton–Cotes-based approach, demonstrating superior effectiveness over methods considering regenerative effects alone. Liang et al. [17] advanced NIM by introducing variable time-delay parameters, enabling stability prediction under low radial immersion conditions. Niu et al. [18] generalized the Runge–Kutta method derived from the second-kind Volterra integral equation to enhance computational performance. Zhang et al. [19] established the superiority of Simpson’s rule over 1stSDM and 2ndFDM in convergence speed and accuracy. To address variable-speed milling challenges, Ding et al. [20] proposed an NIM based on ODE precise solutions, while Niu et al. [21] concurrently proposed a variable-step NIM by unifying spindle speed and time delay within the Floquet theory framework. Ozoegwu et al. [22] later contributed 3rd and 4th vector NIM for high-precision solutions to milling stability integrals. Mei et al. [23] developed two initialization methods for Milne–Simpson’s scheme using linear multistep theory, numerically validating their impact on convergence rates. To enhance accuracy and efficiency, Dong et al. [24] developed an updated NIM leveraging Hermite interpolation. Concurrently, Li et al. [25] employed Newton–Cotes rules to establish response relationships between successive time points via integral equations—distinct from conventional discrete methods that directly integrate state–space equations. Lou et al. [26] applied the Cotes formula to compute fitted DDE solutions at discretization points, demonstrating superior computational accuracy and efficiency over both the 1stSDM and 1stFDM benchmarks. Further advancing the field, Xia et al. [27] introduced an NIM using Lagrange interpolation schemes, later refining it into a hybrid Lagrange–Simpson approach [28]. This technique constructs high-order integration formulas through Lagrange polynomials while strategically applying Simpson’s rule to minimize local discretization errors. Finally, Zhan et al. [29] presented a method combining 1/3 and 3/8 Simpson formulas, achieving exceptional prediction accuracy with computational efficiency.

Numerical integration methods often offer high computational efficiency and precision. However, achieving further simultaneous enhancement in both precision and efficiency, or establishing a favorable trade-off between them, remains an ongoing research challenge. This study introduces a method that synergistically combines Simpson’s and Hermite’s rules, thereby achieving a balanced integration of accuracy and computational efficiency. Furthermore, regarding the accuracy assessment of stability lobe diagrams (SLDs), current practices predominantly rely on qualitative approaches. To address this, the present work proposes a quantitative accuracy assessment method based on the mean squared error (MSE) between the predicted and reference stability limits across relevant spindle speeds.

2. Mathematical Model and Algorithm

The milling dynamics incorporating regenerative effects are governed by state–space formulations [11]:

$$\mathbf{M} \ddot{\mathbf{w}} + \mathbf{C} \dot{\mathbf{w}} + \mathbf{K} \mathbf{w} = a \mathbf{K}_c(t) [\mathbf{w}(t) - \mathbf{w}(t - T)] \quad (1)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ represent the mass, damping, and stiffness matrices, $\mathbf{K}_c(t)$ denotes the time-periodic coefficient matrix with period T , reflecting the regenerative effect in milling processes, a indicates the axial depth of cut and $\mathbf{u}(t)$ corresponds to the displacement state vector that characterizes the system's dynamic response. Equation (1) describes the 2nd DDE for milling dynamics. To transform it into the 1st DDE, define Equation (2)

$$\begin{aligned} \mathbf{p}(t) &= \mathbf{M} \dot{\mathbf{w}}(t) + \mathbf{C} \mathbf{w}(t) / 2 \\ \mathbf{x}(t) &= \begin{bmatrix} \mathbf{w}(t) & \mathbf{p}(t) \end{bmatrix}^T \end{aligned} \quad (2)$$

Equation (3) is derived by inserting Equation (2) into Equation (1). The $\mathbf{A}_0$ is a constant matrix representing the time-invariant properties of the system, and $\mathbf{B}(t)$ is a periodic matrix determined by the dynamic cutting force, considering the regenerative effect, satisfying $\mathbf{B}(t) = \mathbf{B}(t + T)$. The derivation process of $\mathbf{A}$ and $\mathbf{B}(t)$ is detailed in Appendix A.

$$\dot{\mathbf{x}}(t) = \mathbf{A} \mathbf{x}(t) + a \mathbf{B}(t) [\mathbf{x}(t) - \mathbf{x}(t - T)] \quad (3)$$

The solution of Equation (3) is represented by Equation (4).

$$\mathbf{x}(t) - e^{\mathbf{A}(t-t_0)} \mathbf{x}(t_0) = a \int_{t_0}^t \left\{ e^{\mathbf{A}(t-\xi)} \mathbf{B}(\xi) [\mathbf{x}(\xi) - \mathbf{x}(\xi - T)] \right\} d\xi \quad (4)$$

As illustrated in Figure 1, T can be divided into free vibration and forced vibration intervals and T_f and T_c denote the two durations, where t_0 and t_1 represent the starting and ending points of free vibration, respectively. The termination moment can be mathematically expressed by Equation (5) (at this stage, the $\mathbf{B}(\xi)$ in Equation (4) vanishes). To accurately characterize the cutting region, T_c is discretized into m subintervals, each with a duration of $h = T_c/m$. The discrete time points can be determined as $t_i - (t_0 + T_f) = (i - 1)h$, where $i = 1, \dots, m + 1$. Consequently, Equation (5) can be reformulated as Equation (6). For simplicity, $\mathbf{x}(t_i)$ is written as $\mathbf{x}_i$, $\mathbf{x}(t_i - T)$ is written as $\mathbf{x}_{i-T}$ and $\mathbf{B}(t_i - T)$ is written as $\mathbf{B}_{i-T}$.

$$\mathbf{x}(t_0 + T_f) - e^{\mathbf{A}T_f} \mathbf{x}(t_0) = 0 \quad (5)$$

$$\mathbf{x}_1 - e^{\mathbf{A}T_f} \mathbf{x}_{m+1-T} = 0 \quad (6)$$

Defining $\mathbf{G}_{i,n} = \mathbf{x}_i - e^{\mathbf{A}(i-n)h} \mathbf{x}_n$, (where $i > n$), Equation (4) can be rewritten as Equation (7).

$$\mathbf{G}_{i,i-2} = a \int_{t_{i-2}}^{t_i} \left\{ e^{\mathbf{A}(t_i-\xi)} \mathbf{B}(\xi) [\mathbf{x}(\xi) - \mathbf{x}(\xi - T)] \right\} d\xi \quad (7)$$

Simpson's formula leads to the derivation of Equation (8) from Equation (7), for $i = 3, \dots, m + 1$.

$$\mathbf{G}_{i,i-2} = \frac{a}{3} [e^{2\mathbf{A}h} \mathbf{B}_{i-2} (\mathbf{x}_{i-2} - \mathbf{x}_{i-2-T}) + 4e^{\mathbf{A}h} \mathbf{B}_{i-1} (\mathbf{x}_{i-1} - \mathbf{x}_{i-1-T}) + \mathbf{B}_i (\mathbf{x}_i - \mathbf{x}_{i-T})] \quad (8)$$

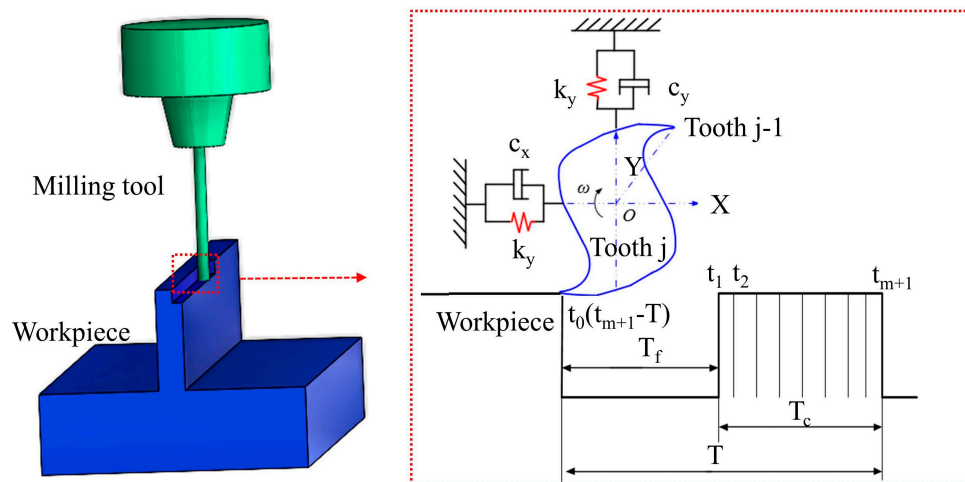


Figure 1. Tooth-passing period discretization mapping in milling process.

Based on reference [24], Equation (9) presents the approximate derivation process of the Hermite integral algorithm. By reformulating Equation (4) into the form of Equation (10), there is a specific case when $i = 2$ can be correspondingly expressed as Equation (11).

$$\int_{x_i}^{x_{i+1}} f(x) dx = \frac{h}{2}(f_i + f_{i+1}) + \frac{h^2}{12}(f'_i - f'_{i+1})$$

$$f'_i = \frac{-3f_i + 4f_{i+1} - f_{i+2}}{2h}$$

$$f'_{i+1} = \frac{-f_{i+1} + f_{i+2}}{2h}$$

$$\int_{x_i}^{x_{i+1}} f(x) dx = \frac{5h}{12} f(x_i) + \frac{2h}{3} f(x_{i+1}) - \frac{h}{12} f(x_{i+2})$$

$$\mathbf{G}_{i,i-1} = a \int_{t_{i-1}}^{t_i} \left\{ e^{\mathbf{A}(t_i - \zeta)} \mathbf{B}(\zeta) [\mathbf{x}(\zeta) - \mathbf{x}(\zeta - T)] \right\} d\zeta \quad (9)$$

$$\mathbf{G}_{2,1} = \frac{ah}{12} [5e^{\mathbf{A}h} \mathbf{B}_1 (\mathbf{x}_{1-T} - \mathbf{x}_{1-T}) + 8\mathbf{B}_2 (\mathbf{x}_2 - \mathbf{x}_{2-T}) - e^{-\mathbf{A}h} \mathbf{B}_3 (\mathbf{x}_3 - \mathbf{x}_{3-T})] \quad (11)$$

Based on Equations (6), (8) and (11), we establish the discrete dynamic mapping between the current state and the previous cycle, as formulated in Equation (12).

$$\left(\mathbf{H} - \frac{ah}{12} \mathbf{N} \right) \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_{m+1} \end{bmatrix} = \left(-\frac{ah}{12} \mathbf{N} + \mathbf{F} \right) \begin{bmatrix} \mathbf{x}_{1-T} \\ \mathbf{x}_{2-T} \\ \vdots \\ \mathbf{x}_{m+1-T} \end{bmatrix} \quad (12)$$

where the state transition matrix Φ is defined as Equation (13)

$$\Phi = \left(\mathbf{H} - \frac{ah}{12} \mathbf{N} \right)^{-1} \left(-\frac{ah}{12} \mathbf{N} + \mathbf{F} \right) \quad (13)$$

with

$$\begin{aligned}
 \mathbf{H} &= \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 & 0 \\ -e^{A_h} & \mathbf{I} & 0 & 0 & 0 \\ -e^{2A_h} & 0 & \mathbf{I} & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & -e^{2A_h} & 0 & \mathbf{I} \end{bmatrix} \\
 \mathbf{N} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 5e^{A_h}\mathbf{B}_1 & 8\mathbf{B}_2 & -e^{-A_h}\mathbf{B}_3 & 0 & 0 \\ 4e^{2A_h}\mathbf{B}_1 & 16e^{A_h}\mathbf{B}_2 & 4\mathbf{B}_3 & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 4e^{2A_h}\mathbf{B}_{m-1} & 16e^{A_h}\mathbf{B}_m & 4\mathbf{B}_{m+1} \end{bmatrix} \\
 \mathbf{F} &= \begin{bmatrix} 0 & 0 & 0 & e^{A_f T_f} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
 \end{aligned} \tag{14}$$

Floquet analysis [30] reveals that stability depends on the eigenvalue λ of state transition matrix Φ .

Remark: For singular configurations of Φ encountered in numerical implementation, the regularization framework detailed from reference [15] provides an effective resolution strategy.

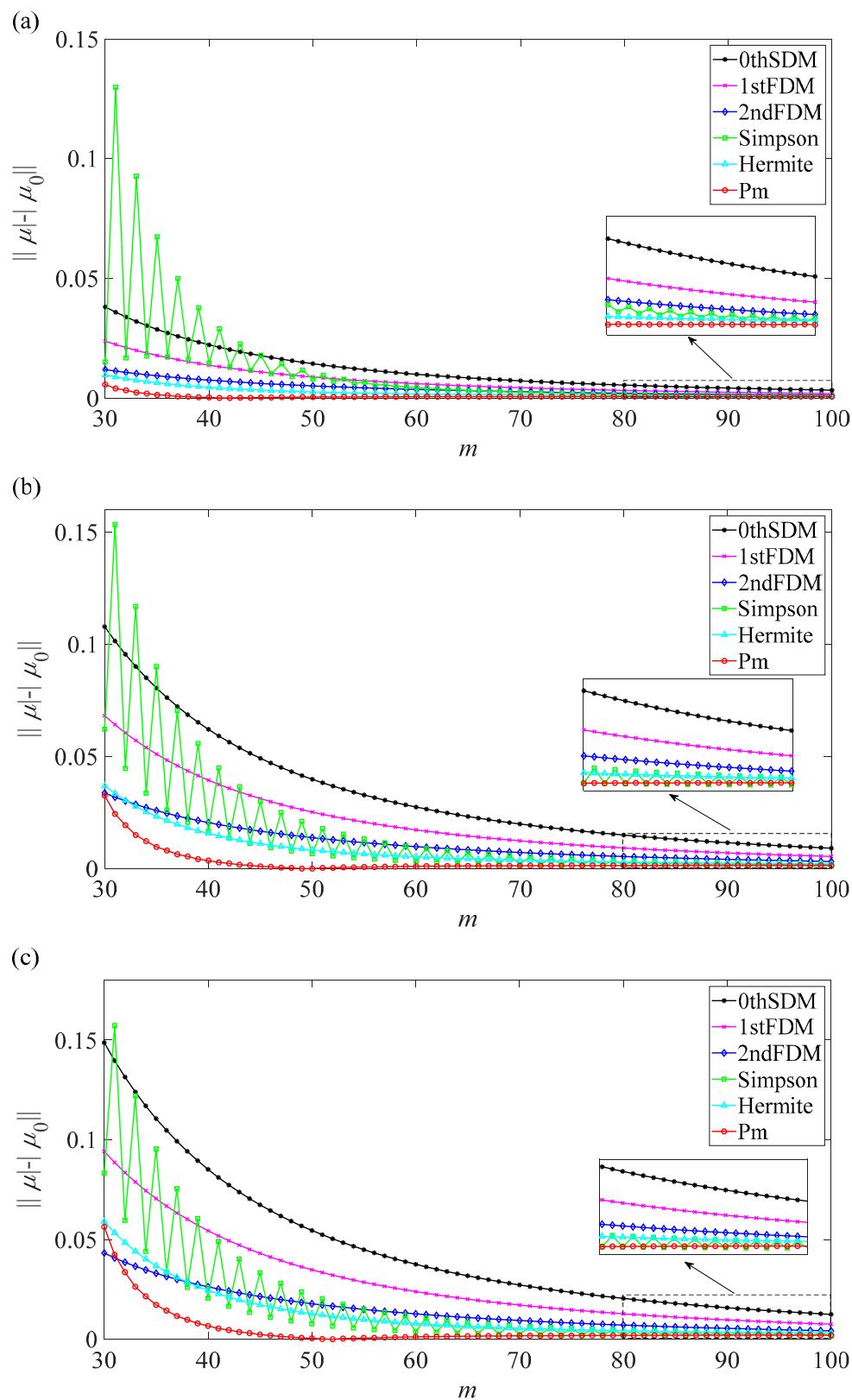
3. Convergence Rate

Convergence verification is performed using a One-DOF milling model example. The relevant parameters are provided in Appendix A. The analysis is performed between the proposed method and five established approaches in terms of convergence rate: 0thSDM [11], 1stFDM [12], 2ndFDM [14], Simpson's method [19], and Hermite's method [24]. For clarity, the proposed method is labeled as Pm in all figures below. All simulations are performed using MATLAB (v2024), on a desktop computer with a 12th Gen Intel i5 CPU and 16 GB RAM. The complete set of parameters used in the simulations is listed in Table 1 [13].

Figure 2 presents the convergence of different methods at different time interval number m . The spindle speed n is set to 5000 r/min, and four axial depths of cut of 0.2, 0.5, 0.7 and 1 mm are investigated. The horizontal axis denotes m . The vertical axis denotes the absolute error between critical and reference eigenvalue moduli, defined as $||\mu| - |\mu_0||$, where $|\mu|$ is the eigenvalue modulus computed by the evaluated method, and $|\mu_0|$ is the reference eigenvalue modulus calculated using the 1stSDM [12] at $m = 200$. The reference eigenvalue moduli $|\mu_0|$ corresponding to these depths of cut are 0.8192, 1.0726, 1.2197, and 1.4040 (Figure 3).

Table 1. Relevant parameters.

Parameters	Values
Natural frequency f_n (Hz)	922
Damping ratio ζ	1.1%
Modal mass m_t (Kg)	0.03993
Tangential cutting force coefficient K_t (N/m ²)	6×10^8
Radial cutting force coefficient K_r (N/m ²)	2×10^8
Number of teeth	2



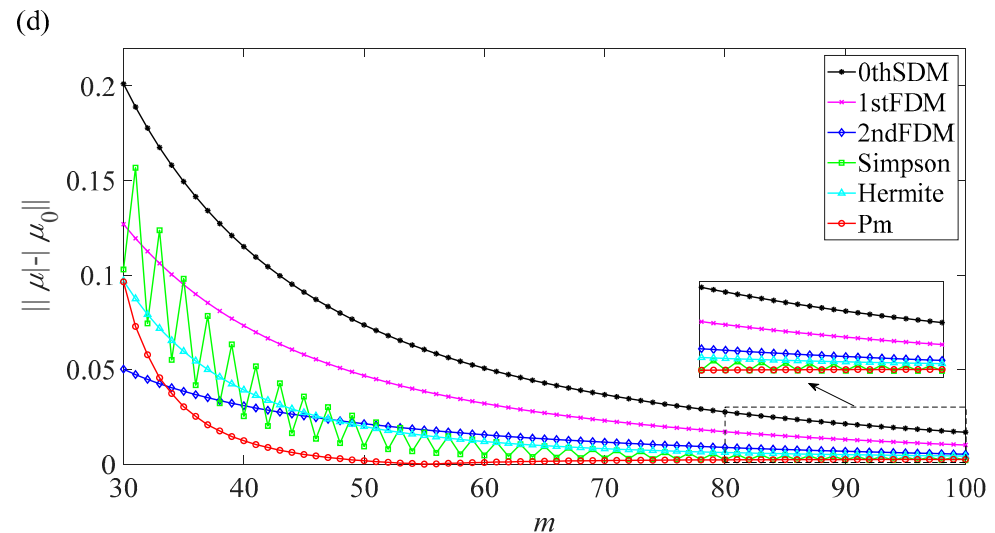


Figure 2. Comparisons of the convergence at different m of different methods: (a) $a = 0.2$, $|\mu_0| = 0.8192$; (b) $a = 0.5$, $|\mu_0| = 1.0726$; (c) $a = 0.7$, $|\mu_0| = 1.2197$; and (d) $a = 1$, $|\mu_0| = 1.4040$.

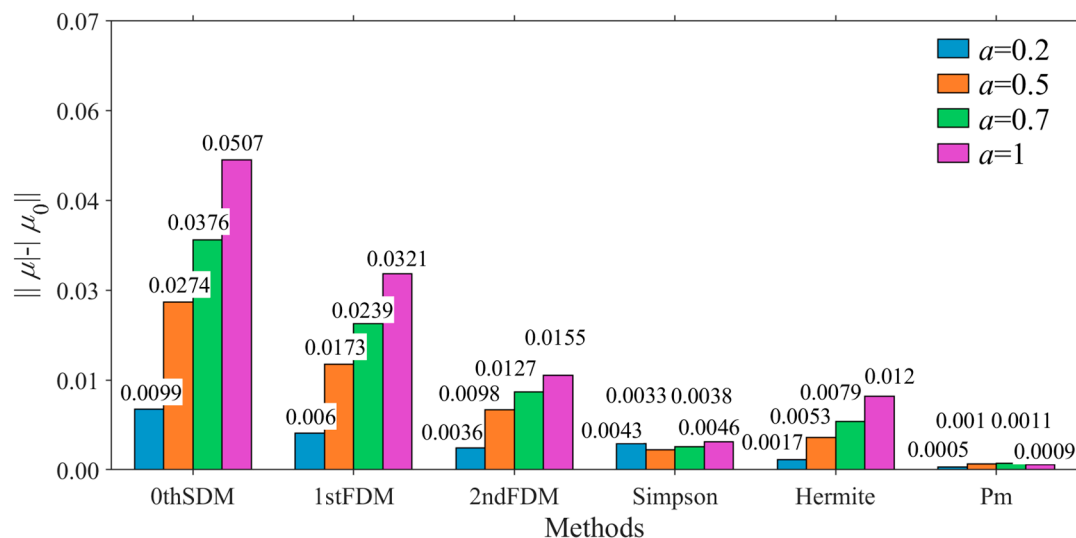


Figure 3. Comparisons of $||\mu| - \mu_0||$ under four axial depths of cut at $m = 60$.

4. Stability Lobe Diagrams Analysis

4.1. One-DOF Model Example

For the One-DOF milling model, SLDs are generated using the 0thSDM, 1stFDM, 2ndFDM, Simpson's method, Hermite's method, and the proposed method to compare accuracy. Parameters included: spindle speed (5000–10,000 r/min), axial depth of cut (0–10 mm), and radial immersion ratios aD (the ratio of radial depth of cut to tool diameter) of 0.05, 0.1, 0.5, and 1 (down-milling). A 200×100 grid spanned the spindle speed and axial depth plane. Given that SLDs accuracy requires sufficiently large m , the stability boundaries are acquired by the 2ndFDM at $m = 200$ served as the reference curves. Computational efficiency is evaluated by comparing the time for stability-boundary calculations.

Figures 4–7 present the comparative results of the SLDs generated by relevant approaches, compared with the reference curve under the One-DOF milling model. These figures reveal that all methods exhibit progressive improvement in accuracy as m increases. Furthermore, under equivalent m , the method achieves significantly higher

precision than the 0thSDM, 1stFDM, 2ndFDM, and Simpson's method, though it remains slightly less accurate than Hermite's method.

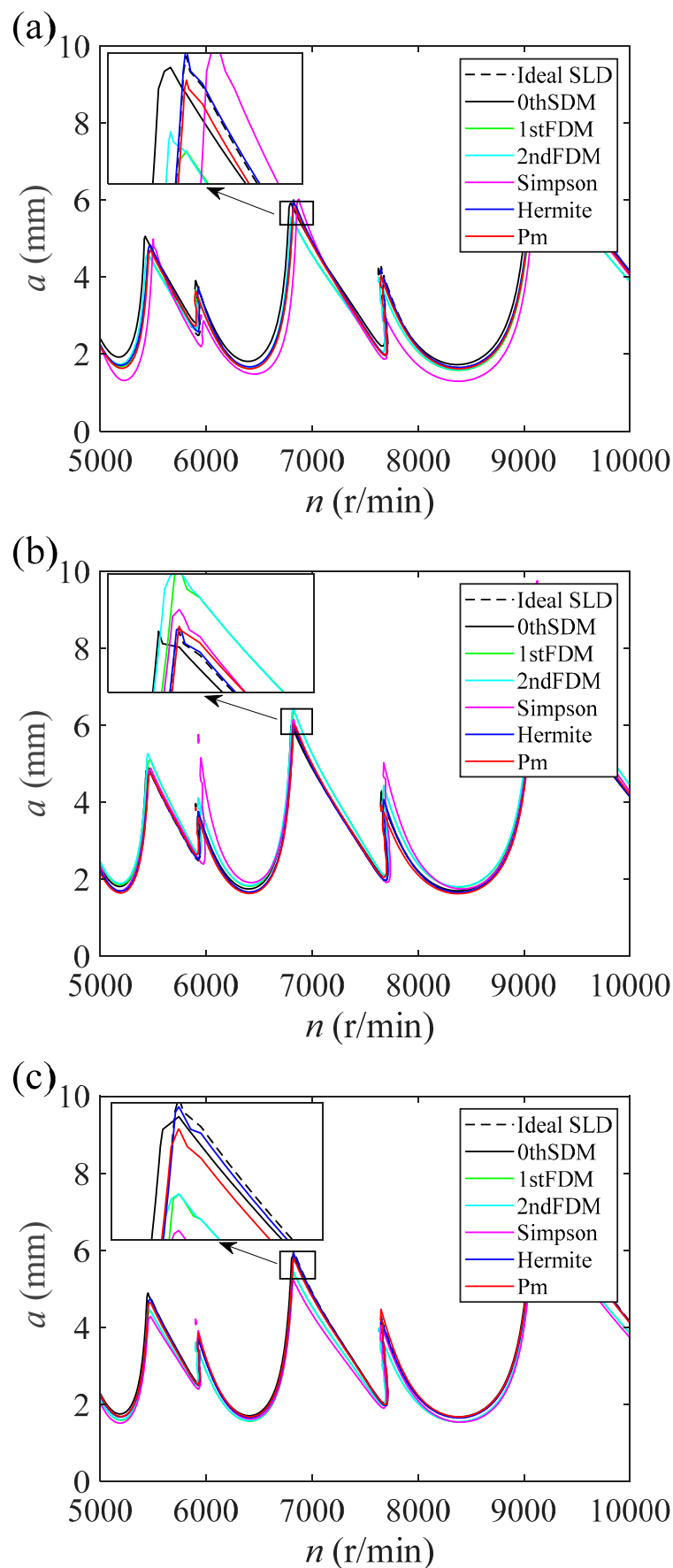


Figure 4. Comparisons of the accuracy of SLDs at aD of 0.05: (a) $m = 30$; (b) $m = 40$; and (c) $m = 50$.

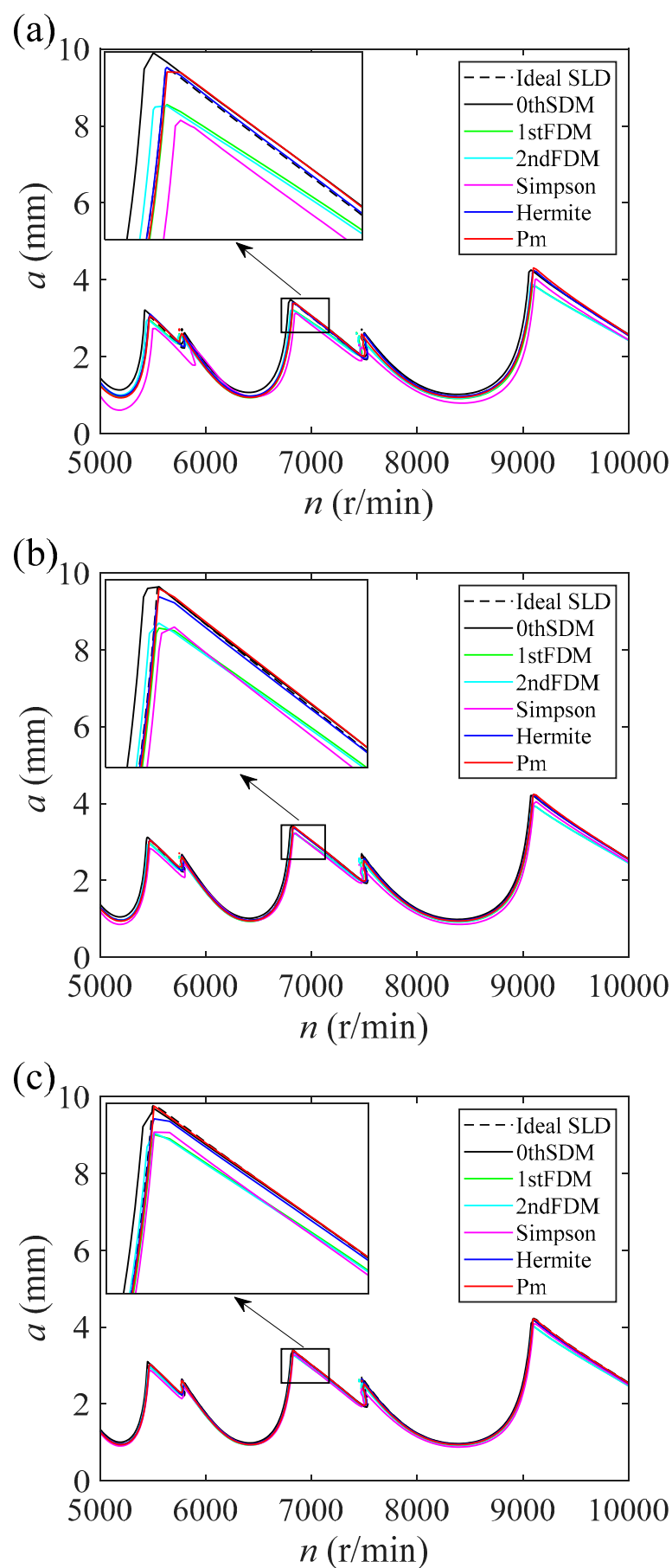


Figure 5. Comparisons of the accuracy of SLDs at aD of 0.1: (a) $m = 30$; (b) $m = 40$; and (c) $m = 50$.

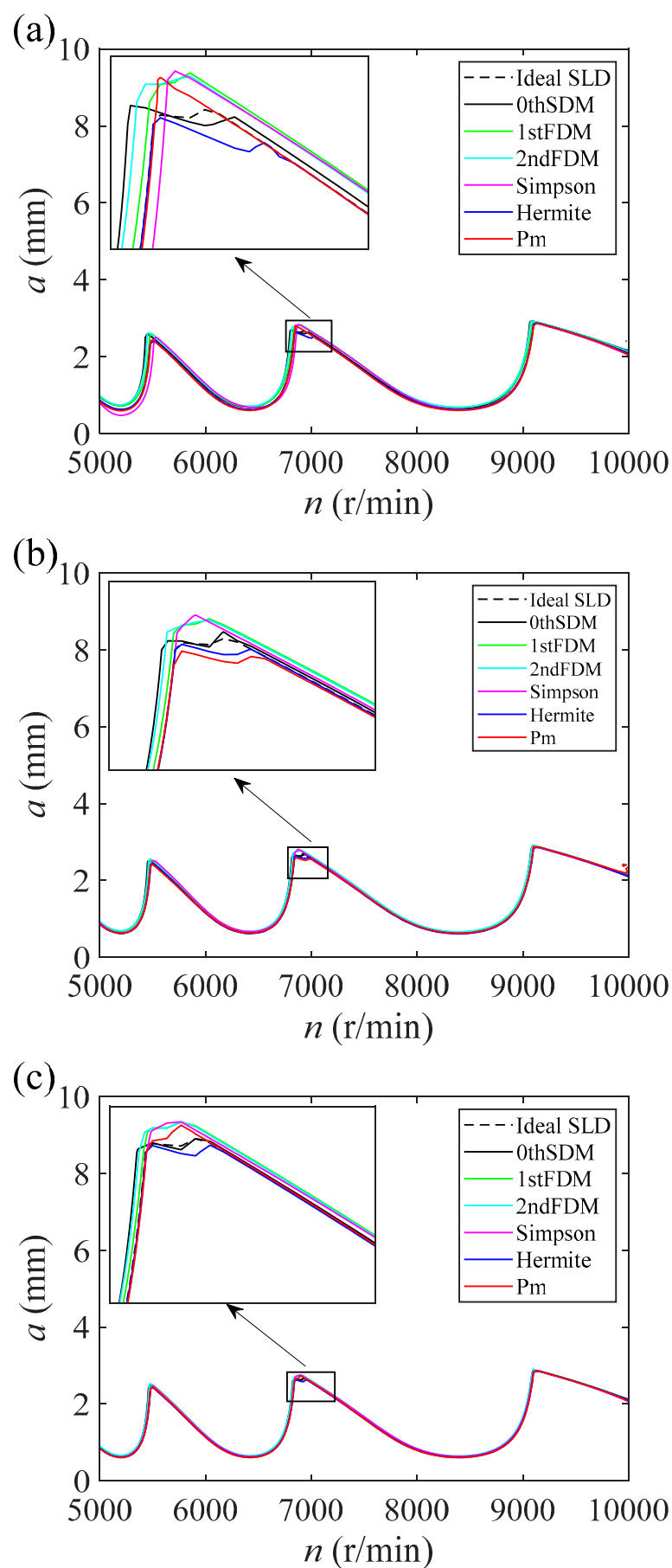


Figure 6. Comparisons of the accuracy of SLDs at aD of 0.5: (a) $m = 30$; (b) $m = 40$; and (c) $m = 50$.

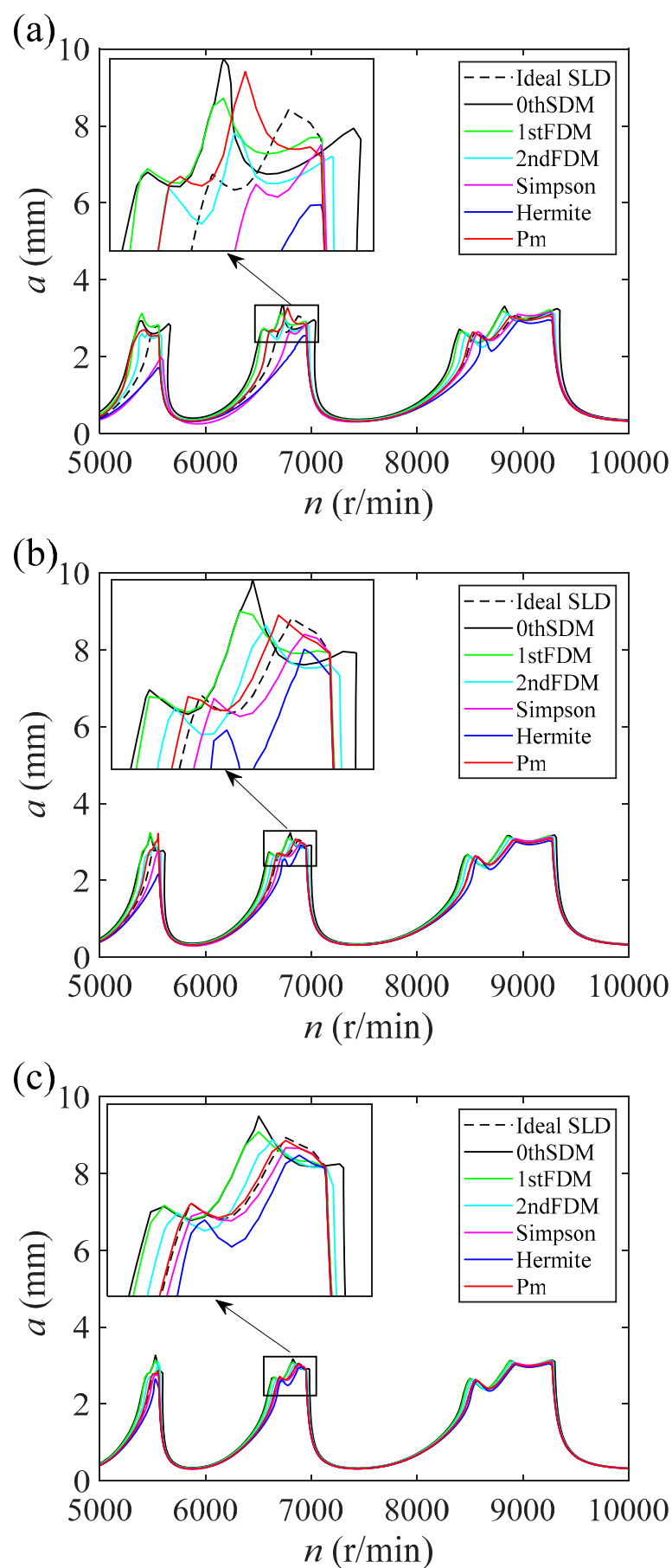


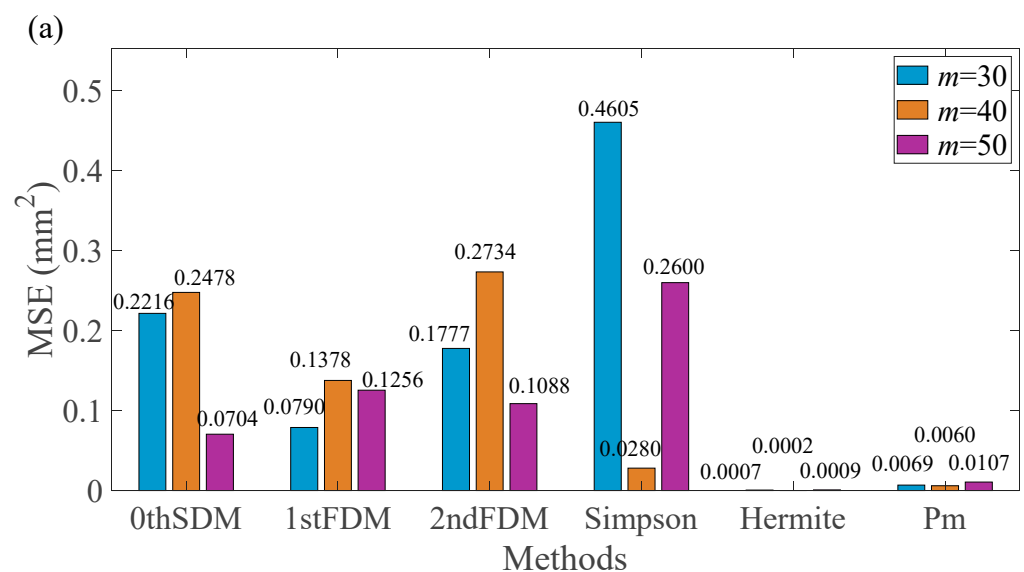
Figure 7. Comparisons of the accuracy of SLDs at aD of 1: (a) $m = 30$; (b) $m = 40$; and (c) $m = 50$.

Traditionally, the accuracy assessment of SLDs has primarily relied on qualitative analysis methods. To address this limitation, this study proposes a quantitative framework for assessing the accuracy of SLDs. Specifically, the critical depth of cut values at N discrete spindle speeds within a specified operational range are selected as reference benchmarks (denoted as y_i). These reference values are obtained using the 2ndFDM with $m = 200$. The corresponding values predicted by the six methods under evaluation (denoted as $\hat{y}_i$) are then compared with these references. The mean squared error (MSE) is employed as the accuracy metric. Taking $N = 5$ as an illustrative example, the MSE is calculated as follows:

$$MSE_{\text{methodA}} = \frac{1}{5} \sum_{i=1}^N (\hat{y}_i^A - y_i)^2 \quad (15)$$

where a smaller MSE value indicates better agreement with the reference data, thus enabling the validation of the precision of the evaluated methods. The MSE metric eliminates subjective judgment, precisely quantifies accuracy differences between methods, detects subtle deviations that are imperceptible to visual inspection, and ensures objectivity and precision throughout the evaluation process.

Figure 8 presents a quantitative accuracy comparison based on MSE with the analysis regions that are consistent with prior qualitative studies. For the first three aD, spindle speeds of 6800, 6900, 7000, 7100, and 7200 r/min are analyzed. For aD = 1, the analyzed spindle speeds are adjusted to 6600, 6700, 6800, 6900, and 7000 r/min. The results demonstrate that the proposed method achieves significantly lower MSE values compared to the 0thSDM, 1stFDM, 2ndFDM, and Simpson's method across all tested conditions. At aD = 1, the MSE values of the proposed method are 0.1676, 0.0029, and 0.0002 mm², which is lower than that of Hermite's method (0.3898, 0.1124 and 0.0170 mm²). The MSE of the method are slightly higher than that of Hermite's method at the other three aD. Overall, the proposed method demonstrates superior accuracy to the 0thSDM, 1stFDM, 2ndFDM, and Simpson's method, but is marginally less accurate than Hermite's method.



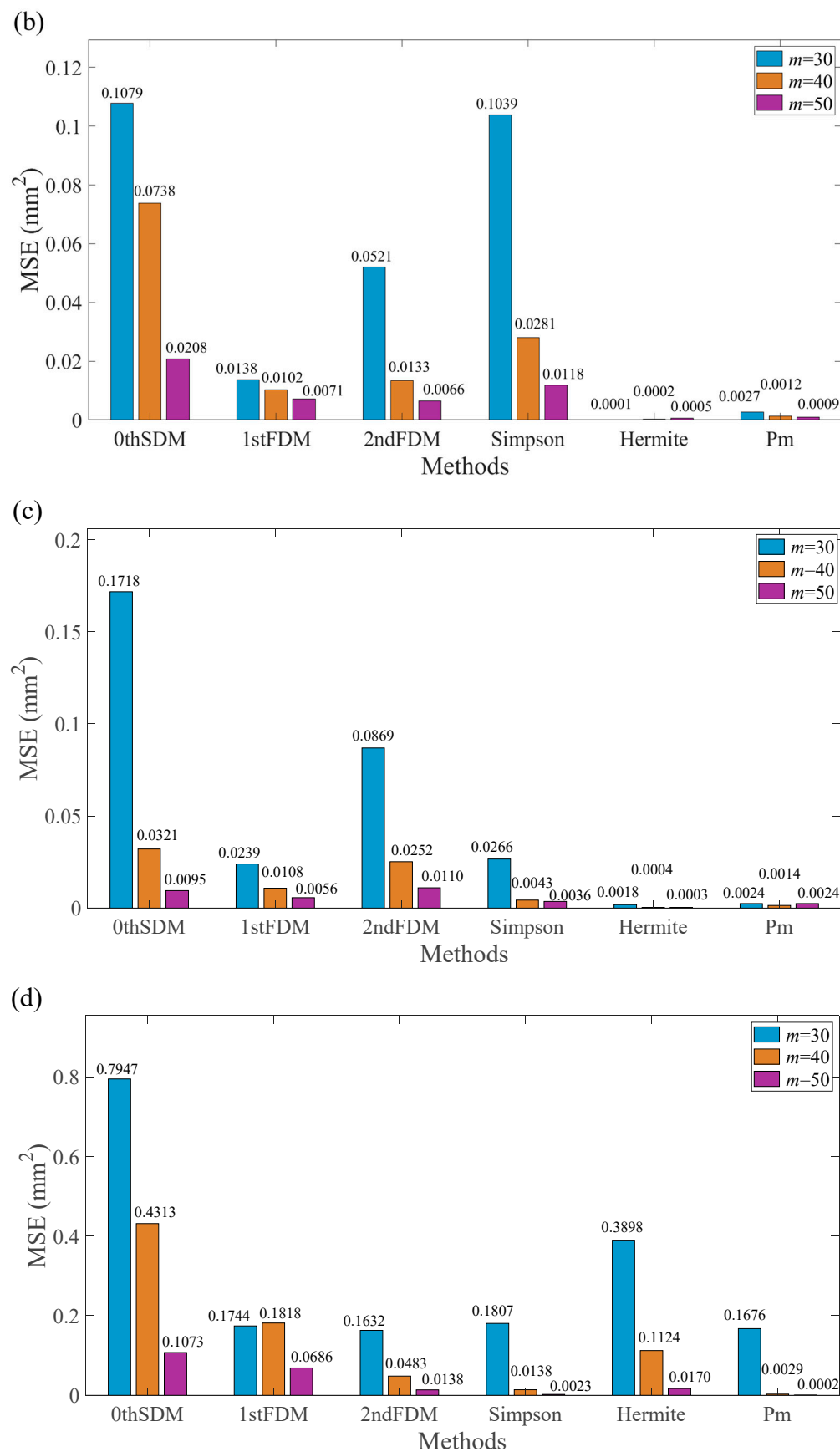
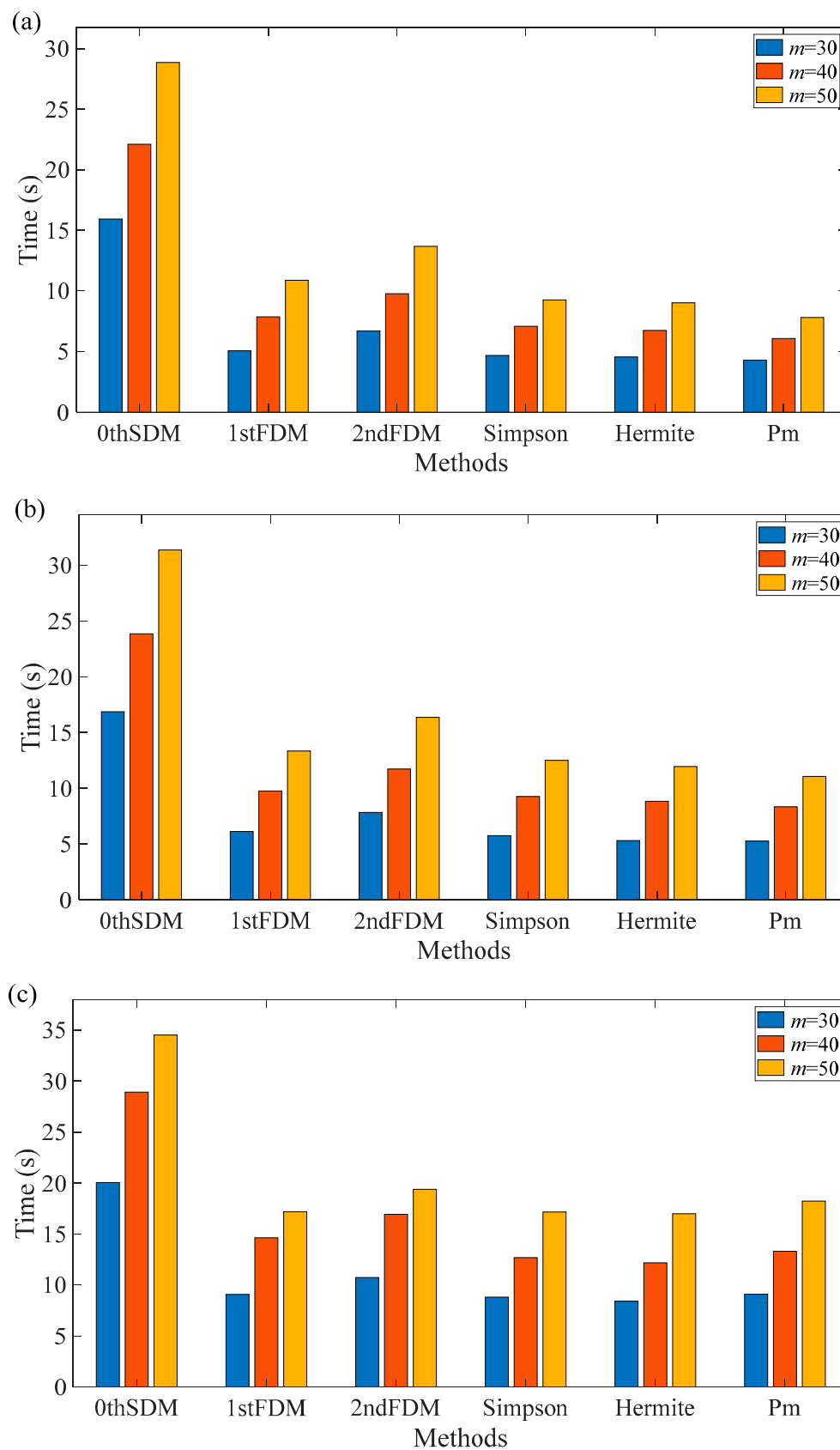


Figure 8. Comparisons of MSE of SLDs: (a) $aD = 0.05$; (b) $aD = 0.1$; (c) $aD = 0.5$; and (d) $aD = 1$.

Figure 9 compares computation times across methods. At $aD = 0.05$, 0.1 , and 1 , the proposed method achieves the lowest computational cost. At $aD = 0.5$, it significantly

outperforms the 0thSDM, 1stFDM, 2ndFDM, and Simpson's method in efficiency, though it remains less efficient than Hermite's method.



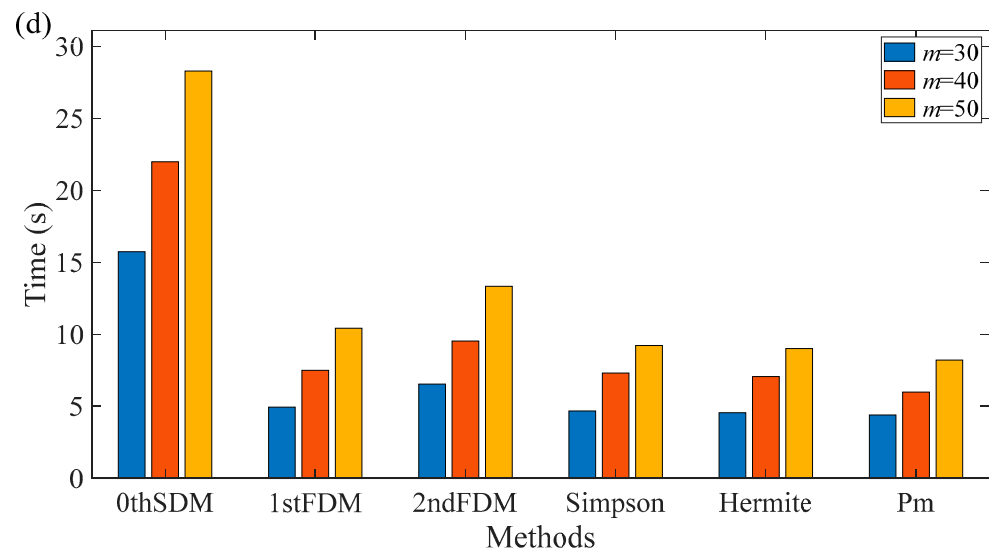


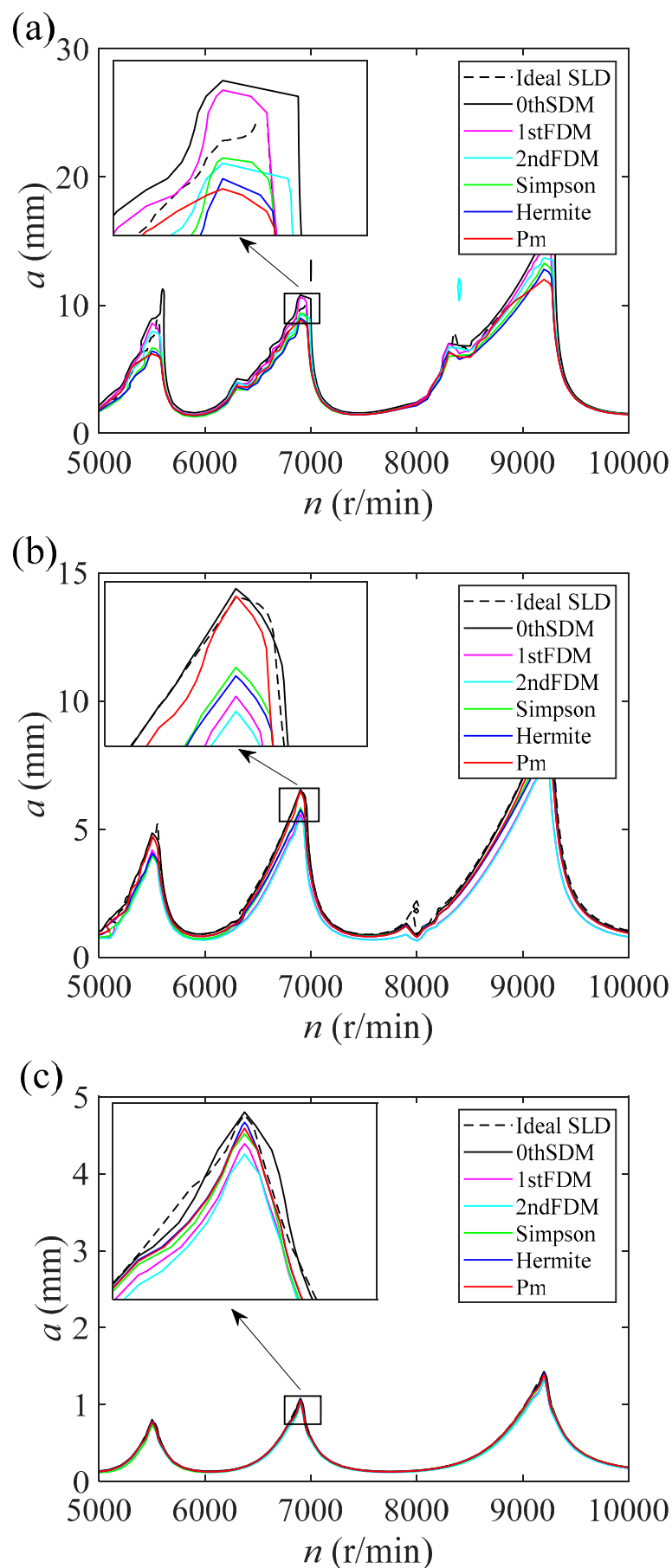
Figure 9. Comparisons of computational efficiency of SLDs: (a) $aD = 0.05$; (b) $aD = 0.1$; (c) $aD = 0.5$; and (d) $aD = 1$.

When $aD = 0.05$, the computation time of the proposed method at $m = 30, 40$, and 50 corresponds to 27%, 85%, 64%, 92%, and 94%; 28%, 77%, 62%, 86%, and 90%; and 27%, 72%, 57%, 84%, and 87% of that required by the other five methods, respectively. For $aD = 0.1$, the corresponding percentages are 31%, 86%, 67%, 92%, and 99%; 35%, 85%, 71%, 90%, and 94%; and 35%, 83%, 68%, 89%, and 93%. At $aD = 0.5$, the percentages become 45%, 100%, 85%, 103%, and 108%; 46%, 91%, 79%, 105%, and 109%; and 53%, 106%, 94%, 106%, and 107%. Finally, for $aD = 1$, the values are 28%, 89%, 67%, 94%, and 97%; 27%, 80%, 63%, 82%, and 85%; and 29%, 79%, 62%, 89%, and 91%.

4.2. Two-DOF Model Example

The principles for analyzing the accuracy of the Two-DOF model remain consistent. The relevant parameters are provided in Appendix A. Here, we analyze the case $m = 40$ as an exemplar. Figure 10 compares the SLDs generated by different methods with the reference curve for $m = 40$. The results show that the method generally achieves higher accuracy than the 0thSDM, 1stFDM, 2ndFDM, and Simpson's method. However, no advantage is observed over the 0thSDM at aDs of 0.1 and 0.5, nor over the 1stFDM at an aD of 0.05. The accuracy of the method is generally comparable to that of Hermite's method.

Quantitative comparisons in Figure 11 further reveal that the proposed method demonstrates superior performance to the 0thSDM, 1stFDM, 2ndFDM, and Simpson's method, with MSE values of 0.2901 mm^2 ($aD = 0.05$); 0.0589 mm^2 ($aD = 0.1$); 0.0002 mm^2 ($aD = 0.5$); and $6.8 \times 10^{-6} \text{ mm}^2$ ($aD = 1.0$), though with two notable exceptions: 1stFDM achieves the lowest MSE of 0.2240 mm^2 at $aD = 0.05$, while 0thSDM outperforms the others at both $aD = 0.1$ (0.0039 mm^2) and $aD = 0.5$ (0 mm^2). Compared to Hermite's method, the method demonstrates superior accuracy at aDs of 0.05 (0.2901 mm^2 vs. 0.5234 mm^2) and 0.1 (0.0589 mm^2 vs. 0.2949 mm^2), but exhibits marginally higher MSE at aDs of 0.5 (0.0002 mm^2 vs. 0.0001 mm^2) and 1.0 ($6.8 \times 10^{-6} \text{ mm}^2$ vs. $4.0 \times 10^{-7} \text{ mm}^2$).



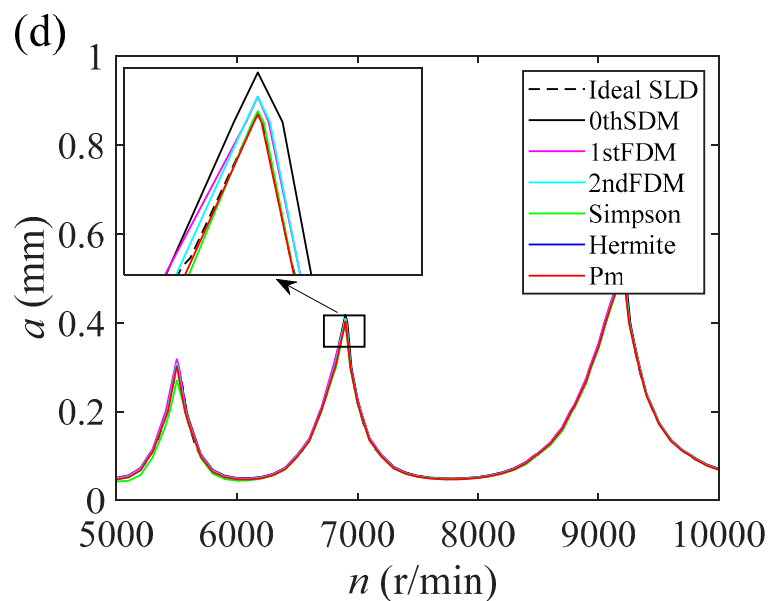
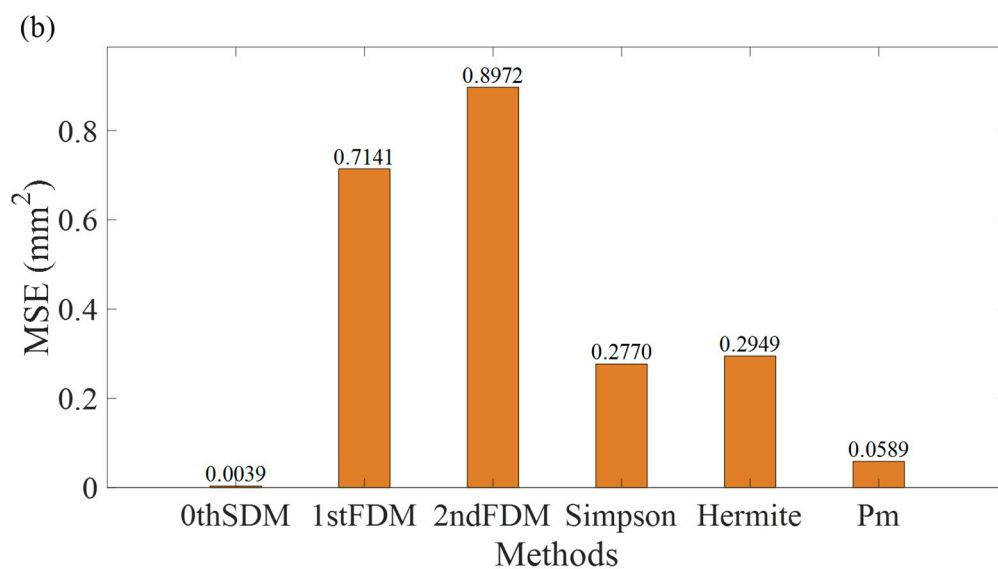
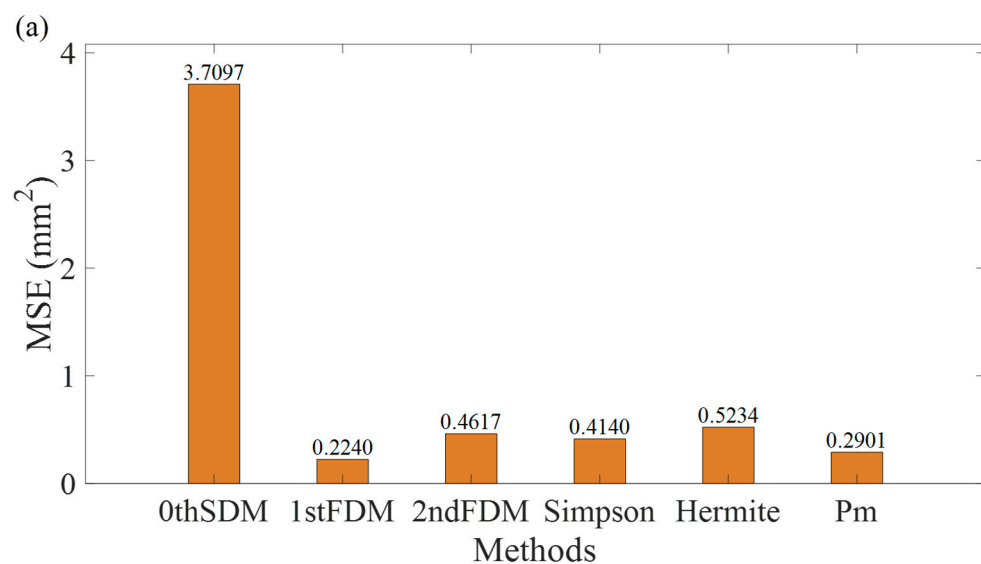


Figure 10. Comparisons of SLDs at $m = 40$: (a) $aD = 0.05$; (b) $aD = 0.1$; (c) $aD = 0.5$; and (d) $aD = 1$.



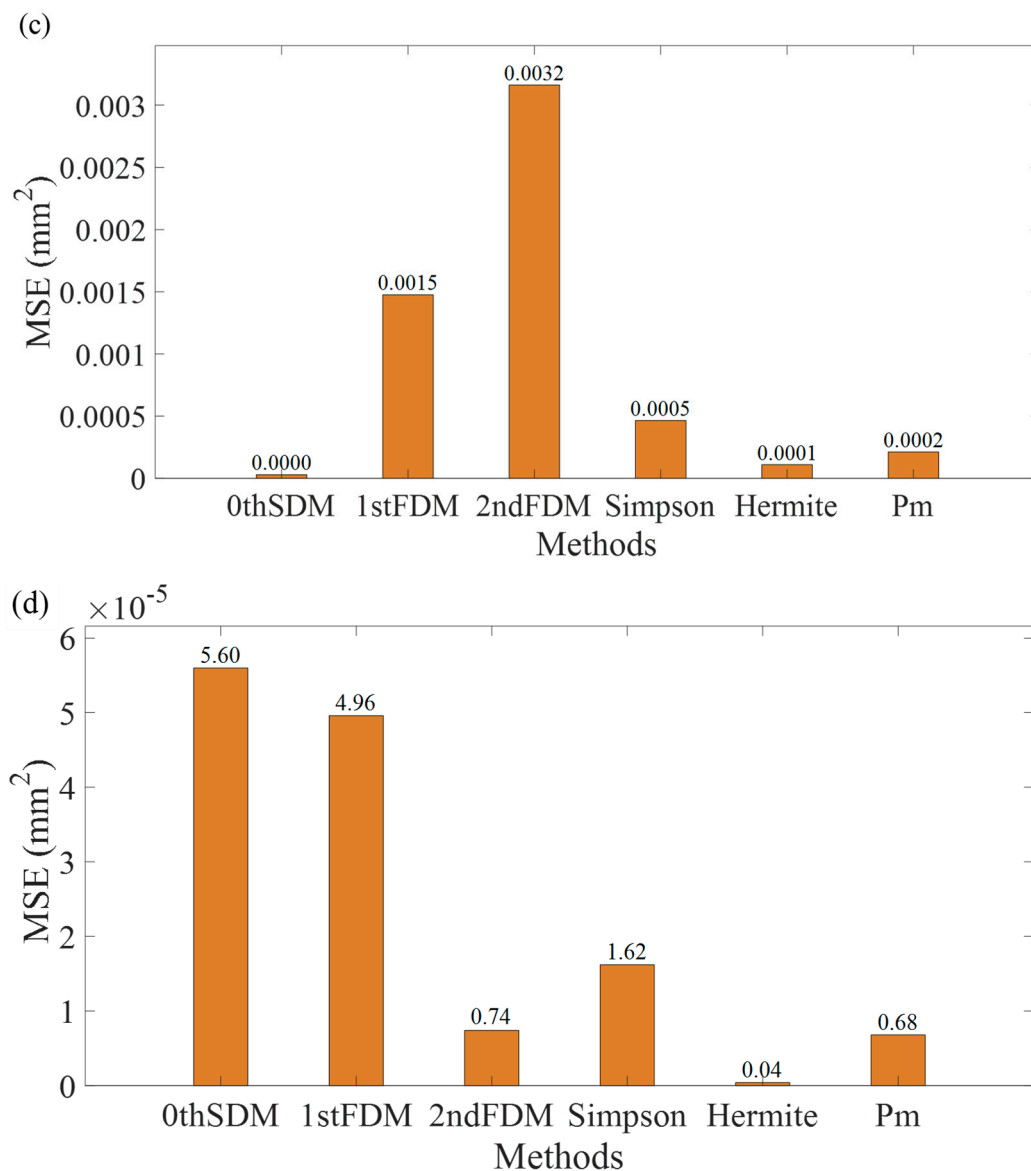


Figure 11. Comparisons of MSE of SLDs at $m = 40$: (a) $aD = 0.05$; (b) $aD = 0.1$; (c) $aD = 0.5$; and (d) $aD = 1$.

Furthermore, Figure 12 demonstrates that this method achieves the highest efficiency among all methods across all four aD in the Two-DOF model, with the only exception being at $aD = 1$, where it yields marginally longer computation times than Simpson's method. When $m = 40$, the computation time of the proposed method at $aD = 0.05, 0.1, 0.5$ and 1 corresponds to 30%, 61%, 57%, 84%, and 76%; 32%, 67%, 59%, 89%, and 79%; 37%, 71%, 63%, 91%, and 82%; and 58%, 93%, 88%, 107%, and 98% of that required by the other five methods, respectively.

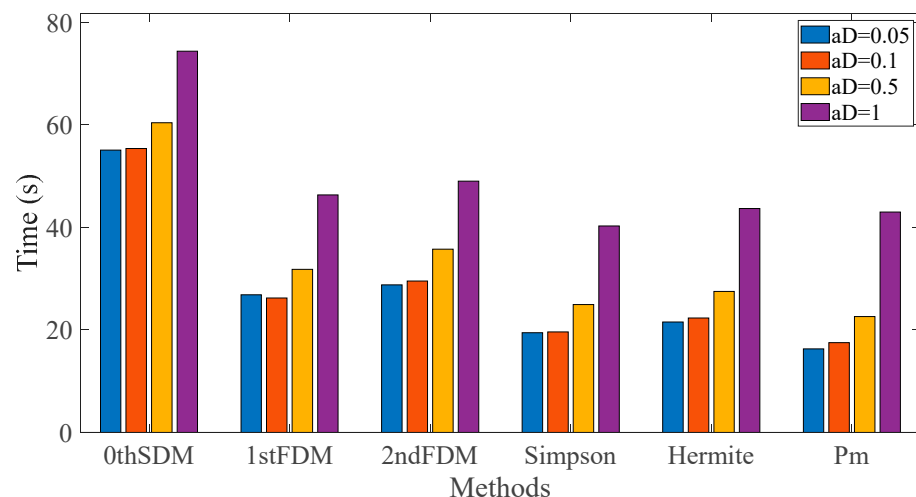


Figure 12. Comparisons of efficiency at $m = 40$.

Comparative assessment under both the One-DOF and Two-DOF models across four immersion ratios demonstrates that this proposed method achieves competitively high accuracy and computational efficiency, positioning it as a good balanced approach among the evaluated techniques.

5. Experimental Validation

This section details experimental investigations encompassing modal analysis, milling force coefficient identification, and machining trials. Given that chatter predominantly manifests along the wall-thickness direction in thin-walled workpieces, the study focuses on the first-order tool modes (X and Y directions) and the first two workpiece modes (Y direction). Experiments employed a four-flute solid carbide end mill with a 10 mm diameter, 100 mm overhang length, and 30° helix angle. Machining was performed on a Ti-6Al-4V (TC4) rectangular workpiece (80 mm × 50 mm × 4 mm). The milling model is depicted in Figure 13. All tests are conducted on a DOOSAN DNM-415 vertical machining center (Doosan Machine Tools (Yantai) Co., Ltd., Yantai, Shandong, China) under dry cutting conditions.

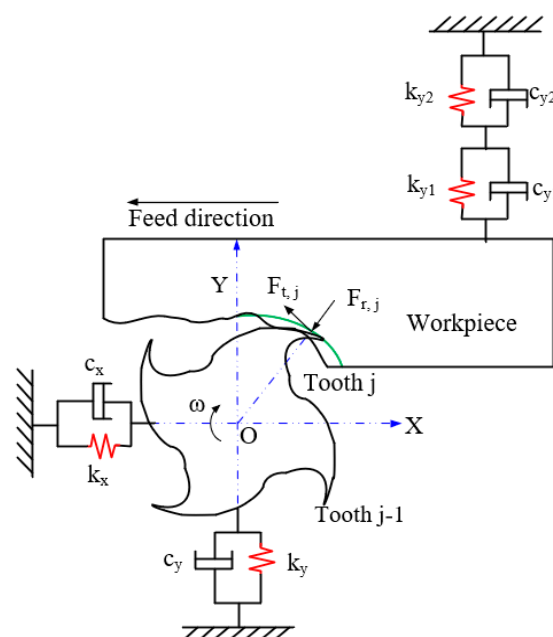
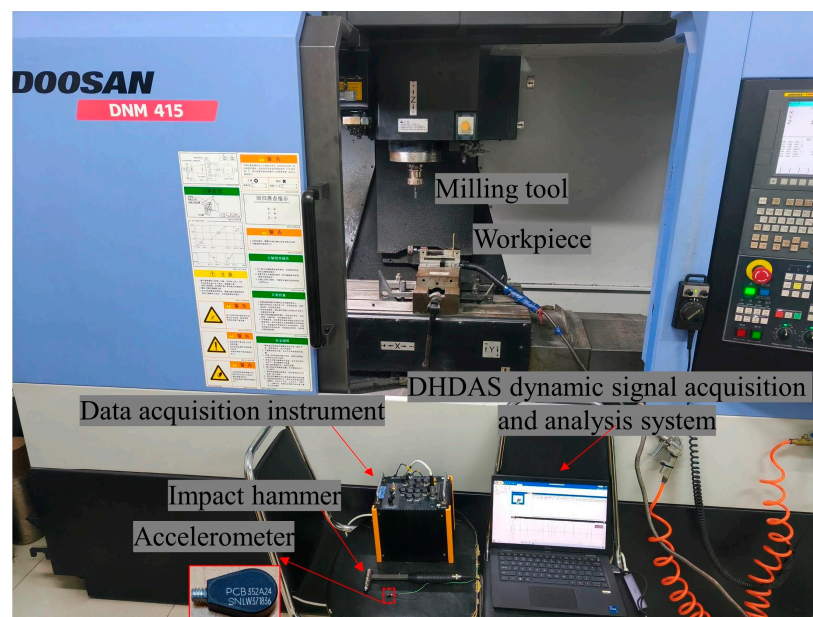


Figure 13. Four-DOF milling dynamic model.

5.1. Experimental Modal Analysis

During modal testing, impact excitation is applied to the workpiece and milling tool using an instrumented hammer. Accelerometers mounted on both components recorded vibration responses, with data acquired through a DHDAS dynamic signal acquisition system (Jiangsu Donghua Testing Technology Co., Ltd., Taizhou, Jiangsu, China). Response data are saved after each measurement point excitation. Following the predefined measurement point numbering sequence, frequency response functions (FRFs) are generated within DHDAS's modal analysis interface. Modal parameters are subsequently extracted through FRF curve fitting. The experimental configuration is documented in Figure 14, with the identified parameters summarized in Table 2.

**Figure 14.** Field modal test setup.**Table 2.** Modal parameters of manufacturing system.

Systems	Natural Frequency (Hz)	Damping Ratio (%)	Stiffness (N/m ²)
Cutter X (Mode no. 1)	1938	4.2%	1.45×10^8
Cutter Y (Mode no. 1)	1933	3.9%	1.41×10^8
Workpiece Y (Mode no. 1)	834	0.8%	4.94×10^6
Workpiece Y (Mode no. 2)	2446	1.4%	4.72×10^7

5.2. Milling Force Coefficient Determination Experiments

The rapid calibration method [31] is adopted to acquire milling force coefficients. For slot milling experiments, time-averaged force components in X and Y directions follow Equation (16):

$$\begin{cases} \overline{F_x} = \frac{Na}{4} K_r f_z + \frac{Na}{\pi} K_{re} \\ \overline{F_y} = \frac{Na}{4} K_t f_z + \frac{Na}{\pi} K_{te} \end{cases} \quad (16)$$

As shown in Equation (16), coefficients are determinable through feed variation experiments. Since the cutting force coefficients depend solely on the workpiece and the tool

itself and are independent of specific machining parameters, relatively conservative cutting parameters are selected. The experimental parameters are as follows in Table 3.

Table 3. Cutting force coefficients identification experiments.

No.	Spindle Speed (r/min)	Axial Depth of Cut (mm)	Radial Depth of Cut (mm)	Feed Rate (mm/min)
1	1000	1	10	80
2	1000	1	10	160
3	1000	1	10	240
4	1000	1	10	320
5	1000	1	10	400

To measure the dynamic cutting forces during the milling process, a DOOSAN DNM 415 machining center was used as the processing platform. A Kistler 9159A dynamometer (Kistler Group, Winterthur, Switzerland) was mounted on the machine table. The vise holding the workpiece was fixed onto the top plate of this dynamometer, ensuring that the cutting forces acting on the workpiece could be directly measured. The output signals from the dynamometer were conditioned by a charge amplifier and subsequently recorded synchronously by a data acquisition system (Figure 15).

During the experimental process, the average milling forces in the X and Y directions under five sets of cutting parameters were measured using the dynamometer. Linear regression was performed with the feed per tooth (f_z) as the independent variable and the average milling force components F_y and F_x as the dependent variables, respectively. The cutting force coefficients K_t and K_r were subsequently obtained by converting the slopes of the resultant linear fits according to the established formula. The calibrated results are $K_t = 12.6 \times 10^8 \text{ N/m}^2$ and $K_r = 4.9 \times 10^8 \text{ N/m}^2$.



Figure 15. Experimental setup for milling force coefficients measurement.

5.3. Experimental Results and Discussion

Using parameters derived from Sections 5.1 and 5.2, the SLD at $m = 40$ and $aD = 0.1$ is generated within the spindle speed range of 2000–5000 r/min, which is commonly used in milling operations, as shown in Figure 16. To validate the method, many machining trials spanning the SLD space are conducted, with four characteristic operating conditions

(A–D) selected for detailed analysis in Table 4: points A, C, and D are within the stable region versus B beyond the stability boundary. The radial depth of cut was uniformly set to 1 mm, corresponding to $aD = 0.1$ as defined when generating the SLD. Moreover, the typical feed per tooth range for titanium alloy milling is 0.01–0.03 mm/z; the selected value of 0.02 mm/z lies at the mid-range of this interval. Consequently, the cutting speeds are 240, 280, 280, and 320 mm/min, respectively.

Table 4. Four sets of cutting experiments.

No.	Spindle Speed (r/min)	Axial Depth of Cut (mm)	Radial Depth of Cut (mm)	Feed Rate (mm/min)
A	3000	0.3	1	240
B	3500	0.4	1	280
C	3500	0.1	1	280
D	4000	0.4	1	320

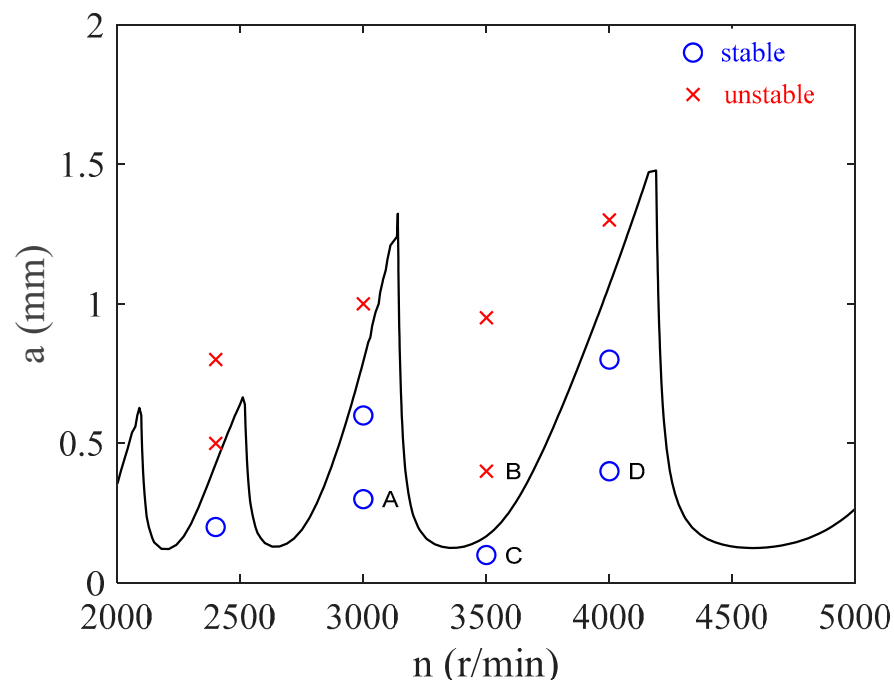


Figure 16. The SLD generated by the proposed method and nine operational conditions selected for experimental validation based on SLD (Including Characteristic conditions A–D).

Comprehensive measurements included milling forces and frequency spectra. The cutting force waveforms in Conditions A, C, and D exhibit relatively smooth profiles, whereas Condition B demonstrates pronounced fluctuations, as evidenced in Figure 17. The results also show that the dominant frequencies under Condition A are stable harmonics at 200 Hz ($4\omega_r$), 800 Hz ($16\omega_r$), and 1200 Hz ($24\omega_r$), where ω_r represents the spindle rotational frequency; those under Condition C at 233 Hz ($4\omega_r$), 699 Hz ($12\omega_r$), and 1166 Hz ($20\omega_r$); and those under Condition D at 266 Hz ($4\omega_r$), 800 Hz ($12\omega_r$), and 1333 Hz ($20\omega_r$). Condition B exhibited definitive chatter signatures: non-harmonic components at 1122 Hz and 1355 Hz flanking harmonics 233 Hz ($4\omega_r$) and 1166 Hz ($20\omega_r$) with frequency difference $\Delta f = 233$ Hz ($4\omega_r$), confirming instability.

The integrated analysis of time-domain milling forces and corresponding frequency spectra reveals close correspondence between experimental observations and SLD predictions. This multi-modal validation confirms the efficacy of the method in predicting machining stability boundaries across diverse operating conditions.

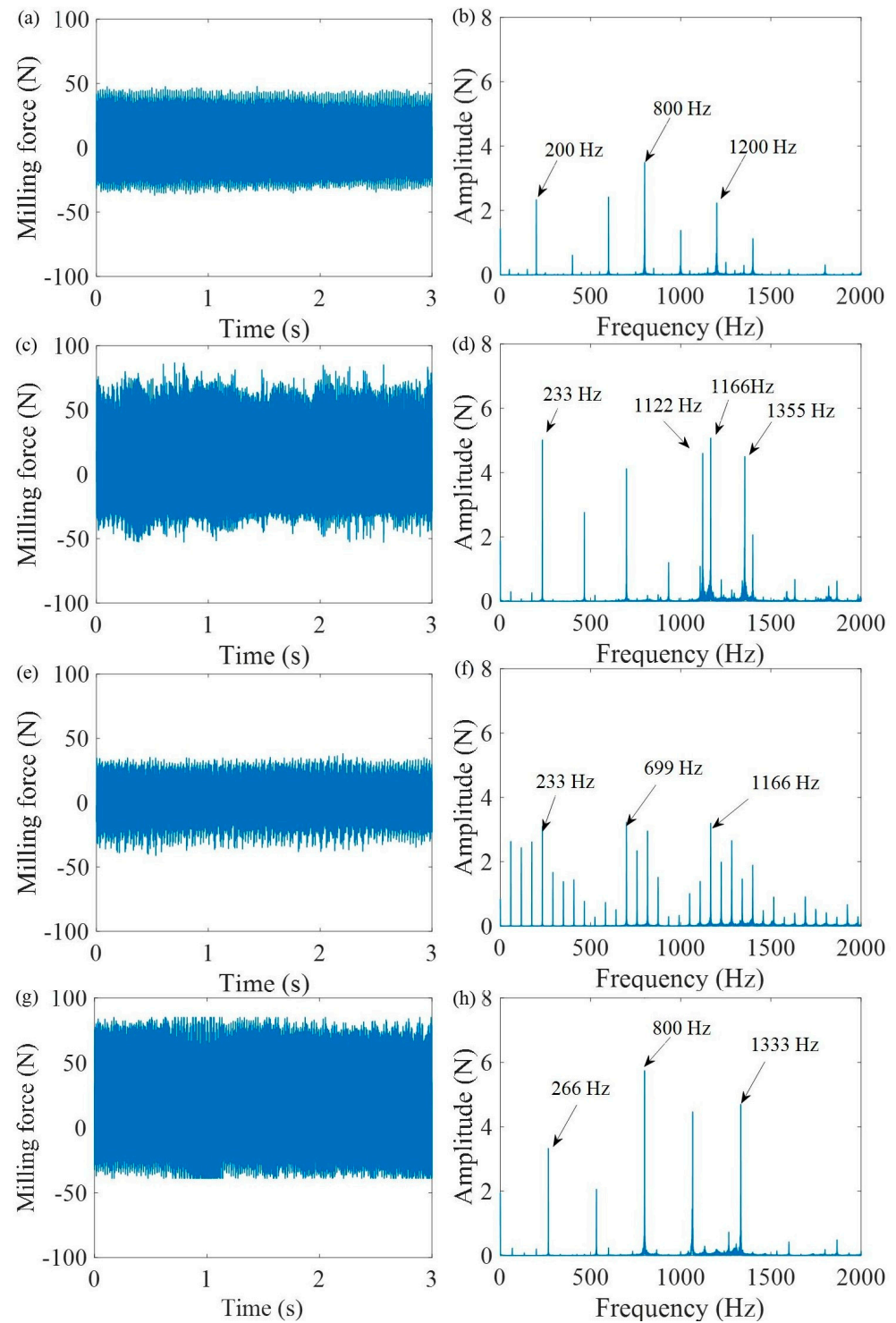


Figure 17. The results obtained from cutting experiments: subfigures (a–h) show the time domain and frequency domain diagrams of the milling forces at points A, B, C, and D.

6. Conclusions

This paper proposes an enhanced numerical integration technique combining Simpson's method and Hermite's method for milling stability prediction. It is compared in detail with the existing methods and experimentally validated. Here, some important conclusions are summarized:

- (1) This work establishes a quantitative accuracy assessment framework for SLDs, utilizing the critical depth of cut values at N spindle speeds as reference benchmarks

- (2ndFDM, $m = 200$). The mean squared error (MSE) quantifies deviations across six methods, with lower values indicating higher predictive fidelity.
- (2) The method demonstrates superior convergence and computational efficiency. Overall, it achieves a significantly lower MSE than 0thSDM, 1stFDM, 2ndFDM, and Simpson's method. Isolated exceptions occur at the Two-DOF model, where 1stFDM ($aD = 0.05$) and 0thSDM ($aD = 0.1, 0.5$) yield the marginally lowest MSE. In One-DOF systems, it exhibits marginally higher MSE than Hermite's method ($aD = 0.05, 0.1$ and 0.5) and achieves significantly lower MSE ($aD = 1$); for Two-DOF systems, it exhibits lower MSE at $aD = 0.05$ and 0.1 , though it has marginally higher values at $aD = 0.5$ and 1.0 . Thus, the proposed method is good.
 - (3) Except the cutting force waveform, which exhibits pronounced fluctuations, Condition B exhibited definitive chatter signatures: non-harmonic components at 1122 Hz and 1355 Hz, flanking fundamental harmonics 233 Hz ($4\omega_r$) and 1166 Hz ($20\omega_r$) with frequency difference $\Delta f = 233$ Hz ($4\omega_r$), confirming instability. The analysis of milling force signals and the corresponding frequency spectra validates the method's effectiveness.
 - (4) The proposed method is outperformed by the existing techniques in certain One-DOF and Two-DOF scenarios—a fact which warrants further research and analysis. Moreover, a limitation of this study is its neglect of the workpiece's time-varying modal parameters. Accordingly, future work will focus on integrating the proposed numerical integration-based stability prediction method with these time-varying dynamics.

Author Contributions: Methodology, X.W., Y.X., G.S., Z.H., P.Z., B.L. and J.D.; Validation, X.W.; Writing—original draft, X.W.; Writing—review & editing, Y.X., G.S., Z.H., P.Z., B.L. and J.D.; Funding acquisition, Y.X. and G.S. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Key Research and Development Program of Shandong Province (2025CXGC010701, 2024JMRH0207), the Open Fund Project of Shandong Key Laboratory of Advanced Motors and Electromechanical Systems (HLZDSY2025006), the Natural Science Foundation of Shandong Province (No. ZR2025MS740), the Shandong Province Science and Technology-based Small and Medium-sized Enterprises Innovation Capability Enhancement Project (2025TSGCCZZB0939) and the Major Innovation Project of Qilu University of Technology (Shandong Academy of Sciences) (2024ZDZX04).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data will be made available upon reasonable request.

Conflicts of Interest: Author Zhaoting Hou was employed by Dezhou Hengli Electrical Machinery Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Appendix A

The derivation process of **A** and **B**(t) is as follows, from Equation (A1) into Equation (A8). Equation (A1) can be derived from **p**(t) in Equation (2).

$$\dot{\mathbf{w}}(t) = \mathbf{M}^{-1}\mathbf{p}(t) - \frac{1}{2}\mathbf{M}^{-1}\mathbf{C}\mathbf{w}(t) \quad (\text{A1})$$

Equation (A2) can be obtained by differentiating **p**(t) with respect to time.

$$\dot{\mathbf{p}}(t) = \mathbf{M} \ddot{\mathbf{w}}(t) + \frac{1}{2} \mathbf{C} \dot{\mathbf{w}}(t) \quad (\text{A2})$$

Equation (A3) is obtained by substituting Equation (1) into Equation (A2).

$$\dot{\mathbf{p}}(t) = -\frac{1}{2} \mathbf{C} \dot{\mathbf{w}}(t) - \mathbf{K} \mathbf{w}(t) + a \mathbf{K}_c(t) [\mathbf{w}(t) - \mathbf{w}(t-T)] \quad (\text{A3})$$

Equation (A4) is derived by substituting Equation (A1) into Equation (A2).

$$\dot{\mathbf{p}}(t) = -\frac{1}{2} \mathbf{C} \mathbf{M}^{-1} \dot{\mathbf{p}}(t) + \frac{1}{4} \mathbf{C} \mathbf{M}^{-1} \mathbf{C} \dot{\mathbf{w}}(t) - \mathbf{K} \mathbf{w}(t) + a \mathbf{K}_c(t) [\mathbf{w}(t) - \mathbf{w}(t-T)] \quad (\text{A4})$$

Equation (A5) is obtained by integrating Equation (A1) and Equation (A4).

$$\left\{ \begin{array}{l} \dot{\mathbf{w}}(t) = \mathbf{M}^{-1} \dot{\mathbf{p}}(t) - \frac{1}{2} \mathbf{M}^{-1} \mathbf{C} \dot{\mathbf{w}}(t) \\ \dot{\mathbf{p}}(t) = -\frac{1}{2} \mathbf{C} \mathbf{M}^{-1} \dot{\mathbf{p}}(t) + \frac{1}{4} \mathbf{C} \mathbf{M}^{-1} \mathbf{C} \dot{\mathbf{w}}(t) - \mathbf{K} \mathbf{w}(t) + a \mathbf{K}_c(t) [\mathbf{w}(t) - \mathbf{w}(t-T)] \end{array} \right\} \quad (\text{A5})$$

Equation (A6) is further expressed as Equation (A5).

$$\begin{bmatrix} \dot{\mathbf{w}}(t) \\ \dot{\mathbf{p}}(t) \end{bmatrix} = \begin{bmatrix} -\mathbf{M}^{-1} \mathbf{C} / 2 & \mathbf{M}^{-1} \\ \mathbf{C} \mathbf{M}^{-1} \mathbf{C} / 4 - \mathbf{K} & -\mathbf{C} \mathbf{M}^{-1} / 2 \end{bmatrix} \begin{bmatrix} \mathbf{w}(t) \\ \mathbf{p}(t) \end{bmatrix} + a \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_c(t) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{w}(t) - \mathbf{w}(t-T) \\ \mathbf{p}(t) - \mathbf{p}(t-T) \end{bmatrix} \quad (\text{A6})$$

Therefore, A and B(t) can be expressed by Equations (A7) and (A8).

$$\mathbf{A} = \begin{bmatrix} -\mathbf{M}^{-1} \mathbf{C} / 2 & \mathbf{M}^{-1} \\ \mathbf{C} \mathbf{M}^{-1} \mathbf{C} / 4 - \mathbf{K} & -\mathbf{C} \mathbf{M}^{-1} / 2 \end{bmatrix} \quad (\text{A7})$$

$$\mathbf{B}(t) = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_c(t) & \mathbf{0} \end{bmatrix} \quad (\text{A8})$$

The coefficient matrixes for the One-DOF milling system can be described as follows:

$$\mathbf{A} = \begin{bmatrix} -\xi \omega_n & \frac{1}{m_t} \\ m_t (\xi \omega_n)^2 - m_t \omega_n^2 & -\xi \omega_n \end{bmatrix} \quad (\text{A9})$$

$$\mathbf{B}(t) = \begin{bmatrix} 0 & 0 \\ -h(t) & 0 \end{bmatrix} \quad (\text{A10})$$

$$h(t) = \sum_{j=1}^N g(\phi_j(t)) \sin(\phi_j(t)) \left[K_t \cos(\phi_j(t)) + K_r \sin(\phi_j(t)) \right] \quad (\text{A11})$$

$$\phi_j(t) = \frac{2\pi n}{60} t + \frac{(j-1)2\pi}{N} \quad (\text{A12})$$

$$g(\phi_j(t)) = \begin{cases} 1 & \text{if } \phi_{st} < \phi_j(t) < \phi_{ex} \\ 0 & \text{otherwise} \end{cases} \quad (\text{A13})$$

The coefficient matrixes for the two-DOF milling system can be described as follows:

$$\begin{aligned}
h_{xx}(t) &= \sum_{j=1}^N g(\phi_j(t)) \sin(\phi_j(t)) \left[K_t \cos(\phi_j(t)) + K_r \sin(\phi_j(t)) \right] \\
h_{xy}(t) &= \sum_{j=1}^N g(\phi_j(t)) \cos(\phi_j(t)) \left[K_t \cos(\phi_j(t)) + K_r \sin(\phi_j(t)) \right] \\
h_{yx}(t) &= \sum_{j=1}^N g(\phi_j(t)) \sin(\phi_j(t)) \left[-K_t \sin(\phi_j(t)) + K_r \cos(\phi_j(t)) \right] \\
h_{yy}(t) &= \sum_{j=1}^N g(\phi_j(t)) \cos(\phi_j(t)) \left[-K_t \sin(\phi_j(t)) + K_r \cos(\phi_j(t)) \right]
\end{aligned} \tag{A14}$$

$$\mathbf{M} = \begin{bmatrix} m_t & \\ & m_t \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 2m_t \zeta & \\ & 2m_t \zeta \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} m_t \omega_n^2 & \\ & m_t \omega_n^2 \end{bmatrix} \tag{A15}$$

References

- Quintana, G.; Ciurana, J. Chatter in machining processes: A review. *Int. J. Adv. Manuf. Technol.* **2011**, *51*, 363–376.
- Mishra, R.; Singh, B. Prediction of milling chatter using SBLMD-ANN. *J. Mech. Sci. Technol.* **2022**, *36*, 877–882.
- Yang, Y.Q.; Wu, D.H.; Liu, Q. Chatter stability prediction of milling considering nonlinearities. *Proc. Inst. Mech. Eng. Part B-J. Eng. Manuf.* **2021**, *235*, 862–876.
- Liu, Y.; Cui, N.; Chen, H.; Yan, X.; Zhao, C.; Bao, G.Q.; Shan, Y.; Yin, Z.Q. Effect of temperature on milling stability of thin-walled parts. *Proc. Inst. Mech. Eng. Part B-J. Eng. Manuf.* **2023**, *237*, 414–423.
- Nair, V.S.; Rameshkumar, K.; Saravanamurugan, S. Chatter identification in milling of titanium alloy using machine learning approaches. *Int. J. Progn. Health Manag.* **2024**, *15*, 1–15.
- Wang, C.X.; Zhang, Y.X.; Hu, J.W. Adaptive milling chatter identification based on sparse dictionary considering noise estimation and critical bandwidth analysis. *J. Manuf. Process.* **2023**, *106*, 328–337.
- Griskevicius, M.; Kharka, V.; Kilic, Z.M. Chatter mitigation in turning slender components using viscous fluids. *J. Manuf. Mater. Process.* **2024**, *8*, 128–139.
- Sheikhi, M.R.; Gürgen, S.; Li, J. Vibration suppression and surface quality enhancement in milling thin-walled structures using shear thickening fluids. *J. Manuf. Process.* **2024**, *131*, 2219–2229.
- Altintas, Y.; Weck, M. Chatter stability of metal cutting and grinding. *CIRP Ann.* **2008**, *53*, 619–642.
- Altıntaş, Y.; Budak, E. Analytical prediction of stability lobes in milling. *CIRP Ann.* **2008**, *44*, 357–362.
- Inspurger, T.; Stépán, G. Semi-discretization method for delayed systems. *Int. J. Numer. Methods Eng.* **2002**, *55*, 503–518.
- Inspurger, T.; Stépán, G. Updated semi-discretization method for periodic delay-differential equations with discrete delay. *Int. J. Numer. Methods Eng.* **2004**, *61*, 117–141.
- Ding, Y.; Zhu, L.M.; Zhang, X.J.; Ding, H. A full-discretization method for prediction of milling stability. *Int. J. Mach. Tools Manuf.* **2010**, *50*, 502–509.
- Ding, Y.; Zhu, L.M.; Zhang, X.J.; Ding, H. Second-order full-discretization method for milling stability prediction. *Int. J. Mach. Tools Manuf.* **2010**, *50*, 926–932.
- Ding, Y.; Zhu, L.M.; Zhang, X.J.; Ding, H. Numerical integration method for prediction of milling stability. *J. Manuf. Sci. Eng.* **2011**, *133*, 031005.
- Zhang, X.J.; Xiong, C.H.; Ding, Y.; Feng, M.J.; Xiong, Y.L. Milling stability analysis with simultaneously considering the structural mode coupling effect and regenerative effect. *Int. J. Mach. Tools Manuf.* **2012**, *53*, 127–140.
- Liang, X.G.; Yao, Z.Q.; Luo, L.; Hu, J. An improved numerical integration method for predicting milling stability with varying time delay. *Int. J. Adv. Manuf. Technol.* **2013**, *68*, 1967–1976.
- Niu, J.B.; Ding, Y.; Zhu, L.M.; Ding, H. Runge-Kutta methods for a semi-analytical prediction of milling stability. *Nonlinear Dyn.* **2014**, *76*, 289–304.
- Zhang, Z.; Li, H.G.; Meng, G.; Liu, C. A novel approach for the prediction of the milling stability based on the Simpson method. *Int. J. Mach. Tools Manuf.* **2015**, *99*, 43–47.
- Ding, Y.; Niu, J.B.; Zhu, L.M.; Ding, H. Numerical integration method for stability analysis of milling with variable spindle speeds. *J. Vib. Acoust.* **2016**, *138*, 011010.

21. Niu, J.B.; Ding, Y.; Zhu, L.M.; Ding, H. Stability analysis of milling processes with periodic spindle speed variation via the variable-step numerical integration method. *J. Manuf. Sci. Eng.* **2016**, *138*, 114501.
22. Ozoegwu, C.G. High order vector numerical integration schemes applied in state space milling stability analysis. *Appl. Math. Comput.* **2016**, *273*, 1025–1040.
23. Mei, Y.G.; Mo, R.; Sun, H.B.; He, B.B.; Wan, N. Stability prediction in milling based on linear multistep method. *Int. J. Adv. Manuf. Technol.* **2019**, *105*, 2677–2688.
24. Dong, X.F.; Qiu, Z.Z. Stability analysis in milling process based on updated numerical integration method. *Mech. Syst. Signal Process.* **2019**, *137*, 106435.
25. Li, W.T.; Wang, L.P.; Yu, G. An accurate and fast milling stability prediction approach based on the Newton-Cotes rules. *Int. J. Mech. Sci.* **2020**, *177*, 105469.
26. Lou, W.D.; Qin, G.H.; Zuo, D.W. Investigation on Cotes-formula-based prediction method and its experimental verification of milling stability. *J. Manuf. Process.* **2021**, *64*, 1077–1088.
27. Xia, Y.; Wan, Y.; Luo, X.C.; Liu, Z.Q.; Song, Q.H. An improved numerical integration method to predict the milling stability based on the Lagrange interpolation scheme. *Int. J. Adv. Manuf. Technol.* **2021**, *116*, 2111–2123.
28. Xia, Y.; Wan, Y.; Su, G.S.; Du, J.; Zhang, P.R.; Xu, C.H. An improved numerical integration method for prediction of milling stability using the Lagrange-Simpson interpolation scheme. *Int. J. Adv. Manuf. Technol.* **2022**, *120*, 8105–8115.
29. Zhan, D.N.; Jiang, S.L.; Li, S.K.; Sun, Y.W. A hybrid multi-step method based on 1/3 and 3/8 Simpson formulas for milling stability prediction. *Int. J. Adv. Manuf. Technol.* **2022**, *120*, 265–277.
30. Farkas, M. *Periodic Motions*; Springer: Berlin/Heidelberg, Germany, 1994.
31. Wang, M.H.; Gao, L.; Zheng, Y.H. An examination of the fundamental mechanics of cutting force coefficients. *Int. J. Mach. Tools Manuf.* **2014**, *78*, 1–7.

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