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Supplementary Material: On the Asymptotic Distribution of Ridge Regression Estimators using Training and Test Samples

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Abstract: This document contains supplementary material for the simulation exercise described in Section 4. The tables presented correspond to all the specifications considered by the authors. This includes 4 specifications of the precision parameter δ $(0.1, 0.25, 0, 5, 1)$, 4 sample sizes n $(25, 50, 250, 500)$, 3 values of the prior β^p $\left((\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})', (\sqrt{2}, \sqrt{2})', (\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}})'\right)$ and 3 values of the training sample proportion τ $(0.5, 0.7, 0.9)$.

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1. Prior 1: $\beta^p = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})'$

1.1. Training Sample Proportion $\tau = 0.5$

Table S1. Estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ using TSLS and ridge estimator for $\beta^p = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})'$. The proportion of sample used in the training sample is set to $\tau = 0.5$. The estimator with lower combined MSE value is highlighted in bold.

			$\hat{\beta}_1$			$\hat{\beta}_2$			$(\hat{\beta}_1, \hat{\beta}_2)$
δ	n	Estimator	Bias	SD	MSE	Bias	SD	MSE	MSE
0.10	25	TSLS	0.013	0.231	0.054	0.630	1.515	2.693	2.747
		Ridge	0.061	0.142	0.024	0.668	0.540	0.739	**0.763**
	50	TSLS	0.006	0.190	0.036	0.546	1.430	2.343	2.379
		Ridge	0.039	0.091	0.010	0.628	0.576	0.726	**0.736**
	250	TSLS	-0.000	0.081	0.007	0.202	1.515	2.337	2.344
		Ridge	0.013	0.046	0.002	0.458	0.595	0.564	**0.566**
	500	TSLS	-0.000	0.041	0.002	0.060	0.766	0.591	0.592
		Ridge	0.008	0.030	0.001	0.342	0.379	0.260	**0.261**
0.25	25	TSLS	0.007	0.216	0.047	0.326	1.144	1.414	1.461
		Ridge	0.059	0.146	0.025	0.500	0.517	0.517	**0.542**
	50	TSLS	0.002	0.147	0.022	0.180	1.089	1.219	1.241
		Ridge	0.036	0.093	0.010	0.395	0.406	0.321	**0.331**
	250	TSLS	-0.001	0.047	0.002	-0.002	0.297	0.088	**0.091**
		Ridge	0.008	0.044	0.002	0.143	0.274	0.095	0.097
	500	TSLS	0.000	0.032	0.001	0.000	0.188	0.035	**0.036**
		Ridge	0.005	0.031	0.001	0.092	0.199	0.048	0.049
0.50	25	TSLS	0.002	0.198	0.039	0.055	0.747	0.561	0.600
		Ridge	0.056	0.150	0.026	0.255	0.454	0.271	**0.296**
	50	TSLS	-0.000	0.113	0.013	0.005	0.403	0.162	0.175
		Ridge	0.030	0.100	0.011	0.159	0.282	0.105	**0.116**
	250	TSLS	-0.001	0.045	0.002	-0.001	0.130	0.017	**0.019**
		Ridge	0.007	0.044	0.002	0.054	0.136	0.021	0.023
	500	TSLS	0.000	0.032	0.001	0.000	0.091	0.008	**0.009**
		Ridge	0.005	0.031	0.001	0.036	0.097	0.011	0.012
1.00	25	TSLS	-0.002	0.162	0.026	-0.004	0.244	0.060	0.086
		Ridge	0.047	0.154	0.026	0.082	0.218	0.054	**0.080**
	50	TSLS	-0.000	0.106	0.011	0.001	0.153	0.023	**0.035**
		Ridge	0.029	0.103	0.011	0.055	0.148	0.025	0.036
	250	TSLS	-0.001	0.045	0.002	-0.000	0.064	0.004	**0.006**
		Ridge	0.011	0.046	0.002	0.022	0.067	0.005	0.007
	500	TSLS	0.000	0.032	0.001	-0.000	0.045	0.002	**0.003**
		Ridge	0.008	0.032	0.001	0.015	0.048	0.003	0.004

1.2. Training Sample Proportion $\tau = 0.7$

Table S2. Estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ using TSLS and ridge estimator for $\beta^p = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})'$. The proportion of sample used in the training sample is set to $\tau = 0.7$. The estimator with lower combined MSE value is highlighted in bold.

			$\hat{\beta}_1$			$\hat{\beta}_2$			$(\hat{\beta}_1, \hat{\beta}_2)$
δ	n	Estimator	Bias	SD	MSE	Bias	SD	MSE	MSE
0.10	25	TSLS	0.013	0.232	0.054	0.630	1.514	2.690	2.744
		Ridge	0.100	0.138	0.029	0.677	0.403	0.621	**0.650**
	50	TSLS	0.006	0.189	0.036	0.542	1.428	2.333	2.368
		Ridge	0.062	0.100	0.014	0.646	0.548	0.717	**0.731**
	250	TSLS	-0.000	0.081	0.007	0.204	1.511	2.323	2.330
		Ridge	0.021	0.044	0.002	0.498	0.385	0.396	**0.398**
	500	TSLS	-0.000	0.041	0.002	0.060	0.763	0.585	0.587
		Ridge	0.014	0.032	0.001	0.401	0.349	0.282	**0.283**
0.25	25	TSLS	0.008	0.216	0.047	0.322	1.156	1.440	1.486
		Ridge	0.101	0.145	0.031	0.549	0.467	0.519	**0.550**
	50	TSLS	0.002	0.147	0.022	0.179	1.085	1.209	1.231
		Ridge	0.062	0.098	0.013	0.461	0.342	0.329	**0.343**
	250	TSLS	-0.001	0.047	0.002	-0.002	0.297	0.088	**0.091**
		Ridge	0.016	0.045	0.002	0.215	0.263	0.116	0.118
	500	TSLS	0.000	0.032	0.001	0.000	0.188	0.035	**0.036**
		Ridge	0.009	0.032	0.001	0.143	0.211	0.065	0.066
0.50	25	TSLS	0.002	0.198	0.039	0.053	0.751	0.566	0.605
		Ridge	0.099	0.158	0.035	0.338	0.362	0.245	**0.280**
	50	TSLS	-0.000	0.113	0.013	0.005	0.402	0.162	0.175
		Ridge	0.057	0.104	0.014	0.239	0.264	0.127	**0.141**
	250	TSLS	-0.001	0.045	0.002	-0.000	0.130	0.017	**0.019**
		Ridge	0.014	0.045	0.002	0.088	0.147	0.029	0.032
	500	TSLS	0.000	0.032	0.001	-0.000	0.091	0.008	**0.009**
		Ridge	0.009	0.032	0.001	0.057	0.108	0.015	0.016
1.0	25	TSLS	-0.002	0.163	0.026	-0.004	0.244	0.060	**0.086**
		Ridge	0.090	0.164	0.035	0.144	0.221	0.070	0.105
	50	TSLS	0.000	0.106	0.011	0.001	0.153	0.023	**0.035**
		Ridge	0.054	0.107	0.014	0.095	0.155	0.033	0.047
	250	TSLS	-0.001	0.045	0.002	-0.000	0.064	0.004	**0.006**
		Ridge	0.018	0.048	0.003	0.035	0.073	0.007	0.009
	500	TSLS	0.000	0.032	0.001	-0.000	0.045	0.002	**0.003**
		Ridge	0.013	0.034	0.001	0.024	0.053	0.003	0.005

1.3. Training Sample Proportion $\tau = 0.9$

Table S3. Estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ using TSLS and ridge estimator for $\beta^p = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})'$. The proportion of sample used in the training sample is set to $\tau = 0.9$. The estimator with lower combined MSE value is highlighted in bold.

			$\hat{\beta}_1$			$\hat{\beta}_2$			$(\hat{\beta}_1, \hat{\beta}_2)$
δ	n	Estimator	Bias	SD	MSE	Bias	SD	MSE	MSE
0.10	25	TSLS	0.013	0.230	0.053	0.633	1.511	2.683	2.736
		Ridge	0.244	0.206	0.102	0.685	0.257	0.535	**0.637**
	50	TSLS	0.006	0.189	0.036	0.546	1.429	2.340	2.376
		Ridge	0.161	0.165	0.053	0.670	0.293	0.535	**0.588**
	250	TSLS	-0.000	0.081	0.007	0.201	1.513	2.331	2.338
		Ridge	0.046	0.060	0.006	0.558	0.313	0.410	**0.416**
	500	TSLS	-0.000	0.041	0.002	0.062	0.757	0.576	0.578
		Ridge	0.030	0.042	0.003	0.496	0.268	0.318	**0.321**
0.25	25	TSLS	0.008	0.216	0.047	0.323	1.162	1.455	1.502
		Ridge	0.247	0.211	0.105	0.618	0.260	0.449	**0.555**
	50	TSLS	0.002	0.147	0.022	0.177	1.082	1.202	1.224
		Ridge	0.166	0.162	0.054	0.565	0.265	0.389	**0.443**
	250	TSLS	-0.001	0.047	0.002	-0.002	0.298	0.089	**0.091**
		Ridge	0.046	0.060	0.006	0.371	0.241	0.196	0.201
	500	TSLS	0.000	0.032	0.001	0.000	0.188	0.035	**0.036**
		Ridge	0.027	0.042	0.002	0.281	0.231	0.132	0.135
0.50	25	TSLS	0.002	0.198	0.039	0.055	0.744	0.556	0.596
		Ridge	0.253	0.218	0.111	0.494	0.249	0.306	**0.417**
	50	TSLS	0.000	0.112	0.013	0.005	0.402	0.162	**0.174**
		Ridge	0.174	0.168	0.058	0.420	0.234	0.232	0.290
	250	TSLS	-0.001	0.045	0.002	-0.001	0.131	0.017	**0.019**
		Ridge	0.041	0.060	0.005	0.189	0.183	0.069	0.074
	500	TSLS	0.000	0.032	0.001	0.000	0.091	0.008	**0.009**
		Ridge	0.023	0.040	0.002	0.124	0.144	0.036	0.038
1.00	25	TSLS	-0.002	0.163	0.026	-0.004	0.244	0.059	**0.086**
		Ridge	0.252	0.225	0.114	0.330	0.241	0.167	0.282
	50	TSLS	-0.000	0.106	0.011	0.001	0.153	0.023	**0.035**
		Ridge	0.168	0.172	0.058	0.242	0.204	0.100	0.158
	250	TSLS	-0.001	0.045	0.002	-0.000	0.064	0.004	**0.006**
		Ridge	0.042	0.062	0.006	0.075	0.097	0.015	0.021
	500	TSLS	0.000	0.032	0.001	-0.000	0.045	0.002	**0.003**
		Ridge	0.027	0.044	0.003	0.049	0.071	0.007	0.010

2. Prior 2: $\beta^p = (\sqrt{2}, \sqrt{2})'$

2.1. Training Sample Proportion $\tau = 0.5$

Table S4. Estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ using TSLS and ridge estimator for $\beta^p = (\sqrt{2}, \sqrt{2})'$. The proportion of sample used in the training sample is set to $\tau = 0.5$. The estimator with lower combined MSE value is highlighted in bold.

			$\hat{\beta}_1$			$\hat{\beta}_2$			$(\hat{\beta}_1, \hat{\beta}_2)$
δ	n	Estimator	Bias	SD	MSE	Bias	SD	MSE	MSE
0.10	25	TSLS	0.013	0.231	0.053	0.630	1.511	2.680	2.733
		Ridge	0.060	0.173	0.033	0.866	0.799	1.388	**1.421**
	50	TSLS	0.006	0.189	0.036	0.546	1.434	2.354	2.390
		Ridge	0.038	0.110	0.013	0.844	0.812	1.370	**1.384**
	250	TSLS	-0.000	0.081	0.007	0.203	1.514	2.333	2.340
		Ridge	0.009	0.047	0.002	0.557	0.733	0.848	**0.850**
	500	TSLS	-0.000	0.041	0.002	0.060	0.763	0.585	0.587
		Ridge	0.005	0.031	0.001	0.373	0.527	0.418	**0.419**
0.25	25	TSLS	0.008	0.218	0.047	0.325	1.162	1.456	1.504
		Ridge	0.055	0.179	0.035	0.624	0.728	0.919	**0.954**
	50	TSLS	0.002	0.147	0.022	0.178	1.081	1.201	1.223
		Ridge	0.031	0.102	0.011	0.477	0.578	0.561	**0.573**
	250	TSLS	-0.001	0.047	0.002	-0.002	0.298	0.089	**0.091**
		Ridge	0.004	0.044	0.002	0.130	0.288	0.100	0.102
	500	TSLS	0.000	0.032	0.001	0.000	0.188	0.035	**0.036**
		Ridge	0.003	0.030	0.001	0.082	0.193	0.044	0.045
0.50	25	TSLS	0.002	0.199	0.040	0.052	0.749	0.564	0.604
		Ridge	0.046	0.161	0.028	0.277	0.505	0.332	**0.360**
	50	TSLS	-0.000	0.113	0.013	0.005	0.403	0.162	0.175
		Ridge	0.022	0.101	0.011	0.157	0.321	0.128	**0.138**
	250	TSLS	-0.001	0.045	0.002	-0.001	0.130	0.017	**0.019**
		Ridge	0.006	0.044	0.002	0.051	0.136	0.021	0.023
	500	TSLS	0.000	0.032	0.001	-0.000	0.091	0.008	**0.009**
		Ridge	0.005	0.031	0.001	0.035	0.097	0.011	0.012
1.00	25	TSLS	-0.002	0.162	0.026	-0.004	0.244	0.060	0.086
		Ridge	0.039	0.157	0.026	0.075	0.230	0.058	**0.084**
	50	TSLS	-0.000	0.106	0.011	0.001	0.153	0.023	**0.035**
		Ridge	0.026	0.103	0.011	0.052	0.150	0.025	0.037
	250	TSLS	-0.001	0.045	0.002	-0.000	0.064	0.004	**0.006**
		Ridge	0.010	0.046	0.002	0.021	0.068	0.005	0.007
	500	TSLS	0.000	0.032	0.001	0.000	0.045	0.002	**0.003**
		Ridge	0.008	0.033	0.001	0.015	0.048	0.003	0.004

2.2. *Training Sample Proportion* $\tau = 0.7$

Table S5. Estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ using TSLS and ridge estimator for $\beta^p = (\sqrt{2}, \sqrt{2})'$. The proportion of sample used in the training sample is set to $\tau = 0.7$. The estimator with lower combined MSE value is highlighted in bold.

			$\hat{\beta}_1$			$\hat{\beta}_2$			$(\hat{\beta}_1, \hat{\beta}_2)$
δ	n	Estimator	Bias	SD	MSE	Bias	SD	MSE	MSE
0.10	25	TSLS	0.012	0.234	0.055	0.628	1.517	2.696	2.751
		Ridge	0.098	0.176	0.041	0.935	0.648	1.294	**1.335**
	50	TSLS	0.006	0.190	0.036	0.547	1.426	2.332	2.368
		Ridge	0.057	0.126	0.019	0.904	0.751	1.381	**1.401**
	250	TSLS	-0.000	0.081	0.007	0.203	1.515	2.335	2.342
		Ridge	0.015	0.068	0.005	0.646	1.089	1.604	**1.609**
	500	TSLS	-0.000	0.041	0.002	0.056	0.758	0.578	0.580
		Ridge	0.008	0.033	0.001	0.467	0.504	0.473	**0.474**
0.25	25	TSLS	0.008	0.216	0.047	0.327	1.167	1.469	1.516
		Ridge	0.093	0.179	0.041	0.720	0.655	0.947	**0.988**
	50	TSLS	0.002	0.147	0.021	0.178	1.084	1.207	1.228
		Ridge	0.051	0.110	0.015	0.581	0.527	0.615	**0.630**
	250	TSLS	-0.001	0.047	0.002	-0.002	0.297	0.088	**0.091**
		Ridge	0.010	0.043	0.002	0.224	0.302	0.141	0.143
	500	TSLS	0.000	0.032	0.001	0.001	0.188	0.035	**0.036**
		Ridge	0.005	0.030	0.001	0.138	0.213	0.065	0.065
0.50	25	TSLS	0.001	0.200	0.040	0.053	0.751	0.567	0.607
		Ridge	0.082	0.175	0.037	0.390	0.478	0.381	**0.418**
	50	TSLS	0.000	0.112	0.013	0.005	0.403	0.162	**0.175**
		Ridge	0.045	0.105	0.013	0.258	0.329	0.175	0.188
	250	TSLS	-0.001	0.045	0.002	-0.001	0.131	0.017	**0.019**
		Ridge	0.011	0.044	0.002	0.086	0.149	0.030	0.032
	500	TSLS	0.000	0.032	0.001	0.000	0.091	0.008	**0.009**
		Ridge	0.008	0.032	0.001	0.057	0.108	0.015	0.016
1.00	25	TSLS	-0.002	0.163	0.026	-0.004	0.244	0.060	**0.086**
		Ridge	0.080	0.168	0.035	0.143	0.244	0.080	0.115
	50	TSLS	0.000	0.106	0.011	0.001	0.153	0.023	**0.035**
		Ridge	0.049	0.108	0.014	0.093	0.160	0.034	0.048
	250	TSLS	-0.001	0.045	0.002	-0.000	0.064	0.004	**0.006**
		Ridge	0.017	0.048	0.003	0.035	0.073	0.007	0.009
	500	TSLS	0.000	0.032	0.001	-0.000	0.045	0.002	**0.003**
		Ridge	0.012	0.034	0.001	0.024	0.053	0.003	0.005

2.3. Training Sample Proportion $\tau = 0.9$

Table S6. Estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ using TSLS and ridge estimator for $\beta^p = (\sqrt{2}, \sqrt{2})'$. The proportion of sample used in the training sample is set to $\tau = 0.9$. The estimator with lower combined MSE value is highlighted in bold.

			$\hat{\beta}_1$			$\hat{\beta}_2$			$(\hat{\beta}_1, \hat{\beta}_2)$
δ	n	Estimator	Bias	SD	MSE	Bias	SD	MSE	MSE
0.10	25	TSLS	0.012	0.232	0.054	0.630	1.519	2.705	2.759
		Ridge	0.216	0.285	0.127	1.020	0.503	1.293	**1.420**
	50	TSLS	0.006	0.189	0.036	0.548	1.433	2.353	2.389
		Ridge	0.140	0.218	0.067	0.979	0.503	1.210	**1.277**
	250	TSLS	-0.000	0.081	0.007	0.203	1.516	2.340	2.346
		Ridge	0.034	0.068	0.006	0.767	0.525	0.863	**0.869**
	500	TSLS	-0.000	0.041	0.002	0.060	0.762	0.585	**0.587**
		Ridge	0.020	0.045	0.002	0.650	0.468	0.642	0.644
0.25	25	TSLS	0.008	0.216	0.047	0.326	1.161	1.454	1.500
		Ridge	0.211	0.278	0.122	0.874	0.510	1.024	**1.146**
	50	TSLS	0.002	0.147	0.022	0.178	1.084	1.206	1.227
		Ridge	0.133	0.202	0.059	0.769	0.472	0.814	**0.873**
	250	TSLS	-0.001	0.047	0.002	-0.002	0.297	0.088	**0.090**
		Ridge	0.032	0.056	0.004	0.457	0.359	0.338	0.342
	500	TSLS	0.000	0.032	0.001	-0.000	0.188	0.035	**0.036**
		Ridge	0.018	0.035	0.002	0.323	0.297	0.193	0.194
0.50	25	TSLS	0.002	0.199	0.040	0.053	0.749	0.564	**0.603**
		Ridge	0.215	0.265	0.117	0.636	0.431	0.590	0.706
	50	TSLS	-0.000	0.113	0.013	0.004	0.403	0.162	**0.175**
		Ridge	0.145	0.193	0.058	0.524	0.373	0.413	0.471
	250	TSLS	-0.001	0.045	0.002	-0.001	0.131	0.017	**0.019**
		Ridge	0.033	0.053	0.004	0.205	0.210	0.086	0.090
	500	TSLS	0.000	0.032	0.001	-0.000	0.091	0.008	**0.009**
		Ridge	0.019	0.036	0.002	0.127	0.151	0.039	0.041
1.00	25	TSLS	-0.002	0.163	0.026	-0.003	0.244	0.060	**0.086**
		Ridge	0.240	0.261	0.126	0.369	0.323	0.241	0.367
	50	TSLS	0.000	0.106	0.011	0.001	0.153	0.023	**0.035**
		Ridge	0.159	0.191	0.062	0.262	0.252	0.132	0.194
	250	TSLS	-0.001	0.045	0.002	-0.000	0.064	0.004	**0.006**
		Ridge	0.040	0.060	0.005	0.076	0.100	0.016	0.021
	500	TSLS	0.000	0.032	0.001	-0.000	0.045	0.002	**0.003**
		Ridge	0.026	0.043	0.003	0.050	0.072	0.008	0.010

3. Prior 3: $\beta^p = (\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}})'$

3.1. Training Sample Proportion $\tau = 0.5$

Table S7. Estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ using TSLS and ridge estimator for $\beta^p = (\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}})'$. The proportion of sample used in the training sample is set to $\tau = 0.5$. The estimator with lower combined MSE value is highlighted in bold.

			$\hat{\beta}_1$			$\hat{\beta}_2$			$(\hat{\beta}_1, \hat{\beta}_2)$
δ	n	Estimator	Bias	SD	MSE	Bias	SD	MSE	MSE
0.10	25	TSLS	0.012	0.230	0.053	0.630	1.499	2.645	2.698
		Ridge	0.059	0.198	0.043	0.969	0.998	1.935	**1.978**
	50	TSLS	0.005	0.190	0.036	0.543	1.429	2.338	2.374
		Ridge	0.037	0.132	0.019	0.939	0.994	1.869	**1.888**
	250	TSLS	-0.000	0.081	0.007	0.202	1.514	2.333	2.339
		Ridge	0.007	0.056	0.003	0.576	0.911	1.162	**1.165**
	500	TSLS	-0.000	0.041	0.002	0.059	0.763	0.585	0.587
		Ridge	0.004	0.034	0.001	0.373	0.638	0.545	**0.546**
0.25	25	TSLS	0.008	0.217	0.047	0.324	1.161	1.453	1.500
		Ridge	0.051	0.198	0.042	0.676	0.877	1.227	**1.269**
	50	TSLS	0.002	0.147	0.022	0.176	1.081	1.200	1.221
		Ridge	0.028	0.117	0.014	0.505	0.820	0.928	**0.942**
	250	TSLS	-0.001	0.047	0.002	-0.002	0.297	0.088	**0.090**
		Ridge	0.004	0.044	0.002	0.126	0.293	0.101	0.103
	500	TSLS	0.000	0.032	0.001	0.000	0.188	0.035	**0.036**
		Ridge	0.003	0.030	0.001	0.080	0.193	0.044	0.044
0.50	25	TSLS	0.002	0.199	0.040	0.054	0.746	0.559	0.599
		Ridge	0.043	0.188	0.037	0.284	0.602	0.443	**0.480**
	50	TSLS	0.000	0.112	0.013	0.005	0.401	0.161	0.173
		Ridge	0.021	0.104	0.011	0.153	0.346	0.143	**0.154**
	250	TSLS	-0.001	0.045	0.002	-0.001	0.131	0.017	**0.019**
		Ridge	0.006	0.044	0.002	0.050	0.136	0.021	0.023
	500	TSLS	0.000	0.032	0.001	-0.000	0.091	0.008	**0.009**
		Ridge	0.005	0.031	0.001	0.035	0.097	0.011	0.012
1.00	25	TSLS	-0.002	0.163	0.026	-0.004	0.244	0.059	**0.086**
		Ridge	0.037	0.162	0.028	0.072	0.242	0.064	0.091
	50	TSLS	-0.000	0.106	0.011	0.001	0.153	0.023	**0.035**
		Ridge	0.025	0.104	0.011	0.051	0.151	0.025	0.037
	250	TSLS	-0.001	0.045	0.002	-0.000	0.064	0.004	**0.006**
		Ridge	0.010	0.046	0.002	0.021	0.068	0.005	0.007
	500	TSLS	0.000	0.032	0.001	-0.000	0.045	0.002	**0.003**
		Ridge	0.008	0.033	0.001	0.015	0.048	0.003	0.004

3.2. Training Sample Proportion $\tau = 0.7$

Table S8. Estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ using TSLS and ridge estimator for $\beta^p = (\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}})'$. The proportion of sample used in the training sample is set to $\tau = 0.7$. The estimator with lower combined MSE value is highlighted in bold.

			$\hat{\beta}_1$			$\hat{\beta}_2$			$(\hat{\beta}_1, \hat{\beta}_2)$
δ	n	Estimator	Bias	SD	MSE	Bias	SD	MSE	MSE
0.10	25	TSLS	0.012	0.232	0.054	0.629	1.516	2.693	2.747
		Ridge	0.090	0.222	0.058	1.050	0.895	1.903	**1.961**
	50	TSLS	0.006	0.189	0.036	0.546	1.425	2.328	2.363
		Ridge	0.051	0.152	0.026	1.000	0.972	1.944	**1.970**
	250	TSLS	-0.000	0.081	0.007	0.205	1.511	2.325	2.332
		Ridge	0.012	0.074	0.006	0.686	1.196	1.902	**1.908**
	500	TSLS	-0.000	0.041	0.002	0.058	0.764	0.588	**0.589**
		Ridge	0.006	0.035	0.001	0.489	0.614	0.615	0.617
0.25	25	TSLS	0.008	0.216	0.047	0.324	1.160	1.451	1.498
		Ridge	0.085	0.217	0.054	0.786	0.850	1.340	**1.394**
	50	TSLS	0.002	0.147	0.022	0.178	1.082	1.202	1.223
		Ridge	0.046	0.127	0.018	0.629	0.718	0.910	**0.928**
	250	TSLS	-0.001	0.047	0.002	-0.003	0.297	0.088	**0.091**
		Ridge	0.008	0.042	0.002	0.225	0.318	0.151	0.153
	500	TSLS	0.000	0.032	0.001	-0.000	0.188	0.035	**0.036**
		Ridge	0.005	0.030	0.001	0.136	0.215	0.065	0.066
0.50	25	TSLS	0.002	0.199	0.040	0.053	0.748	0.562	0.602
		Ridge	0.077	0.190	0.042	0.412	0.561	0.485	**0.527**
	50	TSLS	-0.000	0.113	0.013	0.005	0.402	0.161	**0.174**
		Ridge	0.041	0.108	0.013	0.263	0.364	0.201	0.215
	250	TSLS	-0.001	0.045	0.002	-0.001	0.130	0.017	**0.019**
		Ridge	0.011	0.044	0.002	0.086	0.149	0.030	0.032
	500	TSLS	0.000	0.032	0.001	-0.000	0.091	0.008	**0.009**
		Ridge	0.008	0.031	0.001	0.056	0.108	0.015	0.016
1.0	25	TSLS	-0.002	0.162	0.026	-0.003	0.244	0.060	**0.086**
		Ridge	0.076	0.171	0.035	0.142	0.256	0.086	0.121
	50	TSLS	0.000	0.106	0.011	0.001	0.153	0.023	**0.035**
		Ridge	0.048	0.108	0.014	0.093	0.162	0.035	0.049
	250	TSLS	-0.001	0.045	0.002	-0.000	0.064	0.004	**0.006**
		Ridge	0.017	0.048	0.003	0.034	0.074	0.007	0.009
	500	TSLS	0.000	0.032	0.001	-0.000	0.045	0.002	**0.003**
		Ridge	0.012	0.034	0.001	0.024	0.053	0.003	0.005

3.3. Training Sample Proportion $\tau = 0.9$

Table S9. Estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ using TSLS and ridge estimator for $\beta^p = (\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}})'$. The proportion of sample used in the training sample is set to $\tau = 0.9$. The estimator with lower combined MSE value is highlighted in bold.

			$\hat{\beta}_1$			$\hat{\beta}_2$			$(\hat{\beta}_1, \hat{\beta}_2)$
δ	n	Estimator	Bias	SD	MSE	Bias	SD	MSE	MSE
0.10	25	TSLS	0.013	0.231	0.053	0.626	1.509	2.670	2.723
		Ridge	0.169	0.332	0.139	1.140	0.797	1.936	**2.075**
	50	TSLS	0.005	0.191	0.036	0.543	1.433	2.350	2.386
		Ridge	0.109	0.263	0.081	1.092	0.814	1.856	**1.937**
	250	TSLS	-0.000	0.081	0.007	0.202	1.514	2.334	2.341
		Ridge	0.025	0.076	0.006	0.821	0.736	1.215	**1.221**
	500	TSLS	-0.000	0.041	0.002	0.061	0.765	0.589	**0.591**
		Ridge	0.014	0.047	0.002	0.689	0.580	0.811	0.813
0.25	25	TSLS	0.007	0.217	0.047	0.324	1.162	1.454	**1.501**
		Ridge	0.167	0.307	0.123	0.953	0.733	1.446	1.569
	50	TSLS	0.002	0.147	0.022	0.179	1.088	1.215	1.237
		Ridge	0.105	0.223	0.061	0.828	0.625	1.076	**1.137**
	250	TSLS	-0.001	0.047	0.002	-0.002	0.298	0.089	**0.091**
		Ridge	0.027	0.058	0.004	0.486	0.425	0.417	0.421
	500	TSLS	0.000	0.032	0.001	-0.000	0.188	0.035	**0.036**
		Ridge	0.015	0.034	0.001	0.333	0.327	0.218	0.219
0.50	25	TSLS	0.002	0.199	0.040	0.054	0.748	0.562	**0.602**
		Ridge	0.185	0.292	0.119	0.678	0.584	0.801	0.920
	50	TSLS	-0.000	0.112	0.013	0.005	0.402	0.162	**0.175**
		Ridge	0.127	0.207	0.059	0.555	0.454	0.514	0.573
	250	TSLS	-0.001	0.045	0.002	-0.001	0.131	0.017	**0.019**
		Ridge	0.031	0.052	0.004	0.209	0.222	0.093	0.097
	500	TSLS	0.000	0.032	0.001	-0.000	0.090	0.008	**0.009**
		Ridge	0.018	0.035	0.002	0.129	0.154	0.040	0.042
1.00	25	TSLS	-0.002	0.162	0.026	-0.003	0.243	0.059	**0.085**
		Ridge	0.229	0.269	0.125	0.379	0.360	0.273	0.398
	50	TSLS	-0.000	0.106	0.011	0.001	0.153	0.023	**0.035**
		Ridge	0.155	0.202	0.065	0.269	0.278	0.150	0.214
	250	TSLS	-0.001	0.045	0.002	-0.000	0.064	0.004	**0.006**
		Ridge	0.039	0.060	0.005	0.076	0.101	0.016	0.021
	500	TSLS	0.000	0.032	0.001	-0.000	0.045	0.002	**0.003**
		Ridge	0.026	0.043	0.003	0.050	0.072	0.008	0.010