

Article

Surface–Volume Integral Formulation for Evaluating Magnetization Losses in CORC[®] Cables

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Abstract

Modeling the electromagnetic (EM) behavior of CORC[®] cables presents significant computational challenges due to the coexistence of thin superconducting tapes and thick structural formers. This creates a strongly multiscale problem, making traditional FEM-based approaches cumbersome, as they require extremely fine meshes to accurately resolve the different geometric scales. Integral Equation Methods (IEMs), on the other hand, are well-suited for magnetization loss analysis in multiscale superconducting structures, as they avoid modeling non-EM-active parts of the domain. This greatly reduces the effort involved in meshing the computational domain. In this work, we propose an IEM that couples surface and volumetric models to perform transient nonlinear analysis of CORC[®]-like superconducting cables.

Keywords: high temperature superconductors (HTS); AC losses; integral equations

1. Introduction

The second generation of High Temperature Superconductors (HTS), based on REBCO and arranged in Conductor-on-Round-Core (CORC[®]) wires, is attracting growing interest in electrical engineering. Their applications extend beyond power distribution to high-energy physics, including magnets for nuclear fusion devices and particle accelerators [1,2]. In this context, numerical analysis plays a crucial role throughout both the design and operational phases: it reduces prototyping costs and enables the management or even prevention of disruptive events, such as quenching, through advanced monitoring strategies [3,4].

In the literature, two main approaches are proposed for the electromagnetic (EM) analysis of superconducting CORC[®] cables: Finite Element Methods (FEMs) and Integral Equation Methods (IEMs) [5]. IEMs have been applied to REBCO coils under 2D assumptions in [6,7]. As shown in Figure 1, a typical CORC[®] cable consists of two main components: thin HTS tapes helically wound around a thick copper (Cu) former. The strong geometric disparity between the two is evident; the HTS tapes have a thickness of ~0.1 mm, with a REBCO layer of about 1 μm [8,9], while the Cu former has a typical radius of ~2.8 mm. This scale separation makes CORC[®] cables a clear case of multiscale modeling.

Standard FEM approaches, based on magnetic field formulations (e.g., the H -formulation), have been applied to CORC[®] cables by modeling both the HTS layer and the Cu former as bulk conductors [10–12]. While effective, this approach is computationally demanding and requires careful meshing of the computational domain. To reduce complexity, alternative formulations have been proposed in both the FEM and IEM frameworks, where the HTS layer is represented as a surface in 3D under the assumption of uniform



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current density across its thickness [5,13]. A hybrid strategy, using surface discretization for thin parts and volumetric discretization for the thick domains, has proven to be a practical solution for the EM analysis of such multiscale structures [14]. Notably, this approach has already been employed within the IEM framework to investigate the influence of passive conducting structures in magnetic confinement fusion devices [15,16].

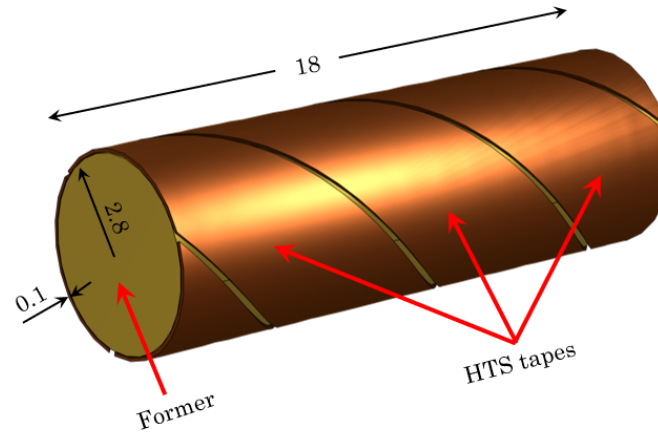


Figure 1. Schematic representation of CORC® cable including HTS tape and former. Units in mm.

In this work, the surface–volume IEM is extended to analyze the EM response of CORC® cables, particularly the AC losses, subjected to a time-varying external field, taking into account the nonlinear behavior of the HTS material. Different loss mechanisms occur in REBCO tapes, including coupling losses and hysteresis (magnetization) losses [17,18]. In the following analysis, only the AC losses associated with current loops in the tapes are considered. These losses are induced by an external sinusoidal magnetic field.

2. Materials and Methods

In this section, the formulation of the electromagnetic problem adopted for the analysis of CORC® cables is presented. The model is derived from the magneto-quasistatic approximation of Maxwell’s equations:

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t} \quad (1)$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \mathbf{J}(\mathbf{r}, t), \quad (2)$$

where $\mathbf{E}(\mathbf{r}, t)$ is the electric field, $\mathbf{J}(\mathbf{r}, t)$ is the current density, $\mathbf{B}(\mathbf{r}, t)$ is the magnetic induction, $\mathbf{H}(\mathbf{r}, t)$ the magnetic field, and $\mathbf{r} = [x, y, z]^T \in \Omega \subset \mathbb{R}^3$ is the position vector.

The H and $H - \phi$ formulations, derived from Equations (1) and (2), are the most widely used approaches for solving the EM problem in superconducting cables and are implemented in several commercial softwares [19,20]. They rely upon the solution in terms of $\mathbf{H}(\mathbf{r}, t)$ and the magnetic scalar potential $\phi(\mathbf{r}, t)$ within the computational domain.

Hereafter, an integral equation (IE) approach derived from the potential-based Faraday’s law coupled with the charge continuity equation is considered. If a time-varying incident electric field $\mathbf{E}_{inc}(\mathbf{r}, t)$ exists in Ω , these equations are expressed as follows:

$$\mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mathbf{A}(\mathbf{r}, t)}{\partial t} - \nabla V(\mathbf{r}, t) + \mathbf{E}_{inc}(\mathbf{r}, t), \quad (3)$$

$$\nabla \cdot \mathbf{J}(\mathbf{r}, t) = 0 \quad (4)$$

where $\mathbf{A}(\mathbf{r}, t)$ is the magnetic scalar potential satisfying $\nabla \times \mathbf{A} = \mathbf{B}$, and $V(\mathbf{r}, t)$ is the electric scalar potential. The current density is related to the magnetic scalar potential through an integral expression:

$$\mathbf{A}(\mathbf{r}, t) = \mu_0 \int_{\Omega} \frac{\mathbf{J}(\mathbf{r}', t)}{4\pi \|\mathbf{r} - \mathbf{r}'\|} d\mathbf{r}'. \quad (5)$$

By substituting Ohm's law $\mathbf{E} = \rho \mathbf{J}$, where ρ is the resistivity, and Equation (5) into Equation (3), a system of two equations with the unknowns $\mathbf{J}$ and V is obtained, leading to the current–potential integral formulation detailed in [21]. The main advantage of this approach lies in the fact that only the electromagnetically active regions of Ω , i.e., the conductive domains, need to be modeled, since the current density $\mathbf{J}$ is nonzero exclusively within these regions.

2.1. Modeling Electrical Resistivity

The electromagnetic behavior of superconducting materials is modeled using a nonlinear form of Ohm's law. Various expressions have been proposed in the literature, typically based on a power-law formulation [22]:

$$\mathbf{E} = \underbrace{\frac{E_c}{J_c} \left(\frac{\|\mathbf{J}\|}{J_c} \right)^{n-1}}_{\rho(\|\mathbf{J}\|)} \mathbf{J}, \quad (6)$$

where E_c and J_c are the critical electric field and the critical current density, while $n > 20$ is an integer exponent. For HTS materials, the magnetic flux density ($\mathbf{B}$) typically affects both the n -value and the critical current density J_c [23]. In particular, for the REBCO layer in CORC[®] cables, J_c is commonly expressed as a function of $\mathbf{B}$ as follows:

$$J_c(\mathbf{B}) = \frac{J_{c0}}{\left(1 + \sqrt{(kB_{\parallel})^2 + B_{\perp}^2} / B_c\right)^b}, \quad (7)$$

where the parameters have the same meaning as in [24]. It is worth noting that Equation (7) can be extended to account for strain and temperature dependence, as discussed in [13,25–27]. Such extensions are particularly useful for describing the behavior of CORC[®] cables under bending or during quenching events.

For the normal core of CORC[®] cables, typically made of copper, a linear Ohm's law is employed. It is worth noting that temperature effects can also be incorporated in the core; however, in this preliminary study, which focuses solely on the electromagnetic problem, thermal effects are neglected.

2.2. Numerical Solution

Based on the integral formulation described in Section 2, a numerical scheme is developed by subdividing Ω into finite elements. As stated in Section 1, in practical applications, Ω can be divided into multiple subdomains with different geometric scales, such as thick volumetric regions (denoted as Ω_v) and thin volumetric regions, which can be represented as surfaces in $\mathbb{R}^3$ (denoted as Ω_s). In the following, Ω_s is constructed by extracting the mid-surface of the corresponding thin volumetric domain.

After partitioning Ω_s with N_s triangular or quadrilateral elements, and Ω_v with N_v tetrahedra or hexahedra, the unknowns $\mathbf{J}$ and V are expressed in terms of vector ($\mathbf{w}(\mathbf{r})$) and piecewise constant ($\psi(\mathbf{r})$) basis functions:

$$\mathbf{J}(\mathbf{r}, t) = \sum_{k=1}^{N_{f/e}} i_k(t) \mathbf{w}_k(\mathbf{r}), \quad V(\mathbf{r}, t) = \sum_{h=1}^{N_{v/s}} u_h(t) \psi_h(\mathbf{r}). \quad (8)$$

In Equation (8), N_f is the number of faces of the volumetric mesh, and N_e is the number of edges of the surface mesh. It is worth mentioning that normalized RWG basis functions [28] are used for surface current density, and a rescaling process is applied to account for the thickness of the thin regions. By Galerkin testing Equations (3) and (4), the following nonlinear Differential Algebraic Equation (DAE) is assembled:

$$\begin{bmatrix} \mathbf{L} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \mathbf{i}(t) \\ \mathbf{u}(t) \end{bmatrix} = - \begin{bmatrix} \mathbf{R}(\mathbf{i}) & \mathbf{D}^\top \\ \mathbf{D} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{i}(t) \\ \mathbf{u}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{\text{inc}}(t) \\ \mathbf{0} \end{bmatrix}. \quad (9)$$

A detailed description of the evaluation of matrix entries can be found, e.g., in [29].

Since the computational domain is divided into Ω_s and Ω_v , the matrices and vectors in (9) are correspondingly expanded as follows:

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_v & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_s(\mathbf{i}) \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} \mathbf{L}_v & \mathbf{L}_{vs} \\ \mathbf{L}_{vs}^\top & \mathbf{L}_s \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{D}_v & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_s \end{bmatrix}, \quad (10)$$

and

$$\mathbf{i}(t) = \begin{bmatrix} \mathbf{i}_v(t) \\ \mathbf{i}_s(t) \end{bmatrix}, \quad \mathbf{u}(t) = \begin{bmatrix} \mathbf{u}_v(t) \\ \mathbf{u}_s(t) \end{bmatrix}, \quad \mathbf{e}_{\text{ext}}(t) = \begin{bmatrix} \mathbf{e}_{\text{inc},v}(t) \\ \mathbf{e}_{\text{inc},s}(t) \end{bmatrix}. \quad (11)$$

It is worth noting that $\mathbf{D}_s$ and $\mathbf{D}_v$, the discrete counterparts of the divergence operator, are sparse and can therefore be stored efficiently in memory. In contrast, the inductance matrices $\mathbf{L}_v$, $\mathbf{L}_{vs}$, and $\mathbf{L}_s$, arising from the integral expression in Equation (5), are dense. To manage the quadratic memory scaling, low-rank representations, such as hierarchical matrices, are typically employed. A detailed discussion of these techniques is beyond the scope of this work, but the interested reader is referred to [30].

The coupling between surface and volumetric elements is handled via the inductance matrix $\mathbf{L}_{vs} \in \mathbb{R}^{N_f \times N_e}$. For CORC[®] cables, the volumetric domain Ω_v corresponds to the Cu former, a linear material, making $\mathbf{R}_v$ a constant sparse matrix. The HTS tapes are represented by Ω_s , where nonlinearity appears in the resistance matrix $\mathbf{R}_s(\mathbf{i})$. This matrix is also sparse, but its entries depend on the current vector $\mathbf{i}$ (i.e., the current density) at a given time, according to the law defined in Equation (6).

The algorithmic procedure used to solve the EM problem with the proposed approach can be summarized as follows:

1. Define the geometry and generate the mesh for Ω_v and Ω_s ;
2. Construct and store the time-independent matrices $\mathbf{R}_v$, $\mathbf{D}_k$, $\mathbf{L}_k$ (for $k = v, s$), and $\mathbf{L}_{vs}$, using a suitable data-sparse representation;
3. Define the mapping $\mathbf{i} \mapsto \mathbf{R}_s(\mathbf{i})$ for constructing the resistance matrix;
4. Specify the external field excitation;
5. Solve the nonlinear time-dependent problem.

For a given geometry and material configuration, steps 1–3 can be carried out offline, with the corresponding quantities stored in memory, while the remaining steps are performed online. Indeed, if the external excitation changes, the system matrices do not need to be recomputed from scratch. The procedure can be easily extended to consider also a nonlinear resistivity for Ω_v .

The DAE in Equation (9) can be recast into the state-space form commonly used in control theory:

$$\mathbf{E} \frac{d\mathbf{x}}{dt} = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}_u \mathbf{u}, \quad (12)$$

where $\mathbf{x} = [\mathbf{i}, \mathbf{u}]^\top$:

$$\mathbf{E} = \begin{bmatrix} \mathbf{L} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{A}(\mathbf{x}) = - \begin{bmatrix} \mathbf{R}(\mathbf{x}) & \mathbf{D}^\top \\ \mathbf{D} & \mathbf{0} \end{bmatrix}, \quad \mathbf{B}_u = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}. \quad (13)$$

Solving the nonlinear time-dependent problem expressed by Equation (12) requires the additional evaluation of the Jacobian matrix $\mathcal{H}_s(\mathbf{i})$, as described in [21]. This matrix is sparse, and the mapping $\mathbf{i} \mapsto \mathcal{H}_s(\mathbf{i})$ can be constructed through a procedure analogous to the one used in step 3 [21]. In what follows, the time integration is performed with a fixed time step τ using an implicit Euler scheme; thus, no adaptive time stepping is employed. By discretizing Equation (12) using the Euler method, we obtain the following state update equation:

$$\left[-\mathbf{A}(\mathbf{x}_{k+1}) + \frac{1}{\tau} \mathbf{E} \right] \mathbf{x}_{k+1} = \frac{1}{\tau} \mathbf{E} \mathbf{x}_k + \mathbf{B}_u \mathbf{u}_{k+1}, \quad (14)$$

where $\mathbf{x}_k$ is the solution vector at k -th time step. The tolerance of the nonlinear solver ($TolNL$), based on the Newton–Raphson (N-R) method, is iteratively adjusted at time t using the following rule:

$$TolNL = \beta \left\| \frac{1}{\tau} \mathbf{E} \mathbf{x}(t-1) + \mathbf{B}_u \mathbf{u}(t) \right\|, \quad (15)$$

where $\beta = 10^{-2}$ in this case, while the maximum number of N-R iterations is set to 100. At a given time step, the nonlinear solver also requires the solution of the following system for a given right-hand-side vector $\mathbf{f}$ [31]:

$$\begin{bmatrix} \tau^{-1} \mathbf{L} + (\mathbf{R} + \mathcal{H}) & \mathbf{D}^\top \\ \mathbf{D} & \mathbf{0} \end{bmatrix} \tilde{\mathbf{x}} = \mathbf{f}, \quad (16)$$

where, in this case:

$$\mathcal{H} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathcal{H}_s \end{bmatrix}. \quad (17)$$

The system expressed by Equation (16) is solved with the iterative GMRES method, with a tolerance of 10^{-4} , exploiting the Algebraic Multigrid (AMG) preconditioner [32]. It is worth mentioning that hierarchical matrix-based solvers [33] can be conceived in this step, and will be the subject of future research activities.

In contrast to the IE, the FEM-based H and $H - \phi$ formulations introduced in Section 2 present several challenges in meshing the computational domain. First, Ω must include not only the entire CORC[®] cable but also a surrounding air region, where the electromagnetic fields extend. Second, generating a volumetric mesh for the entire Ω becomes problematic when modeling a single REBCO layer of thickness $\sim 1 \mu\text{m}$. To address this, the literature often employs a homogenization approach, in which the thickness of the superconducting tapes is artificially increased (typically by a factor of 100) with proper resistivity scaling, thereby making the meshing process computationally feasible [11,34].

The coupling strategy between the surface and volume discretizations used in the IE approach, along with the associated system matrices, is illustrated in Figure 2.

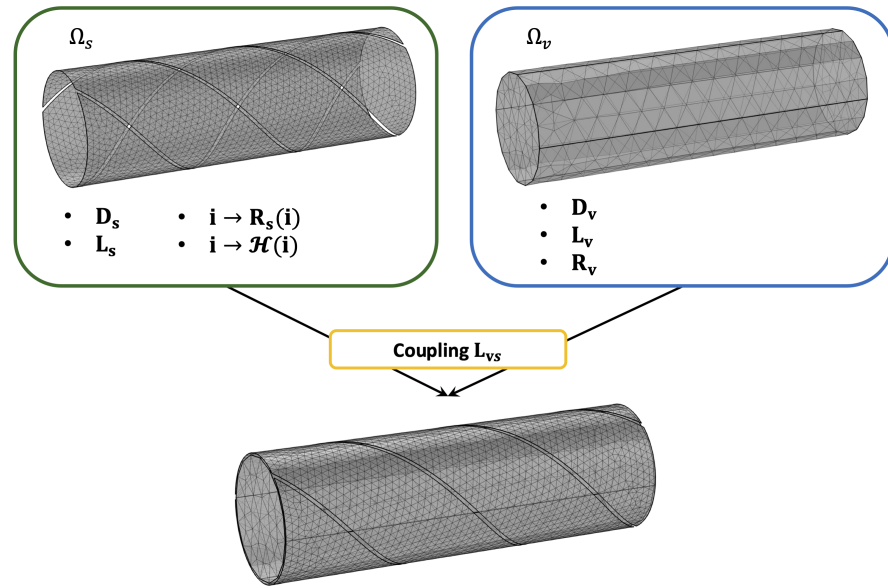


Figure 2. Coupling strategy of surface–volume discretizations for the CORC[®] cable using the IE approach.

Electrical Resistivity Reconstruction

At a given time, following Equation (6), a uniform current density within each mesh cell of Ω_s is required and is obtained through Equation (8), corresponding to a linear map $\mathbf{i} \mapsto \mathbf{J}$ described by a sparse matrix [35]. Under the previous assumption, it is also possible to build the magnetic flux density within each element, as required e.g., in Equation (7), using the analytical expression given by [36]:

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \mathbf{J}(t) \times \mathbf{n}_s W_s(\mathbf{r}), \quad (18)$$

where $\mathbf{n}_s$ is the normal surface vector and $W_s(\mathbf{r})$ is the analytical form of the surface integral detailed in [36].

2.3. Losses Calculation

In this study, the EM analysis is conducted to evaluate the instantaneous AC losses of the superconducting structure, defined as follows:

$$P_{AC}(t) = \int_{\Omega} \mathbf{E}(\mathbf{r}, t) \cdot \mathbf{J}(\mathbf{r}, t) d\mathbf{r}. \quad (19)$$

From the numerical point of view, the integral is evaluated with a quadrature rule, approximating the integrand function with its value at the center of each mesh element ($\mathbf{r}_e$):

$$P_{AC}(t) \approx \sum_{h=1}^{N_{v/s}} \mathbf{E}(\mathbf{r}_{e,h}, t) \cdot \mathbf{J}(\mathbf{r}_{e,h}, t) V^h, \quad (20)$$

where V^h is the measure (area/volume) of the h -th (surface/volumetric) element. For sinusoidal excitations, such as incident fields or transport currents with electrical period T and amplitude A_0 , the AC magnetization losses are computed as follows:

$$Q = 2 \int_{T/2}^T P_{AC}(t), dt, \quad (21)$$

Analytical power-law models expressing $Q = Q(A_0)$ are usually extracted to characterize the superconducting cables [37]. Following [19], the integral in Equation (21) is evaluated during the second half of the first excitation period to disregard transient effects.

3. Results

In this section, the proposed IE approach, implemented in MATLAB R2025a using MEX-Fortran functions, is tested by first solving a benchmark model and comparing the results with those obtained from a commercial FEM software. Then, the AC losses caused by a time-varying external field are computed for a CORC[®] cable. To simplify the analysis, it is assumed that n and J_c are field-independent in both cases. In what follows, the mesh data (coordinates, connectivity) generated with COMSOL[®] Multiphysics 6.3 are imported into the MATLAB environment using proprietary functions. The simulation was performed on a machine Intel[®] Xeon[®] w5-2465X @ 3.10 GHz, 16 core, 32 thread with 128 GB RAM.

3.1. HTS Plate Above Conductive Substrate

We consider the geometry shown in Figure 3 inspired from benchmark#2 listed in [38], corresponding to a thin HTS disk above a conductive substrate made of iron ($\rho = 8.93 \cdot 10^{-8} \Omega\text{m}$), embedded in a uniform vertical magnetic flux density with sinusoidal trend $\mathbf{B}_{inc}(t) = B_0 \sin(2\pi f_0 t) \hat{\mathbf{z}}$ with $B_0 = 50 \text{ mT}$ and $f_0 = 20 \text{ Hz}$, corresponding to $T = 1/f_0 = 50 \text{ ms}$. The incident electric field is constructed analytically from Equation (1), resulting in the following:

$$\mathbf{E}_{inc}(\mathbf{r}, t) = B_0 \pi f_0 \cos(2\pi f_0 t) [y, -x, 0]^T. \quad (22)$$

The power law of Equation (6) is used for the HTS disk with $n = 30$, $E_c = 0.1 \text{ mV/m}$, and $J_c = 2.5 \cdot 10^{10} \text{ A/m}^2$.

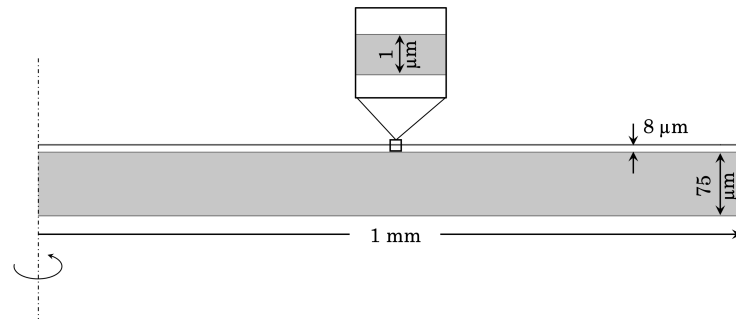


Figure 3. Two-dimensional axisymmetric geometry of the benchmark model.

The proposed surface–volume IE framework, based on a full 3D model of the geometry, is compared against the $H - \phi$ formulation implemented in the FEM software COMSOL[®] Multiphysics. The IE discretization employs $N_s = 5414$ triangular elements for the HTS disk and $N_v = 1146$ tetrahedral elements for the substrate. Exploiting the axial symmetry of the problem, a 2D mesh with 3418 elements is used for the $H - \phi$ FEM formulation. The conductive structure is embedded in an air domain with a circular shape and a radius of 6 mm.

Within the time interval $[0, T]$, the computed AC losses for the HTS disk are presented in Figure 4, together with the instantaneous absolute error, computed as follows:

$$\epsilon(t) = |P_{AC,H-\phi}(t) - P_{AC,J-V}(t)|. \quad (23)$$

The trend of $\epsilon(t)$ is reported by considering three levels of discretization for the HTS, with $N_s = 630$, $N_s = 1844$, and $N_s = 5414$ elements. Moreover, the root mean squared error (RMSE) is computed as follows:

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{k=1}^N (P_{AC,H-\phi}(t_k) - P_{AC,J-V}(t_k))^2}, \quad (24)$$

which is also reported in Table 1 for the three cases.

Table 1. Root mean squared error (RMSE) for three discretization levels.

Discretization	RMSE
$N_s = 630$	$1.22 \cdot 10^{-5}$
$N_s = 1844$	$1.06 \cdot 10^{-5}$
$N_s = 5414$	$6.25 \cdot 10^{-6}$

Figure 5 compares the norm of the current density obtained with the two methods, showing good agreement between the two approaches.

Looking at Figure 5a,c, it can be noticed that in the core of the HTS layer, the inversion of the current density vectors appears and is caught by both modeling strategies.

The effect of the amplitude of the incident flux density is now examined through the evaluation of the magnetization loss density Q expressed by Equation (21) using three values for B_0 : 25 mT, 40 mT, 50 mT, 80 mT, and 100 mT. Figure 6 confirms a good agreement between the proposed strategy and the reference solution computed with the $H - \phi$ approach. Moreover, a straight line in the log-log plane is observed for $Q = Q(B_0)$, allowing the construction of a power-law expression for the loss density in the following form:

$$Q = 1.2 \cdot 10^{-10} \left(\frac{B_0}{B_c} \right)^{2.53}, \quad (25)$$

where $B_c = 35$ mT.

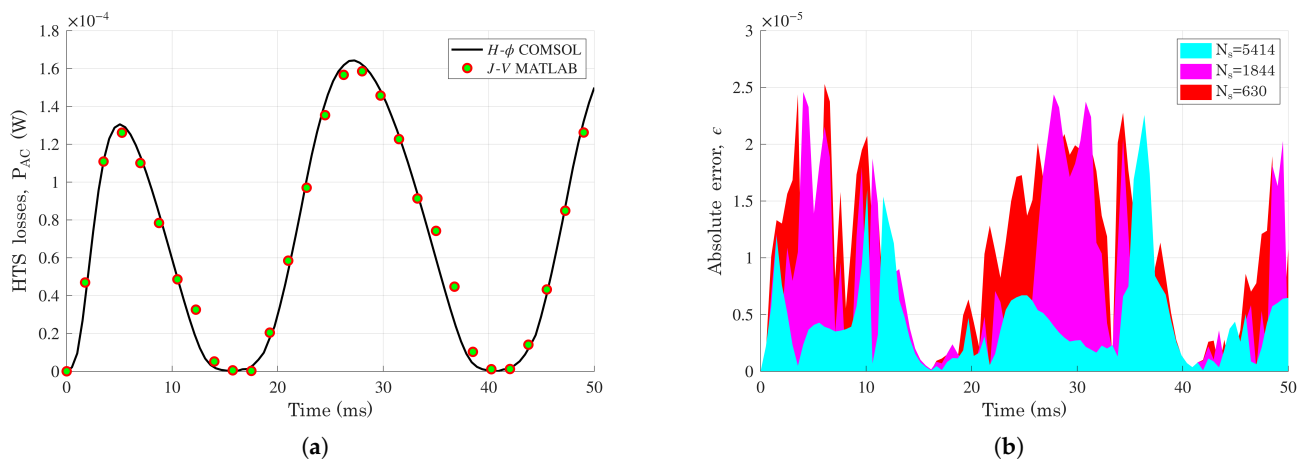


Figure 4. (a) AC losses HTS disk for the benchmark model (b); absolute error.

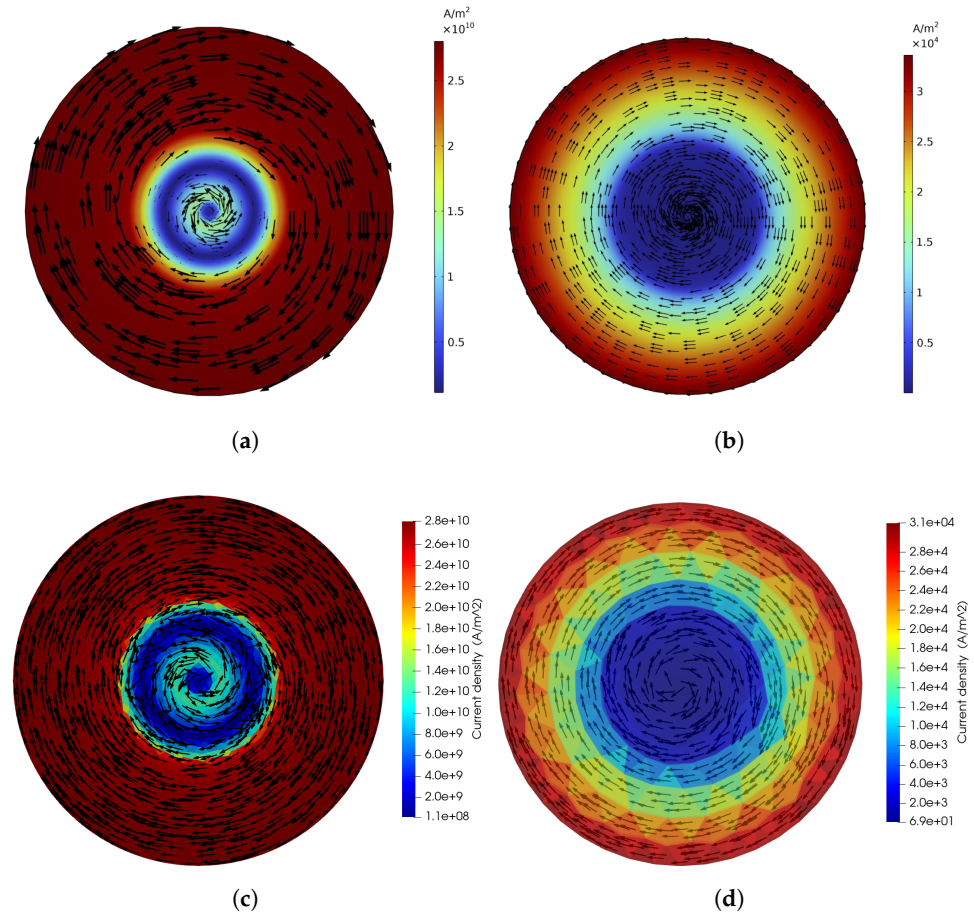


Figure 5. Top view of the current density norm at $t = 50$ ms. (a) HTS disk with $H - \phi$. (b) Iron substrate with $H - \phi$. (c) HTS disk with $J - V$. (d) Iron substrate with $J - V$.

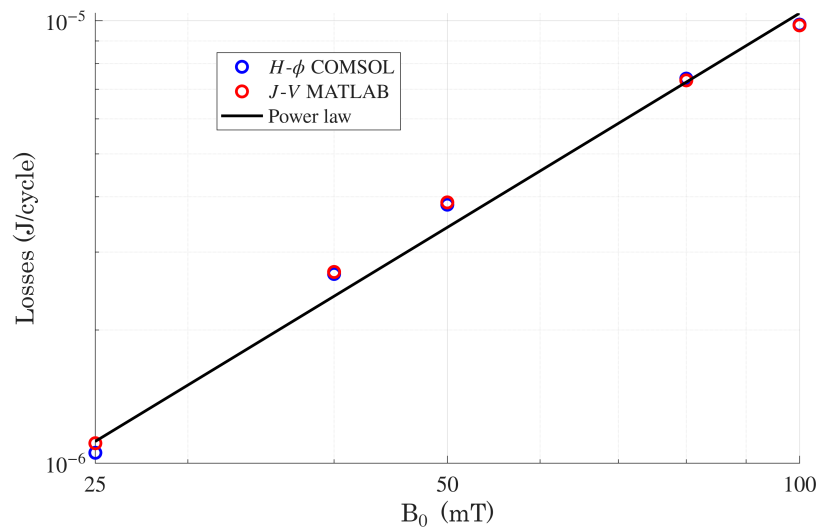


Figure 6. Magnetization loss density for the benchmark model under varying peak value of the external field. The extracted power-law expression of Equation (25), is also included.

3.2. CORC[®] Cable

The CORC[®] cable model illustrated in Figure 1 is analyzed in this section. The cable is characterized by three HTS tapes of $1 \mu m$ thickness, whose resistivity is given in Equation (7) with $n = 30$, $E_c = 0.1$ mV/m, and $J_c = 2.5 \cdot 10^{10}$ A/m². The cylindrical former is made

of Cu with a constant resistivity $\rho = 1.67 \cdot 10^{-8} \Omega\text{m}$. The cable is subjected to an incident electric field, expressed as follows:

$$\mathbf{E}_{inc}(\mathbf{r}, t) = B_0 \pi f_0 \cos(2\pi f_0 t) [-z, 0, x]^T, \quad (26)$$

with $B_0 = 50 \text{ mT}$ and $f_0 = 20 \text{ Hz}$. The transient analysis is solved for one electric period corresponding to $T = 1/f_0 = 50 \text{ ms}$. A triangular mesh with $N_s = 5622$ elements is used for the HTS, while a tetrahedral mesh of $N_v = 3576$ cells is used for the Cu.

The trend of AC losses due to the HTS material and the Cu within the interval $[0, T]$ is reported in Figure 7. It can be noticed that the AC losses are comparable in this case due to the high value of the current density induced in the former. A detailed map of the current density in HTS tapes and the former is illustrated in Figure 8 for the time instant $t = 50 \text{ ms}$. The current density in the HTS tapes is predominantly concentrated near their boundaries, consistent with results reported in the literature [39].

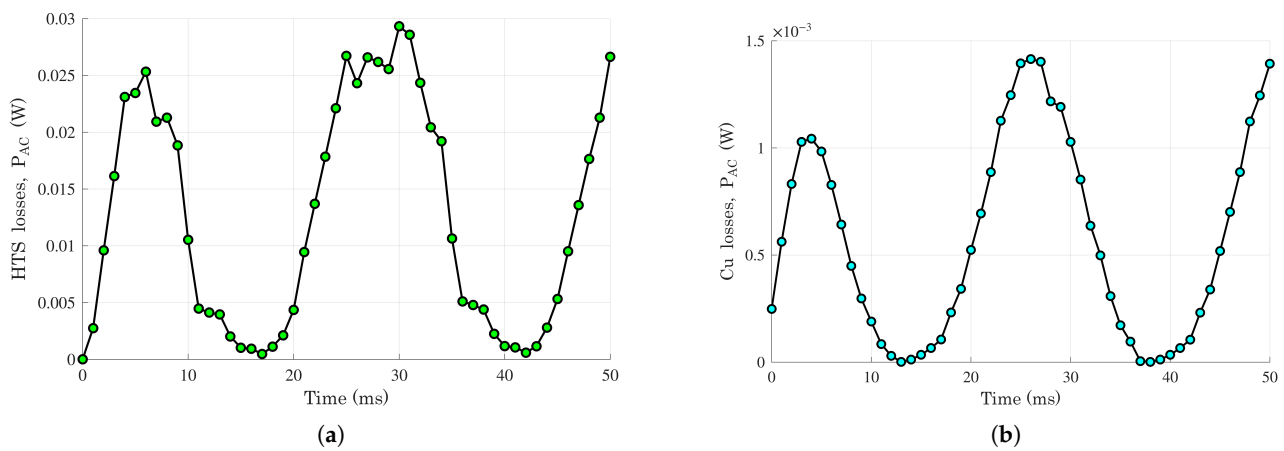


Figure 7. AC losses of CORC® cable computed with the $J - V$ formulation. (a) HTS tapes. (b) Cu former.

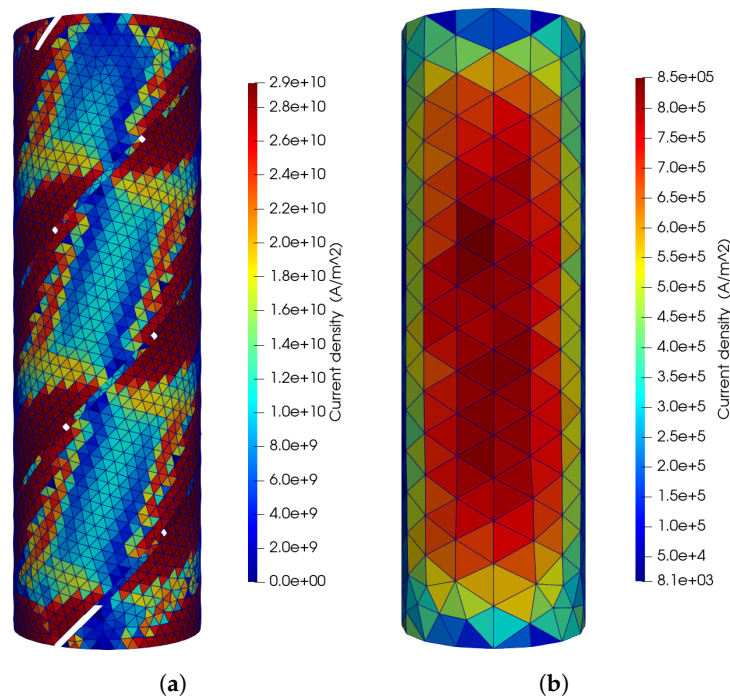


Figure 8. Current density norm at $t = 50 \text{ ms}$ computed with the $J - V$ formulation. (a) HTS tapes. (b) Cu former.

To conclude, a summary of the computational resources, expressed in terms of the solution time, is provided in Table 2. It can be observed that the time required to build the time-independent system matrices and the excitation is negligible compared to that needed for solving the transient nonlinear problem. The latter is carried out using non-optimized proprietary MATLAB scripts, and the code efficiency lies beyond the scope of this study. It is worth mentioning that a major bottleneck is observed in the number of calls of the maps $\mathbf{R}_s(\mathbf{i})$ and $\mathcal{H}_s(\mathbf{i})$ and the solution of the system of Equation (16). In the future, we are planning to explore hierarchical matrix-based solvers and model order reduction techniques to speed up the simulation time.

Table 2. Computational burden of $J - V$ formulation for CORC[®] cable example.

Task	Time
Evaluation of $\mathbf{D}_s, \mathbf{L}_s, \mathbf{e}_{inc,s}$	3.2 s
Evaluation of $\mathbf{D}_v, \mathbf{L}_v, \mathbf{R}_v, \mathbf{e}_{inc,v}$	3.1 s
Evaluation of $\mathbf{L}_{vs}$	4.7 s
Time integration	1 days, 9 h

4. Conclusions

An IEM coupling of surface and volumetric models for the transient nonlinear analysis of CORC[®] cables is presented. By following an IE approach and thereby avoiding the meshing of non-conductive domains, the modeling effort required to solve the electromagnetic problem is significantly reduced. The proposed method, possibly enhanced through acceleration techniques such as hierarchical matrices, is demonstrated to be effective for addressing multiscale problems involving thin superconducting layers and bulk normal materials. Future work will extend the approach to include thermal effects, with the aim of modeling quenching phenomena.

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