

## Article

# Analysis of Low-Frequency Sound Absorption Performance and Optimization of Structural Parameters for Acoustic Metamaterials for Spatial Double Helix Resonators

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**Abstract:** Low-frequency noise absorbers often require large structural dimensions, constraining their development in practical applications. In order to improve space utilization, an acoustic metamaterial with a spatial double helix, called a spatial double helix resonator (SDHR), is proposed in this paper. An analytical model of the spatial double-helix resonator is established and verified by numerical simulations and impedance tube experiments. By comparing the acoustic absorption coefficients of the spatial double-helix resonator, it is shown that the results of the analytical model, the numerical model, and the experiments are in good agreement, proving the accuracy of the theoretical model. The effects of different structural parameters on the peak sound absorption coefficient and resonance frequency are quantitatively revealed. The impedance variation law of the model is obtained, and the resistance and reactance distributions at the resonance frequency are analyzed. In the optimization model, the Back Propagation (BP) network is used to construct the mapping between the structural parameters and the resonance frequency and sound absorption coefficient, and this is used as the constraints of the equation, which is combined with Wild Horse Optimization (WHO) to establish the BP-WHO optimization model to minimize the volume of the spatial double helix resonator. The results show that, for a given noise frequency, the optimized structural parameters enhance the space utilization without affecting the performance of the space double helix resonator.

**Keywords:** spatial double helix resonator; acoustic metamaterials; sound absorption; parameter optimization

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## 1. Introduction

The study and prediction of the acoustic properties of sound-absorbing materials have attracted much attention in noise control technology. Especially for low-frequency noise attenuation, it has an essential role in fields such as industrial and environmental noise control. However, the design of sound-absorbing materials is a complex subject [1] that encompasses multiple vital areas, such as multiphysics field coupling, thermo-viscous effects, and nonlinear acoustics. Conventional acoustic materials, such as various types of inorganic fibers and foams, have good acoustic performance in the mid- and high-frequency ranges but are less effective in the low-frequency range and require much space [2–4]. Therefore, in this paper, we propose an acoustic metamaterial with high space utilization and low-frequency noise attenuation capability named spatial double helix

resonator (SDHR), which can be used in future scenarios such as industrial production noise and equipment operation noise.

The emergence of acoustic metamaterials provides a cutting-edge and effective low-frequency noise attenuation method consisting of artificial periodic structures capable of significantly manipulating acoustic waves [5–7]. Acoustic metamaterials offer excellent acoustic wave manipulation capabilities compared to conventional acoustic materials [8,9]. The more common acoustic metamaterial structures capable of low-frequency acoustic absorption are locally resonant acoustic wave crystals [10–12], thin-film bulk acoustic resonators [13,14], metamaterial plates [15,16], quarter-wave resonators [17], Helmholtz resonators [18], and pipeline silencers, among others [19]. However, one challenge acoustic metamaterials face in practical applications is that they often require large form factors in low-frequency noise attenuation, significantly limiting their development prospects.

In recent years, spatially coiled acoustic metamaterials have gradually entered the public eye [20–23]. The propagation properties of sound waves are altered by creating complex coiled structures to achieve desired acoustic properties. As a prototype of its development, quarter-wavelength tubes (QWTs) have a solid ability to absorb or reflect sound waves at specific frequencies. In its development, Howard [24] proposed a quarter-wavelength tube with three geometrical shapes, and it was found through testing that the noise reduction effect of the flared mouth-shaped adaptive quarter-wavelength tube was good. Similarly, with a spatially convoluted structure, Palma [25] designed a broadband modular metasurface based on the expansion of its frequency range through numerical optimization to introduce a tailored phase shift in the acoustic field in order to achieve an ultrathin design with extraordinary refractive and reflective properties. Sun [26] proposed a soft-walled, space-convoluted metamaterial utilizing soft walls to construct internal channels, which achieves excellent low-frequency acoustic absorption performance through strongly coupled frictional dissipation and deformation dissipation, and it also achieves an acoustic absorption coefficient of 0.97 at 615 Hz, demonstrating its efficient acoustic absorption at sub-wavelength thicknesses and a wide range of bandwidth properties. Zheng [27] designed acoustic metamaterials with  $C_{\infty v}$  symmetry by introducing spatially coiled structures, realizing sub-wavelength Dirac linear dispersion and topological valley spin states, breaking through geometrical limitations of phonon topology insulators, and providing theoretical foundations for robust acoustic topology transmission in low-frequency bands. Wang [28] introduced a new type of low-frequency absorber composed of perforated plates and curled channels, which realizes the low-frequency absorption of deep sub-wavelength thickness by adjusting the folding number of the channels and broadens the absorption bandwidth by combining the units with different folding numbers to achieve an absorption coefficient of more than 0.8 at 300–400 Hz. Man [29] presented Mie resonance-based metamaterials (MRMMs), which are designed using spatial coiling and self-similar fractal techniques to realize artificial metamaterials for very low-frequency airborne sound and demonstrates their significant advantages in forming band structures, multi-frequency bandgap properties, and radiated field control. It is a contribution to acoustic filtering devices and acoustic wavefront shaping applications. Wu [30] proposed a curled, space-type acoustic metamaterial to optimize the sound quality of bladeless fans and then reduce the noise of bladeless fans by 4.9 dB without affecting the wind speed, effectively suppressing the aerodynamic noise.

The spatially coiled metamaterial structure is highly compatible and easy to combine with other components, which can further enhance the acoustic absorption effect. Liu [31] proposed an acoustic absorption structure based on microperforated plates and impedance-matched coiled cavities, which significantly broadens the low-frequency band at a deep sub-wavelength thickness. Liu [32] proposed an ultra-broadband acoustic metamaterial with near-perfect continuous sound absorption in the range of 380–3600 Hz by coupling the multistep resonance properties of the cavity and the broadband absorption properties of the micro-perforated plate with an average absorption coefficient of more than 90% at a thickness of only 7.2 cm, which has a wide range of applications in the

automotive industry. Boccaccio [33] proposed a hybrid acoustic absorber of deep sub-wavelength parallel-aligned microperforated plates and Archimedean spirals, realizing broadband acoustic absorption in the frequency range of 400–2800 Hz with a total thickness of less than a  $1/28$  wavelength with an absorption coefficient of more than 60%. Li [34] proposed a compact broadband absorber constructed from a detuned resonator to achieve an effective absorption band of 299–623 Hz with a thickness of only 52 mm through a spatial coiling concept.

In practice, the volume of acoustic materials needs to be as small as possible due to space constraints. Hence, the geometry and structure of acoustic materials need to be designed more flexibly to achieve the best attraction effect. Yan [35] designed and optimized different back cavity heights of perforated panels using a particle swarm optimization algorithm, and the absorption coefficients were all above 0.9 in 1140–3000 Hz. Wang [36] proposed a composite material model with variable cross-section cavities, optimized the parameters of the acoustic absorption model with an artificial fish swarm algorithm, and improved the particle swarm optimization algorithm. Jia [37] optimized the design of broadband low-frequency acoustic performance of the acoustic structure of the Helmholtz resonator embedded in the gap by using differential for optimization algorithm, and the average acoustic absorption coefficient was 0.86 in the frequency range of 170–380 Hz. Sun [38] optimized the design of multi-frequency perfect acoustic absorption super-surface model parameters using a sequential quadratic programming algorithm to obtain perfect acoustic absorption at the target frequency of urban substation noise. Overall, optimizing the geometrical parameters of the acoustic model is to utilize the available space more efficiently and achieve the best acoustic absorption performance in the target space.

In low-frequency noise management, acoustic metamaterials sound absorption capacity and optimization must be further developed. Therefore, in this study, a different subwavelength structure with double helix characteristics is proposed with good sound absorption performance and good space utilization in the low-frequency range. This paper is organized as follows: after the above introduction of the prior art, an analytical model and a numerical simulation model based on thermo-viscous acoustics are constructed based on the impedance principle in Section 2. In addition, the Back Propagation-Wild Horse Optimization (BP-WHO) model is proposed, where the BP network structure is designed to achieve the mapping between the structural parameters and the resonant frequency, the peak of the acoustic absorption coefficients, and to act as the equation constraints. In Section 3, the proposed spatial double helix resonator is experimented with to obtain the sound absorption coefficients and is compared with the analytical model and numerical simulation model to verify the accuracy of the theoretical model. Based on the calculation results of the model, the variation rules of resistance and reactance are analyzed. As detailed in Section 4, the BP network was first trained to describe its accuracy and used as a basis to obtain the law of influence of the structural parameters on the resonant frequency and peak sound absorption coefficient. The structural parameters of the spatial double helix resonator are optimized by combining with the WHO optimization model, and the pre- and post-optimization models are compared to verify the feasibility of the optimized model. Finally, Section 5 summarizes the conclusions.

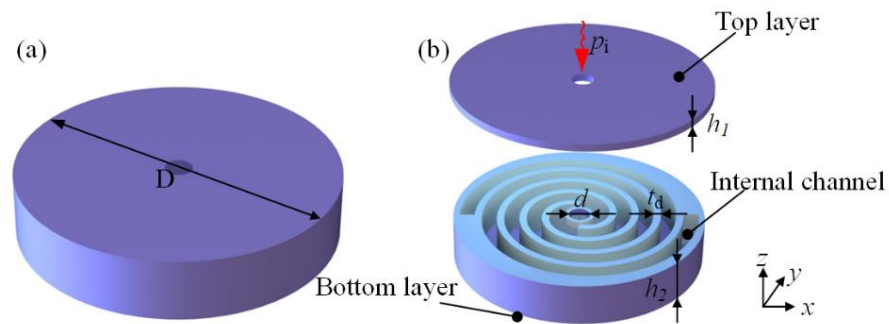
## 2. Theoretical Analysis

### 2.1. Analytical Model

The proposed spatial double helix resonator model is shown in Figure 1. Each cell consists of two parts, the cover plate, and the double helix structure, with heights  $h_1$  and  $h_2$ , respectively. The front plate is a circular perforated thin plate with a diameter of  $D$  and a center hole with a diameter of  $d$ . The internal helical channel is enclosed by two helices with radius  $r$  and length  $L$ , denoted as Equation (1), and the thickness of the helical layer is  $t_a$ . The harmonic plane wave is the incident at the top, denoted by  $p_i$ . The sound wave is assumed to be completely reflected at the wall.

$$\begin{cases} r = a + b\theta \\ L = \int_0^\theta \sqrt{b^2 + (a + b\theta)^2} d\theta \end{cases} \quad (1)$$

where  $r$  is the distance from the origin to any point on the helix,  $\theta$  is the angle measured from the polar axis, and  $a$  and  $b$  are constants where  $a$  determines the starting point of the helix and  $b$  determines the pitch of the helix.



**Figure 1.** (a) SDHR geometry model. (b) SDHR structure dimensional parameters.

According to the theory [39], the formula used to calculate the inlet hole loss is based on an acoustic model of a circular cross-section region using an equivalent diameter, where the impedance of the plane wave  $p_i$  after passing through the circular cross-section and at the QWT connection is

$$Z_{Leff} = -jZ_{eff} \cot(k_{eff} L_{eff}), \quad (2)$$

where  $Z_{eff} = \rho_{eff} c_{eff}$  is the effective impedance,  $k_{eff}$  is the effective wave number of the tube, and  $L_{eff}$  is the effective length. According to the Low Reduced Frequency (LRF) model,  $c_{eff} = \omega / k_{eff}$  is the effective speed of sound and  $\rho_{eff}$  is the effective density, which are expressed as

$$\rho_{eff} = \frac{k_{eff} Z_{eff}}{\omega} = \frac{k_0 Z_0}{\omega} \frac{1}{\psi_v} = \frac{\rho_0}{\psi_v}, \quad (3)$$

where  $k_0 = \omega / c_0$  is the wave number in air,  $\omega$  is the angular frequency,  $c_0$  is the speed of sound in air,  $\rho_0$  is the air density, and  $Z_0 = \rho_0 c_0$  is the air-specific acoustic impedance. The complex beam  $k_{eff} = \sqrt{k_0^2 (\gamma - (\gamma - 1) \psi_h / \psi_v)}$  and specific acoustic impedance  $Z_{eff} = \sqrt{Z_0^2 / (\psi_v (\gamma - (\gamma - 1) \psi_h))}$  in a narrow tube;  $\gamma$  is the specific heat rate; and  $\psi_v$  and  $\psi_h$  are the viscous function associated with the temperature change due to compression and expansion and the thermal function associated with the energy loss due to viscous friction, respectively, which can be obtained by solving a fully linearized system of Navier–Stokes equations [22].

$$\psi_i = k_i^2 \sum_{m=0}^{\infty} \left[ (\alpha_m m_0)^{-2} \left( 1 - \frac{\tan(\alpha_m W / 2)}{\alpha_m W / 2} \right) + (\beta_m m_0)^{-2} \left( 1 - \frac{\tan(\beta_m H / 2)}{\beta_m H / 2} \right) \right], \quad (4)$$

where  $i = v, h$  corresponds to the viscous and thermal fields, respectively;  $\alpha_m = \sqrt{k_i^2 - (2m_0 / W)}$ ,  $\beta_m = \sqrt{k_i^2 - (2m_0 / H)}$ , and  $m_0 = (m + 1/2)\pi$ ; and  $W$  and  $H$  are the length and width of the rectangular tube (for a square tube  $W = H$ ). The system has a viscous wave number  $k_v^2 = -j\omega\rho_0 / \mu$  and a thermal wave number  $\alpha_m = \sqrt{k_i^2 - (2m_0 / W)}$

, where  $\mu$  is the dynamic viscosity,  $C_p$  is the specific heat at constant other, and  $\kappa$  is the thermal conductivity.

The total impedance of the space spiral resonator consists of the perforated thin plate and the bent QWT. The impedance of the perforated thin plate is defined by the Johnson–Champoux–Allard (JCA) model because a plane wave passes through the holes of the perforated thin plate and part of the wave is reflected, and the bending QWT impedance should be modified to be the quarter-wavelength impedance, whose impedance is defined by the LRF model [40]. Therefore, the impedance of the spatial spiral resonator is defined as follows: in a special case, when two quarter-wavelength resonators of the same length and cross-section are combined in a coiled manner (Figure 1), the circuit model can be utilized to compute the second quarter-wavelength resonator with its surface acoustic impedance, and since the two quarter-wavelength resonators have the same cross-section, the total impedance of the spatial double spiral resonator can be expressed as [41]

$$Z_{tot} = \frac{1}{\phi_{inlet}} \left[ Z_{dJCA} \frac{-jZ_{in} \cot(k_{dJCA}) + Z_{dJCA}}{Z_{in} - jZ_{dJCA} \cot(k_{dJCA}t_d)} \right], \quad (5)$$

where  $Z_{in} = -jZ_{eff} \cot(k_{eff} L_{tot})$  and  $L_{tot} = L_{eff1} + L_{eff2}$ ;  $Z_{dJCA} = \rho_{JCA} c_{JCA}$  and  $k_{dJCA} = \omega / c_{JCA}$  are the impedance and complex wave number of the circular inlet with thickness  $t_d$  and diameter  $d$ , respectively;  $c_{JCA} = \sqrt{K_{JCA} / \rho_{JCA}}$ ,  $\rho_{JCA}$ , and  $K_{JCA}$  are the effective sound velocity, effective density, and bulk modulus, respectively; and  $\phi_{inlet} = A_{hole/plate}$  is the perforation ratio between the pore area and the plate area. The complex-valued effective density  $\rho$  and bulk modulus  $K$  are expressed as follows:

$$\rho_{JCA} = \frac{\alpha_{\infty} \rho_0}{\phi} + \frac{\sigma}{j\omega} \sqrt{1 + \frac{4j\alpha_{\infty}^2 \mu \rho_0 \omega}{\sigma^2 \lambda_v^2 \phi^2}}, \quad (6)$$

$$K_{JCA} = \frac{\gamma P_0 / \phi}{\gamma - (\gamma - 1) \left[ 1 + \frac{8\mu}{j\rho_0 \omega \text{Pr} \lambda_{th}} \sqrt{1 + \frac{j\rho_0 \omega \text{Pr} \lambda_{th}}{16\mu}} \right]^{-1}}, \quad (7)$$

where  $\alpha_{\infty}$  is tortuosity,  $\phi$  is porosity,  $\sigma$  is air flow resistivity,  $\mu$  is the dynamic viscosity,  $P_0$  is atmospheric pressure,  $\gamma$  is the ratio of specific heats, Pr is the Prandtl number, and  $\lambda_v$  and  $\lambda_{th}$  are the viscous and thermal wavelengths, respectively. The JCA parameters are generally used to characterize porous materials, and perforated plates are viewed as possessing the following characteristics:  $\alpha_{\infty} = 1$ ,  $\phi = 1$ ,  $\lambda_{th} = \lambda_v = d/2$ , and  $\sigma$ . These are denoted as follows:

$$\sigma = \frac{1}{\phi_{inlet}} \left[ \left( \frac{t_d}{d} + 1 \right) \frac{2\sqrt{2\mu\omega\rho_0}}{\phi} \frac{1}{t_d} \right]. \quad (8)$$

The corresponding sound absorption coefficients are expressed as follows:

$$\alpha = \frac{4 \text{Re}(Z_{tot})}{|Z_{tot}|^2 + 2 \text{Re}(Z_{tot}) + 1}. \quad (9)$$

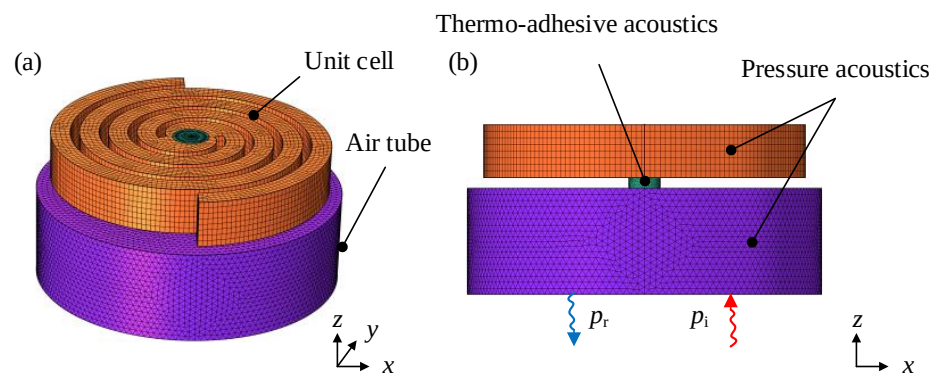
## 2.2. Numerical Model

Acoustic finite element simulations have been widely used in the design of acoustic metamaterials to improve the research efficiency while reducing the cost of experiments [42]. Figure 2 presents the numerical model and meshing of a spatial double helix resonator based on the finite element method (implemented in COMSOL Multiphysics 6.2) for evaluating the absorption coefficients of the spatial double helix resonator. The numerical

model consists of a unit cell and an air tube, which acts as a medium for the propagation of sound waves and interacts with the unit cell and the entire resonator. The combination of the unit cell and the air tube can accurately simulate the propagation and absorption of sound waves in this acoustic system. The sound pressure amplitude of the air tube is 1 Pa, and its sound pressure distribution is controlled by the Helmholtz equation [43].

$$\nabla \cdot \left( -\frac{1}{\rho_0} \nabla p \right) - \frac{\omega^2}{\rho_0 c_0^2} p = 0, \quad (10)$$

where  $p$  is the total acoustic pressure,  $\omega$  is the angular frequency, and  $\rho_0$  and  $c_0$  are the air density and speed of sound, respectively. The air tube is defined as the background pressure field with an incident plane wave  $p_i$ .



**Figure 2.** SDHR finite element modeling and meshing. (a) 3D view; (b) main view.

The spatial double helix resonator is numerically simulated using the pressure acoustics module and the thermo-viscous acoustics module of COMSOL Multiphysics 6.2. During the simulation, normal plane wave radiation is defined at the initial position of the air tube, and the sample is placed at the rigid termination of the tube. The tube and the resonator walls are rigid structures, and hard acoustic field boundary conditions are used. The unit cell and air tube shown in Figure 2b use a pressure acoustic module, and the perforated thin plate aperture channel uses a thermo-viscous acoustic module. The frequency range is defined as 200 Hz–600 Hz. A finite element mesh containing 29,854 cells is delineated, as shown in Figure 2. SDHR finite element modeling and meshing. The necessary physical parameters of the air are shown in Table 1.

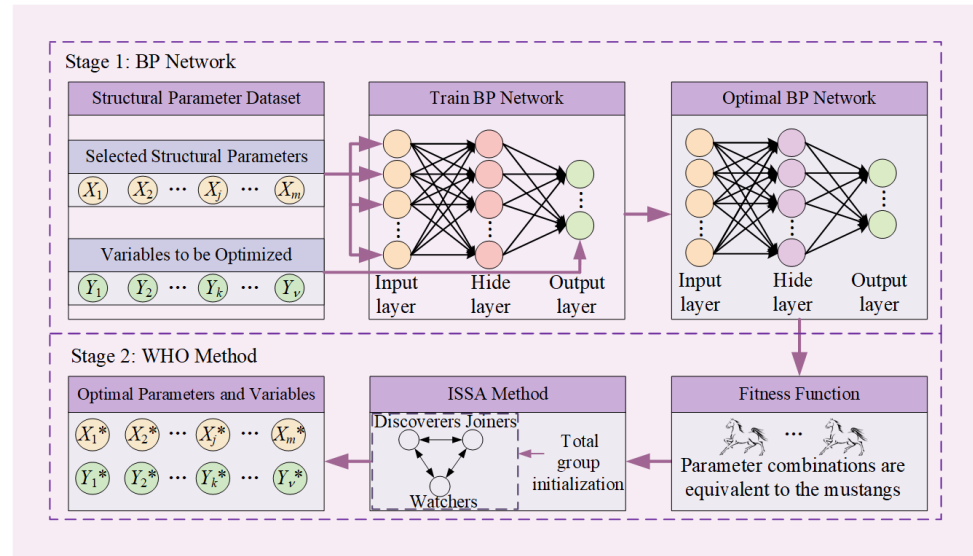
**Table 1.** Physical parameters of air.

| Parameters                                                                | Value                 |
|---------------------------------------------------------------------------|-----------------------|
| Bulk viscosity/(Pa·s)                                                     | $1.11 \times 10^{-5}$ |
| Dynamic viscosity/(Pa·s)                                                  | $1.85 \times 10^{-5}$ |
| Density/(Kg·m <sup>-3</sup> )                                             | 1.21                  |
| Thermal conductivity/(W·m <sup>-1</sup> ·K <sup>-1</sup> )                | $2.63 \times 10^{-2}$ |
| Heat capacity at constant pressure/(J·Kg <sup>-1</sup> ·s <sup>-1</sup> ) | 1005.71               |
| Speed of sound/(m·s <sup>-1</sup> )                                       | 343.00                |
| Ratio of specific heat                                                    | 1.4                   |

### 2.3. Optimization Models

The overall framework of the optimization model proposed in this paper is shown in Figure 3. In the first stage, a BP network is used in the optimization to construct the mapping relationship between the structural parameters and the acoustic performance of the resonator. The nonlinear relationship between the structural parameters and the target

acoustic performance is accurately captured through training to provide effective constraints for subsequent optimization. The second stage uses the constraints from the first stage combined with the Mustang optimization algorithm to minimize the size of the resonator while maintaining the acoustic performance.



**Figure 3.** The overall optimization framework.

The sound absorption performance of the SDHR is mainly governed by the three parameters of the Archimedean spiral curve, namely the polar angle  $\theta$ , and the constants  $a$  and  $b$ . In the BP model, the structural parameters are taken as inputs, and the frequencies and peak sound absorption coefficients are taken as outputs. Therefore, the BP network contains two nodes in the output layer and three nodes in the input layer. In addition, 10 hidden layer nodes are set up. The number of neuron nodes in the input layer is related to the number of variables affecting the sound absorption performance. The dataset used for the BP network was obtained from 2000 subsamples of the analytical model, of which 75% was used for training, 15% for validation, and the rest for testing. After training, the relationship between structural parameters and acoustic performance was obtained:

$$\begin{cases} \alpha_{\max} = f_1(\theta, a, b) \\ f_s = f_2(\theta, a, b) \end{cases} \quad (11)$$

where  $\alpha_{\max}$  is the peak sound absorption coefficient of the space spiral resonator and  $f_s$  is the resonance frequency.

In practice, the resonance frequency and the absorption coefficient are the primary design objectives used as constraints in the design equations. It was assumed that the actual noise frequency is  $Q_1$ , and the maximum value of the desired absorption coefficient is greater than  $Q_2$ . Minimization of the volume of the double helix structure  $\Gamma$  is the optimization objective. To improve the quality of the initial solution and to maintain the diversity of the algorithm population, the range of the structural parameters is restricted, as shown in Table 2. The fitness function was constructed as shown in Equation (12), with an equality constraint and an inequality constraint, which is denoted as Equation (13). If the constraint is not satisfied, the objective function adds a penalty term  $o = o_1 + \beta \sum_i c_{eq}(x_i)^2$ , where  $\beta = 1000$  is the penalty factor that guides the optimization algorithm to find a solution that meets the constraint.

$$o_1 = \min \Gamma(\theta, a, b), \quad (12)$$

$$\begin{cases} f_s = Q_1 \\ lb_1 \leq \theta \leq ub_1 \\ lb_2 \leq a \leq ub_2 \\ lb_3 \leq b \leq ub_3 \\ \alpha_{\max} \geq Q_2 \end{cases} \quad (13)$$

**Table 2.** Constrained range of the optimized parameters.

| $ub_1$ | $ub_2/(\text{mm})$ | $ub_3/(\text{mm})$ | $lb_1$ | $lb_2/(\text{mm})$ | $lb_3/(\text{mm})$ |
|--------|--------------------|--------------------|--------|--------------------|--------------------|
| $6\pi$ | 4                  | 2.5                | $4\pi$ | 1                  | 2                  |

WHO randomly assigns the initial structural parameters and sets the ratio parameter of the main design parameter to the auxiliary design parameter, which is initialized based on the formula of the adaptive parameter.

$$\text{TRD} = 1 - i \times \left( \frac{1}{\text{Max}_{\text{iter}}} \right), \quad (14)$$

where  $i$  and  $\text{Max}_{\text{iter}} = 1000$  denote the current and maximum number of iterations, respectively. The tuning behavior of the auxiliary design parameters can be expressed as

$$X_j^i = \begin{cases} 2Z \cos(2\pi ZR) \times (S^j - X_j^i) + S^j, & r \geq PC \\ \text{Average}(X_j^{i,A}, X_j^{i,C}), & \text{Otherwise} \end{cases} \quad (15)$$

where  $Z$  denotes the adaptive parameter,  $R$  is a random number generated within the range of  $-2$  and  $2$ , and  $X$  is the position of the auxiliary design parameter, which is optimized by cross-optimizing with the main design parameter. The updated position of the main design parameters can be defined as follows:

$$\begin{aligned} S^i &= \begin{cases} 2Z \cos(2\pi ZR) \times (WH - S^i) + WH, & r \geq 0.5 \\ 2Z \cos(2\pi ZR) \times (WH - S^i) - WH, & \text{Otherwise} \end{cases} \\ S^i &= \begin{cases} X_j^i \cos(X_j^i) < \cos(S^i) \\ S^i & \text{Otherwise} \end{cases} \end{aligned} \quad (16)$$

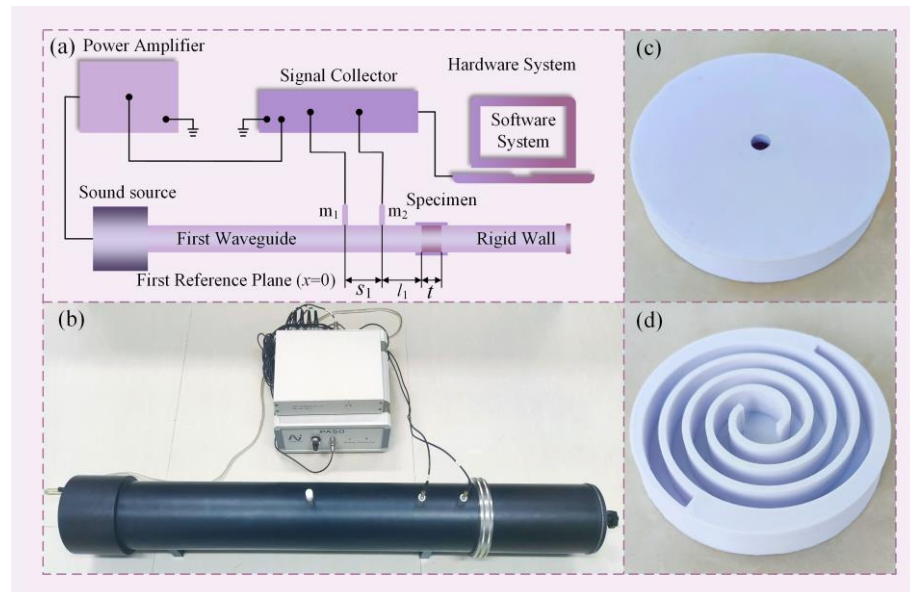
where  $WH$  is the optimal solution location of the objective function.

### 3. Validation and Analysis

The experimental principle of this paper is shown in Figure 4a. A two-channel acoustic test system was used. The impedance tube had a diameter of 100 mm, and two microphones, labeled  $m_1$  and  $m_2$ , were placed between the sound source and the sample. The distance between the two microphones was  $s_1 = 80$  mm, the distance between the sample and microphone  $m_2$  was  $l_1 = 50$  mm, and the front surface of the sample was the first reference plane ( $x = 0$ ). The SDHR sample was made of resin by stereolithography (STL), with a layer thickness of 0.05 mm, a nozzle diameter of 0.2 mm, and a repeat positioning accuracy of  $\pm 0.01$  mm, as shown in Figure 4c,d. The test sample was placed at the end of the impedance tube and held in place by a rigid wall. The complex acoustic transfer function between microphones  $m_1$  and  $m_2$  was  $H_{12}$  and corrected for microphone amplitude and phase response mismatch, and the complex reflection coefficient under plane wave excitation was expressed as [44]

$$R = \frac{H_{12} - e^{-jk_0 s}}{e^{jk_0 s} - H_{12}} e^{2jk_0 x_1}, \quad (17)$$

where  $k_0$  is the number of acoustic waves, and, finally, the absorption coefficient under planar excitation was evaluated using the relation  $\alpha = 1 - |R^2|$ .

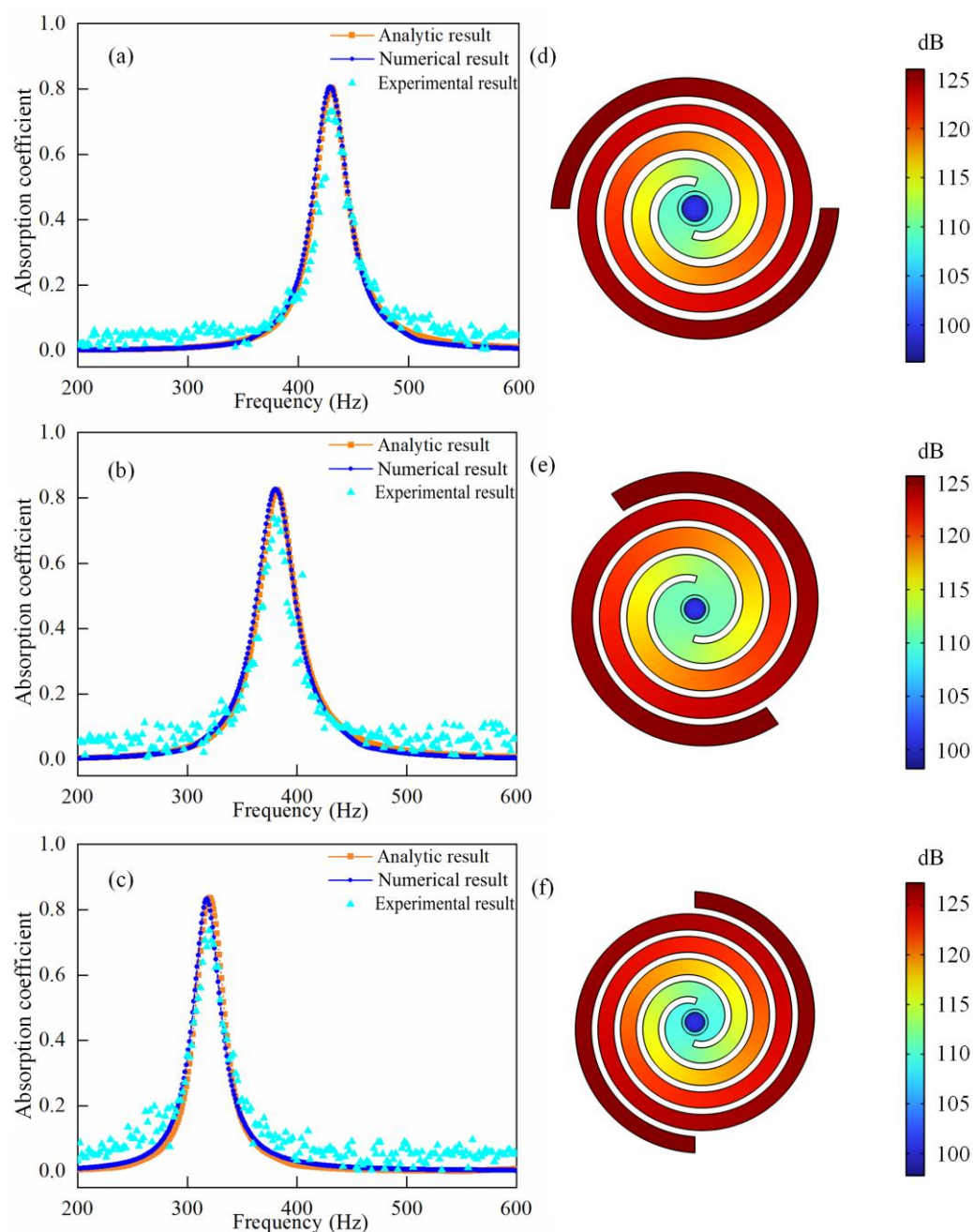


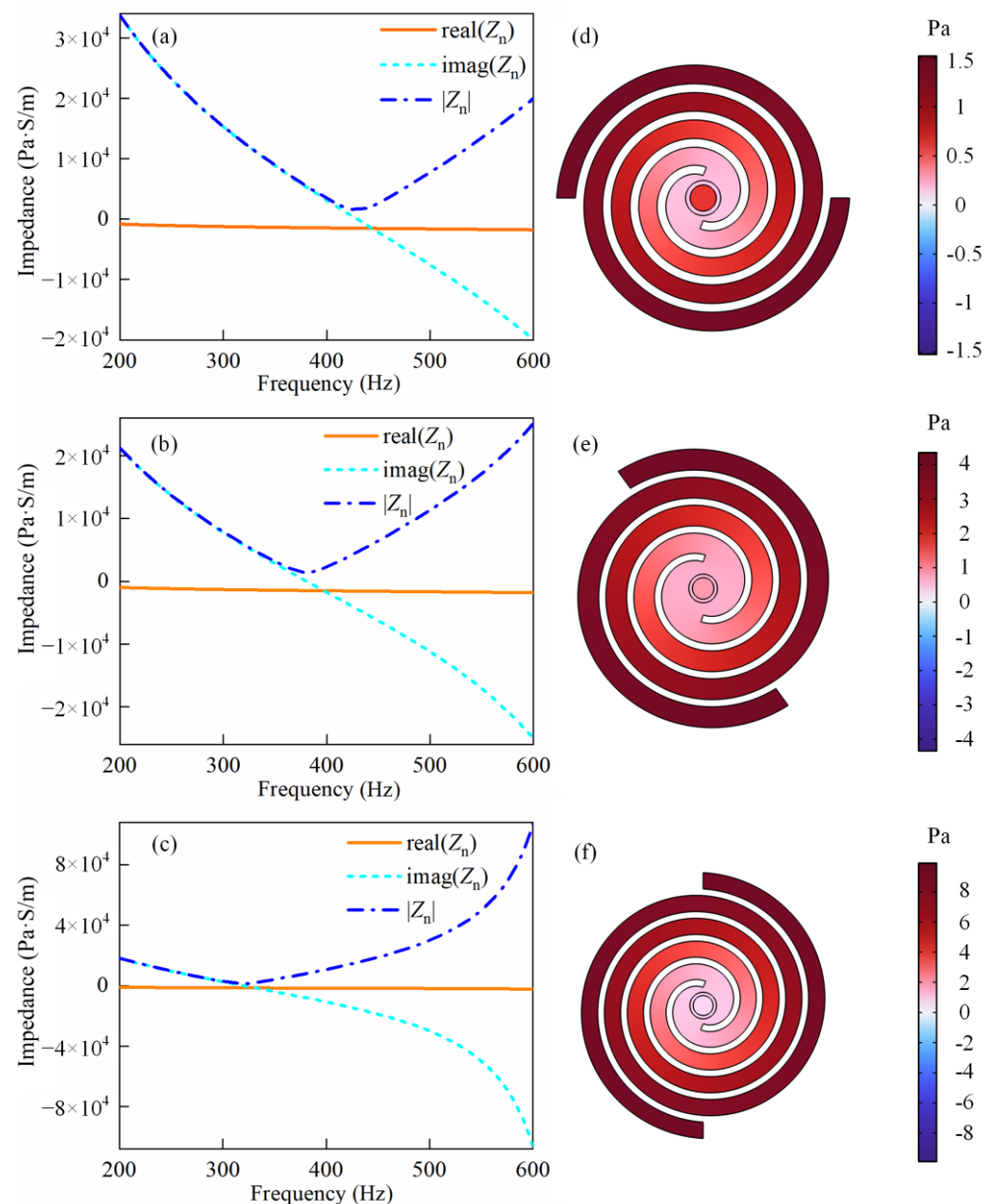
**Figure 4.** Th test system components and SDHR samples. (a) Schematic diagram of the experimental system composition; (b) experiment equipment; (c) SDHR sample; and (d) the inside of the SDHR sample.

In order to verify the accuracy of the model more deeply, a comparison of the sound absorption performance of the analytical, numerical, and experimental models was carried out. Table 3 shows the three groups of different structural parameters of the SDHR. The sound absorption coefficients corresponding to the three different sets of structural parameters are shown in Figure 5a–c, respectively. The results show that the analytical and numerical models were well-matched. The peak absorption coefficients of the analytical model were 0.803, 0.823, and 0.833, corresponding to the frequencies located at 430 Hz, 382 Hz, and 320 Hz, respectively, and the peak absorption coefficients of the numerical model were 0.806, 0.827, and 0.835, corresponding to the frequencies located at 429 Hz, 381 Hz, and 318 Hz, with an error of no more than 0.17%. The peak values of the sound absorption coefficients of the experimental model decreased relative to the previous two, which were 0.730, 0.736, and 0.737, corresponding to the frequencies located at 431 Hz, 379 Hz, and 320 Hz, respectively. The distribution of the sound pressure level of the SDHR at the resonance frequency is shown in Figure 5d–f. The maximum value of the sound pressure level occurred internally due to the resonance generated by the double-helix channel, which indicates that the double-helix resonator performed satisfactorily in noise reduction. An ideal rigid wall was assumed in the analytical and numerical modeling. In contrast, some acoustic waves penetrates the material in the experiments, leading to acoustic leakage because the experimental samples were not completely rigid. Due to these defects, the experimental results showed fluctuating contours instead of the relatively smooth curves in the simulation. In order to investigate the noise reduction principle of the model further, the acoustic impedance at the entrance of the model and the model acoustic pressure map were extracted, as shown in Figure 6. It can be clearly seen that the acoustic impedance at the resonance frequency was 1518.4 Pa·S/m, 1464.3 Pa·S/m, and 1390.4 Pa·S/m, respectively, and the corresponding acoustic pressure plots are shown in Figure 6d–f, where the acoustic pressure at the inlet was close to 0 Pa. Due to the thermo-viscous loss, the acoustic impedance and acoustic pressure at the inlet were non-zero.

**Table 3.** Structural parameters of the space double helix resonators.

| Groups | $D/\text{mm}$ | $d/\text{mm}$ | $h_1/\text{mm}$ | $h_2/\text{mm}$ | $t_a/\text{mm}$ | $a/\text{mm}$ | $b/\text{mm}$ | $\theta$ |
|--------|---------------|---------------|-----------------|-----------------|-----------------|---------------|---------------|----------|
| 1      | 67            | 4             | 2               | 12              | 2               | 2             | 2             | $5\pi$   |
| 2      | 80            | 4             | 2               | 12              | 2               | 4             | 2.2           | $4.7\pi$ |
| 3      | 80            | 4             | 2               | 12              | 2               | 2             | 2.2           | $5.5\pi$ |

**Figure 5.** The SDHR transmission coefficients: (a) Group 1; (b) Group 2; and (c) Group 3. The structural sound pressure level distributions: (d) Group 1; (e) Group 2; and (f) Group 3.

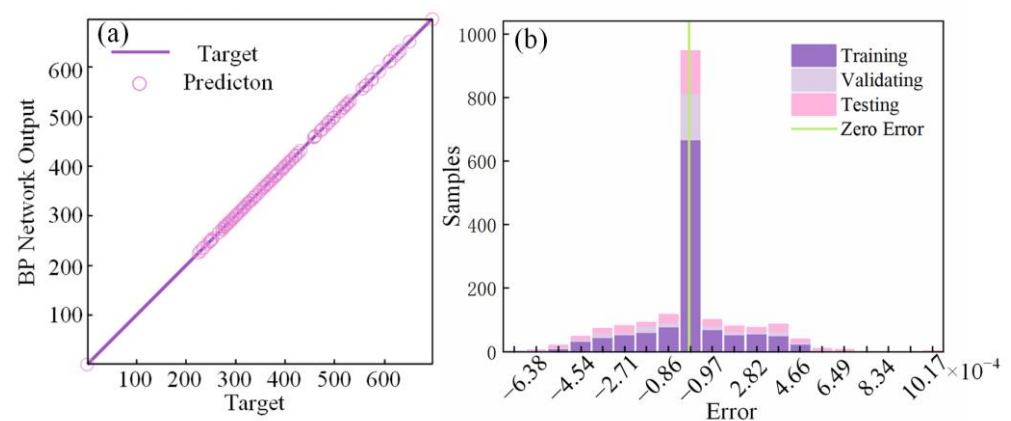


**Figure 6.** Impedance at the SDHR inlet: (a) Group 1; (b) Group 2; and (c) Group 3. The structural sound pressure distribution: (d) Group 1; (e) Group 2; and (f) Group 3.

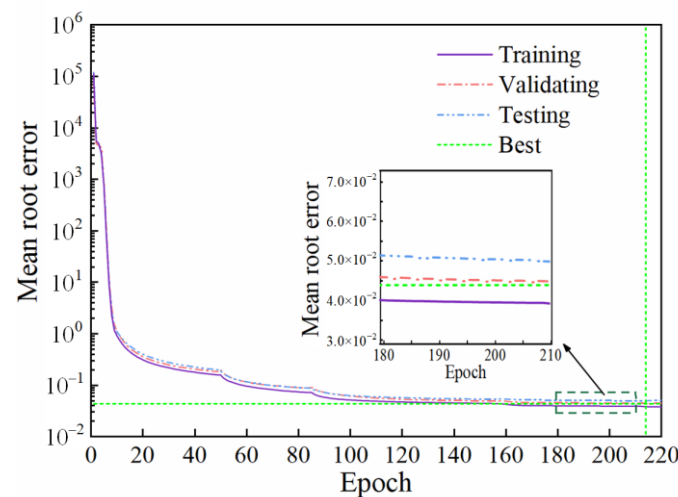
## 4. Forecasting and Optimization

### 4.1. BP Network with Structural Parameter Effects

In the optimization model, the accuracy of the results of the BP network is critical. Its regression state is shown in Figure 7a and was achieved by comparing and analyzing the predicted values with the real values of 100 different parameter combinations in the test set. The results show that the regression performance of the prediction model was very excellent, and its maximum fitting coefficient  $R^2$  reached 0.9998. The errors of the training set, validation set, and test set were further analyzed. Figure 7b shows a normal distribution, and the error value was minimal, indicating that the model prediction accuracy met the expected requirements. The mean square error (MSE) functions of the model training set, validation set, and test set are shown in Figure 8. The model converged quickly in the first 10 training epochs, and the MSE value decreased slowly in the subsequent training, stabilizing after 216 training epochs with a final training epoch of 220. The MSEs for training, testing, and validation were  $4.02 \times 10^{-2}$ ,  $5.06 \times 10^{-2}$ , and  $4.71 \times 10^{-2}$ , respectively.

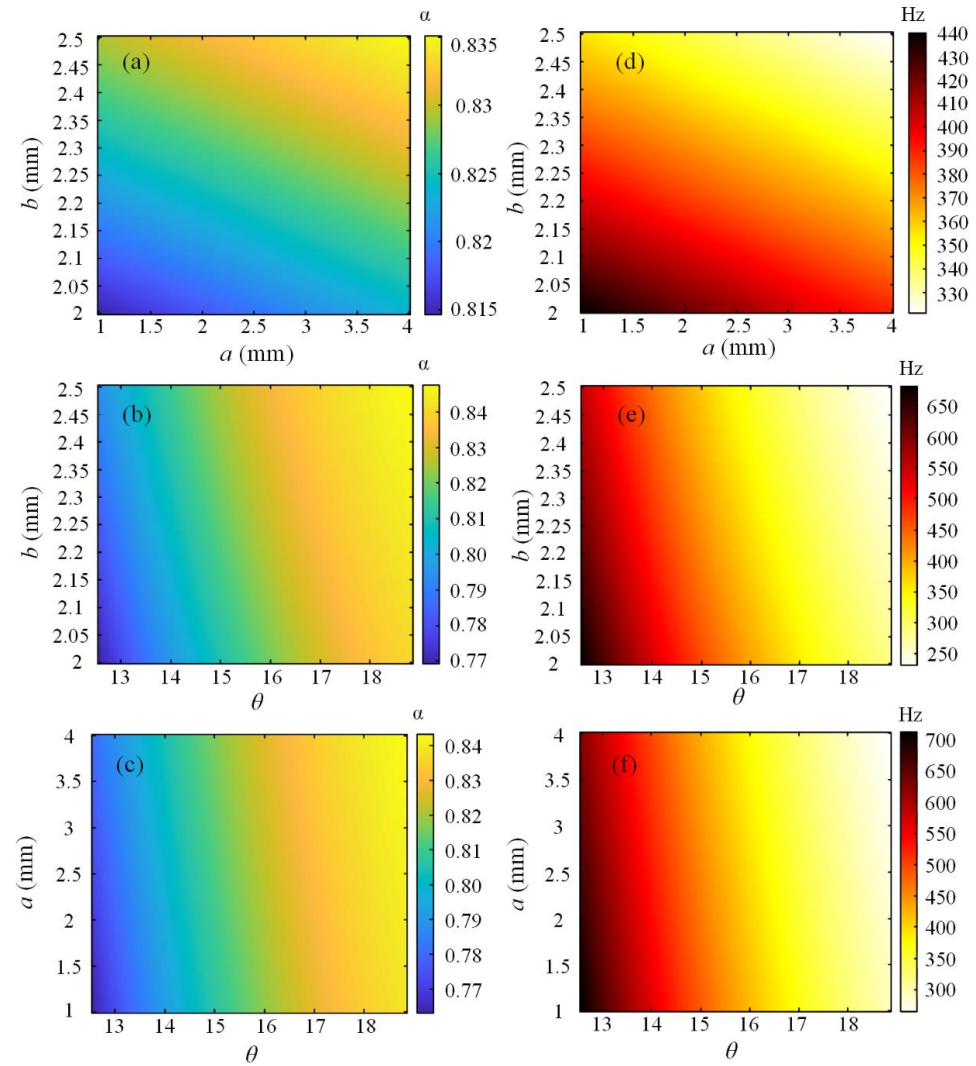


**Figure 7.** BP network prediction accuracy. (a) Regression state; (b) error distribution.



**Figure 8.** The mean square error (MSE) map.

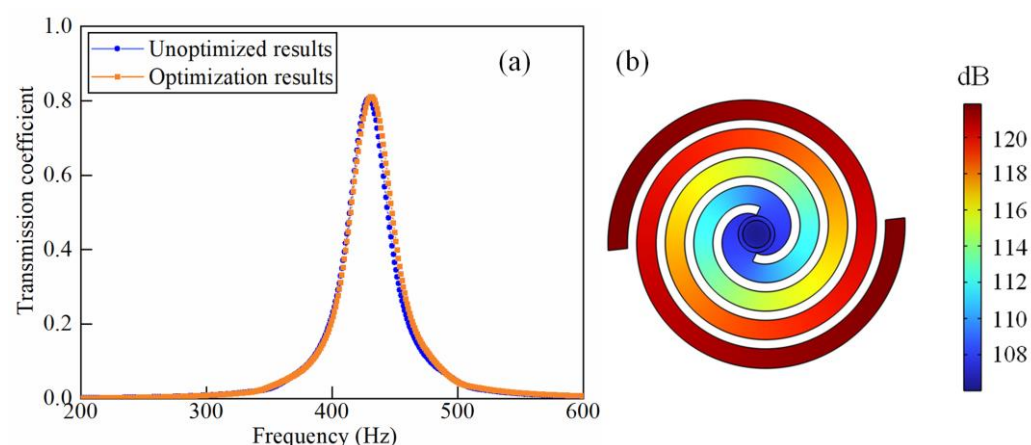
The structural parameter directly effects the maximum value of the sound absorption coefficient  $\alpha_{\max}$  and the resonance frequency. Therefore, the influence pattern was calculated as shown in Figure 9. Fixing the pole angle  $\theta$  constant, the increase of parameters  $a$  and  $b$  increases the peak value of the absorption coefficient and corresponding frequency, and the effect of parameter  $b$  is slightly larger than that of parameter  $a$ . When  $a$  and  $b$  reach the maximum at the same time, the peak value of the absorption coefficient reaches 0.835, and the corresponding frequency is 440 Hz. Fixing  $a$  as constant, the increase of the pole angle  $\theta$  and  $b$  increases the peak value of the absorption coefficient, the corresponding frequency is increased, and the effect of  $\theta$  is more significant than  $b$ . When  $\theta$  and  $b$  reach the maximum, the peak absorption coefficient is 0.846, and the corresponding frequency is 680 Hz. Fixing  $b$  as constant, the increase of  $\theta$  and  $a$  increases the peak absorption coefficient and increase the corresponding frequency, and the effect of  $\theta$  is much larger than  $a$ . When  $\theta$  and  $a$  reach the maximum value, the peak absorption coefficient is 0.843, and the corresponding frequency is 710 Hz. In summary, increasing any of the structural parameters positively affects the acoustic performance, and its corresponding frequency decreases.



**Figure 9.** A three-parameter map of the optimum sound absorption coefficient. (a) Fixing the pole angle  $\theta$  as constant; (b) fixing  $a$  as constant; (c) fixing  $b$  as constant and its frequency; (d) fixing the pole angle  $\theta$  as constant; (e) fixing  $a$  as constant; and (f) fixing  $b$  as constant.

#### 4.2. Optimization Results

The WHO optimization results corresponded to the structural parameters  $\theta' = 5.035\pi$ ,  $a' = 1$ , and  $b' = 2$ . Compared with the original structural parameters, the structural parameters were reduced to minimize the overall occupied space. After a numerical modeling of the optimized structural parameters, the comparison of sound absorption coefficients before and after optimization were obtained, as shown in Figure 10a, and the sound pressure level distribution is shown in Figure 10b. Compared with the original spatial spiral resonator, satisfactory noise reduction performance was maintained while improving space utilization. The optimization results show that the structural parameters of the spatial spiral resonator can be optimized in a targeted manner to improve space utilization and maximize the noise reduction effect after clarifying the noise frequency.



**Figure 10.** (a) Comparison of the original and optimized sound absorption coefficient factors. (b) Optimized structure SPL distribution.

## 5. Conclusions

In this work, a spatial double helix resonator acoustic absorption structure in which the quarter-wavelength resonator was coiled in the shape of a helix is proposed. The corresponding analytical and numerical models were established to analyze the acoustic properties, and the validity of the models was verified experimentally. By constructing the BP-WHO optimization model, the space utilization of the spatial spiral resonator at specific frequencies was optimized. The main conclusions are as follows.

(1) The accuracy of the theoretical model was verified by comparing the analytical model, theoretical model, and experimental results. The impedance matching of the double helix resonator was realized at the resonant frequency, which proves that the double helix resonator can absorb noise effectively.

(2) The relationship between the structural parameters and the acoustic performance of the spatial double helix resonator was predicted by the BP neural network. Increasing any one of the structural parameters positively affects the acoustic performance, and its corresponding frequency decreases subsequently.

(3) It was verified that the BP-WHO optimization model can effectively optimize the combination of structural parameters, and the optimized structural parameters improve space utilization without affecting the acoustic performance of the spatial double helix resonator.

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