



Article

Laser-Spot Step-Heating Thermography for Non-Destructive Evaluation of Thermal Diffusivity in Apples

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Abstract

In this work, thermal imaging is employed to study the opto-thermal response of apples (*Malus domestica* Borkh.), assessing their post-harvest evolution through the estimation of thermal diffusivity. A non-destructive experimental procedure based on mid-wave infrared (MWIR) thermal camera (3–5 μm) and localized heating with a visible laser is developed, enabling spatially and temporally resolved surface temperature measurements. Temperature fields are recorded at different time points and radial distances from the heated spot. A theoretical model based on Fourier thermal diffusion equation is formulated to describe the spatio-temporal evolution of surface temperature. After validation on a reference sample, the method is applied to Golden and Red Delicious apples over a 28-day storage period at room temperature. Red Delicious apple exhibits higher mean diffusivity values without significant temporal changes, whereas a progressive increase in diffusivity is observed for Golden Delicious apples. These results show that thermal diffusivity is sensitive to post-harvest physiological changes in apple tissue and may be associated with intrinsic properties such as tissue density and water content. By relating laser-induced temperature fields to the estimation of thermal diffusivity, this approach enables the non-destructive, quantitative assessment of thermal diffusivity, showing potential for fruit maturity and quality assessment, which are of high importance in agri-food monitoring applications.



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1. Introduction

Non-destructive techniques for monitoring food quality, safety and security are highly desirable in the modern food industry and the agricultural sector. As food processing faces challenges in the context of the Fourth Industrial Revolution [1], researchers have been working on time- and cost-efficient, user-friendly and rapid ways to monitor quality at different points of the production chain [2]. Imaging-based non-destructive approaches

such as multispectral, hyperspectral and thermal imaging [3–7] are able to relate the information obtained from direct analysis of bioactive compound profile to the stage of ripeness, freshness, and shelf life of plant food. Specifically, thermal imaging, i.e., monitoring temperature, is fundamental in agri-food processes, as the chemical and structural properties of foods define their thermal properties. Thermal imaging is based on mapping the infrared radiation, which is naturally emitted as heat by an object due its temperature; therefore, unlike near-infrared imaging/spectroscopy, thermal imaging does not require an external light source, and nor does it depend on the ambient lightning. It allows for rapid, non-destructive and contactless measurements of temperature distribution of the whole sample over time. Modern thermal imaging instrumentation allows for repeatable, simple and long-term monitoring in different environments. The simplicity and practicability of the measurement protocols and data analysis in thermal imaging have already paved the way to industrial applications from medicine to the agricultural sector [7,8]. Furthermore, when needed, thermal imaging can be complemented with additional, more expensive and sophisticated non-destructive techniques to go beyond the temperature monitoring of the sample's surface: nowadays, researchers can retrieve precise material information from unique spectral signatures with hyperspectral imaging [9] or with acoustic vibration technology [10]. Finally, state-of-the-art photonics [11] and data processing integrated with super-resolution artificial intelligence algorithms [12] are expected to further boost the quality of thermal images.

While raw thermal images show the sample's surface temperature, data analysis in thermal imaging allows for the extraction of a more quantitative and material-specific thermal parameter: thermal diffusivity. This physically meaningful property is related to heat transport mechanisms, which are governed by food density, water content and tissue structure. Unlike purely empirical features extracted from thermal images, thermal diffusivity is a material property with a well-defined physical meaning related to heat transport in the tissue. As a result, diffusivity-based metrics are inherently more interpretable and potentially more suitable for comparison across different measurement setups, fruit varieties, and environmental conditions. Retrieving thermal diffusivity from thermal imaging experiments via model-based fitting shows great promise for food quality control over time.

In the agri-food sector, thermal imaging has a long history of proposed applications in assessing fruit quality, maturity, and the presence of defects, due to its ability to capture spatially resolved temperature patterns without physical contact [8,13–15]. Most existing approaches rely on empirical features extracted from thermal images—such as temperature contrasts, cooling rates, or statistical descriptors—which are subsequently correlated with quality attributes through chemometric or machine-learning techniques [14–17]. While these methods have demonstrated promising performance, they generally lack a direct connection to fundamental thermophysical properties of the biological tissue.

In this context, the development of thermographic methods that can directly measure the thermophysical properties of biological tissues is of particular interest. Specifically, thermography allows for the estimation and monitoring of thermal diffusivity, which is intrinsically connected with heat transport in food. Recent work by Lee et al. [18] demonstrated that thermal responses measured during fruit heating and cooling can be correlated with moisture content in apples; however, the proposed approach relies on empirical diffusivity-related indices rather than on the direct estimation of a fundamental thermophysical property. A model-based determination of thermal diffusivity would therefore represent a significant step forward, enabling a quantitative and interpretable link between thermal measurements and fruit physiological state.

In the literature, several thermography techniques have been used to measure the thermal diffusivity of solids, among which lock-in thermography and pulsed thermography are the most widely employed [19–21]. These approaches rely on focused laser excitation—either harmonically modulated or delivered as a high-power, short-duration pulse—while the resulting surface temperature evolution is recorded with an infrared camera. Although these methods provide accurate diffusivity measurements, they require expensive, delicate, and potentially hazardous equipment, which is not suitable for in situ measurements in industrial agri-food environments. In this context, step-heating thermography, which uses continuous-wave (CW) laser excitation over a finite time interval combined with temperature monitoring over time through an infrared camera, represents a promising alternative [22]. This approach enables a significantly simpler and more cost-effective experimental setup that can be readily implemented with minimal technical expertise. The accuracy of step-heating thermography for thermal diffusivity estimation across a wide range of materials, including polymers, metals, glass, and graphite, has been demonstrated by Salazar et al. [22].

In this framework, the present work proposes a laser-spot thermography approach to quantitatively measure the thermal diffusivity of apples. Localized surface heating is achieved by means of a continuous-wave laser applied for 1 min, followed by a natural cooling phase of equal duration after the laser is switched off. The temporal evolution of the surface temperature during both heating and cooling stages is monitored using a mid-wave infrared camera. A theoretical model based on Fourier thermal diffusion equation is developed, providing a fully analytical description of the transient thermal response during laser excitation and subsequent cooling. Thermal diffusivity is retrieved through analytical fitting of the experimental data using two distinct approaches. The validity of the proposed model is first assessed on a reference polymer sample, and subsequently applied to two varieties of apples, i.e., Red Delicious and Golden Delicious. The results suggest time-dependent changes in both the apple matrix and skin associated with dehydration, which may be related to the reduced material density and, consequently, thermal diffusivity. Finally, differences in the mean thermal diffusivity between the two apple varieties are reported, indicating that intrinsic fruit characteristics play a key role in determining their thermal response.

2. Materials and Methods

2.1. Materials

The evaluation of the thermal diffusivity of apples (*Malus domestica* Borkh.) was conducted on two varieties commonly cultivated in Italy: Golden Delicious and Red Delicious. A total of 6 apples (3 fruits in each cultivar) were sourced from local retail markets because they represent fruits typically available to consumers under conventional post-harvest conditions. More precisely, the apples had the Melinda consortium brand (Trento, Italy), which guarantees the origin and adherence to strict production, harvesting and storage standards. However, information regarding orchard management practices, harvest timing, and the physiological ripeness of the apples at acquisition was not accessible. At the time of acquisition, no instrumental measurements of maturity proxies such as firmness, total soluble solids (TSS), or color index were performed. Nevertheless, to reduce variability, fruits were chosen to be as uniform as possible in terms of size (diameter range: 72–74 mm for Golden Delicious and 77–80 for Red Delicious), weight (range: 180–200 g for Golden Delicious and 180–220 for Red Delicious), and absence of visible external defects (e.g., bruises, cuts, or signs of rot). Apples showing any mechanical or biological damage were excluded. No additional sorting procedures were applied beyond these criteria. As a reference sample, a 6 mm thick commercial poly(vinyl chloride) (PVC) specimen

supplied by RS Pro (RS Components, Corby, UK) was used. The thermal diffusivity of the polymer sample was independently measured using a Modified Transient Plane Source (MTPS) technique with a TCi instrument provided by C-Therm Technologies (Fredericton, NB, Canada).

2.2. Thermal Imaging Experimental Set-Up

A schematic representation of the experimental thermography setup is provided in Figure 1. A continuous-wave (CW) laser source (405 nm, 12 mW) is focused onto the sample surface by means of a 150 mm focal-length lens, producing a localized hot spot. The laser beam has a Gaussian profile and is focused on the sample surface to a radius a , obtained from the spatial temperature distributions by fitting Equation (4), as shown below in Section 2.3.1. The corresponding irradiated area ($a^2\pi$) yields an average power density at the sample surface of approximately 0.5 W cm^{-2} . The laser irradiation is applied for 60 s, after which the beam is switched off to allow the specimen to cool naturally to an ambient temperature. Surface temperature measurements are acquired using a mid-wave infrared (MWIR) camera (FLIR X6981, 3–5 μm , kindly provided by Teledyne FLIR Solutions, Limbiate, MB, Italy) positioned at an angle of 30° with respect to the surface normal, due to experimental constraints imposed by the in-plane laser geometry, which would partially obscure the field of view at normal incidence. According to the literature, angular emissivity effects at this angle are negligible [23]. As the oblique viewing geometry introduces a geometric foreshortening along the vertical direction, spatial temperature profiles were extracted exclusively along the horizontal axis, perpendicular to the plane of incidence, which is not affected by projection effects.



Figure 1. Schematic representation of the IR thermography setup.

Surface emissivity, ambient temperature, and reflected temperature were set as input parameters for thermographic data processing. An emissivity value of 1 was assumed to provide a consistent apparent temperature scale across all measurements. Since the thermal diffusivity extraction is based on the spatial and temporal evolution of the temperature field and on normalized temperature profiles, the assumed emissivity affects only the absolute temperature levels and is not expected to introduce a significant bias in the estimated diffusivity values. Reflected temperature was measured at the start of each measurement session using the standard reflector method recommended in the FLIR X6981 user manual and ISO 18434-1:2008 [24], averaging the apparent temperature over a crumpled aluminum foil placed adjacent to the sample, facing the camera at the same distance and angle, with emissivity set to 1. Measurements on apple samples were conducted over a 28-day period (Days 0, 4, 8, 12, 15, 18, and 28), during which the apples were stored at room temperature

(25 °C ± 1 °C) to allow progressive natural maturity. The same fruits were repeatedly measured throughout the entire period at two predetermined locations on the fruit surface, which were consistently measured throughout all experimental sessions. Three apples were measured for each cultivar. For each measurement day, the reported thermal diffusivity corresponds to the average of the two locations, and data are reported as the mean ± SD.

2.3. Step-Heating Thermography for Thermal Diffusivity Estimation

2.3.1. Thermography Diffusivity Estimation Background: Theoretical Model and Analytical Solution

The estimation of the thermal diffusivity relies on an analytical solution of the heat diffusion equation describing the spatio-temporal evolution of the temperature field generated by a laser spot with a Gaussian spatial profile. The sample is modelled as a homogeneous and isotropic, opaque, thermally thick medium, and the thermal properties are assumed constant. Heat is assumed to be deposited at the surface ($z = 0$) for a finite duration τ , which is modelled as a temporal step function. The heat diffusion equation is expressed as:

$$\nabla^2 T - \frac{1}{D} \frac{\partial T}{\partial t} = -\frac{w}{k} f(t) \quad (1)$$

where D is the thermal diffusivity, k the thermal conductivity, w the peak volumetric power density, and $f(t)$ the temporal shape of the laser excitation (step-like over the interval $[0, \tau]$).

Applying the Laplace transform in time and a double Fourier transform in the x - and y -directions (see Appendix A), the equation becomes:

$$\left(-(\delta_x^2 + \delta_y^2) + \frac{\partial^2}{\partial z^2} \right) \bar{\bar{T}} - \frac{s}{D} \bar{\bar{T}} = -\frac{\bar{\bar{q}}}{k} \left(\frac{1 - e^{-s\tau}}{s\tau} \right) \quad (2)$$

Next, we consider $\delta^2 = \delta_x^2 + \delta_y^2$, a is the beam spot size, and $q_{\max}$ the maximum surface energy density deposited by the laser beam. Enforcing the boundary condition imposed by the surface heat flux yields the transformed temperature field:

$$\bar{\bar{T}} = q_{\max} \frac{a^2 \pi}{2k\tau} e^{-\frac{\delta^2 a^2}{8}} \cdot \frac{1 - e^{-s\tau}}{s \sqrt{\frac{s}{D} + \delta^2}} \cdot e^{-\sqrt{\frac{s}{D} + \delta^2} \cdot z}. \quad (3)$$

The inversion of the Laplace and Fourier transforms leads to two compact expressions for the surface temperature $T(r, t)$, corresponding to the heating phase ($t < \tau$) and the subsequent cooling phase ($t \geq \tau$):

$$\begin{cases} T_1(r, t) = \frac{q_{\max} a^2}{4\tau k} \int_{-\infty}^{+\infty} J_0(\delta r) \cdot e^{-\frac{\delta^2 a^2}{8}} \cdot \text{erf}(\sqrt{\delta^2 D t}) d\delta \\ T_2(r, t) = \frac{q_{\max} a^2}{4\tau k} \int_{-\infty}^{+\infty} J_0(\delta r) \cdot e^{-\frac{\delta^2 a^2}{8}} \cdot \left[\text{erf}(\sqrt{\delta^2 D t}) - \text{erf}(\sqrt{\delta^2 D (t - \tau)}) \right] d\delta \end{cases} \quad (4)$$

These expressions constitute the forward model used to fit the measured thermographic data and estimate the thermal diffusivity. All intermediate steps and mathematical details are provided in Appendix A.

2.3.2. Thermal Diffusivity Computation

Thermal diffusivity was estimated by fitting the measured spatio-temporal surface temperature distributions to the analytical solution of the heat diffusion equation derived in Section 2.3.1. (Equation (4)). The sample was modelled as a homogeneous, isotropic, opaque, and thermally thick medium, with heat deposited at the surface ($z = 0$) by a

Gaussian laser beam for a finite duration $\tau = 60$ s. The surface was assumed to be planar and laterally infinite in the x–y directions, consistent with the analytical formulation.

Convective and radiative heat losses at the surface were neglected during the fitting time window. As reported by Salazar et al. [22] for opaque and thermally thick samples, the surface temperature evolution induced by laser step heating is dominated by heat diffusion into the bulk, while convective and radiative losses have a negligible influence over the time scales relevant to the present analysis. The fitting procedure included both the heating phase ($t \leq 60$ s) and the cooling phase ($t \geq 60$ s) over a total measurement window of 120 s, to ensure a robust estimation of D and reduce sensitivity to the assumed spot size.

The fitting was performed by minimization of the root mean square error (RMSE) of the residuals between the experimental and modelled temperature profiles, defined as:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N [T_{exp,i} - T_{model,i}]^2}$$

where $T_{exp,i}$ and $T_{model,i}$ are the experimental and modelled temperatures, respectively. The thermal diffusivity D (and, when applicable, the laser spot radius a) were treated as free parameters. Initial conditions correspond to a uniform temperature equal to the ambient temperature prior to laser excitation.

2.3.3. Temporal Evolution in Thermographic Imaging

Figure 2a shows a representative thermographic image of the hot spot generated on a 6 mm thick reference specimen, whose thickness was selected to ensure thermally thick behavior under the adopted experimental conditions. This material was used as a homogeneous reference sample to validate measurement and fitting procedures, as its thermal diffusivity ($D \sim 1.7 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$) fall within the same order of magnitude as those reported for apples [7,8]. The corresponding spatial temperature distribution extracted along a selected line is shown in Figure 2b. The spatial profile at fixed time is well approximated by a Gaussian distribution, with a pronounced maximum at the beam incidence point and a smooth radial decay.

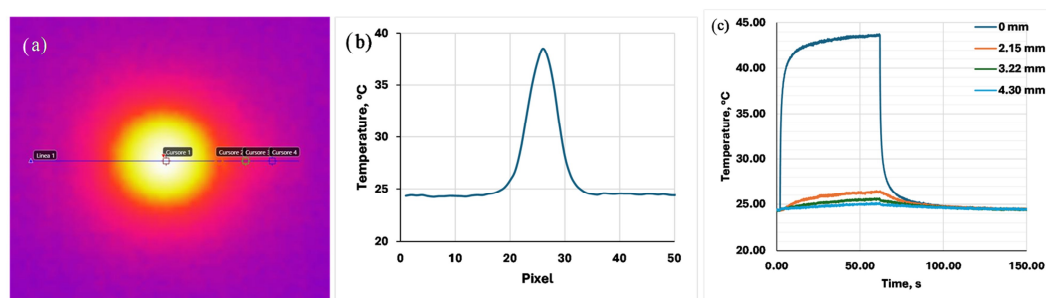


Figure 2. (a) Representative thermographic image of the hot spot generated on the polymer sample surface displaying a horizontal line passing through the center of the spot and four points located at distances of 0 mm, 2.15 mm, 3.22 mm, 4.30 mm from the spot center; (b) corresponding spatial temperature profile along the horizontal line; (c) temperature evolution over time at various distances from the center of the spot (0 mm, 2.15 mm, 3.22 mm, 4.30 mm).

Figure 2c illustrates the temporal evolution of the temperature at different radial distances from the center of the heated spot (0 mm, 2.15 mm, 3.22 mm, 4.30 mm). The temporal evolution (Figure 2c) reveals the expected monotonic increase during the heating phase, characterized by a steep rise at short times followed by a plateau around 60 s. At the center of the spot, the laser excitation produces a maximum temperature rise of

approximately $\Delta T \approx 19.5^\circ\text{C}$ at the end of the heating phase ($t = 60$ s). After the laser is switched off, the temperature undergoes a fast initial drop followed by a slower long-term decay as the system returns to ambient conditions.

The noise level of the thermographic signal, estimated from temperature fluctuations in steady-state conditions, is approximately $\pm 0.05^\circ\text{C}$. Repeated measurements performed after complete thermal equilibration of the sample to ambient temperature showed variations in the measured temperature profiles within the experimental noise level, indicating the good repeatability of the thermographic measurements.

2.4. Spatio-Temporal Temperature Profiling Calculations

Following the theory developed in Section 2.3, Equation (4) is used to calculate the temporal and radial profiles of temperature for an opaque and thermally thick sample illuminated by a focused laser beam for $\tau = 60$ s. Figure 3 reports the radial profiles of temperature with varying input parameters, namely the thermal diffusivity D , the time instant t from the onset of laser excitation, and the spot size a . As shown in Figure 3a, at the end of the heating phase ($t = 60$ s), variations in D primarily affect the temperature distribution at larger radial distances from the beam center, where the temperature tends to remain nearly constant. Conversely, at an intermediate time during heating ($t = 6$ s, Figure 3b), the radial profile preserves its overall shape, while the temperature peak increases systematically with increasing diffusivity. During the cooling phase (Figure 3c, $t = 66$ s), changes in D affect both the maximum temperature at the center of the spot—whose value decreases as D increases—and the width of the radial distribution, which becomes progressively broader for higher diffusivity values. These behaviors are consistent with the physical expectation that materials with higher thermal diffusivity redistribute heat more rapidly and over larger distances from the excitation region [18,25]. Figure 3d,e further illustrates the temporal evolution of the radial profile during heating and cooling, respectively, for a fixed thermal diffusivity of $1 \times 10^{-7} \text{ m}^2/\text{s}$. During the heating phase (Figure 3d), the spatial shape of the profile remains essentially unchanged, while the peak temperature increases with time due to the cumulative heat input. After the laser source is switched off (Figure 3e), the peak temperature decreases and the profile progressively broadens, reflecting radial heat diffusion. Finally, Figure 3f shows the influence of the spot size a on the radial temperature distribution. As expected, larger values of a yield broader Gaussian-like profiles, while the maximum temperature remains essentially unchanged.

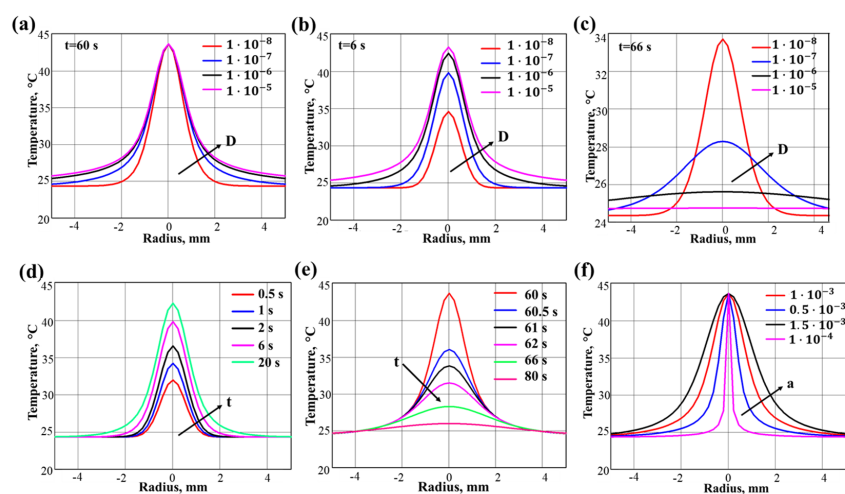


Figure 3. Calculations of the radial temperature profiles obtained from Equation (4) with varying (a–c) thermal diffusivity D (m^2/s), (d,e) time instants (s), (f) spot size a (m). Unless otherwise specified, the calculations have been performed with the parameters $D = 1 \times 10^{-7} \text{ m}^2/\text{s}$, $a = 10^{-3} \text{ m}$, $t = 60$ s.

3. Results

3.1. Thermal Diffusivity Extraction for Reference Polymer Sample

To obtain reliable thermal diffusivity values from thermographic measurements, Equation (4) must be fitted to the experimental $T(r,t)$ profiles with D treated as a free parameter. Two distinct approaches were studied for the analytical fitting of the experimental data: (i) fitting of the temporal temperature profile; (ii) fitting of the radial temperature profile. To ensure a more consistent diffusivity estimate, multiple temporal (radial) profiles recorded at different time instants (radial positions) were fitted simultaneously.

In the first approach the temporal evolution of temperature was modelled at different radial distances from the center of the hot spot (0 mm, 2.15 mm, 3.22 mm, 4.30 mm), as shown in Figure 2a,c. In this case, a fixed value of the spot size a was used, determined by applying a Gaussian fit to the spatial temperature profile across a horizontal line (Figure 2a,b) after 2 s of laser excitation. The Gaussian fit was carried out using the least squares method, while the thermal diffusivity (D) was obtained by minimizing the root mean square error (RMSE) between experimental and modelled temperature profiles. To improve the fitting accuracy, the experimental and theoretical temperature profiles were corrected by subtracting the initial temperature at each location and then normalized by dividing by the similarly corrected profile at the center of the spot (0 mm), according to Equation (5):

$$\tilde{T}(r, t) = \frac{T(r, t) - T_0(r)}{T(r = 0, t) - T_0(r = 0)} \quad (5)$$

The RMSE was therefore computed using these normalized temperature profiles. This normalization removes amplitude-related effects (e.g., local emissivity variations and laser power fluctuations), allowing the fitting to rely exclusively on the spatial and temporal dynamics of heat diffusion.

Figure 4 reports the experimental temporal temperature profiles (Figure 4a–c) obtained on the reference sample and the corresponding normalized profiles (Figure 4d) at the various radial locations, together with the fitted curves and the RMSE plot (Figure 4e) used to determine the thermal diffusivity. The fitting yielded to a thermal diffusivity value of $1.74 \times 10^{-7} \text{ m}^2/\text{s}$ and a size of the hot spot equal to $1.12 \times 10^{-3} \text{ m}$. Although the analytical model reproduces the overall qualitative behavior, clear discrepancies are evident between the experimental and fitting profiles, particularly for radial distances $> 3 \text{ mm}$. These deviations appear both during the heating and cooling phases and may be related to an inaccurate estimation of the spot size.

To quantify the influence of the fitting approach on the estimated thermal diffusivity, Table 1 reports a comparison of D values obtained by fitting exclusively the heating portion, exclusively the cooling portion, or the combined heating–cooling profiles. The table also shows the variations of the estimated diffusivity with $\pm 10\%$ variations in the assumed spot size a . These results clearly indicate that inclusion of the cooling phase or the full transient evolution is essential to reduce the sensitivity of D to the spot size and stabilize the estimation, providing a more reliable determination of the thermal diffusivity.

While fitting only the temporal evolution provides a useful preliminary estimate, the sensitivity to the assumed spot size and the limited spatial information motivated the implementation of a second fitting approach in which the spatial temperature distribution along a horizontal line (indicated in Figure 2a) was modelled. In this case, both the thermal diffusivity and the value of the spot size a were treated as fitting parameters and a larger number of experimental data points were incorporated to enhance the accuracy and reliability of the fitting process. Specifically, the temperature profiles were fitted at 29 time points spanning both the heating and cooling phases. Figure 5 presents the results obtained using this method on the reference sample, showing the experimen-

tal radial temperature profiles together with the corresponding fitting curves, as well as the RMSE plots for the determination of the spot size and thermal diffusivity. This approach produced a thermal diffusivity value of $1.79 \times 10^{-7} \text{ m}^2/\text{s}$ and a spot size value of $8.78 \times 10^{-4} \text{ m}$. Despite the small variation in the thermal diffusivity values yielded by the two approaches, a more reliable estimation of the spot size a is obtained with the second method, arising from the simultaneous determination of D using multiple spatial profiles. The spatial approach reproduces the experimental profiles within $\sim 1\%$ of the total temperature variation (minimum RMSE $\approx 0.188 \text{ }^\circ\text{C}$) compared to $\sim 3\%$ average deviation for the temporal fitting (minimum RMSE ≈ 0.031) on normalized profiles, indicating a more reliable fitting when spatial information is considered. This indicates better agreement between the model and experimental data, providing a more precise and robust determination of the thermal diffusivity and spot size.

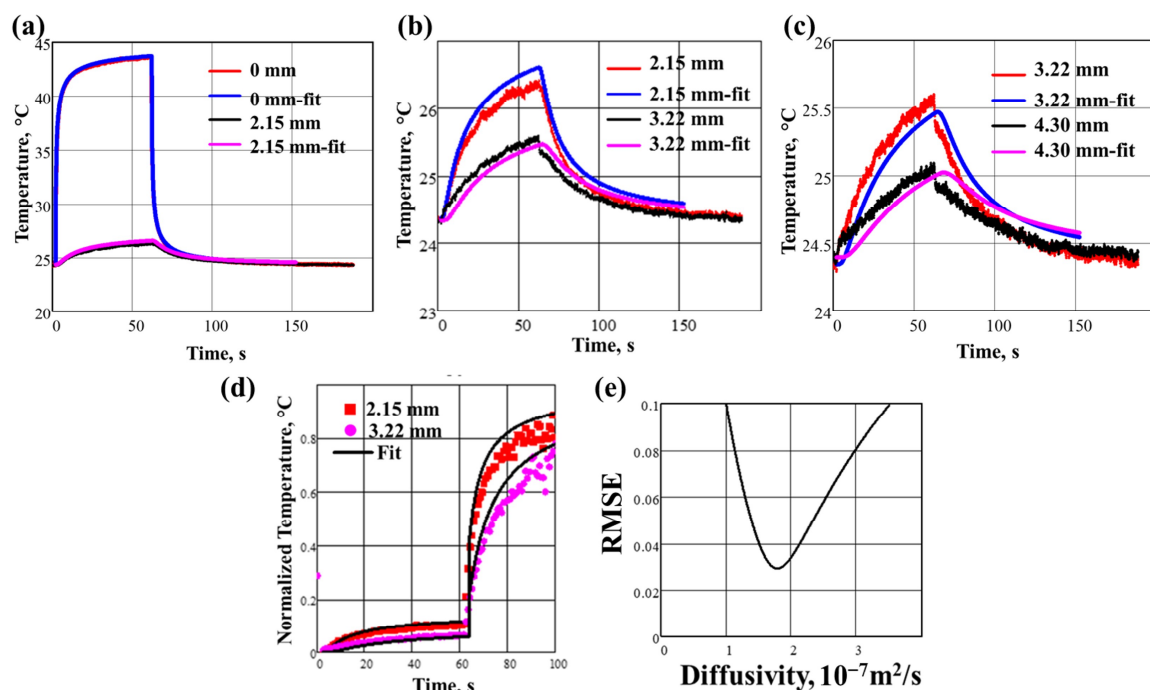


Figure 4. (a–c) Experimental temporal temperature profiles at different radial distances from the hot spot center and associated analytical fitting curves. (d) Normalized temperature profiles with corresponding fitting curves. (e) RMSE plot for the thermal diffusivity estimation.

Table 1. Thermal diffusivity D (m^2/s) for the polymer sample as a function of the assumed spot size using the temporal fitting approach. The nominal spot size is $a = 1.12 \text{ mm}$, determined from the Gaussian fit of the spatial temperature profile after 2 s of laser excitation, while the other values correspond to $\pm 10\%$ variations.

Spot Size a	Heating Only (D) (m^2/s)	Cooling Only (D) (m^2/s)	Heating + Cooling (D) (m^2/s)
1.12 (nominal)	1.38×10^{-7}	1.74×10^{-7}	1.74×10^{-7}
1.01 (−10%)	1.80×10^{-7}	1.76×10^{-7}	1.76×10^{-7}
1.23 (+10%)	1.08×10^{-7}	1.73×10^{-7}	1.71×10^{-7}

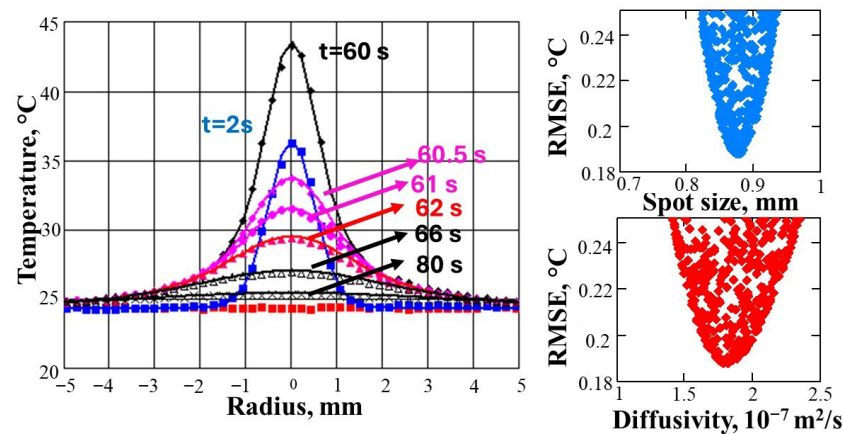


Figure 5. Radial temperature profiles of the polymer sample with fitting curves and RMSE plots for spot size and thermal diffusivity. Colors of the radial profiles correspond to the experimental data points and fitted curves, and arrows indicate the corresponding time instants.

3.2. Thermal Diffusivity Extraction for Golden Delicious and Red Delicious Apples

The experimental procedure developed in this work was applied to measure the thermal diffusivity for Golden Delicious and Red Delicious apples over a 28-day period, showcasing its potential for the non-destructive assessment of fruit maturity and quality. Figure 6a gives representative thermographic images, illustrating the evolution of the hot spot on a Red Delicious apple during the heating and cooling phases at different time instants. Figure 6b shows the experimental radial temperature profiles along with the corresponding fitting curves obtained using the second modelling approach, as well as the RMSE plots for both the spot size and the thermal diffusivity. The fits exhibit excellent agreement with the measured temperature distributions, confirming the robustness of the thermal diffusivity estimation procedure.

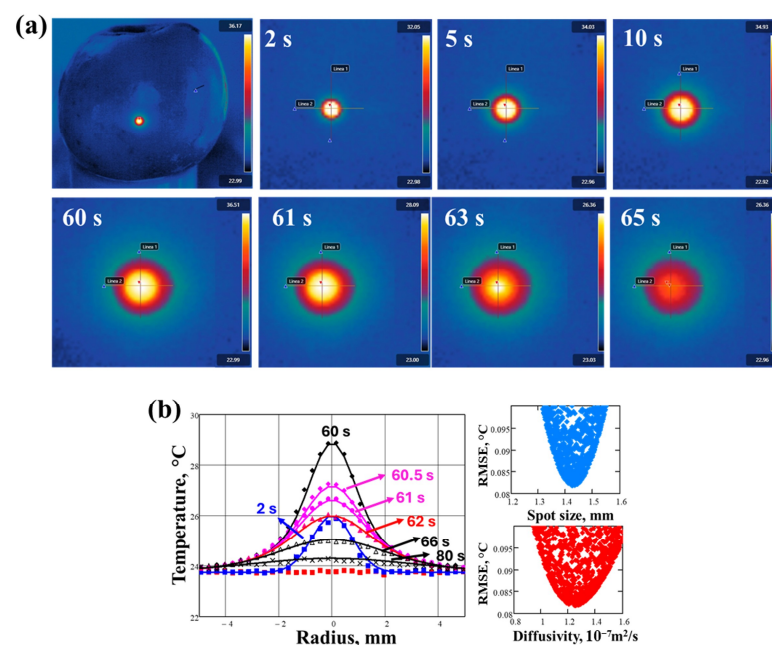


Figure 6. Thermographic characterization of a Red Delicious apple, showing (a) representative thermal images of the laser-induced hot spot evolution during heating and cooling at different time instants, and (b) radial temperature profiles with corresponding fitting curves, along with RMSE plots used to determine the spot size and thermal diffusivity. Colors of the radial profiles correspond to the experimental data points and fitted curves, and arrows indicate the corresponding time instants.

Figure 7 illustrates the temporal evolution of thermal diffusivity throughout the 28-day storage period, accompanied by photographic images of the apples on the first and last day of measurements. Each data point represents the average of measurements taken at two surface locations. The results, summarized in Table 2, reveal variations in thermal diffusivity that depend on both storage time and apple variety.

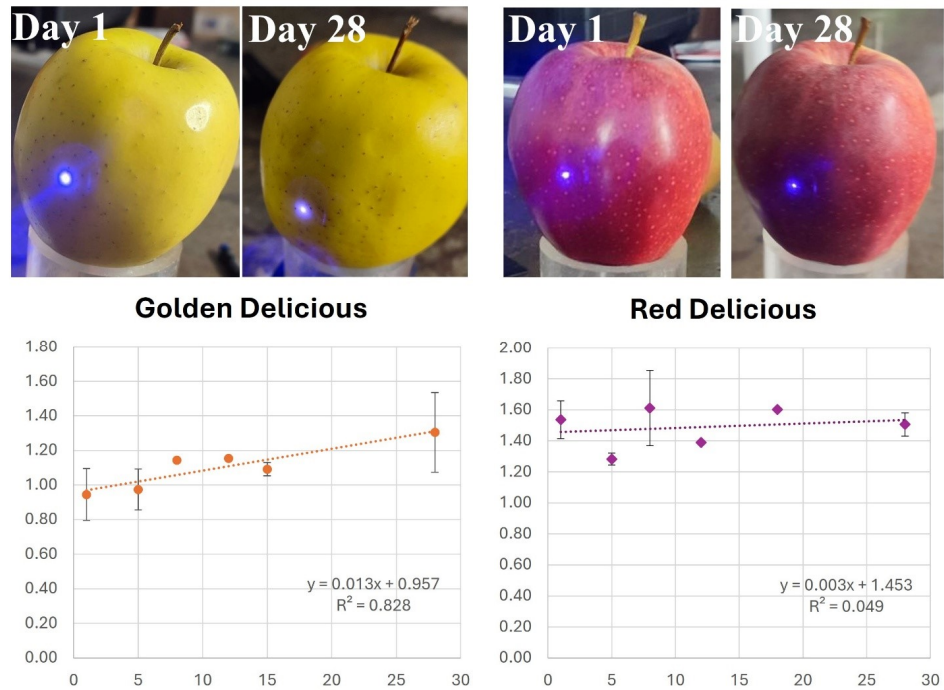


Figure 7. Temporal evolution of thermal diffusivity for Golden Delicious and Red Delicious apples over 28 days of room-temperature storage ($25 \pm 1^\circ\text{C}$). Photographs above each plot show the apple samples on the first and last day of measurement.

Table 2. Thermal diffusivity results for Golden Delicious and Red Delicious apples over 28 days of room-temperature storage ($25 \pm 1^\circ\text{C}$). Values are reported as the mean $\pm$ standard deviation, calculated from measurements at two surface locations for each data point. Thermal diffusivity values are expressed in $\times 10^{-7} \text{ m}^2/\text{s}$. Sampling time points do not fully coincide between cultivars; missing entries are indicated by a “–”.

Variety	Day 0	Day 4	Day 8	Day 12	Day 15	Day 18	Day 28	Average
Golden Delicious	0.95 ± 0.15	0.97 ± 0.12	1.14 ± 0.01	1.15 ± 0.01	1.09 ± 0.04	–	1.31 ± 0.23	1.10 ± 0.13
Red Delicious	1.54 ± 0.12	1.28 ± 0.04	1.61 ± 0.24	1.39 ± 0.01	–	1.60 ± 0.01	1.51 ± 0.07	1.49 ± 0.13

For the Golden Delicious apple, the thermal diffusivity values fluctuate between $0.95 \times 10^{-7} \text{ m}^2/\text{s}$ and $1.31 \times 10^{-7} \text{ m}^2/\text{s}$, with an average value of $(1.10 \pm 0.13) \times 10^{-7} \text{ m}^2/\text{s}$. Linear regression of the temporal evolution of diffusivity yielded a positive trend (coefficient of determination $R^2 \sim 0.83$), indicating an increase in thermal diffusivity over the 28-day storage period at room temperature. This observation is consistent with a visual inspection of the fruit, which revealed localized bruising and darkening of the peel, suggesting structural and compositional changes associated with progressive ripening and tissue aging [26].

In contrast, the Red Delicious apple exhibits higher diffusivity values, ranging from $1.28 \times 10^{-7} \text{ m}^2/\text{s}$ to $1.61 \times 10^{-7} \text{ m}^2/\text{s}$, with an average of $(1.49 \pm 0.13) \times 10^{-7} \text{ m}^2/\text{s}$. In this case, no significant temporal trend was identified, in agreement with the external

appearance of the fruit, which remained largely unchanged, without evident signs of shriveling or degradation.

These results indicate that thermal diffusivity is sensitive to post-harvest storage and may be associated with physiological changes occurring in apple tissue, potentially reflecting alterations related to progressive ripening, senescence, and physiological deterioration. Furthermore, the observed difference in mean diffusivity between the two varieties suggests that intrinsic fruit characteristics, such as tissue density and water content, may play a role in their thermal behavior. Although these findings suggest the potential applicability of this method in the agri-food sector, further investigations involving a larger number of samples are necessary to fully validate its effectiveness. Furthermore, it is necessary to determine other physiological parameters in order to verify the association between thermal diffusivity and physical attributes such as physiological loss in weight (%), tissue density (g/cm^3), and fruit decay (%).

4. Discussion

The theoretical developments and experimental results presented in this work build upon and extend the framework established by Salazar et al. [22], who demonstrated that laser-spot step-heating thermography can yield accurate thermal diffusivity values in solids. A key initial step involves solving the heat diffusion equation under Gaussian laser excitation to obtain an analytical expression for the spatio-temporal evolution of the surface temperature. The analytical formulation developed in this research assumes adiabatic boundary conditions, neglecting heat losses by convection and radiation. This assumption is justified by the findings of Salazar et al. [22], which show that such losses are negligible for thermally thick samples at room temperature. Under these assumptions, the resulting formulation (Equation (4)) is consistent with the structure of the temperature field proposed by Salazar et al. for thermally thick, opaque samples. However, whereas Salazar et al. derived analytical expressions valid only during the heating stage, the model developed here provides a unified analytical description that captures both the heating and the cooling phases, explicitly accounting for the evolution of the surface temperature after the laser is switched off. This extension is particularly relevant for low-diffusivity materials, where the contribution of the cooling stage contains critical information for the accurate identification of the thermal diffusivity. The numerical results in Figure 3 confirm that radial temperature profiles during cooling (Figure 3c) exhibit enhanced sensitivity to the diffusivity: as D increases, both the central peak and the profile width evolve more markedly than during the heating stage (Figure 3b). A similar trend was already noted by Salazar et al. for the heating regime, where the spatial decay of temperature at long radial distances becomes increasingly sensitive to variations in D , improving the ability to discriminate among different diffusivity values. The analysis performed in the present work confirms this behavior and shows that the cooling regime amplifies such sensitivity, thus offering a more favorable condition for parameter estimation.

The increased information content of the cooling phase is corroborated experimentally. As reported in Table 1, when fitting only the heating portion of the temporal temperature evolution, the estimated diffusivity deviates by approximately 20% from the value retrieved using cooling alone or combined heating–cooling profiles. Moreover, the heating-only fit exhibits a strong dependence on the assumed spot size: a $\pm 10\%$ variation in the parameter a induces variations exceeding 20% in the inferred diffusivity. Conversely, when the fitting includes the cooling phase or the full transient evolution changes in the spot size affect D by less than 2%, demonstrating that the cooling regime stabilizes the estimation process and reduces parameter correlations. This result is consistent with the theoretical prediction

from Figure 3, where the curvature and spatial broadening of the radial profiles during cooling provide more distinctive signatures of the thermal diffusivity [27].

Building on these observations, the second fitting strategy, which simultaneously considers the spatial temperature distribution along a horizontal line and multiple time points covering both heating and cooling phases, provides a more robust estimation of the thermal diffusivity [28]. By treating both the spot size a and the diffusivity D as fitting parameters, this approach reduces uncertainties associated with an assumed spot size and leverages the additional information contained in the radial temperature profiles. This capability is particularly advantageous when dealing with small laser beams and irregular biological surfaces, where slight misalignment or defocusing of the laser can otherwise affect the apparent spot size and propagate errors into the estimation of D [29].

The developed method was applied to Golden Delicious and Red Delicious apples to evaluate their thermal diffusivity over a 28-day storage period at room temperature. The interpretation of the obtained diffusivity values requires consideration of the composite nature of the fruit. Apples are inherently multilayered systems comprising a thin epidermal skin and the underlying flesh [30,31]; the estimated diffusivity therefore represents an effective thermal diffusivity of the combined layers within the volume affected by the laser heating. Although the laser beam is assumed to be absorbed at the surface in opaque, thermally thick samples, the depth reached by the thermal wave through diffusion is determined by the heating duration τ , as described by the thermal diffusion length $\mu = \sqrt{D\tau}$ [32]. For diffusivity values in the range of 1.0×10^{-7} – 1.5×10^{-7} m²/s, as obtained in this study, and for a heating time of 60 s, μ spans approximately 2–3 mm, which largely exceeds the typical apple skin thickness (40–80 μ m depending on the cultivar [33]). Under these conditions, the thermal field extends well beyond the epidermis into the underlying flesh, providing a spatially averaged estimate of the fruit's thermophysical behavior. The choice of a 60 s heating interval was therefore intended to probe both skin and pulp and ensure that the resulting diffusivity reflects the combined behavior of the two tissues. Within this framework, the retrieved diffusivity should be interpreted as an effective parameter representative of the composite skin–flesh system within the thermal diffusion length probed by the experiment. The single-layer model cannot resolve multi-layer effects, curvature-induced heat spreading, or local heterogeneities, which may bias the absolute value of D . Nevertheless, since the thermal diffusion length largely exceeds the skin thickness under the present experimental conditions, the estimated diffusivity remains primarily sensitive to the bulk thermophysical response of the fruit, which is the focus of this study.

The absolute diffusivity values measured in this study are consistent with those reported in the literature for other apple varieties. Published data include 1.370×10^{-7} – 1.520×10^{-7} m²/s for Gala apples [34], 1.299×10^{-7} – 1.319×10^{-7} m²/s for Fuji apples [35]. Although these varieties differ from those investigated here, the diffusivities obtained in the present work fall within the expected thermophysical range for apple tissues. Notably, the average diffusivity of $(1.10 \pm 0.13) \times 10^{-7}$ m²/s measured for the Golden Delicious sample aligns well with the value reported by Lisowa et al. [36] at 25 °C for the same cultivar, further supporting the plausibility and reliability of the retrieved values.

The results obtained for Golden Delicious apple indicate a progressive increase in diffusivity over the 28-day storage period. This trend may be associated with structural and compositional changes occurring during post-harvest ripening, including a possible reduction in moisture content. As suggested by Lee et al. [18], a lower moisture content reduces the fruit's capacity to retain heat locally and facilitate faster heat propagation, thereby increasing the measured diffusivity. This aligns with observations that fruits with higher moisture content generally exhibit greater volumetric heat capacity [37]. Since

thermal diffusivity, D , is defined as $D = k/\rho c$, where k is thermal conductivity and ρc is volumetric heat capacity, a reduction in ρc due to moisture loss is expected to contribute to an increase in D . Other factors, such as microstructural changes, cellular degradation, and variations in thermal conductivity, may also contribute to the observed diffusivity evolution. Conversely, Red Delicious apples exhibited higher and substantially more stable diffusivity values throughout the measurement period. The absence of a clear temporal trend, coupled with the lack of visible ripening symptoms, suggests that this sample underwent only limited physiological changes during room-temperature storage. This behavior is coherent with the cultivar-dependent thermal responses the findings of Lee et al. [18]. In this context, the higher average diffusivity observed in Red Delicious relative to Golden Delicious may reflect intrinsic varietal attributes, such as tissue structure and water content, suggesting the need for complementary measurements of mass loss or moisture content to enable a quantitative assessment of the relative contribution of water loss to the observed diffusivity evolution. It should also be noted that the observed differences in thermal diffusivity between cultivars may be partially confounded by unknown initial maturity or handling history, as the samples were purchased from retail sources. While the trends reported here provide valuable preliminary insights, future studies involving controlled harvests with known maturity stages and standardized post-harvest handling will be essential to fully disentangle cultivar-specific thermal responses from variability due to the initial fruit condition.

These results provide preliminary evidence of the potential of the developed method for the non-destructive assessment of fruit quality, and they motivate future studies aimed at full validation through a broader experimental campaign involving a larger number of samples across multiple cultivars. By enabling the estimation of thermal diffusivity from laser-induced temperature fields, the method offers a fundamentally quantitative approach that can be directly related to intrinsic quality parameters, making it particularly relevant for agri-food monitoring. In previous work, Lee et al. [18] used infrared thermography to monitor the spatio-temporal thermal response of apples subjected to controlled heating and cooling, demonstrating a correlation with moisture content as an intrinsic quality parameter. However, their analysis relied on phenomenological thermal indices, including minimum, maximum, and mean temperatures extracted from heating and cooling sequences of the entire fruit, to infer cultivar-specific differences and moisture-related properties. The present study advances this conceptual foundation by directly estimating the physical thermal diffusivity from localized laser-induced temperature fields. This approach provides a parameter with clear thermophysical meaning, rather than relying on empirical descriptors of thermal response. Furthermore, the present model incorporates the full analytical description of heat diffusion under localized excitation, thus providing a more robust and physically grounded quantitative metric. Placed within the broader context of thermal imaging for agri-food quality assessment, this model-based strategy represents a significant advancement. Traditional thermography relies primarily on global surface temperature patterns combined with image analysis or chemometric tools to classify defects, estimate ripeness, or detect bruising, treating thermal features largely as empirical indicators [38]. By contrast, the framework introduced in this study extracts a fundamental heat-transport parameter with high interpretability, effectively bridging the gap between empirical thermal indicators and the underlying physics of heat diffusion. This makes the method a scientifically rigorous complement to existing imaging-based techniques and strengthens its potential for non-destructive, quantitative quality evaluation. Furthermore, as demonstrated by Salazar et al. [22], inexpensive laser sources and infrared cameras can be combined to create low-cost setups that can reliably measure thermal diffusivity. These systems can be assembled and operated with minimal technical expertise, highlight-

ing the practical feasibility of implementing the present method in industrial agri-food monitoring applications.

These considerations define clear directions for future developments. Since moisture content was identified by Lee et al. [18] as a primary factor influencing the thermal properties of apples, future research should explore direct correlations between measured diffusivity, moisture content, and physiological maturity, thereby evaluating the potential of diffusivity as an indicator for fruit freshness and ripening stage. Moreover, the integration of diffusivity measurements with advanced imaging and machine learning techniques offers an avenue for enhancing predictive power [37]. The combination of localized thermophysical parameters with artificial neural networks or other AI algorithms applied to thermal or RGB images could provide a comprehensive framework for automated sorting, quality grading, and freshness monitoring along the distribution chain.

5. Conclusions

The results presented in this study offer promising preliminary evidence for the potential of the proposed method in non-destructive assessment of fruit maturity and quality. By enabling the estimation of thermal diffusivity from laser-induced temperature fields, the approach provides a robust and inherently quantitative tool that can be directly linked to intrinsic quality attributes, making it particularly relevant for agri-food monitoring applications.

These findings lay the groundwork for future research, highlighting the need for broader experimental validation involving a larger number of samples across multiple cultivars. Given that moisture content has been identified by Lee et al. [18] as a key factor influencing the thermal properties of apples, future studies should aim to establish direct correlations between measured diffusivity, moisture content, and physiological maturity, thereby assessing the potential of thermal diffusivity as a reliable indicator of freshness and ripening stage.

Furthermore, integrating diffusivity measurements with advanced imaging techniques and artificial intelligence—such as neural networks applied to thermal or RGB imagery—may significantly enhance predictive capabilities. This combined approach could provide a comprehensive framework for automated sorting, quality grading, and freshness monitoring throughout the distribution chain.

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Appendix A

To calculate the thermal diffusivity of the target material, the spatial and temporal variations of the temperature were analyzed using an analytical model derived from the heat diffusion equation.

To determine the temperature field within the sample at any given time, the heat diffusion equation must be solved while accounting for the specified boundary conditions. For simplification, the thermal properties are assumed constant with respect to temperature.

The heat diffusion equation is expressed as:

$$\nabla^2 T - \frac{1}{D} \frac{\partial T}{\partial t} = -\frac{w}{k} f(t) \quad (\text{A1})$$

where D is the thermal diffusivity, k is the thermal conductivity, w is the peak volumetric power density and $f(t)$ is a function representing the variation of power density with time.

Applying the Laplace transform in time:

$$\bar{f} = f(s) = \int_0^\infty f(t) e^{-st} dt \quad (\text{A2})$$

For a pulse of duration τ , the transformed source term becomes:

$$\bar{w} = w \cdot f(s) = w \int_0^\tau e^{-st} dt = q(x, y, z) \cdot \left(\frac{1 - e^{-s\tau}}{s\tau} \right) \quad (\text{A3})$$

where $q(x, y, z)$ is the energy density deposited during the pulse, as schematically represented in Figure A1.

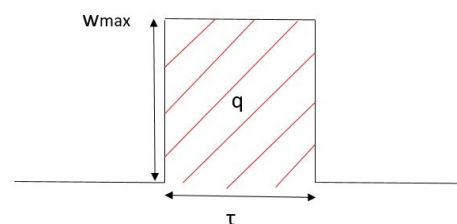


Figure A1. The total amount of energy deposited during a pulse τ at a power $w_{\max}$.

Therefore, the heat diffusion equation in the Laplace domain takes the form:

$$\nabla^2 \bar{T} - \frac{s}{D} \bar{T} = -\frac{q(x, y, z)}{k} \cdot \left(\frac{1 - e^{-s\tau}}{s\tau} \right) \quad (\text{A4})$$

To solve the equation in the spatial domain, the Fourier transform is applied with respect to x and y . The Fourier transform of a one-dimensional spatial function is defined as:

$$\bar{f} = F(p) = \int_{-\infty}^{+\infty} f(x) e^{-2\pi i p x} dx \quad (\text{A5})$$

where p is the spatial frequency in the x -direction. Introducing the substitution $\delta_x = 2\pi p$, the differential operators in Fourier space can be written as:

$$\frac{\partial}{\partial x} = i\delta_x, \quad \frac{\partial^2}{\partial x^2} = -\delta_x^2 \quad (\text{A6})$$

In two spatial dimensions, the Fourier transform is given by:

$$\bar{\bar{f}} = F(p, q) = \iint_{-\infty}^{+\infty} f(x, y) e^{2\pi i (px + qy)} dx dy \quad (\text{A7})$$

where p and q are the spatial frequencies corresponding to the x - and y -directions, respectively.

Applying the Fourier transform with respect to both x and y , the heat diffusion equation yields:

$$\left(-(\delta_x^2 + \delta_y^2) + \frac{\partial^2}{\partial z^2}\right) \bar{T} - \frac{s}{D} \bar{T} = -\frac{\dot{q}}{k} \left(\frac{1 - e^{-s\tau}}{s\tau}\right) \quad (\text{A8})$$

Assuming a Gaussian profile of the laser beam in the transverse x - y plane, the spatial distribution of the energy density is given by:

$$q(x, y) = q_{\max} \cdot e^{-2\left(\frac{x^2 + y^2}{a^2}\right)} \quad (\text{A9})$$

The one-dimensional Fourier transform of the heat source in the x -direction is given by:

$$\dot{q} = \int_{-\infty}^{+\infty} q_{\max} e^{-\frac{2x^2}{a^2}} e^{i\delta_x x} dx \quad (\text{A10})$$

Combining the exponential terms and completing the square in the exponent, the expression becomes:

$$\begin{aligned} \dot{q} &= \int_{-\infty}^{+\infty} q_{\max} e^{-\frac{2}{a^2} \left(x^2 - \frac{a^2}{2} i\delta_x x\right)} dx \\ &= \int_{-\infty}^{+\infty} q_{\max} e^{-\frac{2}{a^2} \left[x^2 - 2\frac{a^2}{4} i\delta_x x + \left(\frac{a^2}{4} i\delta_x x\right)^2 - \left(\frac{a^2}{4} i\delta_x x\right)^2\right]} dx \\ &= \int_{-\infty}^{+\infty} q_{\max} e^{-\frac{2}{a^2} \left[\left(x - \frac{a^2}{4} i\delta_x\right)^2 + \frac{a^4}{16} \delta_x^2\right]} dx \\ &= q_{\max} e^{-\frac{(\delta_x a)^2}{8}} \int_{-\infty}^{+\infty} e^{-\frac{2}{a^2} x^2} dx \end{aligned} \quad (\text{A11})$$

The Gaussian integral evaluates as:

$$\int_{-\infty}^{+\infty} e^{-\frac{2}{a^2} x^2} dx = \sqrt{\frac{a^2 \pi}{2}} \quad (\text{A12})$$

Therefore, the one-dimensional result is:

$$\dot{q} = q_{\max} \sqrt{\frac{a^2 \pi}{2}} e^{-\frac{(\delta_x a)^2}{8}} \quad (\text{A13})$$

The full heat source is assumed to be separable and symmetric in x and y , leading to the following integral that can be evaluated independently in x and y using the previous result (A13):

$$\ddot{q} = \iint_{-\infty}^{+\infty} q_{\max} e^{-\frac{2(x^2 + y^2)}{a^2}} e^{i(\delta_x x + \delta_y y)} dx dy = q_{\max} \frac{a^2 \pi}{2} e^{-\frac{(\delta_x^2 + \delta_y^2) a^2}{8}} \quad (\text{A14})$$

To model the conditions under which a laser irradiates a three-dimensional, opaque material, the power source is assumed to be confined to the plane $z = 0$, with no energy deposition occurring for $z > 0$.

Therefore, for $z > 0$, the heat diffusion equation simplifies to the homogeneous form:

$$\nabla^2 T - \frac{1}{D} \frac{\partial T}{\partial t} = 0 \quad (\text{A15})$$

In the Laplace–Fourier domain, and introducing $\delta_x^2 + \delta_y^2 = \delta^2$, the heat equation becomes:

$$\begin{aligned} (-\delta^2) \bar{\bar{T}} - \frac{s}{D} \bar{\bar{T}} &= 0 \\ \left(-\delta^2 + \frac{\partial^2}{\partial z^2}\right) \bar{\bar{T}} - \frac{s}{D} \bar{\bar{T}} &= 0 \\ \frac{\partial^2 \bar{\bar{T}}}{\partial z^2} - \left(\frac{s}{D} + \delta^2\right) \bar{\bar{T}} &= 0 \end{aligned} \quad (\text{A16})$$

The general solution is:

$$\bar{\bar{T}} = A e^{-\left(\sqrt{\frac{s}{D} + \delta^2}\right)z} + B e^{\left(\sqrt{\frac{s}{D} + \delta^2}\right)z} \quad (\text{A17})$$

The second term is discarded, leaving:

$$\bar{\bar{T}} = A e^{-\left(\sqrt{\frac{s}{D} + \delta^2}\right)z} \quad (\text{A18})$$

At the surface $z = 0$, the boundary condition is defined by the heat flux:

$$\Phi = -k \frac{dT}{dz} \Big|_{z=0} = \ddot{q} \cdot \left(\frac{1 - e^{-s\tau}}{s\tau} \right) \quad (\text{A19})$$

Solving for A :

$$\begin{aligned} k \sqrt{\frac{s}{D} + \delta^2} \cdot A &= q_{\max} \frac{a^2 \pi}{2} e^{-\frac{\delta^2 a^2}{8}} \left(\frac{1 - e^{-s\tau}}{s\tau} \right) \\ A &= \frac{q_{\max} \frac{a^2 \pi}{2} e^{-\frac{\delta^2 a^2}{8}} \left(\frac{1 - e^{-s\tau}}{s\tau} \right)}{k \sqrt{\frac{s}{D} + \delta^2}} \end{aligned} \quad (\text{A20})$$

Thus, the complete temperature field in the Laplace–Fourier domain is:

$$\bar{\bar{T}} = q_{\max} \frac{a^2 \pi}{2k\tau} e^{-\frac{\delta^2 a^2}{8}} \cdot \frac{1 - e^{-s\tau}}{s \sqrt{\frac{s}{D} + \delta^2}} \cdot e^{-\sqrt{\frac{s}{D} + \delta^2} \cdot z} \quad (\text{A21})$$

The inverse Laplace transform of the temperature at $z = 0$ yields the following time-domain expressions, involving the error function (erf):

- For $t < \tau$

$$\ddot{T}_1(\delta, t) = \frac{q_{\max} a^2 \pi}{2\tau k \delta} \cdot e^{-\frac{\delta^2 a^2}{8}} \cdot \text{erf}\left(\sqrt{\delta^2 D t}\right) \quad (\text{A22})$$

- For $t \geq \tau$

$$\ddot{T}_2(\delta, t) = \ddot{T}_1(t) - \ddot{T}_1(t - \tau) = \frac{q_{\max} a^2 \pi}{2\tau k \delta} \cdot e^{-\frac{\delta^2 a^2}{8}} \left[\text{erf}\left(\sqrt{\delta^2 D t}\right) - \text{erf}\left(\sqrt{\delta^2 D (t - \tau)}\right) \right] \quad (\text{A23})$$

The inverse Fourier transform is:

$$T(x, y) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} \ddot{T}(\delta) e^{-i(\delta_x x + \delta_y y)} d\delta_x d\delta_y \quad (\text{A24})$$

Assuming radial symmetry, the following coordinate transformations are introduced:

$$\begin{cases} x = r \cos \varphi \\ y = r \sin \varphi \end{cases} \quad \begin{cases} \delta_x = \delta \cos \theta \\ \delta_y = \delta \sin \theta \end{cases} \quad (\text{A25})$$

The temperature field is then given by:

$$\begin{aligned} T(r) &= \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} \ddot{T}(\delta) e^{-i(\delta \cos \theta r \cos \varphi + \delta \sin \theta r \sin \varphi)} \delta d\delta d\varphi \\ &= \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \ddot{T}(\delta) \delta d\delta \int_0^{2\pi} e^{-i(\delta r \cos(\varphi - \theta))} d\varphi \\ &= \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} 2\pi J_0(\delta r) \ddot{T}(\delta) \delta d\delta \\ &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} J_0(\delta r) \ddot{T}(\delta) \delta d\delta \end{aligned} \quad (\text{A26})$$

where $J_0(\delta r)$ denotes the Bessel function of the first kind of zero order.

Substituting the expression for $\ddot{T}_1(\delta, t)$, the temperature field at the surface for $t < \tau$ becomes:

$$\begin{aligned} T_1(r, t) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} J_0(\delta r) \frac{q_{\max} a^2 \pi}{2\tau k \delta} \cdot e^{-\frac{\delta^2 a^2}{8}} \cdot \text{erf}(\sqrt{\delta^2 D t}) \delta d\delta \\ &= \frac{q_{\max} a^2}{4\tau k} \int_{-\infty}^{+\infty} J_0(\delta r) \cdot e^{-\frac{\delta^2 a^2}{8}} \cdot \text{erf}(\sqrt{\delta^2 D t}) \delta d\delta \end{aligned} \quad (\text{A27})$$

Finally, the temperature field at the surface is described for both time regimes by the following expressions:

$$\begin{cases} T_1(r, t) = \frac{q_{\max} a^2}{4\tau k} \int_{-\infty}^{+\infty} J_0(\delta r) \cdot e^{-\frac{\delta^2 a^2}{8}} \cdot \text{erf}(\sqrt{\delta^2 D t}) \delta d\delta \\ T_2(r, t) = \frac{q_{\max} a^2}{4\tau k} \int_{-\infty}^{+\infty} J_0(\delta r) \cdot e^{-\frac{\delta^2 a^2}{8}} \cdot \left[\text{erf}(\sqrt{\delta^2 D t}) - \text{erf}(\sqrt{\delta^2 D (t - \tau)}) \right] \delta d\delta \end{cases} \quad (\text{A28})$$

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