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Exploring the Dynamics of ZAR/USD Exchange Rate Volatility Using the fGARCH and First-Order Beta-Skew-T-EGARCH Models

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Abstract

This study investigates and explores the volatility dynamics of the South African rand against the US dollar (ZAR/USD) using the Family GARCH (fGARCH) model and the First-Order Beta-Skew-T-Generalised Autoregressive Conditional Heteroskedasticity (Beta-Skew-T-EGARCH) model. Currency volatility across the globe, uncertainties, and instability in emerging markets have become increasingly consequential for trade flows, investment allocation, and macroeconomic management. The ZAR/USD serves as a benchmark of South Africa's economic wealth and vulnerability to external shocks and is one of the most valued, significant, and heavily traded pairings of emerging market currencies. Simple standard GARCH (sGARCH) is one of the most useful models for exchange rate volatility; however, the sGARCH model has some limitations: it fails to accommodate or allow the long memory effects, skewness distribution, and leverage dynamics consistently observed in emerging-market currency returns. This study addresses these limitations by using the fGARCH model, which includes the most popular GARCH models and Beta-Skew-T-EGARCH for daily ZAR/USD returns ranging from 5 January 2000 to 1 October 2024. Five innovation distributions are used for evaluation and comparison under fGARCH and sGARCH, namely generalised hyperbolic (GH), generalised error (GED), skewed Student's *t* (SSTD), skewed generalised error (SGED), and Student's *t* (STD), with model fitness criteria assessed using the Shibata criterion (SIC), Hannan–Quinn criterion (HQ), Bayesian information criterion (BIC), and Akaike information criterion (AIC), choosing the specification with the lowest overall penalty. It is found that the fGARCH(1,1) model fitted to return-frequency data under the SSTD achieves the lowest AIC, outperforming sGARCH. The study also includes an analysis among covariates, which are day, month, trend, oil, and platinum; the trend variable is a statistically significant predictor, with $p = 0.007$, showing a positive influence on ZAR/USD volatility. The Beta-Skew-T-EGARCH model with two components divides volatility into long-run and short-run components, which is found to deliver a superior fit over the one-component variant, evidenced by a lower BIC (3.068435) and a higher log-likelihood (-748.464826). The two components confirm that the model captures declining conditional volatility, whereas the one-component model sustains persistence in the evaluated estimates.



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1. Introduction

1.1. Overview

The continued instability of the South African rand against the US dollar has placed exchange rate volatility at the centre of economic and financial analysis for policymakers, investors, and market participants operating in and around South Africa (Moyo et al., 2025). For a commodity-exporting economy with a floating exchange rate, movements in the ZAR against reserve currencies directly affect export earnings, import costs, debt-servicing pressures, and the South African Reserve Bank's capacity to maintain its inflation-targeting mandate. The ZAR/USD exchange rate is one of the most actively traded currency pairs in emerging financial markets and a major financial indicator in Africa, particularly in South Africa (López-Herrera et al., 2025). It influences trade competitiveness, global investment flows, inflation, and monetary policy decisions. As South Africa remains strongly integrated into global financial markets, its exchange rate has become highly sensitive to domestic economic conditions and international financial shocks.

Eniayewu et al. (2024) note that the value of the rand and other currencies has fluctuated both domestically and internationally in recent times because of changes in the scale of domestic shocks. This has become a problem and a focal area because the rand is widely traded across the world, often as a currency representing emerging markets. The accurate characterisation of this volatility is therefore not simply an academic exercise but an operational requirement for sound monetary and financial risk management (Acheampong et al., 2020). Currency fluctuations indicate the level of uncertainty surrounding a country's macroeconomic environment and shape investment decisions, trade balances, and growth trajectories. In South Africa specifically, where the economy depends heavily on platinum, gold, and coal exports priced in US dollars, the ZAR/USD rate acts as an amplifier of global commodity price cycles, political risk premiums, and reversals in capital flows. This makes its volatility dynamics qualitatively different from those of advanced-economy currency pairs (Epaphra, 2016). Policymakers rely on rigorous volatility estimates to calibrate monetary responses, allocate foreign exchange reserves effectively, and formulate fiscal strategies that stabilise financial markets against external shocks (Epaphra, 2016).

In quantitative terms, exchange rate volatility refers to the dispersion of currency returns over a specific interval. For ZAR/USD daily returns, this measure became exceptionally large during the 2008 global financial crisis, the 2015 China-induced commodity shock, and the 2020 COVID-19 pandemic. These three structural breaks fall within this study's sample period and provide strict tests of any volatility model's adequacy (Mechri et al., 2018; Mieg, 2022).

Modelling exchange rate dynamics has received sustained attention in international economics, yet the empirical literature presents competing views on the determinants of currency volatility in developing economies. A key area of disagreement concerns the roles of persistent economic shocks, unobserved structural factors, and nonlinear adjustment mechanisms, which standard models may fail to capture (Oyinlola & Adedeji, 2021). Controlling currency volatility is particularly difficult when fluctuations arise from sudden reversals in global risk appetite rather than domestic fundamentals, as is often the case for the ZAR (Acheampong et al., 2020).

Maingo et al. (2025) examine the importance of using complex econometric models, such as the GARCH model, to assess and manage financial market volatility and associated risks. The study applied fGARCH and sGARCH but did not include other models, such as First-Order Beta-T-EGARCH. This study applies both frameworks to daily ZAR/USD returns spanning January 2000 to October 2024, with the following specific objectives: to characterise the volatility persistence, asymmetric shock responses, and distributional properties of ZAR/USD returns under the fGARCH family across five innovation distri-

butions; to assess covariate effects of calendar variables, oil prices, and platinum prices on conditional volatility; and to determine whether the additional parameterisation of the two-component Beta-Skew-T-EGARCH model justifies its complexity relative to the one-component variant, using likelihood-based model selection criteria.

Despite the extensive work in the literature based on volatility exchange rate modelling, limited studies have compared the performance of fGARCH without First-Order Beta-Skew-TGARCH models in modelling and forecasting ZAR/USD exchange rate volatility, although [Liao et al. \(2020\)](#) conducted research on hybrid neural network Markov switching Beta-t-EGARCH exchange rate volatility forecasting, and [Maingo et al. \(2025\)](#) modelled the volatility of the Johannesburg stock market using the Family GARCH model for the All Share Index. The comparison undertaken in this study contributes to the organisation of the literature by evaluating how well the selected models capture asymmetry, heavy tails, and volatility persistence.

The findings provide useful insights for researchers, policymakers, markets, and financial market participants concerned with exchange rate risk management. The selection of the First-Order Beta-Skew-T-EGARCH and fGARCH models is motivated by both the empirical properties of the ZAR/USD exchange rate returns and the limitations of simpler volatility modelling approaches. Preliminary analysis shows pronounced volatility clustering, high persistence, asymmetry, and heavy-tailed behaviour in the return distribution. These features cannot be adequately captured by constant-variance models or symmetric GARCH specifications with normally distributed innovations.

This paper's remaining sections are structured as follows. A summary of the relevant theoretical and empirical literature is presented in Section 1.2, and the research hypotheses are presented in Section 1.3. The econometric approach used in the analysis is discussed in Section 2, together with a description of the data. Section 3 outlines the results. Section 4 covers the implications of the results regarding the aims of the study and the current body of literature. The paper concludes in Section 5, which summarises the main findings, emphasises their ramifications, and offers recommendations for further research.

1.2. Literature Review

The ZAR/USD exchange rate experiences high volatility due to economic, political, and global factors. This instability creates challenges for investors, policymakers, and businesses involved in international trade, as it affects financial planning, hedging strategies, and overall economic stability. Many currencies have shown greater stability than the ZAR/USD exchange rate ([Rapoo et al., 2021](#)). Despite the importance of the ZAR/USD exchange rate, existing models often fall short in capturing its complex volatility dynamics. Traditional econometric models often struggle to capture the complexities of exchange rate dynamics, particularly asymmetry, long memory, and heavy tails in financial time series data.

In modern financial theory, uncertainty is regarded as a fundamental factor that shapes financial markets and investment decisions. Recently, most asset-pricing models have suggested that the risk premium depends on the covariance between an asset's expected future return and one or more benchmark portfolios, such as the market portfolio or the growth rate of consumption ([Chou & Kroner, 1992](#)). Over the years, volatility forecasting has gained significance in financial literature reviews because of the global financial crisis, and the foreign exchange market has emerged as highly volatile and incredibly liquid compared to other financial markets globally ([Benlagha & El Omari, 2022](#)).

Financial time series forecasting has evolved from classical linear models, such as the Autoregressive (AR), Moving Average (MA), Autoregressive Moving Average (ARMA), and Autoregressive Integrated Moving Average (ARIMA) families [Makridakis et al. \(1982\)](#),

toward nonlinear specifications that explicitly model time-varying variance. Building on Engle (1982)'s Autoregressive Conditional Heteroskedasticity (ARCH) framework, which permits the conditional variance to evolve as a function of past squared residuals, Bollerslev (1986) proposed the Generalised ARCH (GARCH) extension. By adding both lagged squared residuals and lagged conditional variances to the variance equation, GARCH enhances it and allows for a more economical and empirically robust representation of the volatility clustering, leptokurtosis, and persistence that characterise high-frequency financial returns (Chou & Kroner, 1992). GARCH-family models have since become the standard workhorses for volatility measurement in equity, commodity, and foreign exchange markets globally (Benlagha & El Omari, 2022).

Forecasting involves designing a mathematical model to predict the stock's future value and trend. Anticipating the future worth of stock enables an investor to arrive at informed conclusions about whether or not to purchase, sell, or retain it (Liahmad et al., 2021). Forecasting financial time series has been a persistent challenge and is a field of significant study within the finance industry. The GARCH model, as introduced by Bollerslev (1986), provides an important foundation for the econometric modelling of time-varying volatility. The fGARCH-family models have been extensively applied to emerging-market exchange rates, although applications to the ZAR/USD specifically using the full fGARCH framework are limited.

An investigation of the use of monetary fundamentals to forecast exchange rate volatility was carried out by Eniayewu et al. (2024): the usefulness of monetary fundamentals (growth in the money supply and interest rates) in forecasting daily fluctuations in exchange rates in Nigeria and South Africa was investigated using a GARCH-MIDAS approach. The GARCH-MIDAS model was used based on the data at hand. Monetary growth has an impact on the South African exchange rate, and interest rates are a major factor in explaining exchange rate volatility. The analysis shows that monetary fundamentals have a major role in influencing exchange rate volatility and finds that exchange rate volatility persists over the long term in both nations.

Celestin et al. (2025) investigated the use of GARCH models for volatility forecasting in high-frequency trading, utilising data spanning from 2020 to 2024, while maintaining a focus on volatility persistence. The study's primary goal was to evaluate how well traditional and improved GARCH variants, such as EGARCH and TGARCH, capture the dynamics of market volatility. The results also point to the persistence of volatility over time. Similarly, using Bayesian SARMA-GARCH with Skewed Error Distributions, Mosanawe and Makatjane (2026) examined dynamic volatility and tail risk in the BTC, BWP, and ZAR exchange rates and found that ZAR/USD demonstrates highly persistent volatility; however, a comparison between the fGARCH and other models, like First-Order Beta-T-EGARCH, has not yet been conducted.

Gbolagade et al. (2022) examined how effective GARCH models are in capturing the dynamics of volatility associated with Brent crude oil prices. The research utilised secondary data sourced from investing.com, ranging from 1 January 2010 to 27 April 2022. In order to choose the most effective model for predicting future volatility, GARCH models were utilised to assess fluctuations in the Brent crude oil prices. Several distributions were used to evaluate the goodness of fit and stationarity of the daily prices and returns, including Gaussian and generalised error distributions and Student's *t*.

The analysis indicated that the price of crude oil experienced a decline from 2014 to 2016, followed by a substantial decrease in 2020 due to COVID-19. With a Student's *t*-distribution, the exponential GARCH(1,1) model had the lowest BIC, AIC, and HQIC values, according to the results, suggesting that it was the best model based on these

metrics. As a result, the study suggests predicting crude oil stock price volatility using the exponential GARCH model.

Dana (2016) used the ARCH/GARCH family of models to estimate stock return volatility in the Jordanian stock market. The study focused on identifying key aspects of financial returns, including clustering volatility, leptokurtosis, and the leverage effect, by analysing daily returns from the Amman Stock Exchange (ASE) over the period of 1 January 2005 to 31 December 2014. Both symmetric models (GARCH and ARCH) and the asymmetric EGARCH model were estimated to examine the dynamics of return volatility. Robust evidence of persistent clustering volatility and leptokurtic return innovations was provided by the empirical results, which demonstrated that the symmetric ARCH/GARCH specifications effectively described the volatility behaviour of the ASE. Nevertheless, there was no statistically significant evidence of an asymmetry or leverage effect in the market found using the EGARCH model.

Al-Besher and Al-Najjar (2026) investigated the forecasting performance of several GARCH-family models and a long short-term memory (LSTM) neural network in modelling the returns and volatility of the Tadawul All Share Index over the period 2000–2022. Model accuracy was evaluated using forecasting measures such as the root mean square error (RMSE), mean absolute error (MAE), and related performance metrics. The findings revealed that the LSTM model was more successful in capturing the nonlinear behaviour of market volatility, whereas the GJR-GARCH model produced the most accurate return forecasts among the conventional GARCH specifications.

Sucarrat (2013) notes that the Beta-Skew-T-EGARCH model remains an underexplored volatility model in the literature. Exploring the dynamics of the ZAR/USD using the fGARCH and First-Order Beta-Skew-T-EGARCH contributes largely to the existing literature. The Beta-Skew-T-EGARCH model's conditional volatility accommodates changes in the market while also managing outliers in the data (Sucarrat, 2013). In financial time series, most volatility models fail to capture the heavy tails that are found in different exchange rates (Nugroho et al., 2025). This indicates that a comparison of fGARCH and Beta-Skew-T-EGARCH is necessary to capture volatility and provide insights into how to manage and handle the persistent volatility as well as uncertainties.

Blazsek and Villatoro (2015) compared GARCH and Beta-T-EGARCH over three sub-periods around the 2008 financial crisis(US) and found that the Beta-T-EGARCH specification provided superior out-of-sample density forecasts in the post-crisis period, even when in-sample fit statistics were comparable. This evidence is directly relevant to ZAR/USD modelling: the 2008 global financial crisis constitutes the most severe volatility episode in our sample, and a model that performs better in post-crisis periods offers more reliable forward-looking risk estimates.

A. C. Harvey (2008) expanded conditional score models to accommodate skew distributions, as financial returns often display skewness and fat-tailed distributions. This model combines the conditional distribution's skewness with a leverage effect in the scale's dynamic equation (Muela, 2015). To capture the dynamics of volatility often observed in financial time series, an EGARCH model consisting of heavy-tailed as well as skewed conditional distributions was developed by (A. Harvey & Sucarrat, 2014). Empirical results indicate skewness in the conditional t-distribution across multiple return series, and the proposed model generally outperforms comparable skewed-t GARCH models in terms of model fit. The two-component model gives a better fit in terms of goodness of fit.

Bekar (2023) examined exchange rate risk measurement using daily USD/TRY data from 2005 to 2021. The GJR-GARCH(1,1), EGARCH(1,1), and recently developed Beta-T-EGARCH variants were compared in the study. The findings revealed that the two-component Beta-Skew-T-EGARCH models outperformed alternative specifications, exhibit-

ing a stronger capacity to represent the complex dynamics of exchange rate volatility. The study concluded that skewness, leverage effects, and multiple components lead to more accurate volatility estimation and risk assessment.

The findings of Bekar (2023) provide empirical support for using the Beta-Skew-T-EGARCH model in exchange rate volatility. While the cited study focused on USD/TRY exchange rate volatility, the present study extends this line of research by exploring the dynamics of ZAR/USD using the fGARCH model, because it includes most of the GARCH models and the First-Order Beta-T-EGARCH model.

1.3. Research Hypotheses

Volatility Persistence: Recent studies on exchange rate volatility have found that there is high persistence in volatility. See Maingo et al. (2025), Agara (2026) and Mosanawe and Makatjane (2026).

H1. *The ZAR/USD exchange rate exhibits significant volatility persistence.*

Asymmetric Effects: Studies show that behavioural finance theory and empirical evidence suggest that exchange rates may react to positive and negative shocks in different ways. See Sun (2025) and Kayani et al. (2023).

H2. *The ZAR/USD exchange rate exhibits asymmetric responses to positive and negative shocks.*

H3. *Model Performance: First-Order Beta T-EGARCH provides superior volatility forecasting to fGARCH models.*

2. Methodology

2.1. Data Collection and Processing

The study uses historical rate data from the South African Reserve Bank, which can be accessed at (<https://www.resbank.co.za/en/home/what-we-do/statistics/key-statistics/selected-historical-rates>, accessed on 9 January 2025). The data are sampled at a daily frequency, excluding weekends and public holidays, resulting in observations from 5 January 2000 to 1 October 2024. To obtain the return series, the first difference of the natural logarithm of the exchange rate prices was calculated and then multiplied by 100 to obtain percentage returns.

2.2. Standard GARCH Model

Engle (1982) introduced the ARCH model to comprehend and predict future values based on historical values and past forecast errors. The ARCH model has been critical in this progression because it appears that no structural dynamic economic theory can explain higher-order moments of variation (Chou & Kroner, 1992). By distinguishing between unconditional second-order moments and conditional ones, the ARCH model provides important information, surpassing ordinary linear time series models that rely on the conditional versus unconditional mean. The literature on ARCH-type models is extensive, and the econometric modelling of financial data has garnered significant attention for more than 20 years (Giraitis et al., 2007).

The ARMA-ALLGARCH models are initially fitted to the exchange rate return series data to filter out the ARCH effects before fitting the copula models. Let x_t denote the current exchange rate on day t and x_{t-1} denote the lagged exchange rate on day $t - 1$. r_t denotes the return series, given as

$$r_t = \nabla \log x_t = (1 - B) \log x_t = \log \frac{x_t}{x_{t-1}} \quad (1)$$

A general model is (the mean equation)

$$r_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim \text{Distribution}(0, \sigma^2), \quad \mathbb{E}(\varepsilon_t | \mathcal{F}_{t-1}) = 0, \quad (2)$$

$$\text{with } \mu_t = \sum_{i=1}^p \phi_i r_{t-i} + \sum_{i=1}^q \theta_i \varepsilon_{t-i},$$

where ϕ and θ are parameters to be estimated and ε_t is an error term. The parameter p indicates the autoregressive order, reflecting how past returns influence current returns, while q denotes the moving average order, capturing the impact of past shocks on present returns. Together, these parameters are vital for modelling return dynamics.

Engle (1982) developed ARCH(q) as one possible parameterisation for σ_t^2 to display σ_t^2 as a linear function of past squared values of the process, which takes the following equation:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 = \omega + \alpha(L) \varepsilon_t^2, \quad (3)$$

where $\omega > 0$ is a term that is constant and $\alpha_i \geq 0$ reflects the immediate impact of a shock, while L denotes the lag operation. The financial ARCH(q) model represents the volatility clustering tendency (Chou & Kroner, 1992).

The sGARCH model, as an extension by Bollerslev (1986), is given by

$$\sigma_t^2 = (\omega + \sum_{j=1}^m \zeta_j v_{j-t}) + \sum_{j=1}^q \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2, \quad (4)$$

where the conditional variance is denoted by σ_t^2 , ω is the intercept or constant term, and ε_t^2 is the residual from the mean filtration process or procedure, while ζ_j is the coefficient associated with the exogenous variables or external shocks. Meanwhile, β_j represents volatility persistence, and α_j depicts the short-run impact of shocks. The GARCH order is determined by the (q , p) parameters, representing the GARCH and ARCH models. Additionally, m for pre-lagged external regressors v_j is included (Ghalanos, 2020).

2.3. Family GARCH Model

Hentschel (1995) established the fGARCH model, which is an encompassing model that accommodates various essential symmetric and asymmetric GARCH models as sub-models (Samuel et al., 2023), emphasising their treatment of asymmetry (Hentschel, 1995). This study utilises the fGARCH model because it encompasses many GARCH-type models.

Several of the most well-known GARCH model types, such as Absolute Value GARCH (AVGARCH), Standard GARCH (sGARCH), Exponential GARCH (EGARCH), Nonlinear Asymmetric GARCH (NAGARCH), and Asymmetric Power ARCH (apARCH), are included in the fGARCH model (Ghalanos, 2020). The robust fGARCH(u, v) model is given as

$$\sigma_t^\gamma = \omega + \sum_{j=1}^v \alpha_j \sigma_{t-j}^\gamma (|z_{t-j} - \lambda_{2j}| - \lambda_{1j} \{z_{t-j} - \lambda_{2j}\})^\delta + \sum_{j=1}^u \beta_j \sigma_{t-j}^\gamma. \quad (5)$$

The resilience of the fGARCH model enables various powers of z_t and σ_t to drive the decomposition of the residual's conditional variance equation. Equation (5) shows how the conditional standard deviation is transformed using the Box–Cox method. The absolute value function transforms via δ as the parameter, while γ defines its shape. For asymmetric small shocks, the shifts are determined by λ_{2j} ; for big shocks, the rotations are determined by λ_{1j} . $\gamma = \delta$ can be used to fit the whole fGARCH model, as shown in (Ghalanos, 2020).

The parameter estimate can be used to determine the volatility persistence of the fGARCH model, denoted by $\hat{\rho}$:

$$\hat{\rho} = \sum_{j=1}^q \beta_j + \sum_{j=1}^p \alpha_j \rho_j, \quad (6)$$

where ρ_j , written in Equation (7), represents the projection of z_t associated with the absolute value asymmetry term below the Box–Cox transformation. Persistence is obtained by using the “persistence()” function from the *rugarch1.5 – 5.tar.gz* package (Samuel et al., 2023). For information about nested models and fGARCH, see (Ghalanos, 2020; Hentschel, 1995).

$$\rho_j = \mathbb{E}(|z_{t-j} - \zeta_{2j}| - \zeta_{1j}(z_{t-j} - \zeta_{2j}))^\delta = \int_{-\infty}^{\infty} (|z - \zeta_{2j}| - \zeta_{1j}(z - \zeta_{2j}))^\delta f(z, 0, 1, \dots) dz \quad (7)$$

2.4. Conditional Distributions

2.4.1. Student’s t-Distribution

Bollerslev (1986) proposed the STD as a method to effectively manage extreme leptokurtosis. Its probability density function is

$$f(z) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{\pi v}} \left(1 + \frac{z^2}{v}\right)^{-\frac{v+1}{2}}, \quad (8)$$

where z represents the variable, v stands for the degrees of freedom, and Γ denotes the function of gamma. The STD is centred and symmetric at zero. The distribution approaches normality as the degrees of freedom (v) increase. Distributions with heavier tails are produced by lower values of v , suggesting an increase in extreme values (Dritsaki, 2017).

2.4.2. Skewed Student’s t-Distribution

The SSTD is a modified form of the traditional STD that includes an additional parameter to capture skewness. This parameter allows the distribution to more effectively represent data that are not symmetric. Although its probability density function is mathematically intricate, the distribution can be understood as an adjusted version of the standard Student’s t-distribution that accounts for asymmetry,

$$f(z) = \frac{2}{\sigma B\left(\frac{v}{2}, \frac{1}{2}\right)} \left(1 + \frac{\frac{z-\zeta}{\sigma}}{v}\right)^{-\frac{v+1}{2}}, \quad (9)$$

in which the skewness is determined by ζ , v represents the degrees of freedom, and $B(.,.)$ denotes the Beta function.

2.4.3. Generalised Error Distribution

The exponential power distribution, or GED, is a generalisation of the normal distribution that accommodates various tail thicknesses. It provides flexibility in modelling distributions with lighter or heavier tails by introducing a shape parameter, represented by κ . The following is the expression for the GED’s probability density function:

$$f(z) = \frac{\kappa}{2^{1+1/\kappa}\Gamma(1/\kappa)\sigma} \exp\left(-\frac{1}{2}\left|\frac{z-\mu}{\sigma}\right|^\kappa\right), \quad (10)$$

where the mean is given by μ , the parameter’s scale is represented by σ , and κ denotes the shape parameter, which determines how thick the tails of the distribution are. When $\kappa = 2$, the GED becomes a normal distribution. If κ is less than 2, the tails are thicker, while, if κ is greater than 2, the tails are thinner.

2.4.4. Skewed Generalised Error Distribution

The SGED is an extension of the generalised error distribution (GED) that enables the modelling of both the skewness and kurtosis in financial time series data. This distribution modifies the standard GED to account for asymmetry and variations from conventional models, specifically in terms of excess kurtosis. The SGED's probability density function is as follows:

$$f(z; \mu, \sigma, \lambda, \kappa) = \begin{cases} \frac{c}{\sigma(1+\lambda)} \exp\left[-\frac{1}{2} \left| \frac{z-\mu}{\sigma(1+\lambda)} \right|^\kappa\right], & z < \mu, \\ \frac{c}{\sigma(1-\lambda)} \exp\left[-\frac{1}{2} \left| \frac{z-\mu}{\sigma(1-\lambda)} \right|^\kappa\right], & z \geq \mu, \end{cases} \quad (11)$$

where μ represents the location parameter, which effectively shifts the distribution along the x-axis. The parameter σ serves as the scale parameter, adjusting the distribution's width. Meanwhile, the parameter's scale is denoted by κ , which influences the kurtosis, or "fatness", of the tails' distribution. Finally, the skewness parameter, λ , determines the distribution's asymmetry. Interestingly, the SGED reduces to the typical normal distribution when $\lambda = 0$ and $\kappa = 2$.

2.4.5. Generalised Hyperbolic Distribution

The following equation provides the density function for the generalised hyperbolic distribution (GH):

$$f(x; \lambda, \alpha, \beta, \delta, \mu) = \frac{\alpha e^{\beta(x-\mu)}}{\sqrt{2\pi} \delta^\lambda K_\lambda(\delta \sqrt{\alpha^2 - \beta^2})} \left(\frac{\sqrt{\alpha^2 - \beta^2}}{\sqrt{\delta^2 + (x-\mu)^2}} \right)^{\lambda - \frac{1}{2}} K_{\lambda - \frac{1}{2}} \left(\alpha \sqrt{\delta^2 + (x-\mu)^2} \right), \quad (12)$$

where λ denotes the skewness parameter, the parameter's shape is α , the parameter's scale is β , δ is the parameter's location, and μ is another scale parameter. The modified Bessel function of the second kind refers to $K_\lambda(\alpha\beta)$, specifically of order λ .

2.5. Beta-Skew-T-EGARCH

To account for the volatility or fluctuations in financial returns, the Beta-Skew-T-EGARCH model was developed by [A. Harvey and Sucarrat \(2014\)](#). This approach has merit because it is resistant to volatility spikes and outlier variables. It can handle conditional asymmetry, leverage, and heavy tails ([Müller & Bayer, 2017](#)). Compared to other model types, the model can insert asymmetry and large memory extensions more quickly. A Student-t distribution can describe a heavy-tailed conditional distribution, similarly to the GARCH-t model ([Bollerslev, 1986](#)). In this model, where the dynamic volatility equation uses the conditional score, the Gaussian distribution treats mean data as outliers ([A. Harvey & Sucarrat, 2014](#)). The equation is given by

$$y_t = \exp(\lambda_{t-1}/2) \varepsilon_t, \quad t = 1, \dots, T, \quad (13)$$

where the serially independent variable that has a t_ν -distribution with positive degrees of freedom is denoted by ε_t , ν and $\lambda_{t|t-1}$. Alternatively,

$$y_t = \sigma_{t-1} z_t, \quad \nu > 2, \quad (14)$$

where $z_t = ((\nu - 2)/\nu)^{1/2} \varepsilon_t$ still has a t_ν -distribution, but it is standardised to have unit variance.

The principal feature of the Beta-T-EGARCH model is that λ_{t-1} is a combination of past values of the variable that are linear; see ([A. C. Harvey, 2008](#)).

2.5.1. First-Order Beta-Skew-T-EGARCH

The first-order one-component Beta-Skew-T-EGARCH model is expressed in its martingale difference form as proposed by A. Harvey and Sucarrat (2014). Let y_t represent a financial return at time t , where $t = 1, \dots, n$, and the sample size is represented by n :

$$y_t = \exp(\lambda) \varepsilon_t = \delta_t \varepsilon_t \varepsilon_t \sim st(0, \sigma_\varepsilon^2, \nu, \gamma), \nu > 2 \quad \gamma \in (0, \infty) \quad (15)$$

$$\lambda_t = \omega + \lambda_t^+, \quad (16)$$

$$\lambda^+ = \phi \lambda_{t-1}^+ + \kappa_1 u_{t-1} + \kappa^* \text{sgn}(-y_{t-1})(u_{t-1} + 1) \quad |\phi| < 1. \quad (17)$$

where the volatility of y_t is denoted by σ_t , ε_t is the conditional error, and $\text{sgn}(\cdot)$ is the sign function. The persistence parameter, ϕ , or GARCH parameter, and the intercept of the log-scale, ω , can be regarded as long-term log-volatility; κ_1 represents the ARCH parameter, where a larger value results in a greater response to shocks; γ denotes asymmetry; the leverage parameter is κ^* ; ν represents the degrees of freedom; and u_t defines the conditional score, which is associated with the derivative of the log-density of y_t regarding λ . $\sigma_t \sigma_\varepsilon$ represents the conditional standard deviation, with σ_ε^2 representing the variance of the innovation term ε_t . The log-volatility process, λ_t , is stable if the autoregressive parameter meets the condition $|\phi_1| < 1$.

2.5.2. Two-Component Beta-Skew-T-EGARCH

Two-component volatility models take into account the long-range memory feature by separating volatility into two components: long-term and short-term (Sucarrat, 2013). According to Müller and Bayer (2017), the martingale difference representation of the first-order two-component Beta-Skew-T-EGARCH model is given by

$$y_t = \exp(\lambda_t) \varepsilon_t = \sigma_t \varepsilon_t, \varepsilon_t \sim st(0, \sigma_\varepsilon^2, \nu, \gamma), \nu > 2 \quad \gamma \in (0, \infty) \quad (18)$$

$$\lambda_t^+ = \omega + \lambda_{1,t}^+ + \sigma_{2,t}^+, \quad (19)$$

$$\lambda_{1,t}^+ = \phi_1 \lambda_{1,t-1}^+ + \kappa_1 u_{t-1} + \kappa^* \text{sgn}(-y_{t-1})(u_{t-1} + 1) \quad |\phi| < 1 \quad (20)$$

$$\lambda_{2,t}^+ = \phi_2 \lambda_{2,t-1}^+ + \kappa_2 u_{t-1} \quad |\phi_2| < 1, \phi_1 \neq \phi_2, \quad (21)$$

where the short-term component is denoted as $\lambda_{1,t}^+$, while $\lambda_{2,t}^+$ represents the long-term component. The conditional score u_t accommodates or drives the long- and short-run components; however, in the short-term component, leverage occurs (Sucarrat, 2013). For the short-term component, we use GARCH and ARCH parameters, respectively. ϕ_2 and κ_2 are the long-term component's GARCH and ARCH parameters (Müller & Bayer, 2017). This is consistent with the perspective or view that shocks only have an impact on forecasting volatility in the short-term component (Engle, 1982). However, the model cannot be identified if ϕ_2 equals ϕ_1 .

The model's log-likelihood function is given as

$$\ell(\theta) = \sum_{t=1}^n \ln f_y(y_t), \quad (22)$$

where $\theta = (\phi_1, \phi_2, \gamma, \kappa_2, \nu, \omega, \kappa^*, \kappa_1)$ is the model's parameter vector and $\ln f_y(y_t)$ represents the log-density of y_t (A. Harvey & Sucarrat, 2014):

$$\begin{aligned} \ln f_y(y_t) = & \ln 2 - \ln(\gamma + \gamma^{-1}) + \ln \Gamma\left(\frac{\nu+1}{2}\right) - \frac{1}{2} \ln \pi - \ln \Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2} \ln \nu \\ & - \lambda_{t|t-1} - \frac{\nu+1}{2} \ln \left(1 + \frac{(y_t - \mu)^2}{\gamma^2 \text{sgn}(y_t - \mu) \nu e^{2\lambda_{t|t-1}}}\right), \end{aligned} \quad (23)$$

where the location parameter of y_t is given by μ . The maximum likelihood estimators (MLEs) of $\hat{\theta} = (\hat{\phi}_1, \hat{\phi}_2, \hat{\kappa}^*, \hat{\omega}, \hat{\nu}, \hat{\gamma}, \hat{\kappa}_2, \hat{\kappa}_1)$ are those values that maximise the log-likelihood function $\ell(\theta)$. The calculation of $\hat{\theta}$ requires numerical techniques (Müller & Bayer, 2017).

2.5.3. One- and Two-Component Comparison

This section outlines the statistical procedure for comparing the one- and two-component Beta-Skew-T-EGARCH models to determine which one performs the best. The central question is whether the additional parameters of the two-component model, ϕ_2 and κ_2 , are statistically significant. This is formalised through the hypothesis test:

$$\begin{cases} H_0 : (\phi_2, \kappa_2) = (0, 0); \\ H_1 : (\phi_2, \kappa_2) \neq (0, 0). \end{cases}$$

For the null hypothesis (H_0), the one-component Beta-Skew-T-EGARCH model is used to measure volatility in Equation (16). For the alternative hypothesis (H_1), we adjust the two-component model in (19). The likelihood ratio (LR) statistic is first taken into account to perform this test, and it is provided by

$$LR = 2[\ell(\hat{\theta}_1) - \ell(\hat{\theta}_0)]. \quad (24)$$

In this context, $\hat{\theta}_1$ denotes the maximum likelihood estimates (MLEs) under the alternative hypothesis, while $\hat{\theta}_0$ refers to the restricted MLE vector that corresponds to the null hypothesis (H_0) (Müller & Bayer, 2017).

2.6. Model Selection

Model selection involves choosing the best-fit model based on certain criteria (Wasserman, 2000). We employ the Hannan–Quinn Criterion HQ(p) introduced by (Hannan & Quinn, 1979), the Bayesian information criterion (BIC) proposed by Schwarz (1978) and the Akaike information criterion (AIC) proposed by Akaike (1998) to choose or select the best-fitting model. The AIC balances model complexity and goodness of fit, which is achieved by selecting the model with the fewest parameters that best describes the data (Akaike, 1998). The SIC is similar to the AIC, but it emphasises predictions.

The AIC is given by

$$AIC = 2k - 2\log(l(\theta)) \quad (25)$$

The AIC with a correction for finite sample sizes is known as the Akaike information corrected (AIC_c), which is given by

$$AIC_c = AIC + \frac{2k(k+1)}{n-k-1} \quad (26)$$

The BIC selects a model that maximises the posterior model probability (Yang, 2005) and can be given by

$$BIC = k\log(n) - 2\log(l(\theta)), \quad (27)$$

where k can be defined as the model degrees of freedom and n is the number of observations.

$$HQIC = -2\log(L) + 2k\log(\log(n)) \quad (28)$$

The HQIC strikes a balance between the AIC and BIC.

The SIC is similar to the AIC and is given by

$$SIC = k\log(n) - 2\log(\hat{\sigma}^2), \quad (29)$$

where $\hat{\sigma}^2$ represents the estimated error term, k represents the model's number of free parameters, and the number of observations is represented by n .

2.7. Evaluation of Forecasts

Various statistical measures are employed to test the accuracy of the model, which consist of the root mean squared error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE). The aforementioned measures' formulae are provided by

$$\text{MAE} = \frac{1}{k} \sum_{j=1}^k |y_j - \hat{y}_j| \quad (30)$$

$$\text{MAPE} = \frac{1}{k} \sum_{j=1}^k \left| \frac{y_j - \hat{y}_j}{\hat{y}_j} \right| \quad (31)$$

$$\text{RMSE} = \sqrt{\frac{1}{k} \sum_{j=1}^k (y_j - \hat{y}_j)^2}, \quad (32)$$

where k denotes the observation number, y_j represents the actual value, and $\hat{y}_j$ is the forecast value.

3. Results

3.1. Data Sources

The South African Reserve Bank is the source of the ZAR/USD daily exchange rate data, and the data can be accessed at (<https://www.resbank.co.za/en/home/what-we-do/statistics/key-statistics/selected-historical-rates>, accessed on 9 January 2025), covering 5 January 2000 to 1 October 2024. The five-day trading week is used, excluding weekends and public holidays. The following equation is used to obtain daily log returns for analysis:

$$r_t = 100 \times \log\left(\frac{Y_t}{Y_{t-1}}\right), \quad (33)$$

where Y_t represents the closing price at time t , r_t represents the log return at time t , and Y_{t-1} represents the closing price of the day before.

3.2. Exploratory Data Analysis

Figure 1 represents market prices that have shown an upward trend from 2000 to approximately 2020, with significant fluctuations. This indicates that the market may be experiencing increasing volatility over time.

Based on the density curve, there is unimodality in terms of the data, which reflects that the single peak has the most prevalent range in the marketplace. The normal Q-Q plot displays the market value prices, indicating heavy tails and values that are normally distributed. The box-and-whisker plot of the closing historical prices displays positive or right skewness, indicating that high exchange rates occur more frequently. Time series plots show an upward trend from early 2000 until 2024, although there have been upward and downward trends. Overall, they exhibit an upward trend. There are periods of significant spikes, such as early 2000, 2008 and 2020, triggered by events around the world.

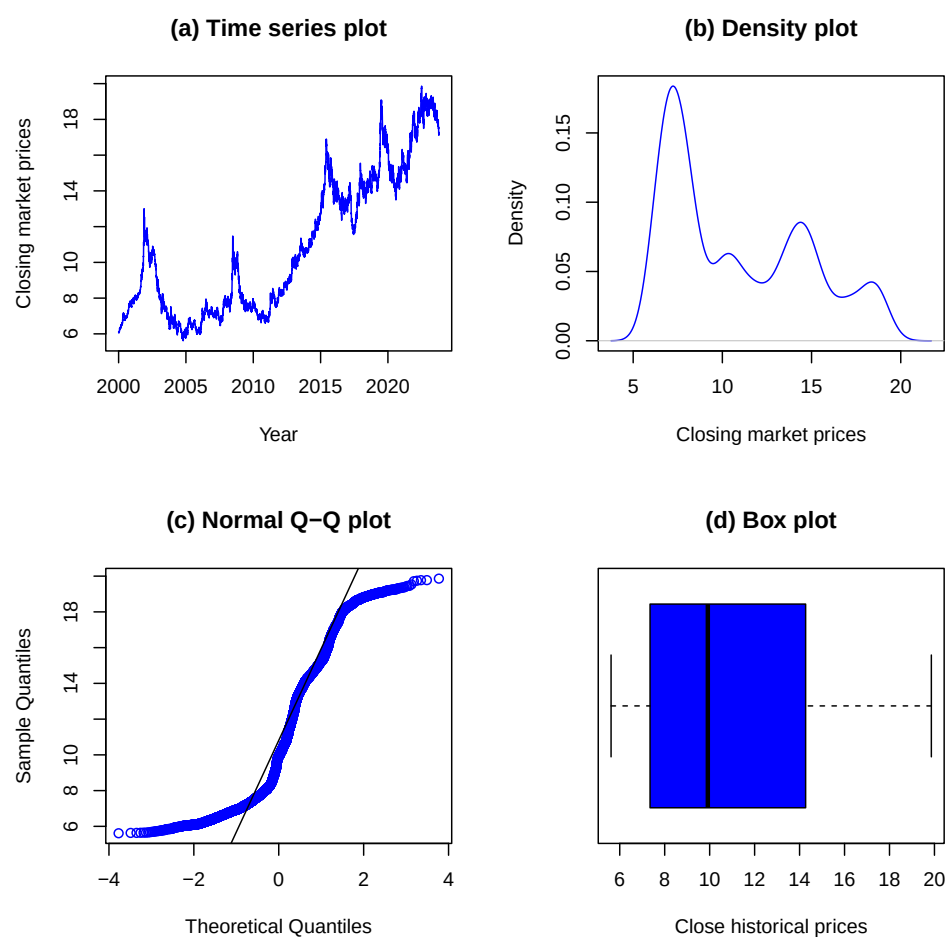


Figure 1. Time series plot.

3.3. Comparative Analysis of ZAR/USD, CHF/ZAR, and ZAR/CNY

Table 1 summarises the descriptive statistics for the log returns of the ZAR/USD, CHF/ZAR, and ZAR/CNY. There is a strong positive trend shown for the ZAR/USD and CHF/ZAR, while the ZAR/CNY moves in the opposite direction, as confirmed by the negative values for the mean, minimum value, and standard deviation. The ZAR has a significantly different relationship with the US dollar and Swiss franc compared to the Chinese yuan.

Table 1. Descriptive statistics of logged values.

	ZAR/USD	CHF/ZAR	ZAR/CNY
Mean	2.3188149	2.239373	−0.3615694
Std Dev	0.3573789	0.491782	0.4142262
Min	1.7207646	1.329195	−1.0294594
Max	2.9859496	3.094767	0.3882511

The study explores the dynamics of the ZAR/USD exchange rate by using the First-Order Beta-Skew-T-EGARCH and fGARCH models, which are applied specifically to the ZAR/USD exchange rate; however, other pairs, like the CHF/ZAR and ZAR/CNY, have been included to facilitate a comparison of the ZAR with other pairs and contextual analysis. This section provides an empirical foundation for interpreting the volatility behaviour of the ZAR/USD exchange rate, as these other exchange rates represent different economic and risk environments involving the ZAR. The interpretation of the ZAR/USD outcomes

is supported by the comparison analysis, which does not expand the model application beyond the main focus of the study.

Table 2 represents the correlation coefficients, which range from -1 to $+1$; these measure the linear relationships between the returns of currency pairs. The ZAR/USD vs. CHF/ZAR ratio is 0.94 , indicating a very strong positive correlation. The ZAR/USD vs. ZAR/CNY ratio is -0.97 , and the CHF/ZAR vs. ZAR/CNY ratio is -0.98 , both showing strong negative correlations.

Table 2. Correlation matrix for exchange rate returns.

	ZAR/USD	CHF/ZAR	ZAR/CNY
ZAR/USD	1.0000000	0.9401065	-0.9733875
CHF/ZAR	0.9401065	1.0000000	-0.9784764
ZAR/CNY	-0.9733875	-0.9784764	1.0000000

3.4. Time Series Plots for Comparative Pairs

Figure 2 represents the log returns for the ZAR/USD vs. CHF/ZAR and ZAR/CNY from 5 January 2002 to 30 September 2024. Figure 2 shows the log returns for the ZAR/USD, ZAR/USD and ZAR/CNY, which are moving in the same direction, showing a similar long-run pattern. Both show an upward trend and slow down over time. Meanwhile, the CHF/ZAR exhibits a downward trend over time, displaying persistent depreciation.

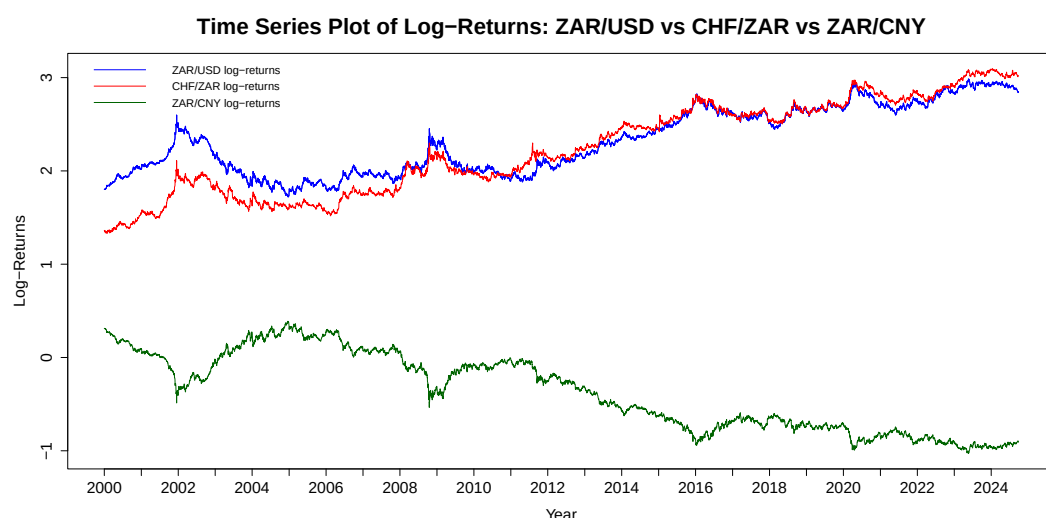


Figure 2. Log returns for ZAR/USD vs. CHF/ZAR vs. ZAR/CNY.

Figure 3 shows the return series for three pairs, and all pairs show a few spikes across the same period: early 2000, 2008, 2015 for China, and 2019, reflecting COVID-19. Across all these patterns, there are periods wherein the fluctuations are large, showing high volatility during the period of 2008, denoting the global financial crisis; the year 2015, representing China's stock market turbulence; and 2020, reflecting COVID-19. There is evidence across various methods.

Figure 4 presents a scatter plot for the ZAR/USD, CHF/ZAR and ZAR/CNY. It reveals a positive correlation between the ZAR/USD and CHF/ZAR returns. The returns for the ZAR/CNY and ZAR/USD, on the other hand, display a strong negative correlation. The data points create a downward-sloping band, indicating that, as the ZAR/USD returns rise, the ZAR/CNY returns tend to decline. Additionally, the scatter plot for the CHF/ZAR and ZAR/CNY returns demonstrates a robust negative correlation between the two.

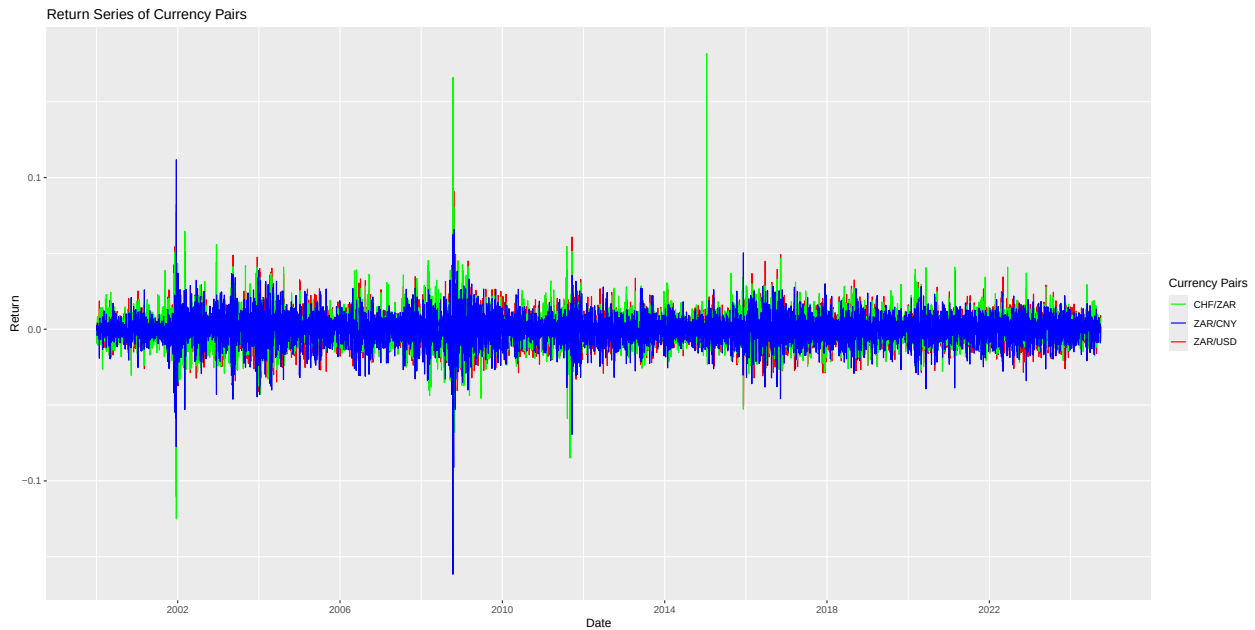


Figure 3. Return series of pairs.

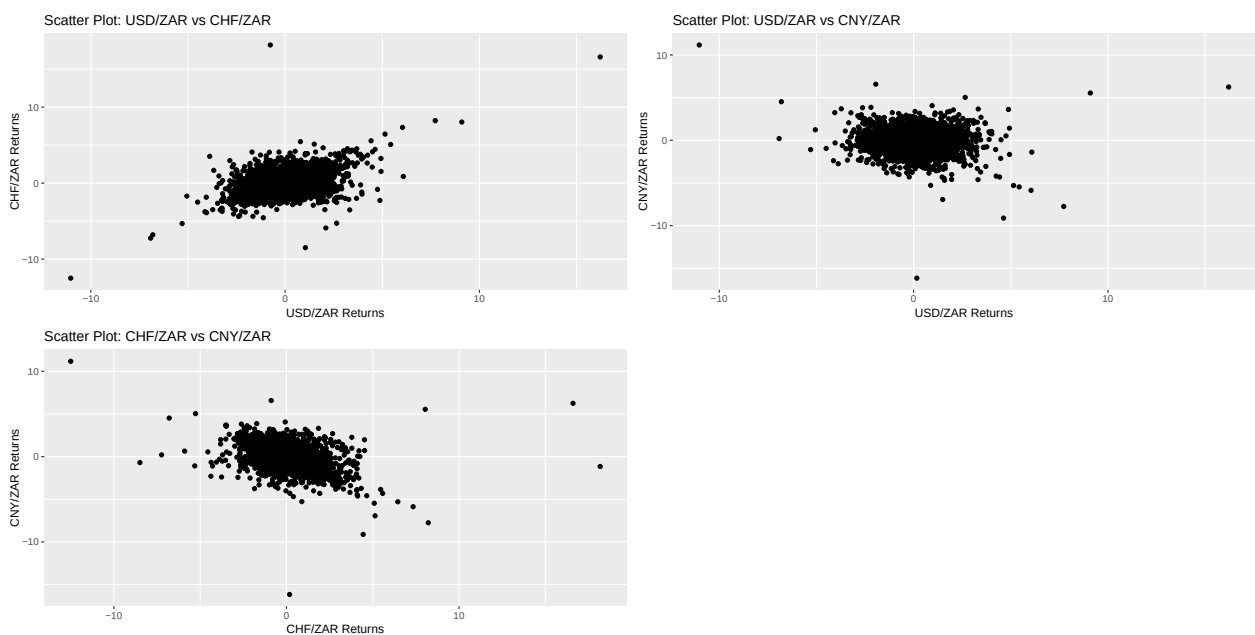


Figure 4. Scatter plots.

Figure 5 illustrates the cumulative log returns for the ZAR/USD, CHF/ZAR and ZAR/CNY exchange rate pairs from 2000 to 2004. The ZAR/USD and ZAR/CNY pairs display comparable long-term upward trends, indicating that both grow at nearly the same rate. This suggests consistent co-movement over the long run. The CHF/ZAR only shows an increase in the early 2000s and a decline around 2010, and this suggests that the rand strengthened along the way against the Swiss franc.

Figure 6 shows a box-and-whisker plot of the studied pairs, wherein RT1 is ZAR/USD, RT2 is CHF/ZAR, and RT3 is ZAR/CNY. Across the three scatter plots, the returns show a positive but moderate correlation. The strongest relationship is observed between the USD/ZAR and CHF/ZAR, while the weakest correlation is between the CHF/ZAR and CNY/ZAR. The presence of scattered points and outliers indicates that the exchange rate returns are volatile. The relationships shown are nonlinear in nature, and, while there

is evidence of co-movement, it is not deterministic. These characteristics support the application of fGARCH models, which are specifically designed to capture the phenomena of volatility clustering and asymmetric behaviour within financial return series.

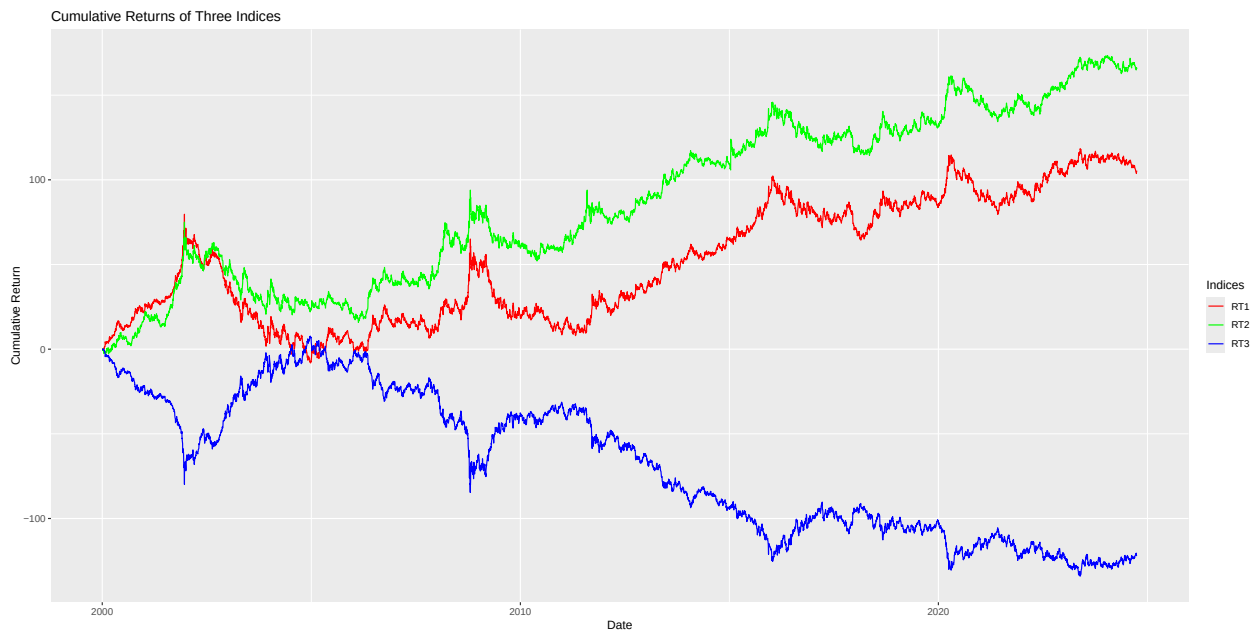


Figure 5. Cumulative log returns.

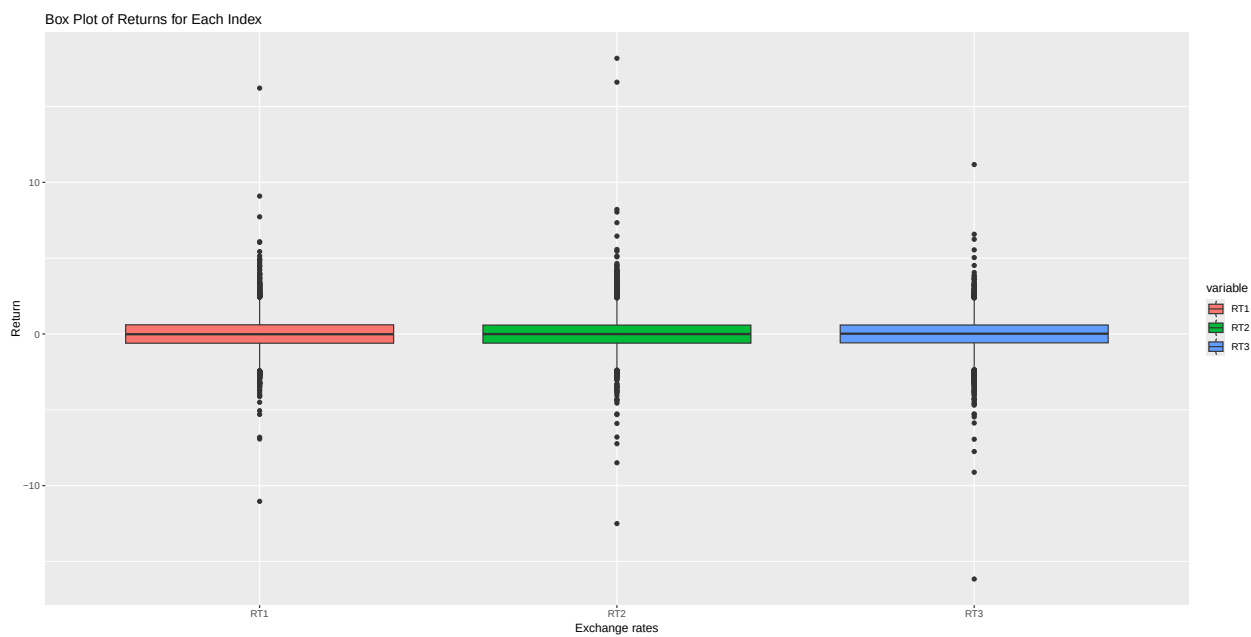


Figure 6. Box-and-whisker plots.

Figure 7 presents the 20-day rolling volatility of the three exchange rate return series. This measures the short-term changes in return and market instability. There are several spikes, similarly to Figures 2 and 3, displaying similar results. In the early 2000s, emerging market stability can be seen, while the 2008–2009 global financial crisis, the 2015–2016 Chinese market crisis and South African rand instability, and the COVID-19 shock caused significant volatility across all series.

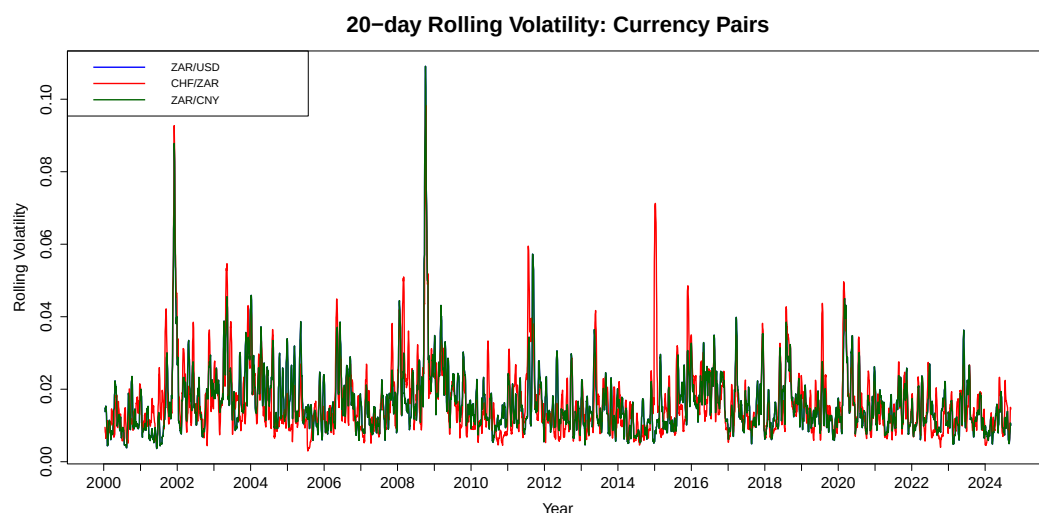


Figure 7. Twenty-day rolling volatility.

3.5. *sGARCH and fGARCH Analysis*

Table 3 shows the ARMA(0,0)-fGARCH(1,1) and ARMA(0,0)-sGARCH(1,1) models; the models are fitted for returns, days and months. The HG (M5) is dominant in sGARCH(1,1) and across all models. The sGARCH(1,1) model (M5) offers the best fit among all information criteria, with the lowest AIC and BIC. The heavy-tailed and skew-flexible GH distribution better captures the specific aspects of the ZAR/USD returns compared to the distributions used in other models. Return-frequency data consistently outperform both daily and monthly data, highlighting the significance of utilising high-frequency series for volatility modelling.

Table 3. ARMA(0,0)-fGARCH(1,1) and ARMA(0,0)-sGARCH(1,1) models with different distributions.

Model	AIC	BIC	SHIBATA	HANNAN-QUINN
ARMA(0,0)sGARCH-(1,1) [M5], returns	−6.3992	−6.3916	−6.3992	−6.3966
ARMA(0,0)fGRACH-(1,1) [M2], returns	−6.3985	−6.3887	−6.3985	−6.3951
ARMA(0,0)fGARCH-(1,1) [M1], months	2.3135	2.3222	2.3135	2.3165
ARMA(0,0)fGARCH-(1,1) [M4], days	2.6391	2.6489	2.6391	2.6425

Regarding the information criteria detailed in Table 3, the most effective fGARCH model is the ARMA(0,0)-fGARCH(1,1) model estimated from return data, namely the SSTD (M2). Among the various fGARCH specifications assessed, the SSTD consistently demonstrates the lowest values for the AIC, BIC, Shibata and Hannan—Quinn, in comparison to other fGARCH models estimated using daily (M1) or monthly (M4) data. This indicates that the fGARCH model fitted to high-frequency return data provides the most accurate depiction of the volatility dynamics within the fGARCH framework.

3.6. *Covariate Effects*

Table 4 summarises the estimated coefficients for covariates with different distributions. mxreg1 represents day, mxreg2 represents month, mxreg3 represents trend, mxreg4 is platinum and mxreg5 is oil. mxreg3 (trend) is identified as the sole significant covariate ($p < 0.05$), which suggests the presence of a strong nonlinear trend. The other covariates (mxreg1, mxreg2, mxreg4 and mxreg5) are statistically insignificant, with $p \approx 0$. The GARCH parameter shows high persistence and moderate ARCH effects for all covariates: ($\beta \approx 0.91$ – 0.96) and ($\alpha \approx 0.01$ – 0.05).

Table 4. Coefficients of covariates in ARMA-ALLGARCH models.

Parameter	Student- <i>t</i>	SGED	Skewed- <i>t</i>	GH	GED
mxreg1 (Day)	0.000013	0.000088	0.000103	0.00008	0.000102
mxreg2 (Month)	−0.000046	−0.000064	−0.000036	−0.000016	−0.000076
mxreg3 (Trend)	0.007071	0.007097	0.007069	0.006455	0.007001
mxreg4 (Platinum)	−0.000045	−0.000022	−0.000079	−0.000055	−0.00020
mxreg5 (Oil)	0.000039	−0.000104	0.000072	0.000050	−0.000107

Although the oil and platinum prices are statistically insignificant across the estimated models, from a theoretical perspective, both variables are expected to influence the South African exchange rate. The insignificant results obtained in this study do not imply that oil and platinum prices are irrelevant determinants of ZAR/USD exchange rate volatility. Similarly, Agara (2026)’s finding regarding the exchange rate and oil indicates a negative relationship, but the conditional mean equation has statistically insignificant direct effects. This indicates that their effects are transmitted through lags and are nonlinear. However, future studies may look into the benefits of incorporating lagged commodity price variables, threshold effects, regime-switching models or nonlinear volatility to investigate the role of commodity prices in expanding exchange rate volatility.

3.6.1. Distribution Properties

Table 5 summarises the distributions of parameters, wherein the SSTD model has the lowest AIC (−6.8749) and the highest log-likelihood (21275.18), which makes it superior to other distributions. The SSTD for mxreg3 exhibits the highest magnitude across all specifications, suggesting that nonlinear time-varying patterns account for approximately 0.7% of the daily return variation. mxreg4 shows a negative coefficient, which suggests that there is an increase in platinum prices that is associated with ZAR depreciation, which contradicts economic theory. This commodity currency exhibits delayed market responses and nonlinear threshold effects.

Table 5. Distribution parameters and fit statistics.

Metric	STD	SGED	SSTD	GH	GED
Skewness	0.000	1.028	1.024	0.200	0.000
Tail shape (λ)	3.017	2.679	3.049	2.000	2.759
Log-likelihood	21,271.05	21,248.48	21,275.18	20,920.82	21,236.75
AIC	−6.8739	−6.8661	−6.8749	−6.5559	−6.8628

3.6.2. Volatility Process Characterisation

The results in Table 6 shows the estimated parameters from the ALLGARCH(1,1) model using two different error distributions. Both models show that $\alpha_1 + \beta_1 \approx 1$, indicating high persistence in volatility. The β_1 (0.95) confirms the strong volatility memory, with shock half-lives of 40–76 trading days.

Table 6. Volatility parameter estimates.

	α_1	β_1	$\alpha_1 + \beta_1$	Half-Life
Student- <i>t</i>	0.032	0.959	0.991	76.4 days
Skewed- <i>t</i>	0.033	0.950	0.983	40.1 days

3.7. Value-at-Risk Computation Using Skewed Student ARMA-ALLGARCH

The Value-at-Risk (VaR) framework employs the ARMA-ALLGARCH specification with skewed Student- t innovations:

$$r_t = \mu + \phi r_{t-1} + \gamma_3 \text{Trend}_t + \epsilon_t \quad (34)$$

$$\epsilon_t = \sigma_t z_t, \quad z_t \sim \text{Skewed-}t(\nu, \lambda) \quad (35)$$

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \eta_1 I_{\{\epsilon_{t-1} < 0\}} \quad (36)$$

wherein r_t denotes the return at time t , μ is the average return, $\phi_1 r_{t-1}$ is an autoregressive term capturing the influence of the previous return, $\gamma_3 \text{Trend}_t$ captures the effect of a time trend or external factor, and the error term (shock) at time t is ϵ_t , representing the unexpected part of the return with the following estimated parameters:

- Degrees of freedom: $\hat{\nu} = 3.0486$;
- Skewness: $\hat{\lambda} = 1.0243$;
- GARCH parameters: $\hat{\alpha}_1 = 0.03297$, $\hat{\beta}_1 = 0.9496$;
- Leverage: $\hat{\eta}_1 = 0.1756$.

3.7.1. Forecasting Conditional Volatility

The one-step-ahead conditional variance is calculated as

$$\hat{\sigma}_{t+1|t}^2 = \hat{\omega} + \hat{\alpha}_1 \epsilon_t^2 + \hat{\beta}_1 \hat{\sigma}_t^2 + \hat{\eta}_1 I_{\{\epsilon_t < 0\}} \quad (37)$$

3.7.2. Computation of Quantiles of Skewed- t Distribution

The VaR at level α is given by

$$\text{VaR}_{t+1}^\alpha = \hat{\mu}_{t+1} + \hat{\sigma}_{t+1|t} \cdot q_\alpha(\hat{\nu}, \hat{\lambda}) \quad (38)$$

where $q_\alpha(\nu, \lambda)$ is the α -quantile of the standardised skewed- t distribution, given by

$$q_\alpha(\nu, \lambda) = \frac{1}{\xi} t_\alpha^{(\nu)} - \frac{\lambda}{\xi} \sqrt{\frac{\nu}{\nu-2}} + \left(t_\alpha^{(\nu)}\right)^2 \left(1 - \frac{1}{\xi^2}\right) \quad (39)$$

with

$$\xi = \sqrt{1 + 3a^2 - a^4}, \quad a = \frac{\lambda}{\sqrt{1 + \lambda^2}}, \quad t_\alpha^{(\nu)} = \text{quantile of standard Student-}t \text{ at level } \alpha.$$

3.8. Summary of the Model

Figure 8 shows the value at risk at 1%. The returns fluctuate over time. In 2008–2009, there was a substantial rise in both the number and frequency of VaR breaches, where the actual returns fell below the model-predicted 1% threshold. Figure 9 shows the conditional standard deviation, which differs significantly over time. Figure 10 shows the series with two conditional SDs superimposed for returns over time. The red lines represent ± 2 conditional standard deviations σ , estimated from the fitted GARCH model. The results indicate that the returns were accurately captured. The years 2008–2009 and early 2020 display spikes because of the global financial crisis and COVID-19. Figure 11 shows the historical and forecast volatility. The volatility declines after major shocks, crises, or events. It is evident that there are spikes in 2008–2009 and 2020. Post-2022, the volatility has been relatively stable.

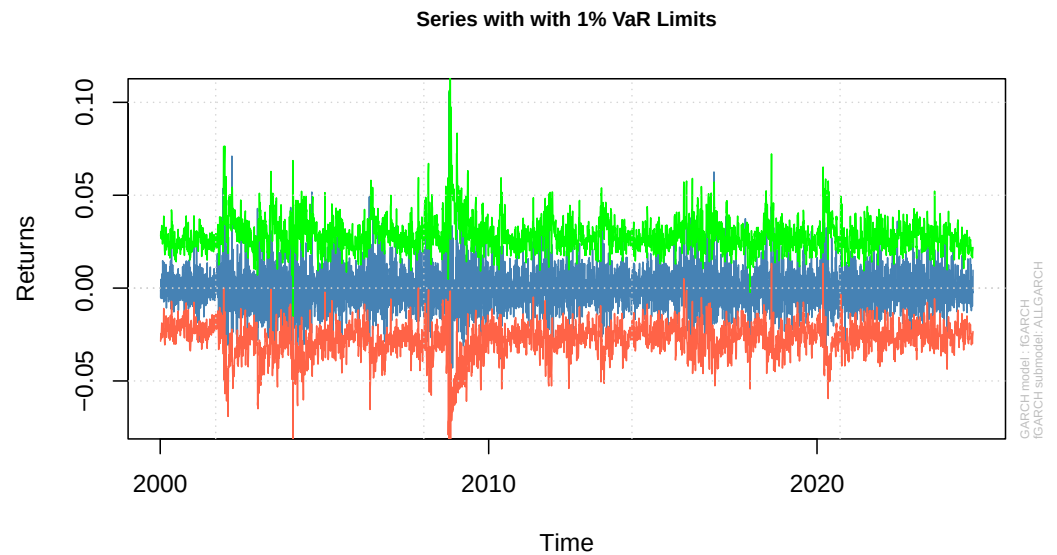


Figure 8. Time series of returns with 1% value at risk (VaR).

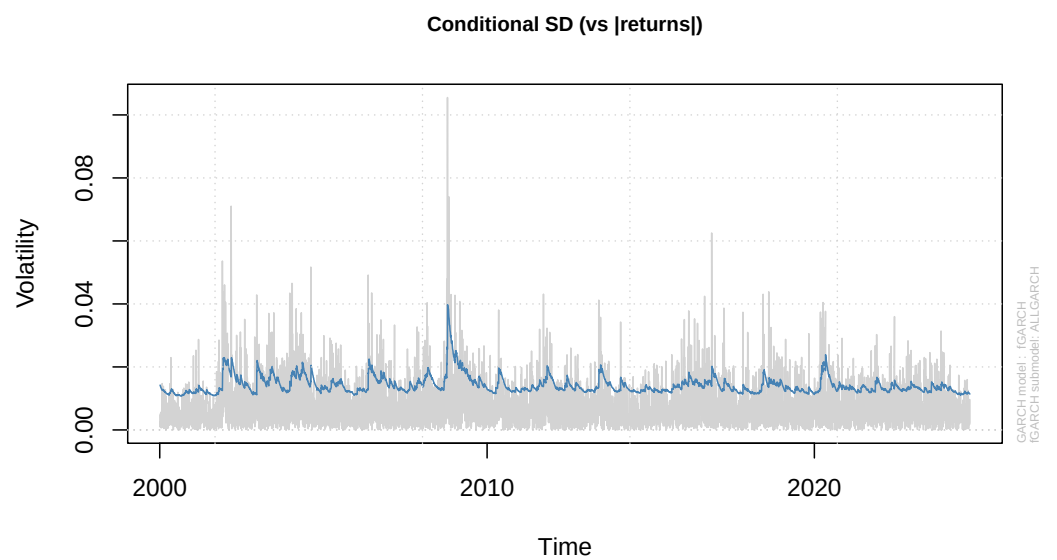


Figure 9. Conditional standard deviation vs. returns.

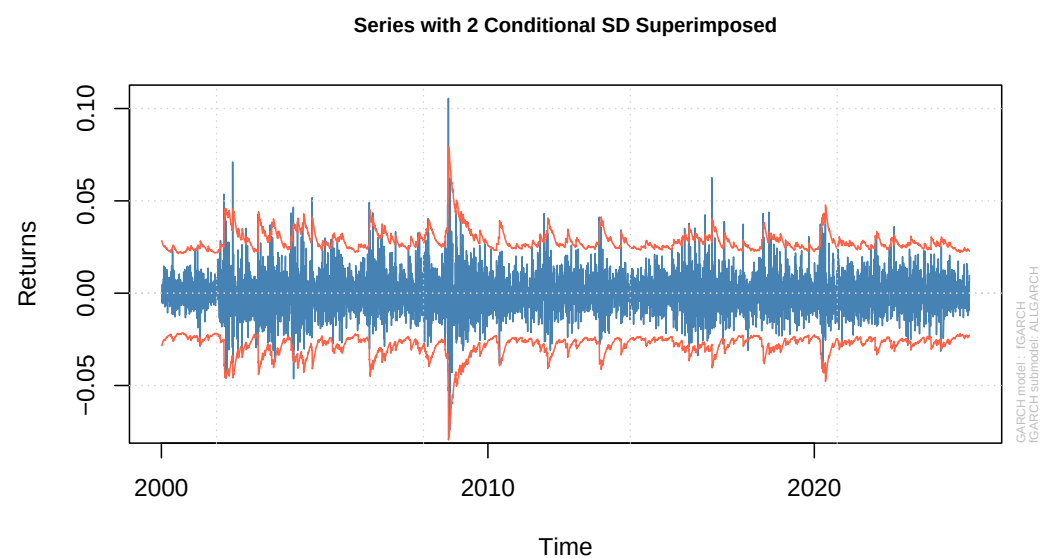


Figure 10. Series with two conditional SDs superimposed.

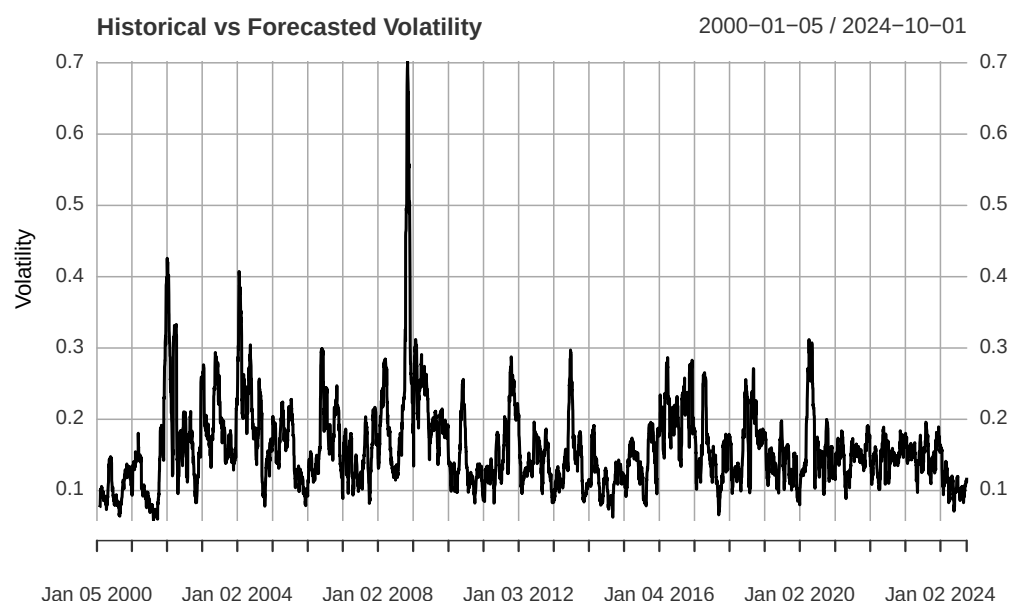


Figure 11. Historical Vs Forecasted Volatility.

3.9. Application of Beta-Skew-T-EGARCH Model

Figure 12 presents the conditional standard deviations from the one- and two-component Beta-Skew-T-EGARCH models for the ZAR/USD daily returns. Both panels register a pronounced spike in 2008–2009, corresponding to the global financial crisis, and a secondary spike in 2020, driven by the COVID-19 pandemic. Post-pandemic, both components trend downward, consistent with the resolution of the acute uncertainty caused by the shock. The two-component model exhibits a notably smoother long-run component alongside a faster-adjusting short-run component, capturing the dual-regime structure of ZAR volatility more effectively than the single-component specification.

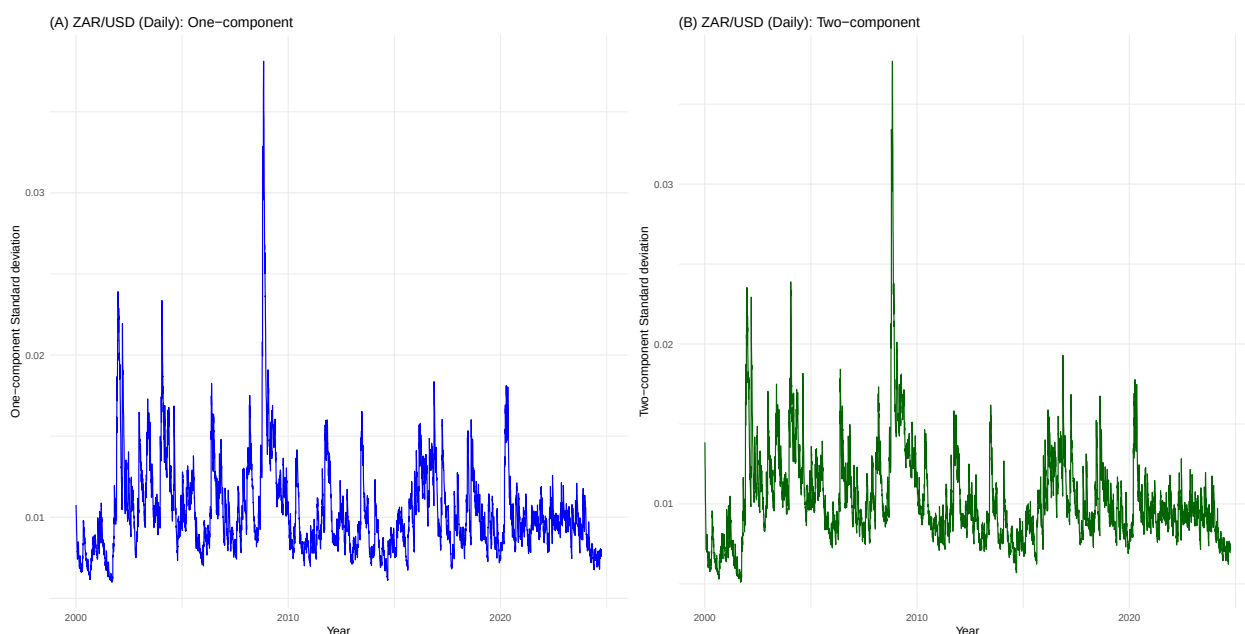


Figure 12. Conditional standard deviations for ZAR/USD daily returns using (A) one-component and (B) two-component Beta-Skew-T-EGARCH model.

3.10. Estimation and Inference

According to Sucarrat (2013), the Beta-T-EGARCH model has two main functions, *teharchSim* and *tegarch*. The first simulates, whereas the other one estimates. Table 7 presents the estimation results for the returns. The long-term log volatility is ω , estimators of ϕ_1 are included for long-term persistence parameters, the long-term response to shocks is estimated by κ_1 , and κ^* reflects the leverage parameter. NaN denotes that the corresponding standard error could not be computed due to numerical instability in the estimation of a highly persistent nonlinear volatility model.

Table 7. Parameter estimates for the *t*-EGARCH model.

Parameter	Estimate	Std. Error
ω	−0.0346497	NaN
ϕ_1	0.999628868	0.000228301
κ_1	0.07788028	NaN
κ^*	−0.000426179	NaN
df	9.99761	NaN
skew	0.9955783	NaN
Log-Likelihood: 19,541.35 (df = 6)		
BIC: −6.3115		

3.11. One-Component and Two-Component Estimates

Table 8 compares the parameter estimates and fit statistics for the one- and two-component Beta-Skew-T-EGARCH models. ϕ is 0.990 for the one-component model and 0.0044 for the two-component model. These results show that there is high persistence in the one-component case and lower persistence in the two-component case. κ yields negative results for one component (−0.0135) and positive for two components (0.0217). These results suggest that a negative κ affects the outcome. The log-likelihood in the one-component case is smaller than in the two-component case, suggesting that two components is a better fit than one component. The BIC in the one-component model is 3.242162, whereas, in the two-component model, it is 3.068434. These indicate that the two-component model gives a better fit, because the BIC is lower and the log-likelihood is higher in the two-component model. Table 9 presents the simulation results. ϕ_1 (0.9987) and ϕ_2 (0.8892) are close to 1, indicating highly persistent volatility. κ_1 and κ_2 are both positive and negative, indicating that there is an influence on volatility shocks. The log-likelihood is −744.86164 and the BIC is 3.07888, indicating that the model offers a reasonable fit for these data. The number of degrees of freedom is 10, which confirms the presence of heavy tails. NaN denotes that the corresponding standard error could not be computed due to numerical instability in the estimation of a highly persistent nonlinear volatility model.

Table 8. Comparison of one- and two-component T-EGARCH models.

Parameter	One-Component Estimate	Std. Error	Two-Component Estimate	Std. Error
ω	0.1070582	NaN	0.004411101	0.05250436
ϕ_1	0.9904859	NaN	0.4667116	0.2475823
κ_1	−0.01351728	NaN	0.02166462	0.0224365
κ^*	0.012825	1.800809×10^{-5}	−0.01217159	0.02236138
df	9.993474	NaN	13.924423	7.693266
skew	1.009158	NaN	0.97733255	0.05894656
Log-Likelihood	−791.896764		−748.464826	
BIC	3.242162		3.068435	

Table 9. Simulation model results (T-EGARCH).

Parameter	Estimate	Std. Error
ω	−0.1358706	NaN
ϕ_1	0.9986547	NaN
ϕ_2	0.8892164	NaN
κ_1	−0.01021127	NaN
κ_2	0.01030438	NaN
κ^*	−0.007006436	NaN
df	10.00108	NaN
$skew$	1.01179862	0.02186357
Log-Likelihood	−744.86164	
BIC	3.07888	

3.12. Forecast Volatility

Table 10 summarises the 7-day-ahead comparison between the one- and two-component models of conditional volatility. The one-component model displays highly persistent volatility from days 1 to 7. The conditional volatility is nearly constant throughout, ranging between 1.162 and 1.160. The conditional volatility in the two-component case is low compared to the one-component case from day 1 to day 7, ranging from 1.024 to 1.005. The findings indicate that the two-component T-EGARCH model is superior to the one-component model in forecasting volatility, as it not only produces lower and more realistic volatility estimates but also reduces forecast uncertainty.

Table 10. Volatility forecasts one- and two-component models.

Day	One-Component		Two-Component	
	σ	St. Dev.	σ	St. Dev.
1	1.161250	1.298509	1.024391	1.107422
2	1.162374	1.299767	1.008021	1.089726
3	1.162053	1.299407	1.006595	1.088184
4	1.161734	1.299051	1.005930	1.087465
5	1.161418	1.298698	1.005619	1.087130
6	1.161106	1.298349	1.005475	1.086973
7	1.160797	1.298003	1.005407	1.086900

3.13. Simulation with 500 Observations

Table 11 presents the estimates and standard errors of the Beta-T-EGARCH model based on 500 simulated observations with set.seed(123). The outcomes for the simulation exhibit statistical significance for most of the parameters, with $\phi_1 = 0.90227794$, $\kappa = 0.90227794$, $\kappa^* = 0.12227762$, $\omega = 0.09851669$. The skew of 0.83289707 is less than 1, indicating that it is to the left, which is consistent with the expected downside of market risk.

Table 11. Estimates and standard errors of Beta-T-EGARCH model.

	ω	ϕ_1	κ_1	κ^*	df	$Skew$
Estimate	0.09851669	0.90227794	0.12227762	0.04601833	6.841992	0.83289707
Std. Error	0.10152282	0.04901111	0.02733901	0.01826780	1.777513	0.04857255
Log-Likelihood: −822.006027						
BIC: 3.362599						

Table 12 displays the variance–covariance matrix of the Beta-T-EGARCH model estimates. The diagonal elements represent the variances of these estimates, which correspond to the standard errors detailed in Table 11. The larger variance, such as (3.1595524081),

associated with the degrees of freedom (df), reflects greater uncertainty in the estimation. The skewness is much lower, indicating a negative skew, which suggests a downside risk. The parameters ϕ and κ are also relatively small, indicating that they are estimated with high precision.

Table 12. Variance–covariance matrix of Beta-TEGARCH estimates.

	ω	ϕ_1	κ_1	κ^*	df	Skew
ω	1.030688×10^{-2}	5.141556×10^{-4}	4.008518×10^{-5}	3.215004×10^{-4}	0.0378534789	−0.0004513878
ϕ_1	5.141556×10^{-4}	2.402089×10^{-3}	$−6.478552 \times 10^{-4}$	6.972156×10^{-5}	−0.0105126630	−0.0005395266
κ_1	4.008518×10^{-5}	$−6.478552 \times 10^{-4}$	7.474213×10^{-4}	9.901035×10^{-5}	−0.0162067679	0.0001274013
κ^*	3.215004×10^{-4}	6.972156×10^{-5}	9.901035×10^{-5}	3.337127×10^{-4}	−0.0052085887	−0.0003590161
df	0.0378534789	−0.0105126630	−0.0162067679	−0.0052085887	3.1595524081	0.0002487455
skew	−0.0004513878	−0.0005395266	0.0001274013	−0.0003590161	0.0002487455	0.0023592926

Figure 13 displays the charts for the simulated returns with one and two components in Panels A and B. Panel A, reflecting one component with 500 observations, indicates that the model does not capture volatility clustering, as the level of variability remains stable over time. Panel B shows the two-component model, which is more appropriate as it accurately captures time-varying volatility, shock persistence, and clustering, which are essential characteristics of financial returns.

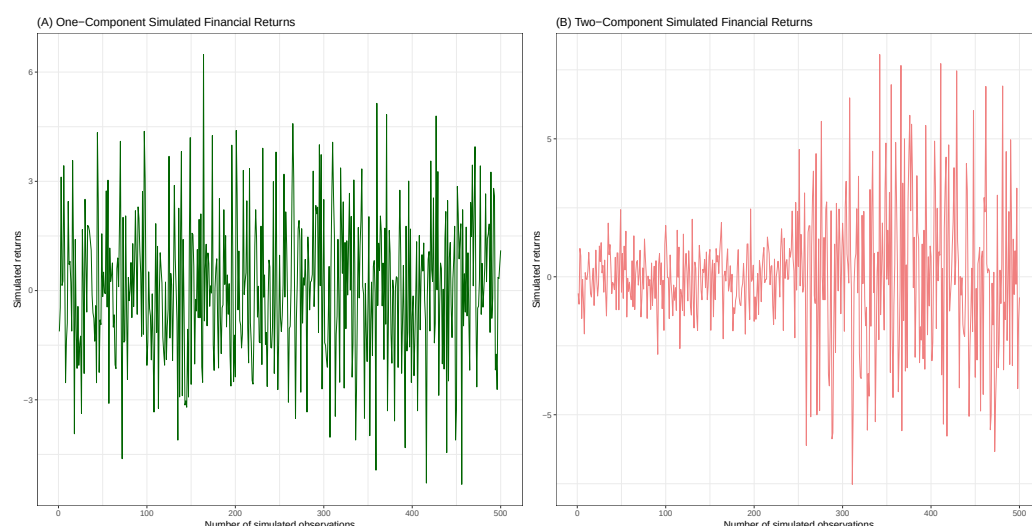


Figure 13. Plots for one-component and two-component simulated returns shown in (A,B).

4. Discussion

The presence of GARCH and ARCH effects demonstrates that past shocks significantly influence future volatility in the ZAR/USD market. This suggests that periods of uncertainty, such as changes in monetary policy, capital flow reversals, or geopolitical events, do not quickly resolve; rather, they persist and affect the rate over extended periods. Persistence in volatility is noticeable for emerging market currencies, which are often vulnerable to sudden shifts in investor confidence. This is consistent with the findings of Agara (2026) regarding Nigeria's inflation dynamics, oil prices, and exchange rate volatility. Meanwhile, an ARCH/GARCH modelling approach was applied by Mosanawe and Makatjane (2026), using Bayesian SARMA-GARCH models with skewed error distributions to analyse the dynamic volatility and tail risk associated with the BTC, BWP, and ZAR exchange rates.

The importance of heavy-tailed and skewed distributions suggests that extreme fluctuations in the ZAR/USD exchange rate occur with greater frequency than would typically

be anticipated under a normal distribution. This phenomenon economically illustrates South Africa's vulnerability to global financial volatility, fluctuations in commodity prices, and the dynamics of international capital markets. Consequently, investors encounter a heightened tail risk when they hold rand-denominated assets.

Remarks on Application of Beta-T-EGARCH

The conditional standard deviation for the one-component and two-component models is compared in Figure 12; both exhibit a notable rise in 2008 and 2009, which is linked to the global financial crisis. Table 8 compares the one-component and two-component models using the log-likelihood and BIC. The two-component model provides a better fit, with the lowest BIC (3.242162) and a high log-likelihood of -748.464826 . The simulation results in Table 9 indicate that there is highly persistent volatility, with $\phi_1 = 0.9987$ and $\phi_2 = 0.8892$, close to 1. This supports the findings of (Bekar, 2023) and other studies regarding the highly persistent volatility.

In Table 10, the forecast volatility for 7 days, ranging from one to two components, is recorded. The one-component level remained high throughout the days, indicating persistent volatility. On the other hand, the two-component case has low conditional volatility. This confirms that the two-component model outperforms the one-component model and therefore reduces uncertainty. This result is consistent with (Bekar, 2023).

As shown in Tables 11 and 12, a simulation of 500 observations was conducted, and the results show statistical significance in most parameters. In Table 12, the covariance matrix has a larger degree of freedom, which suggests that there is uncertainty in the estimation.

Figure 13, with 500 observations, shows that two-component simulated returns are more appropriate, as they accurately capture time-varying volatility. The one-component model shows that the level of variability remains stable over time, but it does not capture volatility clustering. In conclusion, the fGARCH models with five covariates show that the trend variable is statistically significant across all covariates, and, when comparing the performance of the models, Beta-TGARCH emerged as the best-fitting model, as evidenced by the BIC and AIC values and its ability to capture the leverage effect and fat-tailed behaviour in exchange volatility. Moreover, the two-component model performed better than the one-component model.

The performance of the First-Order Beta-Skew-T-EGARCH model is superior to that of traditional GARCH models, suggesting that accounting for asymmetry and heavy-tailed behaviour improves volatility estimation. This result is consistent with the studies by Momoni Mosanawe and Makatjane (2026) and Bekar (2023). For investors, more accurate volatility forecasting facilitate portfolio optimisation and hedging decisions. For financial institutions, improved volatility estimates enhance risk measurement and value-at-risk calculations. The findings can assist policymakers in performing more reliable assessments of exchange rate uncertainty and calculating and formulating monetary as well as exchange rate policies.

Table 13 presents the Ljung–Box test with standardised residuals and square standardised residuals. At lag 1, the test for residual autocorrelation reveals a statistical result of (0.0046334) and a p -value of 0.9457. We accept H_0 and conclude that there is no indication of residual autocorrelation because the value of p is greater than 0.1, 0.05, and 0.01. At lag 1, the result is 1.5289 (test statistic), and the p -value is 0.2163. In conclusion, we cannot reject the null hypothesis of no autocorrelation in the residuals and their squares, suggesting that the model adequately captures the dynamics of ZAR/USD returns.

Table 13. Ljung–Box test results for model diagnostics.

Test	X-Squared	df	p-Value
Standardised Residuals	0.0046334	1	0.9457
Squared Standardised Residuals	1.5289	1	0.2163

5. Conclusions

The effects of all covariates in this study are summarised in Table 4. The five covariates included in the analysis are the daily, monthly, trend, crude oil, and platinum variables. Only the trend variable has a statistically significant predictive effect on the conditional volatility of ZAR/USD, with $p = 0.007$. This effect corresponds to the long-term depreciation of the rand recorded in the 25-year in-sample period, which aligns with South Africa's structural inflation and current account dynamics. The insignificant effects of the remaining four covariates have not yet been fully explained.

From a practical perspective, Table 5 summarises the distribution parameters and fit statistics under the five distributions (STD, SSTD, SGED, GH and GED). A distribution with a lower AIC and higher log-likelihood is superior to others. The SSTD has the highest log-likelihood of 21275.18 and an AIC of -6.8749 , indicating high persistence in volatility, with α and β values close to 1, which is consistent with the results of Maingo et al. (2025) and Agara (2026). Table 6 presents the volatility parameter estimates; Figure 8 illustrates the value at risk at a 1% confidence level. The period from 2008 to 2009 shows a significant improvement due to the global financial crisis, followed by a decrease from 2019 to 2021 due to the COVID-19 pandemic. The returns fluctuate over time to some extent; these events have a remarkable impact on the market.

The results of this study closely align with existing research on exchange rate volatility, particularly regarding emerging market currencies. Previous research has consistently identified key aspects, including volatility clustering, high persistence, asymmetric responses to shocks, and heavy-tailed return distributions (see Mosanawe & Makatjane, 2026). The pronounced volatility persistence observed in the ZAR/USD exchange rate corroborates earlier applications of GARCH-type models to exchange rate data, which indicate that volatility shocks tend to dissipate gradually over time. The study's conclusions have significant theoretical, methodological, practical and policy consequences. The study adds to the current literature by giving additional empirical evidence comparing the performance of the fGARCH and First-Order Beta-Skew-T-EGARCH models in modelling ZAR/USD exchange rate volatility. In terms of methodology, it illustrates the value of sophisticated volatility models that take asymmetry, heavy tails, and volatility persistence into consideration. For investors, financial institutions, and risk managers looking for more accurate volatility forecasts and dependable measurements of exchange rate risk, the findings are practically useful. From a policy standpoint, the findings could help financial regulators and monetary authorities to comprehend exchange rate dynamics and evaluate the possible effects of external economic shocks on financial stability.

The forecasting performance of the fGARCH and First-Order Beta-Skew-T-EGARCH models may be compared in future research with that of deep learning and machine learning strategies, including hybrid econometric–machine learning frameworks, gated recurrent units (GRU), long short-term memory (LSTM), and Transformer-based models.

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Abbreviations

The following abbreviations are used in this manuscript:

ACF	Autocorrelation Function
ADF	Augmented Dickey–Fuller Test
AIC	Akaike Information Criterion
ALLGARCH	All Generalised Autoregressive Conditional Heteroskedasticity Models
ALSI	All Share Index
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroskedasticity
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
BIC	Bayesian Information Criterion
EGARCH	Exponential Generalised Autoregressive Conditional Heteroskedasticity
EUR	Euro
FGARCH	Family Generalised Autoregressive Conditional Heteroskedasticity
GARCH	Generalised Autoregressive Conditional Heteroskedasticity
GBP	Great Britain Pound
HQIC	Hannan–Quinn Information Criterion
KPSS	Kwiatkowski–Phillips–Schmidt–Shin Test
MA	Moving Average
MAE	Mean Absolute Error
MLE	Maximum Likelihood Estimates
PACF	Partial Autocorrelation Function
RMSE	Root Mean Square Error
RSS	Residual Sum of Squares
SSE	Sum of Squared Errors
STI	Straits Time Index
T-EGARCH	Threshold Exponential Generalised Autoregressive Conditional Heteroskedasticity
USD	United States Dollar
ZAR	South African Rand

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