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Calculus and Digital Natives in Rendezvous: *wxMaxima* Impact

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Abstract: This article covers how a computer algebra system (CAS) *wxMaxima* can be explored for teaching single-variable and multivariable calculus to Korean digital natives. We present several examples where *wxMaxima* can handle calculus problems easily, not straightforwardly but still successfully with some human intervention, and unsuccessfully. By soliciting qualitative feedback on students' experience in exploiting the CAS, we gathered a mixed reaction. Although some students commented positively, the majority seemed to be resistant to embracing a new technological tool.

Keywords: symbolic computation; computer algebra system; *wxMaxima*; digital natives; South Korea; calculus

1. Introduction

Each year, college freshmen in South Korea who plan to major in science and engineering are required to enroll in two calculus courses: Single-Variable and Multivariable Calculus, hereby SVC and MVC, respectively. Due to growing up surrounded by technology, they belong to a group called "digital natives". Meanwhile, teaching with technology is not only essential but also has become a necessity during the Fourth Industrial Revolution. With so much software to choose from, we selected a free computer algebra system (CAS) *wxMaxima*. In this article, we discuss some features of this software, particularly in connection to symbolic computation for teaching SVC and MVC. We introduced the CAS into calculus classrooms, and we discuss the students' feedback and reflect on their opinions to improve the pedagogical approach. In this introduction, we briefly cover the main components in this study: calculus, digital natives, and the CAS *wxMaxima*. Prior to embedding and implementing *wxMaxima* into our classrooms, we hypothesized that Korean digital natives would embrace the CAS more positively than migrant, non-digital natives. It turned out that our theory is not entirely correct.

1.1. What Is Calculus?

Calculus is the branch of mathematics that studies how phenomena change, how to measure such changes, and how to utilize those measurements in our lives. It provides the foundation for modeling systems where changes are present. The subject was invented because earlier scientists and mathematicians had a great interest in the physical sciences. In the past, calculus was taught after secondary school level, but nowadays, calculus is also taught and introduced in high school. By the 1930s, calculus became an important part of high school mathematics in the US [1]. By the 1960s, the idea of not teaching calculus in high school would have been unthinkable [2].

Although calculus might be the peak of high school mathematics and many students rush into taking it, many educators and mathematicians are concerned that there are some drawbacks of the action. They argue that many students who take calculus in high school and then enroll in college-level calculus courses tend to view the subject as repetition and thus spend less time on understanding the concepts. Eventually, some struggle and might lose interest in mathematics entirely [3,4]. While it should be an exciting course, calculus has been branded as a barrier or filtering course instead of a pump, particularly for STEM (science, technology, engineering, and mathematics) majors [5–9].

In many universities across North America, the sequence of calculus courses is often offered as a part of the undergraduate curriculum in STEM, particularly during freshman year. SVC is usually split into two-semester, one-year-long courses. The so-called "Calculus 1" usually covers differential calculus, and "Calculus 2" comprises integral calculus and most often encompasses an introduction to differential equations as well as sequences and series. What is known as "Calculus 3" commonly deals with multivariable or vector calculus, and is usually offered during sophomore year.

In South Korea, on the other hand, both SVC and MVC are compressed into the freshmen year. During their first semester in college, i.e., spring (Northern Hemisphere), the freshmen take the so-called "Calculus 1", which comprises both differential and integral calculus, the combined course materials from what is commonly offered in North American universities as two separate courses. After the summer break, freshmen enroll in "Calculus 2" during the fall, their second semester as college students. This course is typically equivalent to "Calculus 3" (MVC) offered by many North American universities.

1.2. Who Are Digital Natives?

The majority of our students, if not all, are "digital natives" or part of the "Internet generation". A term coined by Marc Prensky, a "digital native" is used to describe anyone from the generation of people who grew up in the era of ubiquitous technology, which includes computers and the Internet [10,11]. Helsper and Eynon argued that as educators, we often erroneously overestimate our ability to engage our students with technology due to the existence of a generational gap. It turns out that this is only one factor; other factors could be more important than generational differences, including gender, experience, educational level, and the intensity of use [12].

Indeed, while technology is embedded in young people's lives, their use and skills are not uniform [13,14]. A similar finding among Australian university freshmen confirmed this lack of homogeneity concerning technology [15]. Another study showed that in addition to a limited range of established technologies being utilized by young people, there was no evidence that they adopt radically different learning styles [16].

Although South Korea is one of the world's most technologically advanced and digitally connected countries, several case studies do indeed support the research findings from other countries. For example, Koh and Shin observed a limited degree of and homogeneous pattern in media use among Korean youths [17]. On the other hand, Chung discovered a heterogeneous pattern in multi-tasking behavior, and most Korean college students exhibit an affection-type of dependency on digital technology [18].

The literature review on digital natives presented in this article is far from exhaustive. For more extensive coverage on the topic, the following three references will be helpful. Kivunja provided a literature review to shed some light on theoretical perspectives of how digital natives learn and how we can use that knowledge in facilitating their learning [19]. Palfrey and Gasser offered a sociological portrait of the first generation of digital natives who can seem both extraordinarily sophisticated and strangely confined [20]. Dingly and Seychell extensively covered the topics of digital natives, investigated the paradigm shift between the different generations of digital natives, and analyzed the future trends of technology [21].

1.3. Teaching with Technology

At the beginning of each new academic year, we as teachers and instructors teach younger students than the previous cohorts. Since each new generation seems to attach to technology at even a younger age than earlier generations, teaching with technological tools is no longer an option but has become a necessity. Granted, the current prolonged COVID-19 pandemic has only accelerated this transition process. In particular, teaching and learning mathematics using CAS has intensified during the past two decades. This subsection provides a brief overview of some studies where other software has been

embedded in the mathematics classroom. The following subsection will cover *wxMaxima* in particular, and why we opt for it.

GeoGebra is dynamic mathematics software for all levels of education. As the name suggests, it not only brings together geometry and algebra but also calculus, statistics, and graphing in a single easy-to-use package. Hohenwarter et al. presented applications of *GeoGebra* for teaching calculus at both secondary and college levels [22]. The findings from Saha et al. suggest that using *GeoGebra* enhanced students' academic performance in understanding coordinate geometry [23]. Other studies offered insights on how *GeoGebra* generated not only enjoyment and fun in learning mathematics but also in concretizing abstract concepts [24]. Even though utilizing *GeoGebra* in the mathematics classroom still poses challenges and limitations [25], the software has been found particularly useful in enhancing flipped classroom pedagogy [26].

Another well-known CAS is *Maple*, developed by a Canadian-based software company *Maplesoft*. The selection of the name should not come as a surprise since the maple leaf is the most widely recognized national symbol of Canada. *Maple* has been incorporated into project-based learning for calculus class [27]. The software's implementation in the calculus classroom has been arguably positive [28–31]. Even though many teachers and college instructors have struggled with delivering online teaching during the COVID-19 pandemic, an initiative from Italy has borne a fruitful resolution by combining *Maple* with *Moodle*, a digital learning environment. Teachers employ *Maple* in problem-solving activities, designing student worksheets, increasing students' participation, and providing immediate feedback [32].

A rigorous competitor of *Maple* and *wxMaxima* is *Wolfram Mathematica*, or simply *Mathematica*. The software was formulated by Stephen Wolfram and is currently being developed by Wolfram Research in Champaign, Illinois, US. Although it has been criticized for being a closed source software, it possesses the capabilities of high-performance computing. *Mathematica* is integrated with *WolframAlpha*, a computational knowledge engine developed by the same company. Dimiceli et al. described its benefits and drawbacks for teaching calculus using *WolframAlpha* [33]. Barba-Guaman et al. utilized *Mathematica* to improve reading comprehension in mathematics [34]. Beyond calculus, *Mathematica* enhances students' creativity and academic performance in linear algebra [35], discrete mathematics [36], classical mechanics [37], and economics [38], among others. Table 1 summarizes the literature on teaching with technology.

1.4. Why Do We Opt for wxMaxima?

wxMaxima is one of the graphical user interfaces (GUI) for *Maxima*. *Maxima* is one CAS that specializes in symbolic computation. A CAS is a software that can solve computational problems by rearranging formulas and providing analytical expressions, instead of spitting out numerical values. Both systems are free of charge and released under the terms of the GNU General Public License (GPL). Under this authorization, everyone has the right and freedom to modify and distribute the software as long as its license with them remains unmodified.

Some of the free-of-charge competitors of *wxMaxima* are *Axiom* and *SageMath*. The former lacks a GUI and the latter integrates many CAS packages into a common GUI using a syntax resembling *Python*. Although *SageMath* provides a cell server, the results are often slow to appear. Several commercial programs are more famous than *wxMaxima*, and we need to admit that *wxMaxima* might not be able to compete with them. The so-called *4M* CASs are developed by companies that employ full-time developers and programmers: *Magma*, *Maple*, *Mathematica*, and *Matlab*. The latter initially specialized in numerical computations but the symbolic tools were added later.

For mathematics teachers and college instructors who seek an alternative CAS that is free or charge, adopting *wxMaxima* might be worth a try. In addition to serving as a calculator and symbolic manipulator, it can sketch two- and three-dimensional objects with high-quality figures. Since this CAS is lightweight and straightforward, simple computa-

tions can be completed swiftly. However, a word of caution should be taken into account, as we will see in Section 2. While some problems need to be modified or manipulated for *wxMaxima* to be able to solve them, other problems cannot be solved at all. Indeed, *wxMaxima* possesses many limitations, particularly when it comes to symbolic integration.

Table 1. A list of selected studies where CASs have been assimilated into teaching and learning, not only in mathematics and calculus but also in other subjects.

Software	Creator (Started)	Literature	Topic
GeoGebra	Markus Hohenwarter (2001)	Hohenwarter et al. [22] Saha et al. [23] Celen [24] Wassie and Zergaw [25]	Calculus Coordinate geometry Line and angle Precalculus
Maple	University of Waterloo (1980)	Wu and Li [27] Ningsih and Paradesa [28] Lestiana and Oktanivani [29] Hamid et al. [30] Purnomo et al. [31] Fissore et al. [32]	Calculus Freshmen college math Integral calculus Integral calculus Multivariable calculus Contextualized problems
Mathematica	Wolfram Research (1986)	Dimiceli et al. [33] Barba-Guaman et al. [34] Rahmawati et al. [35] Ivanov et al. [36] Romano and Marasco [37] Říhová et al. [38]	Calculus Computer science Linear algebra Discrete mathematics Classical mechanics Economics
Maxima	Bill Schelter et al. (1976)	Hannan [39] Díaz et al. [40] Ayub et al. [41] Dehl [42] Timberlake and Mixon [43] Senese [44] Žáková [45] Fedriani and Moyano [46]	Calculus Linear Algebra Secondary mathematics Vector calculus Classical mechanics Chemistry Engineering Miscellaneous

The literature offers a glimpse at CASs. Some promising attempts have been shown for teaching calculus and linear algebra using *wxMaxima* [39–42]. In physics, chemistry, engineering, and even business, the software is also gaining popularity [43–46]. Evaluating the impact of symbolic computation in education has also been addressed [47]. In this article, our focus is on the teaching and learning of calculus. It extends our previous discussion and complements the mathematical facet of *wxMaxima* with educational aspects [48,49].

1.5. Theoretical Framework and Research Question

In the preceding subsections, we addressed three points. The object (calculus courses, SVC and MVC), the subject study (Korean digital natives), and the pedagogical tool (CAS *wxMaxima*). Due to the advancement of technology and the majority of our students being digital natives, teaching calculus using technology is no longer an option; it is a necessity for successful learning. By combining these three components, we obtained a Venn-diagram-like relationship. In turn, their intersection formed the theoretical framework for this study, as shown in Figure 1.
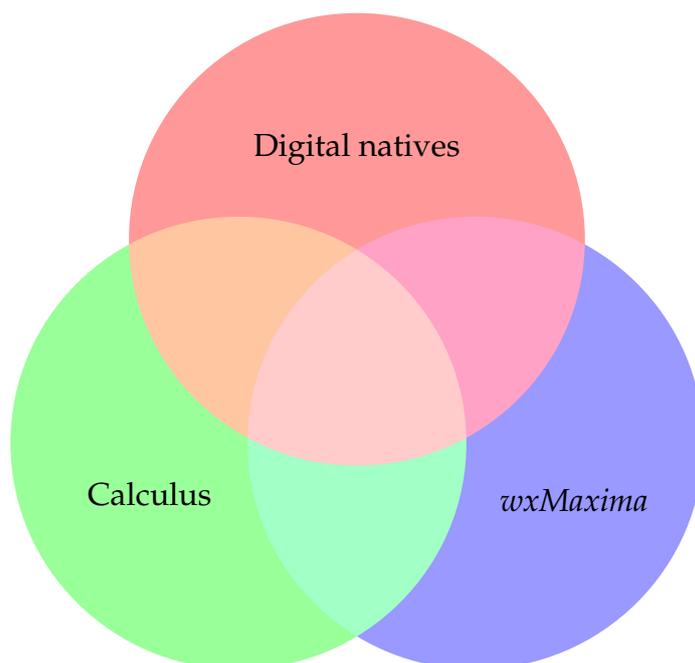
Figure 1. A theoretical framework of teaching calculus to digital natives using CAS *wxMaxima*.

We had the following research questions:

1. What types of symbolic computation can *wxMaxima* perform easily, perform with some manipulation, and not perform?
2. After adopting and implementing *wxMaxima* into calculus teaching and learning, what kind of feedback do we receive as instructors? What can we learn from these students' feedback?

This paper is organized as follows. After this introduction, Section 2 discusses symbolic computation using *wxMaxima*. Through several examples, we show what *wxMaxima* can solve easily and quickly, a type of problem that needs human intervention for *wxMaxima* to be able to solve, and another category of problems with which *wxMaxima* is clueless. Section 3 continues with the educational aspect of *wxMaxima*. It covers research methodology in how we collected data of students' feedback after embedding the CAS into the calculus classrooms. Some findings are also presented. Section 5 concludes this study.

2. Symbolic Computation Using *wxMaxima*

For simple problems, symbolic computation using *wxMaxima* can be performed within seconds. In this section, we consider examples when *wxMaxima* is not only reliable but also struggles to provide immediate outputs. The following examples present a brief overview of the kinds of computations *wxMaxima* can and cannot do.

Example 1 (Easily solved). *The Taylor and Maclaurin series of a function*

$$y = f(x) = 2xe^{-x^2}\cos(3x).$$

Finding Taylor and Maclaurin series using *wxMaxima* are exceptionally fast. The command "`taylor(f(x),x,%pi,2);`" gives the first three nonzero terms of the Taylor series expansion for the function f:

$$e^{-\pi^2}\left[-2\pi + (4\pi^2 - 2)(x - \pi) - (4\pi^3 - 7\pi)(x - \pi)^2 + \cdots\right].$$

By replacing π with 0, the syntax "`taylor(2*x*%e^(-x^2)*cos(3*x),x,0,8);`" yields the first four nonzero terms of the Maclaurin series expansion:

$$2x - 11x^3 + \frac{67}{4}x^5 - \frac{1633}{120}x^7 + \cdots .$$

Admittedly, deriving these series by hand can be tedious. Although every calculus student should master it, *wxMaxima* can be used to check the results or to compute Taylor series for extended functions that would be too arduous for manual computations. Easing the computational load allows the learners to focus more deeply on concepts and patterns.

Taylor and Maclaurin polynomials are employed to approximate functions using the partial sums of the corresponding Taylor and Maclaurin series, respectively. They extend the idea of linearization and are advantageous for understanding asymptotic behavior and the growth of functions, evaluating definite integrals, and solving differential equations.

When it comes to applications, Taylor polynomials have an abundance of them. For example, an equation describing refraction at a spherical interface can be simplified by either a linear or quadratic approximation of the position angle variable. The former is known as the first-order optics or Gaussian optics and the latter is known as third-order optics. The resulting optical theory has become the basic theoretical tool used to design lenses [50].

Evaluating integrals of a rational function can often be easier when the function is decomposed and expressed in a combination of simpler rational functions. This is the idea of partial fraction decomposition.

Example 2 (Easily solved with slightly different outputs). *Partial fraction decomposition of a rational function*

$$P(x) = \frac{2x^2 + 21x - 36}{x^7 + 10x^6 + 37x^5 + 60x^4 + 36x^3}.$$

Using simple commands

"`P:(2*x^2+21*x-36)/(x^7+10*x^6+37*x^5+60*x^4+36*x^3);`"

and "`P1:partfrac(P,x)`"

we obtain the following instant result:

$$P_1(x) = \frac{26}{3(x+3)} + \frac{3}{(x+3)^2} - \frac{6}{x+2} + \frac{35}{4(x+2)^2} - \frac{8}{3x} + \frac{9}{4x^2} - \frac{1}{x^3}.$$

Observe that we can factor the denominator as $x^3(x^2 + 5x + 6)^2$, which explains why we gather the terms x^m, $(x+2)^n$, and $(x+3)^n$, for $m = 1, 2, 3$ and $n = 1, 2$ in the denominators of the partial fraction decomposition. Integrating the original rational function ("`integrate(P,x);`") and its partial fraction decomposition ("`integrate(P1,x);`") using *wxMaxima* produces slightly different outputs, given as follows, respectively:

$$\int P(x)\,dx = \frac{26\log(x+3)}{3} - 6\log(x+2) - \frac{8\log(x)}{3} - \frac{14x^3 + 43x^2 + 11x - 3}{x^4 + 5x^3 + 6x^2}$$

$$\int P_1(x)\,dx = \frac{26\log(x+3)}{3} - 6\log(x+2) - \frac{8\log(x)}{3} - \frac{3}{x+3} - \frac{35}{4(x+2)} - \frac{9}{4x} + \frac{1}{2x^2}.$$

It should be noted that for rational functions that cannot be decomposed, *wxMaxima* often fails to integrate them.

Example 3 (Require some manipulations). *Integral of a beta function:*

$$\int \frac{1}{(x^2 - x^3)^{\frac{1}{3}}}\,dx \qquad and \qquad \int_0^1 \frac{1}{(x^2 - x^3)^{\frac{1}{3}}}\,dx.$$

wxMaxima fails to evaluate both the indefinite and definite integrals. The commands "`integrate(1/(x^2-x^3)^(1/3),x);`" and "`integrate(1/(x^2-x^3)^(1/3),x,0,1);`" produce no output. However, by pulling out the factor $x^{2/3}$ from the integrand and rewriting it as a beta function, using the commands "`integrate(1/(x^(2/3)*(1-x)^(1/3)),x);`" and "`integrate(1/(x^(2/3)*(1-x)^(1/3)),x,0,1);`" we finally acquire a result for both integrals:

$$\int \frac{1}{x^{\frac{2}{3}}(1-x)^{\frac{1}{3}}} \, dx = \ln\left[\frac{\sqrt[3]{x}+\sqrt[3]{1-x}}{\sqrt[3]{x}}\right] - \frac{1}{2}\ln\left[\frac{x-\sqrt[3]{x^2}\sqrt[3]{1-x}+\sqrt[3]{x}\sqrt[3]{(1-x)^2}}{x}\right]$$
$$- \sqrt{3}\, \text{atan}\left[\frac{2\sqrt[3]{1-x}-\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{x}}\right]$$

$$\int_0^1 \frac{1}{x^{\frac{2}{3}}(1-x)^{\frac{1}{3}}} \, dx = \frac{2\pi}{\sqrt{3}} = \text{beta}\left(\frac{1}{3},\frac{2}{3}\right) = \frac{\pi}{\sin\frac{\pi}{3}}.$$

To find the numerical value for the latter, the command "`float((2*pi)/sqrt(3));`" yields 3.627598728468436.

The beta function, which is closely related to the gamma function, finds numerous applications in calculus. Additionally, it is particularly useful for computing and representing the scattering amplitude for Regge trajectories in quantum physics, i.e., the probability amplitude of the outgoing spherical wave relative to the incoming plane wave in a stationary-state scattering process [51–53].

Beyond calculus, the definite integral can be evaluated without *wxMaxima* using contour integration and the residue theorem from complex analysis [54,55]. Let I denote the definite integral; then let $u = 1/x$, we have $dx = -du/u^2$, and it can be written as

$$I = \int_0^1 \frac{1}{(x^2-x^3)^{\frac{1}{3}}} \, dx = \int_\infty^1 \frac{-du/u^2}{(1/u^2-1/u^3)^{\frac{1}{3}}} = \int_1^\infty \frac{du}{u(u-1)^{\frac{1}{3}}}.$$

Substitute $v = u - 1$ and $dv = du$, we can write I further as

$$I = \int_1^\infty \frac{du}{u(u-1)^{\frac{1}{3}}} = \int_0^\infty \frac{v^{-\frac{1}{3}}}{v+1} \, dv.$$

Here $v^{-1/3}$ denotes the positive real number or the principal value of $\exp\left(-\frac{1}{3}\ln v\right)$. The final integral above is improper not only due to its upper limit of integration but also because its integrand has an infinite discontinuity at $v = 0$. Let C_R and C_ρ denote the quasi circles $|z| = R$ and $|z| = \rho$, respectively, where $\rho < 1 < R$. Figure 2 displays an orientation of the contour. It is traced out by moving from $x = \rho$ to $x = R$ along the top of the branch cut for $f(z)$, then around the larger quasi circle C_R and back to $x = R$, next along the bottom of the branch cut for f to $x = \rho$, and finally around the smaller quasi circle C_ρ back to $x = \rho$. We integrate the branch

$$f(z) = \frac{z^{-\frac{1}{3}}}{z+1}, \qquad |z| > 0, \quad 0 < \arg z < 2\pi$$

of the multiple-valued function $z^{-\frac{1}{3}}/(z+1)$ around this simple closed contour with branch cut $\arg z = 0$. For $z = re^{i\theta}$, we can write

$$f(z) = \frac{z^{-\frac{1}{3}}}{z+1} = \frac{e^{-\frac{1}{3}\log z}}{z+1} = \frac{e^{-\frac{1}{3}(\ln r+i\theta)}}{re^{i\theta}+1}.$$

On the upper edge, where $z = re^{i0}$, we have

$$f(z) = \frac{e^{-\frac{1}{3}(\ln r + i0)}}{re^{i0} + 1} = \frac{r^{-\frac{1}{3}}}{r + 1}.$$

On the lower edge, where $z = re^{i2\pi}$, we attain

$$f(z) = \frac{e^{-\frac{1}{3}(\ln r + i2pi)}}{re^{i2\pi} + 1} = \frac{r^{-\frac{1}{3}}e^{-i\frac{2}{3}\pi}}{r + 1}.$$
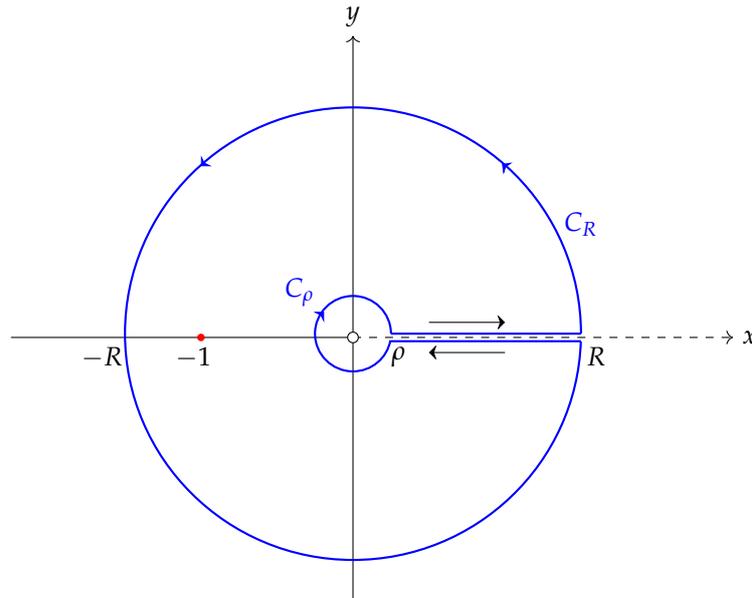

Figure 2. The contour along a branch cut.

Applying the residue theorem yields

$$\int_{\rho}^{R} \frac{r^{-\frac{1}{3}}}{r + 1}\,dr + \int_{C_R} f(z)\,dz + \int_{R}^{\rho} \frac{r^{-\frac{1}{3}}e^{-i\frac{2}{3}\pi}}{r + 1}\,dr + \int_{C_\rho} f(z)\,dz = 2\pi i \operatorname*{Res}_{z=-1} f(z)$$

$$\left(1 - e^{-i\frac{2}{3}\pi}\right) \int_{\rho}^{R} \frac{r^{-\frac{1}{3}}}{r + 1}\,dr = 2\pi i e^{-i\frac{\pi}{3}} - \int_{C_R} f(z)\,dz - \int_{C_\rho} f(z)\,dz.$$

We observe that

$$\left| \int_{C_\rho} f(z)\,dz \right| \le \int_{C_\rho} \left| \frac{z^{-\frac{1}{3}}}{z + 1} \right|\,dz \le \int_{0}^{2\pi} \frac{\rho^{-\frac{1}{3}}}{1 - \rho}\,dz = \frac{2\pi}{1 - \rho}\rho^{\frac{2}{3}} \to 0 \qquad \text{as} \quad \rho \to 0$$

and

$$\left| \int_{C_R} f(z)\,dz \right| \le \int_{C_R} \left| \frac{z^{-\frac{1}{3}}}{z + 1} \right|\,dz \le \int_{0}^{2\pi} \frac{R^{-\frac{1}{3}}}{R - 1}\,dz = \frac{2\pi R}{R - 1} \frac{1}{R^{\frac{1}{3}}} \to 0 \qquad \text{as} \quad R \to \infty.$$

By letting $\rho \to 0$ and $R \to \infty$, we arrive at

$$\left(1 - e^{-i\frac{2}{3}\pi}\right) \int_{0}^{\infty} \frac{r^{-\frac{1}{3}}}{r + 1}\,dr = 2\pi i e^{-i\frac{\pi}{3}}$$

$$\int_{0}^{\infty} \frac{r^{-\frac{1}{3}}}{r + 1}\,dr = 2\pi i \frac{e^{-i\frac{\pi}{3}}}{1 - e^{-i\frac{2}{3}\pi}} = 2\pi i \frac{1}{e^{i\frac{\pi}{3}} - e^{-i\frac{\pi}{3}}} = \frac{2\pi i}{2i \sin\frac{\pi}{3}} = \frac{\pi}{\sin\frac{\pi}{3}} = \frac{2\pi}{\sqrt{3}}.$$

This is the same as the integral *I* and thus we verified the computational result.

Example 4 (Unable to solve). *Improper definite integral with infinite limits of integration:*

$$\int_{-\infty}^{\infty} \frac{1}{(x^2 + 1)(e^x + 1)}\, dx.$$

While wxMaxima *provides no solution for this integral, another CAS, e.g.,* Mathematica, *gives the output:* $\frac{\pi}{2}$.

3. Methodology

We addressed the qualitative aspect of the study by adopting an observational research method. In particular, we were interested in obtaining students' opinions regarding their experience after using the CAS.

3.1. Participant

The participants in this study were the students who were enrolled in two calculus courses offered at the Natural Science Campus at Sungkyunkwan University: Single Variable Calculus (SVC, course code GEDB001) and Multivariable Calculus (MVC, course code GEDB002) for three years from Spring 2016 until Fall 2019, except for the 2017 Academic Year. Each course includes three distinct cohorts, and thus we considered six sections in total. The total number of participants was 284 and their age ranged from 18 to 20 years old. There were 193 students registered in SVC and 91 were in MVC. For unclear reasons, the number of registered students for MVC tends to be fewer than for SVC. The length of the semester is 16 weeks. Weeks 8 and 16 were designated for the midterm and final examination periods, respectively. We employed the convenience sampling technique in selecting the course section due to accessibility and efficiency.

3.2. Measurement

We obtained the students' feedback from the emails that they sent and from the online questionnaire administered by the Academic Affairs Team. Students' comments are accessible through an internal network for faculty and students, known as *Advanced Sungkyunkwan Information Square–Gold Lawn Square (ASIS-GLS)*. The students have the opportunity to deliver feedback twice: before the midterm test and final examination periods, i.e., week 7 and week 15, respectively. In particular, we paid attention to the second part of the questionnaire where the students can write freely while giving feedback on their instructors' teaching. They need to respond to the inquiry, "Please write down your suggestions for the professor to improve the class". The first part of the student teaching evaluation is a typical five-Likert item questionnaire, and we do not cover it here since the statements are generic and do not solicit particular students' opinions on the use of CAS *wxMaxima* during teaching.

3.3. Result

The response rate to the aforementioned open-ended inquiry ranged from 69% to 98%, depending on the semester and course. However, the mean response rate was around 83%. There were only 15 comments about or closely related to *wxMaxima* that we selected. Table 2 summarizes the details of the number of responses in each course.

Table 2. The distribution of participants according to the courses and number of responses.

Course	Number of Sections	Number of Responses			Total	Percentage Average
		Selected	Midterm	Final		
SVC	3	7	137	183	320	82.90%
MVC	3	8	64	89	153	84.07%
Total	6	15	201	272	473	83.27%

Regarding students' feedback, although some wrote positive comments, their perception of the CAS was generally negative. Table 3 displays selected students' teaching evaluations related to *wxMaxima* for both SVC and MVC.

Table 3. Select student feedback on teaching evaluation for SVC and MVC. We included comments related to the CAS *wxMaxima* only.

Course	Semester	Period	Feedback
SVC	Spring 2016	Midterm	*wxMaxima* program is too hard. It's nice to use *wxMaxima*. It would be nice if there was no error by telling us how to use it or the download path. The purpose of using *wxMaxima* is good, but students may be embarrassed to use the program they are not familiar with, so please provide more information on that part. Most of us don't know how to use those programs. With no help to learn a program, I think it is too hard and it needs much time.
		Final	*wxMaxima* is too hard.
	Spring 2018	Midterm	It would be appreciated if you could reduce the drawing of the graph.
	Spring 2019	Midterm	Please tell me carefully the calculus. It is good for taking classes and organizing concepts slowly. You show them all in a graph.
MVC	Fall 2016	Midterm	Using *wxMaxima*. Professor Karjanto teaches us well so I can understand the contents well. *WXMAXIMA* *wxmaxima* program Using computer program to show graph is good. I want more homework solve good problem rather than using computer.
	Fall 2018	Final	It seems that using more graphing software will help you understand.
	Fall 2019	Midterm	The professor directly draws a graph to understand the concept and uploads additional helpful materials to *iCampus*. Was a great help in studying the subject overall.
		Final	I cannot solve the assignment using *wxMaxima* and thus, solve it by hand.

4. Discussion

4.1. Didactic Application

There exist some didactic applications when blending *wxMaxima* with calculus is done properly. First, *wxMaxima* plays a tremendous role in verifying computational results done manually by pen and paper. This feature is essential for saving time, which can be used for course preparation, understanding deeper ideas, fathoming theoretical concepts, and indulging in problem-solving sessions [56]. The opposite is also didactically useful since *wxMaxima* often spits out non-simplified outputs, it can be employed in training students to simplify the expressions by hand.

Second, obtaining the corresponding numerical values of exact expressions is as simple as inputting the command "`float(%)`" or "`numer`". These floating-point functions can be convenient when one wishes to obtain numerical values where outputs are composed of a combination of rational and irrational numbers.

Third, as expressed positively by several students, curve sketching and graphical plots generated by *wxMaxima* enhance their geometrical imagination and comprehension.

Appreciatively, *wxMaxima* produces high-quality drawings that are yet lightweight for further use. Remarkably, three-dimensional plots can be rotated easily without causing any computer memory problems, as is often encountered in other software, e.g., *Matlab*.

Fourth, beyond calculus, *wxMaxima* has plenty of room for further exploration. It can manage data visualizations and represent them with marvelous plots. It can also administer data fitting to either a straight line or a nonlinear curve. It can handle programming features, such as loops, iterations, and decision making [44]. Overall, it is fruitful software for those who would like to try new things.

Indeed, the literature offers abundant didactic benefits of infusing CAS in mathematics classrooms. The fifth educational application comes from a programming point of view. Coding in *wxMaxima* flows logically and thus allows students to carry out simple algorithms independently [57].

Sixth, since students are allowed, even encouraged, to make mistakes and induce judgments by themselves, embracing *wxMaxima* stimulates the interactive learning process through testing, evaluation, decision making, and error correction [45]. Additionally, a wise application of CAS could foster students' abilities in proving, modeling, problem-solving, and communication [58]. We can also amalgamate *wxMaxima* with an active learning methodology such as flipped learning [59].

4.2. Lessons from Students' Feedback

From the students' feedback, we can observe that in general, the response toward *wxMaxima* was negative. Two comments from Spring 2016 that are nearly identical mentioned that the software is too hard to operate. It is unclear whether this particular student had difficulty in downloading and installing the software package or implementing the syntax. In either case, learning might be hampered because the CAS does not enhance learning. For some students, the software installation was not successful, and thus they could not operate the CAS smoothly, as indicated by one comment from SVC in Spring 2016. Another student from the same cohort also commented that although exploiting the software is enlightening and commendable, some students might be reluctant to use *wxMaxima* since they are not familiar with it.

Although we might assume that Korean digital natives grow up with technology, that does not mean that they have seen and used a CAS during their previous stages of education. As one student admitted, most of them did not use any mathematical software to solve mathematical problems during their middle and high school years. Indeed, there is a chain of logical reasoning in this situation. In order to embrace *wxMaxima*, they need to attempt some problems to solve using the CAS. In order to solve calculus problems using *wxMaxima*, they need to learn how to operate and write the syntax input properly. In order to explore *wxMaxima* successfully, they need proper guidance and assistance in administering the software. Thus, as instructors, we can be guides on their side in facilitating the discovery of the usefulness of the CAS to enhance their understanding in learning calculus.

From the selected comments, a natural conclusion would also be that more upfront instruction on how to use the software and more support for using it throughout the course might improve students' dispositions toward its use. In addition to providing detailed technical operations, we could demonstrate each step from the very beginning to enact a smooth user experience: where to download an executable file, how to install it, and how to launch the software accordingly. This would ensure that both *Maxima* and *wxMaxima* are installed properly and successfully. After this essential step, we could display the most basic operations that *wxMaxima* can perform, in the same way we use a calculator to perform arithmetic problems. The tutorial might continue by showing how to evaluate a floating-point approximation, solve elementary calculus problems, and sketch several simple graphs. By doing this, we hope that our Korean digital natives will be more willing to embrace new technology.

The reaction during Fall 2016 for MVC was mixed. Some complimented the adoption of *wxMaxima* while others preferred the traditional way of solving homework by pen and paper instead of using the software. In the subsequent academic years, i.e., 2018 and 2019, what the students picked up from embedding *wxMaxima* into the calculus classroom was mostly related to graph sketching. For some unclear reasons, there was no feedback related to *wxMaxima* during the 2017 academic year. Thus, we excluded any feedback given in 2017 from this study.

As the famous adage says that a picture is worth a thousand words, students can enhance their understanding of calculus concepts by looking at figures instead of simply memorizing formulas and staring at lengthy derivations. The feedback from the last two cohorts suggests that students appreciated the visualizations that *wxMaxima* contributes. By sketching and displaying the graphs produced by *wxMaxima*, students can comprehend better the behavior and characteristics of a particular function, whether 2D or 3D. The graphs help with checking whether the hand or computer calculations are correct. They also train students in observing patterns and irregularities, which in turn connect with important theorems in calculus. By possessing this type of geometric imagination, students will be able to manipulate geometric objects in their minds, e.g., by axis rotation, scaling, and shifting. See [60–62] for the role of visualization in calculus teaching.

4.3. Limitations

This study admits several limitations. First, we only considered the qualitative aspect of students' opinions based on an open-ended inquiry. We have not investigated students' perceptions quantitatively yet. Avowedly, quantitative measurements might deliver better and more objective results than a handful of qualitative data. Furthermore, it would be a promising outcome to examine whether students' attitude towards *wxMaxima*, or any other CAS in general, might reflect their values, self-confidence, enjoyment, motivation, and anxiety in relation to learning calculus or other mathematics subjects. It will be equally interesting also to investigate whether the use of CAS will significantly improve students' academic performances.

Second, the course content in relation to the time availability. Prior to the 2020 academic year, we had a 16-week semester, but only 14 weeks could be used for effective teaching. Weeks 8 and 16 are reserved for the midterm and final examinations, respectively. For MVC, the coverage material seems to be reasonable and instructors do not need to rush when covering the material. We need to cover three chapters before the midterm and the remaining two chapters are for the second half of the semester. Thus, embedding *wxMaxima* into the course seems a promising attempt. On the other hand, the materials in SVC are extremely vast while the time is insufficient. We need to encompass five chapters in the first half of the semester and the remaining four chapters need to be completed before the final exam period. Thus, dedicating one or two sessions for the *wxMaxima* tutorial can be risky timewise and can jeopardize the teaching plan. Hence, splitting the course syllabus into two separate courses might offer better circumstances for blending *wxMaxima* into calculus teaching and learning.

Third, *Maxima* employs command-line applications and thus it can be a little bit hard to manage for beginners. On the other hand, the GUI *wxMaxima* can be quite helpful if one does not wish to depend on the command line entirely. Furthermore, as we have observed in Section 2, *wxMaxima* is not perfect software. When we compare with other commercial CASs such as the *4M*, *wxMaxima* is far from being ideal. The group of open-source developers who work on improving *wxMaxima* is relatively small in number in comparison to the number of professional programmers and scientists who work at large software companies such as *Wolfram* and *Maplesoft*. The latter work full time with handsome salaries, whereas the former are heroes who volunteer during their spare time, sometimes as retirees. Given these dramatic differences in both manpower and financial resources, we foresee that *wxMaxima* will continue to fall behind other giant commercial software. For now, the odds of any chance of catching up are still stacked against *wxMaxima*.

5. Conclusions

In this article, we have covered some aspects of the CAS *wxMaxima*. It can handle some calculus-related problems easily, some with few manipulations, while many are unsuccessfully resolved. Despite this imperfection, we could still utilize *wxMaxima* in mathematics classrooms delicately. However, can we replace far superior software such as *Maple* or *Mathematica* for teaching and learning? If the main concern is cost-related, then the answer to this very question is affirmative. Yet, even if budget is not really an issue, attempting and experimenting with a new CAS might still be worth considering, at least for calculus-linked courses, i.e., PreCalculus, SVC, and MVC. Adopting it will not only allow for both instructors and students alike to explore and learn different CASs but also to identify where *wxMaxima* is particularly powerless and limited. Experiencing these situations and discovering alternative solutions can be a wonderful learning process by itself.

After an encounter with *wxMaxima*, our qualitative study suggests that Korean digital natives pose a diverse reaction. Although some positively acknowledged the advantage of *wxMaxima*—enhancing their understanding, particularly through the plotting features—some questioned whether such an attempt was necessary for calculus teaching and learning. Indeed, several students preferred the traditional way of solving problems from textbooks with a pen and paper instead of using *wxMaxima*. Being freshmen at the university, the Korean digital natives might not have had a chance and prior experience in using CAS when solving mathematics problems. Additionally, the lack of proper introduction might hinder the software integration process. In either case, we as instructors perceived a general reaction of amiable refusal toward embracing this new technological tool. Hence, our initial assumption of digital natives readily welcoming new technology should be addressed properly and investigated further.

After completing this study, we long to stimulate an accelerating debate not only among mathematics educators but also the symbolic computation communities. For the former, a persuasive yet efficient way of integrating *wxMaxima* into mathematics classrooms will be highly sought-after. For the latter, improving the software's effectiveness and capability in handling symbolic computations will certainly be welcomed by many researchers and educators.

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