Debt Puzzle: A Comparative Analysis of Public Debt’s Impact on Production Efficiency across OECD Countries

Usama R. Al-qalawi * and Arqam Al-Rabbaie

Department of Economics, Business School, The Hashemite University, Zarqa 13133, Jordan; rabbai@hu.edu.jo
* Correspondence: alqalawi@hu.edu.jo

Abstract: Debt is a fundamental component of modern economic systems. It serves as a source of financing for government, business, and individual projects. Many earlier studies concentrated on the direct relationship between debt and economic performance using different econometric methodologies. This work investigates the effect of debt on production efficiency, extracted from the estimated production function. Unlike previous econometric approaches, we employ a production stochastic frontier analysis (SFA) on data for 18 OECD countries spanning from Quarter 1, 2015, to Quarter 3, 2021, to capture the short-run effect of debt on the production efficiency and, thus, output growth. The results show that, in the short run, as debt increases by $1 billion, efficiency increases by 0.04%. Additionally, we found that the most indebted countries are the most efficient countries. In our sample, those were the UK and France. Furthermore, the average efficiency for the 18 OECD countries was 70.07.

Keywords: debt; efficiency; growth; stochastic frontier

JEL Classification: H63; O47; E610

1. Introduction

In recent years, financial crises and economic slumps have become more frequent in developed and developing economies, which cause a significant increase in public debt. This has led to a greater focus on the relationship between economic growth and public debt by citizens, media, and policymakers. However, there are only a few empirical studies that explore the relationship between production efficiency and debt in depth.

Country production efficiency refers to how well a country employs its resources (e.g., labor, capital, and land) to produce goods and/or services. An efficient production procedure typically engages in minimizing waste, reducing costs, raising workflow, and maximizing output for a given input. Debt can influence a country’s efficiency of production by supplying the necessary funds for capital investments (Borensztein 1989), especially in technology to enhance production efficiency and capacity. Moreover, debt can fund research and development efforts, leading to product innovation and process improvements that boost efficiency (Geelen et al. 2022). In addition, the practical use of debt can help companies manage business risks by using debt strategically during economic downturns to support production levels and survive challenging periods (Xiao 2011). Conversely, elevated levels of debt might restrict a company’s ability to invest in efficient technologies (Croce et al. 2019) or rationalize operations due to financial constraints imposed by debt servicing that can affect a company’s cash flow and financial flexibility. In addition, high debt service costs may limit funds available for improving production efficiency, improving facilities, or hiring skilled staff. Therefore, financial management on the firm level and country level is essential in order to build debt strategies that enhance production efficiency.

Historically, debt is a concern for most countries. David Hume, a Scottish philosopher and economist, is credited with one of the early prominent writings examining the impact of...
public debt on economic growth and production. David Hume investigated the implications of governmental debt on the economy in his 1752 article “Of Public Credit”, arguing that excessive state borrowing can increase consumption, but can lead to inflation and crowd out private investment. He emphasized the potential detrimental effects of a big public debt on a country’s economy, highlighting the significance of debt management (Hume 1875).

According to the theoretical literature, there is a negative correlation between debt and economic growth. Modigliani (1961) suggests national debt helps the current generation while the burden falls on future generations through a reduction in private capital, and this causes a decline in economic activity. Similarly, Saint-Paul (1992) sets up a mathematical model based on the endogenous neoclassical growth model to investigate the effect of public debt on the economic growth rate. The derived equation shows that an increase in public debt leads to a reduction in the economic growth rate.

Other works of theoretical literature argue that public debt has a positive effect in the short run. Hence, it stimulates aggregate demand and production. However, eventually, an increase in public debt harms economic growth. Since it crowds out private investment because of rising interest rates as a response to inflation. Therefore, it restrains the economy (see Elmendorf and Mankiw 1999; Modigliani 1961; Kumar and Woo 2010; Barro 1990).

The differences in the effect of debt on the production growth of poor versus rich countries were introduced by Cohen (1994). He argued that the marginal productivity of capital in rich countries is lower than in poor ones, suggesting that debt can increase economic growth through productivity.

Moreover, the theoretical literature suggests that the effect of debt on economic growth is likely to be carried out through investment and productivity channels. Thus, reasonably sized debt inflows positively affect economic growth. Hence, economies borrow as long as the marginal product of capital is higher than the world interest rate. But, if the cost of foreign borrowing increases, it leads to a decline in investment and lower long-run economic growth (Pattillo et al. 2002).

The most recent literature review on the relationship between public debt and economic growth was conducted by Rahman et al. (2019). They reviewed 33 articles based on the SCOPUS database. They find that relationships can be positive, negative, and nonlinear. In addition, some studies introduced the nonlinear concept between public debt and economic growth represented by an inverted U-shape (Pattillo et al. 2004; Clements et al. 2003; Kumar and Woo 2010).

Unlike the typical independent variables in the production function as labor and capital, which investigate the effect of inputs on economic growth, this article seeks to contribute to the literature by analyzing the rule of public debt on production efficiency. Therefore, it inserts the debt variable as an independent variable that explains inefficiency in the stochastic production frontier function. Additionally, this study sheds light on a puzzling possible relationship between public debt and efficiency scores and explores whether this relationship has an inverted U-shape.

Moreover, this study aims to extend the econometric methodologies in the past research to SFA. This method allows the creation of efficiency score variables. In addition, the review of past studies reveals that including debt as an input in the production function is rare. More importantly, to the best of our knowledge, the effect of debt on production efficiency has not been considered in past research. In addition, it is worth mentioning that the method applied in this work starts extracting efficiency scores of productions and regressing the effect of debt on production efficiency scores. This regression result shows a puzzle.

The rest of the paper is structured as follows: Section 2 provides a recent survey of the literature on the relationship between public debt and economic growth. Section 3 presents the data and stochastic frontier model specification. Section 4 introduces the empirical results. The Section 5 presents the main conclusions.
2. Literature Review

The voluminous empirical literature on the relationship between debt and economic growth has been extensively studied. One branch of the literature suggests a positive relationship between government debt and economic growth, while the other claims a negative relationship.

Gomez-Puig and Sosvilla-Rivero (2017) examine the harmful effect of public debt on economic growth for 11 countries in the Euro area during the period 1961–2015. The results show a negative impact of debt on economic growth when the debt-to-GDP ratio is between 40% and 50%.

Moreover, for a group of Euro countries, Checherita-Westphal and Rother (2012) implement panel data models to explore the nonlinear relationship of public debt on the per capita GDP growth rate over a prolonged period. The result reveals an inverted U-shape relationship between public debt and economic growth, while the inflection point is 90–100% of the debt/GDP ratio.

In addition, for a large group of advanced economies, Reinhart and Rogoff (2010) explore the relationship between public debt and economic growth on data that cover the period between 1946 and 2009 for 44 advanced countries. The results show that public debt as a percentage of GDP may have a detrimental effect on the growth rate of real GDP.

Égert (2015) put the Reinhart and Rogoff (2010) dataset to empirical econometric testing using nonlinear threshold models. They suggest that finding a negative nonlinear relationship between public debt and economic growth varies concerning the model specifications, the period covered in the study, and across countries. Moreover, Eberhardt and Presbitero (2015) support the existence of a negative relationship between public debt and output growth in a large panel of countries.

Arazmuradov (2016) argues that external debt accounts for a significant portion of a developing country’s GDP. Its efficient usage has an impact on national output production. In addition, the study suggests that DEA efficiency measurements could be used to forecast sovereign debt risk.

In addition, other writers discriminate between the short term and the long term, such as Elmendorf and Mankiw (1999). They argue that there is a short-run positive relationship between public debt and output. This positive relationship is stronger when the economy is below full capacity. However, a negative relationship between government debt and GDP exists in the long run, suggesting an inverted U-shaped relationship between public debt and economic performance.

Moreover, the short-term and long-term effects of government debt on economic growth have been studied by Gómez-Puig and Sosvilla-Rivero (2018). They used data on eleven countries in the Euro area from 1961 to 2013. They employ a Cobb–Douglas-form production function and insert stock debt beside the typical input factors. Thus, to estimate the debt impact on economic growth in the short run and long run, they use an autoregressive distributed lag (ARDL) model that allows the extracting of both short-run and long-run effects. The results vary across individual countries, but they indicate a negative relationship in the long run and a positive relationship in the short run.

Woo and Kumar (2015) explore the long-run relationship between government debt and real per capita GDP in a panel of advanced and emerging countries from 1970 to 2008 using different econometric methods. The results suggest an adverse relationship between debt to GDP ratio and GDP growth in all models.

Afonso and Jalles (2013) investigate the relationship between public debt and economic growth for a panel of 155 developed and developing countries from 1970 to 2008. They adopt different econometrics panel data models to estimate neoclassical growth equations. The results show a statistically significant negative relationship between the debt–GDP ratio and output growth for all econometric specifications. In addition, they used threshold models with the debt–GDP ratio and other explanatory variables and found that countries with a debt–GDP ratio of 90% have a lower economic performance. Moreover, he found that higher debt ratios increase TFP growth.
Baum et al. (2013) explore public debt’s effect on economic performance. They apply a dynamic and non-dynamic threshold panel data model for 12 Euro countries from 1990 to 2010. The results suggest that debt stimulates economic growth in the short run. However, the relation becomes insignificant if the debt-to-GDP ratio increases over 67%, and the relation becomes negative if the debt-to-GDP ratio is over 90%.

Onofrei et al. (2022) explore the relationship between government debt-to-GDP ratio and economic growth in EU economies from 1995 to 2019. In addition, they include fixed capital formation, inflation rate, population growth, and the openness of the economy in the estimated models. They used autoregressive distributed lag models with panel dynamic data estimators such as the mean group estimator, pooled mean group estimator, and dynamic fixed effect model. The results show a negative relationship between the debt-to-GDP ratio and economic growth in the short and long run.

The use of a stochastic frontier model to evaluate the effect of external public debt on production efficiency was introduced by Drine and Nabi (2010). They investigate the nonlinear relationship between external public debt and growth using data from 27 developing countries from 1970 to 2005. They utilize endogenous growth models with overlapping generations. The results confirm the nonlinear effect of public debt on production efficiency.

3. Data and Methodology

This study used data from OECD countries because they share many economic characteristics, such as similar levels of economic development, similar economic challenges, developed financial systems, and institutional quality. This similarity allows for consistent comparisons and conclusions. In addition, OECD countries collect the most extensive and quality data. Moreover, a study on OECD countries has a direct implication for policymaking in order to manage debt while enhancing economic efficiency. Therefore, research on OECD countries tends to be more credible.

The effect of debt on efficiency can vary substantially between developing and developed economies. By concentrating on OECD countries, the study can target advanced economies where debt dynamics and efficiency factors are likely to differ from those in less developed regions. Furthermore, OECD countries represent a substantial portion of the global economy. Understanding how debt affects efficiency in these countries can provide insights into broader economic trends.

In this study, we used the OECD database covering the period from Quarter 1, 2015, to Quarter 3, 2021. The sample includes 18 OECD countries. Those countries are Austria, Belgium, the Czech Republic, Denmark, Estonia, France, Greece, Hungary, Ireland, Latvia, Lithuania, Norway, Poland, Portugal, the Slovak Republic, Spain, Switzerland, and the United Kingdom. We dropped some OECD countries from the sample due to missing data.

Traditionally, there are many methods for estimating efficiency. These methods can be classified into parametric and nonparametric methods. The most common parametric method is the stochastic frontier analysis (SFA). Other parametric methods may include the thick frontier approach (TFA) and the distribution-free approach (DFA). However, data envelopment analysis (DEA) is the most common nonparametric approach. Other nonparametric methods may include the free disposal hull (FDH) and quantile regression. Additionally, there are more kinds of techniques used to estimate efficiency that take new approaches like stochastic non-smooth envelopment of data, that combine some of the features of SFA and DEA; Bayesian stochastic frontier models, that utilize prior distributions of the parameters, and then update this distribution; and machine-learning and artificial intelligence techniques.

These methods differ by the used assumption since each has strengths and weaknesses. The choice between the methods is governed by the setting of the study, the type of data, and the adopted assumptions about the production.

This study employs SFA to measure the production efficiency of a sample of 18 OECD countries. This method can choose a functional form for the production function, which
allows for the testing of the hypotheses about the significance of different inputs. In addition, SFA permits the use of variant kinds of distribution, and then compares their results. Moreover, SFA allows us to use efficiency explanatory variables such as debt and time in our current study to explain the variation in efficiency (Kumbhakar and Lovell 2003).

The use of SFA imposed limitations on this study, such as improper selection of functional form and distribution, inaccurate assumption of independence between error components, false assumption of homogeneous inefficiency distribution between cross-section components, and error due to outliers’ presence.

The stochastic frontier analysis (SFA) technique is a parametric approach first developed and applied by Meeusen and van Den Broeck (1977) and Aigner et al. (1977). This model aims to estimate production or cost functions and measure the efficiency of their cross-section components. The parameters of this method were estimated using the maximum likelihood method (Kumbhakar and Lovell 2003). The difference between actual GDP and ideal GDP serves as a technical efficiency gauge for nations. The model is as follow:

\[ Y_{it} = f(x_{jit}; \beta) \cdot \exp(\varepsilon_{it}) \]  

(1)

\[ Y_{it} \] stands for the country’s i production at time \( t \), and \( x_{jit} \) stands for production input \( j \), while a vector of unknown parameters is represented by \( \beta \). The error phrase \( \varepsilon_{it} \) is partitioned of the letters \( v_{it} \) and \( u_{it} \), where \( v_{it} \) stands for random error and \( u_{it} \) for inefficiency. The random error, \( v_{it} \), is a normally distributed variable with a mean of zero and a variance of \( \sigma_v^2 \), and it is independently and identically distributed. Therefore, the error can be represented as:

\[ \varepsilon_{it} = v_{it} - u_{it} \]  

(2)

The distribution of \( u_{it} \) can be half-normal, truncated normal, exponential, or gamma. In this study, we assume that \( u_{it} \) is nonnegative, with a mean of \( \delta z_{it} \) and a variance of \( \sigma_u^2 \), a zero-truncated normally distributed variable. \( z_{it} \) stands for factors that affect efficiency and \( \delta \) is an unknown coefficient that needs to be decided. Therefore, the following is an example of technological inefficiency:

\[ u_{it} = \delta z_{it} + w_{it} \]  

(3)

where \( w_{it} \) is a truncated normally distributed random variable with zero mean and \( \sigma_w^2 \) variance (Battese and Coelli 1993, 1995). This method assumes that the error terms in the two-stage estimation are dependent, and then are estimated in a single stage. This is expected to improve efficiency considerably. Furthermore, we used a logarithmic production function. Therefore, technical efficiency can be identified as

\[ T E_{it} = E[exp(u_{it}) | \varepsilon_{it}] \]  

(4)

\[ T E_{it} = \left\{ \frac{\theta(r_i - \sigma_s)}{\theta(r_i)} \right\} \exp\left\{ -\mu_{s_{it}} + \frac{1}{2} \sigma_s^2 \right\} \]  

(5)

where \( \theta(\cdot) \) denote the standard normal cumulative distribution and

\[ r_i = \frac{\mu_{s_{it}}}{\sigma_s}, \quad \mu_{s_{it}} = \frac{-\sigma_u^2 \varepsilon_{it} + \delta z_{it} \sigma_v^2}{\sigma_u^2 + \sigma_v^2}, \quad \sigma_s^2 = \frac{\sigma_u^2 \sigma_v^2}{\sigma_u^2 + \sigma_v^2} \]  

(6)

The most used function in this type of literature is the Translog production function. It is an abbreviation for the transcendental logarithmic production function. Therefore, it is possible to compare the outcome of this study with earlier ones. In addition, this study assumes that production (\( Y \)) is a function of labor (\( L \)), capital (\( K \)), and time (\( T \)).

\[ Y_{it} = f(L_{it}, K_{it}, T_{it}) \]  

(7)

where:
Y_{it} is country i’s gross domestic product at quarter t, measured in current US dollars; L_{it} is country i’s total employment at quarter t, measured in persons; K_{it} is country i’s gross fixed capital formation at quarter t, measured in current US dollars; T_{it} is a variable that stands for time.

Stochastic frontier analysis is used to estimate the production function that was previously described. Efficiency is then found for each country quarterly. The optimum practice for each country’s production and production efficiency is then computed. To capture the impact of debt and time on inefficiency, we regress inefficiency scores on debt (D) and time (T) in the second stage. This will allow us to obtain the goal of the study.

To estimate technical efficiency (TE), we first specify the functional form. The most common functional form used in the literature is the Translog function. The Translog Production frontier specification for this study is as follows:

\[
\ln Y_{it} = \beta_0 + \beta_1 \ln L_{it} + \beta_2 \ln K_{it} + \beta_3 T_{it} + 0.5 \left[ \beta_4 (\ln L_{it})^2 + \beta_5 (\ln K_{it})^2 + \beta_6 T_{it}^2 + \beta_7 (\ln L_{it}) (\ln K_{it}) + \beta_8 (\ln L_{it}) (T) + \beta_9 (\ln K_{it}) (T) \right] + u_{it} = \delta_0 + \delta_1 D + \delta_2 T
\]

(8)

(9)

Descriptive statistics for all study variables for each country are provided in Table 1, which shows that France had the highest average production at 3.1 trillion dollars, the highest average capital at 0.71 trillion dollars, and the highest average debt at 3 trillion dollars, over the study period. Additionally, France has the second highest average employee among the sample countries at 26.9 million workers, while the United Kingdom’s average production equals 3.06 trillion dollars, capital equals 0.54 trillion dollars, and debt equals 2.7 trillion dollars, slipping into the second order in these average values. However, it had the highest average number of laborers employed at 32.2 million workers. Additionally, the lowest average production during the study period was produced by Estonia and it was equal to 47 million dollars. Likewise, it has the lowest average employees at 0.7 million workers, the second lowest average capital at 12.8 billion dollars, and the lowest average debt at 4.4 billion dollars. The second lowest mean production was produced by Latvia at 57 billion dollars, and it employed the lowest second number of employees at 0.9 million workers, had the second lowest average capital at 12.6 billion dollars, and had the second lowest mean debt at 14.9 billion dollars.

Table 1. Descriptive statistics for the variable of the study.

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean Y (M$)</th>
<th>Max Y (M$)</th>
<th>Min Y (M$)</th>
<th>S.D</th>
<th>Mean L (M$)</th>
<th>Max L (M$)</th>
<th>Min L (M$)</th>
<th>S.D</th>
<th>Mean K (M$)</th>
<th>Max K (M$)</th>
<th>Min K (M$)</th>
<th>S.D</th>
<th>Mean D (M$)</th>
<th>Max D (M$)</th>
<th>Min D (M$)</th>
<th>S.D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>408,135</td>
<td>415,539</td>
<td>374,027</td>
<td>7.7</td>
<td>35.106</td>
<td>85.276</td>
<td>917.8</td>
<td>199,015</td>
<td>576,043</td>
<td>3,674,483</td>
<td>2889</td>
<td>882,949</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgium</td>
<td>595,724</td>
<td>694,107</td>
<td>531,085</td>
<td>4.7</td>
<td>31.375</td>
<td>64.083</td>
<td>114,586</td>
<td>157,940</td>
<td>561,339</td>
<td>668,068</td>
<td>495,903</td>
<td>3928</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Czech Republic</td>
<td>420,809</td>
<td>484,526</td>
<td>350,088</td>
<td>5.2</td>
<td>31.105</td>
<td>135,660</td>
<td>91,072</td>
<td>13,567</td>
<td>99,472</td>
<td>139,074</td>
<td>82,240</td>
<td>15,460</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Denmark</td>
<td>320,388</td>
<td>390,302</td>
<td>274,859</td>
<td>0.5</td>
<td>30,208</td>
<td>66,459</td>
<td>53,934</td>
<td>9335</td>
<td>151,170</td>
<td>199,369</td>
<td>31,530</td>
<td>17,689</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estonia</td>
<td>47,247</td>
<td>57,447</td>
<td>37,523</td>
<td>9.1</td>
<td>141,835</td>
<td>213,675</td>
<td>91,376</td>
<td>15,740</td>
<td>563,339</td>
<td>668,068</td>
<td>495,903</td>
<td>3928</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>1,830,455</td>
<td>2,057,858</td>
<td>1,578,048</td>
<td>13.9</td>
<td>351,529</td>
<td>413,304</td>
<td>280,427</td>
<td>40,568</td>
<td>1,471,356</td>
<td>1,809,601</td>
<td>1,242,535</td>
<td>158,109</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poland</td>
<td>172,050</td>
<td>189,412</td>
<td>161,102</td>
<td>0.3</td>
<td>36,116</td>
<td>41,486</td>
<td>32,585</td>
<td>2,463</td>
<td>59,273</td>
<td>77,289</td>
<td>49,222</td>
<td>8,665</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slovakia</td>
<td>172,050</td>
<td>189,412</td>
<td>161,102</td>
<td>0.3</td>
<td>36,116</td>
<td>41,486</td>
<td>32,585</td>
<td>2,463</td>
<td>59,273</td>
<td>77,289</td>
<td>49,222</td>
<td>8,665</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>302,385</td>
<td>373,238</td>
<td>27,234</td>
<td>4.8</td>
<td>41,245</td>
<td>76,459</td>
<td>46,024</td>
<td>9766</td>
<td>302,598</td>
<td>366,779</td>
<td>286,392</td>
<td>21,863</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>408,135</td>
<td>415,539</td>
<td>374,027</td>
<td>7.7</td>
<td>35.106</td>
<td>85.276</td>
<td>917.8</td>
<td>199,015</td>
<td>576,043</td>
<td>3,674,483</td>
<td>2889</td>
<td>882,949</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Switzerland</td>
<td>704,921</td>
<td>868,106</td>
<td>529,224</td>
<td>7.7</td>
<td>35.106</td>
<td>85.276</td>
<td>917.8</td>
<td>199,015</td>
<td>576,043</td>
<td>3,674,483</td>
<td>2889</td>
<td>882,949</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Only four OECD countries are scoring above the total average production at 0.76 trillion dollars, the total average labor at 7.7 million workers, and the total average capital at 161 billion dollars. Those countries are France, Poland, Spain, and the United Kingdom; out of those four countries, only Poland was not above the total average of debt at 576 billion dollars.
This descriptive analysis may imply that OECD countries with higher resources and higher access to loans usually have higher production. And, since money in the economic literature does not increase production, we suggest, in this study, that money from loans increases production efficiency, and the increase in efficiency increases production.

**Graphical and Causal Analysis for the Variable of the Study**

To examine the relationship between GDP and the study variables, we sketch three graphs in Figure 1. Figure 1a depicts the relation between capital on the horizontal axis and GDP on the vertical axis. The graph shows a positive relationship between capital and production in OECD countries. Figure 1b shows the relation between labor and GDP, and the graph shows a positive relation between labor and GDP in OECD countries. Figure 1c illustrates the relation between debt and GDP. The graph shows a positive relation between debt and production in OECD countries. All these relations are in line with economic theory. Graphs suggest that GDP increases as capital, labor, and debt increase. However, the figures do not refer to anything about production efficiency. The SFA model extracts efficiency scores from the inputs and output. In addition, the method can see how dose efficiency changes relative to other related variables (input).

---

**Figure 1.** The relationship between production and labor, capital, and debt.
4. Results

Before we performed the SFA technique, we started by examining the causality between the efficiency and its explanatory variables. Therefore, we conducted the Granger causality test to check the direction of the relation between efficiency and its explanatory variables to justify using them as independent variables that explain efficiency in the SFA model. The results are provided in Table 2.

Table 2. Granger causality test.

<table>
<thead>
<tr>
<th>The Null Hypothesis</th>
<th>F-Statistics</th>
<th>p-Value</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time does not Granger-cause efficiency</td>
<td>80.7650</td>
<td>0.0000</td>
<td>Time Granger-causes efficiency</td>
</tr>
<tr>
<td>Efficiency does not Granger-cause time</td>
<td>116.3177</td>
<td>0.0000</td>
<td>Efficiency Granger-causes time</td>
</tr>
<tr>
<td>Debt does not Granger-cause efficiency</td>
<td>31.8007</td>
<td>0.0000</td>
<td>Debt Granger-causes efficiency</td>
</tr>
<tr>
<td>Efficiency does not Granger-cause debt</td>
<td>17.5378</td>
<td>0.0000</td>
<td>Efficiency Granger-causes debt</td>
</tr>
<tr>
<td>Debt does not Granger-cause time</td>
<td>229.3172</td>
<td>0.0000</td>
<td>Debt Granger-causes time</td>
</tr>
<tr>
<td>Time does not Granger-cause debt</td>
<td>297.1135</td>
<td>0.0000</td>
<td>Time Granger-causes debt</td>
</tr>
</tbody>
</table>

We used 8 lags to calculate this result as suggested by VAR lag order selection criteria, and we tried fewer lags and it resulted in almost the same result.

The SFA is applied then. Table 3 reports the results of Equations (8) and (9) using the FRONTIER 4.1 software that uses the maximum likelihood technique (Coelli 1996). By examining the value of $\gamma$, which reflects the variance of the inefficiency component of the error term divided by the entire variance of the error $\sigma_u^2/(\sigma_u^2 + \sigma_v^2)$, which is significant and nearly equal to one, we can validate the adoption of the stochastic frontier model. Additionally, this suggests that the variance of error $\epsilon$ is caused by the inefficiency part rather than measurement error. Furthermore, this suggests that production inefficiencies are necessary in order to account for the output production in the chosen OECD countries. This suggests that decision-makers need to focus more on the elements that contribute to inefficiency.

Table 3. The SFA MLE estimates.

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t-Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>$\beta_0$</td>
<td>2.6136988 ***</td>
<td>0.9427572</td>
</tr>
<tr>
<td>Ln L</td>
<td>$\beta_1$</td>
<td>0.7646317 ***</td>
<td>0.1964224</td>
</tr>
<tr>
<td>Ln K</td>
<td>$\beta_2$</td>
<td>1.2539699 ***</td>
<td>0.1940105</td>
</tr>
<tr>
<td>$0.5 \text{ (Ln L)}^2$</td>
<td>$\beta_3$</td>
<td>−0.0030331</td>
<td>0.0223216</td>
</tr>
<tr>
<td>$0.5 \text{ (Ln K)}^2$</td>
<td>$\beta_4$</td>
<td>−0.0682819 ***</td>
<td>0.0200628</td>
</tr>
<tr>
<td>$\text{Ln (L)} \times \text{Ln (K)}$</td>
<td>$\beta_5$</td>
<td>−0.0214852</td>
<td>0.0202870</td>
</tr>
<tr>
<td>$(T)$</td>
<td>$\beta_6$</td>
<td>−0.0225892 **</td>
<td>0.0109787</td>
</tr>
<tr>
<td>$0.5 \text{ (T)}^2$</td>
<td>$\beta_7$</td>
<td>−0.0001575 ***</td>
<td>0.0000462</td>
</tr>
<tr>
<td>$(\text{Ln (L)}) \times (T)$</td>
<td>$\beta_8$</td>
<td>−0.0051358 ***</td>
<td>0.0009656</td>
</tr>
<tr>
<td>$(\text{Ln (K)}) \times (T)$</td>
<td>$\beta_9$</td>
<td>0.0036250 ***</td>
<td>0.0010350</td>
</tr>
<tr>
<td>Intercept</td>
<td>$\delta_0$</td>
<td>0.3969674 ***</td>
<td>0.0191104</td>
</tr>
<tr>
<td>$T$</td>
<td>$\delta_1$</td>
<td>0.0086423 ***</td>
<td>0.0014451</td>
</tr>
<tr>
<td>D</td>
<td>$\delta_2$</td>
<td>−0.0000004 ***</td>
<td>0.0000000</td>
</tr>
<tr>
<td>Inefficiency variance</td>
<td>$\sigma_u^2$</td>
<td>0.0159345 ***</td>
<td>0.0012827</td>
</tr>
<tr>
<td>$(\sigma_u^2)/(\sigma_u^2 + \sigma_v^2)$</td>
<td>$\gamma$</td>
<td>0.9999999 ***</td>
<td>0.0001414</td>
</tr>
</tbody>
</table>

Note: ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. Authors’ calculation using the FRONTIER 4.1 program.

Cobb-Douglass version of this result are shown in Appendix A.

Almost all the coefficient estimates for level variables, along with the square and product terms, are highly significant. Most coefficient signs are consistent with the economic theories. Additionally, the result of ($\delta_1$) showed that, with the increase in time, the inefficiency in production significantly increases. It shows that the inefficiency scores
increase by 0.8%, and this must ring a bell for OECD countries, and it may suggest that sovereign debt risk increases with time (Arazmuradov 2016).

The coefficient for debt ($\delta$) is small and negative. It means that, as debt increases, production inefficiency decreases. This suggests that an increase in debt by $1$ billion drives falling inefficiency by 0.04%. This result is consistent with many studies such as those by Elmendorf and Mankiw (1999), Drine and Nabi (2010), Checherita-Westphal and Rother (2012), Afonso and Jalles (2013), and Gómez-Puig and Sosvilla-Rivero (2018), which support this conclusion. We may argue that a large amount of debt may improve efficiency. The result is consistent with the economic theories that suggest the use of borrowed capital to finance productive investments can amplify returns on investment. Therefore, debt improves efficiency and productivity, especially in the short run.

Table 4 shows the results of technical efficiency for each country. The UK is the most efficient country, with an average efficiency during the study period of 98.17%. France and Spain followed with technical efficiency scores of 97.4% and 84.54%, respectively. The least efficient countries in production are Hungary and the Czech Republic. Their efficiencies are 52.45%, and 55.51%, respectively.

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>S.D. *</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>0.6734</td>
<td>0.7463</td>
<td>0.5780</td>
<td>0.0493</td>
<td>11</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.7218</td>
<td>0.7979</td>
<td>0.6573</td>
<td>0.0419</td>
<td>6</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>0.5515</td>
<td>0.5741</td>
<td>0.4991</td>
<td>0.0186</td>
<td>17</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.7043</td>
<td>0.7942</td>
<td>0.6250</td>
<td>0.0457</td>
<td>7</td>
</tr>
<tr>
<td>Estonia</td>
<td>0.5575</td>
<td>0.6487</td>
<td>0.4249</td>
<td>0.0597</td>
<td>16</td>
</tr>
<tr>
<td>France</td>
<td>0.9764</td>
<td>0.9999</td>
<td>0.8988</td>
<td>0.0249</td>
<td>2</td>
</tr>
<tr>
<td>Greece</td>
<td>0.8449</td>
<td>0.9782</td>
<td>0.7189</td>
<td>0.0744</td>
<td>4</td>
</tr>
<tr>
<td>Hungary</td>
<td>0.5245</td>
<td>0.6055</td>
<td>0.4459</td>
<td>0.0451</td>
<td>18</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.7642</td>
<td>0.9901</td>
<td>0.4062</td>
<td>0.1713</td>
<td>5</td>
</tr>
<tr>
<td>Latvia</td>
<td>0.5883</td>
<td>0.6716</td>
<td>0.5237</td>
<td>0.0491</td>
<td>14</td>
</tr>
<tr>
<td>Lithuania</td>
<td>0.6050</td>
<td>0.6781</td>
<td>0.5485</td>
<td>0.0356</td>
<td>13</td>
</tr>
<tr>
<td>Norway</td>
<td>0.6879</td>
<td>0.7883</td>
<td>0.5606</td>
<td>0.0595</td>
<td>9</td>
</tr>
<tr>
<td>Poland</td>
<td>0.7023</td>
<td>0.7598</td>
<td>0.6596</td>
<td>0.0259</td>
<td>8</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.6376</td>
<td>0.7117</td>
<td>0.5426</td>
<td>0.0540</td>
<td>12</td>
</tr>
<tr>
<td>Slovak Republic</td>
<td>0.5577</td>
<td>0.6182</td>
<td>0.5272</td>
<td>0.0211</td>
<td>15</td>
</tr>
<tr>
<td>Spain</td>
<td>0.8454</td>
<td>0.8948</td>
<td>0.7279</td>
<td>0.0462</td>
<td>3</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.6877</td>
<td>0.7576</td>
<td>0.6182</td>
<td>0.0408</td>
<td>10</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.9817</td>
<td>0.9995</td>
<td>0.8784</td>
<td>0.0238</td>
<td>1</td>
</tr>
<tr>
<td>All</td>
<td>0.7007</td>
<td>0.9999</td>
<td>0.4062</td>
<td>0.1461</td>
<td>-</td>
</tr>
</tbody>
</table>

*S.D. stands for standard deviation. Authors’ calculation using the FRONTIER 4.1 program.

Moreover, the estimates of the efficiency values’ standard deviation for each country ranged between 0.0186 and 0.1713, while Ireland’s estimates have the highest standard deviation, implying that there is a wide range of values for yearly efficiency; this may indicate the existence of inconsistency in the production and the presence of a risky environment, and these factors harden the process of decision-making for policymakers, especially those that are related with debt. However, the Czech Republic’s estimate shows the lowest standard deviation, and this is because the efficiency values are close to the mean. This shows that the production process is relatively uniform, consistent, and reliable. And all these conditions facilitate the job of policymakers in making debt decisions.

The average of countries’ mean efficiency equals 0.7, and the average of countries’ standard deviation equals 0.049, so we can say that countries with a mean efficiency above 0.7 have a relatively high efficiency, and vice versa. Similarly, countries with an S.D. above 0.049 are having a relatively high fluctuation, and vice versa.

Figure 2 shows the fluctuation in efficiency scores throughout the study for 18 OECD countries. The graph shows the UK, France, Spain, Belgium, Denmark, and Poland have relatively low fluctuations in efficiency scores that are associated with a relatively high
efficiency. This shows that these countries are making relatively wise decisions related to debt. This allows them to score high in efficiency with a low variability. However, the graph for Switzerland, Lithuania, Slovak Republic, Czech Republic, and Hungary shows that these countries have relatively low-efficiency fluctuations associated with relatively low-efficiency scores, which may indicate that policymakers are consistently making relatively bad debt decisions. But an improvement in the decision regarding debt may cause an improvement in efficiency due to consistency.

The graphs of Greece and Ireland show a relatively high fluctuation in efficiency associated with relatively high efficiency scores. This suggests that the decision about debt is consistent most of the time. But, still, there are some risky decisions. The graph for Norway, Austria, Portugal, Estonia, and Latvia shows a relatively high fluctuation in efficiency associated with relatively low-efficiency scores, suggesting that these countries are not making consistent decisions about debt that keeps them with a low efficiency.

5. Conclusions

Our sample data show that the United Kingdom, France, and Spain are the OECD countries that have the highest production, labor, and capital, and, at the same time, they are the countries with the highest debt. Similarly, Estonia, Latvia, and Lithuania are the OECD countries that have the lowest production, labor, and capital. Meanwhile, they have the least debt. This motivates us to examine the impact of debt on production efficiency for the 18 OECD countries. The S stochastic frontier analysis (SFA) is employed to estimate the coefficients of the Translog production function throughout Q1 2015 to Q3 2021. The result shows that the average production efficiency for the 18 OECD countries is 70.07%. Moreover, the findings suggest that debt has a negative impact on inefficiency such that an increase in debt increases efficiency, which, in turn, increases production. Furthermore, time has a positive impact on inefficiency, and this can be explained in this way: as time passes, efficiency decreases, and this may lead to a decrease in production. This implies that OECD countries lose efficiency over time. Furthermore, the survey discovered that the United Kingdom is the most efficient country in terms of production, followed by France, and Spain. Ireland has the highest standard deviation of efficiency, and this may indicate the existence of an inconsistency in the production and the presence of a risky
debt environment, so it may indicate that decision-makers are making inconsistent and risky debt-related decisions. However, the Czech Republic estimate shows the lowest standard deviation and this indicates that the production process is relatively uniform and consistent. And policymakers are making consistent decisions about debt. Additionally, the result found that the UK, France, Spain, Belgium, Denmark, and Poland are making relatively good decisions regarding debt management since they have a relatively high, stable efficiency, while Norway, Austria, Portugal, Estonia, and Latvia are making relatively bad decisions about debt management since they have a relatively low, unstable efficiency.

Future Recommendations

The study can be extended further by taking more kinds and sources of debt, since external or internal debt may have different impacts on efficiency. Moreover, the same sort of analysis can be made with more countries developing and developed with various income levels, and before and after the Covid-19 pandemic for a richer insight into the issue. The reason for this is the lack of studies that connect the effect of debt on production via efficiency.

Author Contributions: U.R.A.-q. and A.A.-R. conceived of the presented idea. A.A.-R. developed the theory, U.R.A.-q. performed the measurements, A.A.-R. and U.R.A.-q. were involved in planning and supervised the work, U.R.A.-q. and A.A.-R. processed the experimental data, performed the analysis, drafted the manuscript, and designed the figures. U.R.A.-q. performed the regression analysis. U.R.A.-q. and A.A.-R. manufactured the samples, all aided in interpreting the results and worked on the manuscript. All authors have read and agreed to the published version of the manuscript.

Funding: In this article: I stress that I don’t have a financial, commercial, legal, or professional relationship with other organizations or the people working with them that could influence our research. In addition, no funding was received to help with preparing this manuscript.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data that support the findings of this study are available from the corresponding author, upon reasonable request.

Conflicts of Interest: In this article, I stress that I do not have a financial, commercial, legal, or professional relationship with other organizations or the people working with them that could influence our research. In addition, no funding was received to help with preparing this manuscript.

Appendix A

The results of the Cobb–Douglas production function are as follows:

Table A1. The SFA MLE estimates.

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Coefficients</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t-Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>B₀</td>
<td>6.69863950  ***</td>
<td>0.12625218</td>
<td>53.05761700</td>
</tr>
<tr>
<td>Ln L</td>
<td>B₁</td>
<td>0.40066739  ***</td>
<td>0.01591290</td>
<td>25.17877300</td>
</tr>
<tr>
<td>Ln K</td>
<td>B₂</td>
<td>0.51897402  ***</td>
<td>0.01253897</td>
<td>41.38887500</td>
</tr>
<tr>
<td>Intercept</td>
<td>δ₀</td>
<td>0.42701870  ***</td>
<td>0.02805990</td>
<td>15.21811300</td>
</tr>
<tr>
<td>T</td>
<td>δ₁</td>
<td>−0.00091854</td>
<td>0.00110304</td>
<td>−0.83273806</td>
</tr>
<tr>
<td>D</td>
<td>δ₂</td>
<td>−0.00000046  ***</td>
<td>0.00000002</td>
<td>−19.54533700</td>
</tr>
<tr>
<td>Inefficiency variance</td>
<td>σ_u²</td>
<td>0.02676073  ***</td>
<td>0.00205386</td>
<td>13.02945600</td>
</tr>
<tr>
<td>(σ_u²)/(σ_u² + σ_v²)</td>
<td>γ</td>
<td>0.87702602  ***</td>
<td>0.03245650</td>
<td>27.02158500</td>
</tr>
</tbody>
</table>

Note: ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. Authors’ calculation using the FRONTIER 4.1 program.

The Cobb-Douglas functional form gives almost similar results.
References
Égert, Balázs. 2015. The 90% public debt threshold: The rise and fall of a stylized fact. Applied Economics 47: 3756–70. [CrossRef]
Onofre, Mihaela, Ionel Bostan, Bogdan Narcis Firtescu, Angela Roman, and Valentina Diana Rusu. 2022. Public debt and economic growth in EU countries. Economies 10: 254. [CrossRef]


**Disclaimer/Publisher’s Note:** The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.