



Universal Logic Expression and the Application of Conditional Probability[†]

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Abstract: In deep integration of universal logic and factor space theory, there has always been a theoretical problem that has not been adequately solved. Many results of the factor space theory are directly described in terms of the probability of statistical results, such as the probability of occurrence and non-occurrence of one event, the probability of simultaneous occurrence (and) of two events, and conditional probability. At present, all statistical results can be expressed in a complete cluster of universal logical operations, with the exception of how conditional probability can be expressed in a complete cluster of universal logical operations. This article focuses on exploring this issue to address a flaw in universal logic. In practical applications, another real mathematical problem has emerged. In the universal logic expression formula, there are generally four independent variables x , y , m , n , and one dependent variable z involved, while the probability statistical results only have x , y , and z , and m , n need to be calculated.

Keywords: universal logic; conditional probability; theory of circular logarithms

1. Introduction

In the artificial intelligence development process, logical reasoning and mathematical expression are two very important aspects. For the development of general artificial intelligence, logic and mathematics need to be deeply integrated with each other. In the deep integration of universal logic [1–3] and factor space theory [4], there are still theoretical issues that have not been resolved. How conditional probability is expressed in a complete set of general logic operations has not yet been explained. This article focuses on exploring this issue to address a flaw in universal logic.

First, the basis of the existing propositional-level theory of universal logic has been supplemented to prove that the complete expression of universal logic operation of conditional probability $C(x, y, h)$ is the complete expression of universal IMP operation $I(x, y, h)$, and the two are completely equivalent.

Secondly, in practical applications, a real mathematical problem has emerged. In universal logic expression formulas, there are generally four independent variables— x , y , m , n —and one dependent variable— z —involved, while in probability statistics, only x , y , and z are obtained. How to calculate m , n based on x , y , and z is a new mathematical challenge. If we know what x , y , and $C(x, y, h) = c$ are, to calculate m as h , we need to calculate $(\min(1 + 0^m, 1 - x^m + y^m))^{1/m} = c$. This is a mathematical puzzle, and Circular Logarithm Theory [5,6] can provide a solution. This leads to a deep fusion of universal logic Theory and Circular Logarithm Theory.

Finally, this article introduces the application of Circular Logarithm Theory in universal logic.



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2. The Basic Model of Universal Logic Operation of Conditional Probability

2.1. Related NT Based Models

(1) NOT operation $N(x) = 1 - x$

(2) AND operation $T(x, y) = \max(0, x + y - 1)$

(3) IMP operation $I(x, y) = \min(1, 1 - x + y)$

The basic logical attributes of $I(x, y)$: $I(0, 0) = 1, I(0, 1) = 1, I(1, 0) = 0, I(1, 1) = 1$;

$I(0, y) = 1, I(1, y) = y, I(x, 0) = 0, I(x, 1) = 1$; when $x \leq y, I(x, y) = 1$.

2.2. Analysis of Basic Logical Attributes of Conditional Probability

The base model of conditional probability can be determined according to its logical properties. There are two events, X and Y , with statistical probabilities of $P(X) = x$ and $P(Y) = y$. When event Y occurs, the conditional probability of event X is $P(X|Y) = P(X \cap Y) / P(Y) = (x \wedge y) / y = c$. As probability values, $x, y, c \in [0, 1]$ are all constrained by the 0, 1 limiting function $\Gamma[*] = \min(1, \max(0, *))$.

Known from $P(Y|X) = C(x, y) = \min(1, T(x, y)/x) = \min(1, \max(0, x + y - 1)/x)$ and $T(x, y) \leq \min(x, y)$. The basic logical characteristics of $C(x, y)$: $C(0, 0) = 1, C(0, 1) = 1, C(1, 0) = 0, C(1, 1) = 1$; $C(0, y) = 1, C(1, y) = y, C(x, 0) = 0, C(x, 1) = 1$; when $x \leq y, C(x, y) = 1$. According to these logical properties, $C(x, y) = I(x, y)$ is a directional IMP operation $P(X) \rightarrow P(Y)$.

Known from $P(X|Y) = C(y, x) = \min(1, T(x, y)/y) = \min(1, \max(0, x + y - 1)/y)$ and $T(x, y) \leq \min(x, y)$. The basic logical attributes of $C(y, x)$ are: $C(0, 0) = 1, C(0, 1) = 1, C(1, 0) = 0, C(1, 1) = 1$; $C(0, x) = 1, C(1, x) = x, C(y, 0) = 0, C(y, 1) = 1$; when $y \leq x, C(y, x) = 1$. Based on these logical properties, $C(y, x) = I(y, x)$ is the IMP operation $P(Y) \rightarrow P(X)$ in the other direction.

Therefore, all the logical properties of universal IMP operations can be used to describe the complete cluster of conditional probability universal logic operations.

3. Universal Logic Operation Complete Cluster of Conditional Probability

3.1. Generation of Complete Cluster of Conditional Probability Logic Operation

According to the needs of the human-machine division of labor, practical agents (artificial intelligence systems) may only need to implement a part of the functions of the low-level Turing machine, not all of them.

The complete cluster of N-type generators is $\Phi(x, k) = x^n$, where $n = -1/\log_2 k, k \in [0, 1]$; $k = 2^{-1/n}, n \in R_+$.

The generated complete cluster of universal NOT operations is $N(x, k) = (1 - x^n)^{1/n}$.

The complete cluster of zero order T-property generators is $F_0(x, h) = x^m$,

where $m = (3 - 4h)/(4h(1 - h)), h \in [0, 1]$; $h = ((1 + m) - ((1 + m)^2 - 3m)^{1/2})/(2m), m \in R$.

The generated complete cluster of universal logic operations is

$C(x, y, h) = (\min(1 + 0^m, 1 - x^m + y^m))^{1/m}$.

The complete cluster of first-order T-type generators is $F(x, h, k) = F_0(\Phi(x, k), h) = x^{nm}$.

The generated complete cluster of universal logic operations is

$C(x, y, h, k) = I(x, y, h, k) = (\min(1 + 0^{nm}, 1 - x^{nm} + y^{nm}))^{1/nm}$.

3.2. Distribution Overview of Complete Cluster of Conditional Probability Logic Operations

Here, we study the first-level universal logic computational model, as shown in Figure 1. As the parameters h and k change, the output value of logical reasoning also changes.

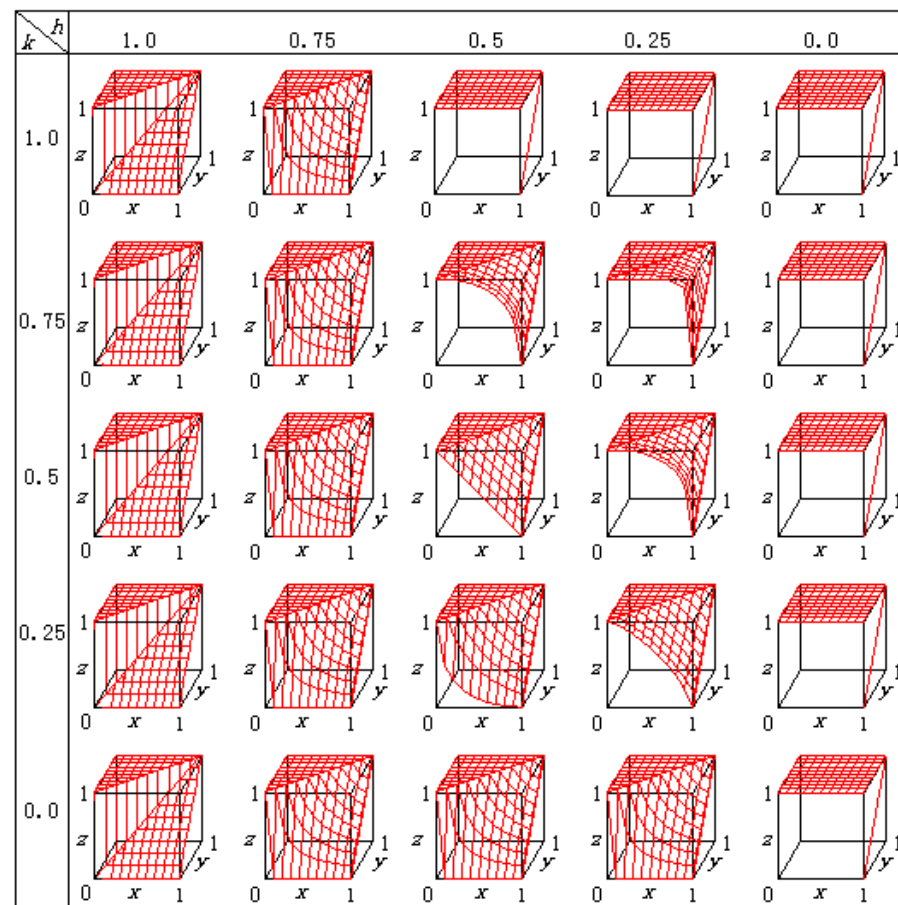


Figure 1. The complete cluster of first-level Conditional probability logic operations.

4. Mathematical Difficulties in Application

4.1. Analysis of Application Challenges

In universal logic expression formulas, four independent variables— x , y , m , n —and one dependent variable— z —are generally involved, and they can all continuously change throughout the entire domain. So, any complete set of universal logic operations provides a global relationship between all variables. The factor space theory, based on the probability statistical results of all observation data in the database, can only be one observation point in the entire global relationship composed of $\langle x_0, y_0, z_0 \rangle$. In order for universal logic to generate universal results, it is necessary to calculate the $\langle m, n \rangle$ corresponding to this observation point $\langle x_0, y_0, z_0 \rangle$ based on the properties of the universal logic formula. Below is an analysis of where this calculation is mathematically difficult.

(1) The complete cluster of Pan African operations is

$$N(x, k) = (1 - x^n)^{1/n}, \text{ where } n = -1/\log_2 k, k \in [0, 1]; k = 2^{-1/n}, n \in \mathbb{R}_+.$$

If, through mathematical statistics, we already know that x and $N(x, k) = z$, $k = (x + z)/2$, $n = -1/\log_2 k$ can be directly calculated.

If $x = z = 0$, then $k = 0$, $n \rightarrow 0$; if $x + z = 1$, then $k = 0.5$, $n = 1$; if $x = z = 1$, then $k = 1$, $n \rightarrow \infty$.

(2) The complete formula of the zero-level universal conditional probability operation is

$$C(x, y, h) = (\min(1 + 0^m, 1 - x^m + y^m))^{1/m}, \text{ Where } m = (3 - 4h)/(4h(1 - h)), h \in [0, 1]; h = ((1 + m) - ((1 + m)^2 - 3m)^{1/2})/(2m), m \in \mathbb{R}$$

If, through mathematical statistics, we already know that x , y , and $C(x, y, h) = c$, to calculate m , it is necessary to calculate $\min(1 + 0^m, 1 - x^m + y^m) = c^m$. The core calculation can be derived from solving equations $y^m - x^m + 1 = c^m$.

If the zero order $m = f_c(x, y, c)$ is calculated, it can be extended to solve first-order problems. Calculate $\min(1 + 0^{nm}, 1 - x^{nm} + y^{nm}) = c^{nm}$.

In general mathematical works, there are no solutions to this type of mathematical problem.

4.2. Solution of Circular Logarithm in Universal Logic

Title: $y^m - x^m + 1 = c^m$.

Among them, $x, y, c \in [0, 1]$ is a known number, and $m \in R$ is an unknown number.

Conditional probability factor (unit $m = 1$)

$$\{X_0\}^{Km} = (x_0^m, y_0^m, c_0^m)^K = \{^m\sqrt{(x^m, y^m, c^m)}\}^K = \{(x, y, c)\}^{Km}$$

Universal logic topology factor (unit body $m \geq 2$)

$$\{X_0^2\}^{Km} = \{(x_0^m y_0^m), (x_0^m c_0^m), (y_0^m c_0^m)\}^K = \{^m\sqrt{(x^m y^m), (x^m c^m), (y^m c^m)}\}^K = \{^m\sqrt{(xy), (xc), (yc)}\}^{Km}$$

$km \in R$ is the unknown shared by (x, y, c) (called Power function). The property attribute $K = (+1, -1, \pm 1, \pm 0)$ sequentially represents $km = (+m)$ positive, $km = (-m)$ reciprocal, $km = (\pm m)$ equilibrium (forward and reverse equilibrium), $km = (\pm 0)$ transformation (forward and reverse transformation). The denominator can be 0, controlled by the property attribute K ; $\{X_0^2\}$ represents the combination of two elements in a group combination, not self-multiplication, squared.

Specifically, for three elements in a polynomial, the first coefficient $A = 1$; therefore, $(x^m, y^m, c^m)^K$ is equivalent to $(x_0, y_0, c_0)^{Km}$, the second coefficient $B = 3$ (probability), $\{X_0\}^{Km} = (1/3)(x_0, y_0, c_0)^{Km}$, the third coefficient $A = 1$ (topology), $\{X_0^2\}^{Km} = (1/3)(x^m y^m, x^m c^m, y^m c^m)^K$ is equivalent to $(1/3)(xy, xc, yc)^{Km}$.

According to Figure 1, $(x^m, y^m, c^m)^K$ the respective probability topological combination is uniformly written as $\{X^m, Y^m, C^m\}$, and $\{X^m$ corresponds to Y^m, C^m combination}; $\{Y^m$ corresponds to $X^m, C^m\}$; $\{C^m$ corresponds to $X^m, Y^m\}$.

Solve:

The axiomatic assumption of circular logarithms: the group combination factor itself is not necessarily 1 except for itself.

$$\text{Circular logarithm rule: } \{(x^m, y^m, c^m)\}^{Km} = (1 - \eta^2)^K (x_0^m, y_0^m, c_0^m)^{Km};$$

the theorem of circular logarithmic isomorphism $m = kNm$ ($N \geq 2$) has been proven. It indicates that the simple conditional probability factor (x^m, y^m, c^m) corresponds to $m = 1$, and the topological (universal logic) complex polynomial (x^m, y^m, c^m) corresponds to $kNm = K(Z \pm S \pm Q \pm N \pm (q = 0, 1, 2, 3, \dots \text{integer}))$ (including the mean factor) and has an isomorphically consistent calculation time.

$$\{(1 - \eta_{[x]}^2)^{Km}, (1 - \eta_{[y]}^2)^{Km}, (1 - \eta_{[c]}^2)^{Km}\} \in (1 - \eta^2)^K \leq 1;$$

derivation process: take one of the three factors as an example:

$$X^m = (1 - Y^m)/(1 - C^m) = [(1 - \eta_{[Y]}^2)/(1 - \eta_{[C]}^2)]^K \cdot \{Y/C\}^{Km}$$

$$= [(1 - \eta_{[Y]}^2) \pm (1 - \eta_{[C]}^2)]^K \cdot \{Y/C\}^{Km} = (1 - \eta_{[X]}^2)^K \cdot \{Y/C\}^{Km};$$

$$\text{and: } (1 - \eta_{[X]}^2)^K = \{0 \text{ or } [0 \text{ to } (1/2) \text{ to } 1] \text{ or } 1\}^{Km};$$

$$\text{similarly, } Y^m = (1 - X^m)/(1 - C^m) = (1 - \eta_{[Y]}^2)^K \cdot \{X/C\}^{Km};$$

$$C^m = (1 - Y^m)/(1 - X^m) = (1 - \eta_{[C]}^2)^K \cdot \{Y/X\}^{Km};$$

$$\text{Solution (1): } (1 - \eta^2)^K = \{(x, y, c)/[(x_0, y_0, c_0)^m]^K = \{0 \text{ or } [(1/2)] \text{ or } 1\}^{Km};$$

$$(1 - \eta^2)^K = (1/2)^{km} \text{ is the zero point of the coordinate center;}$$

$$\text{Solution (2): } km = k(Z \pm S \pm Q \pm N \pm (q = 0, 1, 2, 3, \dots \text{integer}))/t \in R = (1 - \eta^2)^K;$$

Solution (3): $(1 - \eta^2)^K = \{0 \text{ or } (1/2) \text{ or } 1\}^{km}$; the conditional probability $\{X/Y\}^{km}$ structure is in the form of center point $(1/2)$ jump transition;

$(1 - \eta^2)^K = \{[0 \text{ to } (1/2) \text{ to } 1]\}^{km}$; universal logic $\{XY/C\}^{km}$ structure is in the form of a center point $(1/2)$ skip transition.

The shared $m \in R$ satisfying conditional probability and universal logic topological factor with circular logarithm as the base is an arbitrary number value $km \in R$.

$(1 - \eta^2)^K = (1 - \eta^2)^{Km}$ and average value $(x_0^m, y_0^m, c_0^m)^K = (x_0, y_0, c_0)^{Km}$; the table shows three factor spaces $((x, y, c)^{Km}$ linear (conditional probability) and $\{(x_0 y_0), (x_0 c_0), (y_0 c_0)\}^{Km}$ nonlinear (universal logic), shared known number $km = m \in R$, $m = \text{any real number}$ (belonging to R real number), $(1 - \eta^2)^{Km}$ is a shared power function (km), which meets the integration of completeness and compatibility.

Power function (exponential function, history, path integration,...) $m = K (Z \pm S \pm Q \pm N \pm (q = 0,1,2,3,... \text{ integer}))/t$ represents K (property attribute), Z (infinite element), S (any finite element in infinity), Q (element location value, address, region), N/t (calculus order level dynamic $N = 0,1,2,3,... P \leq S$), $(q = 0,1,2,3,... \text{ integer})$ element combination form in order.

Power function m and central zero $(1 - \eta^2)^K = 0$ correspond to the norm logic $(x^m, y^m, c^m)((xy), (xc), (yc))^m$ and can also be combined with the probability topology of $(X, Y, C)^{Km}(XY, CX, CY)^{Km}$, controlling the stability, convergence, and accuracy of all factor spaces.

5. Conclusions

Universal logic is an important logical foundation of artificial intelligence, and Factor space is an important mathematical foundation of artificial intelligence [7,8]. The positive significance of the above examples in this paper, in the field of “ternary number” or univariate cubic (probability topology of asymmetric distribution), is currently missing in traditional mathematics (Kardhan formula only solves symmetric distribution, and the application of the artificial intelligence symmetric algorithm is limited). The combination of normed logic and circular logarithm theory thoroughly solves the logical and arithmetical asymmetry algorithm; fills gaps such as “three-dimensional complex space” and “three-dimensional network hierarchy”; and creates controllable, reliable, and feasible mathematical foundations for a new algorithm of artificial intelligence and the production of a three-dimensional chip architecture.

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