

Article

Nonlinear Adaptive Control Design for Quadrotor UAV Transportation System

Boyū Zhu * and Dazhi Wang

College of Information Science and Engineering, Northeastern University, Shenyang 110004, China; wangdazhi@ise.neu.edu.cn

* Correspondence: 2000691@stu.neu.edu.cn

Abstract: In response to the non-linear and underactuated characteristics of quadrotor UAV suspension transportation system, this paper proposes a novel control strategy aimed at achieving precise position control, attitude control, and anti-swing capabilities. Firstly, a dynamical model required for controller design is established through the Newton-Euler method. In the controller design process, the paper employs the energy method and barrier Lyapunov function to design a double-closed-loop nonlinear controller. This controller is capable of not only accurately controlling the position and attitude angles of the quadrotor UAV suspension transportation system but also effectively suppressing the swing of the payload. Building on this, considering the elastic deformation of the lifting cable, and by analyzing the forces in the Newton-Euler equations, this paper proposes an adaptive control design for the case where the length of the cable connecting the UAV and the payload is unknown. To validate the effectiveness of the proposed control scheme, comparative experiments were conducted in the MATLAB simulation environment, and the results indicate that the method proposed in this paper exhibits superior control performance compared to traditional controllers.

Keywords: quadrotor UAV; position control; anti-swing control; nonlinear adaptive control; barrier lyapunov; energy function



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1. Introduction

Recently, significant progress has been made in unmanned aerial vehicle (UAV) technology, demonstrating its unique advantages in various fields [1–3]. Quadrotor UAVs, in particular, have been widely used in diverse application scenarios due to their excellent adaptability to environmental changes [4–6]. However, in practical applications, one important technical challenge with quadrotor UAVs when performing payload-carrying tasks is the swing of the payload [7–10]. This swing not only affects the flight stability of the UAV but also risks damaging the cargo and potentially endangering the UAV's safety [11,12]. Therefore, researching methods to effectively suppress the payload swing of quadrotor UAVs and enhance the efficiency and safety of transportation has significant theoretical and practical value [13,14].

The quadrotor UAV transportation system is particularly suitable for carrying payloads with high uncertainty. However, this system is complex, featuring multiple degrees of freedom, strong coupling, and underactuation. The quadrotor UAV has six degrees of freedom but only four control inputs. During the operation of the transportation system, the suspended payload further increases the system's underactuated characteristics. Additionally, the quadrotor UAV, the payload, and the cable connecting them are tightly coupled, with complex interactions between their relative displacements and attitude angles, which undoubtedly exacerbates the overall control difficulty of the system. These complexities pose significant challenges in the design and implementation of control strategies for the quadrotor UAV transportation system.

To address these issues, various controllers have been developed. Reference [15] designed a nonlinear model predictive controller for the simplified dynamic model of a

quadrotor UAV payload. In [16], a control design using the nested saturation method was proposed. In reference [17], aerodynamic parameters were identified online using a Kalman filter, followed by rotor speed compensation with an LQR strategy, which significantly improved the quadrotor's control performance in heading and altitude. These studies typically simplify the quadrotor's nonlinear dynamics into linear models for mathematical convenience, although this may introduce approximation errors and limit performance. In [18], a nonlinear control method based on energy analysis is proposed, which achieves simultaneous localization and elimination of payload oscillation for a quadrotor UAV. In [19], a virtual control input was designed for the outer-loop subsystem using an energy-based analysis method. Ultimately, the stability of the closed-loop system was proven based on the LaSalle's invariance principle. In [20] a mathematical model and control strategy suitable for quadrotor UAV transportation systems are proposed, achieving robust control under parametric uncertainties and unmodeled dynamics, as well as precise delivery and swing reduction control for the payload.

In the practical application of the quadrotor UAV transportation system, simplifying the cable connecting the UAV to the payload as a rigid body model is not realistic. During actual flight, due to the relative motion between the UAV and the payload, the cable will undergo elastic deformation, leading to uncertainty in the changes of the cable length. This uncertainty can not only affect the coordinate position of the payload but may also interfere with the system's energy function. When controlling the quadrotor UAV transportation system, this uncertainty can reduce control precision and stability, and may even lead to system instability. Therefore, in order to ensure the safe and efficient operation of the UAV transportation system, it is necessary to thoroughly study the issue of unknown cable lengths and propose corresponding solutions.

An in-depth study of flexible cables in transport systems has been carried out in [21], where each cable is treated as a system of links connected in series and a geometrically nonlinear controller has been designed to enable the precise transport of a rigid body to a predetermined position and to keep all the links aligned in the vertical direction. In [22], a novel bidirectional fuzzy brain emotional learning controller is proposed, which enables precise trajectory tracking for quadrotor UAV in the presence of payload uncertainties. In the referenced documents [23,24], the authors addressed the issue of cable length discrepancies within the system and engineered an adaptive control mechanism to enhance the fidelity of the control scheme. References [25,26] incorporate the cable length into their control strategies. Nevertheless, the swing angles of the payload are not taken into account during the control design process, which may result in reduced performance in anti-swing control measures.

To ensure the precision and stability of the quadrotor UAV transport system, this paper has delved into the problem of anti-swing control and the elastic deformation of the cable. We have developed an adaptive controller that integrates energy functions with barrier Lyapunov functions. This paper addresses the non-linear and underactuated characteristics of the quadrotor UAV transportation system, aiming to achieve precise position control and anti-swing capabilities. Precise position control and effective anti-swing management of the quadrotor UAV suspension transportation system enhance the overall stability and safety of the system, which is crucial for the operation of the UAV and the safety of the payload. Unlike the Lagrange modeling approach used in [23], this paper employs the Newton-Euler method for force analysis in dynamic modeling, which obviates the need for subsequent additional force analysis on the cable. Furthermore, in the design of the controller, this paper introduces an integral barrier Lyapunov function to describe the nonlinear constraints of the system, thereby further enhancing the stability of the system.

The improved precision in control capabilities enable quadrotor UAVs to perform more complex and delicate tasks, such as cargo transportation, aerial rescue, and environmental monitoring in high-altitude or hazardous areas. The adaptive control design ensures that the system maintains good control performance in the face of uncertainties such as unknown cable length, enhancing the system's autonomy and robustness.

The main innovations and contributions of this research can be summarized as follows:

- The paper employs an energy-based approach to successfully achieve anti-swing control of the quadrotor UAV transport system while carrying a payload.
- By utilizing barrier Lyapunov function to describe the system's non-linear constraints, the controller's performance is further optimized, and a double-closed-loop nonlinear controller is designed to achieve stable control of displacement and attitude angle.
- Through the application of adaptive estimation, the length of the cable can be adjusted in real-time, thereby significantly improving the control accuracy of the quadrotor UAV transport system.
- The control stability of the quadrotor UAV transport system has been comprehensively verified by the Lyapunov theory and the LaSalle's invariance theorem.

The structure of this paper is as follows: In Section 2, the dynamical model of the quadrotor UAV transportation system is introduced in detail. Section 3 delves into the design of controllers based on the energy method and the obstacle Lyapunov function, and the asymptotic stability of the control system is proven using the Lyapunov theory and LaSalle's invariance theorem. Section 4 presents the results of simulation experiments. Finally, in Section 5, we summarize the main conclusions of this paper.

2. Dynamic Model of the Quadrotor UAV Transport System

A quadrotor UAV transport system consists of a quadrotor UAV, a cable, and a suspended payload, as shown in Figure 1. In order to describe the attitude and position information of the quadrotor UAV with the suspended payload more accurately, three coordinate systems are established, namely: the inertial reference frame $O_e = \{x_e, y_e, z_e\}$, the body reference frame $O_b = \{x_b, y_b, z_b\}$, and the payload reference frame $O_h = \{x_h, y_h, z_h\}$.

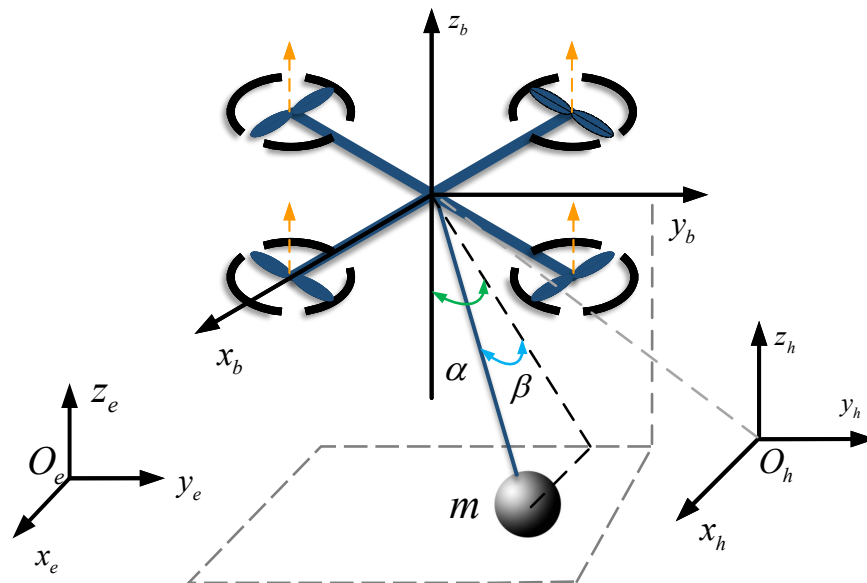


Figure 1. Quadrotor UAV transport system configuration.

The rotational matrix from O_b to O_e is indicated by R_b^e as shown below:

$$R_b^e = \begin{bmatrix} c_\phi c_\theta & s_\phi s_\theta c_\psi - s_\psi c_\phi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ s_\phi c_\theta & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - c_\phi s_\psi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix}, \quad (1)$$

where s_i and c_i are $\sin(i)$ and $\cos(i)$ abbreviations, respectively. And the rotation matrix from O_h to O_e is indicated by R_h^e as shown below:

$$R_h^e = \begin{bmatrix} c_\alpha & 0 & s_\alpha \\ s_\alpha s_\beta & -c_\beta & -s_\beta c_\alpha \\ s_\alpha c_\beta & s_\beta & -c_\alpha c_\beta \end{bmatrix}. \quad (2)$$

The motion of the quadrotor UAV transport system can be accurately represented by two different vectors. Firstly, the vector $R = (x, y, z)^T$ of translational motion represents the position of the system in the three-axis direction with respect to the inertial reference frame. The second is the vector of Euler angles $\Lambda = (\phi, \theta, \psi)^T$, which describes the three Euler angles of the system. The positional information of the payload is represented by two swing angles α and β . The masses of the UAV and the payload are denoted by M and m , respectively, and the tension and length of the sling are denoted by T and L , respectively.

Unification of forces in different reference frames in the quadrotor UAV transport system based on the above rotation matrices.

$$F_e(t) = \| F_b(t) \| \begin{bmatrix} C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\phi S_\theta S_\psi - C_\psi S_\phi \\ C_\phi C_\theta \end{bmatrix}, \quad (3)$$

where $F_e(t) = (F_{ex}, F_{ey}, F_{ez}) \in \mathbb{R}^3$ and $F_b(t) = (F_{bx}, F_{by}, F_{bz}) \in \mathbb{R}^3$ represent the forces on the system in O_e and O_b , respectively. And $\| F_b(t) \|$ is represented as the Euclidean norm of $F_b(t)$. The specific expression of $F_e(t)$ is:

$$\begin{cases} F_{ex}(t) = \| F_b(t) \| (C_\phi S_\theta C_\psi + S_\phi S_\psi), \\ F_{ey}(t) = \| F_b(t) \| (C_\phi S_\theta S_\psi - S_\phi C_\psi), \\ F_{ez}(t) = \| F_b(t) \| C_\phi C_\theta. \end{cases} \quad (4)$$

And the forces $F_h(t) = (F_{hx}, F_{hy}, F_{hz}) \in \mathbb{R}^3$ in O_h can be obtained:

$$F_h(t) = R_h^e(t) F_e(t). \quad (5)$$

The specific expression of $F_h(t)$ is:

$$\begin{cases} F_{hx}(t) = F_{ex}(t) C_\alpha + F_{ez}(t) S_\alpha, \\ F_{hy}(t) = F_{ex}(t) S_\alpha S_\beta - F_{ey}(t) C_\beta - F_{ez}(t) S_\beta C_\alpha, \\ F_{hz}(t) = F_{ex}(t) S_\alpha C_\beta + F_{ey}(t) S_\beta - F_{ez}(t) C_\alpha C_\beta. \end{cases} \quad (6)$$

The positional relationship between the quadrotor UAV and the payload can be represented as:

$$R_h(t) = R_b(t) + L(t) \eta(t), \quad (7)$$

where $R_h(t) = [x_h, y_h, z_h]^T \in \mathbb{R}^3$ is the position of the payload, $R_b(t) = [x_b, y_b, z_b]^T \in \mathbb{R}^3$ is the position of the UAV, and $\eta(t) = [S_\alpha C_\beta, S_\beta, -C_\alpha C_\beta]^T$.

The forces on the UAV can be described as:

$$M \ddot{R}_b(t) = F_e(t) + T(t) \eta(t) - G_b, \quad (8)$$

where $G_b = [0, 0, Mg]^T \in \mathbb{R}^3$, and g is the acceleration of gravity. And the force analysis for the suspended payload is as follows:

$$m \ddot{R}_h(t) = -T(t) \eta(t) - G_b, \quad (9)$$

According to Equations (7)–(9), a specific expression for the tension $T(t)$ can be obtained:

$$T(t) = -\frac{m}{M+m} \left(F_p(t)^T \sigma(t) - L(t) m_q \dot{\eta}(t)^T \dot{\eta}(t) \right), \quad (10)$$

where $F_p(t) = [-F_{ex}, -F_{ey}, F_{ez}]^T$, and $\sigma(t) = [s_\alpha c_\beta, s_\beta, -c_\alpha c_\beta]^T$.

Through the Newton-Euler method, the UAV transport system is analysed in terms of forces, and then after unifying the reference frame, the dynamic equations of the system in the inertial reference frame O_e can be obtained as follows:

$$\begin{cases} M\ddot{R}_b(t) = F_e(t) + T(t)\eta(t) - G_b, \\ \ddot{\alpha} = \frac{2\dot{\alpha}\dot{\beta}S_\beta}{C_\beta} - \frac{F_{ex}(t)C_\alpha + F_{ez}(t)S_\alpha}{ML(t)C_\beta}, \\ \ddot{\beta} = \frac{F_e(t)^T \iota}{ML(t)} - \dot{\alpha}^2 S_\alpha C_\beta, \end{cases} \quad (11)$$

where $\iota = [S_\alpha S_\beta, C_\beta, C_\alpha S_\beta]^T$ is an auxiliary vector.

In the quadrotor UAV system, in order to adjust its attitude angles, the speeds of the individual rotors are typically adjusted to generate an unbalanced torque. This torque enables the UAV to rotate around an axis perpendicular to its body, thereby changing its orientation.

Based on this, the dynamic model of the quadrotor UAV's attitude can be described as:

$$\ddot{\Lambda}(t) = \chi(t) + u(t), \quad (12)$$

where

$$\begin{cases} u(t) = [u_2(t) \ u_3(t) \ u_4(t)]^T, \\ \chi(t) = \left[\frac{\dot{\theta}\psi(I_y - I_z)}{I_x} \quad \frac{\phi\dot{\psi}(I_z - I_x)}{I_y} \quad \frac{\phi\dot{\theta}(I_x - I_y)}{I_z} \right]^T. \end{cases} \quad (13)$$

3. Control Scheme Design

In this section, an in-depth analysis of the energy characteristics of the quadrotor UAV transportation system has been conducted, and a design scheme for a nonlinear anti-sway controller based on the energy method has been proposed. In addition, the control scheme also meticulously considers the issue of cable length changes due to elastic deformation, thus incorporating this key factor into the control strategy of the quadrotor UAV transportation system. The design of the system is based on assumptions, which are presented in the Appendix A. Through this design, it is possible to control the swing of the UAV during payload transportation more accurately, ensuring system stability and transportation efficiency.

3.1. Construction of Energy Functions

This section is dedicated to the design of an anti-swing reduction control scheme for the quadrotor UAV transport system. Addressing the issue of underactuation in the suspended payload, an energy function is constructed to describe the relative motion state between the UAV and the payload, ensuring stable control of both the UAV's movement and the suppression of the payload's swing angle. Furthermore, considering the unknown elastic deformation length of the cable, an adaptive control design is proposed.

Considering the mechanical energy $E_b(t)$ of the quadrotor UAV transport system, which comprises both the system's kinetic energy and potential energy. The kinetic energy of the quadrotor UAV can be defined as:

$$E_b(t) = \frac{1}{2} \dot{R}_b(t)^T M \dot{R}_b(t). \quad (14)$$

Taking the derivative of Equation (14) with respect to time results in.

$$\dot{E}_b(t) = \frac{1}{2} \ddot{R}_b(t)^T M \dot{R}_b(t). \quad (15)$$

The kinetic and potential energy functions of the payload can be defined as:

$$E_h(t) = \frac{1}{2} \dot{R}_h(t)^T m \dot{R}_h(t) + L(t)mg(1 - C_\alpha C_\beta). \quad (16)$$

Taking the derivative of Equation (16) with respect to time results in.

$$\dot{E}_h(t) = m \left(G_b - \frac{m}{M+m} \eta(t) \left(L(t) \dot{\eta}(t)^T \dot{\eta}(t) - \eta(t)^T F_e(t) \right) \right) \dot{R}_b(t). \quad (17)$$

To describe the subsequent control design and stability analysis more clearly, the following auxiliary signals $p_a(t)$, $p_b(t)$ are defined as:

$$\begin{cases} p_a(t) = (k_b \dot{R}_b(t))^T \eta(t), \\ p_b(t) = (k_p e_b(t))^T \eta(t). \end{cases} \quad (18)$$

where $e_b(t) = R_b(t) - R_d$ is the displacement error of the UAV, and $R_d = [R_{dx}, R_{dy}, R_{dz}]^T \in \mathbb{R}^3$ is the target position coordinate of the UAV, and its derivative with respect to time is zero. In Equation (18), k_b and $k_d \in \mathbb{R}^{3 \times 3}$ are two diagonal positive definite gain matrices.

To achieve more precise control over the UAV's flight performance, the total thrust generated by the UAV must be skillfully decomposed into two components: a force $f_\alpha(t)$ parallel to the cable, and a force $f_\beta(t)$ perpendicular to the cable. The primary role of $f_\alpha(t)$ is to counteract the weight of the payload and correct positional deviations in the flight path, while $f_\beta(t)$ is dedicated to minimizing the swinging motion of the payload, ensuring overall stability and operational accuracy. The specific expression for $f_\alpha(t)$ is:

$$f_\alpha(t) = \frac{M+m}{Mk_b + mk_h} \left[-(Mk_b + mk_h)g + \frac{(k_b - k_h)Mm}{M+m} (\dot{\alpha}^2 C_\beta^2 + \dot{\beta}^2) \hat{L}(t) + p_a(t) + p_b(t) \right]. \quad (19)$$

In Equation (19), $\hat{L}(t)$ is used as an estimate for the unknown length of the cable. Meanwhile, the estimation error of the cable length can be defined as $\tilde{L}(t) = L - \hat{L}(t)$, and an adaptive update law can be designed as follows:

$$\dot{\hat{L}}(t) = \frac{1}{\lambda} \text{spec} \left(\frac{(k_b - k_h)Mm}{M+m} (\dot{\alpha}^2 C_\beta^2 + \dot{\beta}^2) (\dot{x}_b S_\alpha C_\beta + \dot{y}_b S_\beta - \dot{z}_b C_\beta C_\alpha) \right), \quad (20)$$

where λ is a positive constant, and the function $\text{spec}(\bullet)$ is defined as:

$$\text{spec}(i) = \begin{cases} 0, & \text{if } \hat{L}(t) = 0 \text{ and } i < 0 \\ i, & \text{else} \end{cases} \quad (21)$$

The controller for the quadrotor UAV transport system is designed with non-linear constraints arising from the UAV's state equations and dynamic characteristics. To improve the controller's performance and ensure the UAV's stable operation under these non-linear constraints, barrier Lyapunov functions are utilized in the controller design process.

Lemma 1. The function $V(x)$, defined with respect to the system $\dot{x} = f(x)$ within an open interval D containing the origin, is considered a barrier Lyapunov function if it meets the following conditions: 1. $V(x)$ is continuous and differentiable within the interval D . 2. $V(x)$ is positive-definite within the interval D . 3. As x approaches the boundary of the interval D , $V(x) \rightarrow \infty$. 4. For all $t \geq 0$ and for any initial condition $x(0) \in D$, there exists a positive number b such that the solution of $\dot{x} = f(x)$ satisfies $V(x) \leq b$ for all t .

The design of the integral barrier Lyapunov function is as follows:

$$V(e_b, \epsilon) = \int_0^{e_b} \frac{\zeta k_c^2}{k_c^2 - (\zeta + \epsilon)^2} d\zeta, \quad (22)$$

where k_c represents the constraint range, and $\varepsilon \in \mathbb{R}^3$, $\varepsilon = e_b(t) + R_d$ represents state constraints.

Based on the constructed energy function of the quadrotor UAV transport system and the designed integral barrier Lyapunov function, the Lyapunov function is designed as:

$$V(t) = k_b E_b + k_h E_h + k_f \int_0^{e_b} \frac{\zeta k_c^2}{k_c^2 - (\zeta + \varepsilon)^2} d\zeta. \quad (23)$$

The derivative of time can be obtained by taking the derivative of time for both sides of $V(t)$ at the same time.

$$\begin{aligned} \dot{V}(t) = & \dot{x}_b \left(k_{bx} F_{ex} + (k_{bx} - k_{hx}) T S_\alpha C_\beta + \frac{e_x k_{cx}^2}{k_{cx}^2 - (e_x + \varepsilon_x)^2} \right) \\ & + \dot{y}_b \left(k_{by} F_{ey} + (k_{by} - k_{hy}) T S_\beta + \frac{e_y k_{cy}^2}{k_{cy}^2 - (e_y + \varepsilon_y)^2} \right) \\ & + \dot{z}_b \left(k_{bz} F_{ez} - (k_{bz} - k_{hz}) T C_\beta C_\alpha + \frac{e_z k_{cz}^2}{k_{cz}^2 - (e_z + \varepsilon_z)^2} - (M k_{bz} + m k_{hz}) g \right). \end{aligned} \quad (24)$$

To ensure that $\dot{V}(t) \leq 0$, it is only necessary to design a controller such that:

$$\begin{cases} k_{bx} F_{ex} + (k_{bx} - k_{hx}) T S_\alpha C_\beta + \frac{e_x k_{cx}^2}{k_{cx}^2 - (e_x + \varepsilon_x)^2} = -k \dot{x}_b, \\ k_{by} F_{ey} + (k_{by} - k_{hy}) T S_\beta + \frac{e_y k_{cy}^2}{k_{cy}^2 - (e_y + \varepsilon_y)^2} = -k \dot{y}_b, \\ k_{bz} F_{ez} - (k_{bz} - k_{hz}) T C_\beta C_\alpha + \frac{e_z k_{cz}^2}{k_{cz}^2 - (e_z + \varepsilon_z)^2} - (M k_{bz} + m k_{hz}) g = -k \dot{z}_b. \end{cases} \quad (25)$$

By organizing the specific expression of T from Equation (10) and the relationship between $F_e(t)$ and $F_b(t)$ from Equation (6), we can obtain:

$$\begin{cases} \Theta_1 = -k_b F_{hx}(t) C_\alpha - k_b F_{hy}(t) S_\alpha S_\beta + k_M F_{hy}(t) S_\alpha C_\beta + \delta(t) L(t) S_\alpha C_\beta, \\ \Theta_2 = -k_b F_{hy}(t) C_\beta - k_M F_{hy}(t) S_\beta + \delta(t) L(t) S_\beta, \\ \Theta_3 = -k_b F_{hx}(t) S_\alpha + k_b F_{hy}(t) S_\beta C_\alpha - k_M F_{hy}(t) C_\alpha C_\beta + \delta(t) L(t) C_\alpha C_\beta, \end{cases} \quad (26)$$

The expression for the controller can be obtained by organizing Equation (26):

$$\begin{cases} F_{hx}(t) = -\frac{1}{k_b} (\Theta_1 C_\alpha + \Theta_3 S_\alpha), \\ F_{hy}(t) = -\frac{1}{k_b} (\Theta_1 S_\alpha S_\beta + \Theta_2 C_\beta - \Theta_3 S_\beta C_\alpha), \\ F_{hz}(t) = \frac{1}{k_M} (\Theta_1 S_\alpha C_\beta - \Theta_2 S_\beta - \Theta_3 C_\alpha C_\beta + \delta(t) L(t)). \end{cases} \quad (27)$$

where Equation (27) is an implicit equation, as shown in Equation (6), where F_{hx} is a function of α and β . And Θ_1 , Θ_2 , and Θ_3 are auxiliary quantities, and their specific expressions are:

$$\begin{cases} \Theta_1 = \frac{k_b k_{cx}^2 e_x}{k_{cx}^2 - (e_x + \varepsilon_x)^2} + k \dot{x}_Q(t), \\ \Theta_2 = \frac{k_b k_{cy}^2 e_y}{k_{cy}^2 - (e_y + \varepsilon_y)^2} + k \dot{y}_Q(t), \\ \Theta_3 = \frac{k_b k_{cz}^2 e_z}{k_{cz}^2 - (e_z + \varepsilon_z)^2} + k \dot{z}_Q(t) - (M k_{bz} + m k_{hz}) g. \end{cases} \quad (28)$$

And

$$\begin{cases} \delta(t) = \frac{(k_b - k_h) m M}{M + m} \dot{\eta}(t)^T \dot{\eta}(t), \\ k_M = \frac{M k_b + m k_h}{m + M}. \end{cases} \quad (29)$$

From the design of the energy function, it is evident that k_b and k_h respectively determine the emphasis placed on the UAV's position and the swing of the suspended

payload within the controller. This means that as the difference between k_h and k_b increases, the influence of the “anti-sway action” within the controller becomes more significant, while if this difference decreases, its impact diminishes accordingly.

Define the attitude angle errors $e_\Lambda(t) = \Lambda(t) - \Lambda_d$, where $\Lambda_d = [\phi_d \ \theta_d \ \psi_d]^T$ represents the desired attitude of the quadrotor UAV. Drawing from the dynamic model of the attitude angles (Equation (12)) and considering the attitude angle errors during the UAV’s flight, we have designed a nonlinear attitude angle controller.

$$\begin{cases} u_2(t) = -\frac{\dot{\theta}\dot{\psi}(I_y - I_z)}{I_x} - k_{pu2}e_\phi(t) - k_{du2}\dot{\phi} - k_{ru2}\phi, \\ u_3(t) = -\frac{\dot{\phi}\dot{\psi}(I_z - I_x)}{I_y} - k_{pu3}e_\theta(t) - k_{du3}\dot{\theta} - k_{ru3}\theta, \\ u_4(t) = -\frac{\dot{\phi}\dot{\theta}(I_x - I_y)}{I_z} - k_{pu4}e_\psi(t) - k_{du4}\dot{\psi} - k_{ru4}\psi. \end{cases} \quad (30)$$

where $k_{pu}, k_{du}, k_{ru} \in \mathbb{R}_+$ are all control gains.

As shown in Figure 2, this is the double closed-loop nonlinear control scheme designed in this paper. The controller integrates energy-based methods and integral barrier Lyapunov functions, achieving precise closed-loop control of the position and attitude angle for the quadrotor UAV transportation system.

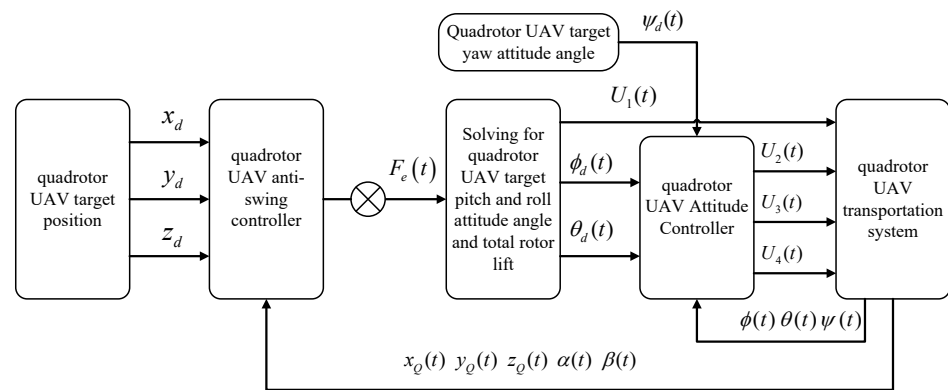


Figure 2. Block diagram of the control system.

3.2. Stability Analysis of Controller

Theorem 1. The quadrotor UAV suspension system described by Equation (11) can effectively achieve the gradual approach of the UAV’s position to a predetermined target point through the controller designed in Equation (27). During this process, the swaying angle of the suspended load is also effectively controlled to ensure that the system stably converges to a balanced state such that

$$\lim_{t \rightarrow \infty} [R_b, \dot{R}_b \alpha, \beta, \dot{\alpha}, \dot{\beta}] = [R_d, 0, 0, 0, 0]. \quad (31)$$

Proof of Theorem 1. The Lyapunov function can be designed as:

$$V(t) = k_b E_b + k_h E_h + k_f \int_0^{e_b} \frac{\zeta k_c^2}{k_c^2 - (\zeta + \varepsilon)^2} d\zeta + \frac{1}{2} \lambda \tilde{L}^2(t). \quad (32)$$

□

By substituting Equation (27) into Equation (32) and organizing the result, we can obtain:

$$\dot{V}(t) = -\dot{R}_b(t)^T k \dot{R}_b(t) + \left(\delta(t) \eta(t)^T \dot{R}_b(t) - \text{spec} \left(\delta(t) \eta(t)^T \dot{R}_b(t) \right) \right) \tilde{L}(t). \quad (33)$$

According to the previous Equations (20) and (21), the following inequality can be obtained.

$$\left(\delta(t)\eta(t)^T \dot{R}_b(t) - \text{spec} \left(\delta(t)\eta(t)^T \dot{R}_b(t) \right) \right) \tilde{L}(t) \leq 0. \quad (34)$$

Thus, it can be concluded that

$$\dot{V}(t) = -\dot{R}_b(t)^T k \dot{R}_b(t) + \left(\delta(t)\eta(t)^T \dot{R}_b(t) - \text{spec} \left(\delta(t)\eta(t)^T \dot{R}_b(t) \right) \right) \tilde{L}(t) \leq 0. \quad (35)$$

From the above results, it can be concluded that $V(t) \in L_\infty$, and based on the fact that $E_b(t)$, $E_h(t)$, $\int_0^{e_b} \frac{\varsigma k_c^2}{k_c^2 - (\varsigma + \varepsilon)^2} d\varsigma$, $\tilde{L}^2(t) \geq 0$, it can be inferred that they are bounded. By utilizing the conclusion that $E_b(t) \in L_\infty$, it can be derived that $\dot{R}_b(t) \in L_\infty$. Then, due to the boundedness of $E_h(t)$ and the unit vector $\eta(t)$, it can be concluded that $\dot{R}_h(t) \in L_\infty$. Given the boundedness of $\dot{R}_b(t)$ and $\dot{R}_h(t)$, it can be inferred that $\dot{\eta}(t) \in L_\infty$. Based on this inference, we can conclude that all signals in the quadrotor UAV transportation system are bounded.

Given the boundedness of all state variables, the Barbalat's lemma can now be employed to prove Theorem 1. For this purpose, the set Ω is defined as follows:

$$\Omega = \{ \dot{V}(t) = 0 \}, \quad (36)$$

Lemma 2. By Barbalat's theorem, assuming that there exists a function $f(t)$ which is uniformly continuous in $[0, \infty) \rightarrow \mathbb{R}$ and that there exists $\lim_{t \rightarrow \infty} \int_0^t f(\tau) d\tau < \infty$, a limit $\lim_{t \rightarrow \infty} f(t) = 0$ exists.

According to Equation (32):

$$E_b(t) = \frac{1}{2} \dot{R}_b(t)^T M \dot{R}_b(t) \leq V(t) \leq V(0). \quad (37)$$

$\dot{R}_b(t)$ is bounded according to Equation (37). Similarly, this also means that the other state quantities are also bounded.

From Equations (32) and (35), there exists $0 \leq V(t) \leq V(0)$, that is, $V(t)$ is bounded. Integration of Equation (35) can be obtained:

$$V(t) - V(0) \leq - \int_0^t \dot{V}(\tau) d\tau. \quad (38)$$

From Equation (38), it can be obtained that:

$$\int_0^t \dot{V}(\tau) d\tau \leq V(0) - \lim_{t \rightarrow \infty} V(t) < \infty. \quad (39)$$

It can be obtained from Equation (39):

$$\int_0^t \dot{V}(\tau) d\tau < \infty. \quad (40)$$

According to Equations (35), (40) and Lemma 2 the following can be obtained:

$$\begin{aligned} \lim_{t \rightarrow \infty} \dot{R}_b(t) &= 0, \\ \lim_{t \rightarrow \infty} \dot{L}(t) &= 0, \\ \lim_{t \rightarrow \infty} \tilde{d}(t) &= 0. \end{aligned} \quad (41)$$

And Equation (11) can be rewritten as:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = R_h^c \begin{bmatrix} F_{hx}(t) \\ F_{hy}(t) \\ \frac{M}{M+m} F_{hz}(t) \end{bmatrix} + \delta(t) L(t) \begin{bmatrix} S_\alpha C_\beta \\ S_\beta \\ -C_\alpha C_\beta \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ Mg \end{bmatrix}. \quad (42)$$

Equation (42) can be regarded as an equation system involving $F_{hx}(t)$, $F_{hy}(t)$, $F_{hz}(t)$, and by solving this equation system, the specific mathematical expressions for $F_{hx}(t)$, $F_{hy}(t)$, $F_{hz}(t)$ can be obtained:

$$\begin{bmatrix} F_{hx}(t) \\ F_{hy}(t) \\ F_{hz}(t) \end{bmatrix} = \begin{bmatrix} Mg S_\alpha C_\beta \\ Mg S_\beta \\ -(M+m)g C_\alpha C_\beta \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ mL(t)\dot{\eta}(t)^T \dot{\eta}(t) \end{bmatrix}. \quad (43)$$

The result can be obtained by substituting Equation (43) into the designed controller (27):

$$\begin{cases} \Theta_1 = k_h mg S_\alpha C_\alpha C_\beta^2 + k_h m S_\beta C_\alpha L(t) \dot{\eta}(t)^T \dot{\eta}(t), \\ \Theta_2 = k_h mg S_\beta C_\alpha C_\beta + k_h m S_\beta L(t) \dot{\eta}(t)^T \dot{\eta}(t), \\ \Theta_3 = -k_b Mg - k_h mg C_\alpha^2 C_\beta^2 - k_h m C_\alpha C_\beta L(t) \dot{\eta}(t)^T \dot{\eta}(t). \end{cases} \quad (44)$$

Take the time derivative of both sides of Equation (44), since $\dot{\Theta}_1, \dot{\Theta}_2, \dot{\Theta}_3 = 0$, the following expression can be obtained:

$$\dot{\alpha} (S_\alpha^2 C_\alpha C_\beta^2 + C_\alpha^3 C_\beta^2 + (\dot{\alpha}^2 C_\beta^2 + \dot{\beta}^2) C_\beta) = 0. \quad (45)$$

Then we intend to employ the method of proof by contradiction to demonstrate that $\dot{\alpha} = 0, \dot{\beta} = 0$. We assume that within some subset of set Ω , there exists a point that does not satisfy the condition of having $\dot{\alpha} \neq 0, \dot{\beta} \neq 0$. Based on Assumption 1, we find that $C_\alpha > 0, C_\beta > 0$. Furthermore, the outcome of Equation (40) clearly leads to the following conclusion:

$$\begin{cases} S_\alpha^2 C_\alpha C_\beta^2 = 0, \\ C_\alpha^3 C_\beta^2 = 0, \\ \dot{\alpha} C_\beta^2 + \dot{\beta}^2 = 0. \end{cases} \quad (46)$$

Based on Equation (46), we can derive the following equation:

$$\begin{cases} \alpha = 0, \\ \beta = 0. \end{cases} \quad (47)$$

And

$$\begin{cases} \dot{\alpha} = 0, \\ \dot{\beta} = 0. \end{cases} \quad (48)$$

The conclusion of Equation (48) conflicts with the assumption that there exists a point in set Ω such that $\dot{\alpha} \neq 0, \dot{\beta} \neq 0$. Therefore, it can be inferred that the point in set Ω does not actually exist, and thus $\dot{\alpha} = 0, \dot{\beta} = 0$ in set Ω . And is also $\ddot{\alpha} = 0, \ddot{\beta} = 0, \alpha = 0, \beta = 0$. Substitute the above result into Equation (36):

$$\begin{cases} \Theta_1 = 0, \\ \Theta_2 = 0, \\ \Theta_3 = -(Mk_b + mk_h)g. \end{cases} \quad (49)$$

Based on Equations (26) and (49), it can be obtained that:

$$e_b(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (50)$$

According to the Barbalat's lemma, prove that $e_b(t)$, $\dot{R}_b(t)$, $\alpha(t)$, $\dot{\alpha}(t)$, $\beta(t)$, and $\dot{\beta}(t)$ in the quadrotor UAV transportation system all asymptotically converge to the equilibrium point.

Consider a Lyapunov function candidate for attitude angle control:

$$V'(t) = \frac{1}{2} \dot{\Lambda}(t)^T k_{du} \dot{\Lambda}(t) + \frac{1}{2} e_{\Lambda}(t)^T k_{pu} e_{\Lambda}(t) + \frac{1}{2} \Lambda(t)^T k_{ru} \Lambda(t) \quad (51)$$

Taking the derivative of Equation (10) with respect to time results in.

$$\begin{aligned} \dot{V}'(t) = & \dot{\phi} \left(\frac{\dot{\theta} \dot{\psi} (I_y - I_z)}{I_x} + u_2(t) + k_{pu1} e_{\phi}(t) + k_{ru2} \phi \right), \\ & + \dot{\theta} \left(\frac{\dot{\phi} \dot{\psi} (I_z - I_x)}{I_y} + u_3(t) + k_{pu3} e_{\theta}(t) + k_{ru3} \theta \right), \\ & + \dot{\psi} \left(\frac{\dot{\phi} \dot{\theta} (I_x - I_y)}{I_z} + u_4(t) + k_{pu4} e_{\psi}(t) + k_{ru4} \psi \right). \end{aligned} \quad (52)$$

Replacing Equation (30) in (52), it can be organized as:

$$\dot{V}'(t) = -\Lambda(t)^T k_{ru} \Lambda(t) \leq 0. \quad (53)$$

By Equation (53), it indicates that $V'(t) \in L_{\infty}$, and the state variables of the system $\Lambda(t)$, $\dot{\Lambda}(t)$, $e_{\Lambda}(t) \in L_{\infty}$. The case $V'(t) = 0$ only occurs when all the state variables are all equal to zero at the same time, so we can prove that the attitude loop subsystem can be asymptotically stable by (53).

4. Results

In this section, to evaluate the control performance of controller 27 for the quadrotor UAV transportation system, numerical simulations were carried out for position control and anti-swing control. A comparison was also made with a nonlinear antiswing control method in [27] and the LQR control method in [28], highlighting the superiority of this control strategy.

The desired attitude of the quadrotor UAV is provided by the position loop control strategy, while the required yaw angle is given by the navigation system.

$$\begin{cases} F(t) = \sqrt{F_{ex}(t)^2 + F_{ey}(t)^2 + F_{ez}(t)^2} \\ \theta_d = \arctan\left(\frac{F_{ey}(t)}{F_{ex}(t)}\right) \\ \phi_d = \arctan\left(\frac{F_{ez}(t)}{F_{ex}(t)}\right) \end{cases} \quad (54)$$

The system parameters are as shown in Table 1.

Table 1. Parameters of the quadrotor UAV transport system.

Parameter	Value
M	0.68 kg
m	0.08 kg
L	0.9 m
g	9.81 m/s ²
k_{bx}, k_{by}, k_{bz}	0.7, 0.6, 0.85
k_{hx}, k_{hy}, k_{hz}	3, 3, 2.6
λ	1.1
k_x, k_y, k_z	0.3, 0.3, 0.55

The designed comparative controller is:

$$\begin{cases} F_{ex}(t) = -k_{px}e_x(t) - k_{ix} \int e_{bx}(t)d\tau - k_{dx}\dot{e}_x(t), \\ \quad + k_{\gamma px}\alpha(t) + k_{\gamma ix} \int \alpha(t)d\tau + k_{\gamma dx}\dot{\alpha}(t), \\ F_{ey}(t) = -k_{py}e_y(t) - k_{iy} \int e_{by}(t)d\tau - k_{dy}\dot{e}_y(t), \\ \quad + k_{\gamma py}\beta(t) + k_{\gamma iy} \int \beta(t)d\tau + k_{\gamma dy}\dot{\beta}(t), \\ F_{ez}(t) = -k_{pz}e_z(t) - k_{iz} \int e_{bz}(t)d\tau - k_{dz}\dot{e}_z(t) - (M + m)g. \end{cases} \quad (55)$$

The gains of the controller are selected as:

$$\begin{cases} k_p = \text{diag}([0.6 \quad 0.6 \quad 0.95]) \\ k_i = \text{diag}([0.02 \quad 0.02 \quad 0.03]) \\ k_d = \text{diag}([0.5 \quad 0.5 \quad 0.5]) \\ k_{\gamma p} = \begin{bmatrix} 0.7 & 0 & 0 \\ 0 & 0.6 & 0 \end{bmatrix}^T \\ k_{\gamma i} = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \end{bmatrix}^T \\ k_{\gamma d} = \begin{bmatrix} 0.6 & 0 & 0 \\ 0 & 0.6 & 0 \end{bmatrix}^T \end{cases} \quad (56)$$

The simulation experiment was divided into three groups, in Case 1 Position Control, the primary focus is on conducting a comparative analysis of different control strategies in terms of their dynamic performance in system displacement control. Case 2 Anti-swing Control emphasized the comparison of the impact of these control strategies on the dynamic performance of system payload swing angle control. Case 3 Robust Control is centered on the evaluation of the robustness performance exhibited by various control strategies.

4.1. Case 1: Position Control

In this case, a stable quadrotor UAV transport system is required to be commanded to reach a specific target point. The initial parameters of the system are as shown in Table 2, with the elastic deformation of the cables taken into consideration. The simulation results are presented in Figures 3–8. To more clearly demonstrate the control performance, error curves for the $R_b(t)$ and the two swing angles $\alpha(t)$, $\beta(t)$ are plotted.

Table 2. Initial parameters of the quadrotor UAV transport system in Case 1.

Parameter	Value
$R_b(0)$	0 m, 0 m, 0 m
R_d	4 m, 3 m, 2 m
$\alpha(0)$	0
$\beta(0)$	0
k_{cx}, k_{cy}, k_{cz}	5, 5, 2

In this case, the quadrotor UAV transportation system is initially in a stable state and now needs to be moved to the target point. Figures 3 and 4 illustrate the positions control and anti-swing control of the quadrotor UAV transportation system while carrying a payload. Figure 5 illustrates the dynamic process of the system's attitude angle control. The change in length of the cable due to elastic deformation is depicted in Figure 6. And the errors in the system's displacement and payload swing angle are shown in Figures 7 and 8, respectively.

Based on the data analysis from Table 3, the control scheme proposed in this paper demonstrates a more stable and rapid displacement control performance compared to both nonlinear antishwing controller and LQR controller.

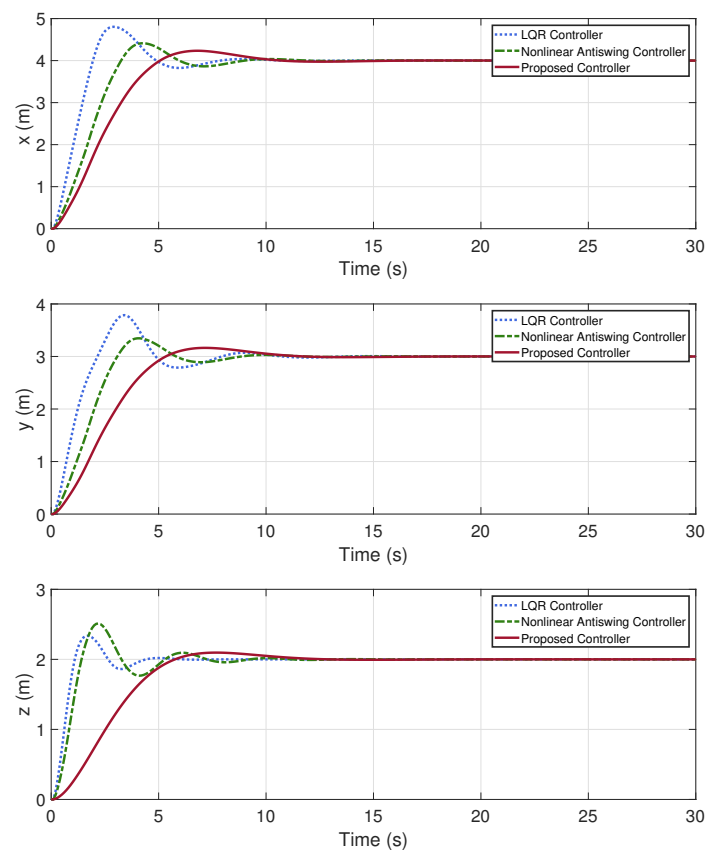


Figure 3. The positions control of the system in Case 1.

Table 3. Positions result of the position control in Case 1.

Simulation Experiments	LQR Control	Nonlinear Antiswing Control	Proposed Control
Adjusting Time of positions	10.4 s	11.1 s	9.6 s
Overshoot of $x_b(t)$	4.81 m	4.40 m	4.23 m
Overshoot of $y_b(t)$	3.86 m	3.35 m	3.16 m
Overshoot of $z_b(t)$	2.34 m	2.51 m	2.09 m

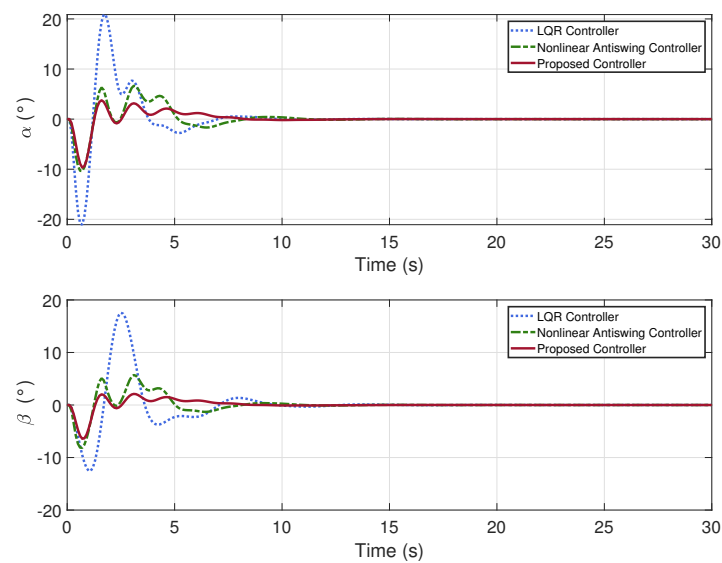


Figure 4. The swing angles control of the system in Case 1.

According to the data from Table 4, the control scheme designed in this paper exhibits superior anti-swing performance compared to both the nonlinear antiswing controller and the LQR controller.

Table 4. Swing angles result of the position control in Case 1.

Simulation Experiments	LQR Control	Nonlinear Antiswing Control	Proposed Control
Adjusting Time of anti-swing	14.6 s	16.8 s	8.3 s
Overshoot of $\alpha(t)$	21.07°	11.37°	9.67°
Overshoot of $\beta(t)$	17.54°	8.31°	6.43°

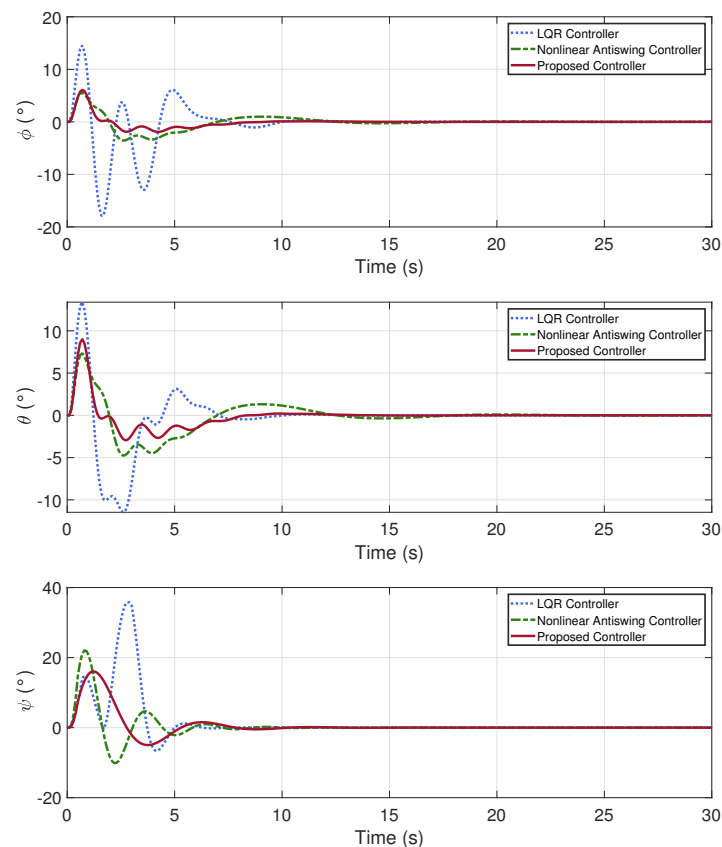


Figure 5. The attitude angles control of the system in Case 1.

In Figure 6, the attitude angles of the quadrotor UAV are maintained within a reasonable range, demonstrating good stability.

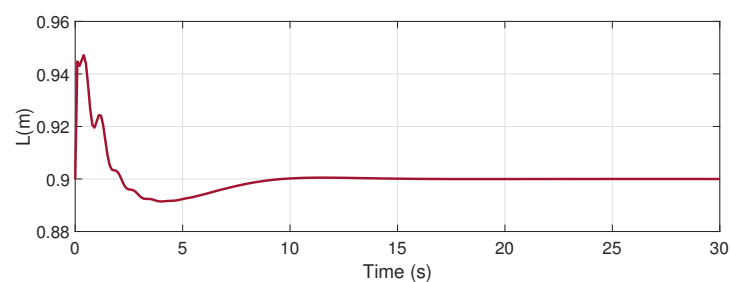


Figure 6. The cable length of the system in Case 1.

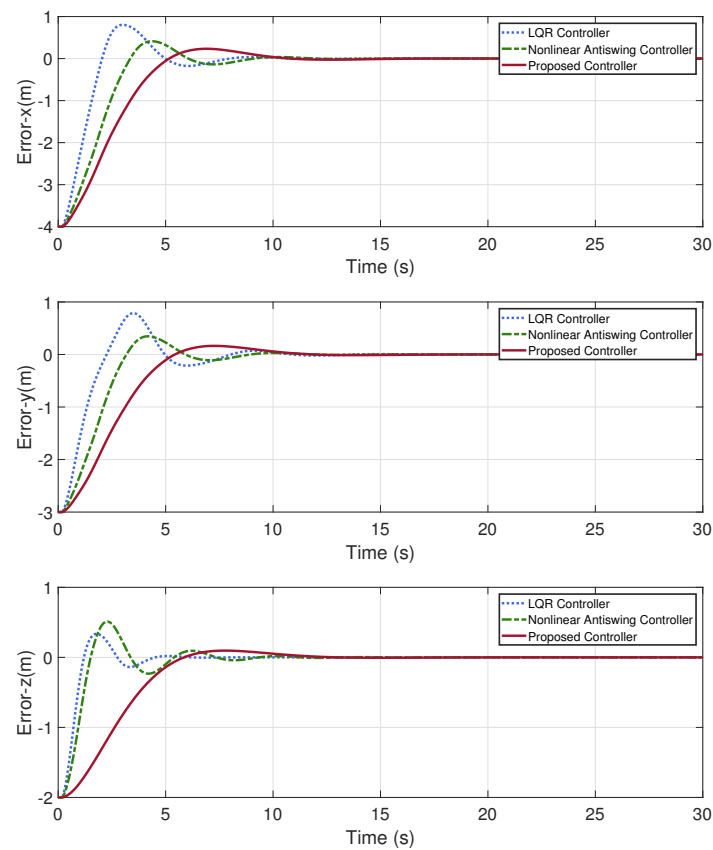


Figure 7. The position errors of the system in Case 1.

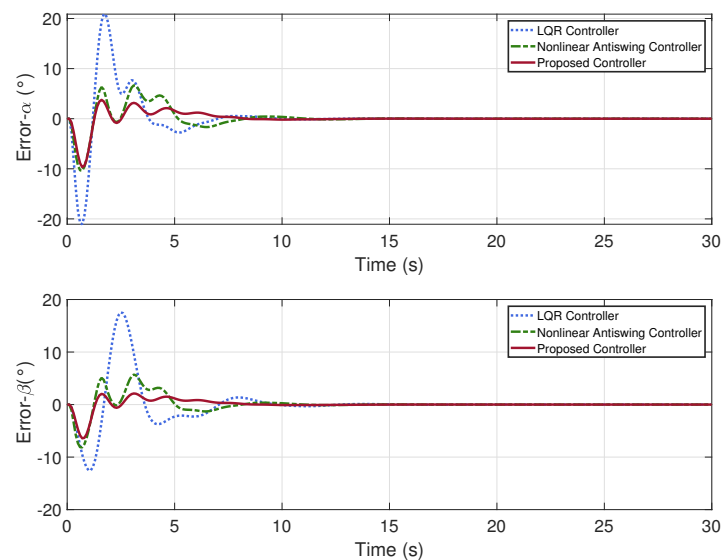


Figure 8. The swing angles error of the system in Case 1.

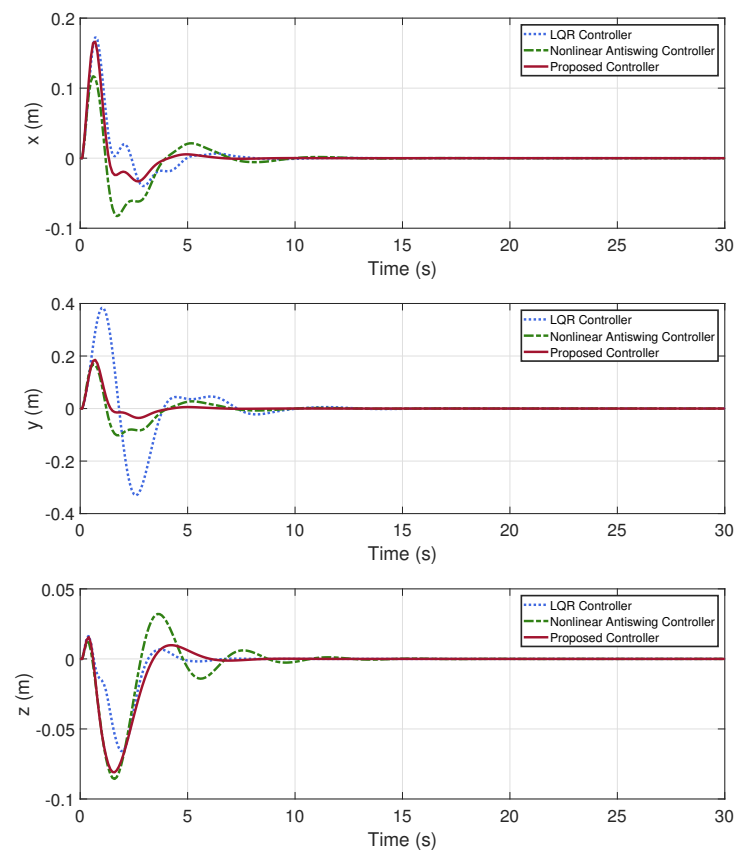
4.2. Case 2: Anti-Swing Control

In this case, the quadrotor UAV transport system with an unstable initial state is required to perform control to suppress the swinging of the payload. The initial parameters of the system are as shown in Table 5, with the elastic deformation of the cables taken into consideration. The simulation results are presented in Figures 9–14, Tables 6 and 7. To more clearly demonstrate the control performance, error curves for the $R_b(t)$ axes and the two swing angles $\alpha(t)$, $\beta(t)$ are plotted.

Table 5. Initial parameters of the quadrotor UAV transport system in Case 2.

Parameter	Value
$R_b(0)$	0 m, 0 m, 0 m
R_d	0 m, 0 m, 0 m
$\alpha(0)$	$\frac{\pi}{8}$
$\beta(0)$	$\frac{\pi}{8}$
k_{cx}, k_{cy}, k_{cz}	0.8, 0.8, 0.75

In this case, the quadrotor UAV transportation system is initially unstable. The current task requires the quadrotor UAV to stabilize the payload angles of the swing. Figures 9 and 10 illustrate the positions control and anti-swing control of the quadrotor UAV transportation system while carrying a payload. Figure 11 illustrates the dynamic process of the system's attitude angle control. The change in length of the cable due to elastic deformation is depicted in Figure 12. And the errors in the system's displacement and payload swing angle are shown in Figures 13 and 14, respectively.

**Figure 9.** The positions control of the system in Case 2.**Table 6.** Positions result of the position control in Case 2.

Simulation Experiments	LQR Control	Nonlinear Antiswing Control	Proposed Control
Adjusting Time of positions	10.6 s	12.3 s	9.1 s
Overshoot of $x_b(t)$	0.17 m	0.12 m	0.16 m
Overshoot of $y_b(t)$	0.38 m	0.17 m	0.19 m
Overshoot of $z_b(t)$	0.02 m	0.15 m	0.02 m

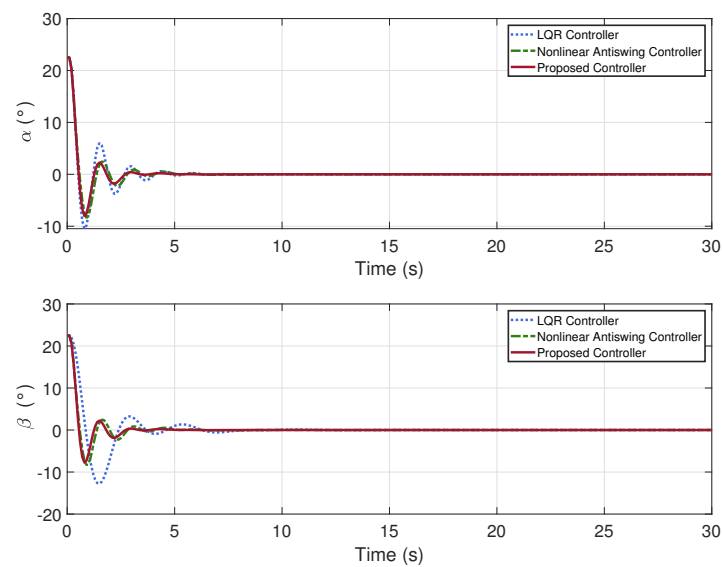


Figure 10. The swing angles control of the system in Case 2.

Table 7. Swing angles result of the position control in Case 2.

Simulation Experiments	LQR Control	Nonlinear Antiswing Control	Proposed Control
Adjusting Time of anti-swing	8.26 s	6.80 s	4.90 s
Overshoot of $\alpha(t)$	10.47°	8.41°	8.03°
Overshoot of $\beta(t)$	12.78°	8.36°	7.72°

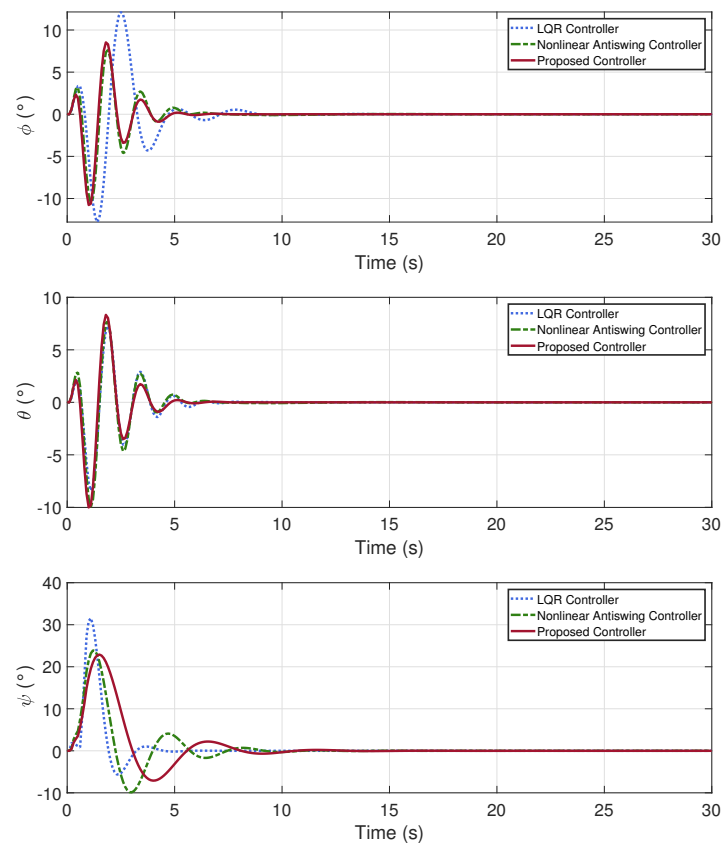


Figure 11. The attitude angles control of the system in Case 2.

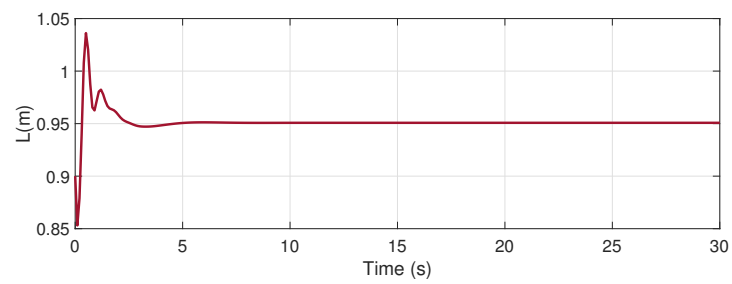


Figure 12. The cable length of the system in Case 2.

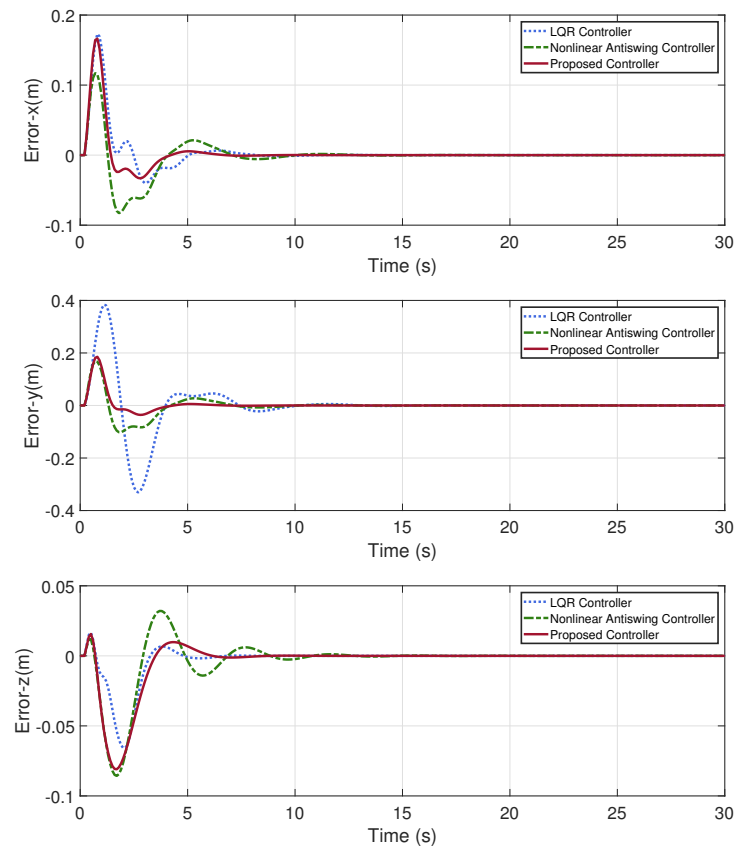


Figure 13. The position errors of the system in Case 2.

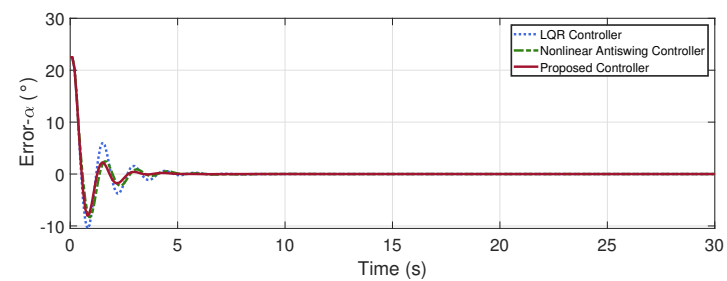


Figure 14. Cont.

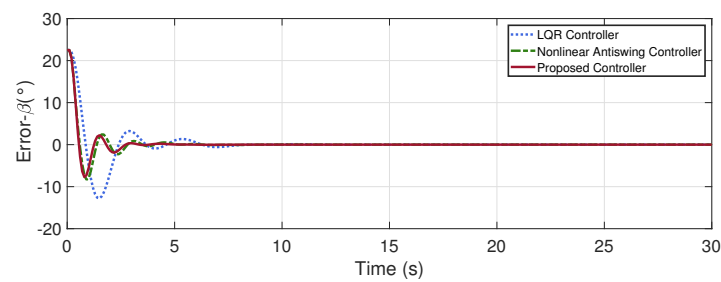


Figure 14. The swing angles error of the system in Case 2.

4.3. Case 3

In this case, once the quadrotor UAV transport system has achieved a state of stability, a disturbance is intentionally injected at the 10.5 s mark to evaluate the robustness of the controller.

The simulation results are presented in Figures 15 and 16. From the simulation results, it can be seen that when the system reaches a steady state, the system as a whole suddenly encounters the same disturbance, and the feedback and suppression of the perturbation are different for different controllers. The outcomes clearly show that the control scheme proposed in this paper exhibits greater robustness compared to both the LQR controller and the nonlinear antiswing controller.

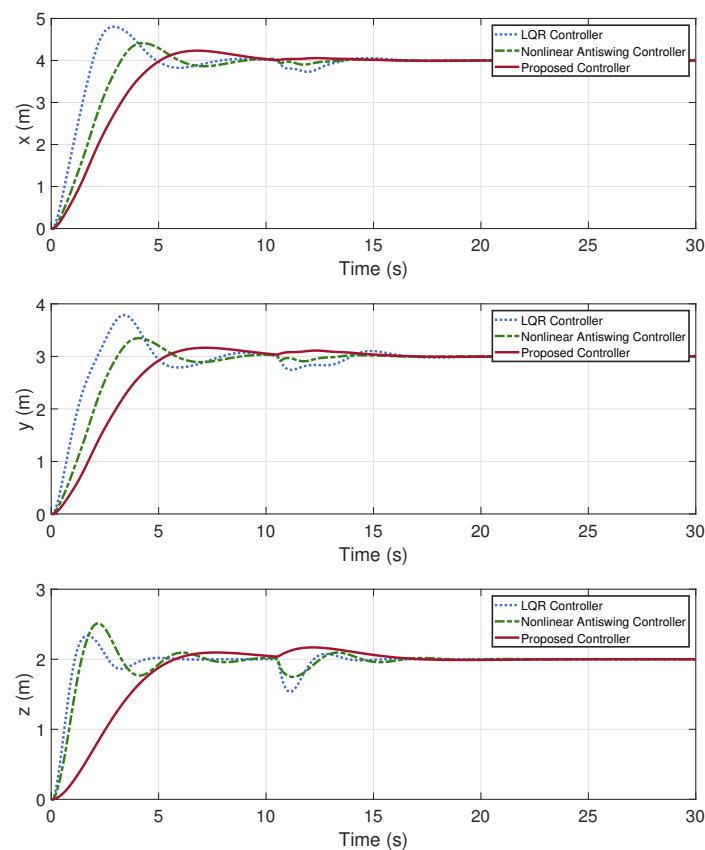


Figure 15. The positions control of the system in Case 3.

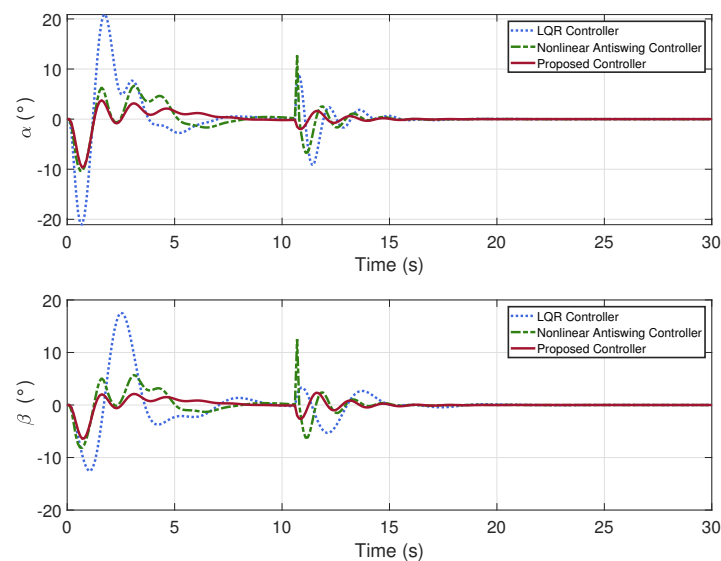


Figure 16. The swing angles control of the system in Case 3.

5. Conclusions

This paper derives the dynamical model of the quadrotor UAV transportation system using Newton-Euler equations and proposes a control strategy based on the energy method and an barrier Lyapunov function. Building on position control and anti-swing control, combined with the force analysis in Newton-Euler equations, this paper introduces an adaptive control design for the unknown cable length connecting the UAV and the payload. The asymptotic stability of the control system is verified using Lyapunov's theory and the LaSalle's invariance principle. Ultimately, compared to traditional controllers, the control scheme proposed in this paper exhibits superior performance in terms of displacement control and anti-swing control.

This paper employs the Newton-Euler modeling method, which clearly reveals the interactions between forces. We conducted independent force analyses on the UAV, cable, and payload, providing convenience for future research, especially in dealing with special cases of specific system components and stable control studies under external disturbances. Additionally, due to elastic deformation, the length of the cable may change, and updating the cable's length in real-time can effectively reduce positioning errors of the payload, ensuring the accuracy and stability of system control. By applying barrier Lyapunov functions, Lyapunov stability theorems, and LaSalle's invariance theorem, we ensured that the designed controller exhibits good precision and stability during the dynamic regulation of system displacement, attitude angle, and payload swing angle.

6. Future Work

Future research will focus on addressing the uncertainty of model parameters in the quadrotor UAV transportation system and the potential external disturbances that the system may encounter during operation. Given the underactuated nature of quadrotor UAV transportation systems, in-depth exploration of controller design with strong robustness against disturbances is a research direction of significant practical application value.

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Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Assumptions

- The suspended payload can be considered as a particle, with its attachment point aligning with the center of mass of the quadrotor UAV. Additionally, the payload remains beneath the UAV at all times, indicating that the swing angles $\alpha \in (-90^\circ, 90^\circ)$, $\beta \in (-90^\circ, 90^\circ)$.
- The quadrotor UAV and its propellers are structurally identical and rigid, with each propeller generating thrust that is mutually aligned.

References

1. Kownacki, C. Control and Position Tracking for UAVs. *Appl. Sci.* **2024**, *14*, 1909. [\[CrossRef\]](#)
2. Sadr, S.; Ali, S.; Moosavian, S.A.A.; Zarafshan, P. Dynamics modeling and control of a quadrotor with swing load. *IEEE J. Robot.* **2014**, *1*, 265897. [\[CrossRef\]](#)
3. Feng, D.; Sun, C.; He, S. Anti-Swing Control for Quadrotor-Slung Load Transportation System with Underactuated State Constraints. *Sensors* **2023**, *23*, 8995. [\[CrossRef\]](#) [\[PubMed\]](#)
4. Lv, Z.-Y.; Wu, Y.; Rui, W. Nonlinear motion control for a quadrotor transporting a cable-suspended payload. *IEEE Trans. Veh. Technol.* **2020**, *69*, 8192–8206. [\[CrossRef\]](#)
5. Godbole, A.R.; Subbarao, K. Nonlinear control of unmanned aerial vehicles with cable suspended payloads. *Aerosp. Sci. Technol.* **2019**, *93*, 105299. [\[CrossRef\]](#)
6. Estevez, J.; Lopez-Guede, J.M.; Garate, G.; Graña, M. A hybrid control approach for the swing free transportation of a double pendulum with a quadrotor. *Appl. Sci.* **2021**, *11*, 5487. [\[CrossRef\]](#)
7. Estevez, J.; Garate, G.; Lopez-Guede, J.M.; Larrea, M. Review of aerial transportation of suspended-cable payloads with quadrotors. *Drones* **2024**, *8*, 35. [\[CrossRef\]](#)
8. Lee, T. Geometric control of quadrotor UAVs transporting a cable-suspended rigid body. *IEEE Trans. Control Syst. Technol.* **2017**, *26*, 255–264. [\[CrossRef\]](#)
9. Labbadi, M.; Muñoz-Vázquez, A.J.; Djemai, M.; Boukal, Y.; Zerrougui, M.; Cherkaoui, M. Fractional-order nonsingular terminal sliding mode controller for a quadrotor with disturbances. *Appl. Math. Model.* **2022**, *111*, 753–776. [\[CrossRef\]](#)
10. Goodarzi, F.A.; Lee, D.; Lee, T. Geometric control of a quadrotor UAV transporting a payload connected via flexible cable. *Int. J. Control. Autom. Syst.* **2015**, *13*, 1486–1498. [\[CrossRef\]](#)
11. Gabellieri, C.; Tognon, M.; Sanalitra, D.; Franchi, A. Force-based Pose Regulation of a Cable-Suspended Load Using UAVs with Force Bias. In Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS 2023, Detroit, MI, USA, 1–5 October 2023.
12. El Houm, Y.; Abbou, A.; Labbadi, M.; Cherkaoui, M. Optimal new sliding mode controller combined with modified supertwisting algorithm for a perturbed quadrotor UAV. *Int. J. Aerosp. Eng.* **2020**, *1*, 9753870. [\[CrossRef\]](#)
13. Zhang, X.; Zhuang, Y.; Zhang, X.; Fang, Y. A Novel Asymptotic Robust Tracking Control Strategy for Rotorcraft UAVs. *IEEE Trans. Autom. Sci. Eng.* **2023**, *20*, 2338–2349. [\[CrossRef\]](#)
14. Li, S.; Sun, Z. A generalized proportional integral observer-based robust tracking design approach for quadrotor unmanned aerial vehicle. *Int. J. Adv. Robot. Syst.* **2022**, *19*, 17298806221117052. [\[CrossRef\]](#)
15. Trachte, J.; Gonzalez, F.; McFadyen, A. Nonlinear models predictive control for a multi-rotor with heavy lung load. *Proc. Int. Conf. Unmanned Aircr. Syst. (ICUAS)*. **2014**, 1105–1110.
16. Nicotra, M.M.; Garone, E.; Naldi, R.; Marconi, L. Nested saturation control of an UAV carrying a suspended load. In Proceedings of the 2014 American Control Conference, Portland, OR, USA, 4–6 June 2014; pp. 3585–3590.
17. Wang, Z.; Chen, Y.; Zheng, X.; Cheng, L. Quadrotor UAV Control with Disturbance Based on Aerodynamic Parameter Estimation. *Inf. Control.* **2018**, *47*, 663–670.
18. Liang, X.; Yu, H.; Zhang, Z.; Liu, H.; Fang, Y.; Han, J. Unmanned aerial transportation system with flexible connection between the quadrotor and the payload: Modeling, controller design, and experimental validation. *IEEE Trans. Ind. Electron.* **2022**, *70*, 1870–1882. [\[CrossRef\]](#)
19. Liang, X.; Zhang, Z.; Yu, H.; Wang, Y.; Fang, Y.; Han, J. Antiswing Control for Aerial Transportation of the Suspended Cargo by Dual Quadrotor UAVs. *IEEE/ASME Trans. Mechatron.* **2022**, *27*, 5159–5172. [\[CrossRef\]](#)
20. Guerrero-Sánchez, M.E.; Mercado-Ravell, D.A.; Lozano, R.; García-Beltrán, C.D. Swing-attenuation for a quadrotor transporting a cable-suspended payload. *ISA Trans.* **2017**, *68*, 433–449. [\[CrossRef\]](#)
21. Goodarzi, F.A.; Lee, T. Dynamics and control of quadrotor UAVs transporting a rigid body connected via flexible cables. In Proceedings of the 2015 American Control Conference (ACC), Chicago, IL, USA, 1–3 July 2015; pp. 4677–4682.
22. Muthusamy, P.K.; Garratt, M.A.; Pota, H.R.; Muthusamy, R. Real-Time Adaptive Intelligent Control System for Quadcopter Unmanned Aerial Vehicles With Payload Uncertainties. *IEEE Trans. Ind. Electron.* **2022**, *69*, 1641–1653. [\[CrossRef\]](#)

23. Yang, S.; Xian, B. Energy-Based Nonlinear Adaptive Control Design for the Quadrotor UAV System With a Suspended Payload. *IEEE Trans. Ind. Electron.* **2020**, *67*, 2054–2064. [[CrossRef](#)]
24. Liang, X.; Li, H. Dynamic feedback linearization trajectory control of four-rotor suspension transportation system. *Acta Autom. Sin.* **2020**, *46*, 1993–2002.
25. Pizetta, I.H.B.; Brandao, A.S.; Sarcinelli-Filho, M. Modelling and control of a pvtol quadrotor carrying a suspended load. In Proceedings of the 2015 International Conference on Unmanned Aircraft Systems (ICUAS), Denver, CO, USA, 9–12 June 2015; pp. 444–450.
26. Liang, X.; Fang, Y.; Sun, N.; Lin, H. Nonlinear hierarchical control for unmanned quadrotor transportation systems. *IEEE Trans. Ind. Electron.* **2018**, *65*, 3395–3405. [[CrossRef](#)]
27. Sun, N.; Wu, Y.; Fang, Y.; Chen, H.; Lin, H. A Nonlinear Antiswing Control for Crane Systems With Double-Pendulum Swing Effects and Uncertain Parameters: Design and Experiments. *IEEE Trans. Autom. Sci. Eng.* **2018**, *15*, 1413–1422. [[CrossRef](#)]
28. Alaimo, A.; Artale, V.; Barbaraci, G.; Milazzo, C.L.R.; Orlando, C.; Ricciardello, A. LQR-PID control applied to hexacopter flight. *SIAM J. Numer. Anal.* **2016**, *9*, 47–56.

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