

Article

Local Transient Simulations of Feed Rod Melting During Floating-Zone Silicon Crystal Growth

Maksims Surovovs , Stanislavs Luka Strozevs  and Janis Virbulis 

Institute of Numerical Modelling, University of Latvia, 3 Jelgavas Street, LV-1004 Riga, Latvia; stanislavs_luka.strozevs@lu.lv (S.L.S.); janis.virbulis@lu.lv (J.V.)

* Correspondence: maksims.surovovs@lu.lv

Abstract

The present study demonstrates the use of a transient multiphase flow model for the description of feed rod melting dynamics during the floating-zone silicon crystal growth process on a local scale. The presented numerical model is verified using a previously introduced analytical model of the thin melt layer on the open melting front. An artificial increase in viscosity on the liquid–gas boundary is used to minimize numerical effects, and good agreement with the analytical model is achieved in the system parameter range that describes a typical growth process. Simulations with a precise interface shape show that a stable solution is achieved in cases with different initial melt distributions, showcasing the re-establishment of melt flow after the thin melt layer breaks. The influence of flow rate fluctuations is investigated, and the obtained stable solution is maintained in the majority of the considered cases.

Keywords: computer simulation; volume of fluid method; OpenFOAM; floating-zone technique; single crystal growth

1. Introduction

High-purity monocrystalline silicon crystals with a diameter up to 8 inches can be obtained with the help of the floating-zone (FZ) technique, which is based on high-frequency electromagnetic inductive heating. The feed rod, which consists of the source material, is melted down, and the liquid silicon travels downwards to create the bulk melt from which the monocrystal is pulled. The growth of crystals with a large diameter is achieved with a needle-eye inductor—a flat disc-like coil with a circular opening in the center, through which the molten silicon passes. This creates a specific molten-zone geometry with a relatively thin liquid bridge at the top of the bulk melt; the latter is much larger and corresponds to the size of the grown crystal. The surface of the feed rod that is exposed to the inductor coil is called the open melting front (OMF). Its shape is greatly influenced by the inductor coil geometry, and small uneven structures appear on it during the growth process. A schematic view of the FZ crystal growth process is shown in Figure 1, which also highlights the typical dimensions of the irregularities on the OMF surface. In other applications of the FZ method, which do not require the use of the needle-eye inductor, the shape of the molten zone is more uniform, and the OMF is not present [1,2]; therefore, simulations of relatively small crystal growth systems do not deal with the OMF and the local structures on it [3–5].

These local structures can significantly influence the melting process due to large differences in electrical conductivity between liquid and solid silicon. Moreover, it was



Academic Editor: Evgeniy N. Mokhov

Received: 8 July 2026

Revised: 10 August 2026

Accepted: 14 August 2026

Published: 21 August 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and

conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

observed during experiments that the local structures can increase in size, in extreme cases reaching the inductor surface, and cause process breakdown due to electric discharge [6,7]. Particularly large structures were observed at the rim of the feed rod, and additional modification of the growth process was required to reduce the possibility of their formation.

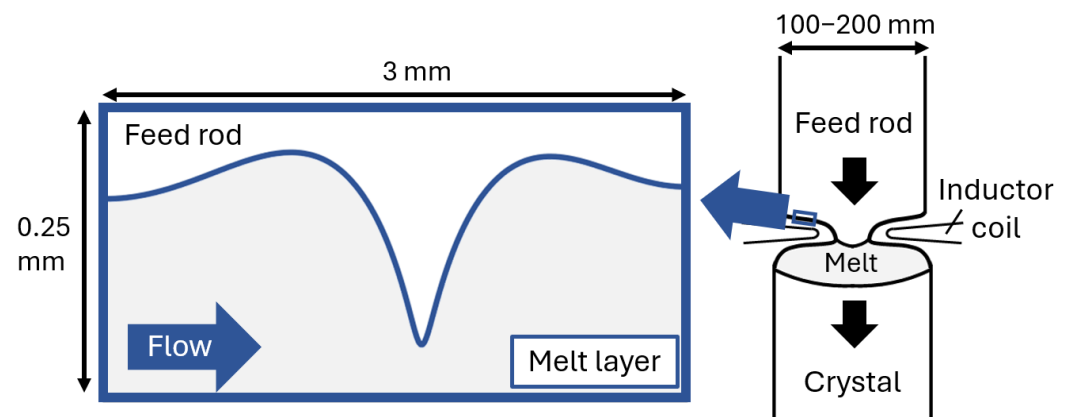


Figure 1. Cross-section schematic view of the floating-zone crystal growth process (right) and the local surface structure domain on the open melting front (highlighted with blue).

Numerical modeling plays a significant role in FZ crystal growth process optimization, and can be used to investigate possible yield improvements. A comprehensive global model of the FZ process for bulk silicon crystal growth was developed in 2003 [8], which allows one to determine the precise shapes of phase boundaries for large industrial FZ technique applications. However, the feed rod melting process is described by a simplified analytical model, which does not include the local irregularities of the surface. The melt layer thickness on the interface is considered to be uniform, with differences in the radial direction being described using specific process parameters [9].

Validation of the OMF shape in simulations can be performed using the experimental measurements of a leftover feed rod obtained at the end of the growth process. For this purpose, the termination of the process is performed quickly to ensure that the OMF shape does not change during the solidification process, and the feed rod rotation is stopped to reduce the influence of unsteady effects [10]. An alternative method of interface validation uses experimental images taken during the growth process. Due to visual obstruction of the molten zone by the high-frequency inductor coil, only a small part of the OMF can be compared, and the calibration of the interface shapes is performed mainly using the bulk melt free surface and the radius of the internal triple point [11]. These comparisons demonstrate the correspondence of the overall shape of phase boundaries, ignoring the local structures that can be seen on the OMF in experimental photos of the process [7,10].

The specific shape of the local irregularities on the OMF was first considered by Bergfelds et al. in 2015 [12]. The developed 2D model described the transient melting and solidification process using the calculated temperature field. Since then, the model has been augmented with melt flow simulations and utilization of moving meshes. The latest model allows one to precisely calculate the quasi-stationary shapes of the local structures based on the heat flux balance at the interface [13]. Using the results obtained by this model, full FZ process simulations were performed and compared to experimental measurements of a standard 4 inch FZ process [10], leading to an improved agreement of the OMF shape.

Due to the quasi-stationary nature of the aforementioned local model, one of its largest drawbacks is the lack of insight regarding the process stability. Transient simulations are required to determine the evolution of the melt layer thickness in time for a given local structure shape. In addition, such simulations would allow one to investigate the process

of the breaking up of the melt layer, for example, when the interface perturbation grows so much that the melt flow is disrupted.

Other technological and industrial applications, such as electromagnetic levitation [14], metallurgical refinement of silicon [15], or electromagnetic stirring of liquid metals [16], were successfully described using multiphase free surface flow models, allowing investigation of the transient melt shape evolution during the process. However, in the context of large-diameter floating-zone silicon crystal growth, multiphase flow models are not utilized, and simulations of melt flow are typically limited to the bulk melt [17,18]. The melting of the feed rod is usually described on the global scale, using a fixed axisymmetric interface shape obtained from measurements [19] or approximated with Bézier curves [20]. Active melting is calculated with the help of effective melt layer thickness [8], while the presence of local structures is neglected.

2. Numerical Model

Simulations were performed using a two-dimensional model based on the volume of fluid method implementation in the OpenFOAM v2212 software [21]. The standard installation of OpenFOAM includes preconfigured solvers with built-in volume of fluid algorithm, such as interFoam, which was selected for simulations. The solver provides a solution algorithm for Navier–Stokes equation in case of two incompressible, isothermal immiscible fluids. Laminar flow is considered due to relatively low Reynolds number $Re \approx 200$. Gravity and surface tension forces are taken into account, and the melt region is defined using a phase fraction field. The physical properties of each cell in the simulation domain are determined using the fraction function α , which requires an additional equation to obtain the shape of the interface between the two phases:

$$\frac{\partial \alpha}{\partial t} + \frac{\partial(\alpha u_j)}{\partial x_j} = 0, \quad (1)$$

where u_j is a component of the melt flow velocity field.

Due to resource-intensive multiphase simulations, a simplified approach is employed for the description of the solid–liquid boundary. The melting of solid silicon is not considered due to significantly slower time scale—melting rate of 2 mm/min compared to melt flow values of 10 cm/s. Instead, constant melt flow boundary conditions are applied to the inlet, which approximate the influx of melt from other regions on the OMF. The solid silicon region is excluded from the simulations, and the solid–liquid interface shape is defined according to the results of the quasi-stationary local model [13]. A constant wetting contact angle condition of 30° is used for the phase fraction field, based on the experimental data of wetting angles during the crystal pulling process [22].

Figure 2 shows the simulation region in case of different initial phase fraction distributions. The simulation domain is 3 mm wide, which is the characteristic length of a single interface perturbation, and 1.7 mm tall, which allows for large melt volume changes.

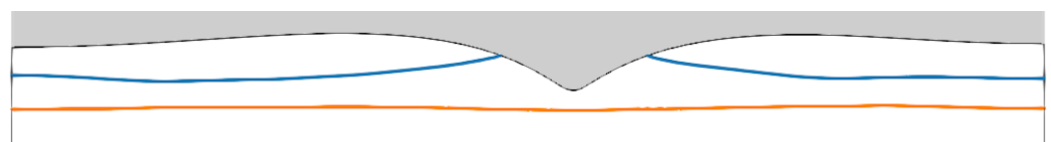


Figure 2. The initial conditions for simulations of specified interface geometry with filled melt layer (orange) and broken layer (blue). Simulation region consists of silicon melt and gas atmosphere, while solid silicon (gray) is excluded.

2.1. Electromagnetic Field

The transient melt flow simulations are coupled with a stationary electromagnetic (EM) solver, because a stationary solution for the EM field is reached much faster than the system geometry changes caused by melt flow can be observed. At the beginning of each time step, the material conductivity field is updated based on the phase fraction field. The EM field calculations are not connected with physical time and are only used to reach the steady-state solution for the specific geometry. The considered model is two-dimensional, which means that the solution can be expressed in terms of the z-component of the magnetic vector potential A_z in the direction perpendicular to the problem domain:

$$\nabla^2 A_z - i\omega\sigma\mu A_z = 0, \quad (2)$$

where ω is the angular frequency of the EM field, σ is the electrical conductivity, and μ is the magnetic permeability.

Simplified boundary conditions of $A_z = 0$ and $A_z = A_0$ are used for the solid silicon surface and the lower domain boundary, respectively. Thus, the lower boundary represents the surface of the inductor coil, and the value A_0 is selected to ensure realistic distance between the inductor and the OMF, using the results of the previously developed quasi-stationary model [13].

Using the fraction field values, it is possible to calculate the induced current density $j_z = -i\omega\sigma A_z$ inside the melt layer, which interacts with the magnetic field $\vec{B}$ to produce the Lorentz force $\vec{f}_L = \vec{j} \times \vec{B}$. The resulting force values are obtained for the specific melt geometry at each time step and added as a term in the Navier–Stokes equation. Lorentz force distribution is obtained in simulations and its effect on the molten silicon layer thickness is shown in Section 3.1.3.

2.2. Liquid–Gas Interface

Due to the very high surface tension of molten silicon, sharp interface changes that occur at the edges of mesh elements can result in unrealistically high velocity values, which are known as spurious currents [23] and are an inherent disadvantage of the volume of fluid method. One of the attempts to mitigate the influence of these unphysical effects within the OpenFOAM framework was the development of an improved solver, *isoAdvector* [24,25], which provides an alternative method of interface compression and sharpening. Another example of numerical stability improvement was introduced with regard to turbulent transition models, in which spurious pressure fluctuations were suppressed using gradient-driven artificial viscosity [26].

Simulations described in this study are performed with the *interFoam* solver without additional interface sharpening. Even though spurious currents were created near the phase boundary, their effect was less significant if the simulations were performed with artificially increased liquid silicon viscosity. To minimize the effect of such a physical parameter change, viscosity was increased only at the liquid–gas interface. The solver was modified to include a mesh cell selection algorithm that increases viscosity in cells that have phase fraction values of $0.01 < \alpha < 0.99$. The impact of such a modification is discussed in Section 3.2.1.

3. Results

3.1. Model Verification

First, simulations were performed with a simplified geometry, which consists of a completely flat solid–liquid interface. This allows the comparison of the obtained results with previous analytical approaches that considered melt layer thickness to be uniform

along the interface and neglected the local structures. According to Ratnieks [9], the fluid film is assumed to change smoothly in the meridional direction, and the flow inside it has a parabolic velocity profile. The integral of the velocity profile is proportional to the fluid discharge at a fixed location, and the melt layer thickness value is derived taking into account the tangential component of gravity. The melt layer thickness can be calculated for any specific position on the interface r :

$$h = \left[\frac{3\eta\rho_S V_F (R_F^2 - r^2)}{2\rho_l^2 g \sin \theta(r) \cdot r} \right]^{1/3}, \quad (3)$$

where η is the dynamic viscosity, ρ is the corresponding phase density, V_F is the feed rod move rate, $\vec{g}$ is the free fall acceleration, R_F is the feed rod radius, and θ is the interface slope angle. Knowing the melt layer thickness, it is possible to calculate the ratio of the induced EM heat between the solid and the liquid region, which allows us to calculate the OMF shape precisely during global crystal growth process simulations.

3.1.1. Outlet Boundary Conditions

Due to the small region size, the surface tension forces significantly influence the shape of the free surface, and additional measures are necessary to prevent return flow, which would not be possible in a realistic system. Therefore, boundary conditions were modified to prohibit return flow using the `inletOutlet` condition. In case of outflow, the calculated velocity values are used. However, if the flow is directed inside the simulation domain, it is overwritten with either a zero value or forced outflow directed out of the simulation domain. This type of boundary condition is used to ensure that the overall melt flow is directed correctly, which would correspond to a realistic situation.

Simulations were performed with different prescribed outflow values, and the evolution of average melt layer thickness is shown in Figure 3. The restriction of back flow alone is not enough to reach a realistic melt layer thickness. Due to the high surface tension, the outflow was too low even in moments without any return flow. A specific velocity value can be found to ensure that the melt layer thickness corresponds to the theoretical value, which was obtained with the previously considered analytical model.

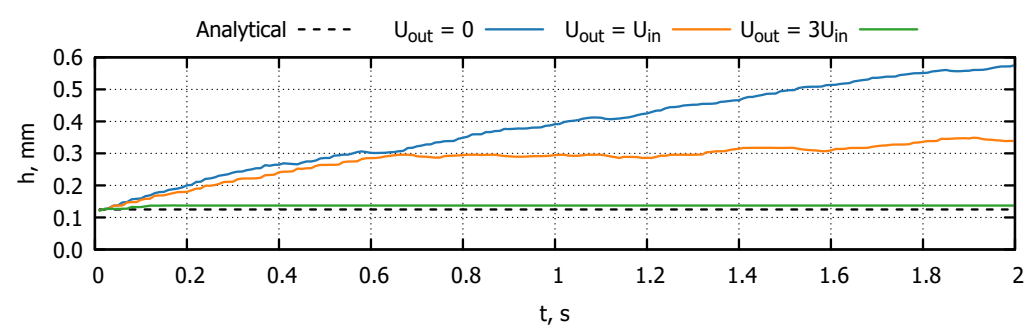


Figure 3. Average melt layer thickness in time with flat interface shape and different outlet boundary conditions. U_{in} corresponds to the average velocity value on the inlet boundary, which is defined using constant flow rate boundary conditions.

3.1.2. Influence of Process Parameters

The analytical value of the melt layer thickness (Equation (3)) depends on the position on the interface r , which is characterized by the interface slope angle θ and the inlet flow rate $Q = V_F(R_F^2 - r^2)$. Due to the limited domain size of the local model, only one specific position can be considered at a time. To evaluate the applicability of the local model to the whole interface, simulations were performed with different θ and Q values. Figure 4 shows

the average melt layer thickness for the parameter range that corresponds to the variance on the interface in a typical crystal growth system. Overall, qualitative agreement with the analytical model is obtained. The largest difference is present in the case of a very high melt flow rate, which would correspond to locations near the center of the OMF. The largest local structures are usually observed near the feed rod rim, meaning that limited accuracy of the model near the center of the system is not critical for the presently considered application.

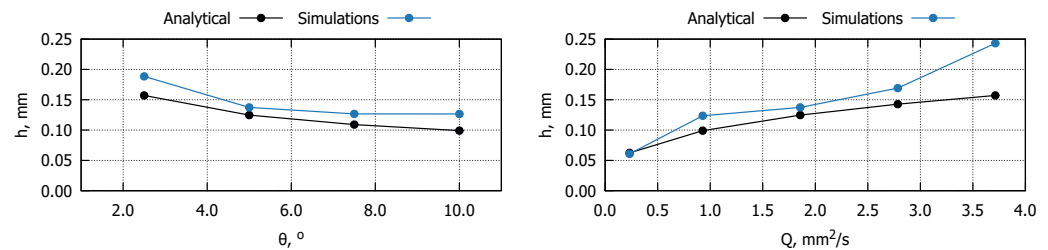


Figure 4. Average melt layer thickness of the stable solution in case of different interface slope angle θ (left) and inlet flow rate Q (right).

3.1.3. Influence of EM Field

To evaluate the influence of the EM field on the melt layer thickness and its stability, simulations were performed with both flat interface geometry and a specified surface perturbation, described in more detail in Section 3.2. Figure 5 shows the distribution of the Lorentz force inside the melt layer at a time step when a stable solution is reached. The induced forces are concentrated near the surface of the melt layer and are directed towards the solid feed rod, acting as a stabilizing pressure force. However, the overall effect of the Lorentz force on the resulting melt layer thickness is rather small, as shown in Figure 6. In the case of a completely flat interface, the difference is $\approx 7\%$, while the values in simulations with specified interface perturbation are almost identical. It can be concluded that the Lorentz force has minimal influence on the flow inside the melt layer, and the applied simplified boundary conditions for the EM simulations are adequate. This result is consistent with the previously published consideration that the impact of the Lorentz force on the fluid film is negligibly small [9,27].

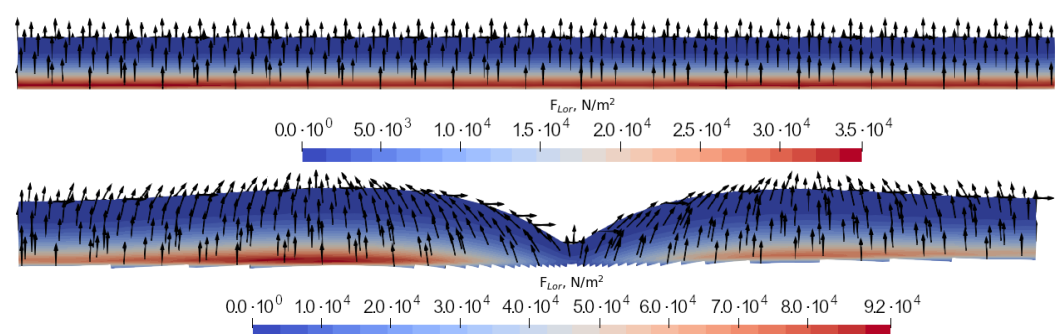


Figure 5. Distribution of the Lorentz force inside the melt layer with arrows representing the direction of the force vectors in simulations with flat interface shape (top) and specified interface perturbation (bottom).

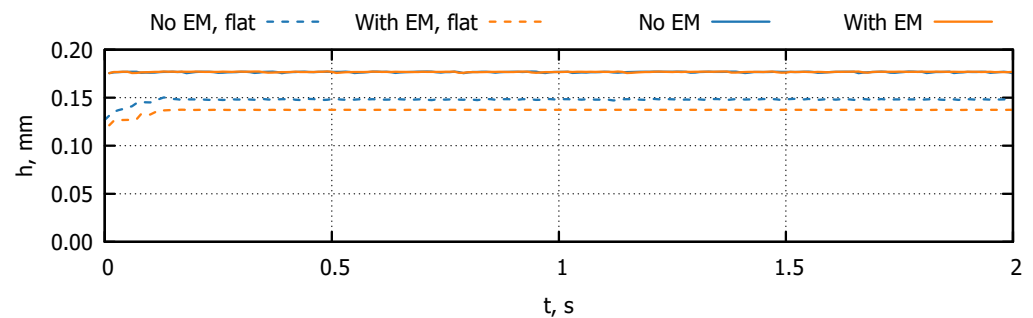


Figure 6. Average melt layer thickness in time in case of simulations without and with EM field.

3.2. Simulations with Specified Interface Geometry

After the simulations with simplistic geometry, in which the optimal boundary conditions were determined, the solid–melt interface shape was specified using the converged solution of the quasi-stationary model. Simulations were performed with both a filled melt layer, to test the stability of the obtained solution, and a broken melt layer, to investigate the process of fluid film reconnection after breaking. The differences in initial phase fraction distribution are shown in Figure 2.

3.2.1. Artificial Increase in Viscosity

As mentioned in Section 2.2, the use of the volume of fluid method in the case of high surface tension of molten silicon results in spurious currents near the liquid–gas interface. These unphysical velocity maxima can significantly influence the stability of the obtained solution and even result in repeated breaking of the melt layer. Initial simulations with a filled melt layer and actual liquid silicon viscosity did not reach a stable solution, as shown by the melt evolution in time in Figure 7 (an animation of the melt shape change is included in Supplementary Materials). Spurious velocity near the interface causes the melt layer to oscillate, and it breaks after $t \approx 0.15$ s. After breaking, the droplet on the left side increases in volume due to constant inflow until it can flow over the perturbation. However, even after the flow is re-established, the melt layer keeps oscillating due to high spurious velocity values near the interface and breaks again. This process is shown in Figure 8, which presents the average melt layer thickness in time. The large amplitude fluctuations characterized by the sharp decrease in the average value and the subsequent gradual increase show moments when the melt layer was broken, and the melt collected on one side of the perturbation.

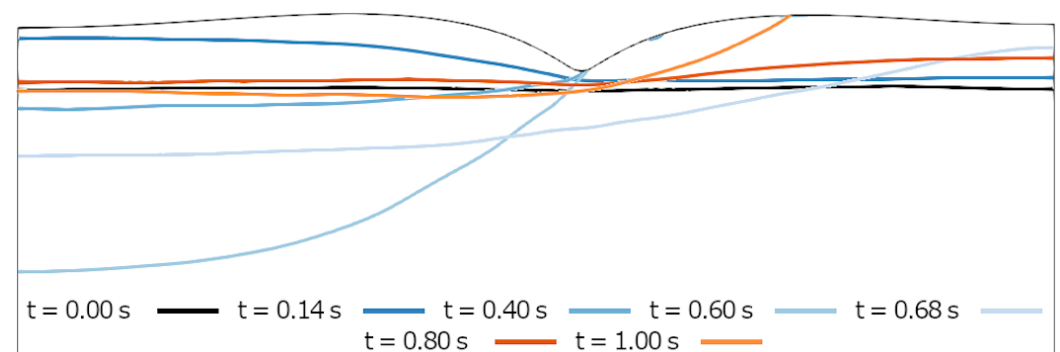


Figure 7. Melt shape at different time steps in case of specified interface geometry with realistic liquid silicon viscosity values.

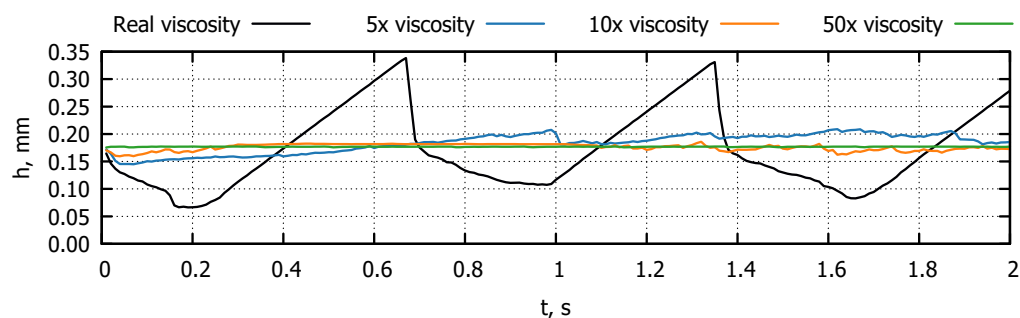


Figure 8. Average melt layer thickness in time in case of specified interface geometry with artificially increased viscosity values on the melt–gas boundary.

To prevent such spurious fluctuations, viscosity was artificially increased in the cells on the melt–gas interface, determined by the phase fraction values of $0.01 < \alpha < 0.99$. Even a small increase in viscosity can disrupt this cyclic process and result in a more stable solution. Simulations with five-times-higher viscosity values do not result in breaking of the fluid film; however, small oscillations caused by spurious velocity can still be observed. To obtain the fully converged melt layer thickness, at least 50-times-higher viscosity is needed. It is worth noting that all of the modified cases result in approximately the same average value, which means that the local increase in viscosity near the melt–gas interface can be an effective tool for increasing stability without compromising the underlying mechanisms that determine the resulting melt layer thickness. Therefore, all further presented simulations were performed with 50-times-increased viscosity on the melt–gas boundary.

3.2.2. Melt Layer Evolution After Breaking

One of the aims of the present study was to investigate the possibility of a stable melt layer configuration after the initial fluid film is broken. The melt layer evolution in time during such an event is shown in Figure 9 (an animation of the melt shape change is included in Supplementary Materials). Due to gravity, the disconnected melt part on the right side of the obstacle flows away, while the melt directly before the interface perturbation increases in size due to the constant influx of liquid silicon from the left side. This process continues steadily until the created droplet can flow over the perturbation, taking into account the constant contact angle boundary condition of 30 degrees. Once the liquid layer reaches the outlet surface, the melt layer thickness rapidly decreases and a stable solution is reached. As shown in Figure 10, the same layer thickness is obtained with different initial conditions, which means that a stable solution exists for a specific set of process parameters, and disruptions of the melt layer do not influence the resulting melt layer shape.

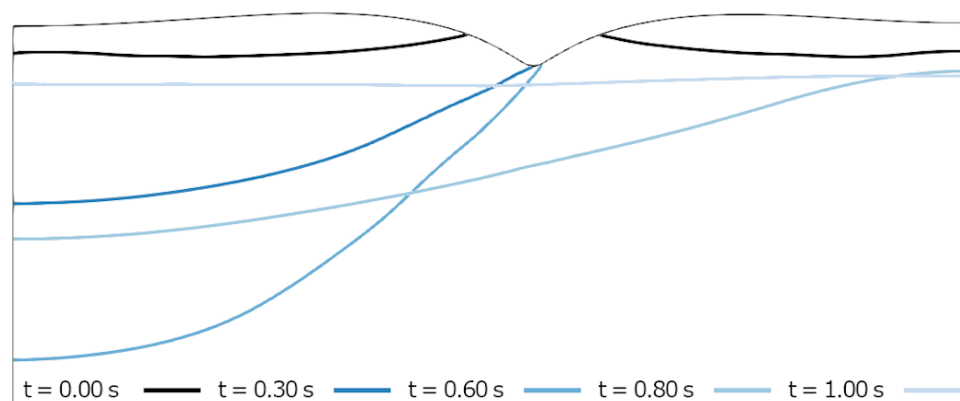


Figure 9. Melt shape at different time steps in case of initial conditions with broken melt layer.

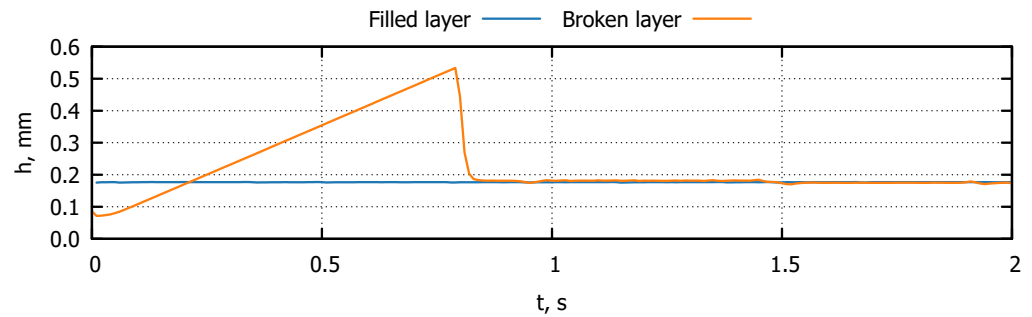


Figure 10. Average melt layer thickness in time in case of specified interface geometry with different initial conditions.

Figure 11 shows the velocity distribution inside the melt layer after the stable solution is reached. The flow velocity maximum of ≈ 10 cm/s is located in the center of the domain, where the melt layer thickness is the smallest. The flow is mainly uniform on the left side of the surface perturbation. However, the obstacle significantly disrupts the flow, resulting in higher velocity values near the surface of the melt and a small region with flow in the opposite direction near the solid surface on the right side of the perturbation.

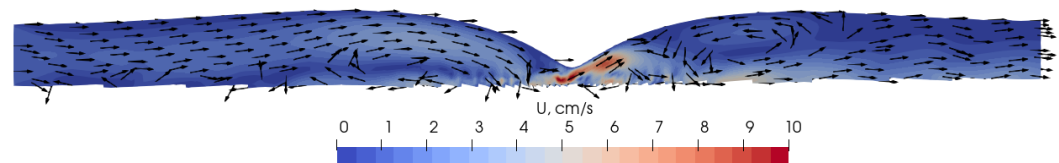


Figure 11. Flow velocity distribution inside the stable melt layer with arrows representing the direction of the velocity vectors.

3.2.3. Influence of Interface Shape

In an attempt to better describe a wider range of positions on the melting interface, as well as to investigate the melt flow dynamics at different points of local structure formation, simulations were performed with different perturbation shapes (expressed as a percentage of the previously used specified surface perturbation). The change in amplitude influences both the position of the peak, and the height of the curves on the sides. All of the simulations were performed with a filled melt layer as the initial condition, and none of the considered geometries resulted in the breaking of the thin fluid film.

The considered interface shapes and the summary of the resulting melt layer thickness are shown in Figure 12. The minimum values are obtained when measuring the distance from the perturbation peak to the melt surface, while the maximum values are observed on the sides, where the curvature of the solid surface is the highest. The interface is positioned at an angle to the gravity vector; therefore, there is a slight asymmetry between the two sides. A higher value of the maximum melt layer thickness was observed on the left side in all of the cases.

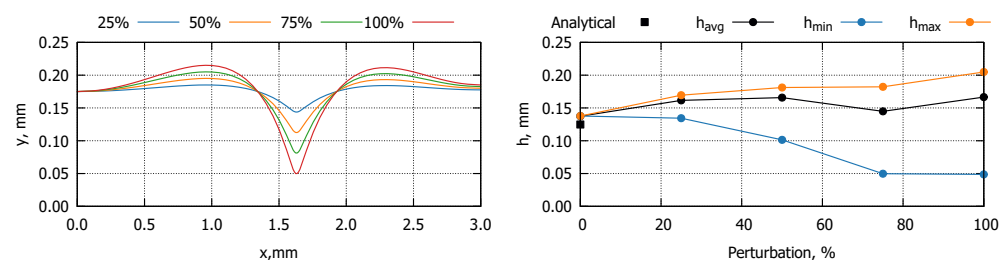


Figure 12. Comparison of different interface shapes described as percentage of specified surface perturbation (**left**) and the resulting average, minimum, and maximum melt layer thickness (**right**).

In the case of a completely flat interface (0%), the melt layer thickness is uniform. A larger height of the solid surface on the sides in the case of a non-flat interface results in a uniform increase in the maximum thickness, while the minimum value becomes smaller due to the peak approaching the melt surface. The change in average thickness is not uniform, which can mainly be attributed to a slight shift of the melt surface. Nevertheless, it generally increases with the increase in the perturbation size, and a stable solution was obtained in all cases.

3.2.4. Investigation of Melt Layer Stability

Previously presented simulation results were all obtained using constant flow rate boundary conditions on the inlet surface. This simplified approach is applicable because the simulation domain is significantly smaller than the total size of the OMF, so the inflow of molten silicon from other areas is averaged in time. However, this average flow rate value can be expected to fluctuate due to repeated breaking and reconnection of the molten silicon layer in parts of the interface that surround the simulation domain. The influence of neighboring regions was introduced by modifying the boundary conditions, and simulations with fluctuating flow rate values were performed to evaluate the stability of the melt layer.

In all of the considered cases the time-averaged flow rate corresponds to the previously used constant value. Harmonic fluctuations with amplitude up to 100% of this value and frequency from 0.5 to 4 Hz were considered for the boundary conditions of the inlet flow rate, as shown in Figure 13. Lower-frequency fluctuations correspond to the outer parts of the OMF, in which there could possibly be only a few neighboring regions with local structures upstream from the simulation domain. Moving closer to the center of the OMF increases the number of such regions, so the frequency of melt layer breaking and reconnection events increases.

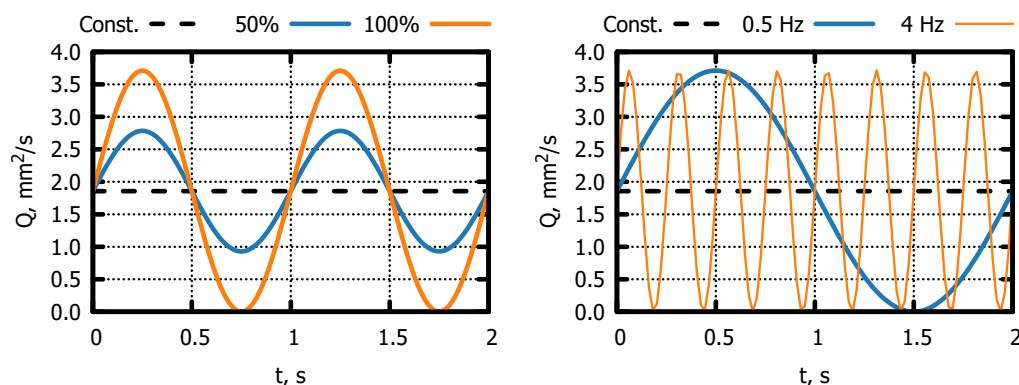
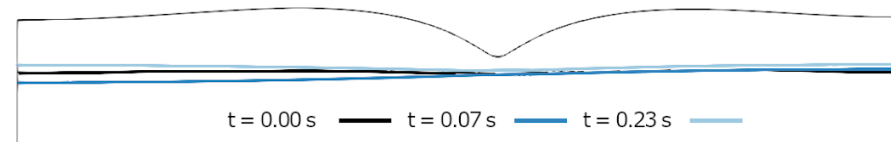


Figure 13. Inlet flow rate values in time in case of different amplitude (left) and frequency (right).

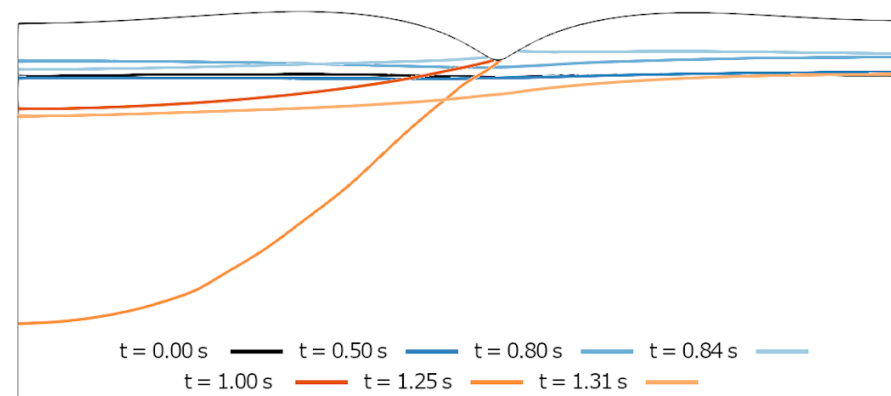
Figure 14 showcases two examples of simulations with large fluctuation amplitude. Animations of the melt shape change for the corresponding cases are included in Supplementary Materials. For high frequency (4 Hz) fluctuations, the change in melt layer thickness is rather small, and the flow is maintained over the whole domain. In the case of 1 Hz frequency, the melt layer thickness is only slightly increased in the period of time with flow rate values greater than average. After 0.50 s, as the flow rate value decreases, so too does the melt layer thickness, until the layer breaks at $t \approx 0.84$ s. The remaining melt droplet on the left side of the perturbation grows as the flow rate increases, and at $t \approx 1.31$ s it flows over the obstacle.

However, this process repeats again at $t \approx 1.8$ s, as can be inferred from the average melt layer thickness in time shown in Figure 15. This cyclic breaking and reconnection

of the melt layer is observed only in the case of large fluctuations with small frequency. For fluctuations with amplitude of 75% and smaller, the flow is maintained throughout the whole simulation time, and the average layer thickness is comparable to the reference case with constant flow rate. The frequency of flow rate fluctuations has no effect on the thickness value averaged in time, and the layer is conserved at all amplitudes in the case of sufficiently high frequency (4 Hz).

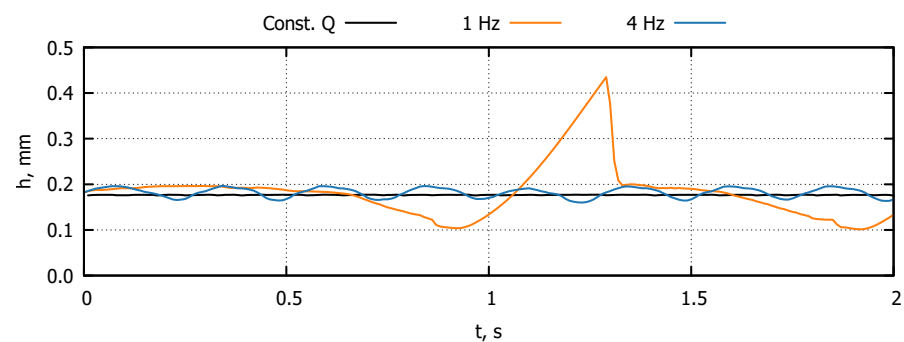


(a) 4 Hz, 100% amplitude

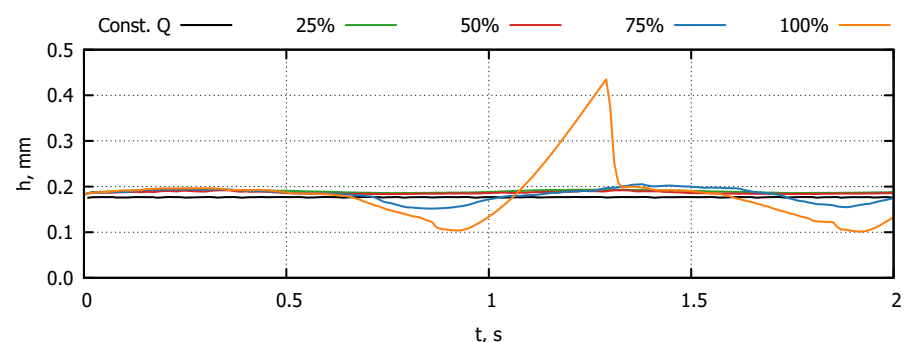


(b) 1 Hz, 100% amplitude

Figure 14. Melt shape at different time steps in case of inlet flow rate fluctuations.



(a) 100% amplitude, different frequency



(b) 1 Hz, different amplitude

Figure 15. Average melt layer thickness in time in case of inlet flow rate fluctuations with different frequency and amplitude.

4. Discussion

The volume of fluid method implementation in OpenFOAM was successfully applied to the thin silicon melt layer to describe the melting dynamics during the floating-zone crystal growth process. Even though this specific application can produce spurious currents due to the very high surface tension of liquid silicon, an artificial increase in viscosity inside the melt near the liquid–gas interface makes it possible to improve the stability of the solution without significant changes in the resulting melt layer thickness.

Forced outflow boundary conditions are required on the outlet surface to ensure correspondence to the analytical model of the melt layer thickness in the case of a flat interface. Qualitative agreement of the melt layer thickness with the previously used analytical model was reached in the system parameter range, which represents the open melting front in a typical crystal growth process.

Transient simulations show that the flow is re-established after the breaking of the film in the case of specified melting interface shape, which corresponds to the realistic geometry obtained in quasi-stationary simulations. A stable solution is reached in cases with different initial conditions and surface perturbation shapes. The average melt layer thickness remains constant even in the case of inlet flow rate fluctuations; the layer is broken only in the case of large fluctuations with low frequency, which are expected to occur mainly in the outer parts of the open melting front.

The main limitations of the presented model are the small domain size, which makes it possible to describe different positions on the interface only effectively through parameter changes and boundary conditions; and the two-dimensional nature, which only allows the use of a symmetrical interface shape and does not describe the azimuthal melt flow changes. In the future, 3D geometry needs to be considered to investigate more complicated situations, which could include irregular shapes of the perturbation and only partial obstruction of the melt flow.

Supplementary Materials: The following supporting information can be downloaded at <https://www.mdpi.com/article/10.3390/cryst16080547/s1>.

Author Contributions: Conceptualization, M.S. and J.V.; methodology, M.S.; software, M.S. and S.L.S.; validation, M.S. and S.L.S.; investigation, M.S.; data curation, M.S.; writing—original draft preparation, M.S.; writing—review and editing, J.V. and S.L.S.; visualization, M.S.; supervision, J.V.; project administration, J.V.; funding acquisition, M.S. All authors have read and agreed to the published version of the manuscript.

Funding: The presented research was performed with financial support from the Project No.: 1.1.1.8/1/24/I/003 Strengthening the Research and Development Capacity of Doctoral Studies at the University of Latvia in the Fields of Smart Specialisation.

Data Availability Statement: The original contributions presented in this study are included in the article/Supplementary Material. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

Abbreviations

The following abbreviations are used in this manuscript:

FZ	Floating zone
EM	Electromagnetic
OMF	Open melting front

References

- Jin, C.; Okano, Y.; Minakuchi, H.; Dost, S. Numerical simulation of thermo-solutal Marangoni convection in a full floating zone with radiative heat transfer under zero gravity. *J. Cryst. Growth* **2021**, *570*, 126204. [\[CrossRef\]](#)
- Kikugawa, N. Recent progress of floating-zone techniques for bulk single-crystal growth. *Crystals* **2024**, *14*, 552. [\[CrossRef\]](#)
- Vieira, L.; Buchovska, I.; Tsiapkinis, I.; Wintzer, A.; Dadzis, K.; Menzel, R. Simulation of crucible-free growth of monocrystalline silicon fibres for mirror suspension in gravitational-wave detectors. *J. Cryst. Growth* **2024**, *629*, 127549. [\[CrossRef\]](#)
- Tsiapkinis, I.; Wintzer, A.; Dadzis, K. Validation of 3D and 2D thermal and electromagnetic models for high-frequency induction heating in crystal growth processes. *J. Cryst. Growth* **2024**, *643*, 127800. [\[CrossRef\]](#)
- Tsiapkinis, I.; Dadzis, K. Development of an open-source heat and mass transfer model for Floating Zone growth. *Int. J. Heat Mass Transf.* **2026**, *260*, 128378. [\[CrossRef\]](#)
- Wünscher, M. Crucible-Free Crystal Growth of Germanium—Experiments and Simulations. Ph.D. Thesis, Technische Universität Berlin, Berlin, Germany, 2011.
- Menzel, R. Growth Conditions for Large Diameter FZ Si Single Crystals. Ph.D. Thesis, Technische Universität Berlin, Berlin, Germany, 2013.
- Ratnieks, G.; Muižnieks, A.; Mühlbauer, A. Modelling of phase boundaries for large industrial FZ silicon crystal growth with the needle-eye technique. *J. Cryst. Growth* **2003**, *255*, 227–240. [\[CrossRef\]](#)
- Ratnieks, G. Modelling of the Floating Zone Growth of Silicon Single Crystals with Diameter up to 8 Inch. Ph.D. Thesis, University of Latvia, Riga, Latvia, 2007.
- Plāte, M.; Dadzis, K.; Krauze, A.; Menzel, R.; Virbulis, J. Validation of a 3D mathematical model for feed rod melting during floating zone Si crystal growth. *J. Cryst. Growth* **2019**, *521*, 46–49. [\[CrossRef\]](#)
- Surovovs, K.; Strozovs, S.L.; Surovovs, M.; Menzel, R.; Ratnieks, G.; Virbulis, J. Calibration of a Melt Flow Model for Silicon Crystal Growth with the Floating Zone Method. *Crystals* **2025**, *15*, 667. [\[CrossRef\]](#)
- Bergfelds, K.; Virbulis, J.; Krauze, A. Modelling of pattern formation during the melting of silicon by a HF EM field. *Magnetohydrodynamics* **2015**, *51*, 397–406. [\[CrossRef\]](#)
- Surovovs, M.; Sjomkane, M.; Virbulis, J.; Ratnieks, G. Numerical modelling of melting front structures during floating zone silicon crystal growth. *J. Cryst. Growth* **2024**, *643*, 127788. [\[CrossRef\]](#)
- Spitans, S.; Jakovics, A.; Baake, E.; Nacke, B. Numerical modeling of free surface dynamics of melt in an alternate electromagnetic field: Part I. Implementation and verification of model. *Metall. Mater. Trans. B* **2013**, *44*, 593–605. [\[CrossRef\]](#)
- Geza, V.; Vencels, J.; Zageris, G.; Pavlovs, S. Numerical study of surface waves generated by low frequency EM field for silicon refinement. *Iop Conf. Ser. Mater. Sci. Eng.* **2018**, *424*, 012049. [\[CrossRef\]](#)
- Dzelme, V. Electromagnetically Driven Liquid Metal Flows with Strong Free Surface Dynamics. Ph.D. Thesis, University of Latvia, Riga, Latvia, 2024.
- Surovovs, K.; Muiznieks, A.; Sabanskis, A.; Virbulis, J. Hydrodynamical aspects of the floating zone silicon crystal growth process. *J. Cryst. Growth* **2014**, *401*, 120–123. [\[CrossRef\]](#)
- Han, X.F.; Liu, X.; Nakano, S.; Harada, H.; Miyamura, Y.; Kakimoto, K. 3D numerical study of the asymmetric phenomenon in 200 mm floating zone silicon crystal growth. *J. Cryst. Growth* **2020**, *532*, 125403. [\[CrossRef\]](#)
- Wünscher, M.; Menzel, R.; Riemann, H.; Lüdge, A. Combined 3D and 2.5D modeling of the floating zone process with Comsol Multiphysics. *J. Cryst. Growth* **2014**, *385*, 100–105. [\[CrossRef\]](#)
- Vieira, L.; Menzel, R.; Holena, M.; Dropka, N. An analysis of elusive relationships in floating zone growth using data mining techniques. *Adv. Theory Simul.* **2025**, *8*, 2400781. [\[CrossRef\]](#)
- Weller, H.G.; Tabor, G.; Jasak, H.; Fureby, C. A tensorial approach to computational continuum mechanics using object-oriented techniques. *Comput. Phys.* **1998**, *12*, 620–631. [\[CrossRef\]](#)
- Satunkin, G. Determination of growth angles, wetting angles, interfacial tensions and capillary constant values of melts. *J. Cryst. Growth* **2003**, *255*, 170–189. [\[CrossRef\]](#)
- Harvie, D.J.; Davidson, M.; Rudman, M. An analysis of parasitic current generation in volume of fluid simulations. *Appl. Math. Model.* **2006**, *30*, 1056–1066. [\[CrossRef\]](#)
- Roenby, J.; Bredmose, H.; Jasak, H. IsoAdvector: Geometric VOF on general meshes. In *Proceedings of the OpenFOAM®: Selected Papers of the 11th Workshop*; Springer: Berlin/Heidelberg, Germany, 2019; pp. 281–296.
- Gamet, L.; Scala, M.; Roenby, J.; Scheufler, H.; Pierson, J.L. Validation of volume-of-fluid OpenFOAM® isoAdvector solvers using single bubble benchmarks. *Comput. Fluids* **2020**, *213*, 104722. [\[CrossRef\]](#)

26. D'Alessandro, V.; Falone, M.; Giammichele, L.; Ricci, R. A robust intermittency equation formulation for transition modeling in Spalart–Allmaras simulations of airfoil flows over a wide range of Reynolds numbers. *Phys. Fluids* **2025**, *37*. [[CrossRef](#)]
27. Bioul, F. Use of Mathematical Expansions to Model Crystal Growth from the Melt Under the Effect of Magnetic Fields. Ph.D. Thesis, UCL-Université Catholique de Louvain, Louvain-la-Neuve, Belgique, 2007.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.