

Article

Nonlinear Dynamic Analysis of Drill-String System Coupling Rock Surface Morphology Evolution and Dry Friction Effect

Pengfei Deng ¹, Jinchao Zhang ¹, Xiaofan Wang ¹, Yiqiao Li ^{1,*}, Luyuan Gong ² and Shengqiang Shen ²

¹ Tianyou Zhan College, School of Mechanical Engineering, Dalian Jiaotong University, Dalian 116028, China; pengfeideng@djtu.edu.cn (P.D.); zhangjinchao2026@outlook.com (J.Z.); xiaofanwang17@gmail.com (X.W.)

² School of Energy and Power Engineering, Dalian University of Technology, Dalian 116024, China; lygong@dlut.edu.cn (L.G.)

* Correspondence: liyiqiao@djtu.edu.cn

Highlights

1. Both rock surface morphology and dry friction effects are considered.
2. Axial vibration undergoes greater attenuation than torsional vibration along the drill string.
3. Stick-slip vibration results in both axial impact and a reduction in vibration frequency.
4. Dry friction parameter variations induce dynamical state bifurcations in the drill-string system.

Abstract

Stick-slip vibration, reversal, axial impact, and dynamic instability are major challenges in deep drilling operations and are closely associated with nonlinear bit-rock interaction. To investigate these phenomena, this study develops a nonlinear axial-torsional coupled dynamic model of a drill-string system by integrating rock surface morphology evolution with a Stribeck dry friction model. The drill string is discretized into a distributed lumped-parameter model with coupled axial and torsional degrees of freedom. A surface morphology matrix is introduced to simulate the rock-cutting process, while the Stribeck friction model is employed to characterise the nonlinear frictional behaviour at the bit-rock interface. Time-domain simulations, bifurcation analysis, and frequency spectrum analysis are performed to investigate the dynamic responses of the system. The results indicate that rock surface morphology evolution significantly influences the contact conditions and frictional behaviour at the bit-rock interface, and together with dry friction induces transitions among steady-state, multi-periodic, and chaotic motions. Stick-slip vibration is accompanied by axial impact, bit bounce, and a reduction in the dominant torsional vibration frequency. In addition, variations in both driving and frictional parameters can trigger dynamic instability and state transitions. The proposed model provides an effective framework for analysing nonlinear drilling dynamics and offers theoretical guidance for drill-string vibration suppression, drilling parameter optimisation, and efficient drilling in complex formations.

Keywords: drilling dynamics; rock surface morphology; stick-slip vibration; Stribeck dry friction; bit-rock interaction

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1. Introduction

Drilling engineering plays a critical role in geological exploration, oil and natural gas extraction, and coal mining [1]. During downhole operations, the slender and flexible drill string is highly prone to complex nonlinear dynamics, such as severe vibrations, rub-impacts, bit bounce, and stick-slip phenomena. These adverse effects frequently lead to the fatigue failure of the drill string or the drill bit [2–4]. Therefore, investigating the dynamic behaviour of drill-string systems is a crucial prerequisite for elucidating the mechanisms driving these dynamics, as well as for predicting and avoiding destructive vibrations [4–6].

The drill-string system, comprising the derrick, top drive, drill pipe, drill collars, stabilisers, and the drill bit, serves as the key structure for transmitting mechanical power and hydraulic energy between surface equipment and the downhole formation [7]. The fundamental configuration and operational principle of a typical drill-string system are illustrated in Figure 1. During the drilling process, the derrick provides structural support, while the top-drive system drives the drill string at a constant feed rate and maintains a consistent rotational speed. The drill pipes and drill collars provide essential guidance and transmit axial force and torque to the polycrystalline diamond compact (PDC) bit, which in turn cuts the rock formation at the bottom of the wellbore [8]. Drilling fluid is circulated through the hollow drill string and bit into the bottom-hole, carrying rock cuttings to the surface through the annulus between the drill string and the wellbore wall [4]. Throughout this operation, the slender drill string is subjected to a combination of forces, including its self-weight, driving forces, mud damping and buoyancy, as well as bit-rock interaction forces. These interactions lead to complex nonlinear dynamic processes and highly intricate vibrations [9–11]. When such vibrations become severe, they may lead to drilling interruptions or the failure of specific drill-string components, while concurrently inducing drill-pipe fatigue and borehole instability. Consequently, establishing high-fidelity dynamic models to investigate the dynamic behaviour of the drill-string system has become a significant and urgent requirement.

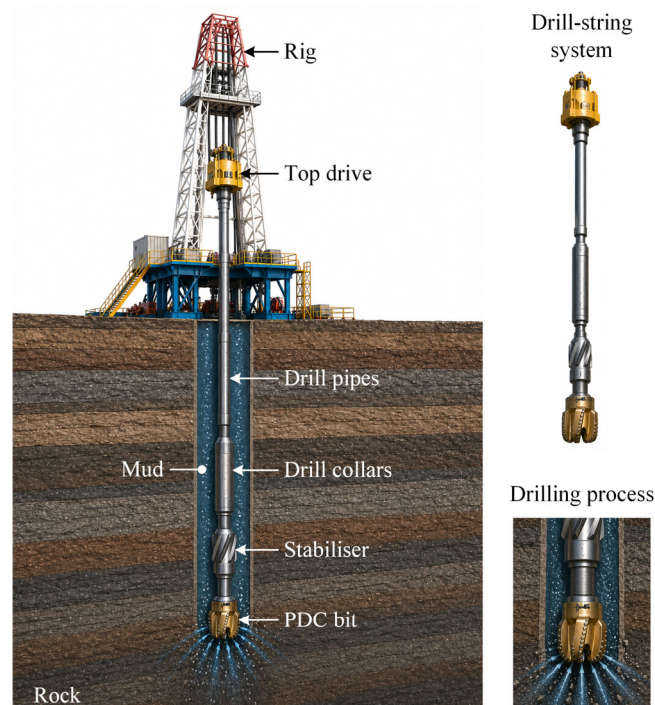


Figure 1. Schematic diagram of the rotary drilling system.

To characterise the complex dynamic behaviour of drill-string systems, existing research on dynamic modelling has primarily focused on two domains: simplified structural modelling of the drill string and PDC bit–rock interaction modelling [12,13]. Notably, the interaction between the bit and the rock is identified as one of the pivotal factors inducing nonlinear vibrations within the system. Rock fragmentation theory dictates that the cutting force exerted on the bit is proportional to the depth of cut (DOC); consequently, considerable academic effort has been devoted to establishing precise DOC models. Detournay et al. [14,15] introduced the Constant Delay (CD) model based on the assumption of a constant average rate of penetration. This model determines the instantaneous DOC by calculating the axial displacement differential between the current bit position and its position in the previous cycle. Considering that the time delay fluctuates with the torsional vibrations of the bit, Insperger et al. [16,17] proposed a state-dependent delay (SDD) model that accounts for axial–torsional coupling. Furthermore, observing that the axial vibration amplitude of the bit often significantly exceeds the feed per cycle, Liu et al. [18] argued that the DOC should incorporate axial displacements over multiple cycles, thereby establishing the Multiple Regenerative (MR) model, which substantially enhanced the computational accuracy of the DOC. Additionally, Liu et al. [19,20] demonstrated that lumped-parameter models for simplified drill-string structures can effectively simulate the propagation of vibration waves within the string, provided that the number of elements is sufficient. Nevertheless, the aforementioned models estimate the DOC indirectly through bit displacements, which fails to capture the authentic geometric evolution of the rock surface during the cutting process. To address this limitation, Deng et al. [11] proposed a more direct approach—the surface morphology (SM) model. This model constructs the geometric profiles of both the bit blades and the rock surface, employing Boolean subtraction to directly determine the depth of cut. The computational fidelity of this model is superior to other approaches, with precision comparable to CD and SDD models and significantly surpassing the MR model.

Apart from accurate DOC calculation, frictional interaction at the bit–rock interface is another fundamental component of the bit–rock interaction model and remains a pivotal factor influencing the dynamic behaviour of the drill-string system [21,22]. Conventional studies predominantly adopt the Coulomb friction model to formulate the governing equations of motion [23–25]. However, the Coulomb model fails to capture the nonlinear velocity dependence of the friction force in low-velocity regimes. Under certain operating conditions characterised by low rotational speeds and high feed rates, the Coulomb model struggles to accurately simulate stick–slip vibration phenomena [26,27]. Experimental and theoretical investigations demonstrate that actual frictional behaviour during drilling typically exhibits pronounced velocity-weakening characteristics, facilitating a smooth transition from static to kinetic friction [28]. To more accurately characterise system dynamics, particularly to capture the abrupt transitions from stick–slip-free to stick–slip regimes and from steady-state motion to multi-periodic or chaotic behaviour, a dry friction model is required to improve the fidelity of bit–rock interaction modelling. Leine et al. [29] proposed a nonlinear friction model dependent on angular velocity, where the friction coefficient transitions between distinct stages according to lubrication conditions. Liu et al. [30] introduced a bit–rock interaction model incorporating Stribeck friction; simulation results indicate that dry friction effects significantly influence the stability of the drill-string system.

The purpose of the study is to create and substantiate a refined model of nonlinear dynamics of a drill string, taking into account the evolution of the rock surface relief and the effect of dry friction, and, based on it, to propose ways to reduce vibrations to improve the reliability and efficiency of drilling in the oil and gas industry. The remainder of this paper is organised as follows: Section 2 elucidates the rock-cutting process and constructs

a lumped-parameter dynamic model of the drill-string system, coupled with the SM and dry friction models. Section 3 analyses the influence of dry friction on the stick–slip vibration characteristics of the system. Section 4 investigates the underlying patterns of how key parameter variations affect the bifurcation and chaotic behaviours of the system. Finally, the primary conclusions derived from this study are presented.

2. Dynamic Modelling of the Drill String Incorporating Dry Friction Effects and Rock Surface Morphology

Building upon the authors' previous research [11,12], the surface morphology (SM) model is utilised to characterise the geometric process of the bit–rock interaction. The cutting forces are derived from the rock fragmentation energy theory, while the Stribeck model is incorporated to represent the dry friction between the drill bit and the rock formation. Furthermore, the drill string is modelled as a lumped-parameter system with coupled axial and torsional degrees of freedom.

2.1. Lumped-Parameter Model of the Drill String with Axial–Torsional Coupling

The simplified configuration of a typical vertical drill-string system is illustrated in Figure 2a. To comprehensively investigate the coupled axial–torsional nonlinear dynamic phenomena, including stick–slip vibration, axial impacts, and bit bounce, the drill string is modelled as a slender beam structure possessing both axial and torsional elasticity. A downward global coordinate x is defined; accordingly, the axial displacement of the drill string at position x is denoted by $u(x, t)$, the angle of twist by $\varphi(x, t)$, the internal axial force by $P(x, t)$, and the internal torque by $T(x, t)$. The drill string is driven by the top-drive system at a constant axial feed rate v_0 and a constant rotational speed ω_0 . The rig is simplified as a spring–damper boundary condition. The reaction force and torque between the top drive and the derrick are denoted by P_0 and T_0 , respectively. At the lower end, the polycrystalline diamond compact (PDC) bit cuts the rock formation, subjecting it to an axial force P_L and a torque T_L . The drilling fluid is circulated downwards through the hollow drill string, is jetted from the bit, and carries the rock cuttings to the surface via the annulus between the drill string and the wellbore wall.

The governing equations of the aforementioned drill-string system constitute a system of two coupled partial differential equations [11], expressed as follows:

$$\begin{aligned}\rho A(x) \frac{\partial^2 u}{\partial t^2} &= \frac{\partial}{\partial x} \left(EA(x) \frac{\partial u}{\partial x} \right) + \frac{\partial^2}{\partial x \partial t} \left(c_{ai} A(x) \frac{\partial u}{\partial x} \right) - c_{ae} A(x) \frac{\partial u}{\partial t} + b \rho g A(x), \\ \rho J(x) \frac{\partial^2 \varphi}{\partial t^2} &= \frac{\partial}{\partial x} \left(GJ(x) \frac{\partial \varphi}{\partial x} \right) + \frac{\partial^2}{\partial x \partial t} \left(c_{ti} J(x) \frac{\partial \varphi}{\partial x} \right) - c_{te} J(x) \frac{\partial \varphi}{\partial t}.\end{aligned}\quad (1)$$

These equations incorporate the force exerted by the derrick and the bit–rock interaction force as the boundary conditions at the upper and lower ends of the drill string, respectively, whilst the effect of the drilling fluid is modelled as linear damping. To determine the dynamic response of the drill-string system, it is necessary to employ the finite element method (FEM) to spatially discretise the drill string into a lumped-mass parameter model, as illustrated in Figure 2b. The drill pipes are discretized into n_p elements of uniform length, and the drill collars into n_c elements; the mass of each respective element is lumped at its two terminal nodes. The two nodes are connected by the stiffness and internal material damping of the drill-string element. The interaction between the drill string element and the drilling fluid is simplified as external damping. Both the top drive and the drill bit are discretised as individual lumped-mass nodes. Following this spatial discretisation, the governing equations of the drill-string system can be reformulated as a system of ordinary differential equations (ODEs), expressed as follows [11]:

$$\begin{aligned}\text{Axial: } \mathbf{M}\ddot{\mathbf{U}}(t) + \mathbf{C}_a\dot{\mathbf{U}}(t) + \mathbf{K}_a\mathbf{U}(t) &= \mathbf{P}(t), \\ \text{Twist: } \mathbf{J}\ddot{\Psi}(t) + \mathbf{C}_t\dot{\Psi}(t) + \mathbf{K}_t\Psi(t) &= \mathbf{T}(t).\end{aligned}\quad (2)$$

Detailed expressions of the matrices in the above equations can be found in the authors' previous work [11].

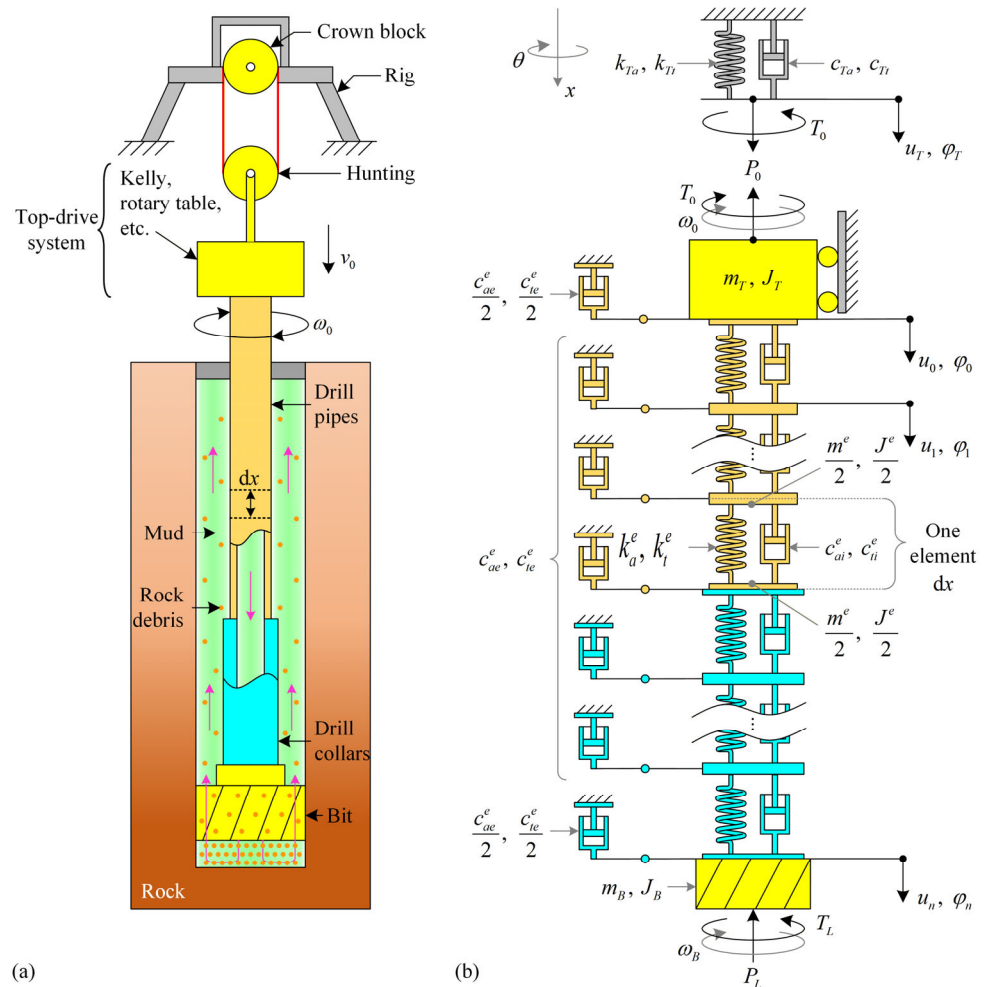


Figure 2. Simplified configuration of the drill-string system and dynamic representation of the drilling process: (a) force analysis of the drill-string system during operation; (b) discretized lumped-mass model of the drill-string system.

2.2. Geometric Process of Rock Cutting Based on Surface Morphology

In Equation (2), the axial force $\mathbf{P}$ and torque $\mathbf{T}$ on the right-hand side represent the interaction forces between the drill bit and the rock. According to rock-cutting theory [11,14,19], the forces exerted on the drill bit during the cutting process are primarily categorised into two components: cutting forces and contact forces. Based on the rock fragmentation energy theory [14,19,31,32], the magnitude of the cutting force acting on a single cutter blade is directly proportional to the instantaneous depth of cut of the drill bit, expressed as $F = \varepsilon d(t)$, where ε denotes the intrinsic specific energy of the rock at the cutting location and $d(t)$ denotes the instantaneous cutting depth. Consequently, investigating the geometric process of rock cutting by the PDC bit constitutes an essential prerequisite for determining the forces acting on the bit. The cutting depth is defined as the distance from the cutter tip to the rock surface, as shown in Figure 3a.

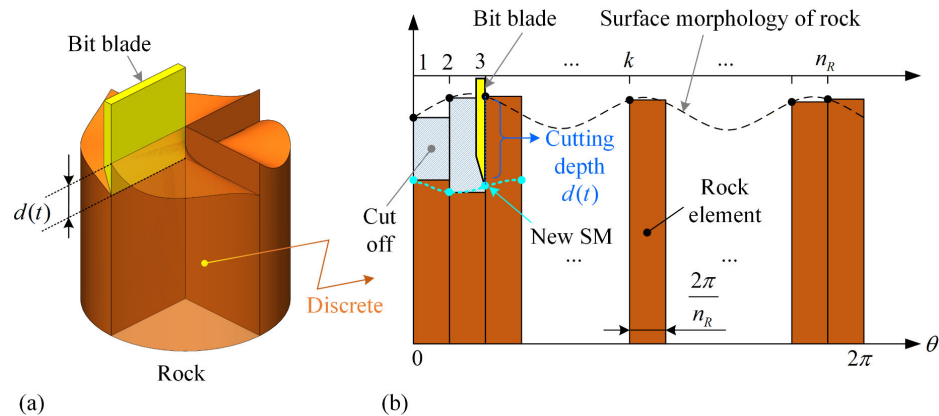


Figure 3. Schematic representation and discretisation of the rock-cutting process: (a) geometric process of rock cutting by a single cutter blade with an instantaneous depth of cut $d(t)$; (b) circumferentially unwrapped two-dimensional discretised profile at a constant radius.

The rock is uniformly divided into n_R segments along the circumferential direction and unfolded into a two-dimensional representation, as shown in Figure 3b. The upper edges of all rock elements form the rock surface morphology, and the height of each rock element is updated in real time as bit blades interact with the rock. The updated surface height of each rock element is referred to as the new rock surface morphology (New SM). As the cutter blade traverses a given rock element, the variation in the rock surface height prior to and following the cutting action is recorded, thereby determining the instantaneous depth of cut $d(t)$, as illustrated in Figure 3b. The instantaneous depth of cut is formulated as follows [11]:

$$d(t) = R(h^+(\varphi(t)) - h^-(\varphi(t) + \frac{2\pi}{n_R})). \quad (3)$$

The instantaneous depth of cut $d(t)$ is determined by calculating the difference between the current rock height h^+ and the post-cut rock height h^- . In this equation, $R(\cdot)$ denotes the ramp function, which ensures the rock height remains unchanged in the absence of rock cutting (for instance, during occurrences of bit bounce or sticking phases).

To represent the rock geometry, the rock domain is spatially discretized, and the rock state matrix $\mathbf{R}$ is defined as follows [11]:

$$\mathbf{R} = \begin{pmatrix} 0 & \frac{2\pi}{n_R} & \cdots & \frac{2\pi(n_R-1)}{n_R} \\ h_0 & h_1 & \cdots & h_{n_R-1} \\ \varepsilon_{R,0} & \varepsilon_{R,1} & \cdots & \varepsilon_{R,n_R-1} \end{pmatrix}, \quad (4)$$

where $h_k = h(\varphi_k)$, $\varepsilon_{R,k} = \varepsilon_R(\varphi_k)$. $\mathbf{R}$ denotes the rock surface morphology matrix. The second row of the matrix constitutes the rock surface morphology.

In Equation (4), the rock formation is partitioned into n_R uniform segments along the circumferential direction, whereby the rock height h and the intrinsic specific energy ε are recorded independently for each discrete angle φ . Each column in the matrix represents the parameters of an individual rock element. When the drill bit cutter engages and cuts a rock element, the height parameter of that element is updated accordingly. It should be noted that in Equation (4), the intrinsic specific energy of each rock element can be individually assigned, enabling the simulation of rocks with non-uniform intrinsic specific energy distribution. Since the focus of this work lies in the influences of driving parameters and friction coefficient on the dynamic characteristics of the system, the values

in the third row of the rock state matrix are set to be identical in the subsequent simulations.

2.3. Drill Bit Dynamics Coupled with Rock Surface Morphology and Dry Friction

A schematic representation of the forces acting on the drill bit blade during the rock-cutting process is illustrated in Figure 4. Specifically, the cutting force F_{lc} is applied to the rake face, thereby exciting coupled axial and torsional vibrations of the bit. Meanwhile, the frictional force F_{lf} exhibits distinct variations in response to changes in the rotational speed.

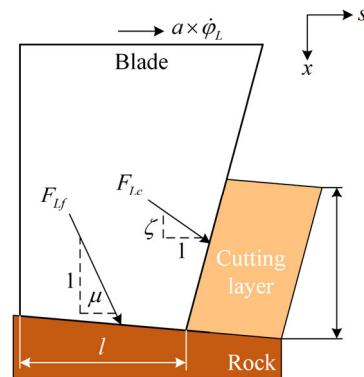


Figure 4. Force analysis of the bit–rock interaction on the drill bit blade.

To better approximate practical conditions and enhance the simulation fidelity of the model, an exponential decay Stribeck friction model is employed in this study. The friction coefficient is formulated as follows:

$$\mu(\dot{\phi}) = \left(\mu_d + (\mu_s - \mu_d) e^{-\frac{|\dot{\phi}|}{\omega_s}} \right) \tanh \frac{\dot{\phi}}{\omega_s} + c_f \dot{\phi}, \quad (5)$$

where μ_s denotes the maximum static friction coefficient; μ_d represents the Coulomb kinetic friction coefficient; ω_s is characteristic Stribeck speed of the Stribeck friction model; and c_f refers to the viscous friction coefficient [31]. Parameters μ_s and μ_d in Equation (5) characterise the nonlinear smooth transition of friction between the static and rotational regimes, and ω_s defines the transition rate from static to kinetic friction, while c_f accounts for the damping effects induced by cuttings and drilling mud during bit rotation. The parameters within Equation (5) encapsulate the frictional responses associated with various rock lithologies. To facilitate subsequent investigations into drill-string dynamics and parametric sensitivity, simulations of the Stribeck friction coefficient as a function of rotational speed and other critical parameters were conducted, the results of which are presented in Figure 5.

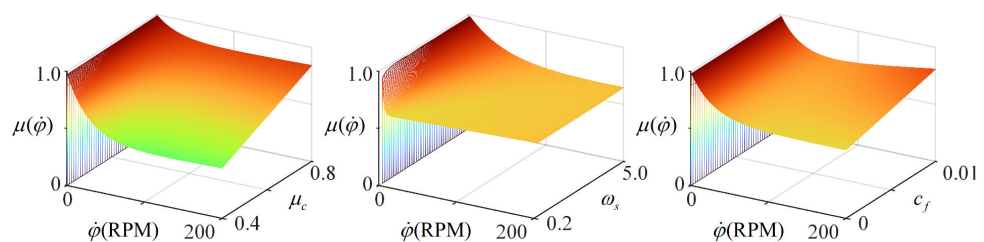


Figure 5. Variations in the Stribeck friction coefficient with respect to bit rotational speed $\dot{\phi}$ and key parameters.

The axial force exerted on the drill bit $P_L(t)$ is determined via the surface morphology (SM) model, as specified in Equation (6). Regarding the torsional torque $T_L(t)$, the frictional components defined by Equation (5) are integrated into the primary formulation. Consequently, the comprehensive expressions for the bit loads are obtained as follows:

$$P_L(t) = \zeta F_{Lc} H(u_L(t)) + F_{Lf} \quad (6)$$

$$T_L(t) = \frac{1}{2} a F_{Lc} H(\dot{\phi}_L(t)) + \frac{1}{2} a \mu(\dot{\phi}_L) F_{Lf \text{sgn}}(\dot{\phi}_L(t)) \quad (7)$$

3. Stick–Slip Vibration Characteristics of the Drill String Subject to Dry Friction

Building upon the dynamic equations formulated in Section 2, this section primarily investigates the dynamic characteristics exhibited by the drill-string system during occurrences of stick–slip vibration. Selected simulation parameters employed for the model in this section are detailed in Appendix A.

3.1. Analysis of Stick–Slip Vibration via Time-Domain Simulation

During drilling operations, an excessive depth of cut can induce bit stalling. As the top-drive system continues to impart rotation, the slender and flexible drill string accumulates elastic strain energy. Upon the fracturing of the rock, the drill bit releases this stored energy at an exceptionally high velocity, thereby engendering stick–slip vibration. Such severe oscillatory behaviour frequently culminates in the fatigue fracture of the drill string, precipitating catastrophic downhole accidents. Consequently, time-domain simulation serves as a widely adopted methodology for analysing these stick–slip phenomena.

Parameter values for this study are drawn from Liu et al. [20,31] and the authors' prior work [11]. To validate the numerical accuracy of the proposed model, comparative time-domain simulations are conducted, as detailed in Appendix B.

To investigate the stick–slip vibration characteristics of the drill-string system, a series of comparative numerical simulations was designed. Driven by an axial feed rate of $v_0 = 20$ m/h and a rotational speed of $\omega_0 = 100$ RPM, the system was simulated to penetrate soft rock (with an intrinsic specific energy of 40 MPa) and hard rock (90 MPa). Regarding the structural discretization, the drill pipe was divided into 20 elements, while the drill collar was discretized into 4 elements. Following the study by Liu [31], the friction parameters were assigned as follows: $\mu_s = 1$, $\mu_d = 0.6$, $\omega_s = 3.0$, and $c_f = 0.002$. The resulting time-domain simulation profiles are illustrated in Figure 6.

As observed in Figure 6, during the drilling of hard rock formations, the drill bit experiences axial and torsional stick–slip vibration. Furthermore, the bit exhibits axial impact vibrations, bit bounce, and backward rotation. Conversely, when drilling through soft rock formations, the axial vibration of the drill bit manifests as high-frequency, low-amplitude periodic oscillations, whilst the torsional vibration presents as low-frequency, low-amplitude periodic oscillations. A comparative analysis of Figure 6(a), (c) reveals that during the phase of accelerated torsional slip, the drill bit initially undergoes an axial bounce, which is subsequently followed by the generation of impact vibrations. As depicted in Figure 6(b), (d), the occurrence of stick–slip vibration within the drill-string system leads to a reduction in the dominant vibration frequencies. Specifically, the axial vibration frequency of the drill-string system decreases from 11.9 Hz to 11.2 Hz, whilst the torsional vibration frequency is reduced from 0.42 Hz to 0.40 Hz. Physically, this phenomenon arises because stick–slip prolongs the duration of the sticking phase, thereby reducing the average rotational velocity and consequently shifting the dominant vibration frequency toward a lower range.

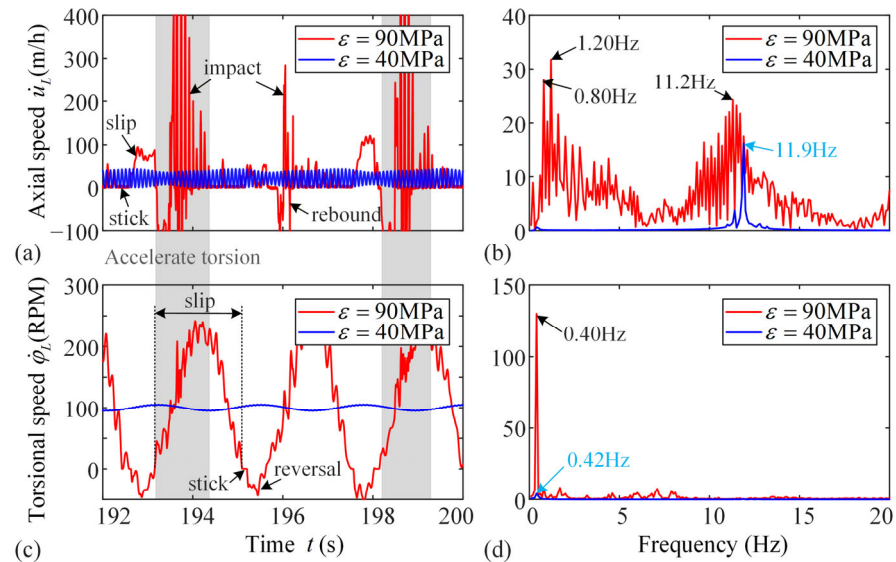


Figure 6. Comparison of the drill bit vibration responses during the drilling of soft and hard rock formations: (a) axial velocity of the drill bit; (b) frequency spectrum of the axial velocity; (c) torsional speed of the drill bit; (d) frequency spectrum of the torsional speed.

Although stick–slip vibration originates from the bit–rock interaction, the consequent structural damage is frequently localised at the drill collar or the lower section of the drill pipe. Furthermore, these vibrations attenuate as they propagate from the drill bit towards the wellhead. Consequently, employing the model proposed herein, the vibration responses at various discrete locations along the drill-string system during the occurrence of stick–slip vibration are investigated. The corresponding time-domain simulation results are presented in Figure 7. Specifically, Node 1 represents the uppermost end of the drill string, corresponding to the top-drive system at the wellhead; Node 25 designates the lowermost end of the drill string, namely the drill bit; and Node 21 denotes the junction interface between the drill pipe and the drill collar.

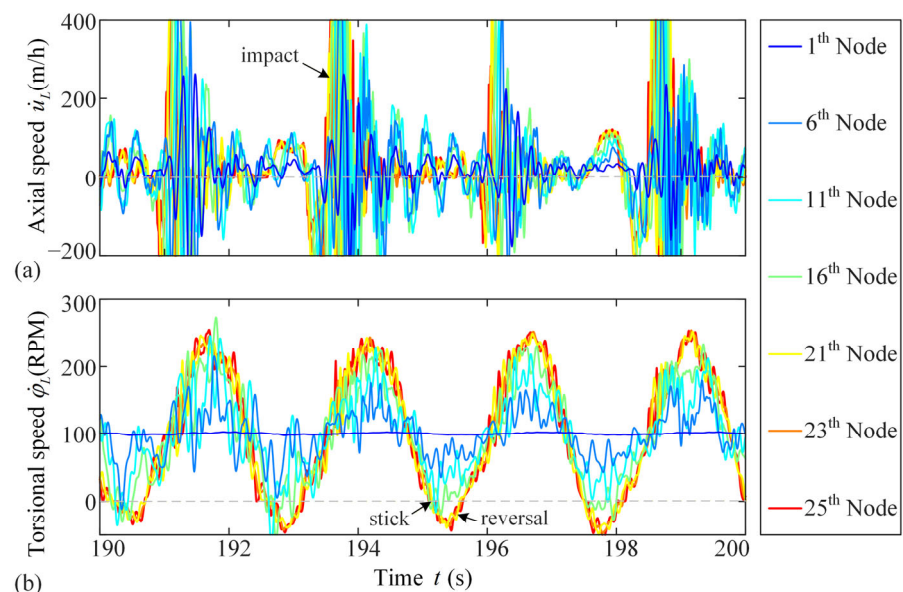


Figure 7. Vibration responses at various discrete nodes of the drill-string system during stick–slip vibration: (a) axial vibration responses; (b) torsional vibration responses.

As is evident from Figure 7, in both the axial and torsional directions, the vibration amplitude progressively attenuates from the drill bit towards the wellhead. Axial impact vibrations are predominantly localised within the drill collar and the lower section of the drill pipe. Furthermore, the sticking phenomenon is primarily concentrated at the lower extremity of the drill collar. These observations are in excellent agreement with the conclusions derived from actual drilling engineering practices [33].

3.2. Influence of Dry Friction on Stick–Slip Vibration

Friction constitutes an integral component of the forces acting on the drill bit and serves as a primary driving mechanism for stick–slip vibration. During drilling operations, varying rock characteristics exhibit distinct frictional behaviours; consequently, a comprehensive investigation into the influence of friction on stick–slip vibration is imperative. In previous studies on drill-string dynamics, the relatively simplified Coulomb friction model has predominantly been employed to describe the frictional forces occurring during the relative sliding between the drill bit surface and the rock formation. To compare the stick–slip vibration characteristics of the drill-string system subjected to Coulomb friction versus the dry friction model proposed herein, a set of comparative time-domain simulations was conducted, with the results illustrated in Figure 8.

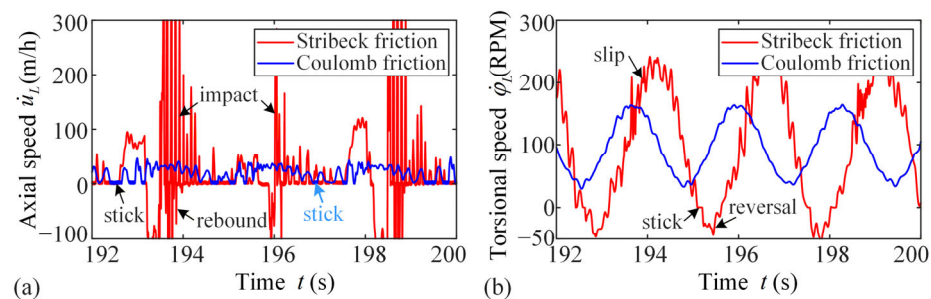


Figure 8. Comparison of the vibration responses of the drill-string system subjected to Coulomb and dry friction: (a) axial velocity; (b) torsional speed.

As is evident from Figure 8, the drill-string system shows markedly different dynamic responses under the Coulomb and Stribeck dry friction models. Axially, both models reproduce stick–slip oscillations, but the Stribeck model predicts significantly larger vibration amplitudes, distinct impact pulses during slip release, and clear bit bounce, whereas the Coulomb model yields a smoother axial response with lower peak velocity and no notable impact or rebound. Torsionally, the discrepancy is more pronounced: the Coulomb model fails to induce torsional stick–slip, with the bit rotating continuously with only small speed fluctuations and no zero-speed sticking or reversal, while the Stribeck model captures complete torsional stick–slip cycles including zero-velocity sticking, pronounced slip with speed overshoot, and distinct reversal. This confirms that torsional stick–slip, axial impacts and bit rebound are inherently coupled phenomena that the simplified Coulomb formulation cannot fully reproduce. Physically, this arises because the Coulomb model's constant friction coefficient cannot capture low-speed velocity-weakening behaviour, while the Stribeck model's negative-slope friction–velocity region introduces a positive feedback mechanism that sustains self-excited vibrations, giving it higher fidelity in characterising bit–rock friction and stick–slip propensity.

Within the dry friction model, the viscous friction coefficient c_f characterises the viscous interactions among the drilling mud, cuttings, and the rock surface. To investigate the impact of the viscous properties of the mud and rock on the stick–slip vibration of the drill-string system, three sets of comparative simulations were conducted, with the values

of c_f assigned as 0.002, 0.004, and 0.010, respectively. The corresponding time-domain simulation results are presented in Figure 9.

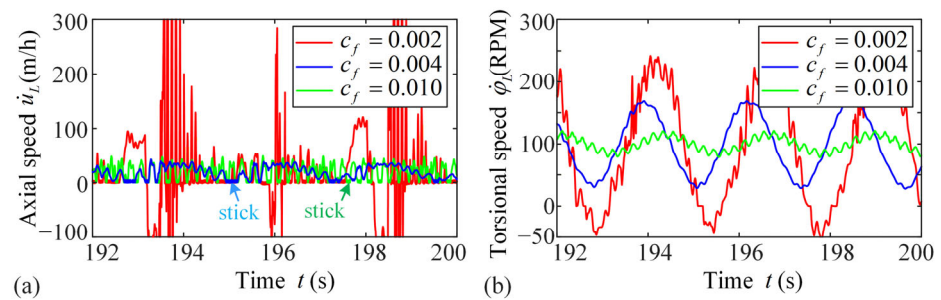


Figure 9. Comparison of the vibration responses of the drill-string system subjected to low- and high-viscosity contacts: (a) axial velocity; (b) torsional speed.

Figure 9 illustrates the vibration responses of the drill bit during the drilling of rock formations characterised by varying viscous damping. As is evident, an increase in viscous damping corresponds to a reduction in both the axial and torsional vibration amplitudes of the drill bit, thereby rendering the system less susceptible to the onset of stick–slip vibration. This phenomenon is attributed to the dissipation of a fraction of the vibration energy by the viscous damping, which consequently enhances the stability of the drill string during the drilling process. The Stribeck velocity ω_s serves as a crucial parameter within the dry friction model, governing the position of the critical transition point between the negative- and positive-slope regions. To investigate the influence of ω_s on the stick–slip vibration of the drill-string system, three sets of comparative simulations were conducted, with the values of ω_s assigned as 1.0, 3.0, and 10.0, respectively. The corresponding time-domain simulation results are depicted in Figure 10.

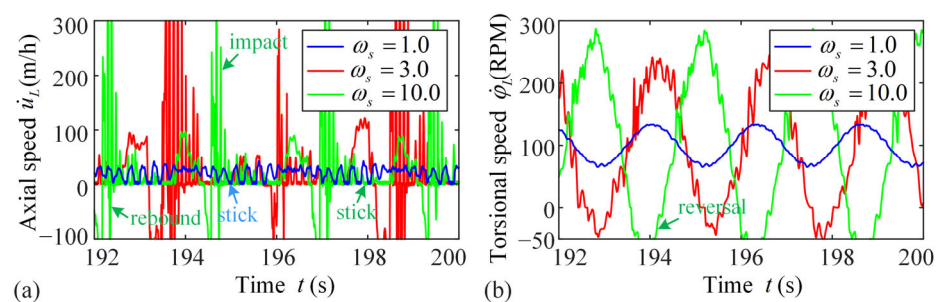


Figure 10. Dependence of the drill-string vibration responses on the Stribeck velocity: (a) axial velocity; (b) torsional speed.

As is evident from Figure 10, when the Stribeck velocity ω_s is set to 1, the drill-string system exhibits stick–slip vibration exclusively in the axial direction. Under this condition, both the axial and torsional vibration amplitudes remain relatively small, and the system is devoid of phenomena such as bit bounce, impact, or backward rotation. Conversely, when the Stribeck velocity is increased to 10, the system experiences axial stick–slip vibration accompanied by bit bounce and impacts. Furthermore, the torsional vibration amplitude becomes substantially larger, featuring the phenomenon of backward rotation, yet is devoid of torsional sticking. This indicates that when the negative-slope region of the dry friction model is relatively broad, it becomes exceedingly difficult for the system to induce torsional sticking phenomena. Physically, a larger Stribeck velocity extends the velocity-

weakening (negative-slope) region of the friction curve, which provides more energy to sustain axial vibration and bit bounce, while reducing the probability of zero-velocity torsional sticking.

4. Influence of Parametric Variations on the Dynamic Characteristics of the Drill-String System

Building on the stick–slip characteristics analysed in Section 3, this section further explores the global dynamic behaviours of the system through bifurcation analysis, identifying critical parameter thresholds for dynamic instability. The drill-string system exhibits typical nonlinear characteristics, where infinitesimal variations in both driving and dry friction parameters can trigger dynamic jumps from steady-state to multi-periodic or even chaotic regimes. This section takes the driving and dry friction parameters as the primary subjects of investigation. By leveraging numerical simulations and bifurcation analysis techniques, the underlying patterns of how parametric variations influence the nonlinear dynamic behaviour and stability of the system are systematically investigated.

4.1. Influence of Driving Parameters on the Dynamic Jump Behaviour of the Drill-String System

The driving parameters of the drill-string system encompass the axial feed rate v_0 and the rotational speed ω_0 of the top-drive system. Variations in these driving parameters profoundly impact the vibration characteristics of the system [12]. To investigate the abrupt transitions in the system's dynamic characteristics induced by these parametric variations, a controlled-variable approach is adopted. Specifically, the axial feed rate of the top-drive system is swept from 0 to 100 m/h, whilst the rotational speed is varied from 20 to 200 RPM. Consequently, bifurcation diagrams illustrating the evolution of the coupled axial–torsional vibrations of the drill bit with respect to the driving parameters—simulated for both soft rock (40 MPa) and hard rock (90 MPa) formations—are obtained, as depicted in Figure 11.

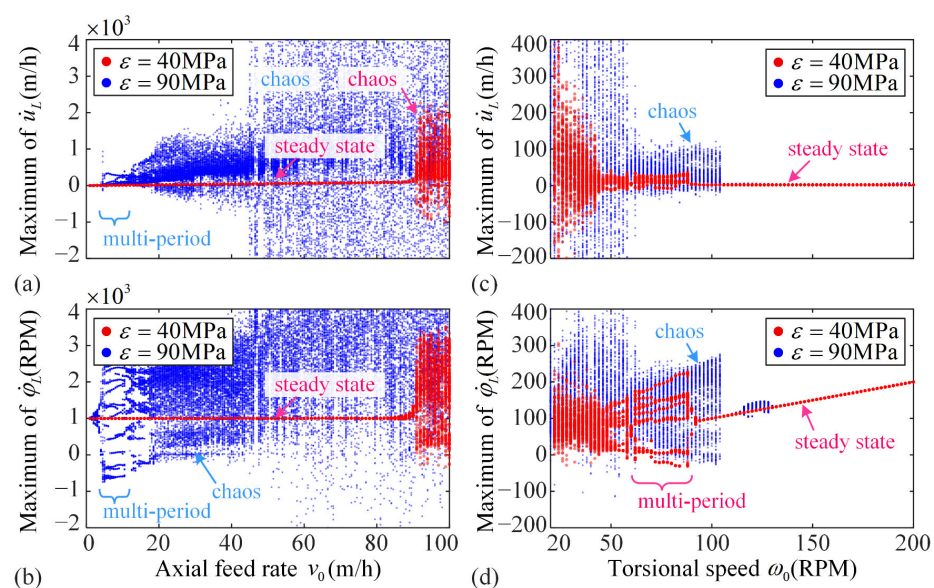


Figure 11. Bifurcation diagrams of the axial and torsional velocity amplitudes versus top-drive speeds during the drilling of two rock formations: (a) axial velocity amplitude versus axial feed rate; (b) torsional velocity amplitude versus axial feed rate; (c) axial velocity amplitude versus rotational speed; (d) torsional velocity amplitude versus rotational speed.

As observed in Figure 11, the drill-string system exhibits a broader range of driving speeds for stable drilling when penetrating soft rock formations. As the axial feed rate increases, the axial and torsional vibrations of the system in hard rock initially undergo a jump from a steady state to multi-period bifurcations, subsequently transitioning into chaos; conversely, for soft rock, the vibrations jump directly from a steady state into chaos. Regarding the influence of rotational speed, an increase in ω_0 causes the vibrations in hard rock to transition from chaos to multi-period bifurcations before eventually converging to a steady state. In contrast, for soft rock, the bit vibrations converge directly from chaos to a steady state. Notably, within the rotational speed interval of 116–128 RPM, the torsional vibration of the bit manifests as multi-periodic oscillations. It should be emphasised that since the drill-string system experiences higher vibration amplitudes and more pronounced dynamic jump behaviours when drilling through hard rock (characterised by a higher intrinsic specific energy), all subsequent simulations are conducted based on a rock intrinsic specific energy of 90 MPa.

4.2. Influence of the Dry Friction Coefficient on the Dynamic Jump Behaviour of the Drill-String System

The influence of the viscous friction coefficient on the dynamic characteristics of the drill-string system is non-negligible. Accordingly, the value of the viscous friction coefficient is varied within the range of 0–0.01. Through numerical simulations, the bifurcation diagrams illustrating the coupled axial–torsional vibrations of the drill bit with respect to the viscous friction coefficient are obtained, as presented in Figure 12.

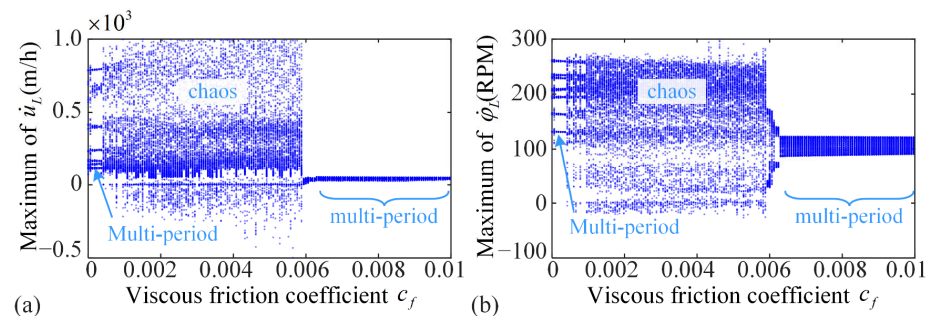


Figure 12. Bifurcation diagrams of the axial and torsional velocity amplitudes versus the rock viscous friction coefficient: (a) axial vibration amplitude; (b) torsional vibration amplitude.

As observed in Figure 12, when the viscous friction coefficient approaches zero, both the axial and torsional vibrations of the drill bit exhibit multi-period states. With an increase in the viscous friction coefficient, the axial vibration amplitude augments, and both vibration modes transition into a chaotic state. Once the viscous friction coefficient surpasses 0.0063, the vibration amplitudes in both directions decrease, and the system undergoes a jump transition from chaos back to a multi-period vibration state.

In addition to the viscous friction coefficient, the Stribeck velocity constitutes another fundamental parameter within the Stribeck friction model. It serves as the critical velocity for the transition from static to kinetic friction, thereby characterising the frictional properties of the bit–rock interface. To investigate its influence on the dynamic characteristics of the drill-string system, the Stribeck velocity is varied across a range of 0.2–5. The resultant variations in the vibration amplitudes, obtained through time-domain simulations, are illustrated in Figure 13.

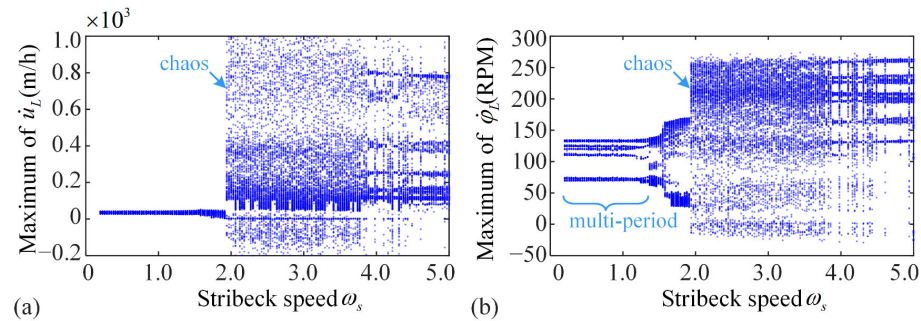


Figure 13. Bifurcation diagrams of the axial and torsional velocity amplitudes versus the Stribeck velocity: (a) axial vibration amplitude; (b) torsional vibration amplitude.

As observed in Figure 13, at relatively low Stribeck velocities, the drill bit exhibits multi-periodic vibration states. Once the Stribeck velocity exceeds a threshold of 1.55, the system transitions into a chaotic motion regime.

Furthermore, the Coulomb friction coefficient is varied across a range of 0.4–0.8. Through time-domain numerical simulations, the bifurcation diagrams illustrating the evolution of the axial and torsional vibrations of the drill bit with respect to the Coulomb friction coefficient are obtained, as presented in Figure 14.

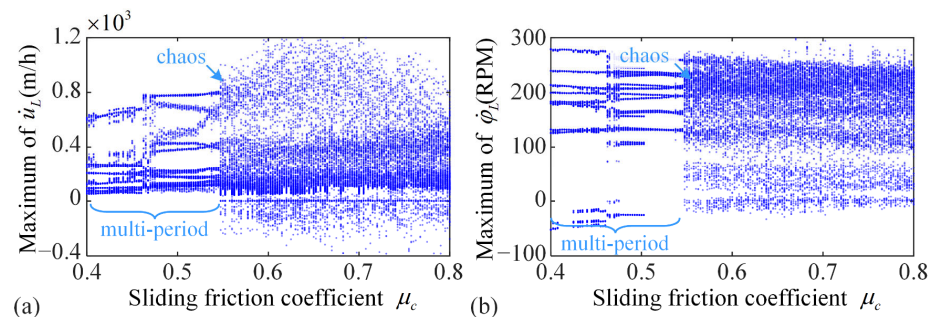


Figure 14. Bifurcation diagrams of the axial and torsional velocity amplitudes versus the Coulomb friction coefficient: (a) axial vibration amplitude; (b) torsional vibration amplitude.

As is evident from Figure 14, an increase in the Coulomb friction coefficient leads to a corresponding augmentation in the axial vibration amplitude of the drill bit. Furthermore, both the axial and torsional vibrations undergo a jump transition from a multi-periodic state into a regime of chaotic motion, with the bifurcation point identified at 0.545. Physically, a higher Coulomb friction coefficient intensifies the velocity-weakening effect at low rotational speeds, which provides more energy for self-excited vibration and promotes the system to enter a chaotic state.

5. Conclusions

Building upon the authors' previous research, a nonlinear axial–torsional coupled dynamic model of a drill-string system was developed by integrating rock surface morphology evolution with a Stribeck dry friction model. Based on this model, the stick–slip vibration characteristics and the nonlinear dynamic transitions of the drill-string system were systematically investigated through time-domain simulations and bifurcation analysis. The primary conclusions are summarised as follows:

1. During drilling in hard rock formations, the drill bit is more susceptible to severe stick–slip vibration, accompanied by axial impact, bit bounce, and backward rotation.

Stick–slip vibration also reduces the dominant vibration frequency, while the vibration amplitude gradually attenuates from the drill bit toward the wellhead.

2. Compared with the conventional Coulomb friction model, the Stribeck dry friction model more accurately reproduces the coupled axial–torsional stick–slip behaviour of the drill-string system. The viscous friction component suppresses vibration by dissipating energy, whereas the Stribeck velocity significantly influences the transition between stick–ing and slipping states.

3. Both driving parameters and frictional parameters have significant influences on the nonlinear dynamics of the drill-string system. Higher axial feed rates, lower rotational speeds, and stronger frictional effects increase the likelihood of dynamic transitions from steady-state or multi-periodic motion to chaotic vibration.

4. The proposed coupled model provides an effective tool for investigating the nonlinear dynamic behaviour of drill-string systems under complex bit–rock interaction. The results offer theoretical guidance for vibration suppression, drilling parameter optimisation, and stable drilling in complex formations.

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Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

The parameters and variables used in this work are listed here. These quantities have been mainly adapted from the earlier study [11].

Table A1. The parameters and variables of this work

Symbol	Quantities	Value	Units
v_0	Nominal axial speed of top drive	20	m/h
ω	Nominal torsional speed of top drive	100	RPM
m_T	Mass of top-drive system	13,200	kg
J_T	Rotary inertia of top-drive system	1100	kg·m ²
k_{Ta}	Axial stiffness of top-drive system	9800	kN/m
k_{Tt}	Torsional stiffness of top-drive system	67.9	kN·m
c_{Ta}	Axial damping of top-drive system	144	kN·s/m
c_{Tt}	Torsional damping of top-drive system	20.4	kN·s·m
m_B	Mass of the drill bit	560	kg
J_B	Rotary inertia of the drill bit	3.1	kg·m ²

r_B	Radius of the drill bit	108	mm
P	Axial force on the drill string	-	N
T	Torque on the drill string	-	N·m
n_p	Number of discretizations of drill pipes	20	-
n_c	Number of discretizations of drill collars	4	-
ε	Intrinsic specific energy of rock	40, 90	MPa
t	Time	-	s
$d(t)$	Instantaneous cutting depth of drill bit blade	-	m
F_{Lc}	Cutting force of drill bit blade	-	N
F_{Lf}	Friction force of drill bit blade	-	N
μ_s	Maximum static friction coefficient	1	-
μ_d	Coulomb kinetic friction coefficient	0.6	-
c_f	Viscous friction coefficient	0.002	-
ω_s	Stribeck speed	3.0	-
ω_δ	Smoothness coefficient of Stribeck model	$\omega_s \times 0.01$	-

Appendix B

To ensure the numerical stability and convergence of the proposed drill-string system dynamical model under the given simulation parameters and verify its accuracy, three sets of drill-string discretization parameters are selected for time-domain simulations, and the results are compared with those of the MR model incorporating Stribeck dry friction, as shown in Figure A1. The computation time of the proposed model is 469 s, while that of the MR model is 15,850 s.

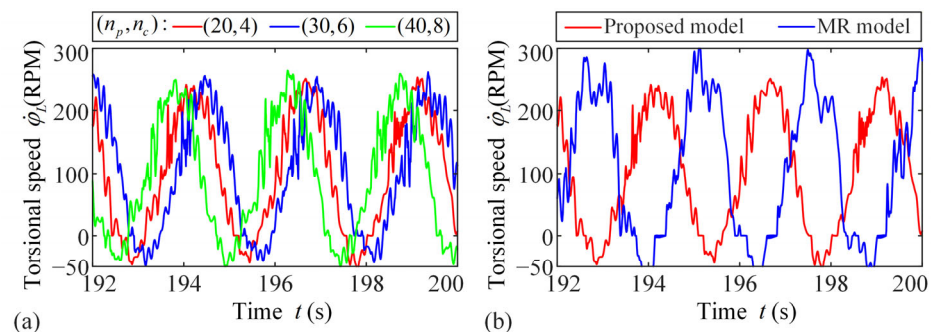


Figure A1. Comparison of time-domain response of drill bit torsional vibration: (a) the results of the proposed model under different discrete parameters of the drill string; (b) results of the proposed model and MR model considering the Stribeck dry friction effect.

As shown in Figure A1, the bit torsional vibration amplitudes of the proposed model under the three sets of discretization parameters are relatively close, with similar waveforms. This indicates that increasing the discretization parameter beyond a certain level has negligible effect on model accuracy, a conclusion that was also summarised in Liu's [20] work. For the same discretization parameter, the bit torsional vibration amplitude of the MR model is significantly larger than that of the SM model. This is attributed to the error in the MR model caused by double-counting the rock-cutting volume, the detailed mechanism of which was fully explained in our previous study [11].

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