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Fractional-Order Backstepping Sliding Mode Control for a Quadrotor UAV

Vicente Borja-Jaimes ^{1,2,*}, Jarniel García-Morales ¹, Jorge Enrique Lavín-Delgado ³, Miguel Beltrán-Escobar ⁴, Jorge Salvador Valdez-Martínez ⁴, Guillermo Ramírez Zúñiga ⁴, Heriberto Adamas-Pérez ² and Antonio Coronel-Escamilla ⁵

¹ Departamento de Ingeniería Electrónica, Centro Nacional de Investigación y Desarrollo Tecnológico (CENIDET), Tecnológico Nacional de México, Interior Internado Palmira s/n, Palmira, Cuernavaca 62490, Morelos, Mexico; jarniel.gm@cenidet.tecnm.mx

² Secretaría de Ciencia, Humanidades, Tecnología e Innovación (SECIHTI), México City 03940, Mexico; heriberto.app@cenidet.tecnm.mx

³ Dirección de Ingeniería en Tecnologías de la Información, Universidad Politécnica del Estado de Guerrero, Puente Campuzano, Carretera Federal Iguala-Taxco km 105, Taxco de Alarcón 40321, Guerrero, Mexico; jorge.lavin@cenidet.edu.mx

⁴ División Académica de Mecánica Industrial, Universidad Tecnológica Emiliano Zapata del Estado de Morelos, Av. Universidad Tecnológica No. 1, Emiliano Zapata 62760, Morelos, Mexico; miguelbeltran@utez.edu.mx (M.B.-E.); jorgevaldez@utez.edu.mx (J.S.V.-M.); guillermoramirez@utez.edu.mx (G.R.Z.)

⁵ División de Ingeniería, Instituto Tecnológico y de Estudios Superiores de Monterrey, Monterrey 64849, Nuevo León, Mexico; antonio.coronel@tec.mx

* Correspondence: vicente.bjp@cenidet.tecnm.mx

Abstract

Quadrotor unmanned aerial vehicles (QUAVs) exhibit strongly coupled nonlinear dynamics and are highly sensitive to disturbances and measurement noise, which can significantly degrade trajectory tracking performance and induce chattering in sliding mode-based controllers. In this work, a fractional-order backstepping sliding mode control (FO-BSMC) strategy is proposed for QUAV trajectory tracking. In contrast to existing fractional-order sliding mode approaches, where the fractional operator is typically introduced into the sliding surface or control law, the proposed methodology incorporates fractional-order behavior directly into the QUAV dynamic model through the Caputo definition, while the Grünwald–Letnikov approximation is adopted for numerical implementation. A conventional integer-order BSMC scheme is also developed, and Lyapunov-based stability analyses are presented for both the conventional BSMC and the proposed FO-BSMC formulations. The fractional order is selected using the PSO algorithm. The performance of both controllers is evaluated under external disturbances, perturbed initial conditions, and measurement noise. Monte Carlo simulations are further conducted to assess the sensitivity of the closed-loop system to initialization uncertainties. The simulation results demonstrate that the proposed FO-BSMC achieves lower tracking errors, faster convergence, improved robustness against external disturbances and measurement noise, and smoother control actions with reduced chattering than the conventional BSMC.

Keywords: quadrotor unmanned aerial vehicle; fractional-order control; backstepping sliding mode control

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1. Introduction

Quadrotor unmanned aerial vehicles (QUAVs) have attracted considerable attention from both academia and industry due to their simple mechanical structure, vertical take-off and landing capability, and high maneuverability [1]. These features have made them suitable for a wide range of applications, including infrastructure inspection, environmental monitoring, precision agriculture, surveillance, and search-and-rescue missions [2]. As the operational scenarios for QUAVs become increasingly complex, developing reliable control strategies that ensure accurate trajectory tracking and robust closed-loop performance remains an active and challenging research topic [3].

The control of QUAVs remains a challenging task due to the strongly nonlinear and underactuated nature of their dynamics. In particular, the translational and rotational motions are intrinsically coupled; so, variations in the vehicle attitude directly affect its position dynamics, increasing the complexity of controller design [4]. Furthermore, practical implementations are inevitably influenced by modeling inaccuracies, external disturbances, and measurement noise, which may deteriorate tracking accuracy and reduce the robustness margins of the closed-loop system [5]. These challenges have motivated the development of robust nonlinear control methodologies capable of handling such adverse conditions [5].

Among the nonlinear control methodologies proposed for QUAVs, backstepping control has received considerable attention because it exploits the cascaded structure of the vehicle dynamics, allowing for the systematic construction of stabilizing control laws [6]. Its application to QUAVs has demonstrated satisfactory tracking performance under nominal conditions. Nevertheless, the performance of backstepping controllers degrades under model uncertainties and external disturbances, since the design strongly depends on the knowledge of the system dynamics and the availability of state measurements [7]. To address these limitations, sliding mode control (SMC) has been integrated with backstepping frameworks, leading to backstepping sliding mode control (BSMC) schemes that combine the recursive stabilization properties of backstepping with the inherent disturbance rejection capability of sliding mode techniques [8].

BSMC strategies have demonstrated remarkable capabilities for improving disturbance rejection and achieving finite-time convergence in QUAV control applications [9]. However, their practical implementation remains affected by the well-known chattering phenomenon associated with the discontinuous nature of the switching action [10]. Excessively large switching gains, commonly adopted to preserve robustness, may excite unmodeled dynamics, amplify measurement noise, and accelerate actuator degradation [11]. Several approaches have been proposed to mitigate this issue, including continuous approximations of the switching function, adaptive gain tuning, and higher-order sliding mode techniques such as the super-twisting algorithm [12]. Recent developments have mainly focused on improving the trade-off between robustness and control smoothness. For instance, in [13], a global fast terminal sliding mode control (GFTSMC) scheme was proposed to improve convergence speed and disturbance rejection, achieving faster steady-state response and reducing tracking errors compared to classical SMC formulations. In [14], an adaptive fast nonsingular terminal sliding mode strategy, combined with a super-twisting reaching law, was introduced to enhance tracking accuracy and attenuate chattering under time-varying disturbances. Along the same direction, the adaptive super-twisting controller introduced in [15] exploited gain adaptation to preserve robustness while generating smoother control actions. Although these approaches demonstrate improved performance in terms of convergence speed, disturbance rejection, and chattering attenuation, they generally require more elaborate controller architectures, additional tuning parameters, or partial knowledge of the disturbance characteristics, which may complicate their practical implementation [16].

The limitations associated with many advanced sliding mode strategies have motivated the exploration of alternative mathematical frameworks capable of improving robustness without significantly increasing controller complexity. In this context, fractional-order calculus (FOC), which generalizes classical differentiation and integration to non-integer orders, has attracted growing interest in the control community [17]. Unlike conventional integer-order models, fractional operators inherently capture memory effects and nonlocal interactions, allowing for a more accurate representation of systems whose behavior depends not only on their current state but also on their past evolution [18]. This additional degree of freedom provides a richer dynamic representation and has been successfully exploited to model and control nonlinear systems affected by uncertainties, disturbances, and partially known dynamics [19].

Recent research on QUAV control has increasingly considered fractional-order extensions of conventional control strategies [20]. In [21], a fractional-order sliding mode control scheme with time-varying state constraints was developed, with the translational and rotational subsystems treated separately to guarantee asymptotic stability and constraint satisfaction under wind disturbances. However, the method relies on accurate constraint definition and does not explicitly address measurement noise or chattering mitigation. In [22], a fractional-order adaptive fixed-time sliding mode fault-tolerant controller was proposed to handle actuator faults and external disturbances simultaneously, ensuring convergence within a predefined time bound. Despite its robustness, the approach introduces multiple adaptive mechanisms and increases parameter tuning complexity. A fractional-order sliding mode controller combined with active disturbance rejection control was presented in [23], achieving improved disturbance rejection and reduced chattering through a dual-loop structure. Nevertheless, the multi-layer architecture increases implementation complexity.

Several works have focused on enhancing convergence properties through terminal sliding mode formulations. In [24], an adaptive fractional-order nonsingular fast terminal sliding mode controller was designed to ensure finite-time convergence under stochastic disturbances, achieving high tracking accuracy. Similarly, in [25], a fractional-order fast terminal backstepping sliding mode control strategy was introduced for QUAV position and attitude regulation, demonstrating improved robustness and convergence speed under disturbances. However, these approaches require complex sliding surface design and adaptive tuning laws, which may limit their practical deployment. Alternative designs include high-order fractional PID–SMC controllers [26], which improve steady-state and transient performance but introduce additional controller parameters, and event-triggered fractional-order sliding mode control strategies [27], which reduce computational burden at the cost of increased implementation complexity due to triggering conditions. Furthermore, improved adaptive fractional-order integral terminal sliding mode controllers [28] have been proposed to enhance robustness against time-varying disturbances. However, they rely on multiple adaptive loops and increased structural complexity.

In parallel, fractional-order backstepping-based approaches have also been investigated. In [29], a neural-network-based fractional-order backstepping controller was proposed to address unknown nonlinear dynamics, achieving improved tracking performance through online approximation. However, the reliance on neural networks increases computational cost and requires training and tuning procedures. From an estimation perspective, fractional-order observer designs have demonstrated notable robustness improvements. In [30], a fractional-order sliding mode observer based on the Caputo definition was developed for actuator fault estimation in QUAVs, demonstrating enhanced robustness to noise, chattering, and initial-condition variations compared to classical observers.

Despite the considerable progress achieved in fractional-order sliding mode control, most existing approaches share a common characteristic: the fractional operator is incorporated into the controller, either through the sliding manifold, the reaching law, or the control algorithm itself, whereas the mathematical model of the plant remains unchanged. As a result, the performance improvements reported in previous studies arise from the combined effect of the modified controller and the fractional formulation, making it difficult to distinguish the contribution of the system dynamics from that of the control strategy.

The present work follows a different perspective. Rather than introducing the fractional operator into the control law, the proposed formulation incorporates fractional dynamics directly into the QUAV model while preserving the original structure of the BSMC. Consequently, the controller synthesis remains unchanged, whereas the state variables used for feedback evolve according to fractional-order dynamics. This separation makes it possible to evaluate the influence of the fractional dynamic model independently of additional modifications to the controller.

Based on this formulation, a fractional-order backstepping sliding mode controller (FO-BSMC) is developed for QUAV trajectory tracking. The fractional dynamics are represented through the Caputo derivative and numerically implemented using the Grünwald–Letnikov approximation. The resulting controller preserves the recursive design of the conventional BSMC while operating with the fractional-order state trajectories generated by the proposed dynamic model. This formulation provides a direct framework for assessing the influence of fractional dynamics on the closed-loop performance without introducing additional modifications into the controller architecture.

The main contributions of this work are summarized as follows:

- A fractional-order dynamic formulation of the QUAV based on the Caputo derivative definition.
- A novel FO-BSMC architecture in which the fractional-order behavior is introduced directly into the QUAV dynamics rather than through fractional sliding manifolds or fractional reaching laws.
- A comparative robustness analysis between the proposed FO-BSMC and the classical integer-order BSMC.
- A theoretical analysis including the Lyapunov stability proof of the conventional BSMC and a fractional Lyapunov–Mittag–Leffler stability proof for the proposed FO-BSMC

The effectiveness of the proposed FO-BSMC strategy is evaluated through a comparative study against the conventional integer-order BSMC. The fractional differentiation order is selected using the Particle Swarm Optimization (PSO) algorithm based on trajectory-tracking performance. Both controllers are assessed under identical operating conditions, considering trajectory tracking in the presence of external disturbances, perturbed initial conditions, and noisy measurements. Furthermore, Monte Carlo simulations are performed to evaluate the robustness of the closed-loop system under variations in the initial conditions. The control performance is quantitatively assessed using the integral absolute error (IAE) and the integral squared error (ISE), while the smoothness of the control signals and the chattering attenuation capability are analyzed qualitatively.

The remainder of this paper is organized as follows: Section 2 presents the fundamental concepts of fractional-order calculus. Section 3 introduces the QUAV mathematical model, the design of the classical BSMC and its Lyapunov-based stability analysis, followed by the proposed FO-SMC formulation together with its corresponding fractional-order stability analysis. Section 4 presents the simulation scenarios and

discusses the obtained results. Finally, Section 5 summarizes the main conclusions of this study.

2. Fractional Calculus Preliminaries

Fractional calculus generalizes classical differential and integral operators by extending their order from integers to real (or even complex) numbers. This generalization provides a flexible mathematical framework capable of capturing memory effects and hereditary properties that naturally arise in a wide class of physical and engineering systems [31]. As a result, fractional-order operators have proven particularly well-suited for modeling and analyzing dynamical systems exhibiting nonlocal or history-dependent behavior [17].

Several equivalent formulations of fractional-order operators have been reported in the literature. Among them, the Riemann–Liouville (RL), Grünwald–Letnikov (GL), and Caputo definitions are the most widely adopted in control-oriented applications [32]. For completeness, the main definitions employed in this work are summarized below [31].

Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a sufficiently smooth function. The RL fractional integral of order $\beta \in \mathbb{R}^+$ is defined as follows:

$${}^{RL}_0 I_t^\beta f(t) = \frac{1}{\Gamma(\beta)} \int_0^t f(\eta)(t-\eta)^{\beta-1} d\eta, \quad t > 0, \quad (1)$$

where $\Gamma(\cdot)$ denotes Euler's Gamma function, given by:

$$\Gamma(z) = \int_0^\infty t^{z-1} \exp(-t) dt. \quad (2)$$

It is worth noting that RL integral in Equation (1) reduces to the classical integral operator when $\beta = 1$.

Based on the same framework, the RL fractional derivative of order $\alpha > 0$ is defined as follows:

$${}^{RL}_0 D_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_0^t f(\eta)(t-\eta)^{m-\alpha-1} d\eta, \quad t > 0, \quad (3)$$

where $m = [\alpha] + 1$, and $[\alpha]$ denote the integer part of α . As expected, when α is 1, the RL derivative is just like the classical derivative. In control applications, the Caputo definition is often preferred due to its compatibility with physically meaningful initial conditions [18]. The Caputo fractional derivative of order α is defined as follows:

$${}^C D_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t f^{(m)}(\eta)(t-\eta)^{m-\alpha-1} d\eta, \quad t > 0, \quad (4)$$

for $\alpha = 1$, this expression coincides with the standard first-order derivative.

Numerical approximation of fractional-order operators

Analytical solutions of fractional-order differential equations are available only for a limited class of problems. Consequently, numerical approximation techniques play a central role in their practical implementation. In this work, the Grünwald–Letnikov (GL) approach is adopted due to its conceptual simplicity and direct suitability for time-domain simulations [30].

The GL fractional derivative of order $\theta \in \mathbb{R}$ is defined as follows:

$${}^{GL}_0 D_t^\theta f(t) = \lim_{h \rightarrow 0} \frac{1}{h^\theta} \sum_{j=0}^{[t/h]} (-1)^j \binom{\theta}{j} f(t-jh), \quad (5)$$

where h denotes the discretization step and $\binom{\theta}{j}$ represents the generalized binomial coefficient. This coefficient can be expressed using the Gamma function as follows:

$$\binom{\theta}{j} = \frac{\Gamma(\theta+1)}{\Gamma(j+1)\Gamma(\theta-j+1)}. \quad (6)$$

For $\theta \in [-1, 0)$, definition in Equation (5) corresponds to a fractional integral, whereas for $\theta \in (0, 1]$, it represents a fractional derivative.

Consider the fractional-order dynamical system:

$${}^G_0D_t^\theta f(t) = g(f(t)). \quad (7)$$

Its numerical implementation using the GL approximation leads to the recursive expression:

$$f(t_k) = g(f(t_{k-1}))h^\theta - \sum_{j=1}^k c_j^\theta f(t_{k-j}) \quad (8)$$

where the coefficients c_j^θ are computed recursively as follows:

$$c_j^\theta = \left(1 - \frac{1+\theta}{j}\right) c_{j-1}^\theta, \quad c_1^\theta = \theta. \quad (9)$$

The summation term in Equation (8) accounts explicitly for the memory effect inherent to fractional-order systems and is commonly referred to as the intrinsic memory contribution.

3. QUAV Modeling and Control Design

This section introduces the nonlinear mathematical model of the quadrotor unmanned aerial vehicle (QUAV) together with the proposed control framework. The conventional backstepping sliding mode controller is first derived, and its closed-loop stability is established using Lyapunov theory. The fractional-order formulation is then developed by incorporating fractional-order dynamics into the QUAV model, followed by the corresponding stability analysis.

3.1. Mathematical Model of the QUAV

This section presents the mathematical model of QUAV employed for control design. The QUAV is modeled as a rigid body operating in three-dimensional space and subject to six degrees of freedom, comprising three rotational and three translational motions. The vehicle dynamics are derived from the Euler–Lagrange formalism under the standard assumptions of rigid-body motion, symmetric mass distribution, and small-angle operation [30].

Let ϕ , θ , and ψ denote the roll, pitch, and yaw angles, respectively, expressed in the body-fixed reference frame, while x , y , and z represent the translational position of the QUAV in the inertia frame. The following second-order nonlinear equations govern the rotational dynamics of the vehicle:

$$\begin{aligned} \ddot{\phi} &= p_1\mu_2 + d_1\dot{\psi}\dot{\theta} + \dot{\psi}\dot{\theta}, \\ \ddot{\theta} &= p_2\mu_3 + d_2\dot{\psi}\dot{\phi} - \dot{\psi}\dot{\phi}, \\ \ddot{\psi} &= p_3\mu_4 + d_3\dot{\phi}\dot{\theta} + \dot{\phi}\dot{\theta}, \end{aligned} \quad (10)$$

where μ_2 , μ_3 , and μ_4 denote the control torques acting along the roll, pitch, and yaw axes, respectively. The nonlinear coupling terms involving the products of the angular velocities arise from the rigid-body rotational dynamics of the adopted QUAV model and describe the inertial coupling between the rotational axes. The coefficients d_1 , d_2 , and d_3 represent inertial coupling terms arising from asymmetries in the vehicle's moments of inertia and account for gyroscopic and cross-axis effects in the rotational dynamics. They are defined as follows:

$$\begin{aligned} d_1 &= (I_{yy} - I_{zz})/I_{xx}, \\ d_2 &= (I_{zz} - I_{xx})/I_{yy}, \\ d_3 &= (I_{xx} - I_{yy})/I_{zz}, \end{aligned} \quad (11)$$

where I_{xx} , I_{yy} , and I_{zz} denote the moments of inertia of the QUAUV about the body-fixed x -, y -, and z -axes, respectively. The scaling coefficients p_1 , p_2 , and p_3 relate the applied control torques to the resulting angular accelerations and are defined as follows:

$$\begin{aligned} p_1 &= 1/I_{xx}, \\ p_2 &= 1/I_{yy}, \\ p_3 &= 1/I_{zz}. \end{aligned} \quad (12)$$

The translational dynamics of the QUAUV are governed by the total thrust force generated by the four rotors and are expressed as follows:

$$\begin{aligned} \ddot{x} &= (S\psi S\phi + C\psi S\theta C\phi) \frac{\mu_1}{m}, \\ \ddot{y} &= (-C\psi S\phi + S\psi S\theta C\phi) \frac{\mu_1}{m}, \\ \ddot{z} &= (C\theta C\phi) \frac{\mu_1}{m} - g, \end{aligned} \quad (13)$$

where μ_1 denotes the collective thrust generated by the four rotors, m is the mass of the vehicle, and g represents the gravitational acceleration. For compactness, $\sin(\cdot)$ and $\cos(\cdot)$ are denoted by $S(\cdot)$ and $C(\cdot)$, respectively.

To facilitate the design of control strategies, the QUAUV dynamics are reformulated in state-space form. The state vector is defined as $X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}]^T = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, x, \dot{x}, y, \dot{y}, z, \dot{z}]^T$. Using this definition, the nonlinear state-space model of the QUAUV can be expressed as follows:

$$\dot{X} = \begin{bmatrix} x_2 \\ d_1 x_4 x_6 + x_4 x_6 + p_1 \mu_2 + \delta_\phi \\ x_4 \\ d_2 x_2 x_6 - x_2 x_6 + p_2 \mu_3 + \delta_\theta \\ x_6 \\ d_3 x_2 x_4 + x_2 x_4 + p_3 \mu_4 + \delta_\psi \\ x_8 \\ \mu_x \mu_1 / m + \delta_x \\ x_{10} \\ \mu_y \mu_1 / m + \delta_y \\ x_{12} \\ (C x_3 C x_1) \mu_1 / m - g + \delta_z \end{bmatrix}, \quad (14)$$

where μ_1 denotes the total thrust input, while μ_2 , μ_3 , and μ_4 correspond to the roll, pitch, and yaw control torques, respectively. The terms δ_i , for $i \in \{\phi, \theta, \psi, x, y, z\}$, represent unknown but bounded external disturbances acting on the acceleration dynamics. These disturbances are assumed to be matched and account for unmodeled aerodynamic effects, wind gusts, actuator imperfections, and parametric uncertainties.

The auxiliary variables μ_x and μ_y represent virtual control inputs defined as follows:

$$\begin{aligned} \mu_x &= (S x_5 S x_1 + C x_5 S x_3 C x_1), \\ \mu_y &= (-C x_5 S x_1 + S x_5 S x_3 C x_1). \end{aligned} \quad (15)$$

The resulting nonlinear state-space model explicitly reveals the underactuated and strongly coupled nature of the QUAUV, as well as the matched structure of the disturbances. This formulation provides a suitable foundation for the development of the backstepping–sliding mode control strategy presented in the following section.

3.2. Backstepping Sliding Mode Control

In this section, a backstepping sliding mode control (BSMC) is developed for the QUAUV trajectory tracking problem. The control objective is to achieve accurate attitude and position trajectory tracking despite the presence of matched disturbances and nonlinear dynamic couplings. The controller is constructed by exploiting the cascaded structure of the QUAUV dynamics through the combination of backstepping design and

sliding mode control principles. Since the derivation procedure is systematic and follows standard BSMC formulations commonly reported for nonlinear aerial systems, only the main design steps and resulting control laws are presented herein for completeness. A more detailed derivation of the classical BSMC methodology can be found in [8,11].

Rotational control design

The rotational control subsystem is designed to achieve accurate tracking of the quadrotor's attitude by regulating the roll, pitch, and yaw dynamics. Let x_{1d} , x_{3d} , and x_{5d} denote smooth desired reference trajectories associated with the roll (ϕ), pitch (θ), and yaw (ψ) angles, respectively. The corresponding tracking errors and their time derivatives are defined as follows:

$$\begin{aligned} e_\phi &= x_{1d} - x_1, & \dot{e}_\phi &= \dot{x}_{1d} - x_2, \\ e_\theta &= x_{3d} - x_3, & \dot{e}_\theta &= \dot{x}_{3d} - x_4, \\ e_\psi &= x_{5d} - x_5, & \dot{e}_\psi &= \dot{x}_{5d} - x_6, \end{aligned} \quad (16)$$

where x_1 , x_3 , and x_5 denote the attitude states corresponding to the roll, pitch, and yaw motions of the QUAV, whereas x_2 , x_4 , and x_6 represent the associated angular velocity states.

The sliding variables for the rotational dynamics are defined as follows:

$$\begin{aligned} s_\phi &= \dot{e}_\phi + \gamma_1 e_\phi, \\ s_\theta &= \dot{e}_\theta + \gamma_1 e_\theta, \\ s_\psi &= \dot{e}_\psi + \gamma_1 e_\psi, \end{aligned} \quad (17)$$

where $\gamma_1 \in \mathbb{R}^+$ is a positive design parameter that determines the convergence rate of the tracking errors toward the sliding manifolds.

To ensure finite-time convergence and robustness against matched disturbances, the following reaching laws are defined as follows:

$$\begin{aligned} \dot{s}_\phi &= -\rho_1 s_\phi - k_1 \text{sign}(s_\phi), \\ \dot{s}_\theta &= -\rho_1 s_\theta - k_1 \text{sign}(s_\theta), \\ \dot{s}_\psi &= -\rho_1 s_\psi - k_1 \text{sign}(s_\psi), \end{aligned} \quad (18)$$

where $\rho_1 > 0$, and $k_1 > 0$ are positive control gains. Substituting the rotational dynamics of the QUAV into the reaching laws and solving for the control inputs yields:

$$\begin{aligned} \mu_2 &= \frac{1}{p_1} [-\rho_1 s_\phi - k_1 \text{sign}(s_\phi) - d_1 x_4 x_6 - x_4 x_6 + \ddot{x}_{1d} + \gamma_1 \dot{e}_1], \\ \mu_3 &= \frac{1}{p_2} [-\rho_1 s_\theta - k_1 \text{sign}(s_\theta) - d_2 x_2 x_6 + x_2 x_6 + \ddot{x}_{3d} + \gamma_1 \dot{e}_3], \\ \mu_4 &= \frac{1}{p_3} [-\rho_1 s_\psi - k_1 \text{sign}(s_\psi) - d_3 x_2 x_4 - x_2 x_4 + \ddot{x}_{5d} + \gamma_1 \dot{e}_5], \end{aligned} \quad (19)$$

where μ_2 , μ_3 , and μ_4 denote the control torques associated with the roll, pitch, and yaw dynamics, respectively. s_ϕ , s_θ , and s_ψ represent the sliding variables corresponding to the roll, pitch, and yaw tracking errors. The positive constants p_1 , p_2 , and p_3 are scaling coefficients relating the applied torques to the angular accelerations, whereas d_1 , d_2 , and d_3 are inertial coupling coefficients. x_2 , x_4 , and x_6 correspond to the angular velocity states associated with the roll, pitch, and yaw motions, respectively. The signals $\ddot{x}_{1d}$, $\ddot{x}_{3d}$, and $\ddot{x}_{5d}$ denote the desired angular accelerations associated with the roll, pitch, and yaw reference trajectories, respectively. The parameter $\gamma_1 > 0$ is a positive design constant, while $\dot{e}_\phi$, $\dot{e}_\theta$, and $\dot{e}_\psi$ denote the derivatives of the roll, pitch, and yaw tracking errors, respectively.

Translational control design

The translational subsystem is controlled through the virtual inputs associated with the inertial-frame motions along the x -, y -, and z -axes. Let x_{7d} , x_{9d} , and x_{11d} denote smooth desired reference trajectories for the horizontal positions x , y , and the altitude z , respectively. The corresponding tracking errors and their time derivatives are defined as follows:

$$\begin{aligned}e_x &= x_{7d} - x_7, & \dot{e}_x &= \dot{x}_{7d} - \dot{x}_7, \\e_y &= x_{9d} - x_9, & \dot{e}_y &= \dot{x}_{9d} - \dot{x}_9, \\e_z &= x_{11d} - x_{11}, & \dot{e}_z &= \dot{x}_{11d} - \dot{x}_{11},\end{aligned}\quad (20)$$

where x_7 , x_9 , and x_{11} denote the position state variables of the QUAUV in the inertial frame along the x -, y -, and z -axes, respectively, whereas $\dot{x}_7$, $\dot{x}_9$, and $\dot{x}_{11}$ represent the corresponding linear velocity state variables.

The sliding variables associated with the translational dynamics are defined as follows:

$$\begin{aligned}s_x &= \dot{e}_x + \gamma_1 e_x, \\s_y &= \dot{e}_y + \gamma_1 e_y, \\s_z &= \dot{e}_z + \gamma_1 e_z,\end{aligned}\quad (21)$$

where $\gamma_1 \in \mathbb{R}^+$ is a positive design parameter.

To guarantee finite-time convergence toward the sliding surfaces, the following reaching laws are imposed:

$$\begin{aligned}\dot{s}_x &= -\rho_1 s_x - k_1 \text{sign}(s_x), \\ \dot{s}_y &= -\rho_1 s_y - k_1 \text{sign}(s_y), \\ \dot{s}_z &= -\rho_1 s_z - k_1 \text{sign}(s_z),\end{aligned}\quad (22)$$

where $\rho_1 > 0$, and $k_1 > 0$ are positive control gains.

By substituting the translational dynamics of the QUAUV into the previous expressions, the virtual control inputs μ_x and μ_y , together with the collective thrust input μ_1 , are obtained as follows:

$$\begin{aligned}\mu_x &= \frac{m}{\mu_1} [-\rho_1 s_x - k_1 \text{sign}(s_x) + \ddot{x}_{7d} + \gamma_1 \dot{e}_7], \\ \mu_y &= \frac{m}{\mu_1} [-\rho_1 s_y - k_1 \text{sign}(s_y) + \ddot{x}_{9d} + \gamma_1 \dot{e}_9], \\ \mu_1 &= \frac{m}{C x_3 C x_1} [-\rho_1 s_z - k_1 \text{sign}(s_z) + g + \ddot{x}_{11d} + \gamma_1 \dot{e}_{11}],\end{aligned}\quad (23)$$

where m denotes the mass of the QUAUV, g is the gravitational acceleration constant, and μ_1 represents the collective thrust generated by the four rotors. The auxiliary variables μ_x and μ_y correspond to the virtual translational control actions. s_x , s_y , and s_z represent the sliding variables along the x -, y -, and z -axes. The signals $\ddot{x}_{7d}$, $\ddot{x}_{9d}$, and $\ddot{x}_{11d}$ denote the desired translational accelerations along the x -, y -, and z -axes, respectively, while x_1 and x_3 represent the roll and pitch attitude state variables of the QUAUV.

3.3. Stability Analysis

In this section, the closed-loop stability of the proposed BSMC scheme for the QUAUV is established using Lyapunov stability theory. The following assumptions are adopted throughout the stability analysis.

Assumption 1. The desired reference trajectories belong to the class C^2 . Moreover, the reference trajectories and their first- and second-order time derivatives are uniformly bounded for all $\forall t > 0$.

Assumption 2. All system state variables are available for measurement.

Assumption 3. The disturbance terms associated with the acceleration dynamics are assumed to be bounded. Specifically, for each controlled degree of freedom:

$$|\delta_i| \leq \bar{\delta}_m, \quad i \in \{\phi, \theta, \psi, x, y, z\} \quad (24)$$

where $\bar{\delta}_m > 0$ is a known positive constant.

Assumption 4. The roll and pitch state variables satisfy:

$$|x_1| \leq \frac{\pi}{2}, |x_3| \leq \frac{\pi}{2}, \forall t \geq 0. \quad (25)$$

Theorem 1. Consider the QUAUV nonlinear system described by Equation (14) under the control laws $\mu_2, \mu_3, \mu_4, \mu_x, \mu_y, \mu_1$ defined in Equations (19) and (23). Under Assumptions 1–4, all sliding variables converge to zero in finite time, and all tracking errors converge asymptotically to zero. Therefore, the closed-loop system is locally stable in the sense of Lyapunov.

Proof. Consider the Lyapunov function:

$$V = \frac{1}{2}(s_\phi^2 + s_\theta^2 + s_\psi^2 + s_x^2 + s_y^2 + s_z^2). \quad (26)$$

The function V is positive definite and radially unbounded with respect to the sliding variables. Hence, it is a valid Lyapunov candidate.

The time derivative of the Lyapunov candidate function along the closed-loop trajectories is given by:

$$\dot{V} = s_\phi \dot{s}_\phi + s_\theta \dot{s}_\theta + s_\psi \dot{s}_\psi + s_x \dot{s}_x + s_y \dot{s}_y + s_z \dot{s}_z. \quad (27)$$

From the QUAUV dynamics presented in Equation (14) and the definition of the sliding variables, the dynamics of each sliding variable can be written as follows:

$$\dot{s}_i = \ddot{e}_i + \gamma_1 \dot{e}_i, \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (28)$$

Substituting the system dynamics together with the proposed control laws into Equation (28) leads to:

$$\dot{s}_i = -\rho_1 s_i - k_1 \text{sign}(s_i) + \delta_i, \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (29)$$

where $\rho_1 > 0$ and $k_1 > 0$ are control gains, while δ_i denotes the external disturbance associated with the corresponding acceleration dynamics introduced in the system model.

Substituting Equation (29) into Equation (27) yields:

$$\dot{V} = -\rho_1 \sum_i s_i^2 - k_1 \sum_i |s_i| + \sum_i s_i \delta_i, \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (30)$$

Using Assumption 3 together with the inequality $|s_i \delta_i| \leq |s_i| \bar{\delta}_m$, it follows that:

$$\dot{V} \leq -\rho_1 \sum_i s_i^2 - \sum_i (k_1 - \bar{\delta}_m) |s_i|, \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (31)$$

By selecting $k_1 > \bar{\delta}_m$. Then, $\dot{V}$ is negative definite and satisfies:

$$\dot{V} \leq -\rho V^{1/2}, \quad \rho > 0, \quad (32)$$

which guarantees finite-time convergence of all sliding variables to zero.

Once $s_i = 0$, the reduced error dynamics satisfy:

$$\dot{e}_i + \gamma_1 e_i = 0, \quad (33)$$

whose solution is:

$$e_i = e_i(0) e^{-\gamma_1 t}, \quad (34)$$

ensuring exponential convergence of all tracking errors to zero.

Thus, the closed-loop QUAUV system is locally stable and achieves asymptotic tracking.

This completes the proof. $\square$

3.4. Fractional-Order Backstepping Sliding Mode Control (FO-BSMC)

This subsection extends the conventional backstepping sliding mode controller presented in Section 3.2 by reformulating the QUAV dynamics within the framework of fractional-order calculus. The objective is to investigate the effect of incorporating fractional-order dynamics into the system model while preserving the original structure of the control law.

Unlike conventional fractional-order sliding mode approaches, where the sliding manifold or the reaching law is directly fractionalized, the proposed methodology incorporates the fractional-order framework into the QUAV dynamic model itself [32]. Consequently, the controller is constructed using the fractional-order states obtained from the numerical solution of the fractional differential equations.

The proposed formulation exploits the memory-dependent behavior inherent to fractional-order dynamics, allowing the closed-loop system to retain information from its past evolution. This characteristic has been shown to improve robustness against uncertainties, measurement noise, and variations in the initial conditions while contributing to smoother closed-loop responses [30].

Among the various definitions of fractional derivatives, the Caputo formulation is adopted in this study due to its suitability for physical systems and control applications. Unlike other formulations, the Caputo derivative allows for the use of classical integer-order initial conditions, facilitating both stability analysis and numerical implementation [30].

The integer-order nonlinear QUAV model presented in Section 3.1 is reformulated using the Caputo fractional derivative definition presented in Section 2. Applying the Caputo derivative definition of Equation (4) to the QUAV model given in Equation (14) results in:

$${}_0^C D_t^\alpha X_f(t) = \begin{bmatrix} x_{2f} \\ d_1 x_{4f} x_{6f} + x_{4f} x_{6f} + p_1 \mu_{2f} \\ x_{4f} \\ d_2 x_{2f} x_{6f} - x_{2f} x_{6f} + p_2 \mu_{3f} \\ x_{6f} \\ d_3 x_{2f} x_{4f} + x_{2f} x_{4f} + p_3 \mu_{4f} \\ x_{8f} \\ \mu_{xf} \mu_{1f} / m \\ x_{10f} \\ \mu_{yf} \mu_{1f} / m \\ x_{12f} \\ (C x_{3f} C x_{1f}) \mu_{1f} / m - g \end{bmatrix} \quad (35)$$

where $D^\alpha(\cdot)$ denotes the Caputo fractional derivative, $X_f = [x_{1f}, x_{2f}, x_{3f}, x_{4f}, x_{5f}, x_{6f}, x_{7f}, x_{8f}, x_{9f}, x_{10f}, x_{11f}, x_{12f}] \in \mathbb{R}^{12}$ denotes the fractional-order state vector obtained from the fractional dynamic model, μ_{2f} , μ_{3f} , and μ_{4f} are the fractional-order version of the roll, pitch and yaw control inputs. μ_{1f} is the thrust fractional order control input. μ_{xf} and μ_{yf} are the fractional-order virtual control inputs. The parameters m , d_i , and p_i with $i = 1, 2, 3$, are the same as the integer-order model of the QUAV, defined in Section 3.1. The superscript α identifies variables associated with the fractional-order formulation and does not denote exponentiation.

It should be emphasized that only the QUAV dynamic model is formulated in the fractional-order sense. Consequently, the subscript f is exclusively used to denote states obtained from the fractional-order system representation. The reference trajectories, tracking errors, and sliding surfaces, remain defined in the conventional integer-order sense.

Numerical solution of the FO-BSMC

For numerical implementation, the Grunwald–Letnikov (GL) algorithm presented in Section 2 is employed to solve the fractional differential equations and obtain the corresponding fractional-order states:

$$X_f(t_k) = \begin{bmatrix} x_{2f} \\ d_1 x_{4f} x_{6f} + x_{4f} x_{6f} + p_1 \mu_{2f} \\ x_{4f} \\ d_2 x_{2f} x_{6f} - x_{2f} x_{6f} + p_2 \mu_{3f} \\ x_{6f} \\ d_3 x_{2f} x_{4f} + x_{2f} x_{4f} + p_3 \mu_{4f} \\ x_{8f} \\ \mu_{xf} \mu_{1f}/m \\ x_{10f} \\ \mu_{yf} \mu_{1f}/m \\ x_{12f} \\ (Cx_{3f}Cx_{1f})\mu_{1f}/m - g \end{bmatrix} h^\alpha - \sum_{j=1}^k c_j^\alpha X_f(t_{k-j}) \quad (36)$$

where $X_f(t_k)$ denotes the fractional-order state vector evaluated at the discrete time instant t_k , h is the integration step, and $0 < \alpha < 1$ is the fractional-order. The recursive structure of the GL approximation naturally incorporates the memory property of fractional-order systems, since the current state depends on the complete history of the previous states through the weighting coefficients $c_j^\alpha = \left(1 - \frac{1+\alpha}{j}\right) c_{j-1}^\alpha$.

Fractional-order rotational control

Consider the fractional-order rotational subsystem of the QUAV. The attitude tracking errors associated with roll, pitch, and yaw motions are defined as follows:

$$\begin{aligned} e_{\phi,1} &= x_{1d} - x_{1f}, \\ e_{\theta,1} &= x_{3d} - x_{3f}, \\ e_{\psi,1} &= x_{5d} - x_{5f}, \end{aligned} \quad (37)$$

where x_{1f} , x_{3f} , and x_{5f} denote the fractional-order attitude state variables corresponding to roll, pitch, and yaw, respectively, while x_{1d} , x_{3d} , and x_{5d} correspond to the desired attitude trajectories.

Since the reference signals are generated by conventional integer-order dynamics, their first- and second-time derivatives are assumed to exist and remain bounded. Thus, the auxiliary velocity-tracking errors are introduced as follows:

$$\begin{aligned} e_{\phi,2} &= \dot{x}_{1d} - x_{2f}, \\ e_{\theta,2} &= \dot{x}_{3d} - x_{4f}, \\ e_{\psi,2} &= \dot{x}_{5d} - x_{6f}, \end{aligned} \quad (38)$$

where x_{2f} , x_{4f} , and x_{6f} denote the fractional-order angular velocity state variables, while $\dot{x}_{1d}$, $\dot{x}_{3d}$, and $\dot{x}_{5d}$ are the classical derivatives of the desired attitude trajectories.

To avoid the computation of fractional derivatives of the desired trajectories, the following auxiliary variables are defined as follows:

$$\begin{aligned} e_{\phi,3} &= \ddot{x}_{1d} - {}^C_0D_t^\alpha x_{2f}, \\ e_{\theta,3} &= \ddot{x}_{3d} - {}^C_0D_t^\alpha x_{4f}, \\ e_{\psi,3} &= \ddot{x}_{5d} - {}^C_0D_t^\alpha x_{6f}, \end{aligned} \quad (39)$$

where $\ddot{x}_{1d}$, $\ddot{x}_{3d}$, and $\ddot{x}_{5d}$ are the classical second derivative of the desired attitude trajectories, and the operator ${}^C_0D_t^\alpha(\cdot)$ denotes the Caputo fractional derivative with $0 < \alpha < 1$.

The attitude sliding manifolds are selected as follows:

$$s_{i,f} = e_{i,2} + \gamma_1 e_{i,1}, \quad i \in \{\phi, \theta, \psi\}, \quad (40)$$

where $\gamma_1 > 0$ is a design parameter.

Using the auxiliary variables introduced in Equations (37)–(39), the following backstepping stabilization functions are defined as follows:

$$\eta_i = e_{i,3} + \gamma_1 e_{i,2}, \quad i \in \{\phi, \theta, \psi\}, \quad (41)$$

It should be emphasized that η_i is an auxiliary variable employed to construct the control law while preserving consistency with the fractional-order system representation.

To enforce convergence toward the sliding manifolds, the following reaching conditions are imposed:

$$\eta_i = -\rho s_{i,f} - k_1 \text{sign}(s_{i,f}), \quad i \in \{\phi, \theta, \psi\}, \quad (42)$$

where $\rho_1 > 0$ and $k_1 > 0$ denote the sliding mode gains.

Substituting the fractional-order QUAV rotational dynamics into Equation (42), the following fractional-order control laws are obtained:

$$\begin{aligned} \mu_{2f} &= \frac{1}{p_1} [-\rho_1 s_{\phi,f} - k_1 \text{sign}(s_{\phi,f}) - d_1 x_{4f} x_{6f} - x_{4f} x_{6f} + \ddot{x}_{1d} + \gamma_1 e_{\phi,2}], \\ \mu_{3f} &= \frac{1}{p_2} [-\rho_1 s_{\theta,f} - k_1 \text{sign}(s_{\theta,f}) - d_2 x_{2f} x_{6f} + x_{2f} x_{6f} + \ddot{x}_{3d} + \gamma_1 e_{\theta,2}], \\ \mu_{4f} &= \frac{1}{p_3} [-\rho_1 s_{\psi,f} - k_1 \text{sign}(s_{\psi,f}) - d_3 x_{2f} x_{4f} - x_{2f} x_{4f} + \ddot{x}_{5d} + \gamma_1 e_{\psi,2}]. \end{aligned} \quad (43)$$

Fractional-order translational control

The translational subsystem is addressed following the same design philosophy adopted for the rotational dynamics. The position tracking errors associated with the Cartesian coordinates are defined as follows:

$$\begin{aligned} e_{x,1} &= x_{7d} - x_{7f}, \\ e_{y,1} &= x_{9d} - x_{9f}, \\ e_{z,1} &= x_{11d} - x_{11f}, \end{aligned} \quad (44)$$

where x_{7f} , x_{9f} , and x_{11f} denote the fractional-order translational state variables, while x_{7d} , x_{9d} , and x_{11d} represent the desired position trajectories.

Since the desired position trajectories are generated by an integer-order reference model, the auxiliary velocity-tracking errors are defined as follows:

$$\begin{aligned} e_{x,2} &= \dot{x}_{7d} - x_{8f}, \\ e_{y,2} &= \dot{x}_{9d} - x_{10f}, \\ e_{z,2} &= \dot{x}_{11d} - x_{12f}, \end{aligned} \quad (45)$$

where x_{8f} , x_{10f} , and x_{12f} correspond to the fractional-order translational velocity state variables, while $\dot{x}_{7d}$, $\dot{x}_{9d}$, and $\dot{x}_{11d}$ represent the classical derivative of desired position trajectories. In addition, the following auxiliary variables are defined as follows:

$$\begin{aligned} e_{x,3} &= \ddot{x}_{7d} - {}^C D_t^\alpha x_{8f}, \\ e_{y,3} &= \ddot{x}_{9d} - {}^C D_t^\alpha x_{10f}, \\ e_{z,3} &= \ddot{x}_{11d} - {}^C D_t^\alpha x_{12f}, \end{aligned} \quad (46)$$

where $\ddot{x}_{7d}$, $\ddot{x}_{9d}$, and $\ddot{x}_{11d}$ are the classical second derivative of the desired position trajectories, and the operator ${}^C D_t^\alpha(\cdot)$ denotes the Caputo fractional derivative.

The translational sliding manifolds are selected as follows:

$$s_{i,f} = e_{i,2} + \gamma_1 e_{i,1}, \quad i \in \{x, y, z\}, \quad (47)$$

where $\gamma_1 > 0$ is a design parameter.

Using the auxiliary variables introduced in Equations (44)–(46), the following stabilization functions are defined as follows:

$$\eta_i = e_{i,3} + \gamma_1 e_{i,2}, \quad i \in \{x, y, z\}. \quad (48)$$

which are employed in the recursive backstepping design.

To guarantee convergence toward the sliding manifolds, the reaching conditions are imposed as follows:

$$\eta_i = -\rho_1 s_{i,f} - k_1 \text{sign}(s_{i,f}), \quad i \in \{x, y, z\}, \quad (49)$$

where $\rho_1 > 0$ and $k_1 > 0$ are sliding-mode gains.

Substituting the fractional-order translational dynamics into Equation (49) yields the virtual control inputs:

$$\begin{aligned}\mu_{xf} &= \frac{m}{\mu_{1f}} [-\rho_1 s_{x,f} - k_1 \text{sign}(s_{x,f}) + \ddot{x}_{7d} + \gamma_1 e_{x,2}], \\ \mu_{yf} &= \frac{m}{\mu_{1f}} [-\rho_1 s_{y,f} - k_1 \text{sign}(s_{y,f}) + \ddot{x}_{9d} + \gamma_1 e_{y,2}].\end{aligned}\quad (50)$$

For the altitude dynamics, the fractional-order thrust input is computed as follows:

$$\mu_{1f} = \frac{m}{Cx_{3f}Cx_{1f}} [-\rho_1 s_{z,f} - k_1 \text{sign}(s_{z,f}) + g + \ddot{x}_{11d} + \gamma_1 e_{z,2}]. \quad (51)$$

The resulting control strategy preserves the recursive structure of the conventional BSMC while exploiting the additional degrees of freedom introduced by the fractional-order state representation. Consequently, memory-dependent dynamics are incorporated into the closed-loop system without modifying the original controller architecture, providing a framework for analyzing the influence of fractional-order state dynamics on QUAV trajectory tracking and robustness.

3.5. Fractional-Order Stability Analysis

This subsection establishes the Mittag–Leffler stability of the closed-loop fractional-order QUAV system under the proposed FO-BSMC. Unlike the integer-order formulation presented in Theorem 1, the controller is synthesized from the fractional-order QUAV model introduced in Section 3.4. Consequently, the closed-loop dynamics are governed by Caputo fractional-order differential equations, and their stability must be analyzed within the framework of fractional-order Lyapunov theory. The proof follows the direct Lyapunov approach together with the Mittag–Leffler stability criterion for nonlinear fractional-order systems. The stability analysis of the proposed FO-BSMC is carried out under Assumptions 1 and 3 introduced in Section 3.3, together with the additional assumptions stated below.

Assumption 6. *The complete fractional-order state vector is assumed to be measurable.*

Assumption 7. *The fractional-order state variables for roll and pitch satisfy:*

$$|x_{1f}| \leq \frac{\pi}{2}, \quad |x_{3f}| \leq \frac{\pi}{2}, \quad \forall t \geq 0. \quad (52)$$

Theorem 2. *Consider the fractional-order QUAV dynamics together with the FO-BSMC laws defined in Equations (43), (50) and (51). Suppose that Assumptions 1, 3, 6, and 7 hold. If the controller parameters satisfy:*

$$\rho_1 > 0, \quad k_1 > \bar{\delta}_m, \quad \gamma_1 > 0, \quad (53)$$

then, the closed-loop sliding dynamics are Mittag–Leffler-stable. Consequently, all rotational and translational tracking errors converge asymptotically to zero.

Proof. For each rotational and translational channel, the sliding manifold is defined as follows:

$$s_{i,f} = e_{i,2} + \gamma_1 e_{i,1}, \quad i \in \{\phi, \theta, \psi, x, y, z\}, \quad (54)$$

where $e_{i,1}$ and $e_{i,2}$ denote the fractional-order tracking errors introduced in Sections 3.4.

Since the proposed controller is derived from the fractional-order QUAV model, the auxiliary variables:

$$\eta_i = e_{i,3} + \gamma_1 e_{i,2}, \quad i \in \{\phi, \theta, \psi, x, y, z\}, \quad (55)$$

represent the fractional evolution associated with the sliding manifold.

By design, the reaching dynamics are imposed as follows:

$$\eta_i = -\rho_1 s_{i,f} - k_1 \text{sign}(s_{i,f}), \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (56)$$

Substituting the proposed FO-BSMC control laws into the fractional-order QUAV dynamics yields the closed-loop sliding dynamics:

$${}_0^C D_t^\alpha s_{i,f} = -\rho_1 s_{i,f} - k_1 \text{sign}(s_{i,f}) + \delta_i, \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (57)$$

where δ_i denotes the bounded external disturbances.

Consider the Lyapunov candidate:

$$V = \frac{1}{2} \sum_i s_{i,f}^2, \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (58)$$

The function V is positive definite and radially unbounded with respect to the sliding variables.

Since the system is described by Caputo fractional derivatives, the classical Lyapunov derivative is replaced by its fractional counterpart. According to the fractional Lyapunov inequality established by Aguila-Camacho et al. [33]:

$${}_0^C D_t^\alpha V \leq \sum_i s_{i,f} {}_0^C D_t^\alpha s_{i,f}, \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (59)$$

Substituting Equation (57) into Equation (59) gives:

$${}_0^C D_t^\alpha V \leq -\rho_1 \sum_i s_{i,f}^2 - k_1 \sum_i |s_{i,f}| + \sum_i s_{i,f} \delta_i, \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (60)$$

Using Assumption 3:

$$|s_{i,f} \delta_i| \leq |s_{i,f}| \bar{\delta}_m, \quad i \in \{\phi, \theta, \psi, x, y, z\}, \quad (61)$$

which leads to:

$${}_0^C D_t^\alpha V \leq -\rho_1 \sum_i s_{i,f}^2 - k_1 \sum_i (k_1 - \bar{\delta}_m) |s_{i,f}|, \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (62)$$

Since the controller gain satisfies $k_1 > \bar{\delta}_m$ both terms on the right-hand side of Equation (62) are strictly negative whenever $s_{i,f} \neq 0$. Therefore:

$${}_0^C D_t^\alpha V \leq 0, \quad \forall s_{i,f} \neq 0, \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (63)$$

The conditions of the fractional Lyapunov direct method are satisfied, implying that the equilibrium of the sliding dynamics is Mittag-Leffler-stable. Hence, ${}_0^C D_t^\alpha V < 0$. From the fractional Lyapunov direct method, the equilibrium $s_{i,f} = 0$ is Mittag-Leffler-stable. Consequently:

$$\lim_{t \rightarrow \infty} s_{i,f} = 0, \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (64)$$

Once the sliding manifold is reached, the condition $s_{i,f} = 0$, implies the reduced-order error dynamics:

$$e_{i,2} = -\gamma_1 e_{i,f}, \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (65)$$

Since $\gamma_1 > 0$, the reduced-order error system is asymptotically stable, yielding:

$$\lim_{t \rightarrow \infty} e_{i,1} = 0, \quad \lim_{t \rightarrow \infty} e_{i,2} = 0, \quad i \in \{\phi, \theta, \psi, x, y, z\}. \quad (66)$$

Therefore, the proposed FO-BSMC guarantees asymptotic trajectory tracking while preserving the Mittag-Leffler stability of the closed-loop fractional-order QUAV system.

This completes the proof. $\square$

4. Simulations Results

This section presents a comparative evaluation of the conventional backstepping sliding-mode controller (BSMC) and the proposed fractional-order backstepping sliding-mode controller (FO-BSMC) for the trajectory-tracking problem of a quadrotor unmanned aerial vehicle (QUAV). The objective is to assess the influence of the fractional-order dynamic formulation on closed-loop performance across different operating conditions, while maintaining the same controller structure and tuning parameters for both approaches.

The proposed FO-BSMC was implemented using the fractional-order QUAV model developed in Section 3.4, where the Caputo fractional derivative was numerically approximated through the Grünwald–Letnikov discretization. Since the fractional differentiation order significantly influences the system dynamics, its value was determined through the Particle Swarm Optimization (PSO) algorithm rather than selected empirically. The optimization process was formulated to minimize the trajectory-tracking error under identical operating conditions while keeping the controller gains fixed. The optimization converged to the fractional order $\alpha = 0.99$, which was subsequently adopted in all simulation scenarios presented in this section.

Both control strategies were implemented in MATLAB R2025a using a fixed integration step equal to 0.001 s, and a total simulation time of 15 s. Unless otherwise stated, identical controller gains were employed for both BSMC and FO-BSMC ($\gamma_1 = 1$, $\rho_1 = 10$, and $k_1 = 20$) to ensure a fair comparison. Moreover, the discontinuous sign function was replaced by a hyperbolic tangent approximation in the control law to obtain smoother control signals and reduce chattering during numerical simulations.

The QUAV parameters employed throughout this study are listed in Table 1. To assess the tracking performance under continuously varying operating conditions, the desired attitude and position references $(\phi_d, \theta_d, \psi_d, x_d, y_d, z_d)$, were defined as $\{0.1 \sin(0.4t), 0.1 \sin(0.4t), \frac{\pi}{6} \sin(0.6t), \sin(t), \cos(t), 0.1t\}$.

The comparative study comprises three simulation scenarios. In the first scenario, trajectory tracking is evaluated in the presence of bounded external disturbances acting on both the translational and rotational dynamics. The second scenario investigates robustness against variations in the initial conditions by perturbing the initial state of the QUAV. To further quantify the influence of the initial conditions on the closed-loop response, Monte Carlo simulations are performed using randomly generated initial states within predefined bounds. Finally, the third scenario evaluates the robustness of both controllers under measurement noise affecting all measurable state variables, with particular emphasis on tracking performance, control smoothness, and chattering attenuation.

The performance of both controllers (BSMC and FO-BSMC) is quantified using standard evaluation metrics, including the integral squared error (ISE) and the integral absolute error (IAE), defined as follows:

$$\begin{aligned} ISE &= \int_0^{T_0} E^2 dt, \\ IAE &= \int_0^{T_0} |E| dt, \end{aligned} \quad (67)$$

where $E = X_d - X$ is the tracking error vector, $X_d \in \mathbb{R}^{12}$ is the desired state vector, $X \in \mathbb{R}^{12}$ represents the actual state vector, and T_0 is the total simulation time.

Table 1. Nominal parameters of the QUAV model [11].

Parameter	Symbol	Value
Mass	m	1.4 kg

Arm length	l	0.28 m
Moment of inertia along the x -axis	I_{xx}	0.0116 kg m ²
Moment of inertia along the y -axis	I_{yy}	0.0116 kg m ²
Moment of inertia along the z -axis	I_{zz}	0.0232 kg m ²

4.1. Case 1—Robustness to External Disturbances

In the first simulation scenario, the disturbance-rejection capability of the conventional BSMC and the proposed FO-BSMC is evaluated by introducing bounded external disturbances to the translational and rotational dynamics of the QUAV. Both controllers are implemented using identical system parameters, controller gains, and reference trajectories to ensure a fair comparison. The simulations start from zero initial conditions. For the proposed FO-BSMC, the optimized fractional order obtained using the PSO algorithm is employed throughout the simulation.

The disturbance applied to each controlled channel is defined as follows:

$$\delta_i = 5 \sin(t), \quad i \in \{\phi, \theta, \psi, x, y, z\}, \quad (68)$$

where δ_i denotes an additive disturbance acting on the corresponding acceleration equation. This disturbance is continuously applied over the entire simulation interval to evaluate the capability of both controllers to maintain accurate trajectory tracking under sustained external perturbations.

Figure 1 compares the attitude and position-tracking responses obtained using the conventional BSMC and the proposed FO-BSMC under Case 1. Both controllers are able to follow the prescribed reference trajectories while maintaining stable closed-loop behavior throughout the simulation. However, the proposed FO-BSMC exhibits smaller tracking deviations, particularly in the translational state variables, resulting in a closer agreement with the reference signals. The rotational responses also present slight improvements with respect to the conventional BSMC, although the differences are less pronounced.

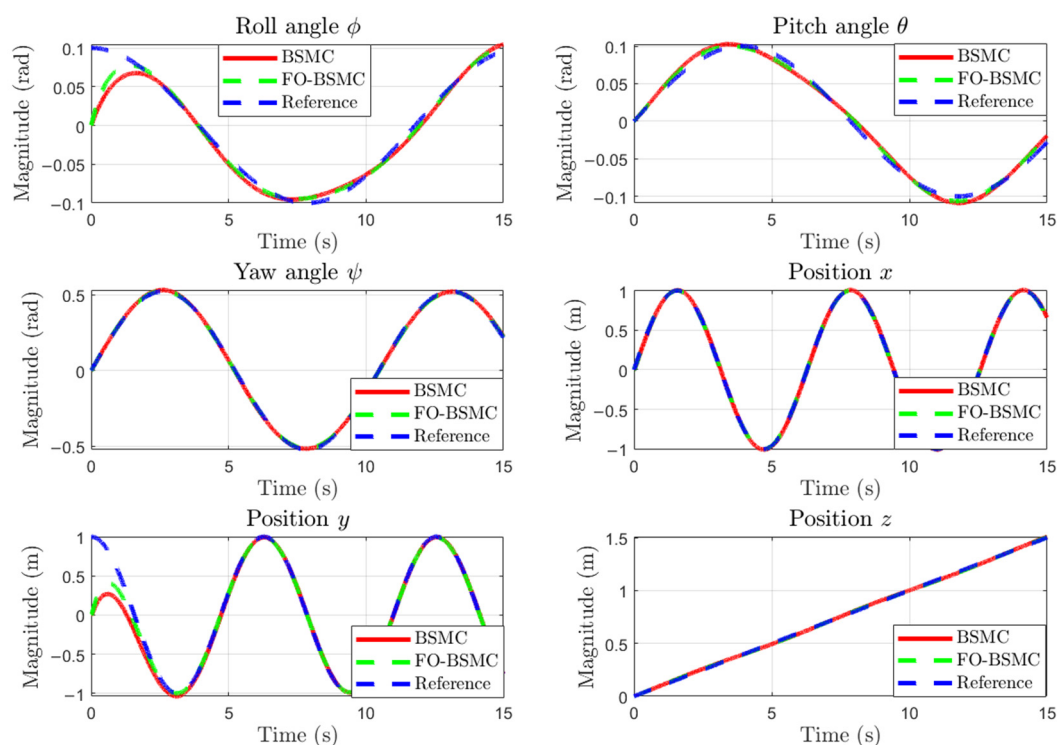


Figure 1. Attitude and position tracking of the QUAV under Case 1.

Figure 2 shows the three-dimensional trajectories generated by the conventional BSMC and the proposed FO-BSMC under Case 1. Both controllers closely follow the desired reference path, a smooth helical trajectory with increasing altitude. However, the proposed FO-BSMC reaches the reference path more quickly, with a shorter transient phase before converging. Once converged, both controllers maintain accurate trajectory tracking with only minor deviations despite continuous external disturbances.

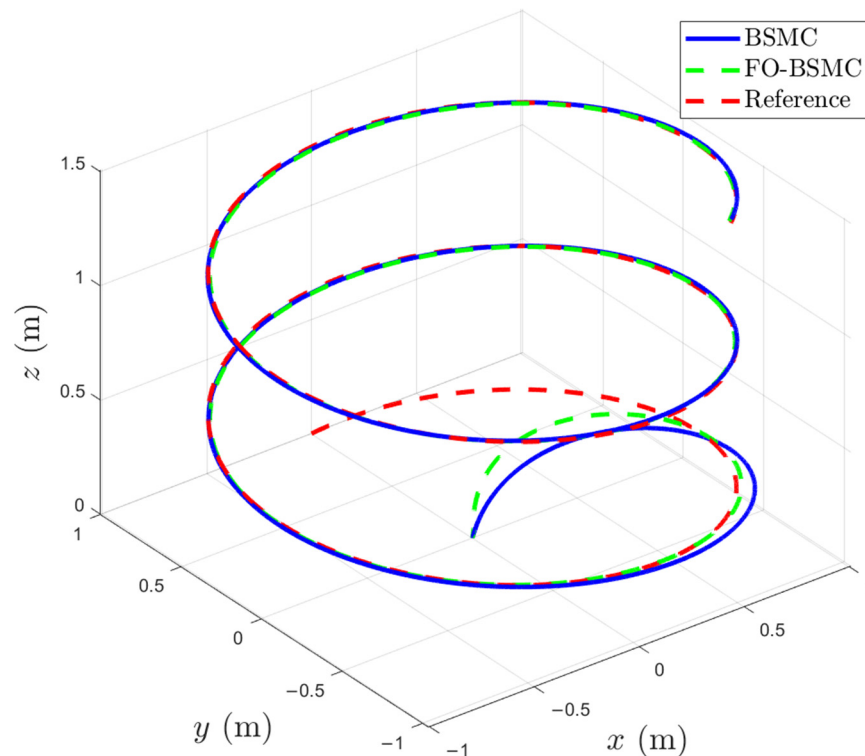


Figure 2. Three-dimensional trajectory of the QUAV under Case 1.

Figure 3 presents the attitude and position tracking errors obtained under Case 1. As observer, the roll error converges more rapidly with the proposed FO-BSMC during the initial transient, whereas both controllers exhibit comparable periodic error profiles once the transient response has vanished. Similar behavior is observed for the pitch and yaw motions, where only slight differences in the error amplitude are noticeable. The most evident improvement is obtained in the translational dynamics. In particular, the error along the x -axis remains confined to a substantially narrower band around zero under the proposed controller, while the conventional BSMC exhibits persistent oscillations throughout the simulation. For the y -axis, the FO-BSMC shortens the transient stage before reaching the periodic tracking regime, whereas, along the z -axis, both controllers produce comparable responses, with the proposed approach exhibiting a slight reduction in the oscillation amplitude.

Figure 4 presents the control inputs generated by the conventional BSMC and the proposed FO-BSMC under Case 1, where the discontinuous switching term is approximated by the hyperbolic tangent function. The thrust and torque commands produced by both controllers exhibit nearly identical time responses, with comparable amplitudes, transient characteristics, and periodic behavior. All control inputs remain continuous and bounded throughout the simulation, and no high-frequency oscillations are observed. As expected, the hyperbolic tangent approximation yields smooth control signals for both controllers while preserving their overall dynamic response.

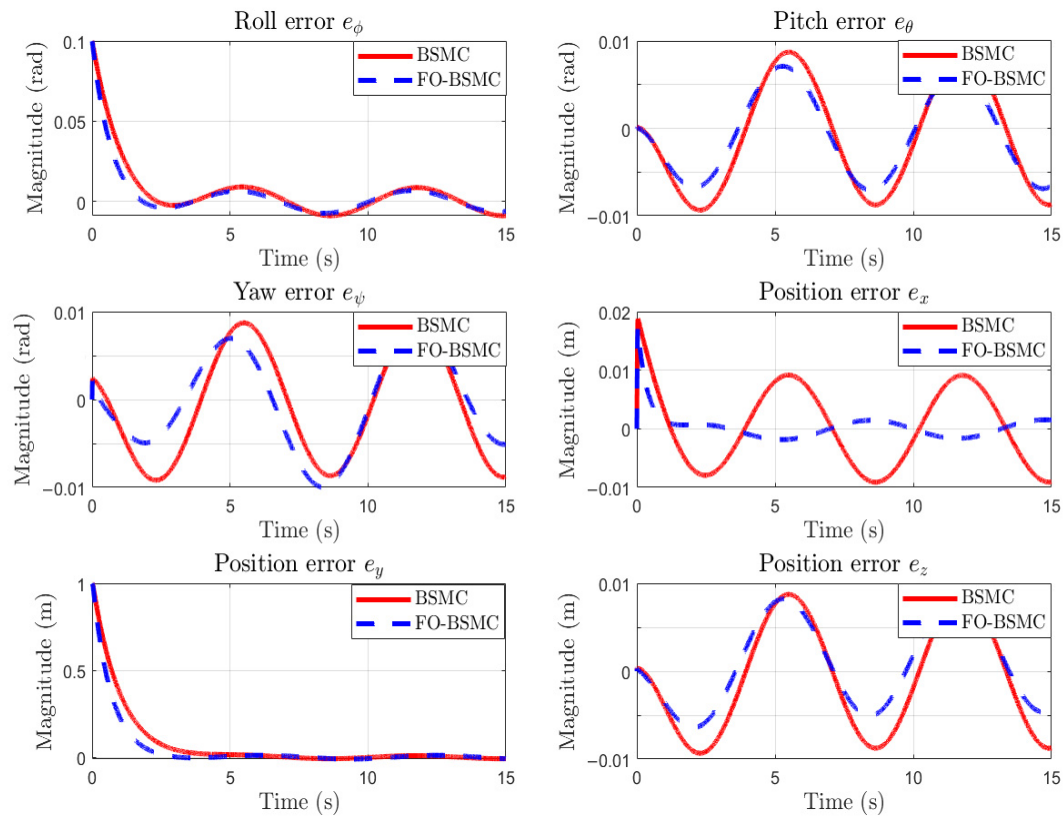


Figure 3. Position and attitude tracking errors of the QUAV under Case 1.

Table 2 presents the tracking performance of both controllers under nominal operation in the presence of external disturbances by means of the ISE and IAE performance indices. Overall, the proposed FO-BSMC consistently achieves lower error values than the conventional BSMC for all attitude and position variables, indicating an improved trajectory-tracking performance under persistent perturbations. The most significant improvement is observed in the translational motion along the x -axis, where both the ISE and IAE are substantially reduced. Noticeable reductions are also obtained for the y -axis position, which represents the most demanding translational channel in this scenario. For the rotational variables, the improvements are more moderate but remain consistent for the roll, pitch, and yaw motions.

Table 2. Tracking error indices (ISE and IAE) under Case 1.

State Variable	Description	BSMC		FO-BSMC	
		ISE	IAE	ISE	IAE
$x_1 = \phi$	Roll angle	0.004998	0.144913	0.003449	0.111396
$x_3 = \theta$	Pitch angle	0.000577	0.082702	0.000348	0.064442
$x_5 = \psi$	Yaw angle	0.000565	0.081873	0.000439	0.070297
$x_7 = x$	Position (x -axis)	0.000685	0.089292	0.000075	0.019954
$x_9 = y$	Position (y -axis)	0.513582	1.058898	0.355186	0.746028
$x_{11} = z$	Position (z -axis)	0.000575	0.082540	0.000379	0.065791

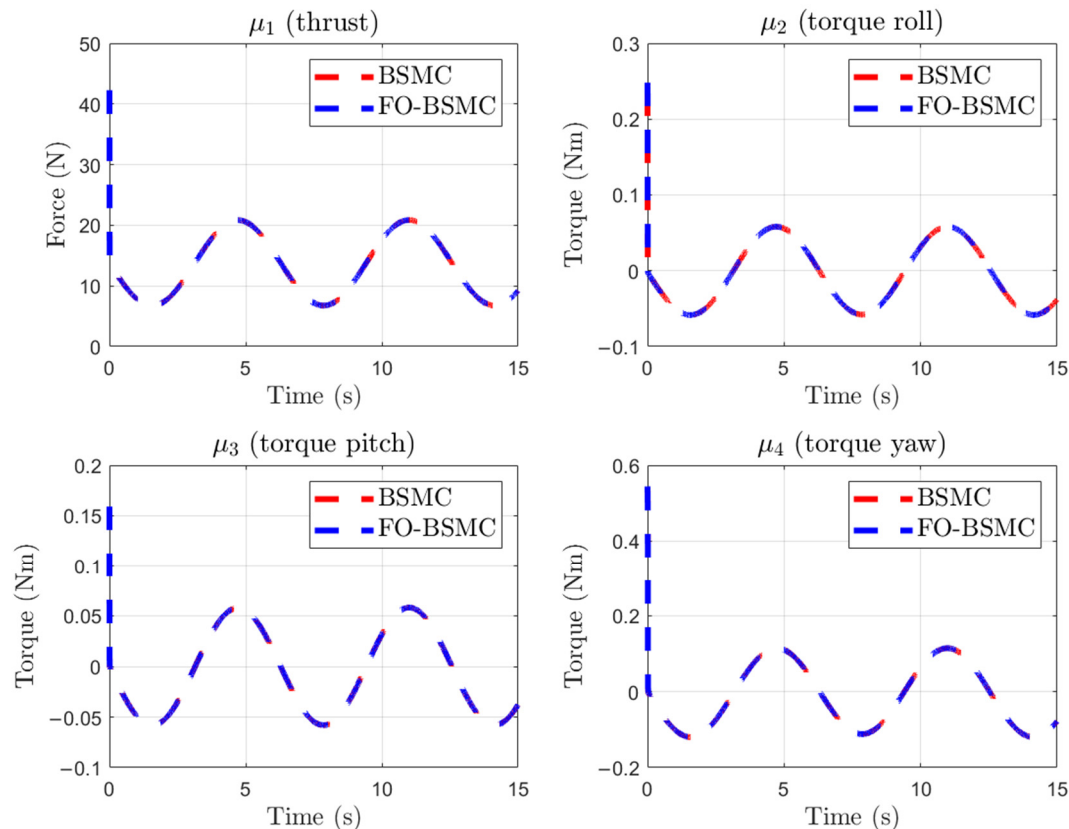


Figure 4. Control inputs using a smooth tanh-based sliding function under Case 1.

4.2. Case 2—Robustness to Initial Condition

The second simulation scenario evaluates the sensitivity of the conventional BSMC and the proposed FO-BSMC to variations in the initial conditions. The initial state vector is selected as $X_0 = [0.75, 0, 0.6, 0, 0.45, 0, 3, 0, -3, 0, 1.5, 0]^T$ representing a large initial tracking error in both the attitude and position variables. All remaining simulation settings are identical to those considered in Case 1, including the controller gains, reference trajectories, and the hyperbolic tangent approximation employed in the sliding-mode term. For the proposed FO-BSMC, the optimized fractional order obtained using the PSO algorithm is employed throughout the simulation.

To complement the deterministic analysis, a Monte Carlo study consisting of 100 independent simulations is performed. In each realization, the initial attitude and position state variables are randomly sampled within predefined operating ranges, while both controllers are evaluated under the same initial conditions. The corresponding ISE and IAE performance indices are computed for every simulation, providing a statistical assessment of the sensitivity of both control strategies to variations in the initial state.

Figure 5 presents the attitude and position responses obtained under Case 2, where both controllers are subjected to large initial tracking errors. The proposed FO-BSMC reaches the reference trajectories more rapidly than the conventional BSMC in both the rotational and translational motions. This difference is particularly evident during the initial transient, where the fractional-order formulation exhibits a faster reduction in the initial tracking error and a shorter settling stage. After the transient response, both controllers accurately follow the prescribed trajectories.

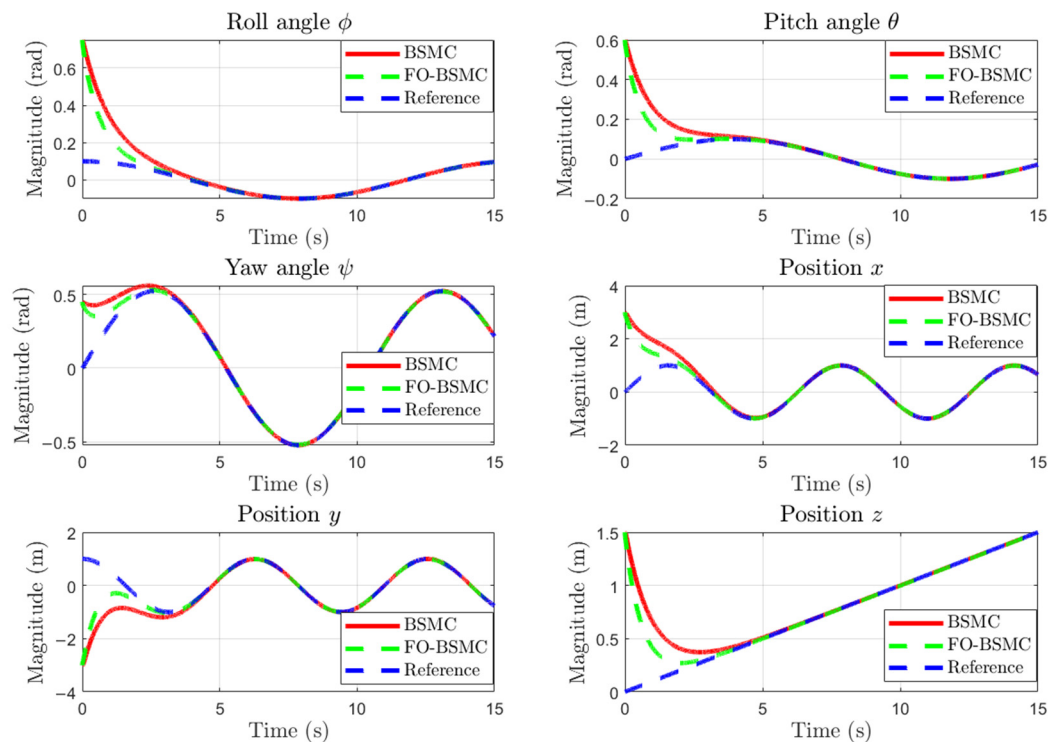


Figure 5. Attitude and position tracking of the QUAUV under Case 2.

Figure 6 illustrates the three-dimensional trajectory of the QUAUV under Case 2. Due to the large initial tracking error, both controllers start from positions considerably displaced from the reference trajectory. The proposed FO-BSMC approaches the reference path more rapidly, reducing the distance to the desired trajectory during the initial stage of motion. In contrast, the conventional BSMC requires a longer transient before reaching the same trajectory. Once the transient has elapsed, both controllers follow the prescribed three-dimensional path with comparable accuracy, although the proposed FO-BSMC reaches the tracking regime earlier.

Figure 7 presents the attitude and position tracking errors obtained under Case 2. For all state variables, the proposed FO-BSMC exhibits a faster decay of the initial tracking errors than the conventional BSMC, resulting in a shorter transient response. This behavior is consistently observed in both the rotational and translational dynamics, where the proposed controller approaches the reference more rapidly during the first seconds of the simulation. As the transient response vanishes, the tracking errors of both controllers converge to values close to zero and remain bounded throughout the remainder of the simulation.

Figure 8 compares the control inputs produced by the BSMC and the proposed FO-BSMC under the conditions of Case 2. The thrust and attitude torque commands remain smooth, continuous, and well-bounded throughout the simulation. Both controllers exhibit nearly identical control amplitudes and similar transient behavior, with only minor differences during the initial response. No oscillatory or chattering phenomena are observed, confirming the effectiveness of the tanh-based sliding function in generating continuous control actions. These results indicate that the superior tracking performance achieved by the FO-BSMC is not accompanied by an increase in control effort, but rather results from a more effective exploitation of the system dynamics through the fractional-order formulation.

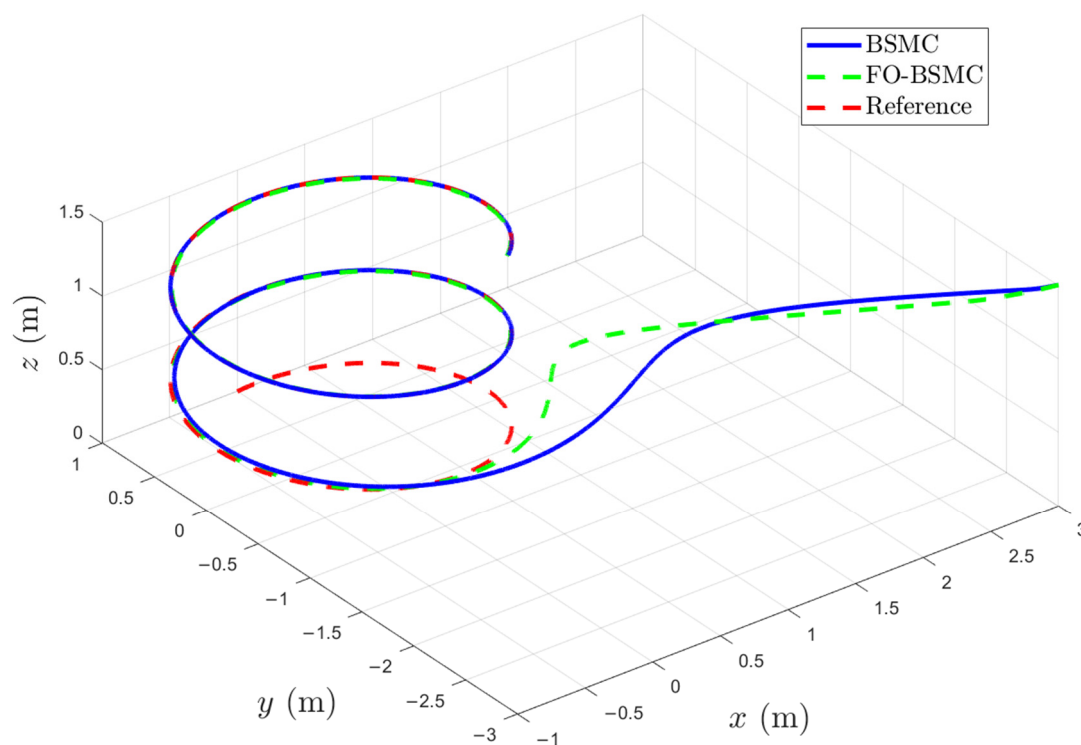


Figure 6. Three-dimensional trajectory of the QUAV under Case 2.

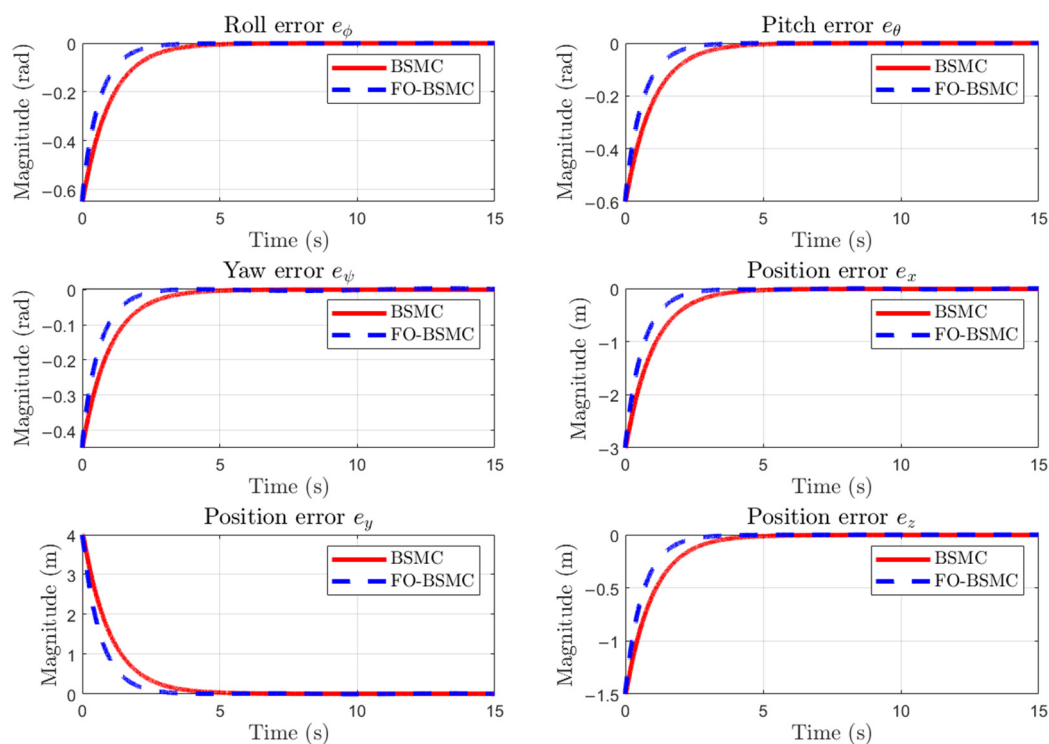


Figure 7. Position and attitude tracking errors of the QUAV under Case 2.

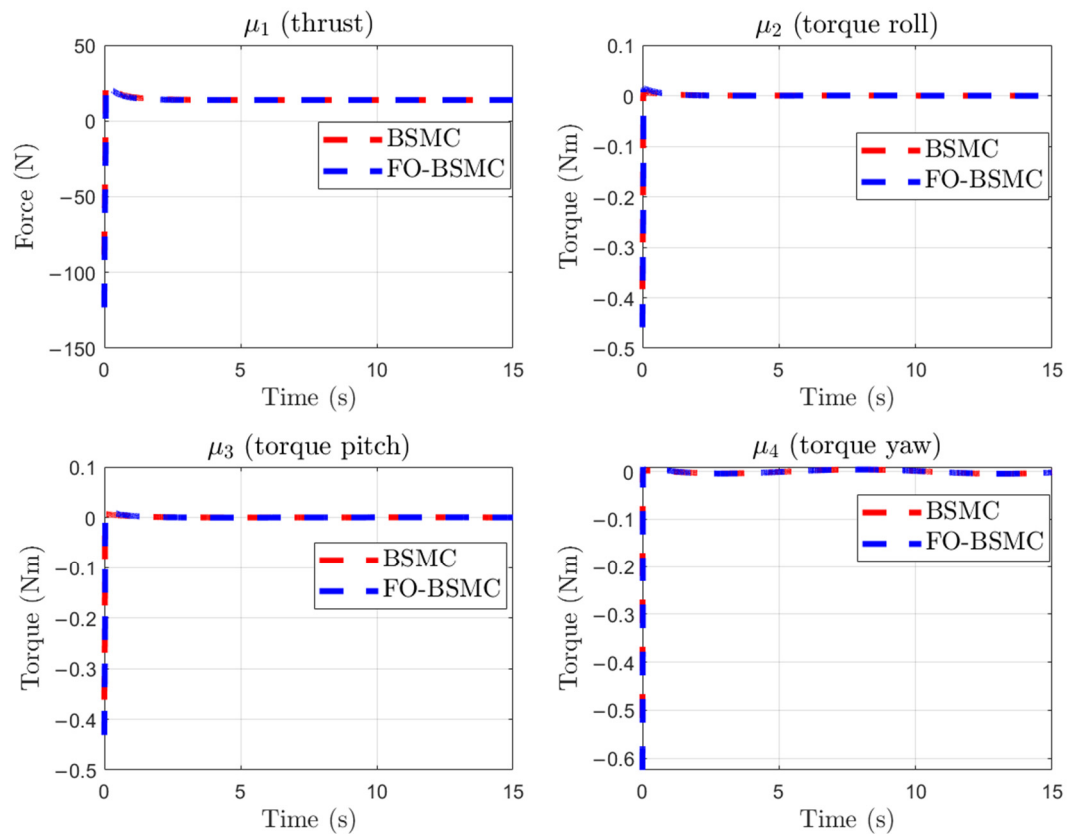


Figure 8. Control inputs using a smooth tanh-based sliding function under Case 2.

Table 3 summarizes the tracking error indices obtained under Case 2. The proposed FO-BSMC consistently produces lower ISE and IAE values than the conventional BSMC for all attitude and position variables. For the rotational dynamics, the reductions observed in the roll, pitch, and yaw responses indicate a more effective attenuation of the initial tracking errors during the transient stage. The same trend is observed in the translational dynamics, where the proposed controller yields lower error indices along all three position axes. The most pronounced improvements are obtained in the x - and y -axis motions, reflecting a faster recovery from large initial state deviations.

Table 3. Tracking error indices (ISE and IAE) under Case 2.

State Variable	Description	BSMC		FO-BSMC	
		ISE	IAE	ISE	IAE
$x_1 = \phi$	Roll angle	0.216405	0.657766	0.126065	0.410548
$x_3 = \theta$	Pitch angle	0.183716	0.606095	0.109141	0.381188
$x_5 = \psi$	Yaw angle	0.101567	0.450927	0.059997	0.296865
$x_7 = x$	Position (x -axis)	4.638866	3.047446	2.791481	1.971956
$x_9 = y$	Position (y -axis)	8.480882	4.121407	5.291136	2.693664
$x_{11} = z$	Position (z -axis)	1.165475	1.526731	0.69861	0.973737

Figure 9 presents the statistical distribution of the ISE obtained from the Monte Carlo analysis under varying initial conditions. Each boxplot summarizes the dispersion of the performance index across multiple simulations for the six evaluated state variables. As observed, the FO-BSMC consistently exhibits lower ISE values than the classical BSMC across all state variables, as evidenced by the downward shift in the corresponding boxplots. In addition to the reduction in median values, the interquartile ranges associated

with the FO-BSMC are generally narrower, indicating a more concentrated distribution and reduced variability in performance.

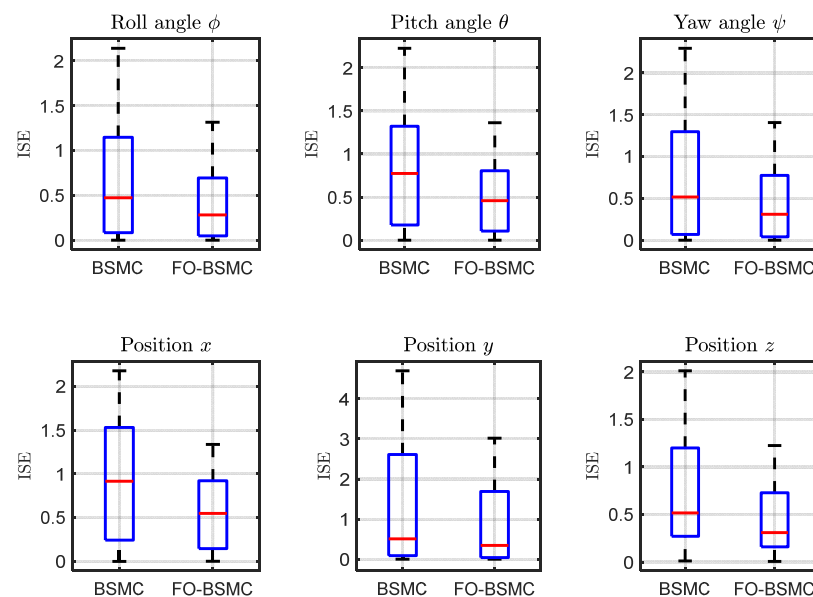


Figure 9. Monte Carlo distribution of ISE for BSMC and FO-BSMC under Case 2.

Figure 10 shows the distribution of the IAE obtained from the Monte Carlo simulations for all evaluated state variables. A consistent trend can be observed across the six variables. The FO-BSMC yields lower accumulated absolute error than the classical BSMC. This is reflected in the position of the boxplots, where the central tendency of the FO-BSMC results is systematically shifted toward smaller values. Moreover, the spread of the distributions is generally reduced, indicating a more uniform behavior across different initial conditions. This behavior suggests that the fractional-order controller not only improves overall tracking accuracy, as measured by accumulated squared error, but also yields a more consistent response across different initial conditions.

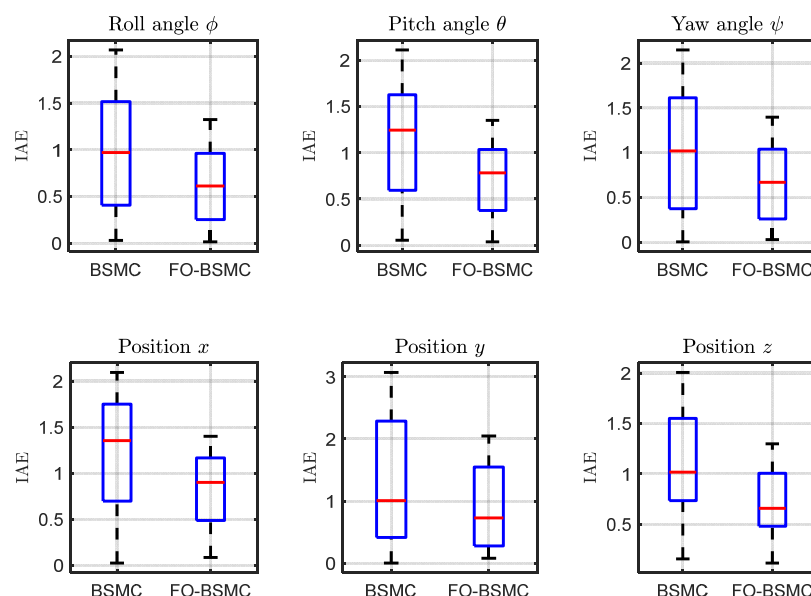


Figure 10. Monte Carlo distribution of IAE for BSMC and FO-BSMC under Case 2.

4.3. Case 3—Robustness to Noise

The third simulation scenario evaluates the robustness of the conventional BSMC and the proposed FO-BSMC when the available state measurements are affected by sensor noise. Unlike the previous cases, measurement uncertainty is introduced into all measurable state variables to emulate realistic operating conditions. Since the control actions are computed directly from the measured states, the injected noise propagates to the control inputs and directly influences the closed-loop response.

Measurement uncertainty is modeled as additive zero-mean Gaussian white noise independently applied to each measurable state variable. The generated noise has a standard deviation of $\sigma = 0.002$, corresponding to a variance of 4×10^{-6} , and remains statistically identical for all measurement channels. To ensure a fair comparison, the same realization of the noise sequence is applied to both the conventional BSMC and the proposed FO-BSMC throughout the simulations.

The QUAV parameters, controller gains, reference trajectories, and optimized fractional order are identical to those adopted in the previous simulation scenarios. Unlike Case 1, no external disturbances are introduced in this case. Consistent with Case 1, all state variables are initialized at zero, allowing the influence of measurement noise on the closed-loop performance to be assessed independently.

Figure 11 illustrates the trajectory-tracking performance of the QUAV under the measurement-noise scenario. As expected, the presence of measurement noise introduces fluctuations in both the attitude and position responses. For the attitude dynamics, the proposed FO-BSMC maintains the roll and pitch responses closer to their corresponding reference trajectories, exhibiting smaller deviations than the conventional BSMC. In the yaw response, both controllers provide comparable tracking performance, although the FO-BSMC preserves a slightly smoother response throughout the simulation. For the translational dynamics, both controllers successfully follow the prescribed reference trajectories despite the measurement disturbances. The differences are more evident in the vertical motion, where the proposed FO-BSMC exhibits smaller tracking deviations than the conventional BSMC, whereas the responses along the x - and y -axes remain similar, with a slight improvement achieved by the fractional-order controller.

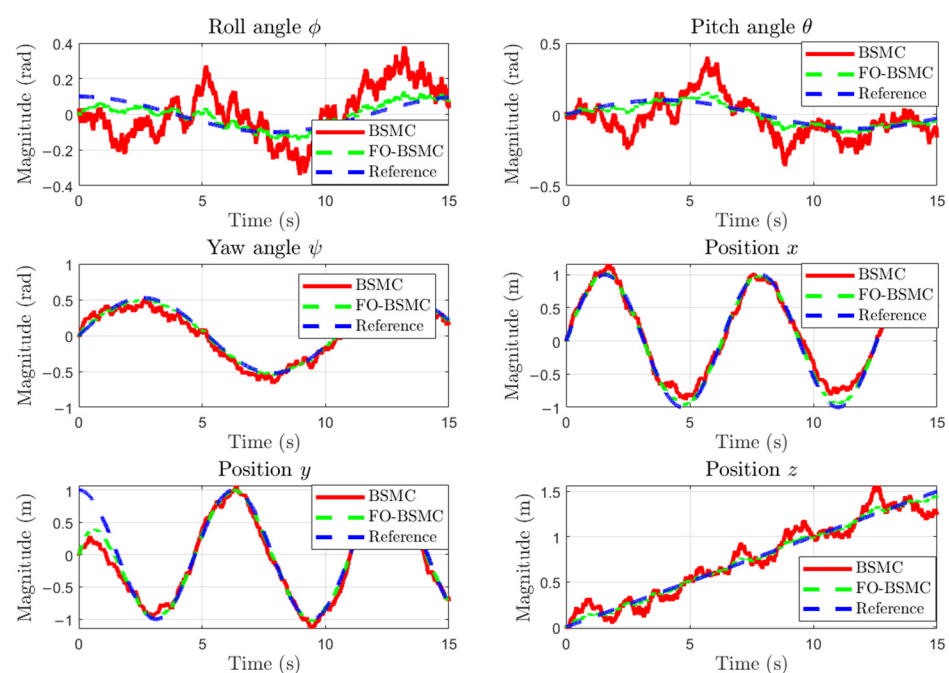


Figure 11. Attitude and position tracking of the QUAV under Case 3.

Figure 12 presents the three-dimensional trajectory-tracking results obtained under Case 3. Both controllers are able to follow the prescribed helical reference trajectory throughout the simulation, preserving its overall geometry despite the presence of measurement noise. Nevertheless, differences can be observed in the deviation from the reference trajectory. The path generated by the conventional BSMC exhibits larger deviations from the desired helix over most of the maneuver, whereas the trajectory produced by the proposed FO-BSMC remains closer to the reference. This difference is particularly noticeable in the upper turns of the helix, where the conventional BSMC departs further from the reference trajectory, while the FO-BSMC follows the desired path more closely.

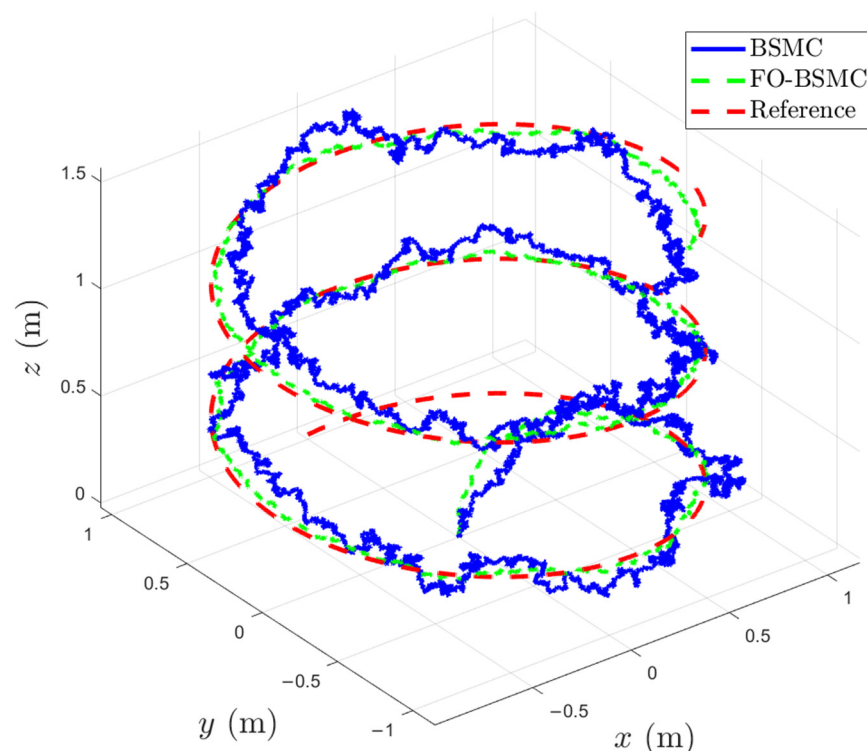


Figure 12. Three-dimensional trajectory of the QUAV under Case 3.

Figure 13 presents the tracking error signals under measurement noise. As expected, all errors fluctuate randomly around zero due to the stochastic perturbations in the feedback signals. Across the attitude variables, the FO-BSMC exhibits a clear reduction in error amplitude, with smoother and more confined variations compared to the classical BSMC, indicating an improved attenuation of noise-induced effects. A similar trend is observed in the z position. However, for the x and y positions, both controllers show comparable error amplitudes, suggesting that the benefit of the fractional-order formulation is less pronounced in these variables. Despite this, the overall error profiles generated by the FO-BSMC remain less oscillatory and more regular. These observations confirm that, in the presence of measurement noise, the fractional-order controller provides a more robust and stable error response.

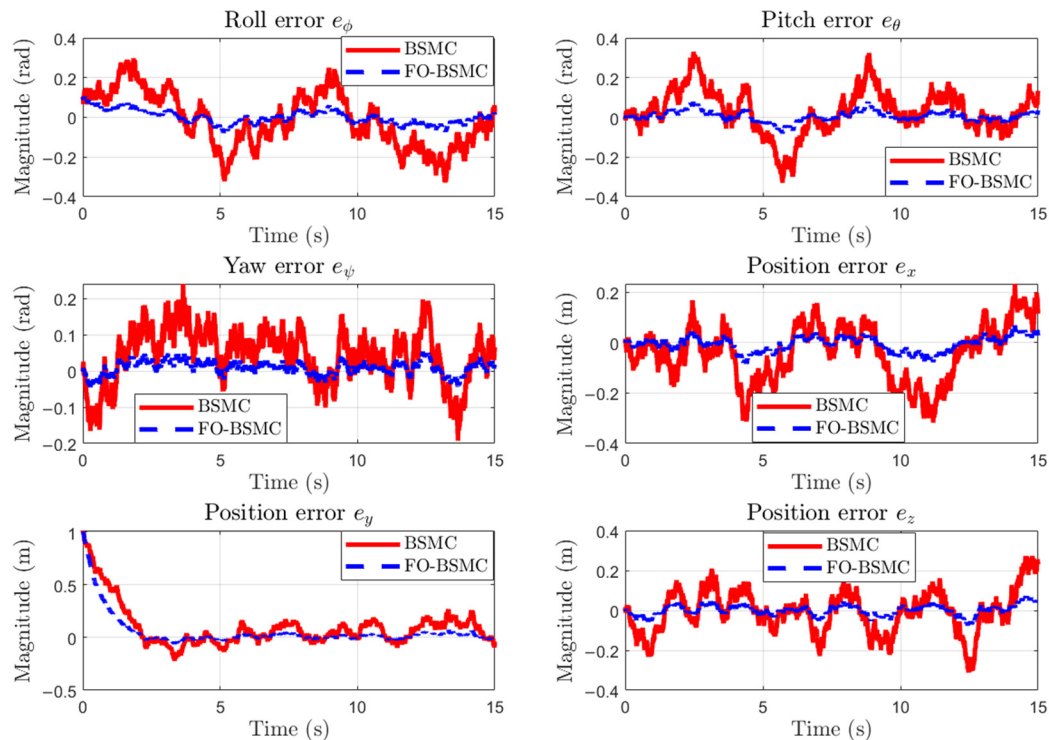


Figure 13. Position and attitude tracking errors of the QUAUV under Case 3.

Figure 14 depicts the control inputs generated by both controllers in the presence of measurement noise. Although the use of the smooth hyperbolic tangent function already mitigates high-frequency switching, noticeable differences can still be observed between the two control strategies. The control signals associated with the classical BSMC exhibit more pronounced oscillations. In contrast, the FO-BSMC produces visibly smoother control inputs, with a significant reduction in the amplitude of the fluctuations, which appears to be on the order of roughly half when compared to the classical approach.

The smoother control profiles observed for the FO-BSMC indicate that the fractional-order structure naturally mitigates the influence of measurement noise, likely due to its inherent memory effect. As a result, the control action exhibits fewer high-frequency oscillations while preserving the desired closed-loop behavior. From a practical perspective, the smoother control profiles produced by the FO-BSMC are expected to reduce actuator wear and unnecessary energy expenditure, making the proposed approach more suitable for real-world implementations where measurement noise and actuator limitations are unavoidable.

Figure 15 compares the control inputs generated by the conventional BSMC and the proposed FO-BSMC using the discontinuous sign function under Case 3. The discontinuous implementation makes the inherent switching behavior of sliding-mode control directly observable. Because measurement noise continuously perturbs the sliding variable, the switching activity is significantly intensified, allowing the chattering characteristics of both controllers to be clearly distinguished. The proposed FO-BSMC produces consistently smaller oscillation amplitudes in all four control channels than the conventional BSMC. It is important to note that this reduction in chattering is only observed in the noisy scenario; under the conditions considered in Cases 1 and 2, both controllers exhibit similar high-frequency switching, making a meaningful visual comparison impractical.

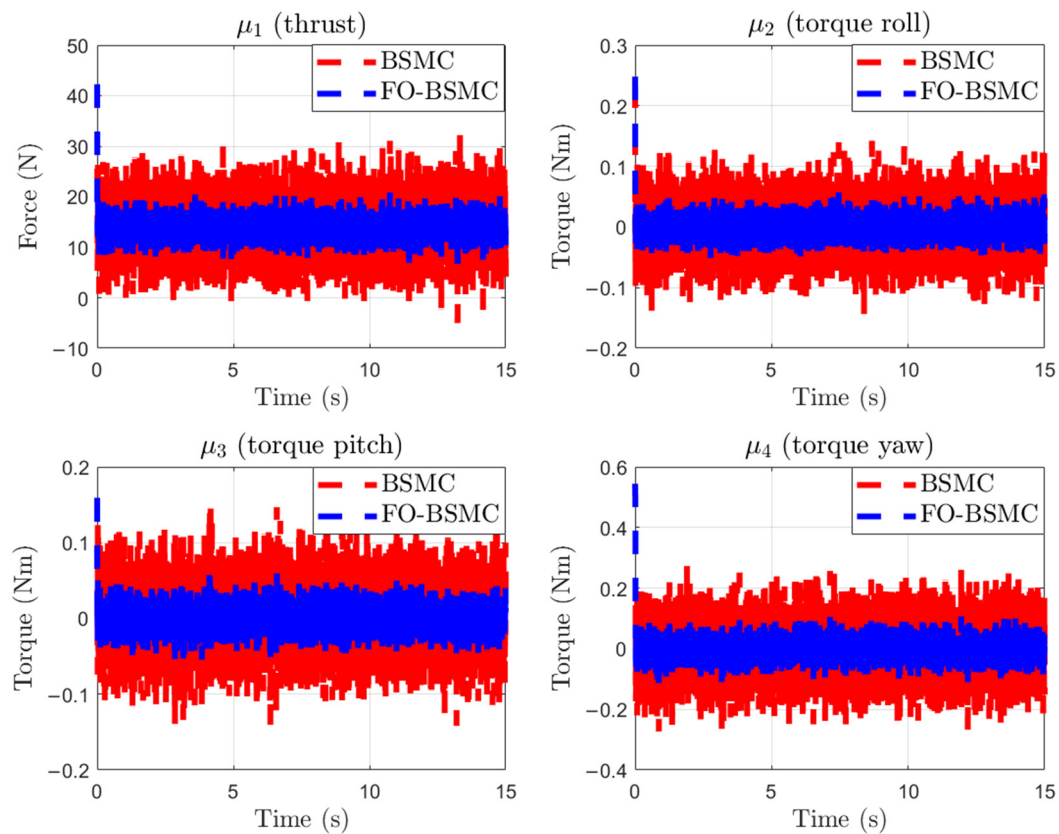


Figure 14. Control inputs using a smooth tanh-based sliding function under Case 3.

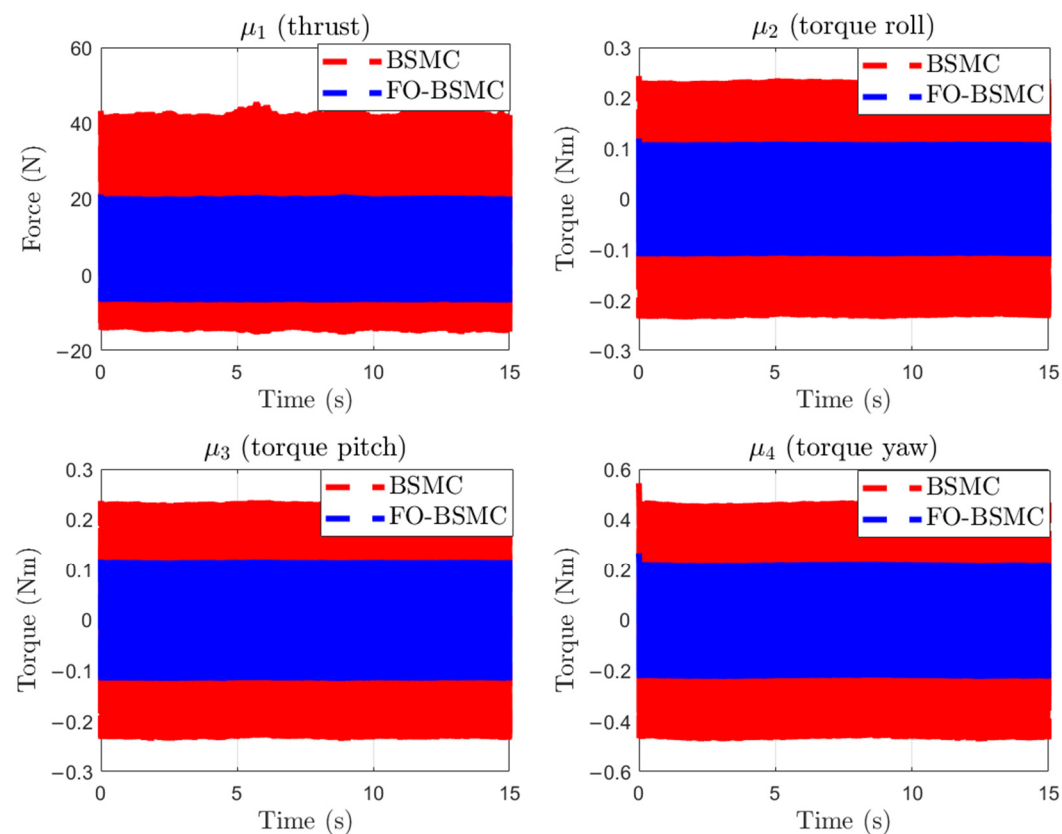


Figure 15. Control inputs using the discontinuous sign-based sliding function under Case 3.

Table 4 summarizes the tracking error indices obtained under Case 3. The proposed FO-BSMC consistently produces lower ISE and IAE values than the conventional BSMC for all attitude and position errors in the presence of measurement noise. For the rotational dynamics, the reductions observed in the roll, pitch, and yaw responses indicate that the proposed controller is less affected by measurement disturbances, resulting in smaller accumulated tracking errors throughout the simulation. A similar trend is observed in the translational dynamics, where the proposed controller also yields lower error indices along the three position axes. The largest improvements are obtained for the x - and z -axis motions, whereas the reductions along the y -axis are comparatively smaller. These results indicate that the proposed fractional-order formulation preserves more accurate trajectory tracking under noisy operating conditions.

Table 4. Tracking error indices (ISE and IAE) under Case 3.

State Variable	Description	BSMC		FO-BSMC	
		ISE	IAE	ISE	IAE
$x_1 = \phi$	Roll angle	0.286669	1.773887	0.018587	0.432239
$x_3 = \theta$	Pitch angle	0.197396	1.312820	0.010623	0.305442
$x_5 = \psi$	Yaw angle	0.111458	1.104432	0.006371	0.259266
$x_7 = x$	Position (x -axis)	0.209387	1.401083	0.013507	0.357956
$x_9 = y$	Position (y -axis)	0.717402	1.961954	0.377345	0.943215
$x_{11} = z$	Position (z -axis)	0.169467	1.245493	0.010892	0.323898

5. Conclusions

This paper presented an FO-BSMC strategy for trajectory tracking of a QUAV. Unlike conventional fractional-order sliding mode approaches, the proposed methodology incorporates the fractional-order behavior directly into the QUAV dynamic model while preserving the original BSMC structure. The fractional order was selected using the PSO algorithm and employed throughout the comparative evaluation.

The comparative study demonstrated that the proposed FO-BSMC provides improved robustness under external disturbances, large initial-condition variations, and measurement noise while maintaining accurate trajectory-tracking performance. Across the evaluated scenarios, the proposed controller consistently reduced the tracking errors and improved the overall closed-loop performance compared with the conventional BSMC. The most significant improvements were observed in the presence of external disturbances and perturbed initial conditions, where the FO-BSMC exhibited faster convergence, lower ISE and IAE values, and a more consistent closed-loop response, as further confirmed by the Monte Carlo analysis. Under measurement noise, the proposed controller also reduced the effect of noise on the tracking errors while producing smoother control actions with reduced chattering.

The obtained results indicate that introducing fractional-order dynamics into the QUAV model constitutes an effective alternative for enhancing the robustness of the conventional BSMC without modifying its original control structure. The observed performance improvements are therefore attributed to the fractional-order dynamic representation rather than to changes in the tracking-error formulation, sliding surfaces, or reaching laws.

Although the proposed formulation has been validated through extensive numerical simulations, experimental verification remains an important step toward assessing its practical applicability. Future work will focus on real-time implementation, adaptive online selection of the fractional order, and the evaluation of the proposed approach

under additional sources of uncertainty, including parameter variations and actuator faults.

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