

Article

Secure Self-Triggered Time-Varying Formation Control for Quadrotor Swarms Against Sequential Multi-Link Scaling Attacks

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Highlights

What are the main findings?

- A novel secure self-triggered control scheme is proposed to achieve STVF for quadrotor swarms against SMLSAs.
- A design method of fully distributed and self-triggered control algorithm for quadrotor swarms is provided.

What are the implications of the main findings?

- The proposed scheme enables fully distributed STVF control without requiring global network information, and is implementable in a self-triggered manner, eliminating continuous neighbor-state monitoring.
- The exponential term of the current triggering instant is embedded in the adaptive coupling-strength update law to bound the coupling gain within a small range, thereby preventing input saturation due to monotonically increasing weights.

Abstract

This paper investigates secure self-triggered time-varying formation control for quadrotor swarm systems against sequential multi-link scaling attacks, which can be implemented in a self-triggered and fully distributed manner. Firstly, based on the outer-loop position and velocity control model of quadrotors, a fully distributed secure time-varying formation control protocol is constructed under sequential multi-link scaling attacks with three characteristics: distributed, sequential, and scalable, and the design criteria for fully distributed secure time-varying formation control are provided. Then, combining the inner and outer-loop control principles of quadrotors, by constructing Euler angle loop controllers and angular velocity controllers, the conversion of the control input of the outer-loop position and velocity to the inner-loop attitude control is achieved, and fully distributed secure time-varying formation control algorithms for quadrotor UAV swarm systems under attacks are proposed. Finally, the effectiveness and applicability of the fully distributed secure consensus method in the formation control of quadrotor UAV swarms is verified through flight experiments using a quadrotor UAV swarm flight test platform. The research results provide a useful reference for the practical application of the fully distributed secure cooperative control theory.

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1. Introduction

The unmanned aerial vehicle (UAV) swarm system, as a typical representative of multi-agent systems, has been widely applied in various civilian and military fields such as fire suppression and rescue [1], target detection [2,3], fire control correction, and swarm attack [4]. For instance, in the conflict between Ukraine and Russia, the deployment and application of various quadrotors have played a significant role in reconnaissance and surveillance, precise strikes, etc. [5]. The quadrotor offensive and defensive confrontations between the two sides have become increasingly intense, even influencing the course of the war and changing the nature of the conflict [6]. Due to the extensive application prospects of UAV swarm systems, the formation control technology of UAV swarm systems has developed rapidly.

Formation control is the foundation for achieving collaborative control of UAV swarm systems, and its core lies in designing appropriate control protocols to enable all individuals in the swarm system to form and maintain desired relative positional relationships according to the predetermined formation configuration [7]. The common formation control includes the leader–follower method [8], behavioral-based method [9], and virtual-structure method [10]. Based on the dynamics analysis of the reference formation and the motion characteristics of the formation individuals, Ren [11] unified the above three formation control methods into a consensus-based formation theory framework, and pointed out that the consensus-based formation control can overcome shortcomings of the above three methods. Based on whether the reference formation changes over time, that is, whether the formation vector is time-varying, the formation can be classified as a fixed formation [12] and a time-varying formation [13]. The time-varying formation exhibits high adaptability and flexibility in meeting task requirements after UAVs form a formation. The formation configuration can be dynamically adjusted according to specific rules, thereby enhancing the robustness of the swarm systems. When dealing with actual task scenarios or complex environments, such as task transitions, obstacle avoidance, real-time formation reconfiguration, and mobile target tracking, etc., the time-varying nature of formation becomes an indispensable capability for the swarm systems, ensuring that the swarm systems can flexibly respond to changing environments and efficiently execute diverse tasks [14].

Research on consensus-based formation control methods for UAV swarm systems mainly focuses on time-varying formation control problems, especially the time-varying formation control problems under various constraint conditions, which have been widely discussed. Since the influence of factors such as the dynamics model of the UAV swarms, the size of the UAV swarms, and the external environment, the formation vector function cannot be arbitrarily specified. For example, in time-varying formations of fixed-wing UAV swarms during circular flight, the radius of the formation vector function must exceed the maximum turning radius of the fixed-wing UAVs. Therefore, it is necessary to determine the feasibility conditions of the time-varying formation for the UAV swarms. Dong et al. [15] proposed a time-varying formation feasibility condition for formation control, that is, it requires the existence of a time-varying formation compensation vector that satisfies the constraints formed by the derivative of the formation vector and the overall system dynamic characteristics. Muslimov et al. [16] proposed a formation control and maintenance method for fixed-wing UAV swarms, which takes into account the dynamic characteristics of fixed-wing UAVs and the constraints on control inputs, and employs the

adaptive backstepping method to generate control inputs, enabling high-precision formation control of the UAV swarms. Li et al. [17] investigated the consensus-based time-varying formation control problem of quadrotor swarms, proposed the feasibility conditions for time-varying formation of quadrotor swarms, gave the overall motion trajectory of the quadrotor swarms, and conducted real flight experiments using an indoor optical motion capture positioning system to verify the effectiveness of the proposed method. Regarding the consensus-based formation control methods, a large number of research results have been achieved in the time-varying formation control of UAV swarms, which can be found in the review literature [18–20]. Finite-time formation and obstacle avoidance of UAV swarms based on biological hierarchical behavior was realized in [21]. It adopts finite-time convergence control to enhance the response speed under disturbances. Ferdinando et al. [22] analyzed and quantified the sampled nonlinear multi-agent system consensus tracking based on the input-to-state stability theory, and addressed the stability degradation caused by quantization errors. However, the above methods only consider the ideal environment without attacks. With the current application of various anti-UAV technologies, the safety of UAV swarms has been greatly threatened. Complex operating environments and potential malicious attacks may cause the communication links of UAV swarm systems to be simultaneously attacked. Such communication link attacks may directly result in incomplete, unavailable, or even interrupted information transmission over communication links, thereby significantly increasing the difficulty of cooperative control of the UAV swarms [23]. However, most existing cooperative control works of the UAV swarms overlook such security threats. Therefore, the formulation of secure cooperative frameworks of the UAV swarms under communication link attacks remains a fundamental challenge.

The security of multi-agent coordination has garnered increasing attention, with a primary focus on two canonical attack paradigms: denial-of-service (DoS) and deception attacks. The seminal work by Persis and Tesi [24] established a foundational framework for DoS attacks by characterizing their duration and frequency. Subsequent research has extended these results to UAV swarm applications. For instance, Yang et al. [25] proposed a resilient fault-tolerant cooperative control method to achieve multi-UAV cooperative control under DoS attacks and actuator faults. Moreover, Fabris and Zelazo [26] discussed the secure-by-design consensus framework, which addresses robustness to structured channel tampering through task objective encoding. Zhao et al. [27] proposed the adaptive compensation paradigm, which designs dynamic coupling gains to counteract specific attack patterns such as sequential link attacks. These proposed control strategies are time-triggered control methods.

It is well known that time-triggered control methods usually lead to a large amount of communication burden and affect the control efficiency. The event-triggered mechanism determines the requirements of information transmission and controller update according to the change in the system states, which can effectively save communication resources [28]. Event-triggered mechanisms for secure time-varying formation (STVF) under DoS attacks have been studied in [29–31]. A periodic control strategy was proposed in [32] for event-based intermittent formation control under deception attacks, where control is active only in selected intervals. Gong et al. [33] developed a two-layered distributed resilient scheme for formation tracking with one-way communications against composite DoS and deception attacks. Moreover, in [34–38], the authors modeled the communication link attack as a sequence scaling attack, and realized dynamic event-triggered limited distributed and fully distributed secure control under the condition of the undirected topology. An event-triggered delayed impulsive control strategy was employed to address the secure consensus of nonlinear multi-agent systems affected by sequential scaling attacks and communication delays in [34], where sequential scaling attacks occur

within the communication channels between agents, while the communication delays arise in the controller-actuator pair. Recently, a sequential multi-link scaling attack (SMLSA) containing three attack properties of distributed, scaling and sequence was proposed in [37], which covers the above-mentioned several types of attacks. The secure control schemes in [36–38] are restricted to undirected graphs and are inapplicable to directed ones. Moreover, most existing UAV formation control methods are only limitedly distributed. For large-scale UAV swarms, when the communication network topology structure information is unknown or frequently changes, the limited distributed control strategies are no longer effective. Therefore, the problem of fully distributed STVF control for UAV swarms under SMLSAs deserves attention and requires in-depth research.

Motivated by the above considerations, this paper mainly discusses the application of fully distributed STVF of quadrotor swarms under SMLSAs, and the effectiveness of the proposed method is verified through the flight experiments of quadrotor UAV swarm formation. This work contributes to the field in three key aspects. To begin with, a secure self-triggered formation protocol is developed for quadrotor swarms operating under directed communication topologies in the presence of SMLSAs, while existing results in [36–38] are restricted to undirected graphs and uniform attack patterns on selected links. Furthermore, unlike the designs reported in [29,35], the proposed framework is fully distributed in nature, as its synthesis requires no global topological knowledge; additionally, it operates in a self-triggered mode, thus freeing each vehicle from continuous neighbor-state sampling for event verification. Moreover, an exponential function of the most recent trigger time is embedded in the adaptive gain dynamics to confine the coupling coefficient within a preset bound, which effectively circumvents actuator saturation arising from unbounded gain growth—a drawback previously observed in [36,37].

The rest of the paper is arranged as follows. Preliminaries and the problem to be addressed are formulated in Section 2. Section 3 derives the secure self-triggered time-varying formation conditions. The detailed control algorithm for quadrotor swarms is presented in Section 4. Section 5 describes the experimental validation through flight tests, and Section 6 concludes the study with final remarks.

2. Preliminaries and Problem Description

2.1. Graph Theory

This paper considers a quadrotor swarm consisting of N quadrotors, and all quadrotors interact with information through a communication network. The communication network between quadrotors can be composed of a general directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, 2, \dots, N\}$ represents the set formed by all quadrotors, $\mathcal{E} \subseteq (\mathcal{V}, \mathcal{V})$ represents the set of all quadrotor communication links, and $\mathcal{A} = [w_{ij}] \in \mathbb{R}^{N \times N}$ represents the adjacency matrix of the communication network. w_{ij} denotes the communication strength between the quadrotors, and $w_{ij} = 1$ if there is information interaction between the two quadrotors, and $w_{ij} = 0$ otherwise. Let $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$ be the Laplacian matrix corresponding to the communication network of the quadrotor swarm system, if $i \neq j$, then $l_{ij} = -w_{ij}$, otherwise $l_{ij} = \sum_{j=1}^N w_{ij}$. The root node is a node with an in-degree of 0, from which there exists a directed path to every other node in the graph [39]. Denote the number of root and non-root nodes in the communication network as r and $\bar{r}$, respectively. Denote the number of root and non-root nodes in the communication network $\mathcal{G}$ as r and $\bar{r}$, respectively, then the Laplacian matrix $\mathcal{L}$ can be decomposed into

$\mathcal{L} = \begin{bmatrix} \mathcal{L}_{rr} & 0 \\ \mathcal{L}_{rr} & \mathcal{L}_{rr} \end{bmatrix}$, where $\mathcal{L}_{rr} \in \mathbb{R}^{r \times r}$, $\mathcal{L}_{rr} \in \mathbb{R}^{r \times r}$ and $\mathcal{L}_{rr} \in \mathbb{R}^{r \times r}$. If a general digraph $\mathcal{G}$ contains at least one directed spanning tree, then $\mathcal{L}_{rr}$ and $\mathcal{L}_{rr}$ are singular and non-singular M-matrices, respectively.

2.2. Sequential Multi-Link Scaling Attacks

In this paper, considering the limited energy of the attacker, the distributed nature of the attack strategy, and the deceptive nature of information transmission, the communication link attack is described as a SMLSA. For any time interval $[t_0, t_e]$, $\forall t_e \geq t_0 \geq 0$, let $\{\mathcal{I}_n\}$ and Δ_n ($n \in \mathbb{N}^+$) be the initiation time and duration of SMLSAs respectively, and within the attack time interval $[\mathcal{I}_n, \mathcal{I}_n + \Delta_n]$, the weight of the communication link is time-invariant, which means that the attacked communication link switches at the beginning and end of the SMLSAs. A finite set $\mathcal{P}$ related to an index set $\mathcal{T} \subset \mathbb{N}$ is represented as all possible communication network topologies under SMLSAs. $s(t) : [0, +\infty) \rightarrow \mathcal{T}$ represents the sequential switching signals within the time interval $[t_0, t_e]$, then the model of SMLSAs can be constructed as [37,38]

$$\bar{w}_{ij}^{s(t)} = \mu_{ij}^{s(t)} w_{ij}, \quad i, j \in \mathcal{V}, \quad (1)$$

where $\mu_{ij}^{s(t)} \geq 0$ and $\bar{w}_{ij}^{s(t)}$ are the attack scaling factor and the equivalent weight of the communication links under SMLSAs. For any time interval $[t_0, t_e]$, the total attack time interval is denoted as $\Pi(t_0, t_e) = \bigcup_{n \in \mathbb{N}^+} [\mathcal{I}_n, \mathcal{I}_n + \Delta_n] \cap [t_0, t_e]$, and the total secure time interval is denoted as $\Omega(t_0, t_e) = [t_0, t_e] \setminus \Pi(t_0, t_e)$. Let $n(t_0, t_e)$ be the total attack number of SMLSAs during the time interval $[t_0, t_e]$, then the total attack number $n(t_0, t_e)$ and the total attack time interval $\Pi(t_0, t_e)$ satisfy the following assumptions.

Assumption 1. For the total attack number $n(t_0, t_e)$ of SMLSAs during the interval $[t_0, t_e]$, there exist positive constants η_1 and ν_1 such that

$$n(t_0, t_e) \leq \eta_1 + \nu_1(t_e - t_0). \quad (2)$$

Assumption 2. For the total attack time interval $\Pi(t_0, t_e)$ of SMLSAs during the time interval $[t_0, t_e]$, then there exist positive constants η_2 and ν_2 such that

$$|\Pi(t_0, t_e)| \leq \eta_2 + \nu_2(t_e - t_0). \quad (3)$$

Similar to [37,38], this paper also considers using a dynamic event-triggered control mechanism to analyze and solve the fully distributed STVF problem for quadrotor swarms under SMLSAs. Let $\{t_i^k, t_i^{k+1}, \dots, t_i^{k+m}\}$ be the triggering sequence of the quadrotors in the time interval $[t_0, t_e]$, where t_i^k represents the k -th triggering instant of the quadrotor i . For the time interval $[t_0, t_e]$, the total attack influence time interval can be expressed as $\bar{\Pi}(t_0, t_e) = \bigcup_{n \in \mathbb{N}^+} [\mathcal{I}_n, \mathcal{I}_n + \Delta_n + \ell_n] \cap [t_0, t_e]$, and the total secure time interval can be expressed as $\bar{\Omega}(t_0, t_e) = [t_0, t_e] \setminus \bar{\Pi}(t_0, t_e)$, where ℓ_n represents the trigger affected time interval of the n -th SMLSA. Based on the analysis of the impact of the attack time interval on the triggering sequence in [33,34], it can be known that the total attack time interval satisfies $\bar{\Pi}(t_0, t_e) \leq \Pi(t_0, t_e) + n(t_0, t_e)\bar{\ell}$, where $\bar{\ell}$ is the maximum trigger affected time interval by SMLSAs. Moreover, the attacked Laplacian matrix can be defined as $\bar{\mathcal{L}}^{s(t)} = [\bar{L}_{ij}^{s(t)}] \in \mathbb{R}^{N \times N}$ with $\bar{L}_{ij}^{s(t)} = \sum_{j=1}^N \bar{w}_{ij}^{s(t)}$, $i = j$ and $\bar{L}_{ij}^{s(t)} = -\bar{w}_{ij}^{s(t)}$, $i \neq j$.

2.3. Model of the Position–Velocity Control in Outer-Loop of Quadrotors

In general, the three directions of North–East–ground in the geographical environment are modeled as the geographic coordinate system $O(X, Y, Z)$. The quadrotor is a typical underactuated control system, it uses four motors to drive the rotor to generate different power combinations to realize its outer-loop position–velocity motion (moving in X, Y, Z directions) and inner-loop attitude motion of yaw, pitch and roll. For the sake of simplicity, the time argument has been omitted in the subsequent sections of this paper. However, in certain specific cases (such as those related to switching signal $s(t)$ and other necessary circumstances), we will explicitly state the time argument t . Let the position and velocity state of the quadrotor in the X, Y, Z directions in the geographic coordinate system as $p_i = [p_{iX}, p_{iY}, p_{iZ}]^T \in \mathbb{R}^3$ and $v_i = [v_{iX}, v_{iY}, v_{iZ}]^T \in \mathbb{R}^3$, respectively, then the motion model of the position–velocity control in the outer-loop of quadrotors can be modeled as

$$\begin{cases} \dot{p}_i = v_i \\ \dot{v}_i = Ge_z - m_i^{-1} F_i \Re_i e_z \end{cases} \quad (i = 1, 2, \dots, N), \quad (4)$$

where G is the gravitational acceleration, $e_z = [0, 0, 1]^T$, m_i is the mass of the quadrotor i , F_i is the resulting total lift of the quadrotor i , and $\Re_i$ is the transformation matrix from the body coordinate system to the geographic coordinate system of the quadrotor i , which can be expressed as

$$\Re_i = \begin{bmatrix} \cos \psi_i \cos \theta_i & \cos \psi_i \sin \phi_i \sin \theta_i - \sin \psi_i \cos \phi_i & \cos \psi_i \cos \phi_i \sin \theta_i + \sin \psi_i \sin \phi_i \\ \sin \psi_i \cos \theta_i & \sin \psi_i \sin \phi_i \sin \theta_i + \cos \psi_i \cos \phi_i & \sin \psi_i \cos \phi_i \sin \theta_i - \cos \psi_i \sin \phi_i \\ -\sin \theta_i & \sin \phi_i \cos \theta_i & \cos \phi_i \cos \theta_i \end{bmatrix}, \quad (5)$$

where ψ_i , ϕ_i and θ_i represent yaw angle, pitch angle and roll angle. Usually, pitch angle and roll angle satisfy with $\phi_i, \theta_i \in (-\pi/2, \pi/2)$ (similar assumptions can be found in [40]).

Let $u_i = [u_{iX}, u_{iY}, u_{iZ}]^T = \dot{v}_i = [\dot{v}_{iX}, \dot{v}_{iY}, \dot{v}_{iZ}]^T \in \mathbb{R}^3$ be the control input in outer-loop in the model of the position–velocity control of quadrotors, then one can obtain that from (4) and (5)

$$\begin{cases} u_{iX} = -m_i^{-1} F_i (\cos \psi_i \cos \phi_i \sin \theta_i + \sin \psi_i \sin \phi_i), \\ u_{iY} = -m_i^{-1} F_i (\sin \psi_i \cos \phi_i \sin \theta_i - \cos \psi_i \sin \phi_i), \\ u_{iZ} = \text{sat}^+(G - m_i^{-1} F_i \cos \phi_i \cos \theta_i), \end{cases} \quad (6)$$

where $\text{sat}^+(x) = \begin{cases} G - \varepsilon, & x \geq G - \varepsilon \\ x, & x < G - \varepsilon \end{cases}$ and ε a tiny positive safety constant with $0 < \varepsilon \ll 1$.

For time-varying formation control of the quadrotor swarm, the outer-loop control input $u_i = [u_{iX}, u_{iY}, u_{iZ}]^T$ is usually designed according to the quadrotor swarm mission requirements. A set of piecewise continuous differentiable functions $h_i = [h_{ip}^T, h_{iv}^T]^T$, $i \in \mathcal{V}$ is used to describe the time-varying formation configurations of the quadrotor swarm, where $h_{ip}^T = [h_{ip}^X, h_{ip}^Y, h_{ip}^Z]^T$ and $h_{iv}^T = [h_{iv}^X, h_{iv}^Y, h_{iv}^Z]^T$.

2.4. Problem Formulation of Secure Self-Triggered Time-Varying Formation Control

To achieve STVF control for a quadrotor swarm under SMLSAs, a secure self-triggered formation protocol is designed. To this end, the local relative formation position error $\Xi_{ip} \in \mathbb{R}^3$ and formation velocity error $\Xi_{iv} \in \mathbb{R}^3$ among quadrotors are defined as

$$\begin{cases} \Xi_{ip} = \sum_{j=1}^N \bar{w}_{ij}^{s(t)} (p_i - p_j - h_{ip} + h_{jp}) \\ \Xi_{iv} = \sum_{j=1}^N \bar{w}_{ij}^{s(t)} (v_i - v_j - h_{iv} + h_{jv}) \end{cases}, i \in \mathcal{V}. \quad (7)$$

A STVF control under an event-triggered mechanism is considered. By integrating the local relative formation position error $\Xi_{ip} \in \mathbb{R}^3$ and formation speed error $\Xi_{iv} \in \mathbb{R}^3$ among the quadrotors, the following secure self-triggered time-varying formation control protocol is designed

$$u_i = u_{i1} + u_{i2}, \quad (8)$$

where

$$u_{i1} = \alpha_p (p_i - h_p) + \alpha_v (v_i - h_v) + \dot{h}_v, \quad (9)$$

$$u_{i2} = d_i (K_1 \Xi_{ip}(t_i^k) + K_2 \Xi_{iv}(t_i^k)). \quad (10)$$

u_{i1} represents the individual motion mode adjustment term of the quadrotor i , where α_p and α_v are the preset scalar values for the mode self-feedback, which are used to adjust the motion mode of the quadrotor itself. $\dot{h}_v$ is the formation compensation quantity introduced by the formation feasibility condition, which is used to eliminate the influence of the derivative of the formation function on the formation control to realize the desired time-varying formation configuration. u_{i2} is the consistent cooperative control term between the quadrotor, where $\Xi_{ip}(t_i^k)$ and $\Xi_{iv}(t_i^k)$ are the local relative formation position error and the local relative formation speed error at the triggering time t_i^k . $K_1, K_2 \in \mathbb{R}^{3 \times 3}$ are the feedback control gains for position error and velocity error respectively, and d_i is the adaptive coupling strength. The adaptive update law is as follows

$$\begin{cases} \dot{d}_i = a e^{-\rho(t-t_i^k)} \bar{h}_i + b v_i(t_i^k), \\ \dot{\bar{h}}_i = \sigma e^{\rho(t-t_i^k)} \Xi_i^T(t_i^k) H \Xi_i(t_i^k), \\ \dot{v}_i = c \Xi_i^T P \Xi_i, \end{cases} \quad (11)$$

where $\Xi_i = [\Xi_{ip}^T, \Xi_{iv}^T]^T$, a, b, c, σ, ρ are positive constants, $\bar{h}_i \in \mathbb{R}$ and $v_i \in \mathbb{R}$ are two coupling strength components of d_i . It should be noted that the initial value of $\bar{h}_i$ needs to satisfy $\bar{h}_i(t_0) > \sqrt{\kappa} a$, where κ is defined in the subsequent dynamic event-triggered condition. Next, the following dynamic event-triggered condition is proposed to determine the next triggering time

$$t_i^{k+1} = \inf \{t > t_i^k \mid g_{i1}(\Xi_i(t), \chi_i(t), t) > 0 \cup g_{i2}(\Xi_i(t_i^k), \Xi_i(t), t) > 0\}, \quad (12)$$

where

$$g_{i1}(\Xi_i(t_i^k), \Xi_i(t), t) \triangleq \hat{d}_i^2 \chi_i^T(t) H \chi_i(t) - \kappa \Xi_i^T(t) H \Xi_i(t) - \vartheta e^{-\pi}, \quad (13)$$

$$g_{i2}(\Xi_i(t_i^k), \Xi_i(t), t) \triangleq (v_i(t) - v_i(t_i^k))^2 \Xi_i^T(t_i^k) H \Xi_i(t_i^k) - \kappa \Xi_i^T(t) H \Xi_i(t) - \vartheta e^{-\pi}. \quad (14)$$

In (13) and (14), $\hat{d}_i = a \bar{h}_i + b v_i$, $\chi_i(t) = \Xi_i(t_i^k) - \Xi_i(t)$, $K = [K_1, K_2]$, κ, ϑ, τ are positive constants. Moreover, the first triggering time is designed as $t_i^1 = t_0 \geq 0$. It should be noted that the local relative position error and local relative velocity error of each quadrotor are available.

The primary goal of this section is to synthesize the gain matrices P , K and H so that quadrotor swarm (4), despite being subject to SMLSAs, can realize fully distributed, secure, self-triggered time-varying formation governed by protocol (8) and event-triggered conditions (12). A rigorous definition of the aforementioned formation control problem is presented as follows.

Definition 1. For any bounded initial positions $p_i(0)$, initial velocities $v_i(0)$, and initial formation vectors $h_i(0)$ of the quadrotor swarm system, if there exist gain matrices P , K and H such that

$$\begin{cases} \lim_{t \rightarrow +\infty} (p_i(t) - p_j(t) - h_{ip}(t) + h_{jp}(t)) = 0 \\ \lim_{t \rightarrow +\infty} (v_i(t) - v_j(t) - h_{iv}(t) + h_{jv}(t)) = 0 \end{cases}, i, j \in \mathcal{V},$$

then quadrotor swarm (4) under SMLSAs achieves fully distributed secure self-triggered time-varying formation control via the control protocol (8) and the dynamic event-triggered condition (12). In this case, the gain matrices P , K and H are designed without relying on network topology information, and the control input evolution does not necessitate continuous neighbor-state acquisition or real-time communication to validate the event-triggered conditions.

Remark 1. It should be noted that the monotonic growth of the time-dependent exponential decay term $e^{-\kappa(t-t_i^k)}$ and sharp reset at triggering instants lead to discontinuous control gains d_i , which causes actuator chattering and rotor wear in actual flight. Developing a smooth transition mechanism to avoid abrupt gain jumps is a promising research topic. In a cooperative quadrotor swarm, the interaction among the quadrotors is mutually reinforcing, meaning that the weights of the communication links are non-negative. However, in an uncooperative quadrotor swarm, there may be a competitive relationship among the quadrotors, where the weights of the communication links are negative. It should be noted that a cooperative quadrotor swarm is considered in this paper. In scenarios where the cooperative relationship among quadrotor swarms is transformed into a competitive one under external attacks, achieving secure formation control becomes a meaningful topic that deserves in-depth study.

3. Secure Self-Triggered Time-Varying Formation Control Criteria

Let the formation reference error of the quadrotor swarm system be $\delta_i = x_i - h_i$, $\Xi = [\Xi_1^T, \Xi_2^T, \dots, \Xi_N^T]^T$ and $x_i = [p_i^T, v_i^T]^T$. Since $\dot{h}_{ip} = h_{iv}$, from (4) and (8), one can obtain

$$\dot{\delta} = (I_N \otimes A)\delta + (D \otimes BK)(\Xi + \chi). \quad (15)$$

where $A = \begin{bmatrix} 0 & 1 \\ \alpha_p & \alpha_v \end{bmatrix}$, $B = [0 \quad 1]^T$, $D = \text{diag}\{d_1, d_2, \dots, d_N\}$, $\delta(t) = [\delta_1^T, \delta_2^T, \dots, \delta_N^T]^T$, $\chi = [\chi_1^T, \chi_2^T, \dots, \chi_N^T]^T$. From (7) and (15), it can be seen that the local relative formation error system among the quadrotor swarm system can be described as

$$\dot{\Xi} = (I_N \otimes A)\Xi + (\bar{\mathcal{L}}^{s(t)} D \otimes BK)(\Xi + \chi), \quad (16)$$

where $\bar{\mathcal{L}}^{s(t)}$ represents the Laplacian matrix corresponding to the communication network under SMLSAs. Based on the root nodes and non-root nodes, the $\bar{\mathcal{L}}^{s(t)}$ is similarly divided to $\bar{\mathcal{L}}^{s(t)} = \begin{bmatrix} \bar{\mathcal{L}}_{rr}^{s(t)} & 0 \\ \bar{\mathcal{L}}_{r\bar{r}}^{s(t)} & \bar{\mathcal{L}}_{\bar{r}\bar{r}}^{s(t)} \end{bmatrix}$. Let $\Xi_R = [\Xi_1^T, \Xi_2^T, \dots, \Xi_r^T]^T$ and

$\Xi_{\bar{R}} = [\Xi_{r+1}^T, \Xi_{r+2}^T, \dots, \Xi_N^T]^T$, the local relative formation error system can be transformed into two local relative formation error subsystems by (16)

$$\dot{\Xi}_R = (I_r \otimes A)\Xi_R + (\bar{\mathcal{L}}_{rr}^{s(t)} D_R \otimes BK)(\Xi_R + \chi_R), \quad (17)$$

$$\dot{\Xi}_{\bar{R}} = (I_{N-r} \otimes A)\Xi_{\bar{R}} + (\bar{\mathcal{L}}_{\bar{r}\bar{r}}^{s(t)} D_{\bar{R}} \otimes BK)(\Xi_{\bar{R}} + \chi_{\bar{R}}) + (\bar{\mathcal{L}}_{r\bar{r}}^{s(t)} D_R \otimes BK)(\Xi_R + \chi_R), \quad (18)$$

where $\chi_R = [\chi_1^T, \chi_2^T, \dots, \chi_r^T]^T$, $\chi_{\bar{R}} = [\chi_{r+1}^T, \chi_{r+2}^T, \dots, \chi_N^T]^T$, $D_R = \text{diag}\{d_1, d_2, \dots, d_r\}$, and $D_{\bar{R}} = \text{diag}\{d_{r+1}, d_{r+2}, \dots, d_N\}$. To obtain the main results of this paper, the following lemmas are required.

Lemma 1 ([41,42]). For the singular M -matrices $\mathcal{L}_{rr}$, there exists a vector $\varphi = [\varphi_1, \varphi_2, \dots, \varphi_r]^T$ with $\varphi_i > 0$ and $\sum_{i=1}^r \varphi_i = 1$ such that $\varphi^T \mathcal{L}_{rr} = 0$. In this case, the following inequality holds

$$\min_{\varphi^T x=0, x^T x=1} x^T (\Phi \mathcal{L} + \mathcal{L}^T \Phi) x \geq \frac{\lambda_2(\Phi \mathcal{L} + \mathcal{L}^T \Phi)}{N} > 0, \quad \text{where} \quad \Phi = \text{diag} \{ \varphi_1, \varphi_2, \dots, \varphi_N \} \quad \text{and} \quad x(t) \in \mathbb{R}^n.$$

Lemma 2 ([41,42]). For the non-singular M -matrix $\mathcal{L}_{rr}$, there exists a positive-definite diagonal matrix $q = \text{diag} \{ q_1, q_2, \dots, q_{N-r} \}$ with $q_i > 0$, $i \in \{1, 2, \dots, N-r\}$, and $[q_1, q_2, \dots, q_{N-r}]^T = \mathcal{L}_{rr}^{-T}(t) \mathbf{1}_{N-r}$ such that $Q \mathcal{L}_{rr} + \mathcal{L}_{rr}^T Q = M > 0$.

Lemma 3. Given positive constant $\alpha, \beta, \eta, \tau$, assume that Assumptions 1 and 2 hold and there exists a continuous nonnegative function V such that

$$\begin{cases} V \leq \beta V + \eta e^{-\tau t}, & t \in [\mathcal{I}_n, \mathcal{S}_n), n \in \mathbb{N}^+, \\ V \leq -\alpha V + \eta e^{-\tau t}, & t \in [\mathcal{S}_n, \mathcal{I}_{n+1}), n \in \mathbb{N}^+. \end{cases}$$

If the attack parameters satisfy $\nu_2 + \nu_1 \bar{\Delta} < \bar{\alpha} / (\bar{\alpha} + 2\beta)$, we have $\lim_{t \rightarrow +\infty} V(t) = 0$.

Proof of Lemma 3. If $V \leq \beta V + \eta e^{-\tau t}$ for $t \in [\mathcal{I}_n, \mathcal{S}_n), n \in \mathbb{N}^+$, we have

$$V(t) \leq e^{\beta(t-\mathcal{I}_n)} V(\mathcal{I}_n) + \eta \int_{\mathcal{I}_n}^t e^{\beta(t-s)-\tau s} ds. \quad (19)$$

Similarly, if $V(t) \leq -\alpha V + \eta e^{-\tau t}$ for $t \in [\mathcal{S}_n, \mathcal{I}_{n+1}), n \in \mathbb{N}^+$, we have

$$V(t) \leq e^{-\alpha(t-\mathcal{S}_n)} V(\mathcal{S}_n) + \eta \int_{\mathcal{S}_n}^t e^{-\alpha(t-s)-\tau s} ds. \quad (20)$$

From (19) and (20), the following recursion is obtained. When $t \in [\mathcal{S}_0, \mathcal{I}_1)$, let $\mathcal{S}_0 = t_0$,

$$V(t) \leq e^{-\alpha(t-t_0)} V(t_0) + \eta \int_{t_0}^t e^{-\alpha(t-s)-\tau s} ds. \quad (21)$$

When $t \in [\mathcal{I}_1, \mathcal{S}_1)$,

$$V(t) \leq e^{\beta(t-\mathcal{I}_1)-\alpha(\mathcal{I}_1-t_0)} V(t_0) + \eta \int_{t_0}^{\mathcal{I}_1} e^{-\tau s + \beta(t-\mathcal{I}_1)-\alpha(\mathcal{I}_1-s)} ds + \eta \int_{\mathcal{I}_1}^t e^{\beta(t-s)-\tau s} ds. \quad (22)$$

When $t \in [\mathcal{S}_1, \mathcal{I}_2)$,

$$\begin{aligned} V(t) &\leq e^{\beta(\mathcal{S}_1-\mathcal{I}_1)-\alpha(t-\mathcal{S}_1+\mathcal{I}_1-t_0)} V(t_0) + \eta \int_{t_0}^{\mathcal{I}_1} e^{-\alpha(t-\mathcal{S}_1+\mathcal{I}_1-s)+\beta(t-\mathcal{I}_1)-\tau s} ds \\ &\quad + \eta \int_{\mathcal{I}_1}^{\mathcal{S}_1} e^{-\alpha(t-\mathcal{S}_1)+\beta(t-s)-\tau s} ds + \eta \int_{\mathcal{S}_1}^t e^{-\alpha(t-s)-\tau s} ds. \end{aligned} \quad (23)$$

Based on the above iterative formula, for $t \in [\mathcal{I}_n, \mathcal{S}_n)$, we can obtain that

$$\begin{aligned} V(t) &\leq e^{\beta(t-\mathcal{I}_n+\mathcal{S}_{n-1}-\mathcal{I}_{n-1}+\dots+\mathcal{S}_1-\mathcal{I}_1)-\alpha(t-\mathcal{S}_{n-1}+\mathcal{I}_{n-1}+\dots-\mathcal{S}_1+\mathcal{I}_1-t_0)} V(t_0) + \eta \int_{\mathcal{I}_n}^t e^{\beta(t-s)-\tau s} ds \\ &\quad + \dots + \eta \int_{\mathcal{I}_1}^{\mathcal{S}_1} e^{\beta(t-\mathcal{I}_n+\mathcal{S}_{n-1}-\mathcal{I}_{n-1}+\dots+\mathcal{S}_1-s)-\alpha(t-\mathcal{S}_{n-1}+\mathcal{I}_{n-1}+\dots-\mathcal{S}_1)-\tau s} ds \\ &\quad + \eta \int_{t_0}^{\mathcal{I}_1} e^{\beta(t-\mathcal{I}_n+\mathcal{S}_{n-1}-\mathcal{I}_{n-1}+\dots+\mathcal{S}_1-\mathcal{I}_1)-\alpha(t-\mathcal{S}_{n-1}+\mathcal{I}_{n-1}+\dots-\mathcal{S}_1+\mathcal{I}_1-s)-\tau s} ds \\ &\leq e^{\beta|\bar{\Pi}(t_0,t)|-\alpha(t-t_0-|\bar{\Pi}(t_0,t)|)} V(t_0) + \eta \int_{\mathcal{I}_n}^t e^{|\bar{\Pi}(s,t)|-\tau s} ds + \dots \\ &\quad + \eta \int_{\mathcal{I}_1}^{\mathcal{S}_1} e^{\beta|\bar{\Pi}(s,t)|-\alpha(t-s-|\bar{\Pi}(s,t)|)-\tau s} ds + \eta \int_{t_0}^{\mathcal{I}_1} e^{\beta|\bar{\Pi}(s,t)|-\alpha(t-s-|\bar{\Pi}(s,t)|)-\tau s} ds \\ &\leq e^{\beta|\bar{\Pi}(t_0,t)|-\alpha(t-t_0-|\bar{\Pi}(t_0,t)|)} V(t_0) + \eta \int_{t_0}^t e^{\beta|\bar{\Pi}(s,t)|-\alpha(t-s-|\bar{\Pi}(s,t)|)-\tau s} ds. \end{aligned} \quad (24)$$

For $t \in [\mathcal{S}_n, \mathcal{I}_{n+1})$, we also can obtain the same result as (24). Then for $\forall t \geq t_0$, we obtain

$$V(t) \leq e^{\beta|\bar{\Pi}(t_0,t)|-\alpha(t-t_0-|\bar{\Pi}(t_0,t)|)} V(t_0) + \eta \int_{t_0}^t e^{\beta|\bar{\Pi}(s,t)|-\alpha(t-s-|\bar{\Pi}(s,t)|)-\tau s} ds. \quad (25)$$

According to $\tilde{\Psi}(t_0, t_e) \leq \Psi(t_0, t_e) + n(t_0, t_e) \bar{\Delta}$, $t_e > t_0 \geq 0$, it can be further derived that by Assumptions 1 and 2

$$\begin{aligned} \mathbb{V}(t) &\leq e^{(\alpha+\beta)(\eta_2+\bar{\Delta}\eta_1)-t(t-t_0)} \mathbb{V}(t_0) + \eta e^{(\alpha+\beta)(\eta_2+\bar{\Delta}\eta_1)} \int_{t_0}^t e^{-t(t-s)-\tau s} ds \\ &\leq e^{(\alpha+\beta)(\eta_2+\bar{\Delta}\eta_1)} e^{-t(t-t_0)} \mathbb{V}(t_0) + o(t), \end{aligned} \quad (26)$$

where $\iota = \alpha - (\alpha + \beta)(\nu_2 + \bar{\Delta}\nu_1)$ and $o(t) = \eta(t - \tau)^{-1} e^{(\alpha+\beta)(\eta_2+\bar{\Delta}\eta_1)} (e^{-\tau t} - e^{-t-\tau t_0})$. Owing to $\nu_2 + \nu_1 \bar{\Delta} < \alpha/(\alpha + \beta)$, one can obtain further that $\iota = \alpha - (\alpha + \beta)(\nu_2 + \bar{\Delta}\nu_1) > 0$ and $\lim_{t \rightarrow +\infty} o(t) = 0$. Hence, it can be obtained that $\lim_{t \rightarrow +\infty} \mathbb{V}(t) = 0$. $\square$

The following theorem provides the design criterion for achieving fully distributed secure self-triggered time-varying formation control for the quadrotor swarm (4) under SMLSAs.

Theorem 1. For any bounded initial positions $p_i(0)$, initial velocities $v_i(0)$, and initial formation vectors $h_i(0)$ of the quadrotor swarm system (4), if the following two conditions hold

- (1) The attack parameters satisfy $\nu_2 + \nu_1 \bar{\ell} < \tilde{\alpha} / (\tilde{\alpha} + 2\bar{\beta})$;
- (2) There exists a gain matrix P such that

$$PA + A^T P - PBB^T P + \alpha P \leq 0, \quad (27)$$

$$PA + A^T P - \bar{\beta} P \leq 0, \quad (28)$$

where $\tilde{\alpha} = \min(\rho, \alpha)$, α , $\bar{\beta}$ represent the nominal convergence rate and divergence rate of the local relative formation error system (16) when it is not attacked and when it is attacked, respectively. Then, it is said that the quadrotor swarm (4) can achieve fully distributed secure self-triggered time-varying formation control under the control protocol (8) and the dynamic event-triggered conditions (12), and the control gains are designed as $K = [K_1, K_2] = -B^T P$ and $H = PBB^T P$.

Proof of Theorem 1. The Lyapunov function is set as $V = V_R + V_{\bar{R}}$, where

$$V_R = \sum_{i=1}^r \varphi_i \left(a e^{-\rho(t-t_i^*)} \bar{h}_i v_i + \frac{1}{2} b v_i^2 \right) + \frac{e^{-\rho(t-t_i^*)} a^2}{2b} \sum_{i=1}^r \varphi_i (\bar{h}_i - \gamma)^2, \quad (29)$$

$$V_{\bar{R}} = \sum_{i=r+1}^N q_{i-r} \left(a e^{-\rho(t-t_i^*)} \bar{h}_i v_i + \frac{1}{2} b v_i^2 \right) + \frac{e^{-\rho(t-t_i^*)} a^2}{2b} \sum_{i=r+1}^N q_{i-r} (\bar{h}_i - \gamma)^2, \quad (30)$$

where t_i^* is the last event-triggered instant with $t_0 \leq t_i^* < t$, ρ is a positive constant, and $\varphi_i \geq 0$, $i \in \{1, 2, \dots, r\}$ and $q_i \geq 0$, $i \in \{1, 2, \dots, r\}$ is defined in Lemmas 1 and 2. By taking the time derivative of V_R yields

$$\begin{aligned} \dot{V}_R &\leq \sum_{i=1}^r \varphi_i \bar{d}_i \dot{v}_i + \frac{\sigma a}{b} \sum_{i=1}^r \varphi_i (\hat{d}_i - a\gamma) \Xi_i^T(t_i^k) H \Xi_i(t_i^k) - \rho a \sum_{i=1}^r \varphi_i e^{-\rho(t-t_i^*)} \bar{h}_i v_i \\ &\quad - \frac{\rho a^2 e^{-\rho(t-t_i^*)}}{2b} \sum_{i=1}^r \varphi_i (\bar{h}_i - \gamma)^2, \end{aligned} \quad (31)$$

where $\bar{d}_i = a e^{-\rho(t-t_i^*)} \bar{h}_i + b v_i$ and $\hat{d}_i = a \bar{h}_i + b v_i$. Note that $\bar{h}_i \geq 0$, $v_i \geq 0$ and $\varphi_i \geq 0$, we obtain that

$$\dot{V}_R \leq \sum_{i=1}^r \varphi_i \bar{d}_i \dot{v}_i + \frac{\sigma a}{b} \sum_{i=1}^r \varphi_i (\hat{d}_i - a\gamma) \Xi_i^T(t_i^k) H \Xi_i(t_i^k) - \frac{\rho a^2 e^{-\rho(t-t_i^*)}}{2b} \sum_{i=1}^r \varphi_i (\bar{h}_i - \gamma)^2. \quad (32)$$

Substituting (17) into (32), (32) can be rewritten as

$$\begin{aligned} \dot{V}_R &\leq b \Xi_R^T(t) (\bar{D}_R \Phi \otimes (PA + A^T P)) \Xi_R(t) + \frac{\sigma a}{b} \sum_{i=1}^r \varphi_i (\hat{d}_i - a\gamma) \Xi_i^T(t_i^k) H \Xi_i(t_i^k) \\ &\quad - 2c \Xi_R^T(t) (\bar{D}_R \Phi \bar{\mathcal{L}}_{rr}^{s(t)} D_R \otimes H) \Xi_R(t) - \frac{\rho a^2 e^{-\rho(t-t_i^*)}}{2b} \sum_{i=1}^r \varphi_i (\bar{h}_i - \gamma)^2. \end{aligned} \quad (33)$$

where $\hat{D}_R = \text{diag}\{\hat{d}_1, \hat{d}_2, \dots, \hat{d}_r\}$, $\bar{D}_R = \text{diag}\{\bar{d}_1, \bar{d}_2, \dots, \bar{d}_r\}$, $\Phi = \text{diag}\{\phi_1, \phi_2, \dots, \phi_r\}$ and $\Xi_R(t_i^k) = [\Xi_1^T(t_i^k), \Xi_2^T(t_i^k), \dots, \Xi_r^T(t_i^k)]^T$. Owing to $d_i = \bar{d}_i + b(u_i(t_i^k) - v_i(t))$, we can obtain

$$\begin{aligned} & -2\Xi_R^T(t) \left(\bar{D}_R \Phi \bar{\mathcal{L}}_{rr}^{s(t)} D_R \otimes H \right) \Xi_R(t_i^k) \\ & = -2\varphi \Xi_R^T(t) \left(\bar{D}_R \Phi \bar{\mathcal{L}}_{rr}^{s(t)} (v_R(t_i^k) - v_R(t)) \otimes H \right) \Xi_R(t_i^k) - 2\Xi_R^T \left(\bar{D}_R \Phi \bar{\mathcal{L}}_{rr}^{s(t)} \bar{D}_R \otimes H \right) (\Xi_R + \lambda) \end{aligned} \quad (34)$$

where $v_R(t_i^k) = \text{diag}\{v_1(t_i^k), v_2(t_i^k), \dots, v_r(t_i^k)\}$ and $v_R(t) = \text{diag}\{v_1(t), v_2(t), \dots, v_r(t)\}$. By the dynamic event-triggered condition (12), it follows

$$\begin{aligned} & -2\Xi_R^T \left(\bar{D}_R \Phi \bar{\mathcal{L}}_{rr}^{s(t)} \bar{D}_R \otimes H \right) \chi_R \\ & \leq \zeta_1 \Xi_R^T \left(\bar{D}_R \Phi \bar{\mathcal{L}}_{rr}^{s(t)} (\bar{\mathcal{L}}_{rr}^{s(t)})^T \Phi \bar{D}_R \otimes H \right) \Xi_R + \zeta_1^{-1} \kappa \Xi_R^T (I_r \otimes H) \Xi_R + \zeta_1^{-1} r \vartheta e^{-\tau}, \end{aligned} \quad (35)$$

$$\begin{aligned} & -2\Xi_R^T \left(\bar{D}_R \Phi \bar{\mathcal{L}}_{rr}^{s(t)} (v_R(t_i^k) - v_R(t)) \otimes H \right) \Xi_R(t_i^k) \\ & \leq \zeta_2 \Xi_R^T \left(\bar{D}_R \Phi \bar{\mathcal{L}}_{rr}^{s(t)} (\bar{\mathcal{L}}_{rr}^{s(t)})^T \Phi \bar{D}_R \otimes H \right) \Xi_R + \zeta_2^{-1} \kappa \Xi_R^T (I_r \otimes H) \Xi_R + \zeta_2^{-1} r \vartheta e^{-\tau}, \end{aligned} \quad (36)$$

where ζ_1 and ζ_2 are the arbitrary positive constants. Due to $2\Xi_R^T \left(\bar{D}_R \Phi \bar{\mathcal{L}}_{rr}^{s(t)} \bar{D}_R \otimes H \right) \Xi_R = \Xi_R^T \left(\bar{D}_R \left(\Phi \bar{\mathcal{L}}_{rr}^{s(t)} + (\bar{\mathcal{L}}_{rr}^{s(t)})^T \Phi \right) \bar{D}_R \otimes H \right) \Xi_R$, let $\bar{\lambda}_1 = \lambda_{\max, \forall t > 0} \left(\Phi \bar{\mathcal{L}}_{rr}^{s(t)} (\bar{\mathcal{L}}_{rr}^{s(t)})^T \Phi \right)$, it follows by substituting (35) and (36) into (34) that

$$\begin{aligned} & -2\Xi_R^T(t) \left(\bar{D}_R \Phi \bar{\mathcal{L}}_{rr}^{s(t)} D_R \otimes H \right) \Xi_R(t_i^k) \\ & \leq \bar{\lambda}_1 (\zeta_1 + b\zeta_2) \Xi_R^T \left(\bar{D}_R^2 \otimes H \right) \Xi_R + (b\zeta_2^{-1} + \zeta_1^{-1}) (\kappa \Xi_R^T (I_r \otimes H) \Xi_R + r\vartheta e^{-\tau}) \\ & \quad - \Xi_R^T \left(\bar{D}_R \left(\Phi \bar{\mathcal{L}}_{rr}^{s(t)} + (\bar{\mathcal{L}}_{rr}^{s(t)})^T \Phi \right) \bar{D}_R \otimes H \right) \Xi_R, \end{aligned} \quad (37)$$

where $\bar{\lambda}_1 = \lambda_{\max, \forall t > 0} \left(\Phi \bar{\mathcal{L}}_{rr}^{s(t)} (\bar{\mathcal{L}}_{rr}^{s(t)})^T \Phi \right)$. From $\dot{h}_i(t) \geq 0$, $\bar{h}_i(0) > \sqrt{\kappa/\varphi} > 0$, and $\hat{d}_i = a\bar{h}_i + bv_i$, one can obtain that $\hat{d}_i \geq a\bar{h}_i > 0$. If the dynamic event-triggered condition (13) has not been activated, then it follows that

$$\hat{d}_i^2 \chi_i^T H \chi_i - \kappa \Xi_i^T H \Xi_i - \vartheta e^{-\tau} \leq 0. \quad (38)$$

Owing to $\chi_i(t) = \Xi_i(t_i^k) - \Xi_i(t)$, we can obtain

$$\Xi_i^T(t_i^k) H \Xi_i(t_i^k) \leq \left(2 + \frac{2\kappa}{\hat{d}_i^2(t)} \right) \Xi_i^T(t) H \Xi_i(t) + \frac{2\vartheta}{\hat{d}_i^2(t)} e^{-\tau}, \quad (39)$$

$$\Xi_i^T(t) H \Xi_i(t) \leq (1 + \varepsilon^{-1}) \Xi_i^T(t_i^k) H \Xi_i(t_i^k) + (1 + \varepsilon) \chi_i^T(t) H \chi_i(t), \quad (40)$$

where ε is any positive constant. By combining inequalities (39) and (40), it is easy to obtain that

$$-\Xi_i^T(t_i^k) H \Xi_i(t_i^k) \leq \left(\frac{\varepsilon \kappa}{\hat{d}_i^2(t)} - \frac{\varepsilon}{1 + \varepsilon} \right) \Xi_i^T(t) H \Xi_i(t) + \frac{\varepsilon \vartheta}{\hat{d}_i^2(t)} e^{-\tau}. \quad (41)$$

From (39) and (41), it follows

$$\begin{aligned} & \frac{\sigma a}{b} \sum_{i=1}^r \varphi_i (\hat{d}_i - a\gamma) \Xi_i^T(t_i^k) H \Xi_i(t_i^k) \\ & \leq -\frac{\sigma a}{b} \left(\frac{\varepsilon a \gamma}{1 + \varepsilon} - 2\hat{d}_{\max} - \frac{\kappa(2\hat{d}_{\max} + \varepsilon a \gamma)}{b^2 \hat{h}_{\min}^2} \right) \Xi_R^T(t) (\Phi \otimes H) \Xi_R(t) + \frac{\sigma a \theta}{b} \sum_{i=1}^r \frac{\varphi_i (2\hat{d}_i + a\gamma \varepsilon)}{\hat{d}_i^2} e^{-\tau}, \end{aligned} \quad (42)$$

where $\bar{h}_{\min} = \min \{ \bar{h}_i(t_0), \forall i, j \in \mathcal{V} \}$. Since $\bar{h}_i(t_0) > \sqrt{\kappa/a}$, $i \in \mathcal{V}$, one has $a^2 \bar{h}_i^2(t_0) \kappa^{-1} - 1 > 0$. Thus, there must exist a positive constant $0 < \varepsilon < \frac{a^2 \bar{h}_i^2(t_0)}{\kappa} - 1$ such that $\frac{\varepsilon}{1+\varepsilon} - \frac{\varepsilon \kappa}{a^2 \bar{h}_{\min}^2} > 0$. Hence, we can conclude that $\hat{\gamma} = \frac{\sigma a}{b} \left(\frac{\varepsilon a \gamma}{1+\varepsilon} - 2\hat{d}_{\max} - \frac{\kappa(2\hat{d}_{\max} + \varepsilon a \gamma)}{a^2 \bar{h}_{\min}^2} \right)$ is a sufficiently large constant by choosing γ . By substituting (37) and (42) into (33), we can obtain that

$$\begin{aligned} \dot{V}_R \leq & -c \Xi_R^T \left((c^{-1} \hat{\Phi} - \bar{\lambda}_1(\zeta_1 + b\zeta_2) \bar{D}_R^2 - \kappa(b\zeta_2^{-1} + \zeta_1^{-1}) I_r) \otimes H \right) \Xi_R \\ & + c \Xi_R^T \left(\bar{D}_R Q \otimes (PA + A^T P) \right) \Xi_R - \frac{\rho a^2 e^{-\rho(t-t_i^*)}}{2b} \sum_{i=1}^r \varphi_i (\bar{h}_i - \gamma)^2 \\ & - c \Xi_R^T \left(\bar{D}_R \left(\Phi \bar{\mathcal{L}}_{rr}^{s(t)} + (\bar{\mathcal{L}}_{rr}^{s(t)})^T \Phi \right) \bar{D}_R \otimes H \right) \Xi_R + \zeta_1, \end{aligned} \quad (43)$$

$$\text{where } \zeta_1(t) = o e^{-\tau t} \quad o = r b \vartheta \left(\frac{\zeta_1 b + \zeta_2}{\zeta_1 \zeta_2} \right) + \frac{r \sigma a \vartheta}{b \bar{h}_{\min}} \left(2 + \frac{a \gamma \varepsilon}{\bar{h}_{\min}} \right).$$

Following a process similar to the derivation of V_R , taking the derivative of $V_{\bar{R}}$ yields

$$\begin{aligned} \dot{V}_{\bar{R}} \leq & c \Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \otimes (PA + A^T P) \right) \Xi_{\bar{R}} - 2c \Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \bar{\mathcal{L}}_{rr}^{s(t)} \bar{D}_{\bar{R}} \otimes H \right) (\Xi_{\bar{R}} + \chi_{\bar{R}}) \\ & - 2c \Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \bar{\mathcal{L}}_{rr}^{s(t)} \bar{D}_{\bar{R}} \otimes H \right) (\Xi_{\bar{R}} + \chi_{\bar{R}}) - \frac{\rho a^2 e^{-\rho(t-t_i^*)}}{2b} \sum_{i=r+1}^N q_{i-r} (\bar{h}_i - \gamma)^2 \\ & + \frac{\sigma a}{b} \sum_{i=r+1}^N q_{i-r} (\hat{d}_i - a\gamma) \Xi_i^T(t_i^k) H \Xi_i(t_i^k). \end{aligned} \quad (44)$$

Owing to $\bar{d}_i = \bar{d}_i + b(u_i(t_i^k) - v_i(t))$, the third term on the right side of (44) can be rewritten as

$$\begin{aligned} & -2c \Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \bar{\mathcal{L}}_{rr}^{s(t)} \bar{D}_{\bar{R}} \otimes H \right) (\Xi_{\bar{R}} + \chi_{\bar{R}}) \\ & = -2cb \Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \bar{\mathcal{L}}_{rr}^{s(t)} (v_R(t_i^k) - v_R(t)) \otimes H \right) \Xi_{\bar{R}}(t_i^k) - 2c \Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \bar{\mathcal{L}}_{rr}^{s(t)} \bar{D}_{\bar{R}} \otimes H \right) (\Xi_{\bar{R}} + \chi_{\bar{R}}). \end{aligned} \quad (45)$$

Based on the dynamic event-triggered conditions (13) and (14), it can be derived that

$$\begin{aligned} & 2\Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \bar{\mathcal{L}}_{rr}^{s(t)} \bar{D}_{\bar{R}} \otimes H \right) \chi_{\bar{R}} \\ & \leq \zeta_1 \Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \bar{\mathcal{L}}_{rr}^{s(t)} (\bar{\mathcal{L}}_{rr}^{s(t)})^T Q \bar{D}_{\bar{R}} \otimes H \right) \Xi_{\bar{R}} + \zeta_1^{-1} \kappa \Xi_{\bar{R}}^T (I_r \otimes H) \Xi_{\bar{R}} + \zeta_1^{-1} r \vartheta e^{-\tau t}, \end{aligned} \quad (46)$$

$$\begin{aligned} & -2\Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \bar{\mathcal{L}}_{rr}^{s(t)} (v_R(t_i^k) - v_R(t)) \otimes H \right) \Xi_{\bar{R}}(t_i^k) \\ & \leq \zeta_2 \Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \bar{\mathcal{L}}_{rr}^{s(t)} (\bar{\mathcal{L}}_{rr}^{s(t)})^T Q \bar{D}_{\bar{R}} \otimes H \right) \Xi_{\bar{R}} + \zeta_2^{-1} \kappa \Xi_{\bar{R}}^T (I_r \otimes H) \Xi_{\bar{R}} + \zeta_2^{-1} r \vartheta e^{-\tau t}, \end{aligned} \quad (47)$$

$$2\Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \bar{\mathcal{L}}_{rr}^{s(t)} \bar{D}_{\bar{R}} \otimes H \right) \Xi_{\bar{R}} \leq \bar{\lambda}_2 \zeta_3 \Xi_{\bar{R}}^T (\bar{D}_{\bar{R}}^2 \otimes H) \Xi_{\bar{R}} + \zeta_3^{-1} \Xi_{\bar{R}}^T (\bar{D}_{\bar{R}}^2 \otimes H) \Xi_{\bar{R}}, \quad (48)$$

where $\bar{\lambda}_2 = \lambda_{\max, \forall t > t_0} \left(Q \bar{\mathcal{L}}_{rr}^{s(t)} (\bar{\mathcal{L}}_{rr}^{s(t)})^T Q \right)$. From (45)–(48), we can obtain that

$$\begin{aligned} & -2\Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \bar{\mathcal{L}}_{rr}^{s(t)} \bar{D}_{\bar{R}} \otimes H \right) (\Xi_{\bar{R}} + \chi_{\bar{R}}) \\ & \leq \bar{\lambda}_2 (\zeta_1 + b\zeta_2 + \zeta_3) \Xi_{\bar{R}}^T (\bar{D}_{\bar{R}}^2 \otimes H) \Xi_{\bar{R}} + \zeta_3^{-1} \Xi_{\bar{R}}^T (\bar{D}_{\bar{R}}^2 \otimes H) \Xi_{\bar{R}} + \zeta_4 \left(\kappa \Xi_{\bar{R}}^T (I_r \otimes H) \Xi_{\bar{R}} + r \vartheta e^{-\tau t} \right), \end{aligned} \quad (49)$$

where $\zeta_4 = b\zeta_2^{-1} + \zeta_1^{-1}$. Similarly, the second term on the right side of (44) can be rewritten as

$$\begin{aligned} & -2\Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \bar{\mathcal{L}}_{\bar{r}\bar{r}}^{s(t)} \bar{D}_{\bar{R}} \otimes H \right) (\Xi_{\bar{R}} + \chi_{\bar{R}}) \\ & \leq \bar{\lambda}_3 (\zeta_1 + b\zeta_2) \Xi_{\bar{R}}^T (\bar{D}_{\bar{R}}^2 \otimes H) \Xi_{\bar{R}} + \zeta_4 \left(\vartheta \Xi_{\bar{R}}^T (I_{N-r} \otimes H) \Xi_{\bar{R}} + (N-r) \vartheta e^{-\tau t} \right) \\ & \quad - \Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} \left(Q \bar{\mathcal{L}}_{\bar{r}\bar{r}}^{s(t)} + (\bar{\mathcal{L}}_{\bar{r}\bar{r}}^{s(t)})^T Q \right) \bar{D}_{\bar{R}} \otimes H \right) \Xi_{\bar{R}}, \end{aligned} \quad (50)$$

where $\bar{\lambda}_3 = \lambda_{\max, \forall t > 0} \left(Q \bar{\mathcal{L}}_{\bar{r}\bar{r}}^{s(t)} (\bar{\mathcal{L}}_{\bar{r}\bar{r}}^{s(t)})^T Q \right)$. Substituting (49) and (50) to (44), we can obtain that

$$\begin{aligned} \dot{V}_{\bar{R}} & \leq c\Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \otimes (PA + A^T P) \right) \Xi_{\bar{R}} - \frac{\rho a^2 e^{-\rho(t-t_i^*)}}{2b} \sum_{i=r+1}^N q_{i-r} (\bar{h}_i - \gamma)^2 + c\Xi_{\bar{R}}^T \left(\zeta_3^{-1} \bar{D}_{\bar{R}}^2(t) + \kappa \zeta_4 I_r \otimes H \right) \\ & \quad + c \left(\Xi_{\bar{R}}^T \left((\bar{\lambda}_4 \bar{D}_{\bar{R}}^2 + \kappa \zeta_4 I_{N-r} - \hat{\gamma} Q) \otimes H \right) \Xi_{\bar{R}} \right) - c\Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} \left(Q \bar{\mathcal{L}}_{\bar{r}\bar{r}}^{s(t)} + (\bar{\mathcal{L}}_{\bar{r}\bar{r}}^{s(t)})^T Q \right) \bar{D}_{\bar{R}} \otimes H \right) \Xi_{\bar{R}} + \zeta_2, \end{aligned} \quad (51)$$

where $\zeta_2(t) = \bar{o} e^{-\tau t}$ with $\bar{o} = Nc\vartheta\zeta_4$, and $\bar{\lambda}_4 = (\bar{\lambda}_2 + \bar{\lambda}_3)(\zeta_2 + b\zeta_3) + \bar{\lambda}_2\zeta_4$. Next, the Lyapunov function is analyzed within the secure time interval and the attack-affected interval, and

$$\text{val, and } \zeta_2(t) = \bar{o} e^{-\tau t} \text{ with } \bar{o} = Nc\vartheta\zeta_4 + \frac{N\sigma a\vartheta}{b\bar{h}_{\min}} \left(2 + \frac{a\vartheta}{\bar{h}_{\min}} \right).$$

When $t \in [\mathcal{I}_n + \Delta_n + \ell_n, \mathcal{I}_{n+1})$, $n \in \mathbb{N}^+$, it is easy to know that $\bar{w}_{ij}^{s(t)} = w_{ij}$ and $\bar{\mathcal{L}}^{s(t)} = \bar{\mathcal{L}}$, by Lemmas 1 and 2 it follows that

$$-c\Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} \left(\Phi \bar{\mathcal{L}}_{rr}^{s(t)} + (\bar{\mathcal{L}}_{rr}^{s(t)})^T \Phi \right) \bar{D}_{\bar{R}} \otimes H \right) \Xi_{\bar{R}} \leq -c\bar{\lambda}_5 \Xi_{\bar{R}}^T (\bar{D}_{\bar{R}}^2 \otimes H) \Xi_{\bar{R}}, \quad (52)$$

$$-c\Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} \left(Q \bar{\mathcal{L}}_{\bar{r}\bar{r}}^{s(t)} + (\bar{\mathcal{L}}_{\bar{r}\bar{r}}^{s(t)})^T Q \right) \bar{D}_{\bar{R}} \otimes H \right) \Xi_{\bar{R}} \leq -c\bar{\lambda}_6 \Xi_{\bar{R}}^T (\bar{D}_{\bar{R}}^2 \otimes H) \Xi_{\bar{R}}, \quad (53)$$

where $\bar{\lambda}_5 = \frac{\lambda_2 (\Phi \bar{\mathcal{L}}_{rr} + \bar{\mathcal{L}}_{rr}^T \Phi)}{r} > 0$ and $\bar{\lambda}_6 = \lambda_{\min}(M) > 0$ with $Q \bar{\mathcal{L}}_{\bar{r}\bar{r}} + \bar{\mathcal{L}}_{\bar{r}\bar{r}}^T Q = M$. Thus, from

(43) and (51), we can obtain that

$$\begin{aligned} \dot{V} & \leq -c\Xi_{\bar{R}}^T \left((c^{-1} \hat{\gamma} Q + \bar{\lambda}_8 \bar{D}_{\bar{R}}^2 - \kappa \zeta_4 I_{N-r}) \otimes H \right) \Xi_{\bar{R}} - \frac{\rho a^2 e^{-\rho(t-t_i^*)}}{2b} \sum_{i=r+1}^N q_{i-r} (\bar{h}_i - \gamma)^2 \\ & \quad + c\Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} Q \otimes (PA + A^T P) \right) \Xi_{\bar{R}} + c\Xi_{\bar{R}}^T \left(\bar{D}_{\bar{R}} \Phi \otimes (PA + A^T P) \right) \Xi_{\bar{R}} + \zeta_1 + \zeta_2 \\ & \quad - c\Xi_{\bar{R}}^T \left((c^{-1} \hat{\gamma} \Phi + \bar{\lambda}_7 \bar{D}_{\bar{R}}^2 - 2\kappa \zeta_4 I_r) \otimes H \right) \Xi_{\bar{R}} - \frac{\rho a^2 e^{-\rho(t-t_i^*)}}{2b} \sum_{i=1}^r \varphi_i (\bar{h}_i - \gamma)^2, \end{aligned} \quad (54)$$

where $\bar{\lambda}_7 = \bar{\lambda}_5 - \bar{\lambda}_1(\zeta_1 + \zeta_2) - \zeta_3^{-1}$ and $\bar{\lambda}_8 = \bar{\lambda}_6 - \bar{\lambda}_4$. Consequently, one can find a collection of positive constants ζ_i ($i = 1, 2, 3$) satisfying both $\bar{\lambda}_7 > 0$ and $\bar{\lambda}_8 > 0$. Then, for $\forall \varpi \geq 0$, we have

$$\begin{aligned} & \bar{\lambda}_7 \bar{d}_i^2 - (\varpi + 1) \varphi_i \bar{d}_i + c^{-1} \hat{\gamma} \varphi_i - 2\kappa \zeta_4 \\ & = \bar{\lambda}_7 \left(\bar{d}_i - \frac{(\varpi + 1) \varphi_i}{2\bar{\lambda}_7} \right)^2 - \frac{(\varpi + 1)^2 \varphi_i^2}{4\bar{\lambda}_7} + c^{-1} \hat{\gamma} \varphi_i - 2\kappa \zeta_4. \end{aligned} \quad (55)$$

One can always find a sufficiently large $\hat{\gamma}$ such that $\hat{\gamma} > \frac{2c\kappa\zeta_4}{\varphi_i} + \frac{c(\varpi+1)^2\varphi_i}{4\bar{\lambda}_7}$, one can obtain

$$\bar{\lambda}_7 \bar{d}_i^2 - \varpi \varphi_i \bar{d}_i + c^{-1} \hat{\gamma} \varphi_i - 2\kappa \zeta_4 \geq \varphi_i \bar{d}_i. \quad (56)$$

Obviously, the following inequalities hold

$$\begin{cases} c^{-1} \hat{\gamma} \Phi + \bar{\lambda}_7 \bar{D}_{\bar{R}}^2 - 2\kappa \zeta_4 I_r \geq \bar{D}_{\bar{R}} \Phi, \\ c^{-1} \hat{\gamma} Q + \bar{\lambda}_8 \bar{D}_{\bar{R}}^2 - \kappa \zeta_4 I_{N-r} \geq \bar{D}_{\bar{R}} Q. \end{cases} \quad (57)$$

Due to $H = PBB^T P$, by substituting (57) into (54), it can be further derived from (27) that

$$\begin{aligned} \dot{V} \leq & -\frac{\rho a^2 e^{-\rho(t-t_i^*)}}{2b} \sum_{i=1}^r \varphi_i (\bar{h}_i - \gamma)^2 - \frac{\rho a^2 e^{-\rho(t-t_i^*)}}{2b} \sum_{i=r+1}^N q_{i-r} (\bar{h}_i - \gamma)^2 \\ & - \alpha c \Xi_R^T (\bar{D}_R \Phi \otimes P) \Xi_R - \alpha c \Xi_R^T (\bar{D}_R Q \otimes P) \Xi_R + \zeta_1 + \zeta_2 \\ & \leq -\tilde{\alpha} V + \zeta_3, \end{aligned} \quad (58)$$

where $\zeta_3 = \zeta_1 + \zeta_2$ and $\tilde{\alpha} = \min(\rho, \alpha)$. When $t \in [\mathcal{I}_n, \mathcal{I}_n + \Delta_n + \ell_n)$, $n \in \mathbb{N}^+$, the communication network under SMLSAs may no longer have a spanning tree, we can derive from (43) and (51) that

$$\begin{aligned} \dot{V} \leq & c \Xi_R^T (\bar{D}_R \Phi \otimes (PA + A^T P)) \Xi_R + c \Xi_R^T (\bar{D}_R Q \otimes (PA + A^T P)) \Xi_R - \frac{\rho a^2 e^{-\rho(t-t_i^*)}}{2b} \sum_{i=r+1}^N q_{i-r} (\bar{h}_i - \gamma) \\ & - c \Xi_R^T \left((c^{-1} \hat{\gamma} \Phi - \bar{\lambda}_1 (\zeta_1 + b\zeta_2 - \zeta_3^{-1}) \bar{D}_R^2 - 2\vartheta \zeta_4 I_r) \otimes PBB^T P \right) \Xi_R - \frac{\rho a^2 e^{-\rho(t-t_i^*)}}{2b} \sum_{i=1}^r \varphi_i (\bar{h}_i - \gamma)^2 \\ & - c \Xi_R^T \left((c^{-1} \hat{\gamma} Q - \bar{\lambda}_4 \bar{D}_R^2 - \kappa \zeta_4 I_{N-r}) \otimes PBB^T P \right) \Xi_R + \zeta_3. \end{aligned} \quad (59)$$

Since $\hat{\gamma}$ is a sufficiently large constant, we can obtain $c^{-1} \hat{\gamma} Q - \bar{\lambda}_4 \bar{D}_R^2(t) - \kappa \zeta_4 I_{N-r} > 0$ and $c^{-1} \hat{\gamma} \Phi - \bar{\lambda}_1 (\zeta_1 + b\zeta_2 - \zeta_3^{-1}) \bar{D}_R^2(t) - 2\kappa \zeta_4 I_r > 0$. Thus, it can be derived from (28) and (59) that

$$\begin{aligned} \dot{V} \leq & \bar{\beta} c \Xi_R^T (\bar{D}_R Q \otimes P) \Xi_R - \frac{\rho a^2 e^{-\rho(t-t_i^*)}}{2b} \sum_{i=1}^N \varphi_{i-r} (\bar{h}_i - \gamma)^2 + \bar{\beta} c \Xi_R^T (\bar{D}_R \Phi \otimes P) \Xi_R + \zeta_3 \\ & \leq 2\bar{\beta} V + \zeta_3. \end{aligned} \quad (60)$$

Based on Lemma 3, we have $\lim_{t \rightarrow +\infty} V(t) = 0$. In view of (29) and (30), one has $\lim_{t \rightarrow +\infty} \Xi_i(t) = 0$ ($i = 1, 2, \dots, N$), which implies $\lim_{t \rightarrow +\infty} (\delta_i(t) - \delta_j(t)) = 0$. Thus, the quadrotor swarm (4) is proven to realize fully distributed secure self-triggered time-varying formation control via the control protocol (8) in conjunction with the dynamic event-triggered conditions (12). $\square$

Remark 2. By considering the attack time interval and the secure time interval, using switching control theory and the Lyapunov stability analysis method, the tolerance of formation control for the quadrotor swarm (4) under SMLSAs is given, that is, the attack parameters satisfy the condition $v_2 + v_1 \bar{\Delta} < \tilde{\alpha}/(\tilde{\alpha} + 2\bar{\beta})$. For the control gains K and H , it can be obtained by setting a certain value for the nominal convergence speed α of the local relative formation reference error system, then solving the divergence rate $\bar{\beta}$ and gain matrix P according to the linear matrix inequality in Theorem 1, and calculating the control gains $K = -B^T P$ and $H = PBB^T P$. In this sense, we can find that the control gains do not depend on the global eigenvalue bounds of all possible attacked Laplacian topologies and the proposed method can be executed in a fully distributed architecture. Note that assumptions 1 and 2 are sufficient prerequisites for swarm systems stability and only apply to bounded regular attacks; all stability conclusions will be invalid if attack duration or frequency exceeds the given limits. The Lyapunov proof in Equations (29)–(60) targets outer-loop formation translation errors and adopts the standard separation principle: the attitude inner-loop is configured with much higher bandwidth to realize fast tracking, which is a common assumption in quadrotor formation control. All simulations and experiments in this work adopt moderate maneuvers and mild disturbances without severe actuator saturation or obvious inner-loop delay.

Theorem 2. The local relative formation error system (16) of the quadrotor swarm (4) does not exhibit Zeno behavior if Theorem 1 holds.

Proof of Theorem 2. Suppose that Zeno behavior exists, namely, $\lim_{k \rightarrow \infty} t_i^k = Z_T > 0$. When $t = t_i^{k+1}$, the dynamic event-triggered mechanism is activated, and we can derive that at least one of the dynamic event-triggered conditions, (13) and (14), is triggered

$$\hat{d}_i^2(t_i^k) \chi_i^T(t_i^k) H \chi_i(t_i^k) \geq \vartheta e^{-\alpha_i^k}, \quad (61)$$

$$\left(v_i(t_i^k) - v_i(t_i^{k-1})\right)^2 \Xi_i^T(t_i^{k-1}) H \Xi_i(t_i^{k-1}) \geq \vartheta e^{-\tau t_i^k}, \quad (62)$$

Note that $\Xi_i, \chi_i, \hat{d}_i$ and v_i are continuous variables and $\lim_{k \rightarrow +\infty} t_i^{k-1} = \lim_{k \rightarrow +\infty} t_i^k = Z_T$, we can conclude that $\lim_{k \rightarrow +\infty} \chi_i(t_i^k) = 0$ and $\lim_{k \rightarrow +\infty} (v_i(t_i^k) - v_i(t_i^{k-1})) = 0$. Then, we can obtain that

$$\lim_{k \rightarrow +\infty} \hat{d}_i^2(t_i^k) \chi_i^T(t_i^k) H \chi_i(t_i^k) = 0, \quad (63)$$

$$\lim_{k \rightarrow +\infty} \left(v_i(t_i^k) - v_i(t_i^{k-1})\right)^2 \Xi_i^T(t_i^{k-1}) H \Xi_i(t_i^{k-1}) = 0. \quad (64)$$

However, it is obvious from (61) and (62) that $\lim_{k \rightarrow +\infty} \vartheta e^{-\tau t_i^k} = \lim_{k \rightarrow +\infty} \vartheta e^{-\tau Z_T} > 0$, which is contradiction to (63) and (64). Therefore, the local relative formation error system (16) of the quadrotor swarm (4) does not exhibit Zeno behavior. $\square$

Remark 3. There are two types of non-Zeno designs in the academic community. One is the fixed strictly positive minimum interval, this method adds a global strictly positive constant lower bound greater than zero in the triggering rule. The advantage is that it is intuitive and easy to understand. However, the fixed constant brings conservatism, and in high-dynamic and maneuverable scenarios, it will sacrifice tracking accuracy. The other is the exponential decay threshold scheme adopted in this paper. The threshold in this paper is always positive, i.e., $\vartheta e^{-\tau} > 0$ and $\lim_{t \rightarrow +\infty} \vartheta e^{-\tau t} = 0$. The Zeno effect requires a sequence of triggering intervals that infinitely approach 0. By using the contradiction method, it can be strictly proved that the exponential constraint in this paper can ensure that any adjacent triggering intervals are greater than 0, which is a classic method widely used in distributed multi-agent event-triggered control [36–38] and does not have any theoretical loopholes.

4. Design of STVF Control Algorithm for Quadrotor Swarm System

This section first introduces the basic principle of outer-loop position–velocity control driven for the quadrotors by inner-loop attitude control. Then, based on the main conclusions of Section 3, the parameter-setting method of the STVF control protocol is discussed. Finally, the specific algorithm steps for achieving fully distributed STVF control for the quadrotor swarm under SMLSAs are presented.

The quadrotor swarm generates the expected position and velocity of each quadrotor in the outer-loop position–velocity control system according to the external task requirements (such as formation control instructions), under the effect of the control protocol. Meanwhile, the inner-loop attitude control system corrects the three attitude angles to drive each quadrotor to track the expected position and velocity states of the individual control. By decomposing the formation control protocol (8) into three control input components u_{ix}, u_{iy} , and u_{iz} , the outer-loop position–velocity control input components u_{ix}, u_{iy} , and u_{iz} from (6) can be transformed into the expected pitch angle $\bar{\theta}_i$, roll angle $\bar{\phi}_i$, and the total lift force F_i in the quadrotor body coordinate system. The specific calculations are as follows

$$\bar{\theta}_i = -\arctan\left((u_{ix} \cos \psi_i + u_{iy} \sin \psi_i)(G - u_{iz})^{-1}\right), \quad (65)$$

$$\bar{\phi}_i = \arctan\left(((u_{iy} \cos \psi_i - u_{ix} \sin \psi_i) \cos \theta_i)(G - u_{iz})^{-1}\right), \quad (66)$$

$$F_i = \frac{m_i (G - u_{iz})}{\cos \theta_i \cos \phi_i}, \quad (67)$$

It should be noted that the quadrotor is assumed not to perform inverted flight; that is, the pitch and roll angles satisfy $\phi_i, \theta_i \in (-\pi/2, \pi/2)$, and similar assumptions can be found in [40]. Since the yaw angle ψ_i is the angle between the quadrotor nose direction and the X-axis direction in the geographic coordinate system, the expected yaw angle $\bar{\psi}_i$ can be designed as

$$\bar{\psi}_i = \begin{cases} \arctan(v_{iY}v_{iX}^{-1}), & v_{iX} > 0 \\ \frac{\pi}{2}, & v_{iX} = 0, v_{iY} > 0 \\ \arctan(v_{iY}v_{iX}^{-1}) + \pi, & v_{iX} < 0 \\ -\frac{\pi}{2}, & v_{iX} = 0, v_{iY} < 0 \\ \text{Hold}, & v_{iX} = 0, v_{iY} = 0 \end{cases} \quad (68)$$

where “Hold” indicates maintaining the heading of the previous moment. To achieve the expected Euler angles through the inner-loop attitude control, the following error feedback control system is constructed:

$$\begin{cases} \dot{\bar{\psi}}_i = k_\psi(\psi_i - \bar{\psi}_i) + \dot{\bar{\psi}}_i, \\ \dot{\bar{\phi}}_i = k_\phi(\phi_i - \bar{\phi}_i) + \dot{\bar{\phi}}_i, \\ \dot{\bar{\theta}}_i = k_\theta(\theta_i - \bar{\theta}_i) + \dot{\bar{\theta}}_i, \end{cases} \quad (69)$$

where k_ψ , k_ϕ and k_θ are three positive feedback control gain constants. It should be noted that the actual attitude angles of the quadrotor, ψ_i , ϕ_i and θ_i , can be obtained through measurement data from on-board gyroscopes, accelerometers, and electronic compasses, etc., and calculated by attitude angle fusion algorithms such as the Kalman filter. The expected attitude angles, $\bar{\psi}_i$, $\bar{\phi}_i$ and $\bar{\theta}_i$, can be calculated by (65), (66) and (68). $\dot{\bar{\psi}}_i$, $\dot{\bar{\phi}}_i$ and $\dot{\bar{\theta}}_i$ can be estimated using the differential tracker proposed in [43].

Define the body coordinate system of the quadrotor as $O(\bar{X}, \bar{Y}, \bar{Z})$. The inner-loop attitude control can be modeled as:

$$\begin{bmatrix} \dot{\bar{\psi}}_i \\ \dot{\bar{\phi}}_i \\ \dot{\bar{\theta}}_i \end{bmatrix} = \begin{bmatrix} 0 & \sin \phi_i / \cos \theta_i & \cos \phi_i / \cos \theta_i \\ 1 & \tan \theta_i / \sin \phi_i & \tan \theta_i / \cos \phi_i \\ 0 & \cos \phi_i & -\sin \phi_i(t) \end{bmatrix} \begin{bmatrix} \omega_{i\bar{X}} \\ \omega_{i\bar{Y}} \\ \omega_{i\bar{Z}} \end{bmatrix}, \quad (70)$$

$$\begin{cases} J_{i\bar{X}} \dot{\omega}_{i\bar{X}} = \omega_{i\bar{Y}} \omega_{i\bar{Z}} (J_{i\bar{Y}} - J_{i\bar{Z}}) + \mathcal{T}_{i\bar{X}} + \nabla_{i\bar{X}}, \\ J_{i\bar{Y}} \dot{\omega}_{i\bar{Y}} = \omega_{i\bar{Z}} \omega_{i\bar{X}} (J_{i\bar{Z}} - J_{i\bar{X}}) + \mathcal{T}_{i\bar{Y}} + \nabla_{i\bar{Y}}, \\ J_{i\bar{Z}} \dot{\omega}_{i\bar{Z}} = \omega_{i\bar{X}} \omega_{i\bar{Y}} (J_{i\bar{X}} - J_{i\bar{Y}}) + \mathcal{T}_{i\bar{Z}} + \nabla_{i\bar{Z}}, \end{cases} \quad (71)$$

where $J_{i\bar{X}}$, $J_{i\bar{Y}}$ and $J_{i\bar{Z}}$ are the moments of inertia of the quadrotor i around the three axes of the body coordinate system, $\omega_{i\bar{X}}$, $\omega_{i\bar{Y}}$ and $\omega_{i\bar{Z}}$ are the corresponding angular velocities, $\mathcal{T}_{i\bar{X}}$, $\mathcal{T}_{i\bar{Y}}$ and $\mathcal{T}_{i\bar{Z}}$ are the control torques, and $\nabla_{i\bar{X}}$, $\nabla_{i\bar{Y}}$ and $\nabla_{i\bar{Z}}$ are the three-direction disturbance torques in the inner-loop attitude control.

Let the expected angular velocities be $\omega_{i\bar{X}}$, $\omega_{i\bar{Y}}$ and $\omega_{i\bar{Z}}$, which can be obtained from (70) as:

$$\begin{bmatrix} \omega_{i\bar{X}} \\ \omega_{i\bar{Y}} \\ \omega_{i\bar{Z}} \end{bmatrix} = \begin{bmatrix} 0 & \sin \phi_i / \cos \theta_i & \cos \phi_i / \cos \theta_i \\ 1 & \tan \theta_i / \sin \phi_i & \tan \theta_i / \cos \phi_i \\ 0 & \cos \phi_i & -\sin \phi_i \end{bmatrix}^{-1} \begin{bmatrix} \dot{\bar{\psi}}_i \\ \dot{\bar{\phi}}_i \\ \dot{\bar{\theta}}_i \end{bmatrix}, \quad (72)$$

where $\dot{\bar{\psi}}_i$, $\dot{\bar{\phi}}_i$ and $\dot{\bar{\theta}}_i$ in the body coordinate system can be obtained from the error feedback control system (69). Since the control input of the inner-loop attitude control model are the angular accelerations in three directions $\mathcal{T}_{i\bar{X}}(t)J_{i\bar{X}}^{-1}$, $\mathcal{T}_{i\bar{Y}}(t)J_{i\bar{Y}}^{-1}$ and $\mathcal{T}_{i\bar{Z}}(t)J_{i\bar{Z}}^{-1}$, a state feedback controller is designed in the form of (71) as

$$\begin{cases} \mathcal{T}_{i\bar{x}} J_{i\bar{x}}^{-1} = k_{\bar{x}} (\bar{\omega}_{i\bar{x}} - \omega_{i\bar{x}}) + \dot{\bar{\omega}}_{i\bar{x}} - \bar{\sigma}_{i\bar{x}}, \\ \mathcal{T}_{i\bar{y}} J_{i\bar{y}}^{-1} = k_{\bar{y}} (\bar{\omega}_{i\bar{y}} - \omega_{i\bar{y}}) + \dot{\bar{\omega}}_{i\bar{y}} - \bar{\sigma}_{i\bar{y}}, \\ \mathcal{T}_{i\bar{z}} J_{i\bar{z}}^{-1} = k_{\bar{z}} (\bar{\omega}_{i\bar{z}} - \omega_{i\bar{z}}) + \dot{\bar{\omega}}_{i\bar{z}} - \bar{\sigma}_{i\bar{z}}, \end{cases} \quad (73)$$

where $k_{\bar{x}}$, $k_{\bar{y}}$ and $k_{\bar{z}}$ are positive feedback control gains, and $\bar{\sigma}_{i\bar{x}}$, $\bar{\sigma}_{i\bar{y}}$ and $\bar{\sigma}_{i\bar{z}}$ are the three-direction compensation terms, which can be estimated by designing an extended state observer, as detailed in [44]. By estimating $\bar{\sigma}_{i\bar{x}}$, $\bar{\sigma}_{i\bar{y}}$ and $\bar{\sigma}_{i\bar{z}}$, the influences of the three-direction disturbance torques $\nabla_{i\bar{x}}$, $\nabla_{i\bar{y}}$ and $\nabla_{i\bar{z}}$ in the inner-loop attitude control can be eliminated. In view of (71)–(73), the outer-loop position and velocity references are achieved by the inner-loop attitude control, with the required lift and torques being implemented through the speed differences among the four rotors. The outer-loop double-integrator simplification in (4) is a common modeling choice in cooperative control research of quadrotor swarms, the aerodynamic drag and nonlinear uncertainties are regarded as three-direction disturbance torques $\nabla_{i\bar{x}}$, $\nabla_{i\bar{y}}$ and $\nabla_{i\bar{z}}$, which are estimated and compensated by an extended state observer embedded in the fast inner attitude loop of each quadrotor. Relying on time-scale separation, the extended state observer inner-loop compensation greatly attenuates aerodynamic uncertainties before they propagate to the outer coordination loop.

Based on the design criteria in Section 3, combined with the principle of the position-attitude transformation in quadrotor control. Algorithm 1 presents the specific design steps for the fully distributed STVF control of the quadrotor swarms.

Algorithm 1: Fully Distributed STVF Control Algorithm for Quadrotor Swarms under SMLSAs.

Step1: Define the modal self-feedback scalars α_p and α_v , and denote the number of quadrotors as N .

Step2: Select the nominal convergence speed α of the local relative formation reference error system, and solve the divergence rate $\bar{\beta}$ and gain matrix P of the local relative formation reference error system under sequential attacks according to the linear matrix inequality in Theorem 1, and calculate the control gains $K = -B^T P$ and $H = PBB^T P$.

Step3: Configure the spatial formation of the quadrotor swarm, and select an appropriate formation vector function $h_i = [h_{ip}^T, h_{iv}^T]^T$ that satisfies the formation feasibility condition $\dot{h}_{ip} = h_{iv}$, $i \in \mathcal{V}$.

Step4: Select the parameters κ , $\hat{v}$, τ of the dynamic event-triggering function as arbitrary positive numbers, and set the initial triggering time of each quadrotor as $t_i^1 = t_0 \geq 0$, $i \in \mathcal{V}$.

Step5: Select the control parameters a, b, c, σ, ρ of the STVF control protocol (11), and set the initial value of the coupling factor component $\bar{h}_i$ as $\bar{h}_i(t_0) > \sqrt{\kappa/a}$, $i \in \mathcal{V}$.

Step6: Set the initial position and velocity $p_i(0)$, $v_i(0)$, $i \in \mathcal{V}$ of each quadrotor.

Step7: Each quadrotor calculates the control input $u_i = [u_{i\bar{x}}, u_{i\bar{y}}, \dots, u_{i\bar{z}}]^T$ of the outer-loop position and velocity control model through the STVF control protocol (8).

Step8: Determine the expected attitude angles $\bar{\psi}_i$, $\bar{\phi}_i$ and $\bar{\theta}_i$ from (65), (66) and (68), and simultaneously estimate the derivatives of the expected attitude angles $\dot{\bar{\psi}}_i$, $\dot{\bar{\phi}}_i$ and $\dot{\bar{\theta}}_i$ using the tracking differentiator proposed in [43].

Step9: Compute the lift F_i by (67), and calculate the control torques $\mathcal{T}_{i\bar{x}}$, $\mathcal{T}_{i\bar{y}}$ and $\mathcal{T}_{i\bar{z}}$ from (65), (66) and (68). The quadrotor achieves the expected lift and control torques through different rotational speeds of the four servos.

Step10: Obtain the expected lift and control torques by controlling the rotational speeds of the four motors of the quadrotor. The desired position and velocity states of the quadrotor $\mathcal{Q}_v$ are achieved through the transformation of lift and control moment. The algorithm ends.

Remark 4. Under the proposed STVF control algorithm, the control input is first obtained from the formation protocol, then translated into desired attitude angles via position-attitude transformation. The inner-loop attitude control subsequently ensures tracking of these angles, effecting the conversion from outer-loop position/velocity inputs to inner-loop attitude responses, and finally achieving secure formation control of quadrotor swarms. We introduce a tiny positive safety constant ε ($0 < \varepsilon \ll 1$) and design a unilateral saturation function only limiting the upper bound of u_{iz} . This design strictly guarantees $G - u_{iz} \geq \varepsilon > 0$, such that the denominator in the virtual-to-attitude mapping can never approach zero, completely avoiding software-level division-by-zero errors and inherent singularities in attitude dynamics. Our study targets conventional commercial/industrial quadrotors equipped with unidirectional electronic speed controllers and single-surface propellers, which are physically incapable of sustained inverted flight for formation coverage missions. The constraint that attitude angles are bounded within $\pm\pi/2$ (no full inversion) is consistent with the hardware and mission characteristics of our target platform. In addition, supporting continuous inverted flight requires a full 3D dynamic model with bidirectional motor thrust, symmetric propeller aerodynamics and redefined inverted body coordinate frames. This introduces strong coupled positive/negative lift terms and drastically complicates Lyapunov stability derivation and convergence proof for multi-agent cooperative control.

Remark 5. The parameters α_p and α_v in the secure formation control protocol serve as self-feedback gains that govern the quadrotor's inherent motion characteristics. For flight safety, α_p and α_v must be chosen such that the eigenvalues of the system matrix A lie in the open left-half plane. Under this condition, once the desired STVF is achieved, the formation center will asymptotically converge to the origin. The parameters a, b, c, σ, ρ are closely tied to the inter-agent coupling strengths. Increasing these gains amplifies the control input, thereby accelerating formation convergence. However, under the event-triggered mechanism, higher gains also lead to more frequent triggering events. Hence, these parameters should be tuned moderately to balance convergence speed and communication overhead. The parameters κ, ϑ, τ determine the dynamic threshold of the event-triggered condition. Larger values of κ, ϑ together with a smaller τ yield a larger threshold and thus lower triggering frequency; conversely, smaller κ, ϑ with larger τ produce a smaller threshold and higher triggering frequency, which accelerates formation convergence at the cost of increased communication burden. Therefore, κ, ϑ, τ should also be chosen moderately, with the constraint $\tilde{h}_i(t_0) > \sqrt{\kappa a}$ as specified. Moreover, by Theorem 1, we have $v_2 + v_1 \bar{\Delta} < \frac{\tilde{\alpha}}{\tilde{\alpha} + 2\beta}$ with $\tilde{\alpha} = \min(\kappa, \alpha)$. Notably, the parameter κ reflects the system's tolerance to adversarial attacks and should be set to its maximum permissible value to enhance robustness.

5. Secure Formation Flight Experiment for Quadrotor Swarm Systems

To verify the effectiveness of the proposed STVF control method, a flight experiment for the quadrotor swarm systems is conducted. The flight experiment platform of the quadrotor swarm systems mainly consists of a satellite navigation real-time kinematic (RTK) positioning system, a quadrotor swarm system, and a ground control station, as shown in Figure 1. The quadrotor is FS450-type, which is developed by the open-source PX4 flight control system. It is mainly composed of an onboard computer, flight control module, onboard data transmission module, and onboard RTK module. The flight control module is developed based on the open-access PX4 flight control system and additional

modules such as the external loop velocity safety time-varying formation control module and the state information reception and processing module have been added according to the research content. The communication link attack information is injected into the onboard data transmission module through the ground control station. For communication link scaling attacks, it can be equivalently regarded as a multiplicative deception attack on the communication link, and the transmitted data of the data transmission module is scaled and tampered with before transmission; for certain communication link interruption attacks, the receiving quadrotor does not receive data transmission from that link, thereby achieving the disruption and impact of communication link attacks on the data transmission of the quadrotor swarm. Then, the damaged data obtained by the quadrotor is used for the update of the external loop control input, and the external loop control input is converted by the flight control module into the quadrotor control signal for internal loop attitude control through position-attitude transformation, thereby driving the quadrotor to move.

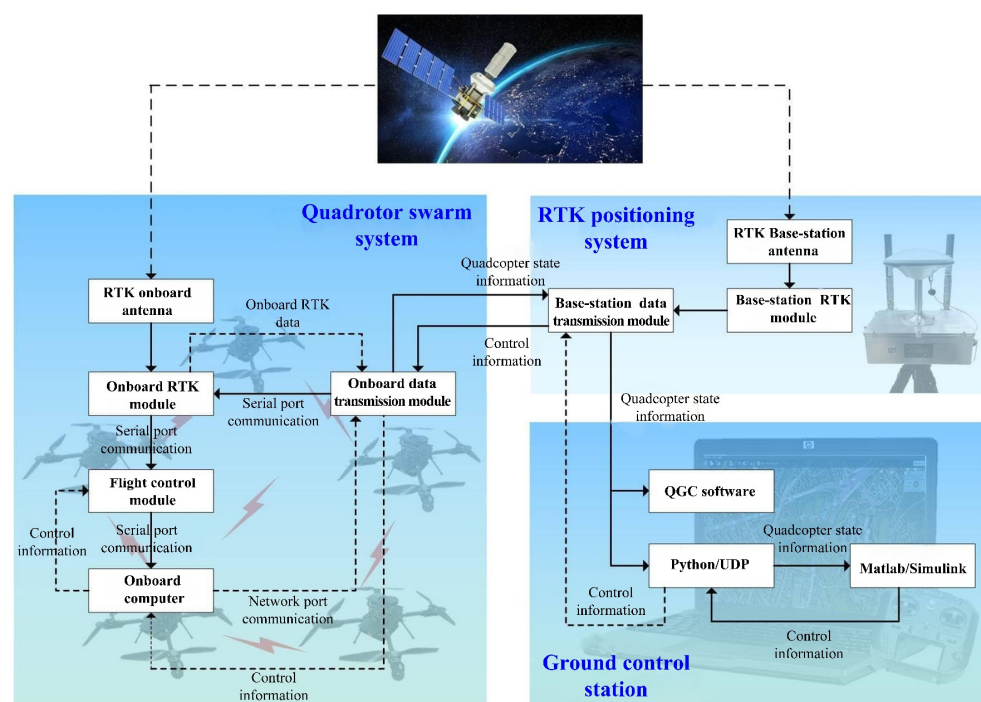


Figure 1. Principle framework diagram of the flight experimental platform for the quadrotor swarm.

A quadrotor swarm consisting of five agents is considered, with the communication topology shown in Figure 2. The formation flight is designed as a time-varying spiral ascent/descent, and the formation configuration is given by

$$h_p(t) = \begin{bmatrix} R_r \cos(R_\omega t + 0.5\pi i) \\ R_r \sin(R_\omega t + 0.5\pi i) \\ H_a \cos(H_\omega t) \end{bmatrix}, \quad h_v(t) = \dot{h}_p(t) = \begin{bmatrix} -R_\omega R_r \sin(R_\omega t + 0.5\pi i) \\ R_\omega R_r \cos(R_\omega t + 0.5\pi i) \\ H_\omega H_a \sin(H_\omega t) \end{bmatrix} \quad (i = 1, 2, \dots, 5),$$

where $R_r = 5\text{m}$, $R_\omega = 0.25\text{rad/s}$, $H_a = 0.4\text{m}$, $H_\omega = 0.1\text{rad/s}$. From this, it can be seen that the desired quadrotor swarm formation performs circular motion in the X-Y horizontal plane with a radius of 5m and an angular velocity of 0.25rad/s . At the same time, the entire formation is undergoing amplitude-symmetric oscillation in the Z-axis direction with an amplitude of 0.4m and an angular frequency of 0.1rad/s .

Consider that the quadrotor swarm is subject to a SMLSA. The equivalent communication topology after the attack is depicted in Figure 2, where certain link weights are scaled, corresponding to multiplicative deception attacks, while some links are completely

interrupted, corresponding to DoS attacks. The attack sequence is periodic with a cycle of 16 s, and the detailed attack pattern and switching signal trajectory per cycle are illustrated in Figure 3. Let the set of topologies under such SMLSAs be denoted as $\{\mathcal{G}, \mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3\}$. By defining $\mathcal{G}_0$ equivalently as $\mathcal{G}$, it follows that the communication topology switches within $\{\mathcal{G}_0, \mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3\}$ during the attack process, as shown in Figure 3.

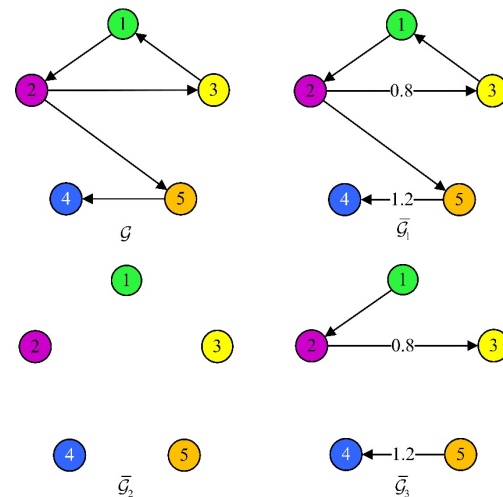


Figure 2. The set of all topologies under SMLSAs.

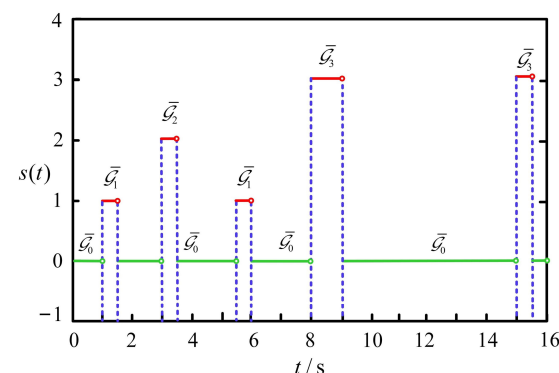


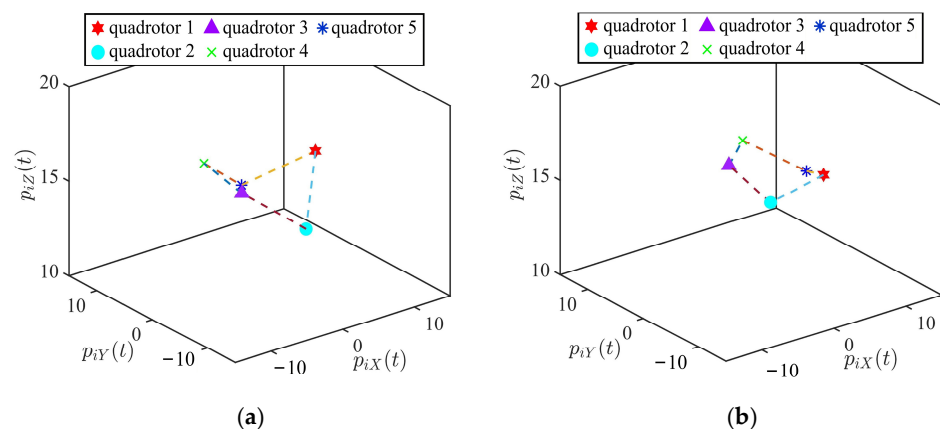
Figure 3. Trajectories of the attack sequence and the switching signal in each cycle of the SMLSAs.

According to Algorithm 1, the parameters of secure formation control protocol (8) are set as $a = b = c = \sigma = 1$, $\rho = 0.5$ and $\omega_i(0) = 1.1$, $i \in \{1, 2, \dots, N\}$. The parameters of the dynamic event-triggering function are set as $\kappa = \vartheta = 1$ and $\tau = 0.4$, which can be easily verified to satisfy the condition $\bar{h}_i(0) = 1.1 > \sqrt{\kappa/a} = 1$. From the previous theoretical analysis, it can be known that α_p and α_v , in the secure formation control protocol (8) are self-feedback quantities for adjusting the movement mode of the quadrotor itself. Considering the safety of the flight experiment, $\alpha_p = -0.02$ and $\alpha_v = -0.03$ are selected. In this case, the eigenvalues of system matrix A are $-0.015 \pm 0.1406i$. Under the condition that the quadrotor swarm realizes the required STVF, the formation center will asymptotically approach the origin. The coordinate origin is set as the central point of the flight field at a height of 15m above the ground. Select $\alpha = 0.5$, and $\beta = 0.441$. According to Theorem 1, the gain matrix can be obtained as $P = \begin{bmatrix} 10.6331 & -2.6617 \\ -2.6617 & 1.4653 \end{bmatrix}$. In this case, it can be calculated that $K = B^T P = [0.3133 \quad 1.2515]$ and $H = PB^T BP = \begin{bmatrix} 0.0981 & 0.3921 \\ 0.3921 & 1.5663 \end{bmatrix}$. Furthermore, through simulation and flight experiments, the maximum triggering influence

time interval can be obtained to verify the satisfaction of the STVF control conditions, i.e., $\nu_2 + \nu_1 \bar{\ell} = 0.298 < \tilde{\alpha} / (\tilde{\alpha} + 2\tilde{\beta}) = 0.362$. The initial positions and velocities of each quadrotor are randomly determined. In this experiment, it is set as

$$\begin{aligned} p_1(0) &= [7.6043 \quad -0.1319 \quad -2.8923]^T, & p_2(0) &= [1.5570 \quad -7.7456 \quad 1.2938]^T, \\ p_3(0) &= [-6.9650 \quad 5.0368 \quad 1.4738]^T, & p_4(0) &= [-4.8482 \quad 4.9316 \quad -2.2525]^T, \\ p_5(0) &= [2.1222 \quad 6.4397 \quad 1.5853]^T, & v_i(0) &= [0 \quad 0 \quad 0]^T \quad (i=1,2,\dots,N). \end{aligned}$$

The simulation and flight experiment results are shown in Figures 4–9. Figure 4 presents the relative positions of the quadrotor swarm in the state space at times 27 s, 35 s, 40 s, and 80 s. Figure 4a–d respectively illustrate the formation process of the five-quadrotor swarm from the disordered state after being attacked to the regular pentagon formation. Figures 5 and 6 respectively show the motion trajectories of the positions and velocities of the five quadrotors during the complete process from takeoff to landing over 115 s in the simulation and flight experiments. From Figures 4–6, it can be seen that the quadrotor swarm under SMLSAs can still form time-varying formations and perform spiral ascending and descending movements centered at the origin. It should be noted that inaccurate tuning of the tracking differentiator under high-dynamic maneuvers will introduce obvious phase delay and tracking deviation when transmitting reference signals from the outer-loop control to the inner-loop attitude control. From Figures 5 and 6, the position and velocity trajectories of quadrotors exhibit observable lag and deviation relative to their desired formation references. During the actual flight experiment, the quadrotor swarms are affected by external disturbances such as wind gusts. From Figures 5 and 6, it can be observed that the states of quadrotor swarms fluctuate more significantly than the expected simulation state. Nevertheless, all such tracking errors or formation control errors stay within the acceptable tolerance range defined for the formation mission and do not damage the overall stability and control performance of the proposed secure formation protocol.



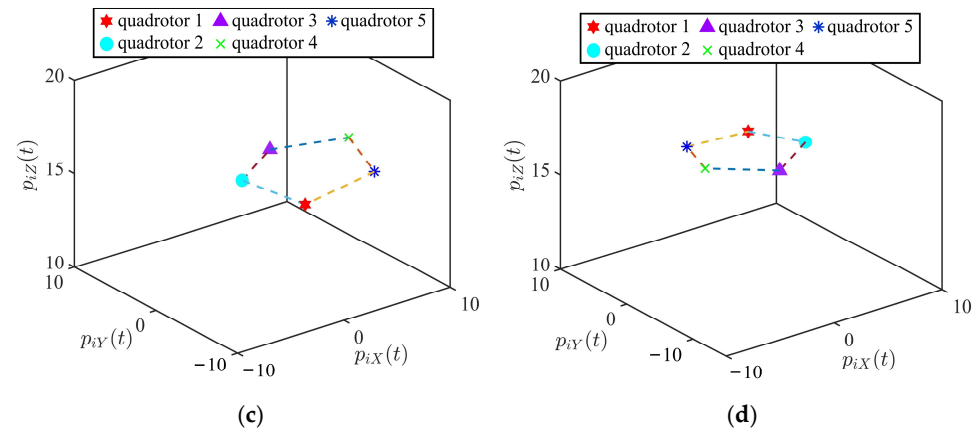


Figure 4. Spatial positions of quadrotor swarm formation at different times under SMLSAs. (a) =27 s; (b) =35 s; (c) t = 40 s; (d) t = 80 s.

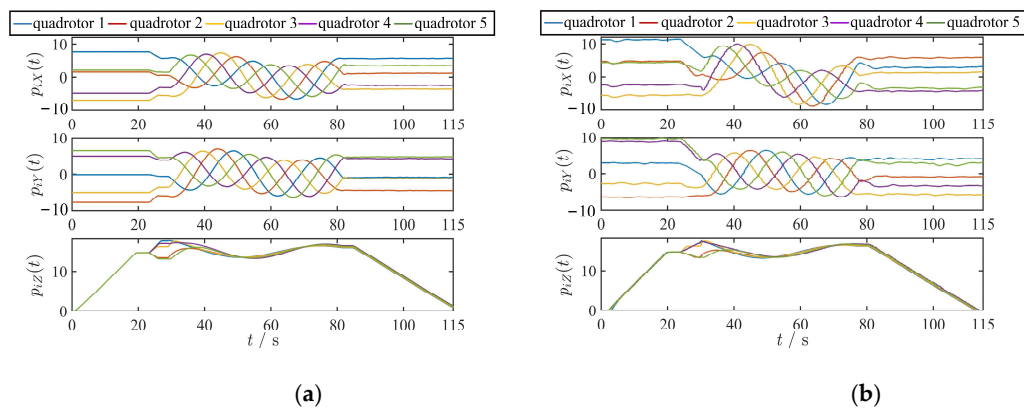


Figure 5. Position trajectories of the quadrotor swarm formation under SMLSAs. (a) Position trajectories in the simulation; (b) position trajectories in the flight experiment.

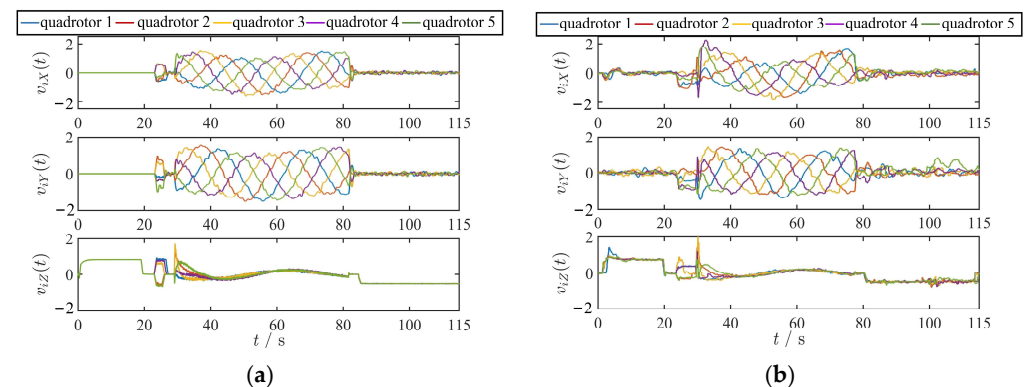


Figure 6. Velocity trajectories of the quadrotor swarm formation under SMLSAs. (a) Velocity trajectories in the simulation; (b) velocity trajectories in the flight experiment.

In this experiment, a dynamic event-triggered control mechanism was adopted. Figure 7 shows the triggering instants of each quadrotor in the time interval [30 s, 42 s] in the simulation and flight experiments. It can be observed that the communication triggering intervals of each UAV are larger than the corresponding sampling periods (60 Hz in simulation and 30 Hz in flight experiments). The quadrotor swarm systems achieve dynamic event-triggered control. It should be noted that the simulation and flight experiments provided the complete process of the quadrotor swarm systems from takeoff to landing,

where $[0\text{ s}, 30\text{ s})$ was the programmed takeoff stage, $[30\text{ s}, 75\text{ s})$ was the autonomous formation control flight stage of the quadrotor swarm systems, and $[75\text{ s}, 115\text{ s})$ was the programmed landing and return stage. Figures 8 and 9, respectively, show the variation curves of coupling strength and control input during the formation flight process of each quadrotor in the time interval $[30\text{ s}, 75\text{ s}]$ under SMLSAs.

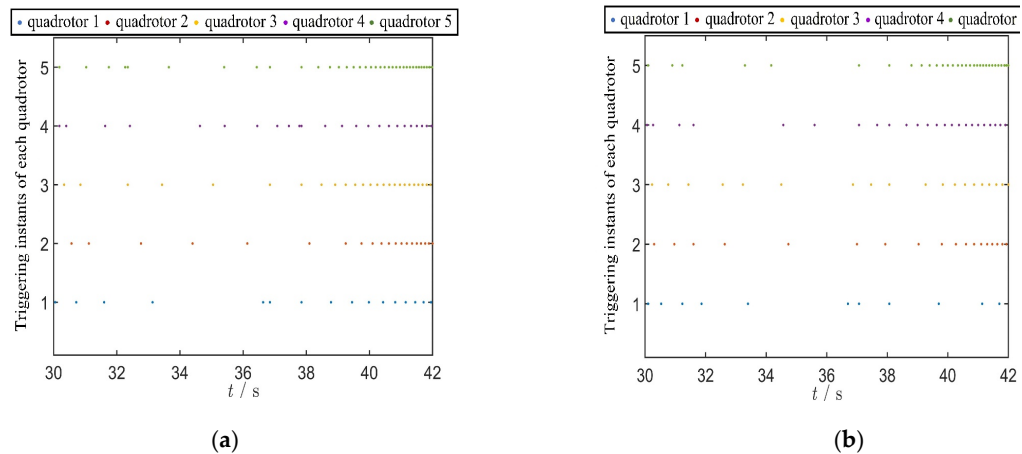


Figure 7. Triggering instants of each quadrotor under SMLSAs. (a) Triggering instants of each quadrotor in the simulation; (b) triggering instants of each quadrotor in the flight experiment.

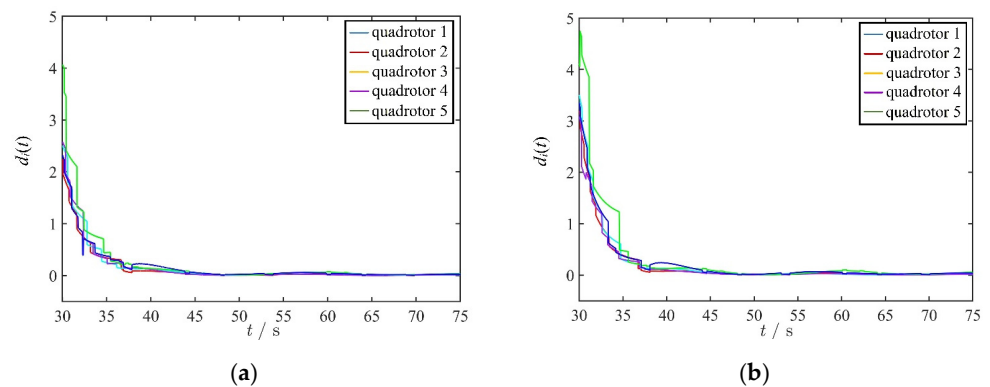


Figure 8. Variation curves of coupling strength for each quadrotor in the time interval $[30\text{ s}, 75\text{ s}]$. (a) Variation curves of the coupling strength of each quadrotor in the simulation; (b) variation curves of the coupling strength of each quadrotor in the flight experiment.

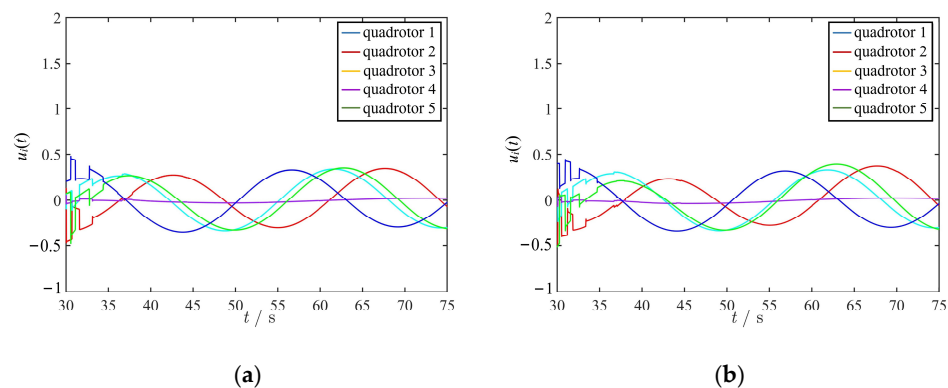


Figure 9. Variation curves of control input for each quadrotor in the time interval [30 s, 75 s]. (a) Variation curves of the control input for each quadrotor in the simulation; (b) variation curves of the control input for each quadrotor in the flight experiment.

It can be seen that the proposed STVF control method for the quadrotor swarm systems can ensure that the coupling strength and control input of each quadrotor remain within a small range and will not experience input saturation and other phenomena. The preceding analysis confirms that, in the presence of SMLSAs, the quadrotor swarm attains STVF control through the secure formation protocol. Moreover, to verify the anti-interference performance of the quadrotor under wind disturbance conditions, an experiment on the quadrotor hovering flight against wind disturbances was conducted. During the experiment (the wind speed was approximately 4.3 m/s), when the quadrotor took off and hovered, the electric fan was set to the 2nd speed to simulate the gusty wind interference. The wind direction was parallel to the plane of the drone. The hovering effect was observed, and the horizontal position data was recorded. The hovering effects under wind disturbance conditions and without wind disturbance were compared, and the experimental results are shown in Figure 10. As shown in Figure 10, when there is no wind disturbance, the hovering trajectory of the quadrotor is concentrated within the range of $U = \{(p_x, p_y) \mid |p_x| < 0.2, |p_y| < 0.2\}$. However, after the fan is turned on, due to the wind disturbance direction being from southeast to northwest, the quadrotor will have certain fluctuations in this direction. Nevertheless, the errors in the p_x, p_y directions can converge within the set U , and the overall drift error is only $U_d = \{(p_x, p_y) \mid -0.3 < p_x < 0.2, -0.5 < p_y < 0.3\}$. It can be seen that the extended state observer adopted can effectively compensate for the external wind disturbance, and the anti-disturbance effect of the quadrotor's horizontal position is satisfactory. The corresponding public video link for this flight experiment is <https://www.bilibili.com/video/BV1gC411b7ke/> (25 Jan. 2026).

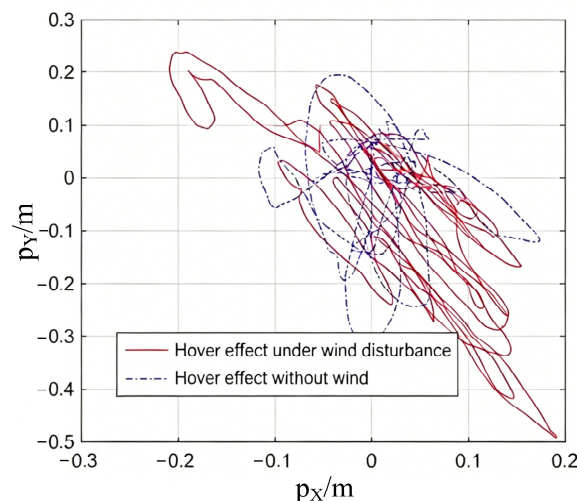


Figure 10. The horizontal movement trajectory of the quadrotor during hovering flight experiment.

6. Conclusions

This paper addresses the problem of fully distributed secure self-triggered time-varying formation control for quadrotor swarms under SMLSAs, and verifies the proposed scheme through experimental flight tests. The outer-loop velocity model is first employed to design a fully distributed secure formation protocol, accompanied by synthesis criteria for self-triggered formation achievement. The inner- and outer-loop control laws are then

coordinated to transform the position–velocity control inputs into inner-loop attitude references, yielding a complete formation control algorithm for the swarm. The efficacy and practicality of the method are subsequently validated by flight experiments using a quad-rotor swarm platform. The future research direction is to integrate bidirectional 3D quad-rotor dynamics and reformulate the swarm cooperative constraints with inverted flight capacity.

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Data Availability Statement: The data supporting the findings of this study are available from the corresponding author upon reasonable request.

DURC Statement: Current research is limited to the field of unmanned aerial vehicle formation control, which has innovative academic value and engineering application significance in cooperative control, and does not pose a threat to public health or national security. Authors acknowledge the dual-use potential of the research involving UAVs cooperative control and confirm that all necessary precautions have been taken to prevent potential misuse. As an ethical responsibility, authors strictly adhere to relevant national and international laws about DURC. Authors advocate for responsible deployment, ethical considerations, regulatory compliance, and transparent reporting, mitigating misuse risks and foster beneficial outcomes.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

UAV	Unmanned aerial vehicle
DoS	Denial of service
SMLSA	Sequential multi-link scaling attack
RTK	Real-time kinematic
STVF	Secure time-varying formation

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