



On the Analytical Treatment of Fractional Elliptic Equations: Applications to Steady-State Heat Conduction [†]

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Abstract

Modeling fractional behaviors is of great importance in the characterization of complex phenomena governed by power-law dynamics, long-range correlations, memory effects, and fractal structures, which are frequently analyzed in scientific and engineering applications. In this work, we propose a fractional formulation to steady-state heat-conduction modeling. The problem is explored on a circular plate, and the adopted approach combines two techniques, namely separation of variables and fractional power-series expansion. Special emphasis is placed on clear and didactic analytical development, with each mathematical step carefully introduced and systematically detailed. Numerical results are also presented and discussed.

Keywords: steady-state heat conduction; circular plate; modeling fractional behaviors

1. Introduction

Computational and analytical tools have boosted the use of Fractional Calculus (FC) as an interdisciplinary framework [1,2]. This development has provided a broad repertoire for modeling numerous problems related to physics and engineering, mainly due to the introduction of nonlocality into the analysis of these phenomena [3–5].

FC is characterized by having interdisciplinary applications, encompassing areas such as Physics, Biology and Medicine, Control and Automation, and Economics, among others [6–14]. Heat conduction is usually modeled by means of the classical Laplace equation, whose generalization can be extended using a fractional framework [15–17]. Studies have also employed the classical Laplace equation to describe steady-state heat conduction in a circular plate, combining Fourier expansion and second-order finite differences [18], as well as integrating the method of separation of variables with fractional power series [19].

This work aims to obtain the analytical solution to a steady-state heat conduction problem applied to a circular plate with unit radius and insulated faces. In addition, it adopts a didactic, accessible, and demonstrative approach regarding the most significant analytical steps employed, as well as the exploration of numerical simulations to provide a deeper understanding of the behavior of this phenomenon within the geometry of interest.

This paper is organized as follows: Section 2 presents the proposed fractional mathematical model. Section 3 depicts numerical results. Section 4 discusses the main findings. Section 5 presents the main conclusions of the study.



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2. Mathematical Model

The Laplace equation describes the phenomenon of steady-state heat conduction. Figure 1 illustrates the circular plate studied in this work, described in polar coordinates ρ and φ .

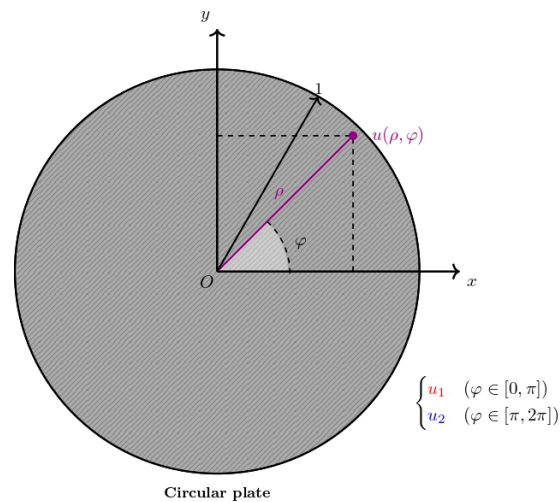


Figure 1. Circular plate with unit radius and insulated faces, with the upper boundary at temperature u_1 and the lower boundary at temperature u_2 .

The classical Laplace equation governing steady-state heat conduction is given as follows:

$$\nabla^2 u(\rho, \varphi) = \frac{\partial^2 u(\rho, \varphi)}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial u(\rho, \varphi)}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 u(\rho, \varphi)}{\partial \varphi^2} = 0 \quad (1)$$

and we consider it subject to the following boundary conditions:

$$u(1, \varphi) = \begin{cases} u_1, & 0 < \varphi < \pi \\ u_2, & \pi < \varphi < 2\pi \end{cases} \quad (2)$$

The governing equation is generalized to its fractional counterpart by employing the Gerasimov–Caputo Fractional Derivative and combining the methods of separation of variables and fractional power series expansions within a fractional framework.

From this perspective, the generalized partial differential equation is

$$\nabla^{2\alpha} u(\rho, \varphi) = D_\rho^{2\alpha} u(\rho, \varphi) + \frac{1}{\rho} D_\rho^\alpha u(\rho, \varphi) + \frac{1}{\rho^2} D_\varphi^{2\alpha} u(\rho, \varphi) = 0 \quad (3)$$

Applying $u(\rho, \varphi) = R(\rho) \times \Phi(\varphi)$, we have

$$D_\rho^{2\alpha} [R(\rho)\Phi(\varphi)] + \frac{1}{\rho} D_\rho^\alpha [R(\rho)\Phi(\varphi)] + \frac{1}{\rho^2} D_\varphi^{2\alpha} [R(\rho)\Phi(\varphi)] = 0 \quad (4)$$

$$\frac{\rho^2}{R(\rho)} D_\rho^{2\alpha} R(\rho) + \frac{\rho}{R(\rho)} D_\rho^\alpha R(\rho) = -\frac{1}{\Phi(\varphi)} D_\varphi^{2\alpha} \Phi(\varphi) = -\lambda \quad (5)$$

Two Ordinary Differential Equations (ODEs) are obtained from Equation (5):

$$\rho^2 D_\rho^{2\alpha} R(\rho) + \rho D_\rho^\alpha R(\rho) + \lambda R(\rho) = 0 \quad (6)$$

$$D_\varphi^{2\alpha} \Phi(\varphi) - \lambda \Phi(\varphi) = 0 \quad (7)$$

The first step consists of analytically solving each ODE separately. Starting with Equation (6), a power-series expansion represented by $R(\rho) = \sum_{n=0}^{\infty} C_n \rho^{n\alpha}$ is employed, along with the fractional derivative operators denoted by $D_\rho^{2\alpha}$ and D_ρ^α .

$$\rho^2 \sum_{n=2}^{\infty} C_n \frac{\Gamma(n\alpha + 1)}{\Gamma[(n-2)\alpha + 1]} \rho^{(n-2)\alpha} + \rho \sum_{n=1}^{\infty} C_n \frac{\Gamma(n\alpha + 1)}{\Gamma[(n-1)\alpha + 1]} \rho^{(n-1)\alpha} + \lambda \sum_{n=0}^{\infty} C_n \rho^{n\alpha} = 0 \quad (8)$$

The next step consists of performing two changes in variables to standardize the starting index of each summation (counters). First, let $n - 2 = m$, such that when $n \rightarrow 2$ and $n \rightarrow \infty$, one obtains $m \rightarrow 0$ and $m \rightarrow \infty$, respectively. Next, let $n - 1 = q$; it is important to note that when $n \rightarrow 1$ and $n \rightarrow \infty$, one obtains $q \rightarrow 0$ and $q \rightarrow \infty$. Rewriting m and q in terms of n , the following equation is obtained:

$$\sum_{n=0}^{\infty} C_{n+2} \frac{\Gamma[(n+2)\alpha + 1]}{\Gamma(n\alpha + 1)} \rho^{n\alpha+2} + \sum_{n=0}^{\infty} C_{n+1} \frac{\Gamma[(n+1)\alpha + 1]}{\Gamma(n\alpha + 1)} \rho^{n\alpha+1} + \lambda \sum_{n=0}^{\infty} C_n \rho^{n\alpha} = 0 \quad (9)$$

It can be observed that the factor ρ^2 multiplying the first summation in Equation (8) leads to $(n\alpha + 2)$ as the power of ρ in Equation (9), while, analogously, the factor ρ multiplying the second summation in Equation (8) leads to $(n\alpha + 1)$ as the corresponding power of ρ in Equation (9).

Performing two additional changes in variables, let $m = n + \frac{2}{\alpha}$ such that when $n \rightarrow 0$ and $n \rightarrow \infty$, one obtains $m \rightarrow \frac{2}{\alpha}$ and $m \rightarrow \infty$, respectively. On the other hand, let $q = n + \frac{1}{\alpha}$, for which, when $n \rightarrow 0$ and $n \rightarrow \infty$, one obtains $q \rightarrow \frac{1}{\alpha}$ and $q \rightarrow \infty$.

In the following derivation, we restrict our analysis to fractional orders satisfying $\alpha = 1/k$, with $k \in \mathbb{N}^+$. Under this assumption, all index shifts arising from the exponent matching procedure remain integer-valued, ensuring the mathematical consistency of the power-series expansion. Rewriting m and q in terms of n , and separating the finite and infinite sums, yields the following:

$$\sum_{n=\frac{2}{\alpha}}^{\infty} \left\{ C_{n-\frac{2}{\alpha}+2} \frac{\Gamma[(n+2)\alpha - 1]}{\Gamma(n\alpha - 1)} + C_{n-\frac{1}{\alpha}+1} \frac{\Gamma[(n+1)\alpha]}{\Gamma(n\alpha)} + \lambda C_n \right\} \rho^{n\alpha} + \sum_{n=\frac{1}{\alpha}}^{\frac{2}{\alpha}-1} C_{n-\frac{1}{\alpha}+1} \frac{\Gamma[(n+1)\alpha]}{\Gamma(n\alpha)} \rho^{n\alpha} + \sum_{n=0}^{\frac{2}{\alpha}-1} C_n \rho^{n\alpha} = 0 \quad (10)$$

For $n \geq \frac{2}{\alpha}$, we have

$$C_{n-\frac{2}{\alpha}+2} = -\frac{C_{n-\frac{1}{\alpha}+1} \frac{\Gamma[(n+1)\alpha] \Gamma(n\alpha - 1)}{\Gamma(n\alpha) \Gamma[(n+2)\alpha - 1]}}{\Gamma(n\alpha) \Gamma[(n+2)\alpha - 1]} - \frac{\lambda C_n \Gamma(n\alpha - 1)}{\Gamma[(n+2)\alpha - 1]} \quad (11)$$

The continuation of the analytical solution depends on the choice of the value of α , since different values of α lead to different solutions for the problem. Therefore, $\alpha = \frac{1}{2}$ is selected.

$$C_{n-2} = -\frac{C_{n-1} \Gamma\left(\frac{n+1}{2}\right) \Gamma\left(\frac{n-2}{2}\right)}{\Gamma\left(\frac{n}{2}\right) \Gamma\left(\frac{n}{2}\right)} - \frac{\lambda C_n \Gamma\left(\frac{n-2}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} \quad (12)$$

Considering $n \geq 4$, the general expression for $R(\rho)$ is obtained.

$$R(\rho) = \sum_{k=2}^{\infty} \frac{\Gamma\left(\frac{k}{2}\right)}{\Gamma\left(\frac{k+2}{2}\right)} \left[-\frac{C_{k+1}\Gamma\left(\frac{k+3}{2}\right)}{\Gamma\left(\frac{k+2}{2}\right)} - \lambda C_{k+2} \right] \rho^{\frac{k}{2}} \quad (13)$$

Next, the solution to the ODE represented by Equation (7) is sought. Using the power-series expansion denoted by $\Phi(\varphi) = \sum_{n=0}^{\infty} C_n \varphi^{n\alpha}$ and the fractional derivative operator represented by $D_{\varphi}^{2\alpha}$:

$$\sum_{n=2}^{\infty} C_n \frac{\Gamma(n\alpha + 1)}{\Gamma[(n-2)\alpha + 1]} \varphi^{(n-2)\alpha} - \lambda \sum_{n=0}^{\infty} C_n \varphi^{n\alpha} = 0 \quad (14)$$

The change in variable $m = n - 2$ is introduced, such that when $n \rightarrow 2$ and $n \rightarrow \infty$, one obtains $m \rightarrow 0$ and $m \rightarrow \infty$, respectively. The solution is then rewritten using summation indices in terms of n .

$$\sum_{n=0}^{\infty} C_{n+2} \frac{\Gamma[(n+2)\alpha + 1]}{\Gamma(n\alpha + 1)} \varphi^{n\alpha} - \lambda \sum_{n=0}^{\infty} C_n \varphi^{n\alpha} = 0 \quad (15)$$

The recurrence formula is given by

$$C_{n+2} = \frac{\lambda C_n \Gamma(n\alpha + 1)}{\Gamma[(n+2)\alpha + 1]} \quad (16)$$

For $n \geq 0$, the generalized solution for $\Phi(\varphi)$ is obtained.

$$\Phi(\varphi) = C_0 + C_1 \varphi^{\alpha} + \frac{\lambda C_0}{\Gamma(2\alpha + 1)} \varphi^{2\alpha} + \frac{\lambda C_1 \Gamma(\alpha + 1)}{\Gamma(3\alpha + 1)} \varphi^{3\alpha} + \dots + \frac{\lambda^3 C_1 \Gamma(\alpha + 1)}{\Gamma(7\alpha + 1)} \varphi^{7\alpha} + \dots \quad (17)$$

$$\Phi(\varphi) = C_0 \sum_{k=0}^{\infty} \frac{(\lambda \varphi^{2\alpha})^k}{\Gamma[(2k)\alpha + 1]} + C_1 \Gamma(\alpha + 1) \varphi^{\alpha} \sum_{k=0}^{\infty} \frac{(\lambda \varphi^{2\alpha})^k}{\Gamma[(2k+1)\alpha + 1]} \quad (18)$$

The equation can be written as $\Phi(\varphi) = C_0 \Phi_0(\varphi) + C_1 \Gamma(\alpha + 1) \varphi^{\alpha} \Phi_1(\varphi)$. So, the terms $\Phi_0(\varphi)$ and $\Phi_1(\varphi)$ can be rewritten as Mittag-Leffler functions. In this work, we substitute $\alpha = \frac{1}{2}$.

$$\Phi(\varphi) = C_0 E_{1,1}(\lambda \varphi) + C_1 \Gamma\left(\frac{3}{2}\right) \sqrt{\varphi} E_{\frac{3}{2}, \frac{1}{2}}(\lambda \varphi) \quad (19)$$

The fundamental condition for the function $\Phi(\varphi)$ is that it must be 2π -periodic, that is, $\Phi(\varphi) = \Phi(\varphi + 2\pi)$. Therefore, by analyzing Equation (19), it follows that $C_1 = 0$, since the term involving $\sqrt{\varphi}$ does not satisfy this requirement.

$$\Phi(\varphi) = C_0 E_{1,1}(\lambda \varphi) = C_0 \sum_{k=0}^{\infty} \frac{(\lambda \varphi)^k}{\Gamma(k+1)} = C_0 e^{\lambda \varphi} \quad (20)$$

Applying the periodicity condition, the function $e^{2\pi\lambda} = 1$ is obtained. In this way, $\lambda = ik$ is written for the application of Euler's identity, noting that k can be expressed as an integer n . In addition, the next step is to write the constant C_0 as $C_0 = c_n + id_n$.

$$C_0 e^{in\varphi} = C_0 [\cos(n\varphi) + i\sin(n\varphi)] = (c_n + id_n) \cos(n\varphi) + (-d_n + ic_n) \sin(n\varphi) \quad (21)$$

Renaming the constants in Equation (21) as A_n and B_n , respectively, a change in variables is also applied to Equation (13) to express the solution $u(\rho, \varphi) = R(\rho) \times \Phi(\varphi)$ as

a Fourier series. This transformation consists of letting $k = n + 2$, such that when $k \rightarrow 2$ and $k \rightarrow \infty$, one obtains $n \rightarrow 0$ and $n \rightarrow \infty$.

Considering the term for $n = 0$, denoted by $A_0 \left[(-1) \frac{3C_3\sqrt{\pi}}{4} + (-1)\lambda C_4 \right] \rho \equiv \frac{a_0}{2} \rho$, the general solution to this problem is

$$u(\rho, \varphi) = \frac{a_0}{2} \rho + \sum_{n=1}^{\infty} [A_n \cos(n\varphi) + B_n \sin(n\varphi)] \frac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n+4}{2}\right)} \left[(-1)C_{n+3} \frac{\Gamma\left(\frac{n+5}{2}\right)}{\Gamma\left(\frac{n+4}{2}\right)} + (-1)\lambda C_{n+4} \right] \rho^{\frac{n}{2}+1} \quad (22)$$

The solution is rewritten as a Fourier series and the boundary conditions are applied.

$$u(1, \varphi) = \frac{a_0}{2} \rho + \sum_{n=1}^{\infty} [a_n \cos(n\varphi) + b_n \sin(n\varphi)] (1)^{\frac{n}{2}+1} = f(\varphi) \quad (23)$$

Therefore, the values of the Fourier series coefficients are $a_0 = (u_1 + u_2)$, $a_n = 0$ and $b_n = \frac{2(u_1 - u_2)}{n\pi}$, where $n = 2k + 1$ (an odd integer). So, substituting and subsequently rewriting k in terms of n , the final solution is obtained as

$$u(\rho, \varphi) = \frac{(u_1 + u_2)}{2} \rho + \sum_{n=0}^{\infty} \frac{2(u_1 - u_2)}{(2n + 1)\pi} \sin[(2n + 1)\varphi] \rho^{n+\frac{3}{2}} \quad (24)$$

3. Numerical Results

This section presents numerical simulation based on the analytical solutions previously obtained.

Simulations were performed for different values of u_1 , u_2 and α . Figures 2–5 illustrate some of the obtained results, whose implications are discussed in Section 4.

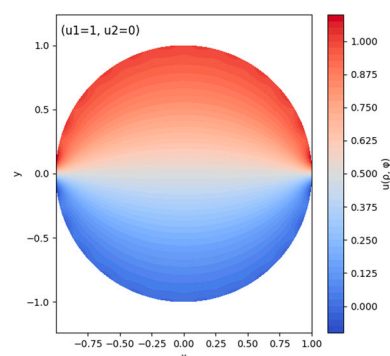


Figure 2. Numerical results for the classical case ($\alpha = 1$) with $u_1 = 1$ and $u_2 = 0$.

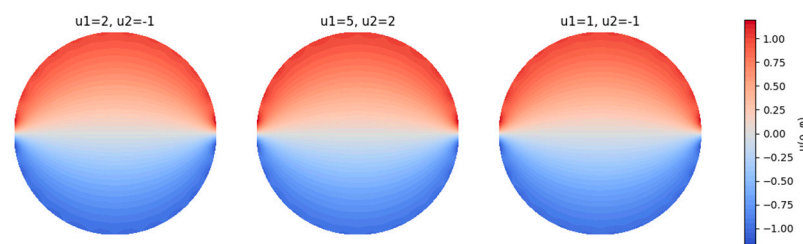


Figure 3. Numerical simulations for the classical case ($\alpha = 1$) with different sign combinations of u_1 and u_2 .

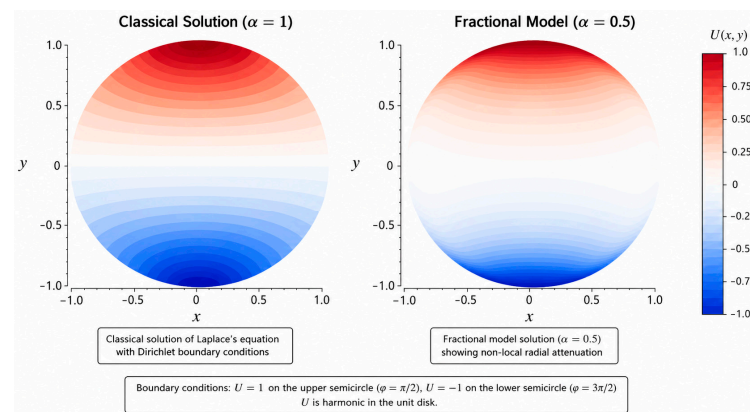


Figure 4. Numerical comparison between the integer-order, on the left side ($\alpha = 1$), and fractional-order on the right side ($\alpha = 0.5$) models.

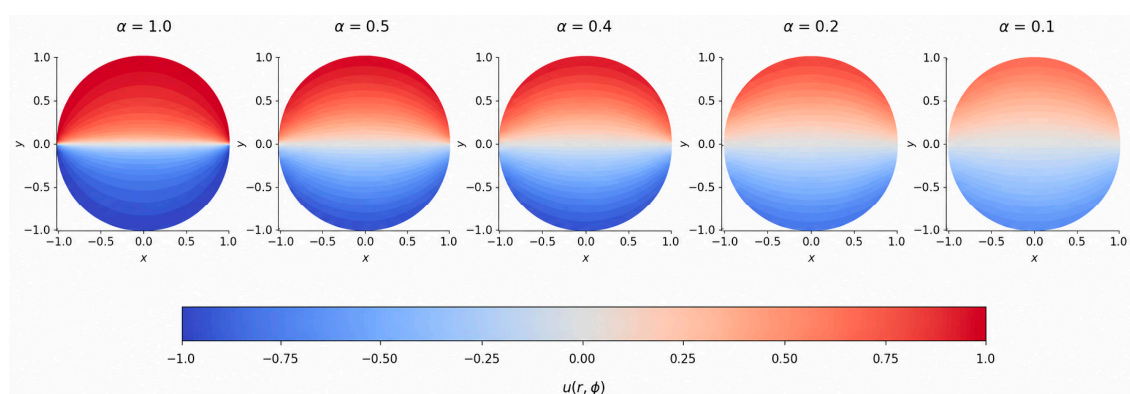


Figure 5. Numerical solutions for different values of α : $\alpha = 1.0, 0.5, 0.4, 0.2$, and 0.1 .

It is important to emphasize that the discontinuity at $\varphi = \pi$ can produce a Gibbs-type phenomenon. In the following section, all major aspects associated with the obtained results will be discussed, both regarding the analytical solution and the numerical simulations.

4. Discussion

The final solution to the problem proposed in this work was described by Equation (24). In this regard, it is emphasized that for $n = 0$, the average between u_1 and u_2 multiplied by the adopted radial value is obtained, corresponding precisely to the mean value for $\rho = 1$. On the other hand, for $n \neq 0$, the discontinuity at $\varphi = \pi$ is highlighted, resulting in overshooting.

Figure 2 illustrates the classical case ($\alpha = 1$) for $u_1 = 1$ and $u_2 = 0$, highlighting the smooth transition between the upper and lower regions of the circular plate due to the discontinuity at $\varphi = \pi$. Indeed, when $\alpha = 1$, a smooth, harmonic behavior is observed, characteristic of a classical local diffusion regime.

The impact of the choice of u_1 and u_2 on the behavior of the solution is also evaluated in Figure 3. It is observed that the positive/negative contrast between these conditions promotes a more pronounced contrast in the simulated behavior.

Figure 4 aims to demonstrate clearly, side by side, the behavior of the phenomenon when treated with integer or classical order ($\alpha = 1$) and with fractional order ($\alpha = 0.5$). It can be observed that, in the classical case, there is local (harmonic) diffusion and the typical behavior associated with the solution of the Laplace equation, with boundary effects penetrating into the interior of the circular plate. In the fractional case, one can observe a boundary-dominated behavior, a more “flattened” interior, and reduced an-

gular propagation, highlighting a nonlocal effect and incorporating dependence on the entire domain.

Figure 5 explores the behavior of the solution for five different values of α , namely $\alpha = 1.0, 0.5, 0.4, 0.2$ and 0.1 . One can observe a transition between the behavior given by the classical Laplace equation solution and the solutions corresponding to different values of α associated with the fractional nonlocal operator. The results for $\alpha = 0.5$ and $\alpha = 0.4$ already show solutions with some “flattening” in the central region of the circular plate, a certain dominance of the boundary, and a loss of a classical harmonic smoothness. For $\alpha = 0.2$ and $\alpha = 0.1$, the solutions become even more concentrated near the boundary of the circular plate, with a more uniform interior and a more evident nonlocal effect.

In summary, as the value of α decreases, the behavior of the solution becomes more constrained by the boundary conditions of the circular plate. It is important to emphasize that fractional differential equations involve a non-integer derivative, which introduces nonlocality into the problem, encompassing an overall thermal, nonlocal spatial interactions effect, resulting in solutions that are more influenced by the global structure and often dominated by the boundary conditions.

5. Conclusions

The fractional-order mathematical modeling proposed in this work fulfilled its purpose of providing a clear and didactic analytical development while also demonstrating sufficient robustness through the numerical simulation results obtained. It is worth emphasizing that the numerical simulations successfully captured the impact of considering different fractional orders within the model, a feature that is not trivial to observe solely from the analytical solution itself, as well as the behavior associated with different values of u_1 and u_2 . In particular, for lower values of α , a smoother transition between the upper and lower boundaries of the circular plate was observed. Ongoing work is currently focused on extending this satisfactorily robust framework to problems involving heat conduction in annular domains, while also opening perspectives for its application to other real-world physical problems.

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